The Spectrum of Global Axion Strings

Mathieu Kaltschmidt
mkaltschmidt@unizar.es

based on 2401.17253 with K. Saikawa, J. Redondo & A. Vaquero

and *if time allows* work in progress with J. Redondo, I. Y. Rybak & A. Drew

2nd General Meeting of the COST Action Cosmic WISPers
Istanbul, September 3rd 2024

Background: G. Pierobon

Departamento de Física Teórica
Universidad Zaragoza

Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza

GOBIERNO DE ARAGON

COSMIC WISPERS

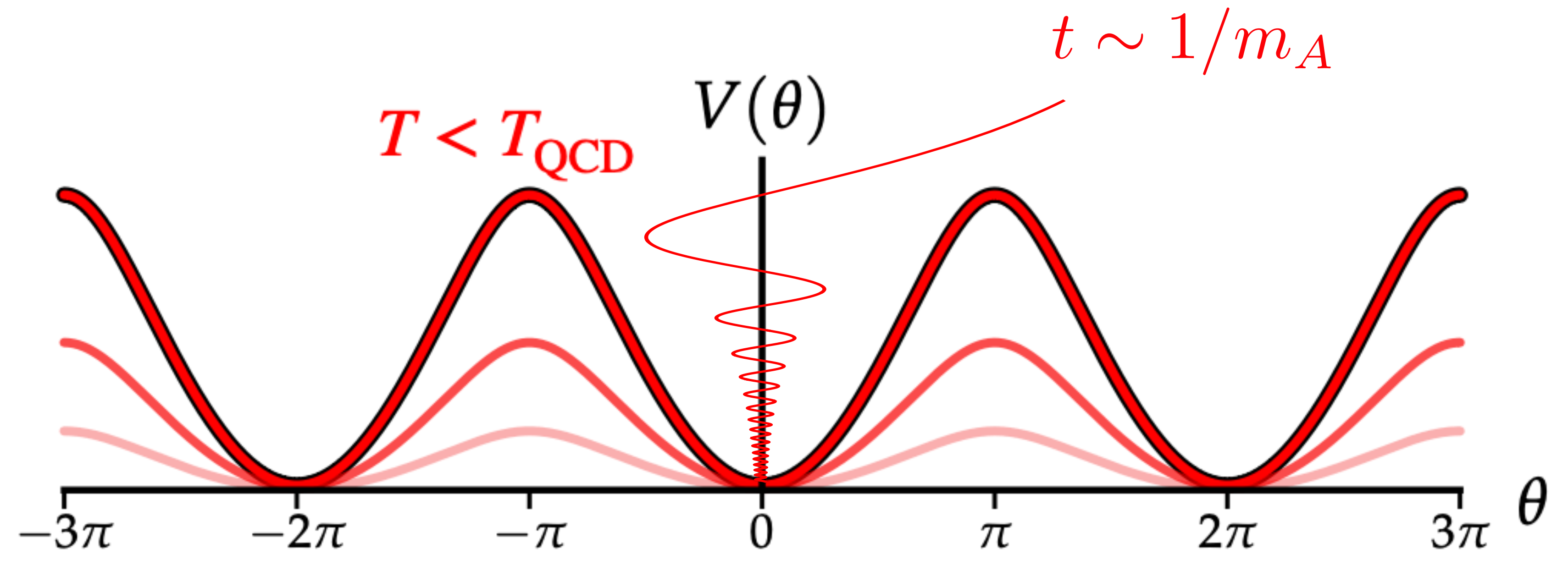
COST EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

QCD Axion

- The QCD Axion
  - Pseudo Nambu-Goldstone boson associated with the spontaneous breaking of the global Peccei-Quinn (PQ) $U(1)$ symmetry at the high-energy scale f_a.
  - Dynamical solution to the strong-CP problem.
  - Suitable candidate for Cold Dark Matter.
- Acquires a mass below the QCD scale.

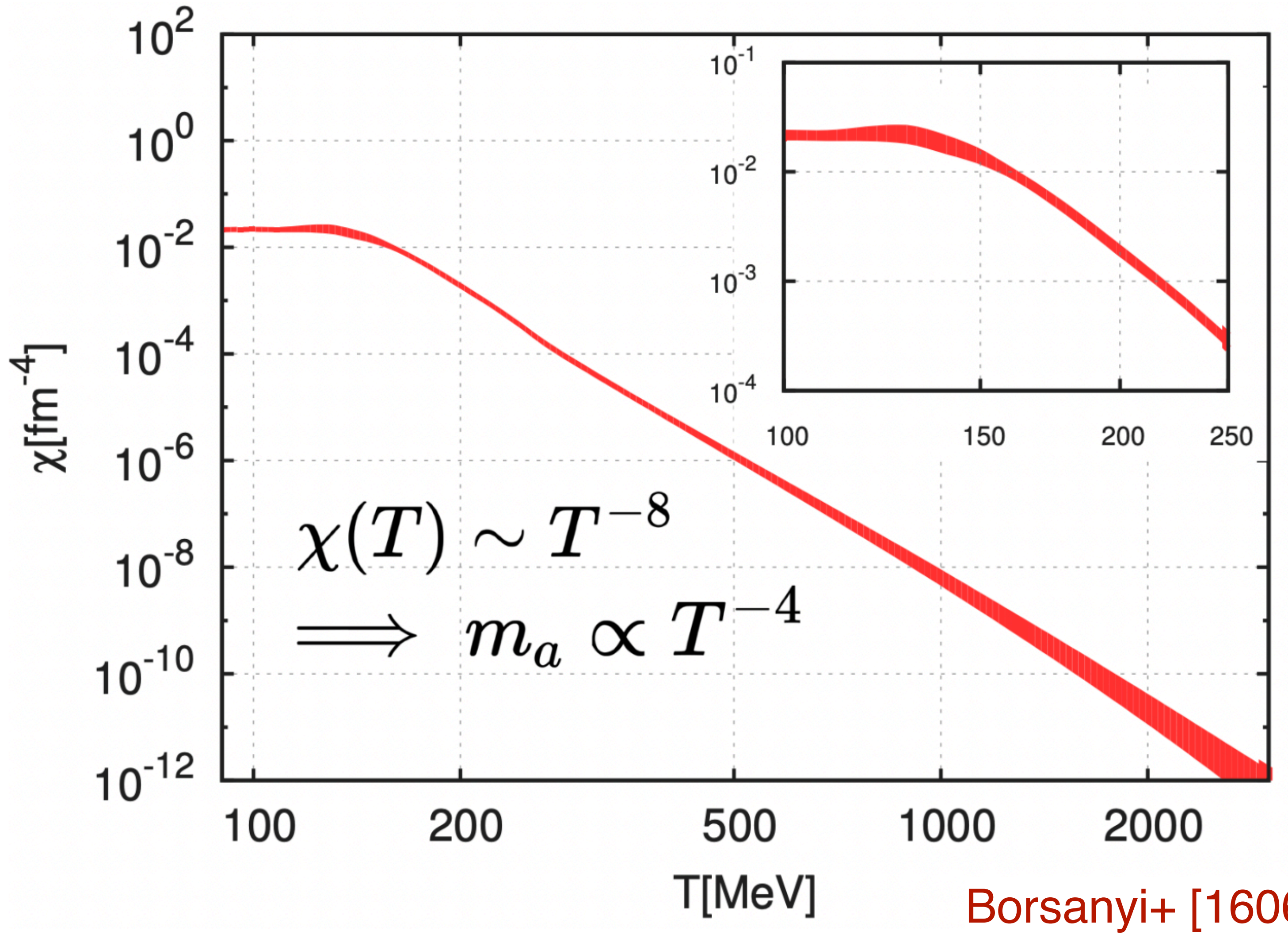
* Throughout this talk, when we refer to the axion, we implicitly mean the **QCD axion** (i.e. solves the strong CP problem)

$$V(\theta) \approx \chi(T)(1-\cos\theta) = m_a^2(T) f_a^2 (1-\cos\theta)$$



O'Hare [2403.17697]

$$m_A|\theta|^2 R^3 = \frac{\rho_A}{m_A} R^3 \equiv N_A$$

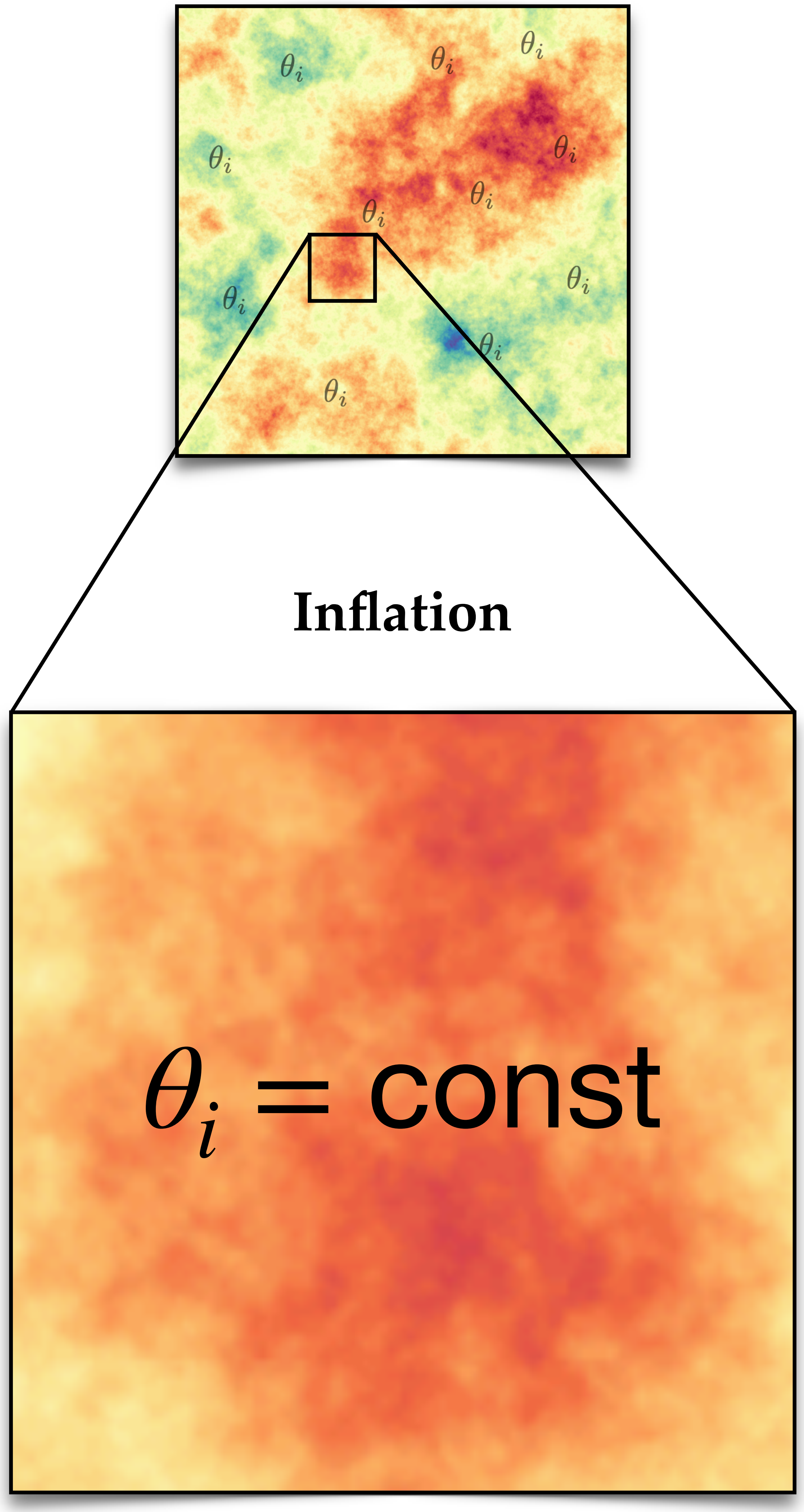


Borsanyi+ [1606.07494]

Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

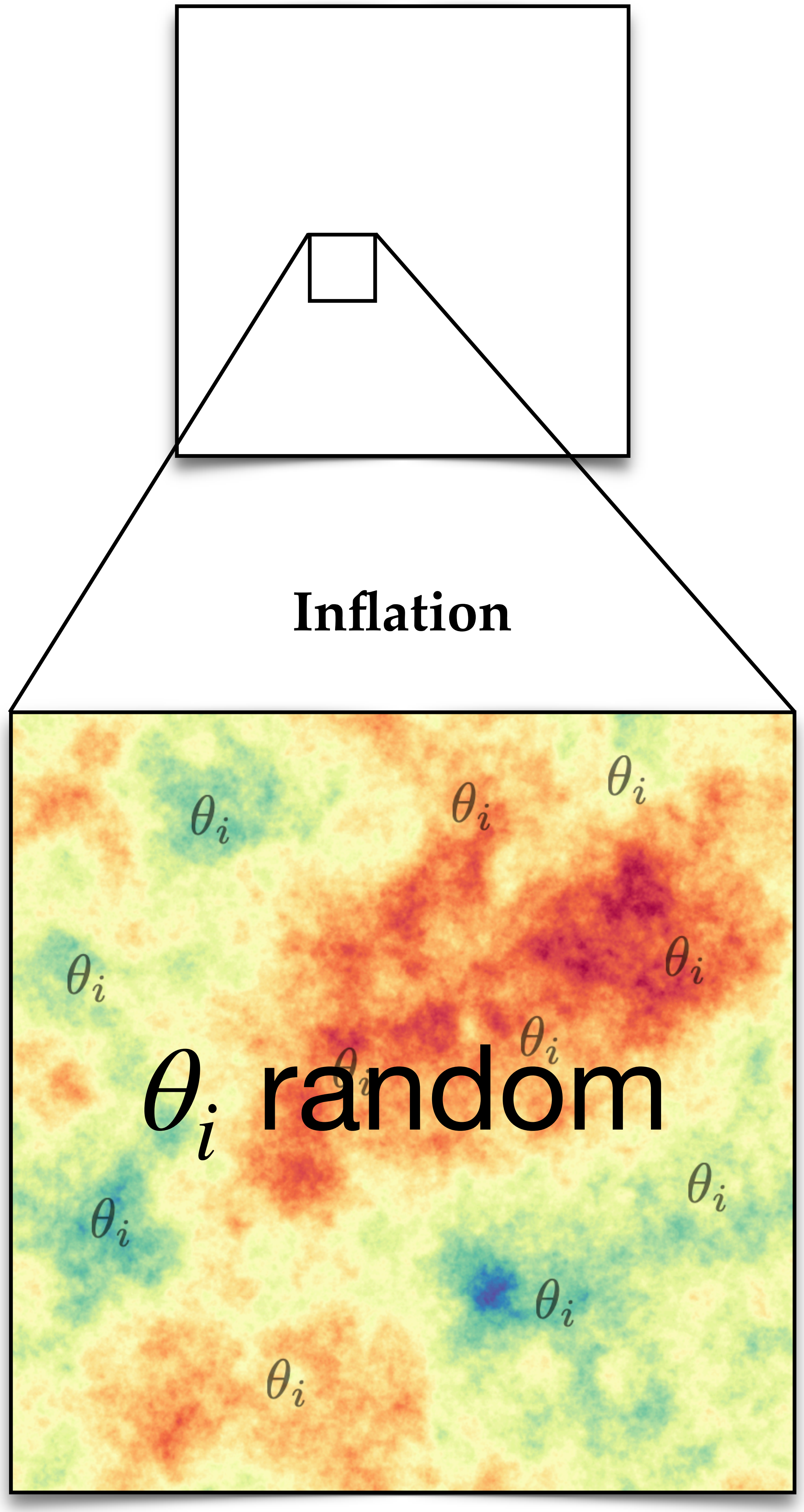
PQ broken **before** and **during** inflation



$$\sqrt{\langle \theta_i \rangle} = ???$$

Post-Inflationary Scenario

PQ broken **after** inflation

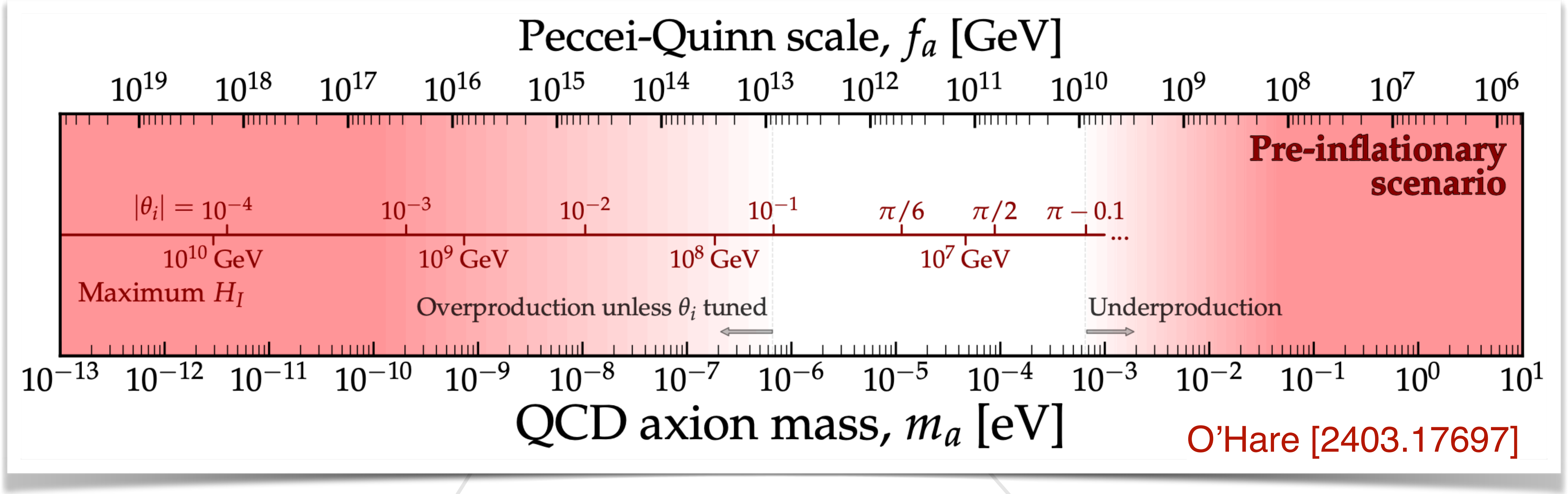


$$\sqrt{\langle \theta_i \rangle} \sim 2$$

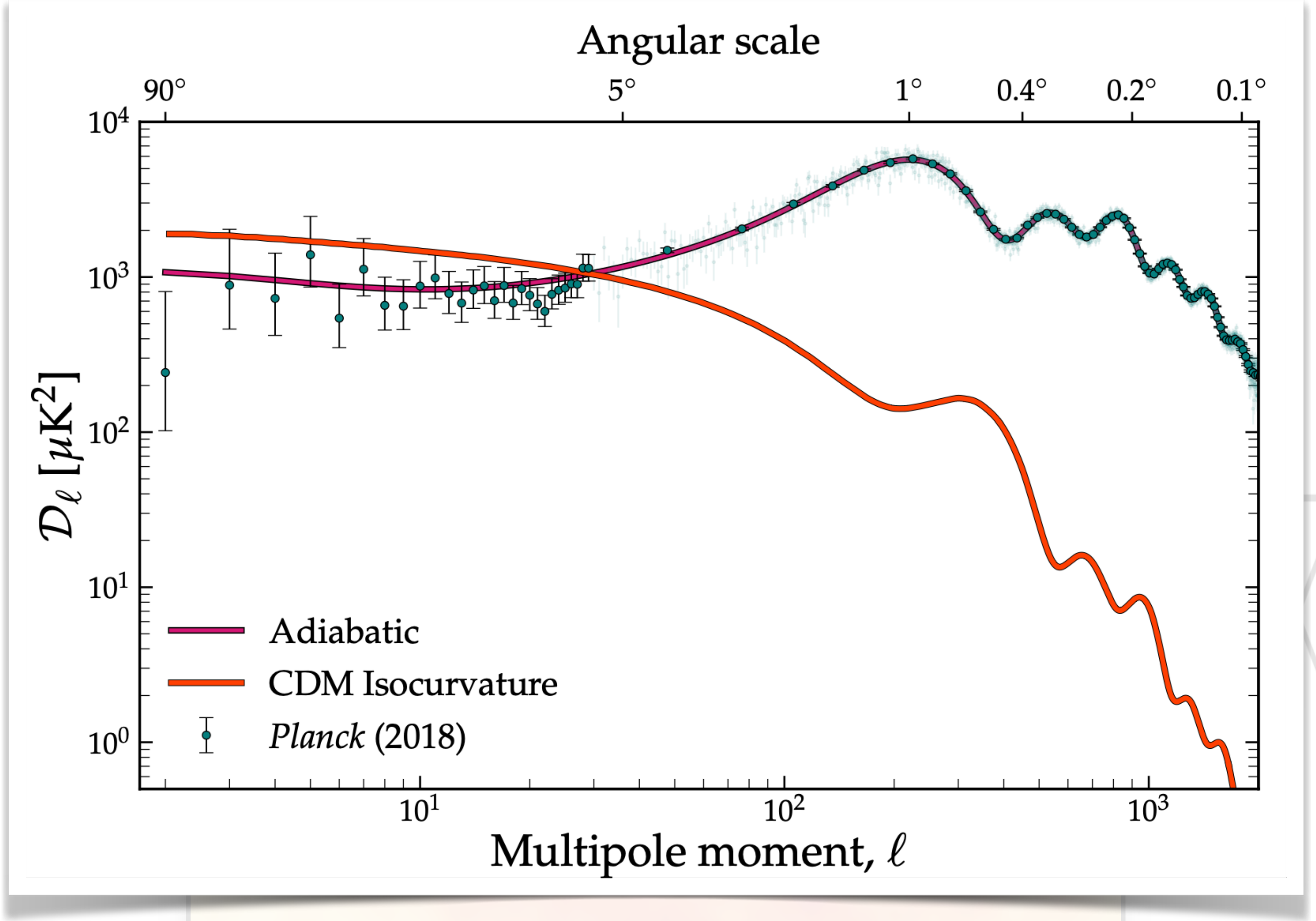
Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation



O'Hare [2403.17697]



PLANCK [1907.12875]

$\sqrt{\langle\theta_i\rangle} = ???$

Post-Inflationary Scenario

PQ broken **after** inflation

$\sqrt{\langle\theta_i\rangle} \sim 2$

Pre- vs. Post-Inflationary Scenario

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation

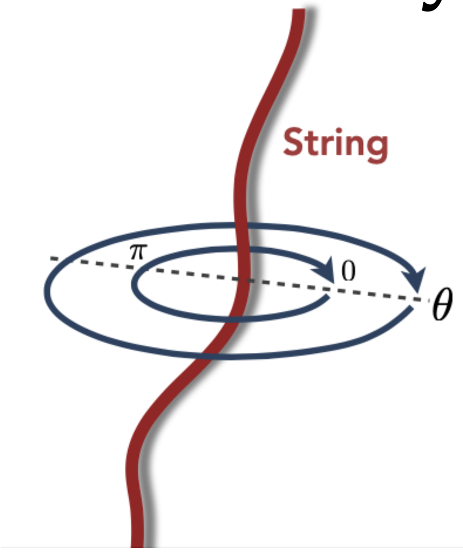
Post-Inflationary Scenario

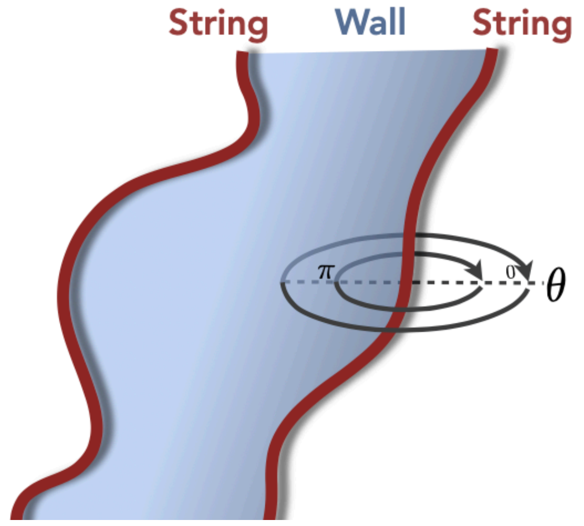
PQ broken **after** inflation

- Allows in principle for precise axion mass prediction

$$\Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ???\mu\mathrm{eV}$$

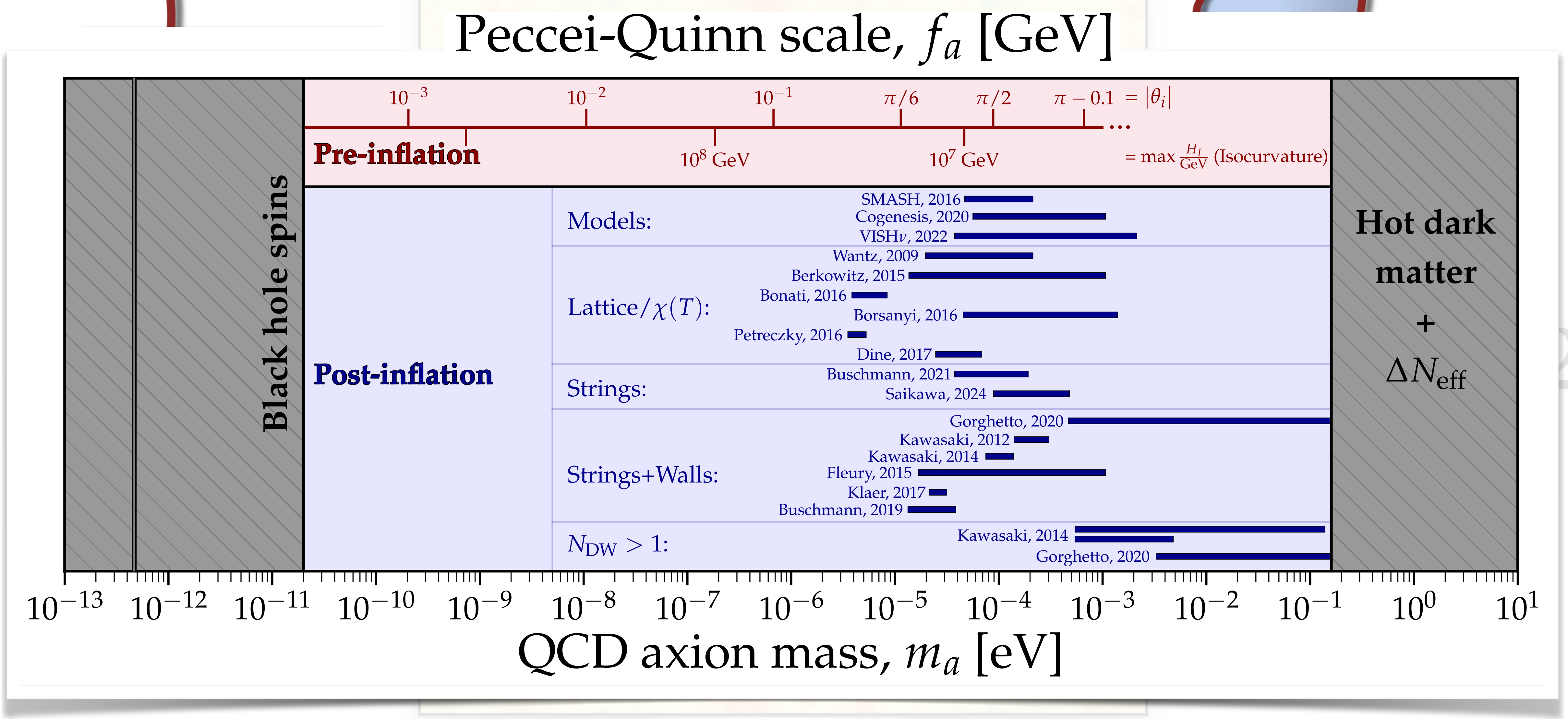
- **Subtlety:** Formation of Strings (+ Domain Walls)



