NICPB KBFI

CONNECTING SCIENCES

# Natural Metric-Affine Inflation


**Antonio Racioppi**

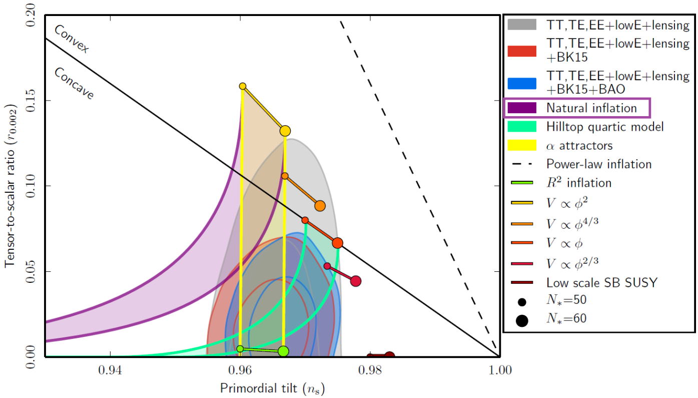
National Institute of Chemical Physics and Biophysics, Tallinn, Estonia


Istanbul, September 4th, 2024

based on
arXiv:2403.18004 (JCAP 06 (2024) 033)
with
A. Salvio (Univ. Rome Tor Vergata & INFN)

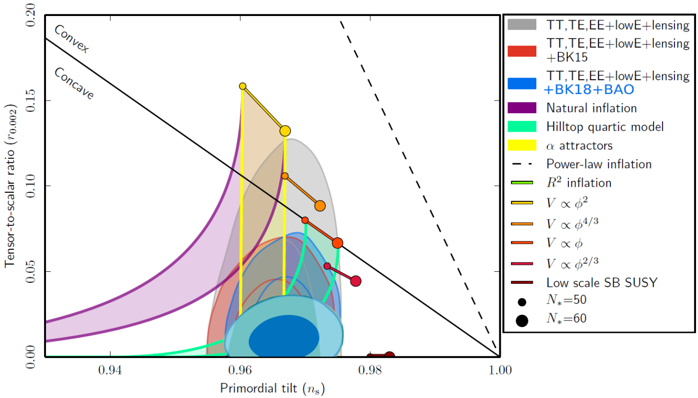

Euroopa Liit
Euroopa Sotsiaalfond
Eesti tuleviku heaks
cost
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY
COSMIC WISPERS CA21106

# • NI vs Planck 2018 data •



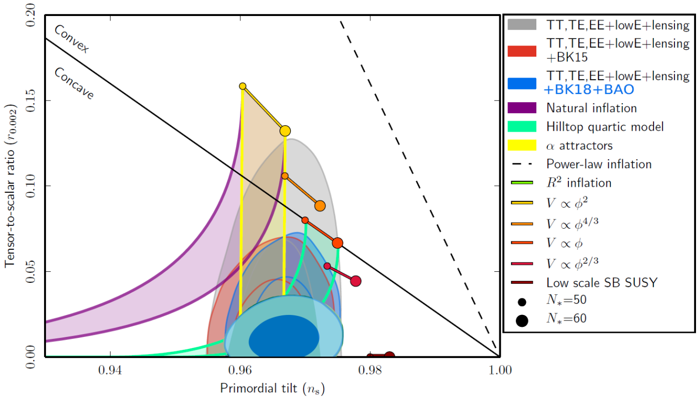

- NI dates back to the 90's (Freese et al., PRL65, 3233)
- inflaton $\phi$ is a PNGB (i.e. ALP) with
  - naturally ($\Lambda \to 0$) flat $V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$
  - naturally protected against radiative corrections
- NI became disfavored after Planck Legacy data

# • NI vs Planck 2018 data •




- NI became strongly disfavored after BICEP/Keck 2018 data
- several proposals to save it by modifying gravity:
  - $\xi[1 + cos(\phi)]R \rightarrow$ OK only at $2\sigma$ (Ferreira et al. 1806.05511)
  - $\xi\phi^n R \rightarrow$ OK only at $2\sigma$ (Bostan, 2209.02434; dos Santos et al., 2312.12286)
  - Palatini $R^2 \rightarrow$ OK! but $(\partial\phi)^4$ (Antoniadis et al., 1812.00847 )
  - probably more?

# • NI vs Planck 2018 data •




- NI became strongly disfavored after BICEP/Keck 2018 data
- several proposals to save it by modifying gravity:
  - $\xi[1 + cos(\phi)]R \rightarrow$ OK only at $2\sigma$ (Ferreira et al. 1806.05511)
  - $\xi\phi^n R \rightarrow$ OK only at $2\sigma$ (Bostan, 2209.02434; dos Santos et al., 2312.12286)
  - Palatini $R^2 \rightarrow$ OK! but $(\partial\phi)^4$ (Antoniadis et al., 1812.00847 )
  - probably more?

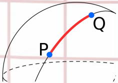
# • Gravity preliminaries •

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

- the metric tensor: $g_{\mu\nu}$
- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

- the metric tensor: $g_{\mu\nu} \to$ distance

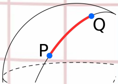



- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$

# Gravity preliminaries

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

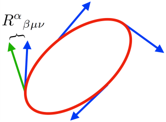
- the metric tensor: $g_{\mu\nu} \rightarrow$ distance



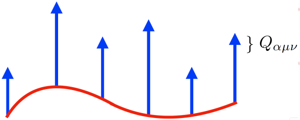

- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

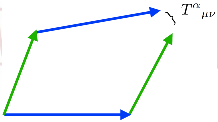
$$\mathcal{A}^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu} \qquad K \rightarrow T, \; L \rightarrow Q$$




curvature




non-metricity

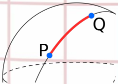



torsion

# • Gravity preliminaries •

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

- the metric tensor: $g_{\mu\nu}$ → distance

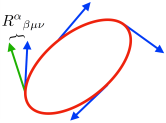



- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$ → parallel transport

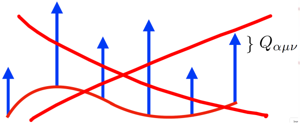
  If Einstein (minimal) theory of gravity:

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

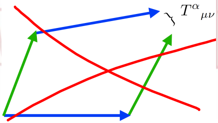
$$\mathcal{A}^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu} + \cancel{K^{\alpha}{}_{\mu\nu}} + \cancel{L^{\alpha}{}_{\mu\nu}}$$




curvature




~~non-metricity~~

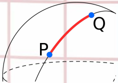



~~torsion~~

# Gravity preliminaries

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

- the metric tensor: $g_{\mu\nu}$ → distance



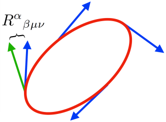

- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$ → parallel transport

  If non-minimal theory of gravity:

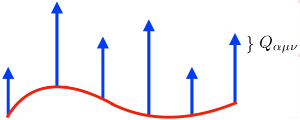
$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

$$\mathcal{A}^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}$$

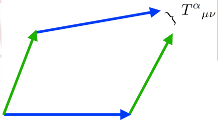
unless $K = 0, L = 0$ by hand




curvature




non-metricity

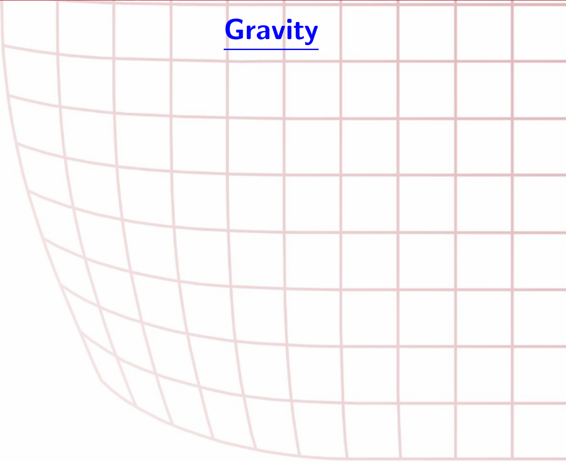



torsion

# • Yang-Mills vs. Gravity analogy •

## Yang-Mills

## Gravity



N.B. Slide not completely exact but close enough ☺

# Yang-Mills vs. Gravity analogy

## Yang-Mills

- gauge field strength:

$$F^i_{\mu\nu} = \partial A^i_\mu - \partial A^i_\nu + g f^{ijk} A^j_\mu A^k_\nu$$

- kinetic term: $F^i_{\mu\nu} F^{i,\mu\nu}$

- dual field strength:

$$\tilde{F}^{i,\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F^i_{\rho\sigma}$$

- ALP-gauge coupling:

$$\boxed{a(\phi)\, F^i_{\mu\nu} \tilde{F}^{i,\mu\nu}}$$

## Gravity

N.B. Slide not completely exact but close enough ☺

# • Yang-Mills vs. Gravity analogy •

## Yang-Mills

- gauge field strength:

$$F^i_{\mu\nu} = \partial A^i_\mu - \partial A^i_\nu + g f^{ijk} A^j_\mu A^k_\nu$$

- kinetic term: $F^i_{\mu\nu} F^{i,\mu\nu}$

- dual field strength:

$$\tilde{F}^{i,\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F^i_{\rho\sigma}$$

- ALP-gauge coupling:

$$\boxed{a(\phi)\, F^i_{\mu\nu} \tilde{F}^{i,\mu\nu}}$$

## Gravity

- Riemann tensor

$$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} = \partial_\mu \mathcal{A}_\nu{}^\rho{}_\sigma - \partial_\nu \mathcal{A}_\mu{}^\rho{}_\sigma + \mathcal{A}_\mu{}^\rho{}_\lambda \mathcal{A}_\nu{}^\lambda{}_\sigma - \mathcal{A}_\nu{}^\rho{}_\lambda \mathcal{A}_\mu{}^\lambda{}_\sigma$$

- curvature term: $\mathcal{R} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}$

$$T^\alpha_{\mu\nu} \neq 0 \Leftrightarrow \mathcal{A}^\lambda_{\alpha\beta} \neq \mathcal{A}^\lambda_{\beta\alpha} \Rightarrow$$

- $\boxed{\text{Holst invariant: } \tilde{\mathcal{R}} \equiv \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\mu\nu\rho\sigma}}$

only other invariant linear in $\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}$

- "ALP-gravity" coupling:

$$\boxed{\beta(\phi)\, \tilde{R}}$$

N.B. Slide not completely exact but close enough ☺

# • MAG NI action •

- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S_{\rm NI} = \int d^4x \sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$V(\phi) = \Lambda^4\left[1 + \cos\left(\frac{\phi}{f}\right)\right]$$

$$\beta(\phi) = \beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

$\frac{M_P^2}{4\beta_0} \to$ Barbero-Irmizzi par.

$\alpha(\phi) = \frac{M_P^2}{2} > 0 \leftarrow$ most minimal $\to$ not 100% OK

# • MAG NI action •

- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S_{\rm NI} = \int d^4x \sqrt{-g_J} \left[ \alpha(\phi) \mathcal{R}_J + \beta(\phi) \tilde{\mathcal{R}}_J - \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V(\phi) \right]$$

$$V(\phi) = \Lambda^4 \left[ 1 + \cos\left(\frac{\phi}{f}\right) \right]$$

$$\beta(\phi) = \beta_0 + \frac{M_P^2}{2} \tilde{\xi} \left( 1 + \cos\left(\frac{\phi}{f}\right) \right)$$

$\frac{M_P^2}{4\beta_0} \to$ Barbero-Irmizzi par.

$$\alpha(\phi) = \frac{M_P^2}{2} + \frac{M_P^2}{2} \xi \left( 1 + \cos\left(\frac{\phi}{f}\right) \right) > 0$$

- no other terms → most general minimal setup without
  - new physical dof's in addition to $h_{\mu\nu}$, $\phi$
  - $(\partial\phi)^4$ terms

# • MAG NI action •

- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S_{\rm NI} = \int d^4x \sqrt{-g_J} \left[ \alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi) \right]$$

- it is possible to integrate out the $\tilde{R}$ term
- obtain an equivalent theory with $Q_{\alpha\mu\nu} \neq 0$, $T^{\alpha}_{\ \mu\nu} = 0$

$$S_{\rm NI} = \int d^4x \sqrt{-g_J} \left[ \alpha\mathcal{R}_J - \left[ 1 + \frac{12(\alpha'\beta + \alpha\beta')^2}{\alpha(\alpha^2 + 4\beta^2)} \right] \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V \right]$$