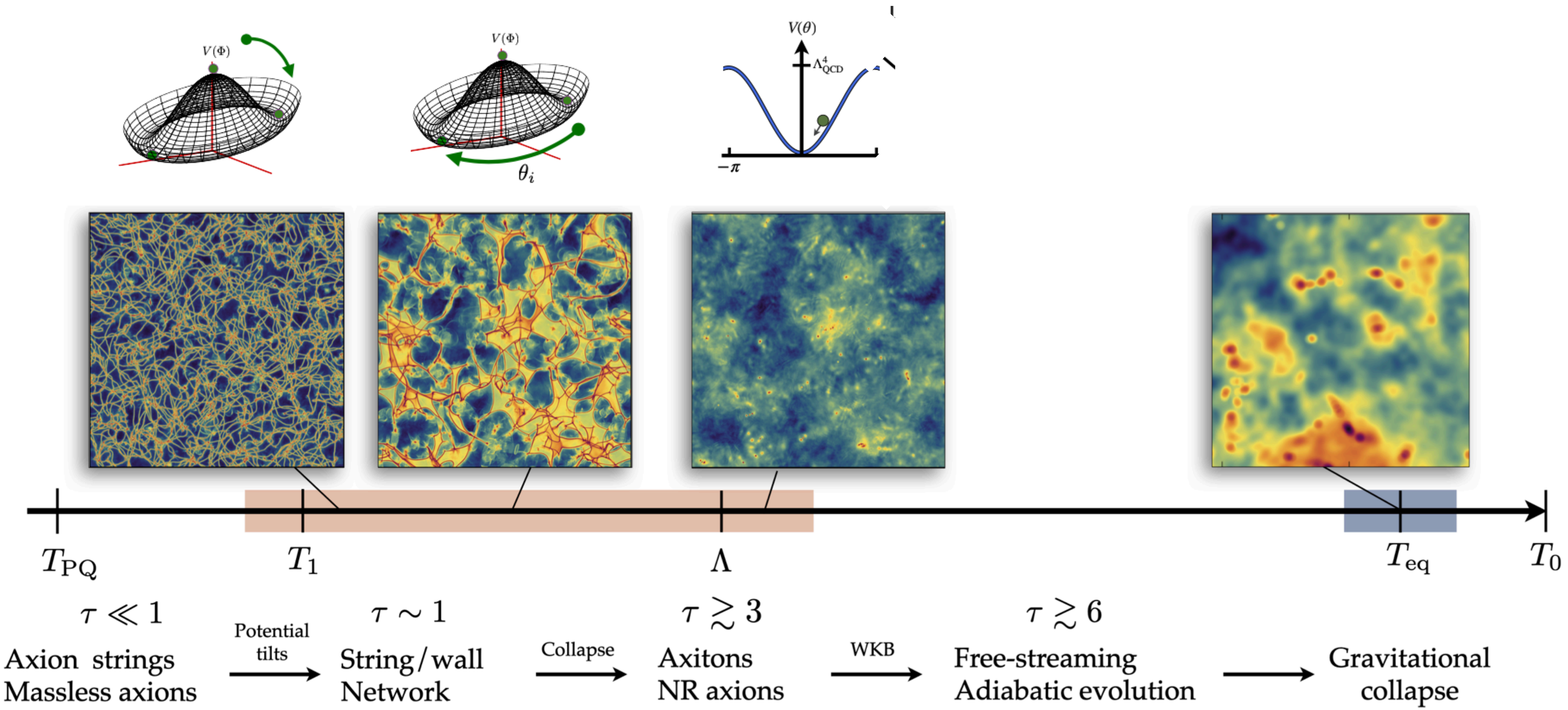


Inflation

O'Hare [2403.17697]



Cosmological Evolution in the Post-Inflationary Scenario



Sorry, no movie in PDF version :-(

O'Hare+ [2110.11014]

How to simulate Axion Strings?

- Solve the classical EOM for a complex scalar field in comoving coordinates, discretised on a lattice:

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda\phi\left(|\phi|^2 - \tau^2\right) = 0$$

- **Tricky:** Simulations require proper resolution of two very different length scales.

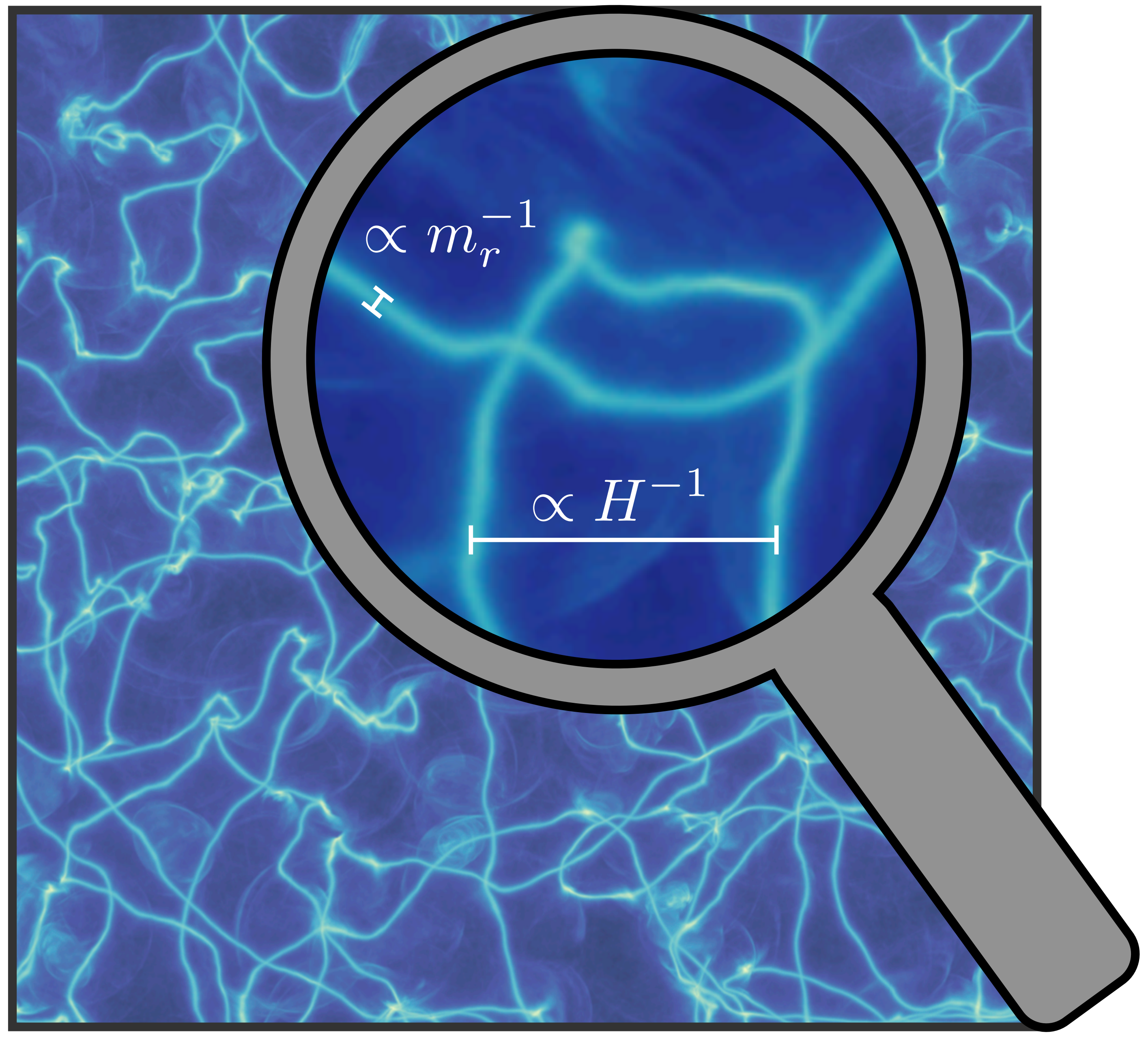
  - String core radius

    $\propto \frac{1}{m_r} \propto \frac{1}{f_a}$, where m_r = radial mass

  - Hubble radius

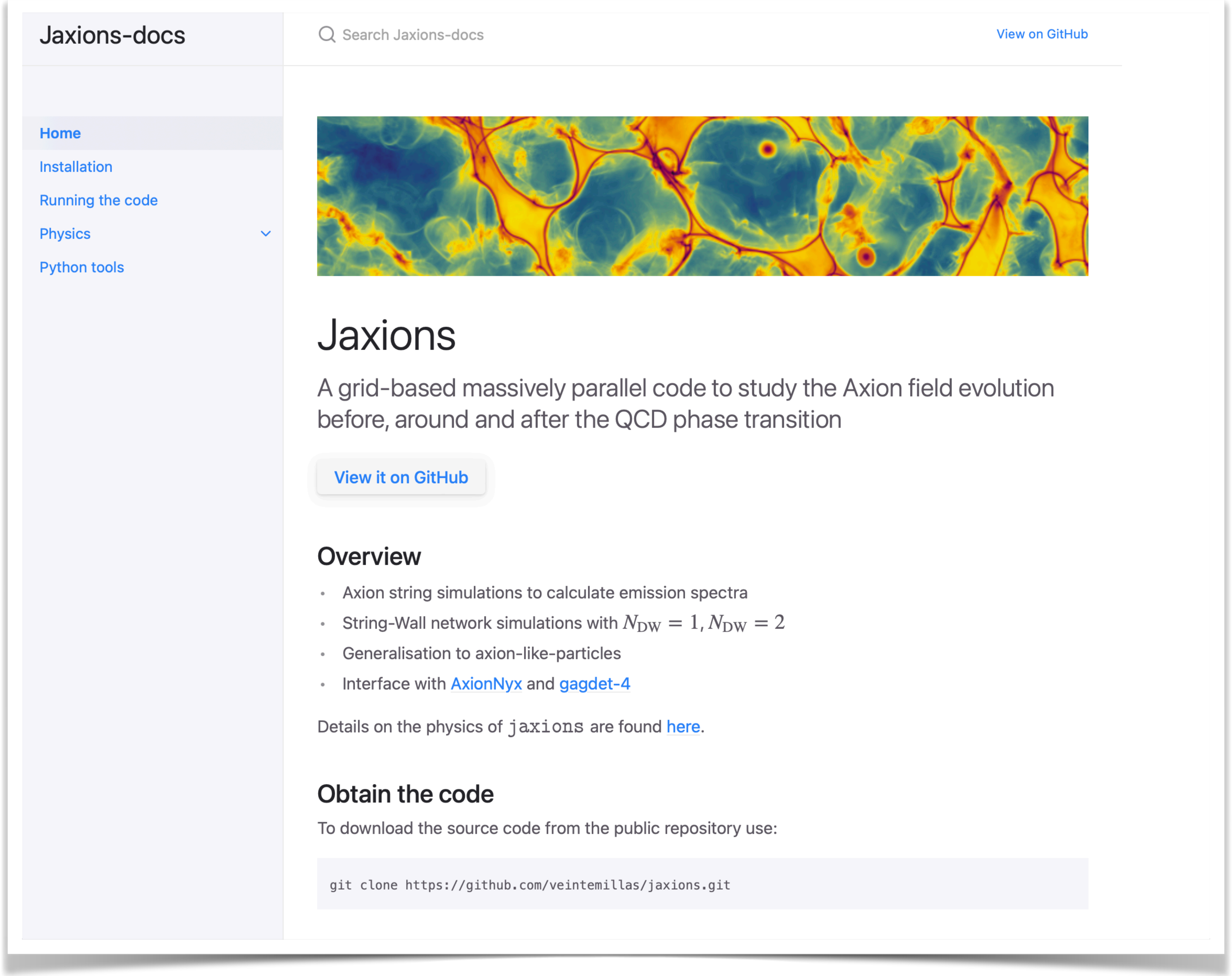
    $\propto \frac{1}{H}$

  - Realistic value: $\frac{f_a}{H_{\text{QCD}}} \approx 10^{30} \implies \log\left(\frac{m_r}{H}\right) \approx 70$



Jaxions Code

- State-of-the-art, highly parallelised C++ code to simulate the evolution of the axion dark matter field in the early Universe.
- Available on Github: https://github.com/veintemillas/jaxions.
- Manual under development and available soon!