- the effect of $T^{\alpha}_{\ \mu\nu} \neq 0$ is moved to the inflaton kinetic term

# • MAG NI action •

- starting Jordan frame action with $Q_{\alpha\mu\nu} \neq 0,\ T^{\alpha}_{\ \mu\nu} = 0$

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha\mathcal{R}_J - \left[1 + \frac{12(\alpha'\beta + \alpha\beta')^2}{\alpha(\alpha^2 + 4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- move to the Einstein frame so that $Q_{\alpha\mu\nu} = T^{\alpha}_{\ \mu\nu} = 0$

$$\boxed{S_{\rm NI} = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]}$$

$$\begin{cases} \frac{d\chi}{d\phi} = M_P\sqrt{\frac{1}{2\alpha}\left[1 + \frac{12(\alpha'\beta+\alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]} \\ U(\chi) = M_P^4\frac{V(\phi(\chi))}{4\alpha^2(\phi(\chi))} \end{cases}$$

- the former effects of $Q_{\alpha\mu\nu}, T^{\alpha}_{\ \mu\nu} \neq 0$ are now moved to $(\partial\chi)^2$
- N.B. symmetry: $\beta \to -\beta \quad \Rightarrow \quad \boxed{\tilde{\xi} > 0,\ \beta_0 \gtreqless 0}$

# • MAG NI action •

- starting Jordan frame action with $Q_{\alpha\mu\nu} \neq 0,\ T^{\alpha}_{\ \mu\nu} = 0$

$$S_{\rm NI} = \int d^4x \sqrt{-g_J} \left[ \alpha \mathcal{R}_J - \left[ 1 + \frac{12(\alpha'\beta + \alpha\beta')^2}{\alpha(\alpha^2 + 4\beta^2)} \right] \frac{\partial_\mu \phi\, \partial^\mu \phi}{2} - V \right]$$

- move to the Einstein frame so that $Q_{\alpha\mu\nu} = T^{\alpha}_{\ \mu\nu} = 0$

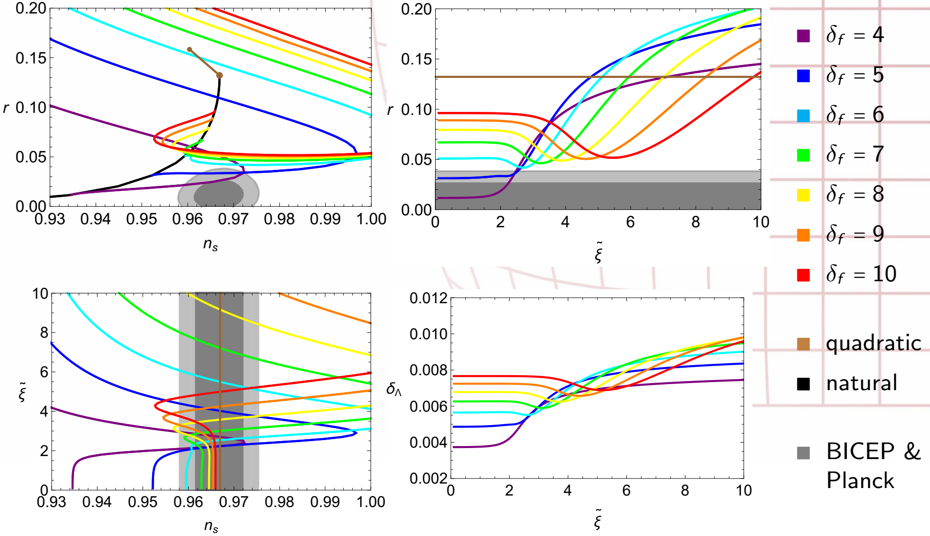
$$\boxed{S_{\rm NI} = \int d^4x \sqrt{-g_E} \left[ \frac{M_P^2}{2} R_E - \frac{\partial_\mu \chi\, \partial^\mu \chi}{2} - U(\chi) \right]}$$

$$\xrightarrow[\text{with } \xi=0]{\text{we start}} \begin{cases} \dfrac{d\chi}{d\phi} = \sqrt{1 + \dfrac{24 M_P^2 (\beta')^2}{(M_P^4 + 16\beta^2)}} \\ U(\chi) = V(\phi(\chi)) \end{cases}$$

- the former effects of $Q_{\alpha\mu\nu},\ T^{\alpha}_{\ \mu\nu} \neq 0$ are now moved to $(\partial\chi)^2$
- N.B. symmetry: $\beta \to -\beta \quad \Rightarrow \quad \boxed{\tilde{\xi} > 0,\ \beta_0 \gtreqless 0}$

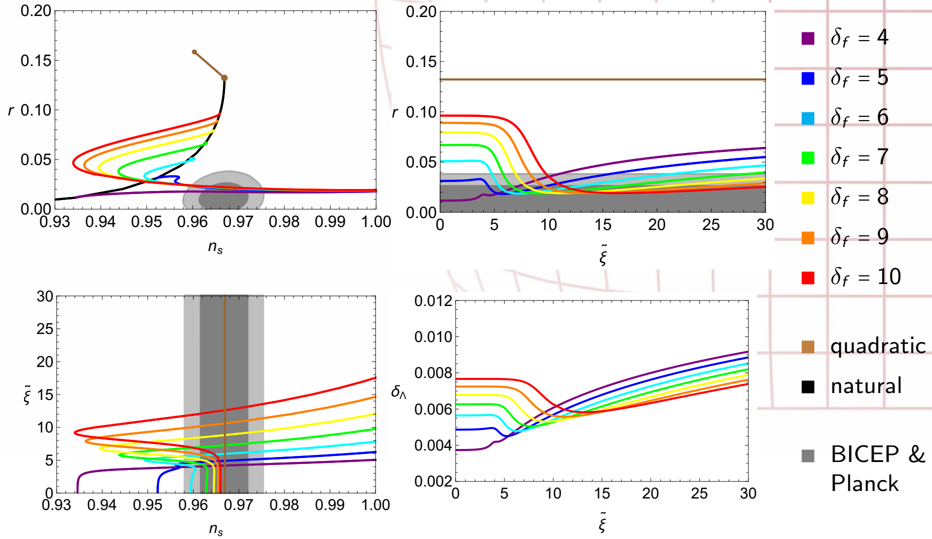
# $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$