Jaxions-docs

Search Jaxions-docs

View on GitHub

Home

Installation

Running the code

Physics

Python tools

Jaxions

A grid-based massively parallel code to study the Axion field evolution before, around and after the QCD phase transition

View it on GitHub

Overview

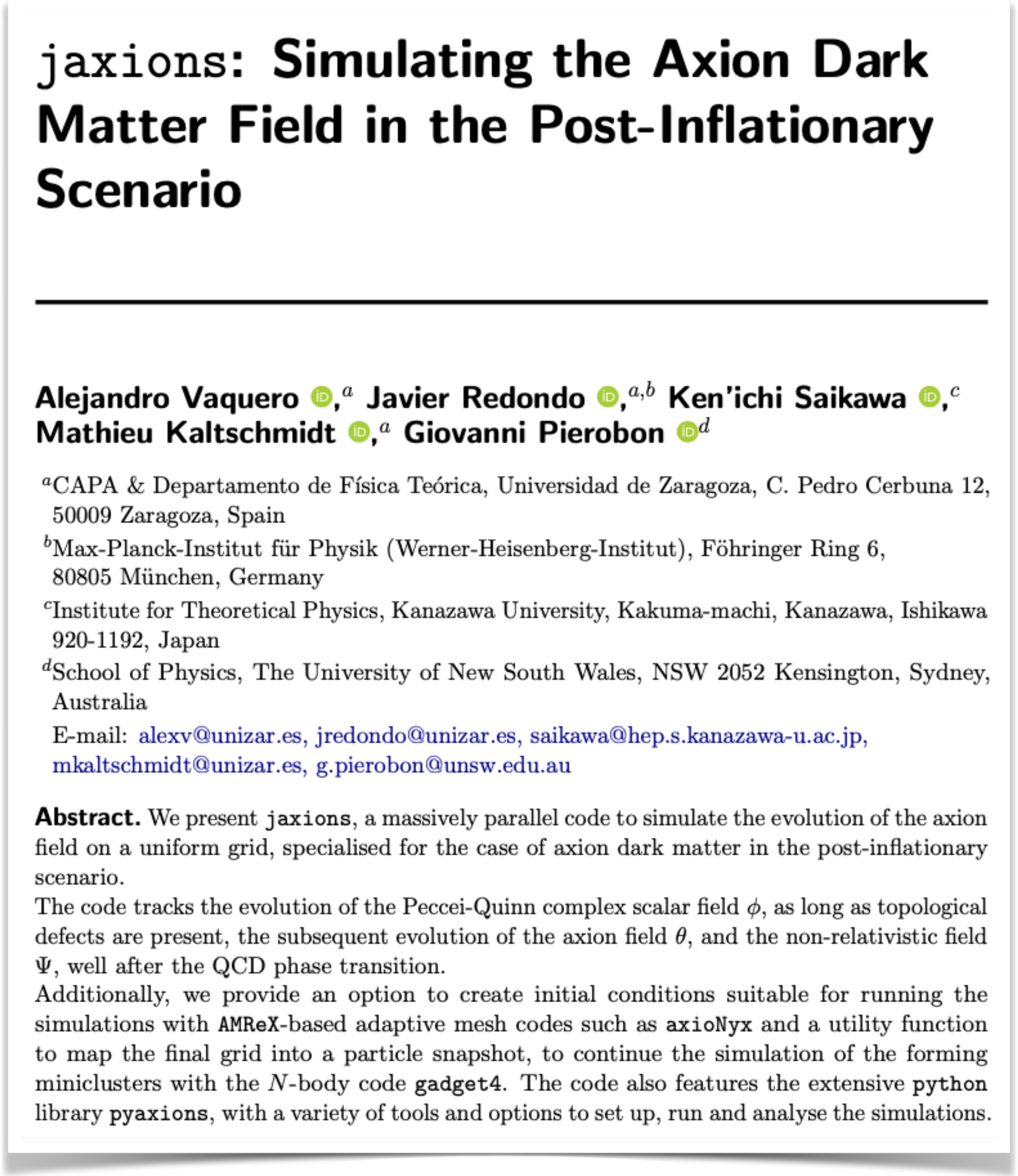
- Axion string simulations to calculate emission spectra
- String-Wall network simulations with $N_{\rm DW} = 1, N_{\rm DW} = 2$
- Generalisation to axion-like-particles
- Interface with AxionNyx and gagdet-4

Details on the physics of `jaxions` are found here.

Obtain the code

To download the source code from the public repository use:

```
git clone https://github.com/veintemillas/jaxions.git
```


jaxions: Simulating the Axion Dark Matter Field in the Post-Inflationary Scenario

Alejandro Vaquero,[a] Javier Redondo,[a,b] Ken'ichi Saikawa,[c] Mathieu Kaltschmidt,[a] Giovanni Pierobon[d]

[a]CAPA & Departamento de Física Teórica, Universidad de Zaragoza, C. Pedro Cerbuna 12, 50009 Zaragoza, Spain
[b]Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
[c]Institute for Theoretical Physics, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan
[d]School of Physics, The University of New South Wales, NSW 2052 Kensington, Sydney, Australia

E-mail: alexv@unizar.es, jredondo@unizar.es, saikawa@hep.s.kanazawa-u.ac.jp, mkaltschmidt@unizar.es, g.pierobon@unsw.edu.au

Abstract. We present `jaxions`, a massively parallel code to simulate the evolution of the axion field on a uniform grid, specialised for the case of axion dark matter in the post-inflationary scenario.
The code tracks the evolution of the Peccei-Quinn complex scalar field ϕ, as long as topological defects are present, the subsequent evolution of the axion field θ, and the non-relativistic field Ψ, well after the QCD phase transition.
Additionally, we provide an option to create initial conditions suitable for running the simulations with **AMReX**-based adaptive mesh codes such as **axioNyx** and a utility function to map the final grid into a particle snapshot, to continue the simulation of the forming miniclusters with the N-body code **gadget4**. The code also features the extensive **python** library **pyaxions**, with a variety of tools and options to set up, run and analyse the simulations.

The Issue of large $\log(m_r/H)$

- Evolution of the string density suggests that the energy density of the system is of order

$$\rho \sim 8\pi\xi \log(m_r/H) H^2 f_a^2$$

This leads to an enhancement by a factor of $\sim \xi \log(m_r/H)$ in comparison to the typical density $H^2 f_a^2$ at QCD temperatures.

- Does this imply an enhancement of the axion abundance (and therefore of the dark matter mass)?

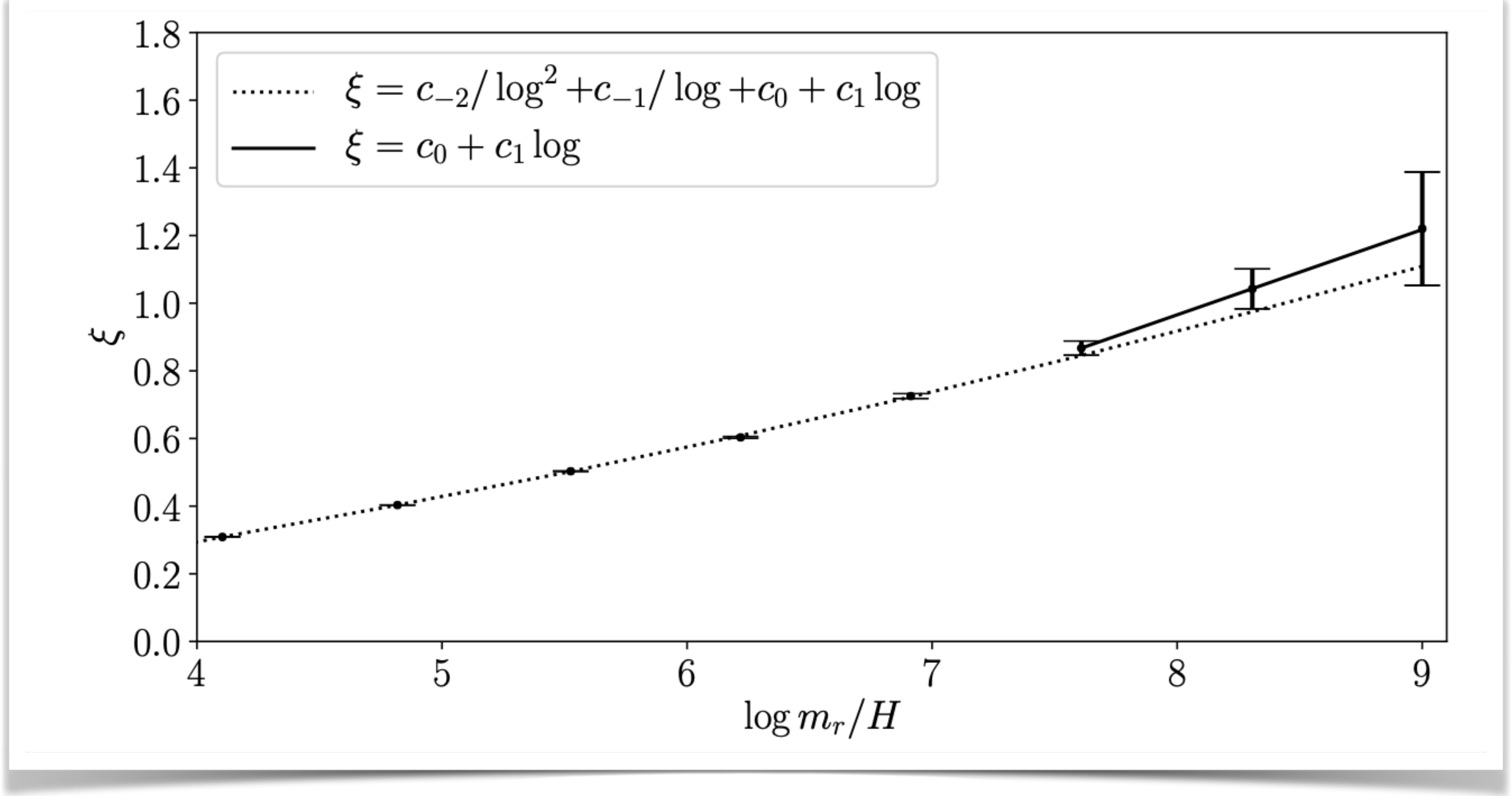
- We need to know how this energy is partitioned into radiated axions (i.e. the axion spectrum).

Logarithmic Growth of String Density

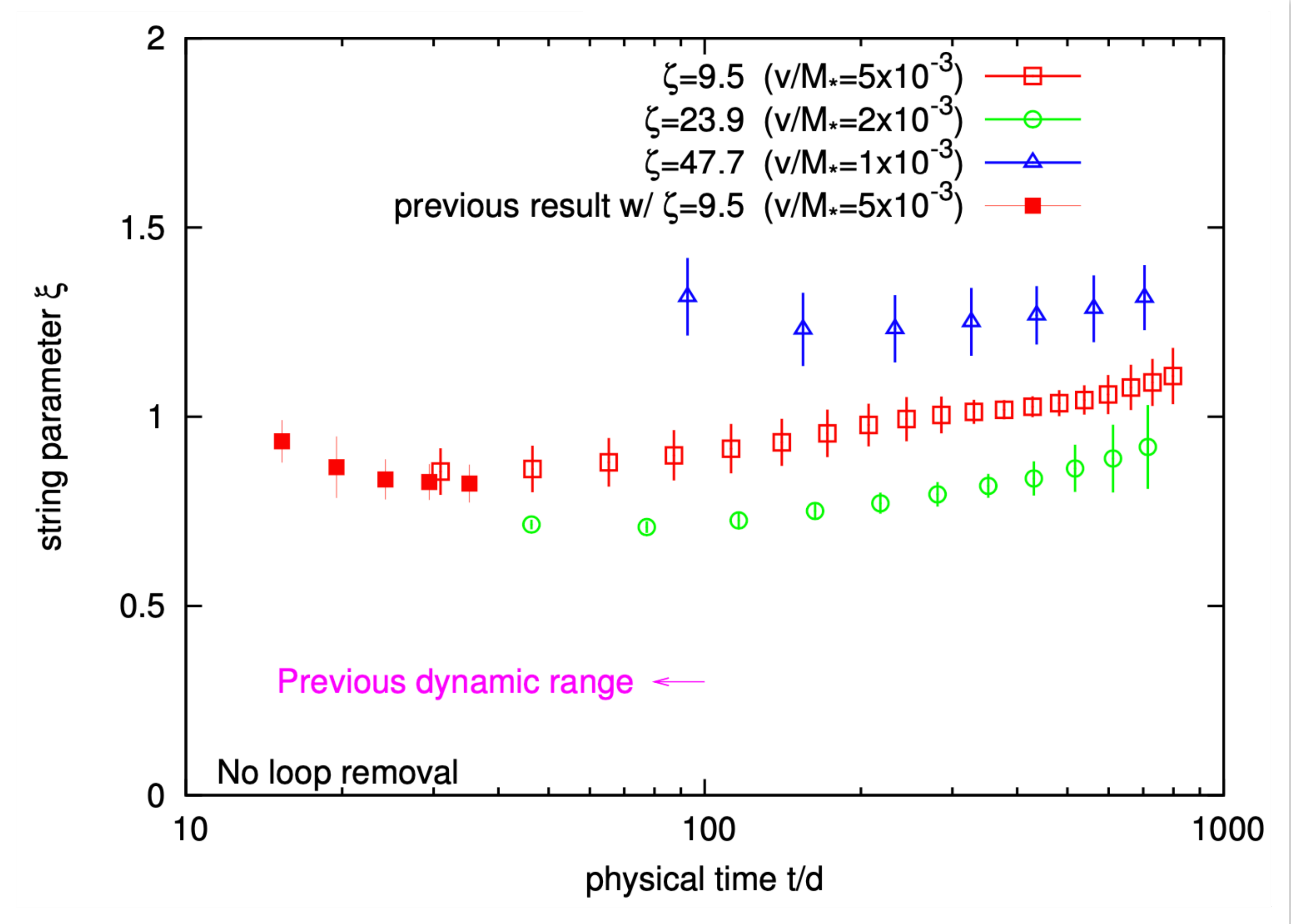
- String density parametrised as

$$\xi = \frac{l_{\mathrm{string}} t^2}{\mathcal{V}} = \frac{\rho_{\mathrm{string}} t^2}{\mu}$$