$\xi = 0 \quad \beta_0 = -2M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



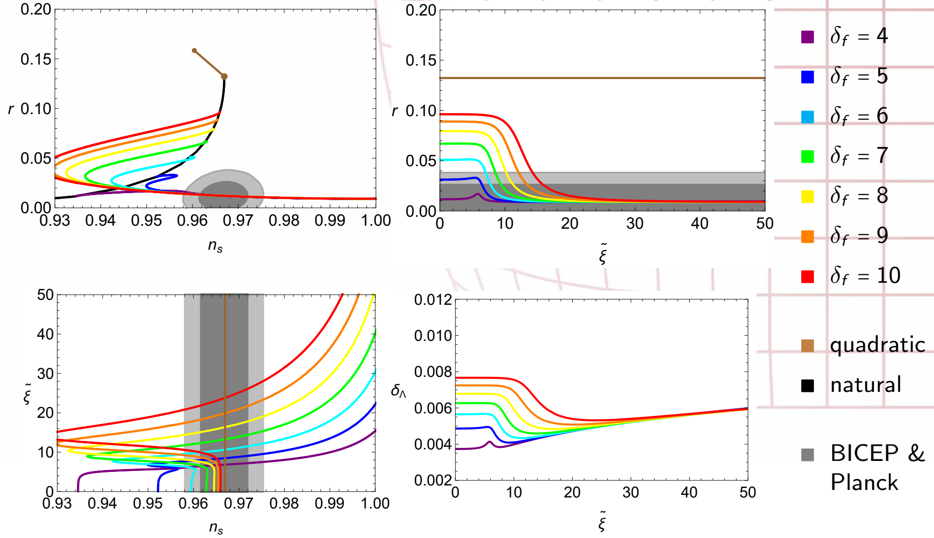
# • $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

$\xi = 0 \quad \beta_0 = -4M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



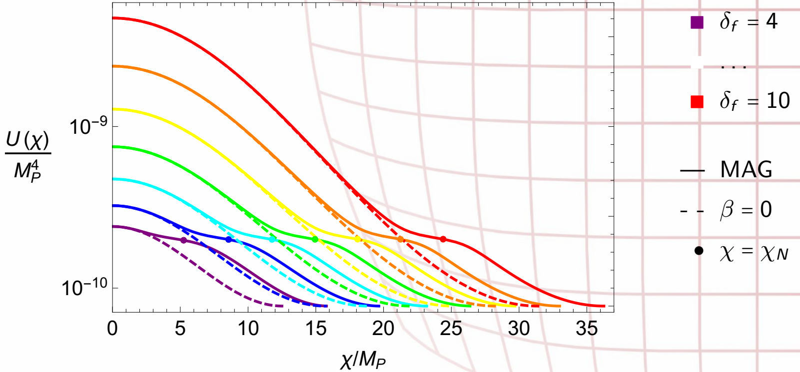
# • $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

$\xi = 0 \quad \beta_0 = -6M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



# • $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

$\xi = 0 \quad \beta_0 = -6M_P^2 \quad N_e = 60 \quad n_s \simeq 0.97 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$

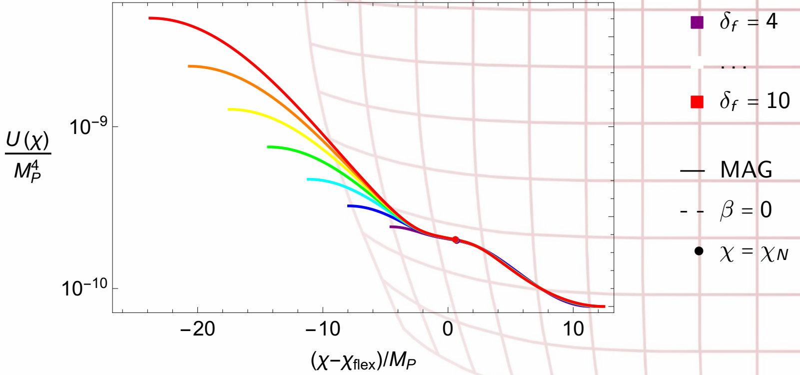



- inflection point inflation!!!

$$\frac{d^2U(\chi)}{d\chi^2} = \frac{d\phi}{d\chi}\frac{d}{d\phi}\left(\frac{d\phi}{d\chi}\frac{dU(\phi)}{d\phi}\right) = 0 \leftarrow \text{possible thanks to } \beta(\phi)\tilde{\mathcal{R}}$$

# • $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

$\xi = 0 \quad \beta_0 = -6M_P^2 \quad N_e = 60 \quad n_s \simeq 0.97 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$




- inflection point inflation!!!

$$\frac{d^2U(\chi)}{d\chi^2} = \frac{d\phi}{d\chi}\frac{d}{d\phi}\left(\frac{d\phi}{d\chi}\frac{dU(\phi)}{d\phi}\right) = 0 \leftarrow \text{possible thanks to } \beta(\phi)\tilde{\mathcal{R}}$$

# • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

- agreement with data $\Rightarrow f, \sqrt{|\beta|} > M_P$
- $|\beta| > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$
- $f > M_P$ more difficult when looking for a UV-completion
- solution: $\xi > 0$
- $\alpha > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$

# • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

- agreement with data $\Rightarrow f, \sqrt{|\beta|} > M_P$
- $|\beta| > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$
- $f > M_P$ more difficult when looking for a UV-completion
- solution: $\xi > 0$
- $\alpha > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$

# • $\xi > 0$ & $\tilde{\xi} > 0$: **low** $f \lesssim M_P$ •

- agreement with data $\Rightarrow f, \sqrt{|\beta|} > M_P$
- $|\beta| > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$
- $f > M_P$ more difficult when looking for a UV-completion
- solution: $\xi > 0$
- $\alpha > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$

# • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

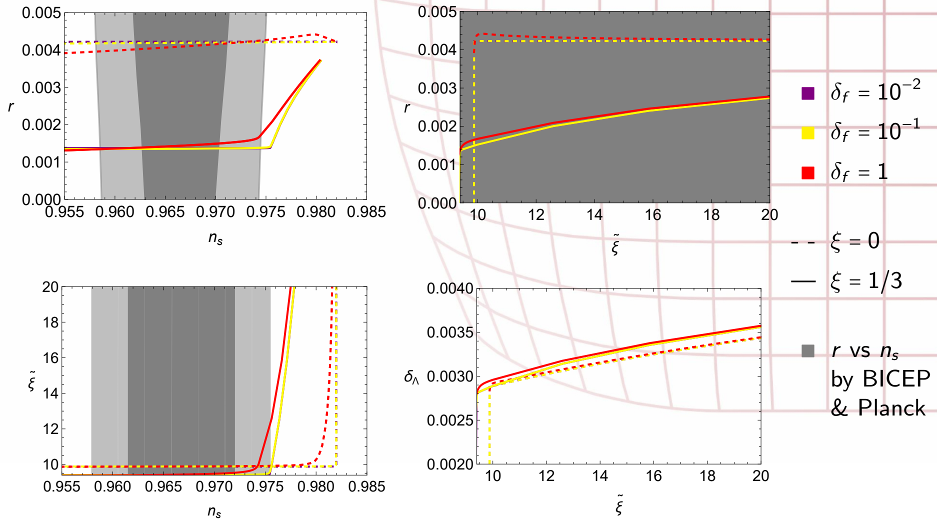
- agreement with data $\Rightarrow f, \sqrt{|\beta|} > M_P$
- $|\beta| > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$
- $f > M_P$ more difficult when looking for a UV-completion
- solution: $\xi > 0$
- $\alpha > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$

## • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

- agreement with data $\Rightarrow f, \sqrt{|\beta|} > M_P$
- $|\beta| > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$
- $f > M_P$ more difficult when looking for a UV-completion
- solution: $\xi > 0$
- $\alpha > M_P^2$ still OK as long as $U < M_P^4 \Leftarrow A_s \simeq 2.2 \times 10^{-9}$

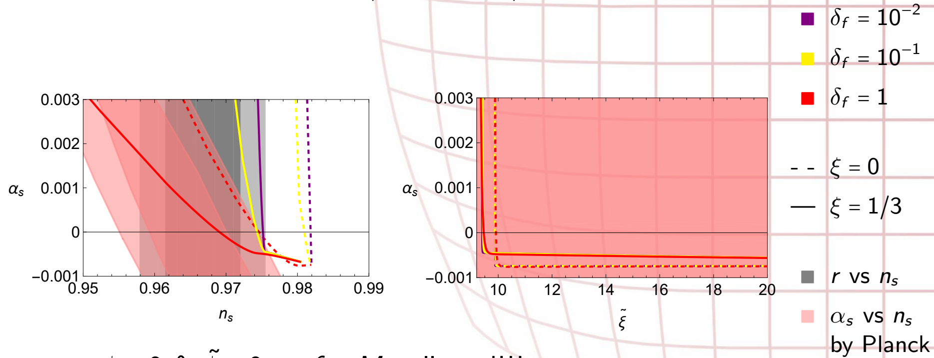
# • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

$\beta_0 = -10 M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



# • $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

$\beta_0 = -10 M_P^2$ $N_e = 60$ $\delta_f = f/M_P$ $\delta_\Lambda = \Lambda/M_P$




- $\xi > 0$ & $\tilde{\xi} > 0 \Rightarrow f \lesssim M_P$ allowed!!!

# • Summary & Conclusions •

- NI strongly disfavored after Planck+BICEP 2018 data

- MAG & allow for torsion
  - new nmc: $\beta(\phi)\tilde{\mathcal{R}}$
  - predictions compatible with data at $1\sigma$
  - but $f > M_P$

- Add an additional nmc: $\alpha(\phi)\mathcal{R}$
  - predictions compatible with data at $1\sigma$
  - $f < M_P$

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

KBFI NICPB

# • Palatini NI action •

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$$

$$\alpha(\phi) = \frac{M_P^2}{2}\left[1+\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right)\right] > 0$$

- notation: $\begin{cases} \mathcal{R} \to \text{curvature from generic } \mathcal{A}_\mu{}^\rho{}_\sigma \\ R \to \text{curvature from Levi-Civita } \Gamma_\mu{}^\rho{}_\sigma \end{cases}$
- Einstein frame: $g^E_{\mu\nu} = F(\phi)g^J_{\mu\nu}$ , $F \equiv \frac{2\alpha}{M_P^2}$

N.B. Palatini $\Rightarrow \mathcal{R}_J = F\,R_E$

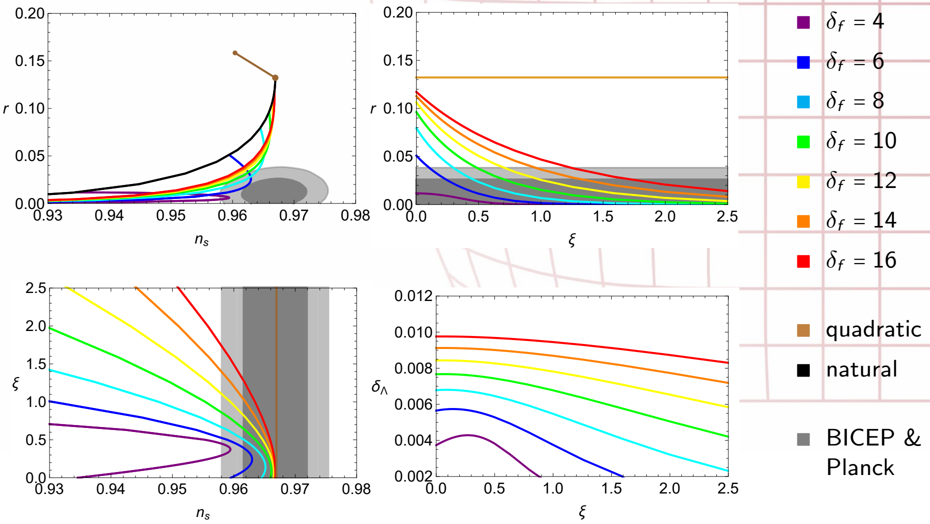
$$S_{\rm NI} = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]$$

$$\frac{d\chi}{d\phi} = \sqrt{\frac{1}{F(\phi)}} = \sqrt{\frac{M_P^2}{2\alpha(\phi)}} \qquad \leftarrow \text{no } \frac{3}{2}\left(\frac{F'}{F}\right)^2 \text{ like in metric}$$

$$U(\chi) = \frac{V(\phi(\chi))}{F^2(\phi(\chi))}$$

# • Palatini results •

$N_e = 60 \qquad \delta_f = f/M_P \qquad \delta_\Lambda = \Lambda/M_P$



# • MAG NI action •

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$\alpha(\phi) = \frac{M_P^2}{2} + \frac{M_P^2}{2}\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right) > 0 \qquad V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$$

$$\beta(\phi) = \beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(1+\cos\left(\frac{\phi}{f}\right)\right) \qquad \frac{M_P^2}{4\beta_0} \rightarrow \text{Barbero-Irmizzi par.}$$

- no other terms → minimal setup without
  - new physical dof's in addition to $h_{\mu\nu}$, $\phi$
  - $(\partial\phi)^4$ terms
- it is possible to integrate out the $\tilde{R}$ term
- obtain an equivalent torsion-less theory with $A^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}$
- performing all the computations . . .