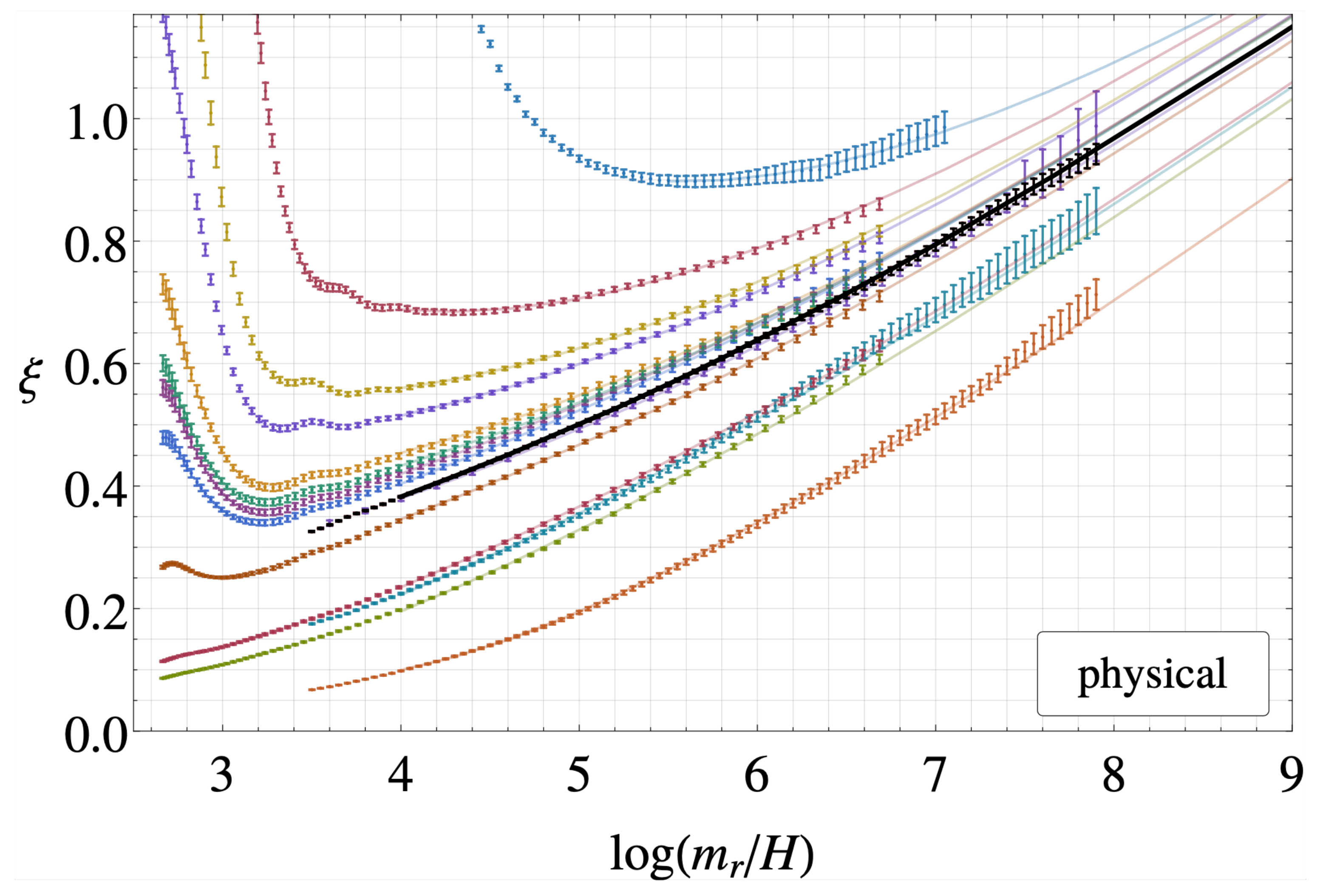
- Most recent simulations observe increase in ξ.



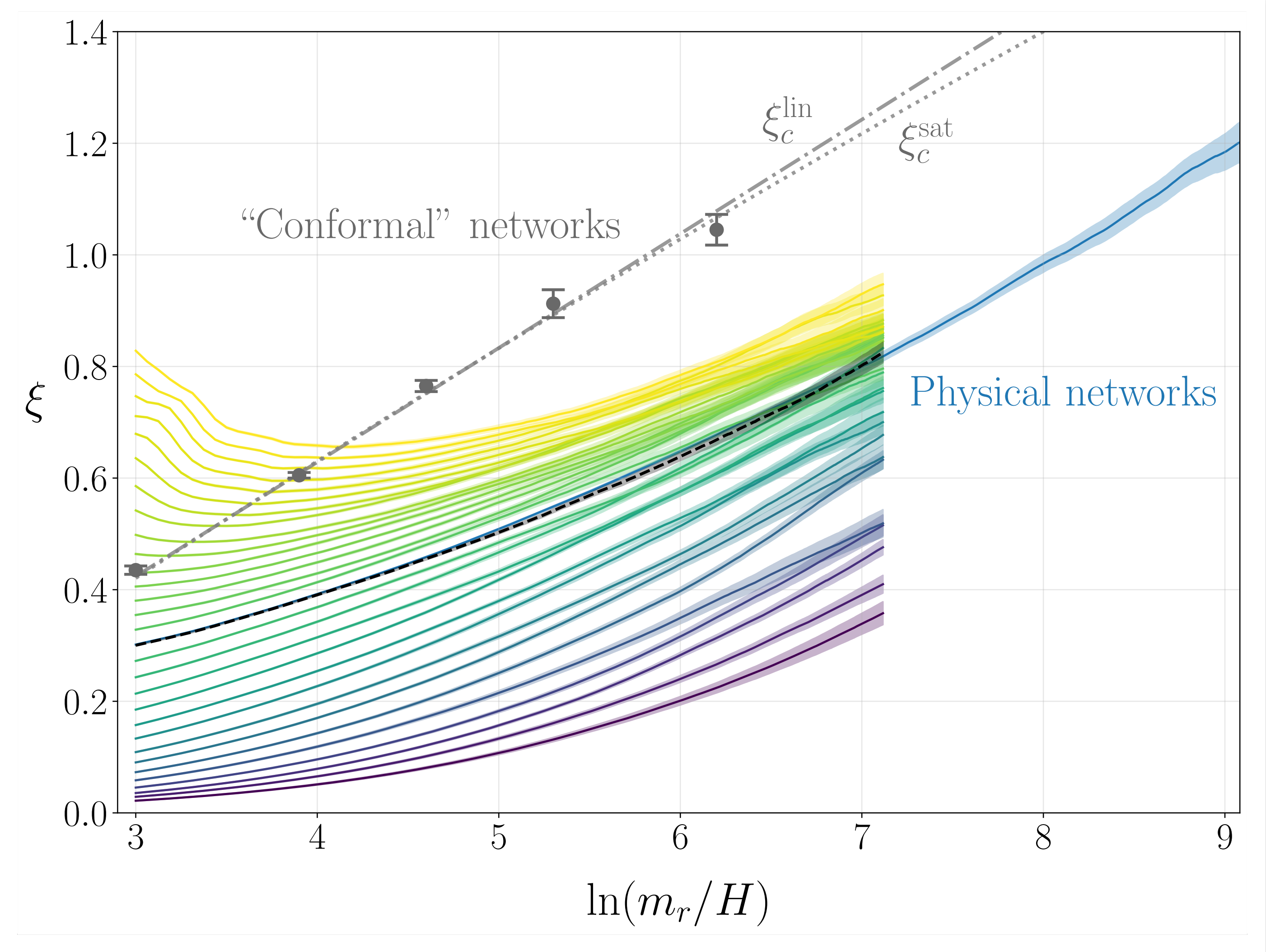
Buschmann+ [2108.05368]



Kawasaki+ [1806.05566]



Gorghetto+ [2007.04990]

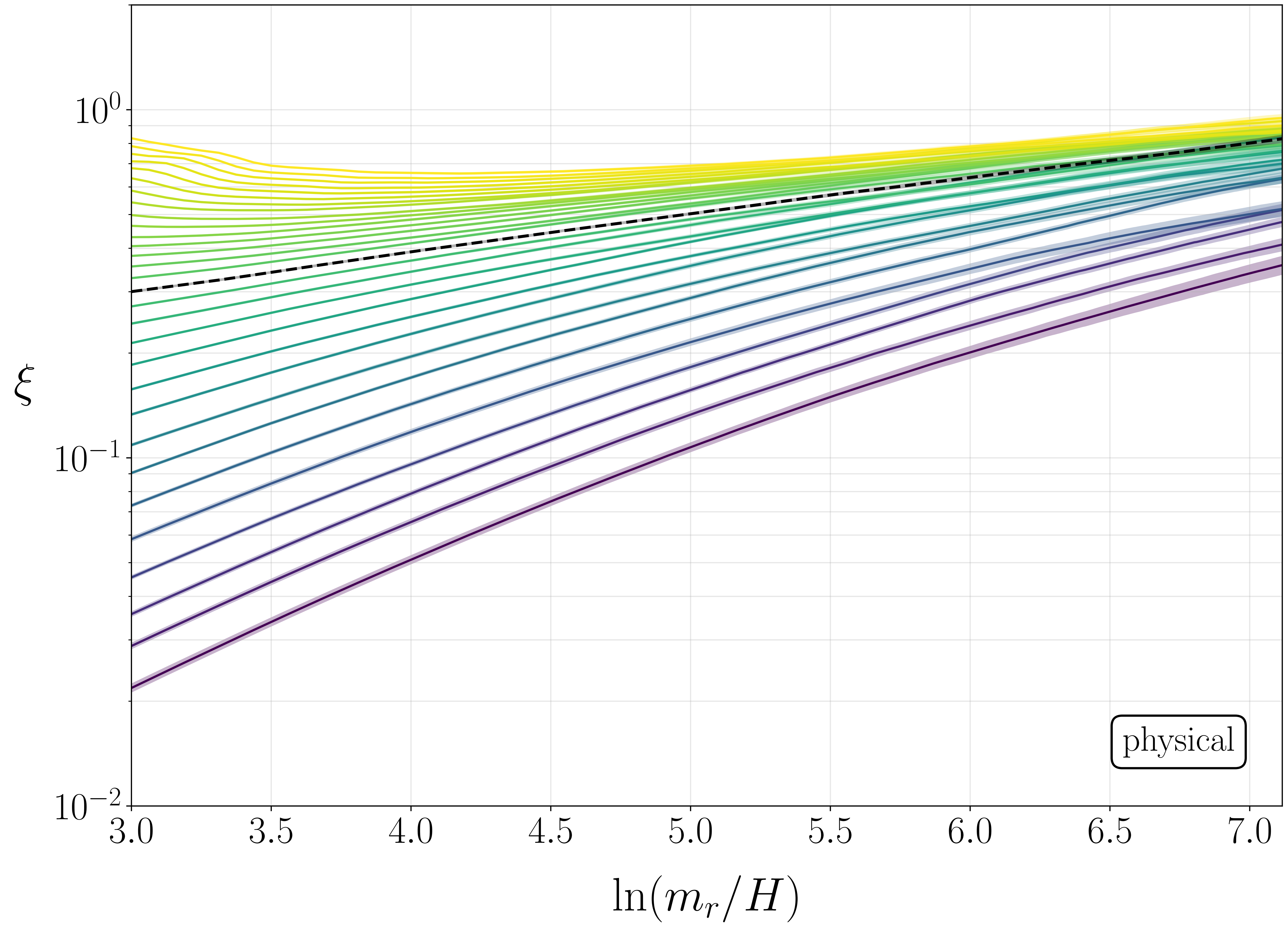


Saikawa, MK+ [2401.17253]

Evolution of String Density

- Model evolution of string network density: $\frac{d\xi}{dt} = \frac{C(x)}{t}(\xi_c(\ell(t)) - \xi(t))$

Klaer & Moore [1912.08058]



Two reasonable fits to the data:

$$\xi^{\text{lin}} = -0.19(3) + 0.205(7)\log(m_r/H)$$

$$\xi^{\text{sat}} = \frac{-0.25(15) + 0.23(6)\log(m_r/H)}{1 + 0.02(4)\log(m_r/H)}$$

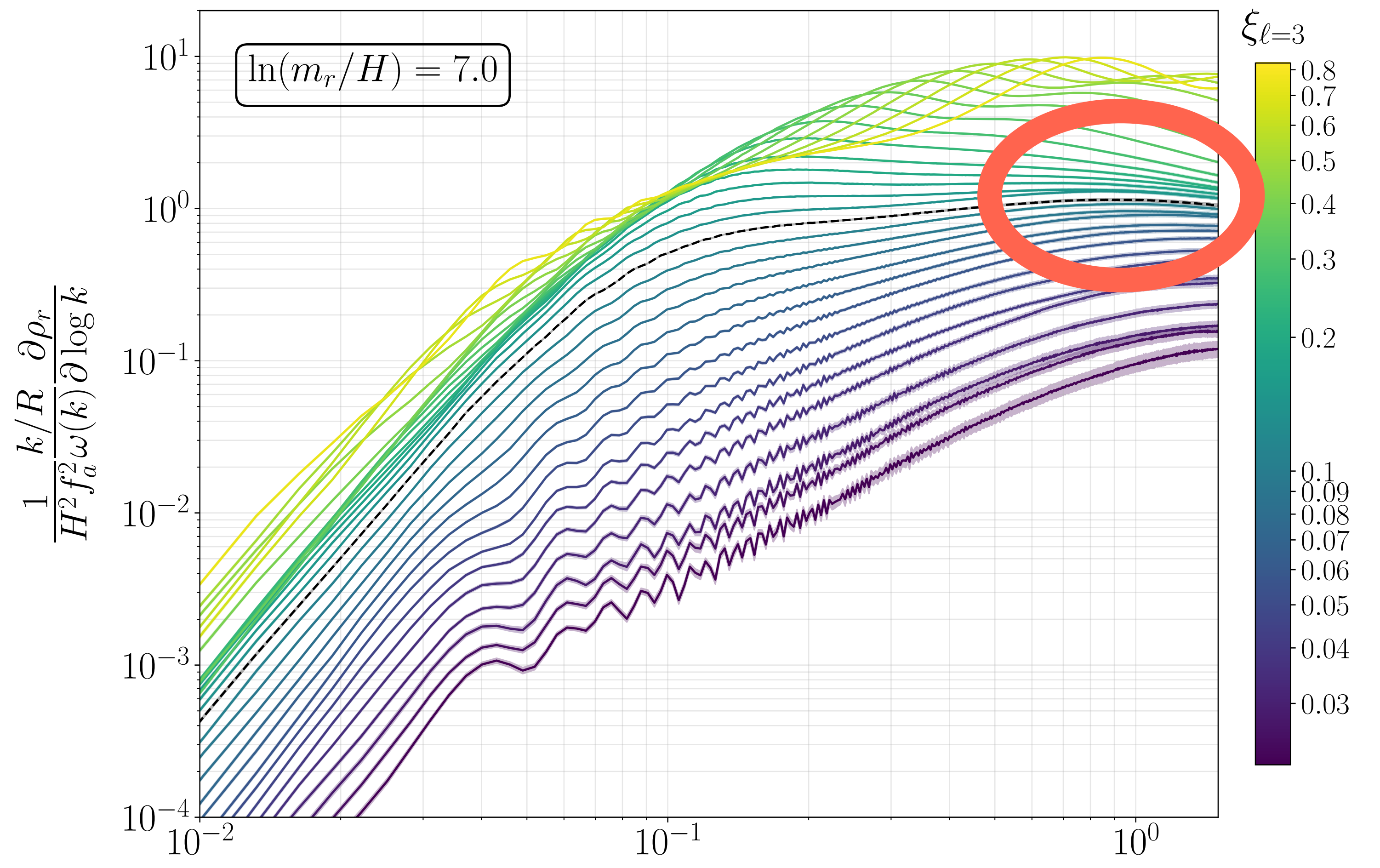
with

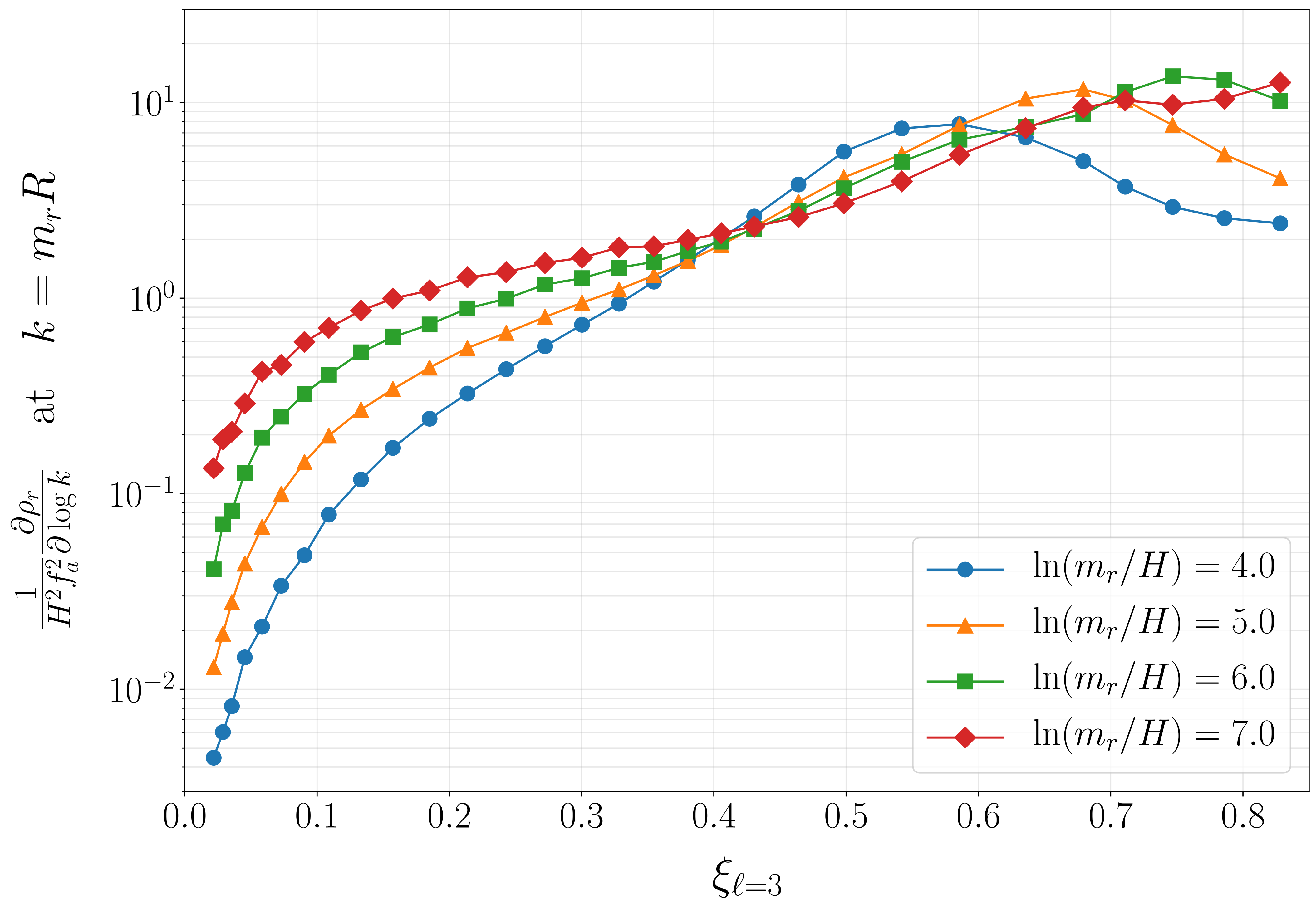
$$\xi^{\text{lin}}(\log = 70) \sim 13.8(5)$$

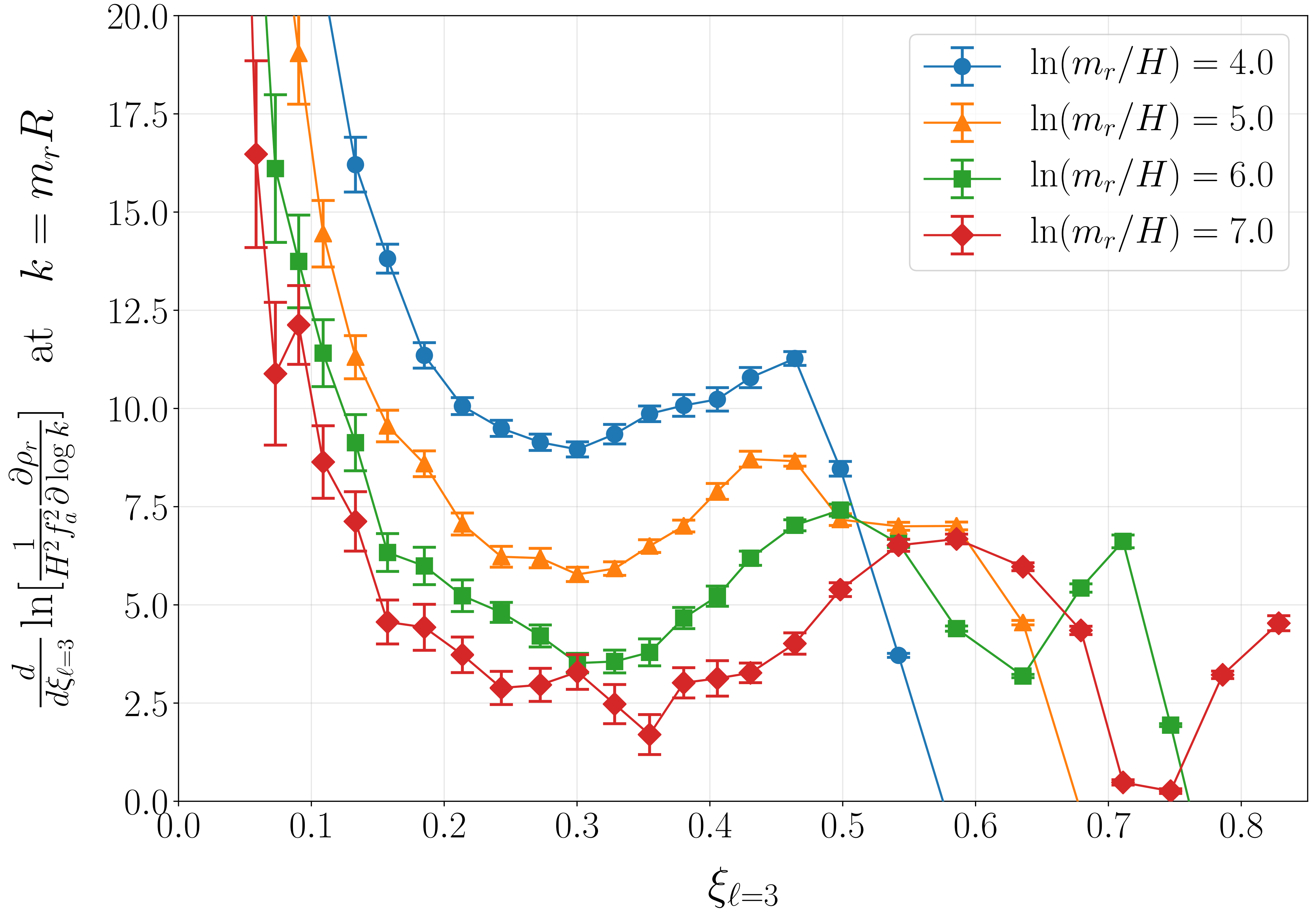
$$\xi^{\text{sat}}(\log = 70) \sim 7(3)$$

Existence of an Attractor?