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha\mathcal{R}_J - \left[1 + \frac{12(\alpha'\beta+\alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- the effect of the torsion is moved to the inflaton kinetic term

# • MAG NI action •

$$S_{\mathrm{NI}} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$\alpha(\phi) = \frac{M_P^2}{2} + \frac{M_P^2}{2}\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right) > 0 \qquad V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$$

$$\beta(\phi) = \beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(1+\cos\left(\frac{\phi}{f}\right)\right) \qquad \frac{M_P^2}{4\beta_0} \to \text{Barbero-Irmizzi par.}$$

- no other terms → minimal setup without
  - new physical dof's in addition to $h_{\mu\nu}$, $\phi$
  - $(\partial\phi)^4$ terms
- it is possible to integrate out the $\tilde{R}$ term
- obtain an equivalent torsion-less theory with $\mathcal{A}^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}$
- performing all the computations ...

$$S_{\mathrm{NI}} = \int d^4x\sqrt{-g_J}\left[\alpha\mathcal{R}_J - \left[1 + \frac{12(\alpha'\beta+\alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- the effect of the torsion is moved to the inflaton kinetic term

# • MAG NI action •

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$\alpha(\phi) = \frac{M_P^2}{2} + \frac{M_P^2}{2}\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right) > 0 \qquad V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$$

$$\beta(\phi) = \beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(1+\cos\left(\frac{\phi}{f}\right)\right) \qquad \frac{M_P^2}{4\beta_0} \to \text{Barbero-Irmizzi par.}$$

- no other terms → minimal setup without
  - new physical dof's in addition to $h_{\mu\nu}$, $\phi$
  - $(\partial\phi)^4$ terms

- it is possible to integrate out the $\tilde{R}$ term
- obtain an equivalent torsion-less theory with $\mathcal{A}^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}$
- performing all the computations . . .

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha\mathcal{R}_J - \left[1 + \frac{12(\alpha'\beta+\alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- the effect of the torsion is moved to the inflaton kinetic term

# • MAG NI action: Einstein frame •

$$S_{\rm NI} = \int d^4x \sqrt{-g_J}\left[\alpha \mathcal{R}_J - \left[1 + \frac{12(\alpha'\beta + \alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- notation: $\begin{cases} \mathcal{R} \to \text{curvature from generic } \mathcal{A}_\mu{}^\rho{}_\sigma \\ R \to \text{curvature from Levi-Civita } \Gamma_\mu{}^\rho{}_\sigma \end{cases}$

- Einstein frame: $g^E_{\mu\nu} = F(\phi) g^J_{\mu\nu}$ , $F \equiv \frac{2\alpha}{M_P^2}$

  N.B. MAG $\Rightarrow \mathcal{R}_J = F\, R_E$

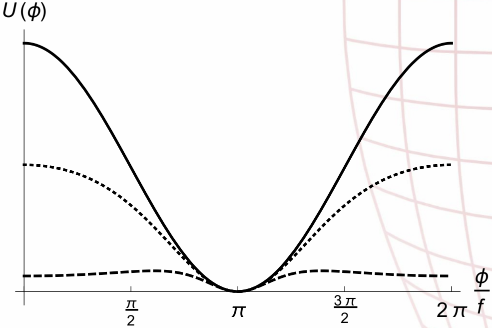
$$S_{\rm NI} = \int d^4x \sqrt{-g_E}\left[\frac{M_P^2}{2} R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]$$

$$\frac{d\chi}{d\phi} = M_P \sqrt{\frac{1}{2\alpha}\left[1 + \frac{12(\alpha'\beta + \alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]} \quad \leftarrow \text{no } \frac{3}{2}\left(\frac{F'}{F}\right)^2 \text{ like in metric}$$

$$U(\chi) = \frac{V(\phi(\chi))}{F^2(\phi(\chi))}$$

- N.B. symmetry: $\beta \to -\beta \quad \Rightarrow \quad \boxed{\tilde{\xi} > 0,\ \beta_0 \gtreqless 0}$

# Jordan frame potential




$$U(\chi) = \frac{\Lambda^4\left(1+\cos\left(\frac{\phi(\chi)}{f}\right)\right)}{\left[1+\xi\left(1+\cos\left(\frac{\phi(\chi)}{f}\right)\right)\right]^2}$$

— $\xi = 0$

· · · $0 < \xi < \frac{1}{2}$

- - $\xi > \frac{1}{2}$

Stationary points:

$$\phi_1 = 0 \qquad U_{\phi\phi}(\phi_1) = \frac{\Lambda^4(2\xi-1)}{f^2(2\xi+1)^3} \rightarrow \begin{cases} \xi < \frac{1}{2} & \text{max} \\ \xi > \frac{1}{2} & \text{min} \end{cases}$$

$$\phi_2 = \pi f \qquad U_{\phi\phi}(\phi_2) = \frac{\Lambda^4}{f^2} \rightarrow \text{(absolute) min}$$

$$\phi_3 = f \arccos\left(\frac{1-\xi}{\xi}\right) \qquad U_{\phi\phi}(\phi_3) = \frac{\Lambda^4(1-2\xi)}{8f^2\xi} \rightarrow \begin{cases} \xi < \frac{1}{2} & \text{NA} \\ \xi > \frac{1}{2} & \text{max} \end{cases}$$

# • Inflationary observables •

- SR parameters

$$\epsilon_U(\chi) = \frac{M_P^2}{2}\left(\frac{U'(\chi)}{U(\chi)}\right)^2$$

$$\eta_U(\chi) = M_P^2\frac{U''(\chi)}{U(\chi)}$$

$$\xi_U^2(\chi) = M_P^4\frac{U'(\chi)U'''(\chi)}{U(\chi)^2}$$

- observables

$$N_e = \frac{1}{M_P^2}\int_{\chi_{\rm end}}^{\chi_N} \mathrm{d}\chi\,\frac{U(\chi)}{U'(\chi)}$$

$$r = 16\epsilon_U(\chi_N)$$

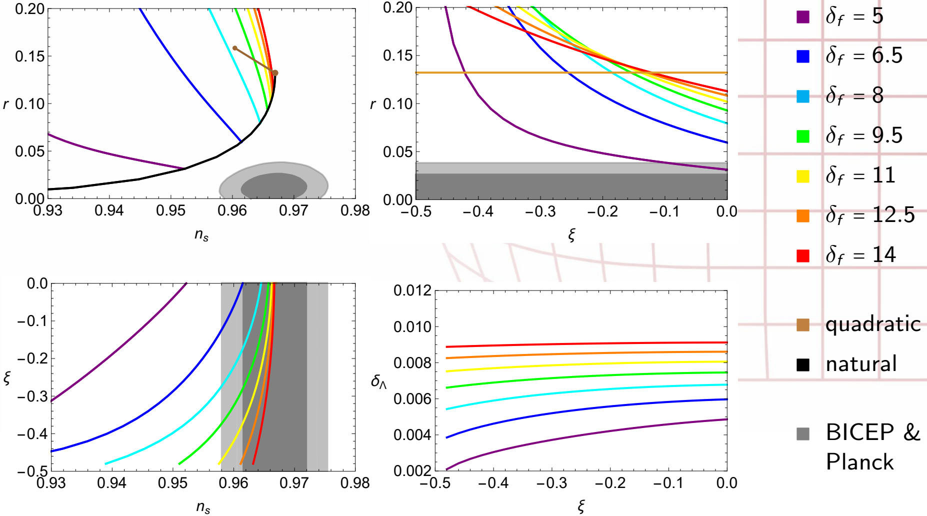
$$n_{\rm s} = 1 + 2\eta_U(\chi_N) - 6\epsilon_U(\chi_N)$$

$$\alpha_{\rm s} \equiv \mathrm{d}n_{\rm s}/\mathrm{d}\ln k = 16\epsilon_U(\chi_N)\eta_U(\chi_N) - 24\epsilon_U^2(\chi_N) - 2\xi_U^2(\chi_N)$$

$$A_{\rm s} = \frac{1}{24\pi^2 M_P^4}\frac{U(\chi_N)}{\epsilon_U(\chi_N)}$$

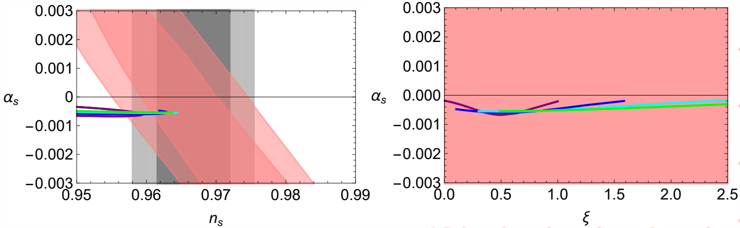
# • Palatini results: $\xi < 0$ •

$\xi < 0 \quad \alpha = 0 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



# Palatini: $\alpha_s$ vs. $n_s$

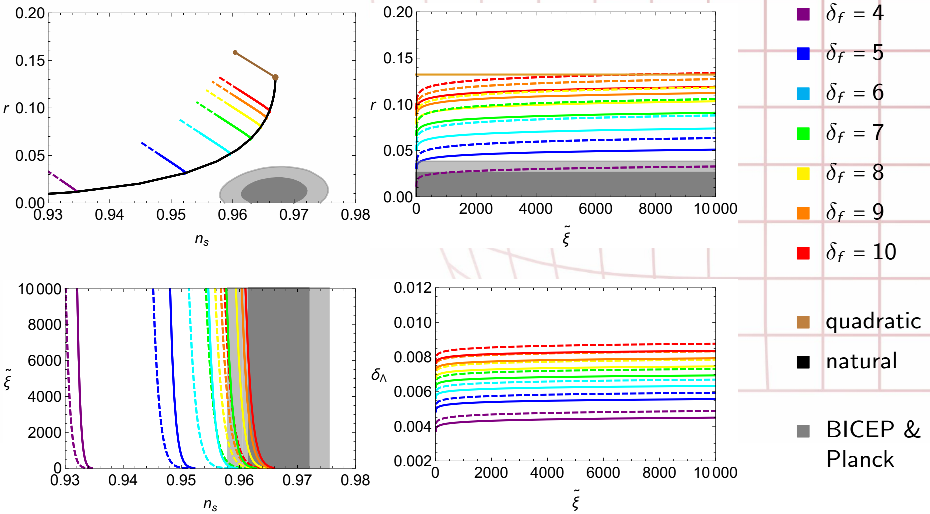
$N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$




- $\delta_f = 4$
- $\delta_f = 6$
- $\delta_f = 8$
- $\delta_f = 10$
- $\delta_f = 12$
- $\delta_f = 14$
- $\delta_f = 16$
- quadratic
- natural
- $r$ vs $n_s$
- $\alpha_s$ vs $n_s$ by Planck

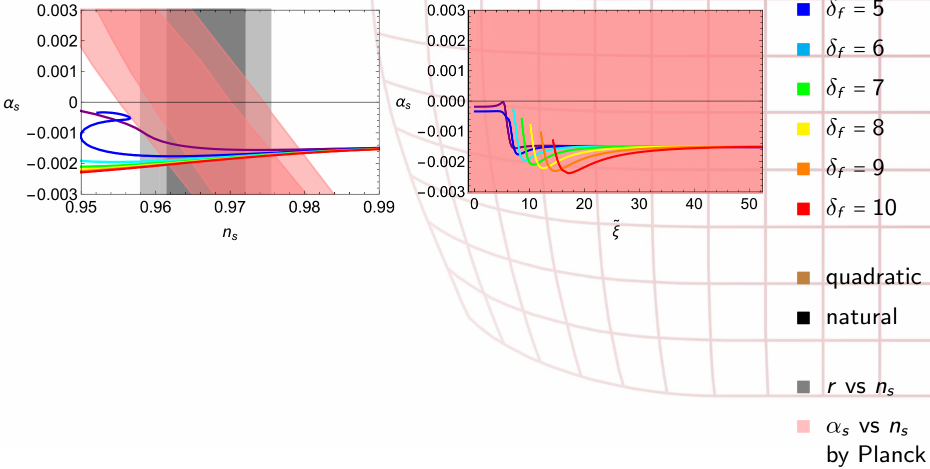
# • $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 > 0$ •