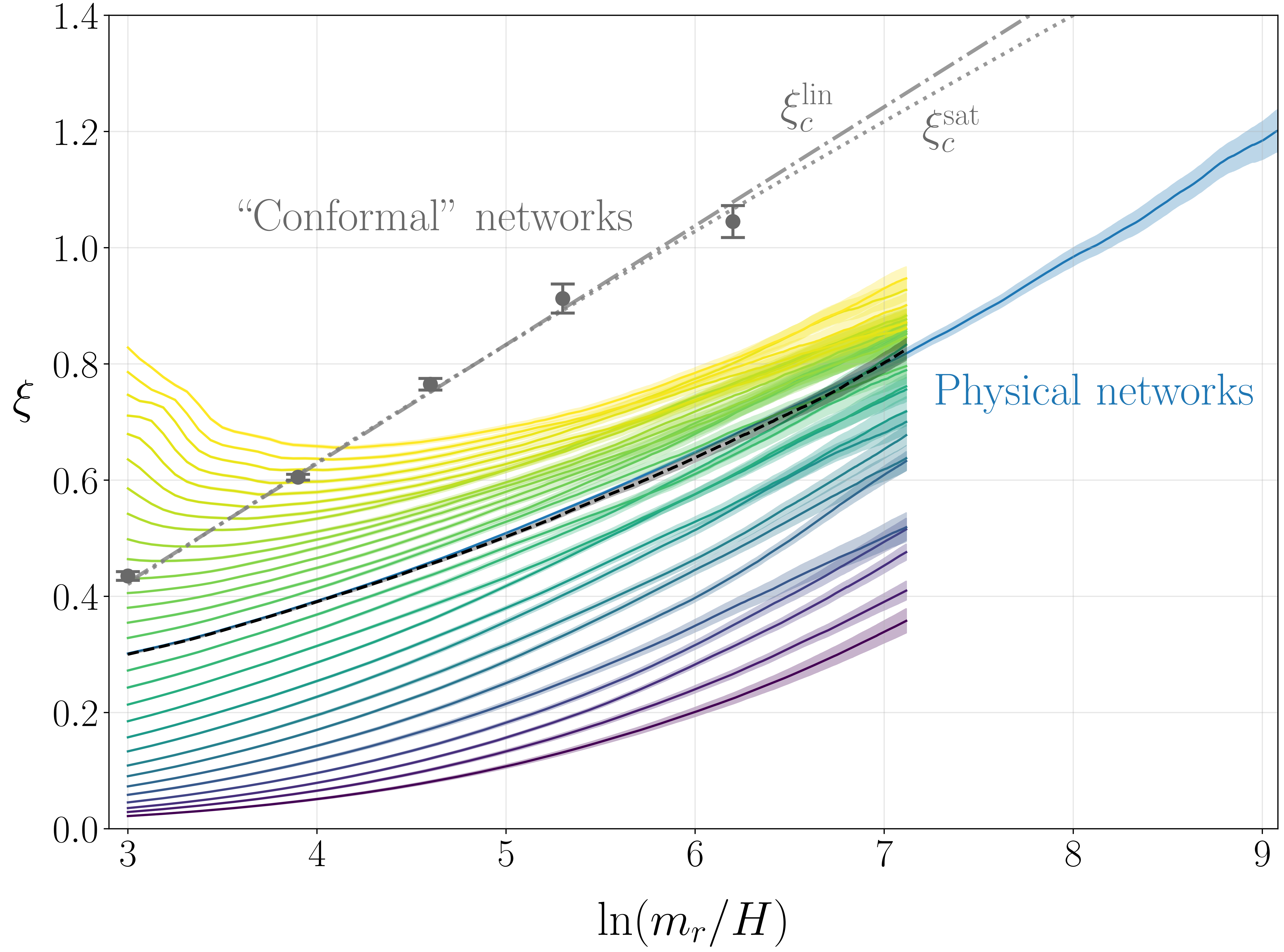
- Convergence behaviour is best observed in radial field spectra.
- Saxions are produced at $k/R \approx m_r$.
- Look for point that is least sensitive to ICs.







Evolution of String Density



Fleury & Moore [1509.00026]
Gorghetto+ [1806.04677; 2007.04990]
Kawasaki+ [1806.05566]
Hindmarsh+ [1908.03522]
Buschmann+ [2108.05368]

Logarithmic growth and “attractor” behaviour compatible with previous findings.

Axion Radiation from Strings

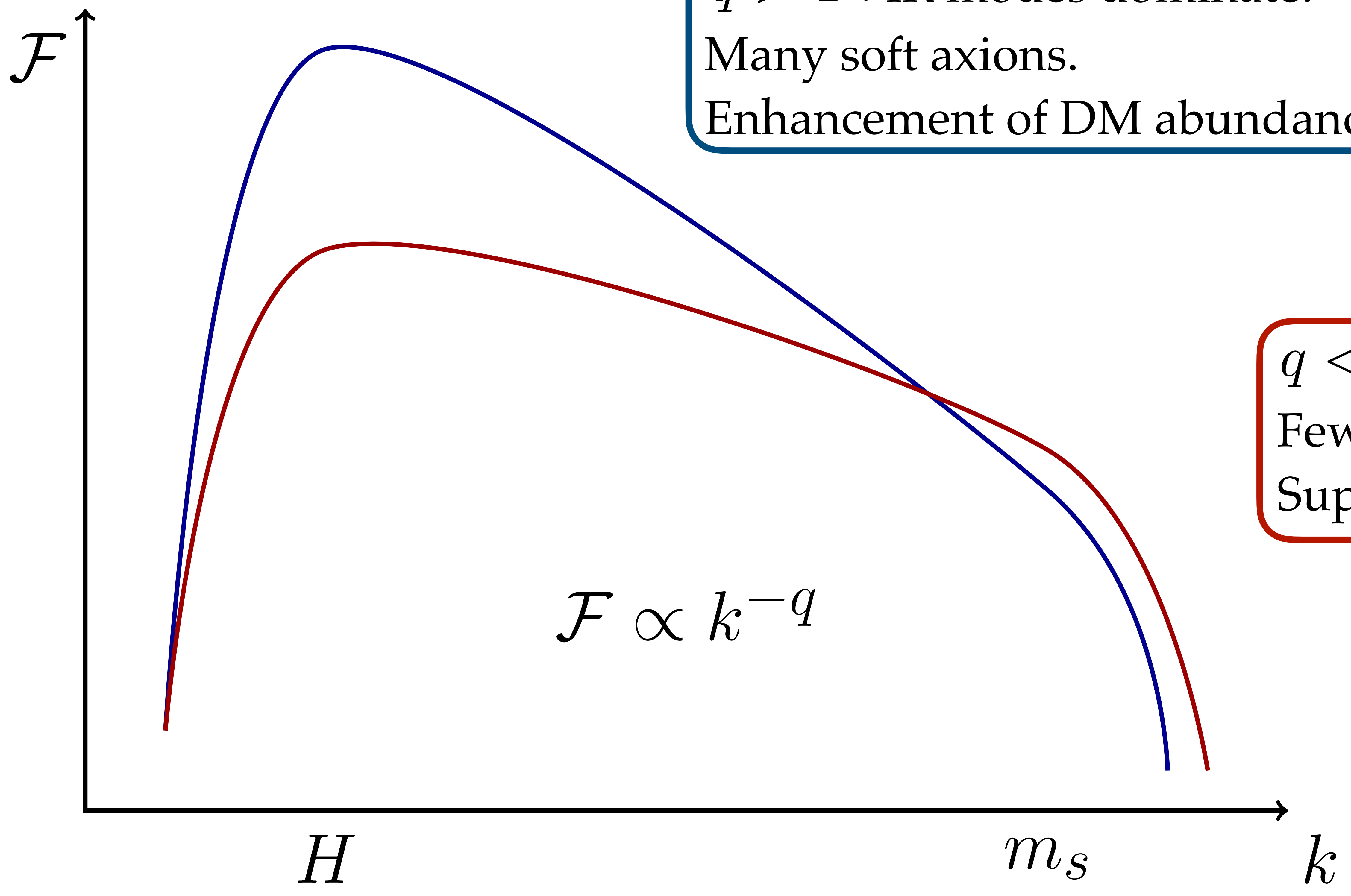
- Differential energy transfer rate:

$$\mathcal{F}\left(\frac{k}{RH},\frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2}\frac{1}{R^3}\frac{\partial}{\partial t}\left(R^4\frac{\partial \rho_a}{\partial k}\right)$$

(R: scale factor)

- Slope is important! Gorghetto+ [1806.04677], Buschmann+ [2108.05368], Saikawa, MK+ [2401.17253]

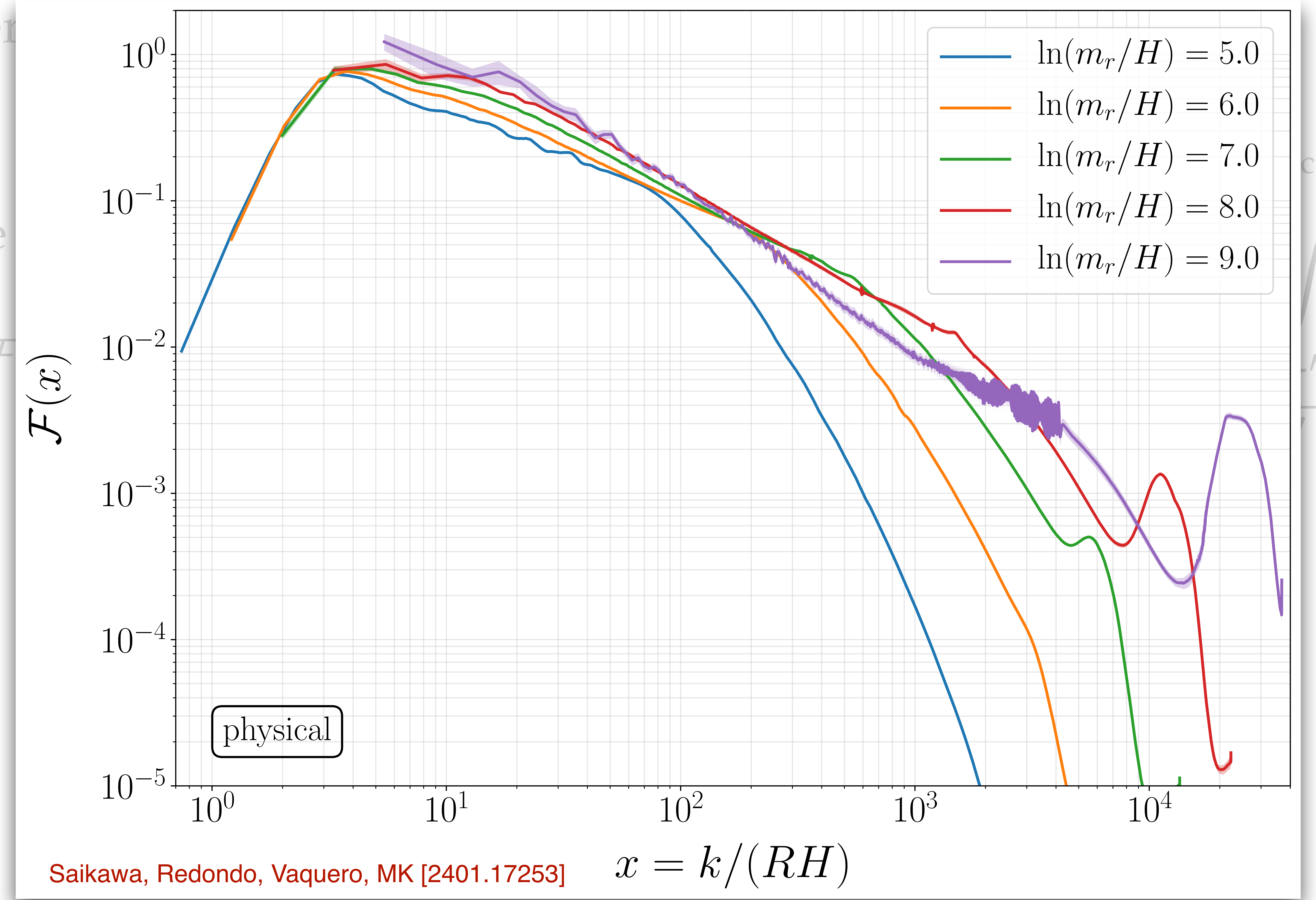
$q > 1$: IR modes dominate.
Many soft axions.
Enhancement of DM abundance.



$$\frac{n_a}{f_a^2 H} = \frac{1}{f_a^2 H}\int \mathrm{d}k \frac{1}{\omega}\frac{\partial \rho_a}{\partial k} = \int^{\tau}\frac{\mathrm{d}\tau'}{\tau'}\int\frac{\mathrm{d}x'}{x'}\mathcal{F}$$

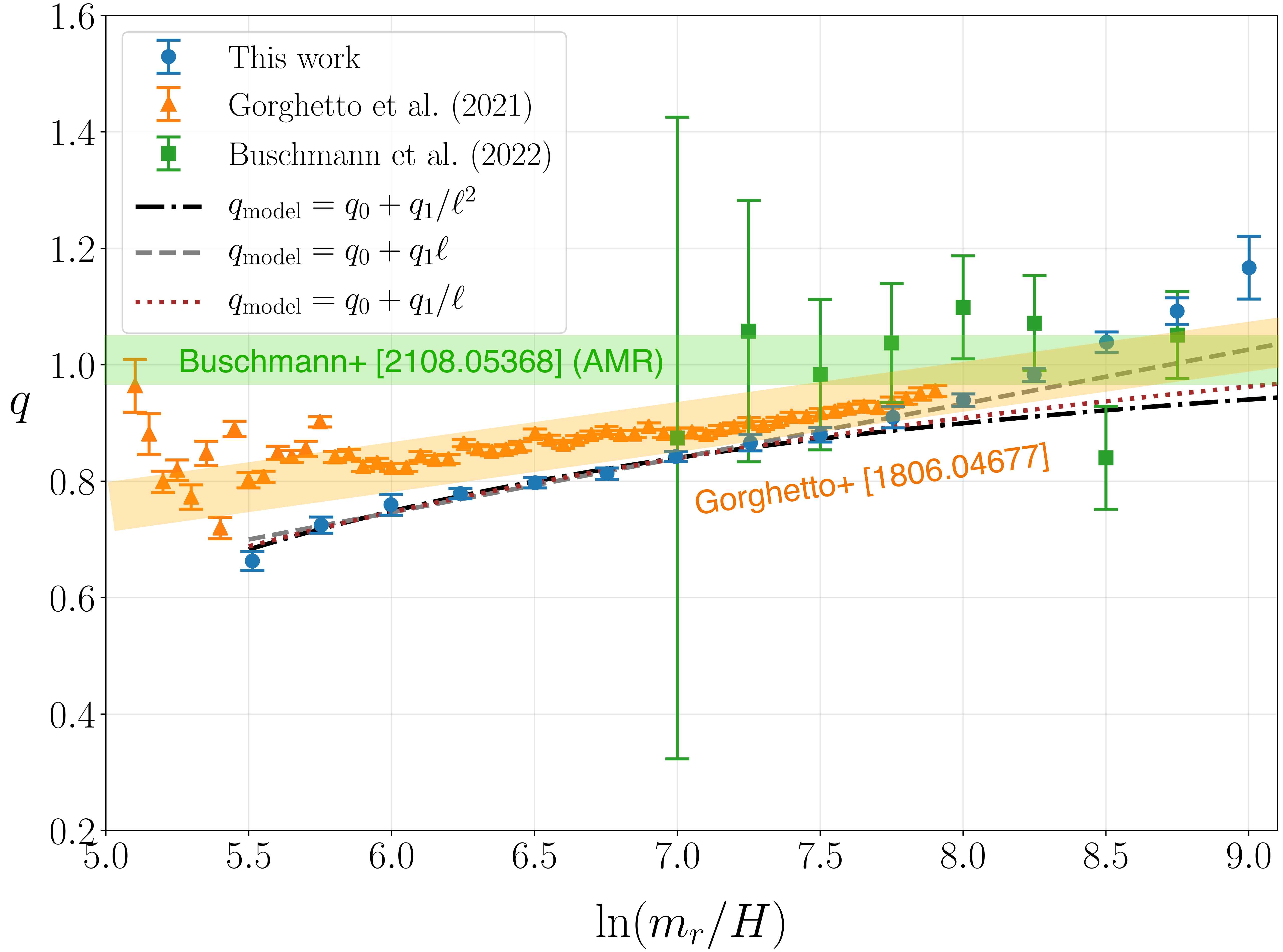
$q < 1$: UV modes dominate.
Fewer hard axions.
Suppression of DM abundance.