$\tilde{\xi} > 0$ $\quad \beta_0 \geq 0$ $\quad N_e = 60$ $\quad \delta_f = f/M_P$ $\quad \delta_\Lambda = \Lambda/M_P$ $\quad$ — $\beta_0 = 2M_P^2$ $\quad$ - - $\beta_0 = 0$



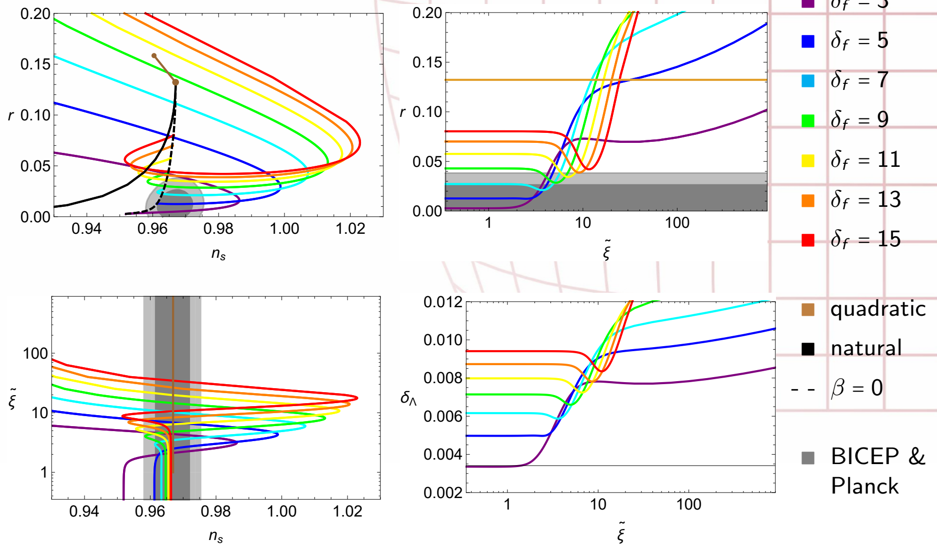
# • MAG $\xi = 0$: $\alpha_s$ vs. $n_s$ •

$\xi = 0$ $\beta_0 = -6M_P^2$ $N_e = 60$ $\delta_f = f/M_P$ $\delta_\Lambda = \Lambda/M_P$



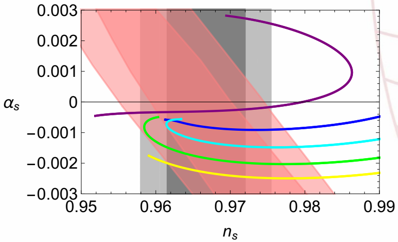
# • $\xi > 0$ & $\tilde{\xi} > 0$ •

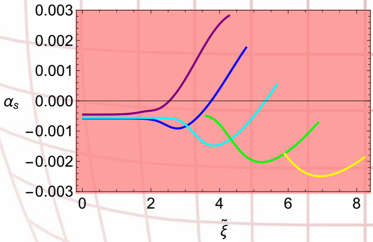
$\xi = 1/3 \quad \beta_0 = -2M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



# • MAG $\xi = 1/3$: $\alpha_s$ vs. $n_s$ •

$\xi = 1/3$ $\beta_0 = -2M_P^2$ $N_e = 60$ $\delta_f = f/M_P$ $\delta_\Lambda = \Lambda/M_P$




- $\delta_f = 3$
- $\delta_f = 5$
- $\delta_f = 7$
- $\delta_f = 9$
- $\delta_f = 11$
- $\delta_f = 13$
- $\delta_f = 15$
- quadratic
- natural
- $r$ vs $n_s$
- $\alpha_s$ vs $n_s$ by Planck

# • Inflaton model completion •

- dark QCD ($SU(3)_f$) with confinement scale $f$
- $\tilde{q}$ mass terms break the axial part of $SU(3)_f$

$$\mathcal{L}_{\rm mass} = \bar{\tilde{q}} M_q \tilde{q} = \bar{\tilde{q}}' \exp\Big(-i\gamma_5 B/(\sqrt{2}f)\Big) M_q \exp\Big(-i\gamma_5 B/(\sqrt{2}f)\Big)\tilde{q}'$$

where $\tilde{q}_i'$ are the Goldstone-free quark fields $\tilde{q}' = \exp\big(i\gamma_5 B/(\sqrt{2}f)\big)\tilde{q}$

- tilde-mesons as PNGBs

$$B \equiv \begin{pmatrix} \frac{\tilde{\pi}^0}{\sqrt{2}} + \frac{\tilde{\eta}^0}{\sqrt{6}} & \tilde{\pi}^+ & \tilde{K}^+ \\ (\tilde{\pi}^+)^\dagger & -\frac{\tilde{\pi}^0}{\sqrt{2}} + \frac{\tilde{\eta}^0}{\sqrt{6}} & \tilde{K}^0 \\ (\tilde{K}^+)^\dagger & (\tilde{K}^0)^\dagger & -\sqrt{\frac{2}{3}}\tilde{\eta}^0 \end{pmatrix}$$

- the lightest acts as the inflaton
- natural inflation potential arising from the $\tilde{q}$ mass terms
- ~~minimal~~ couplings $\tilde{q}$'s with gravity $\Rightarrow$ ~~minimal~~ couplings of $\phi$ with gravity

$$\bar{\tilde{q}} J \tilde{q} \mathcal{R}\,,\ \bar{\tilde{q}} J' \tilde{q} \tilde{\mathcal{R}} \Rightarrow \alpha \mathcal{R}\,,\ \beta \tilde{\mathcal{R}}$$