Axion Radiation from Strings



What can bias the Results?

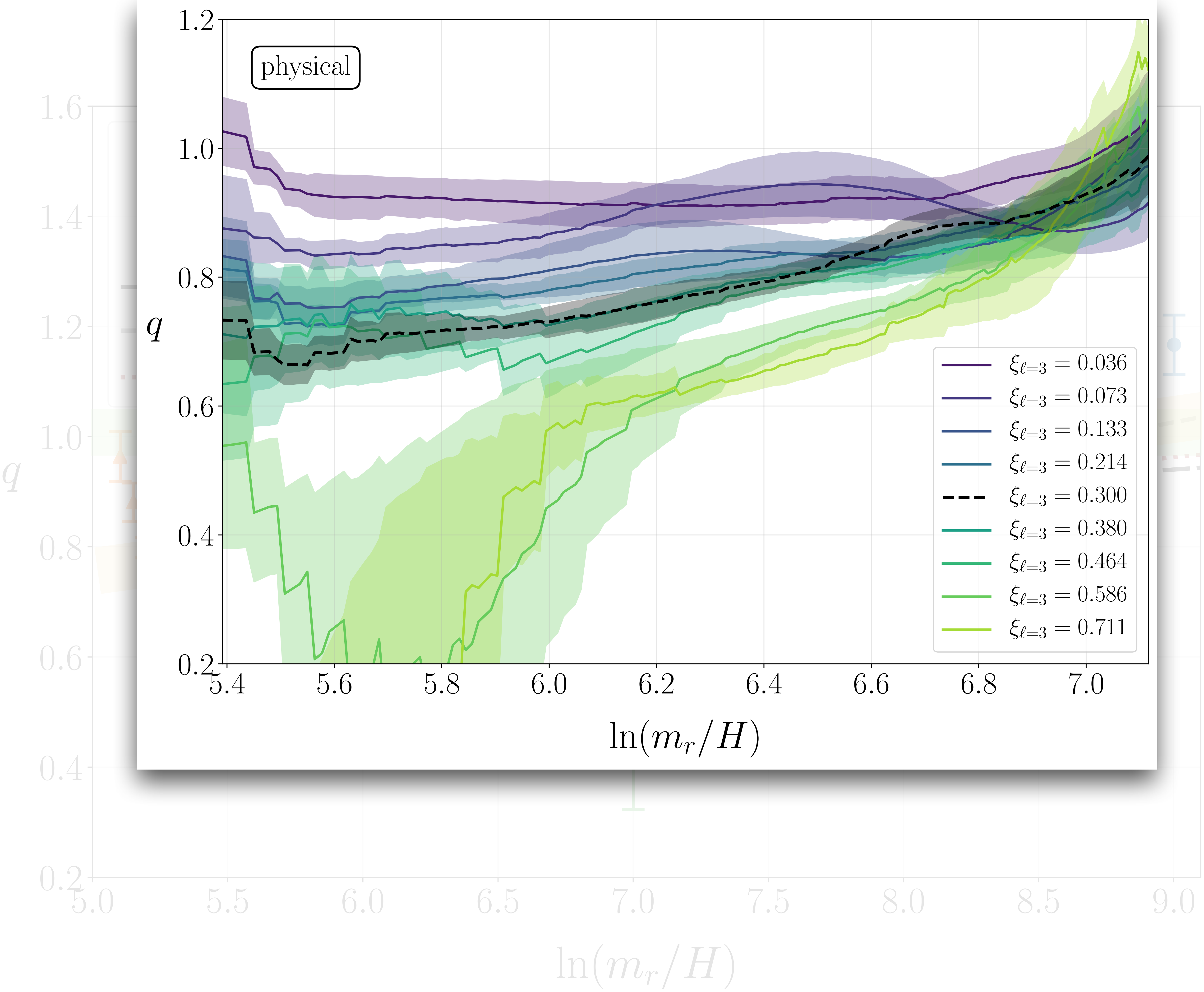
- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



Saikawa, Redondo, Vaquero, MK [2401.17253]

What can bias the Results?

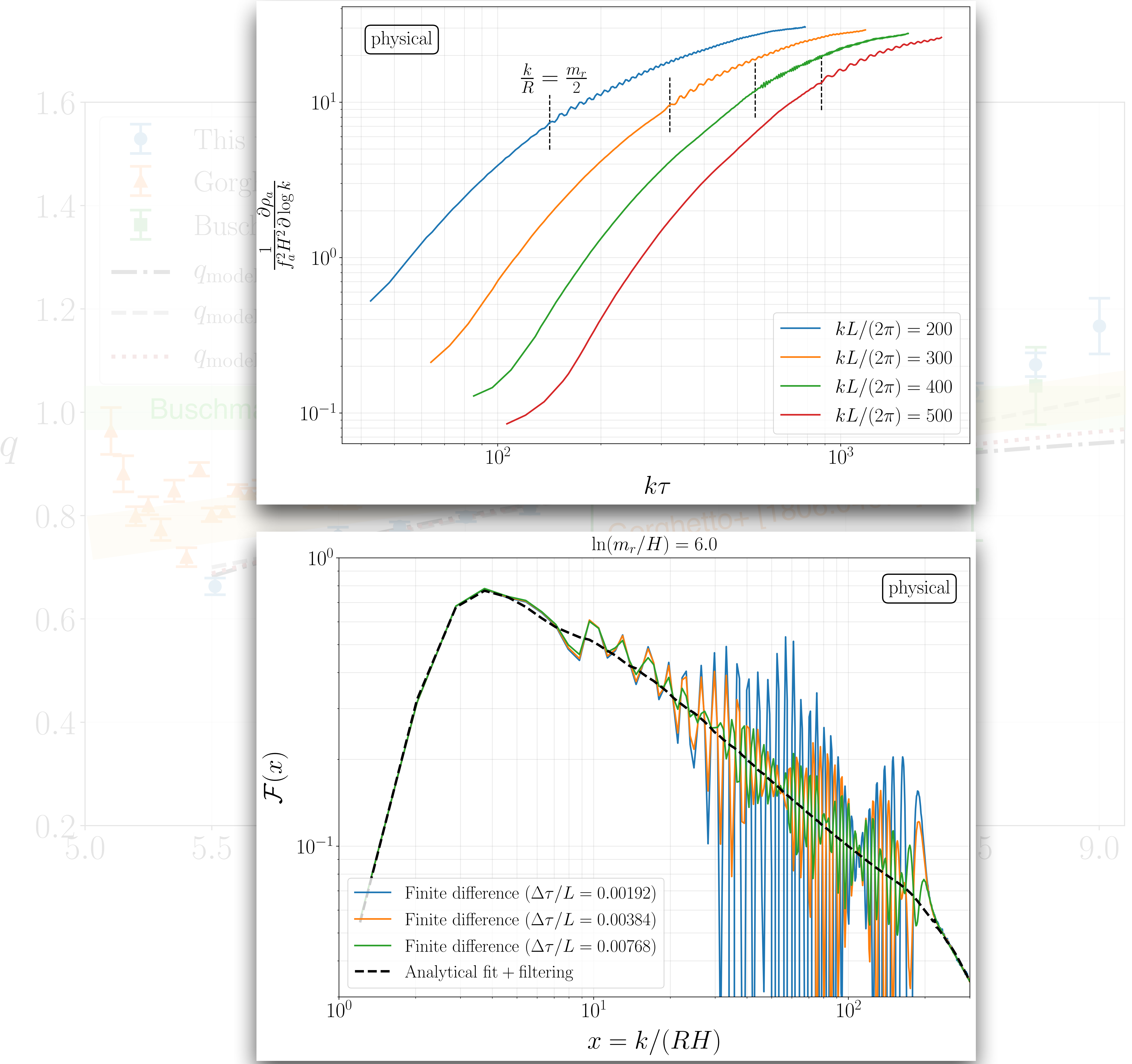
- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



Saikawa, Redondo, Vaquero, MK [2401.17253]

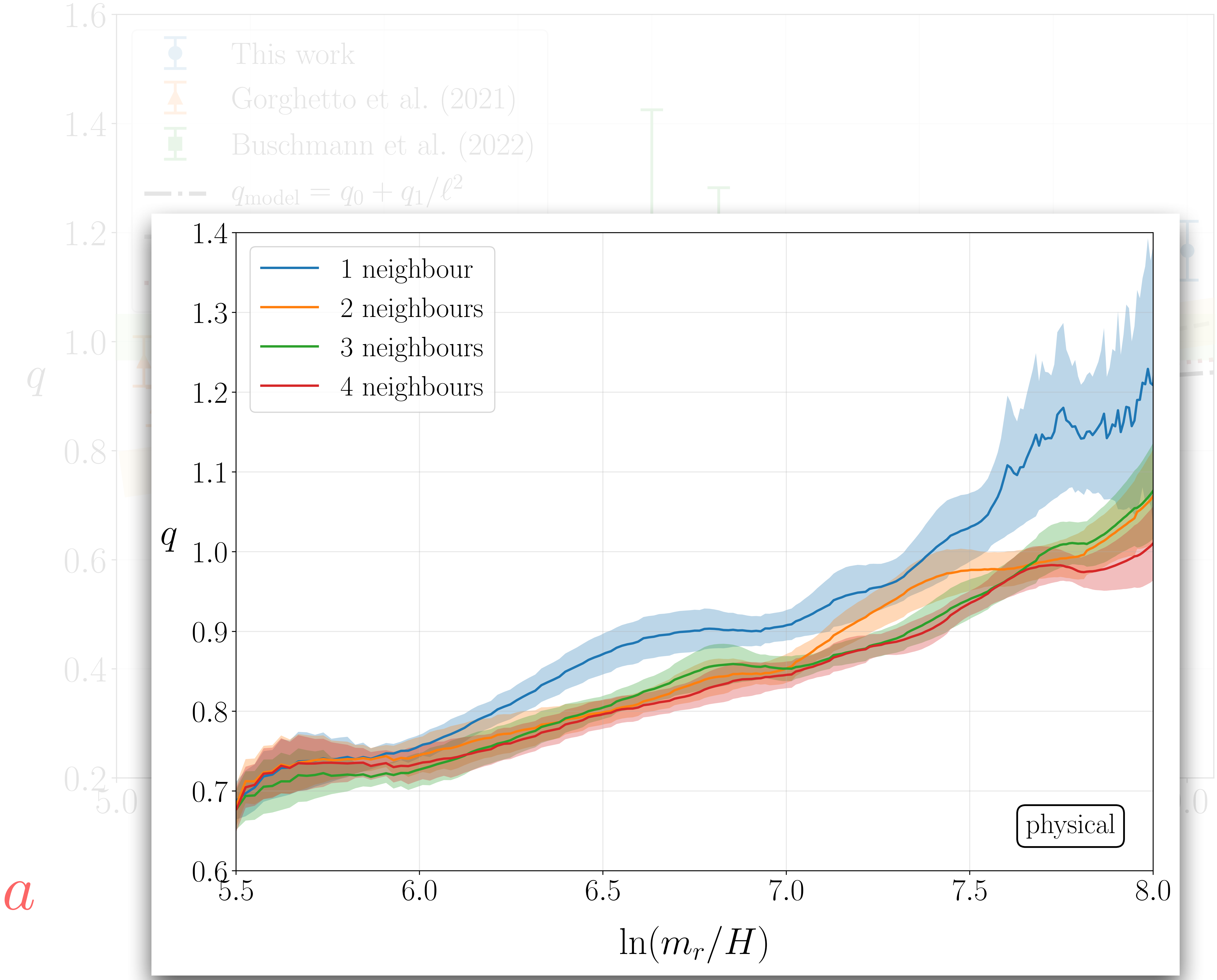
What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



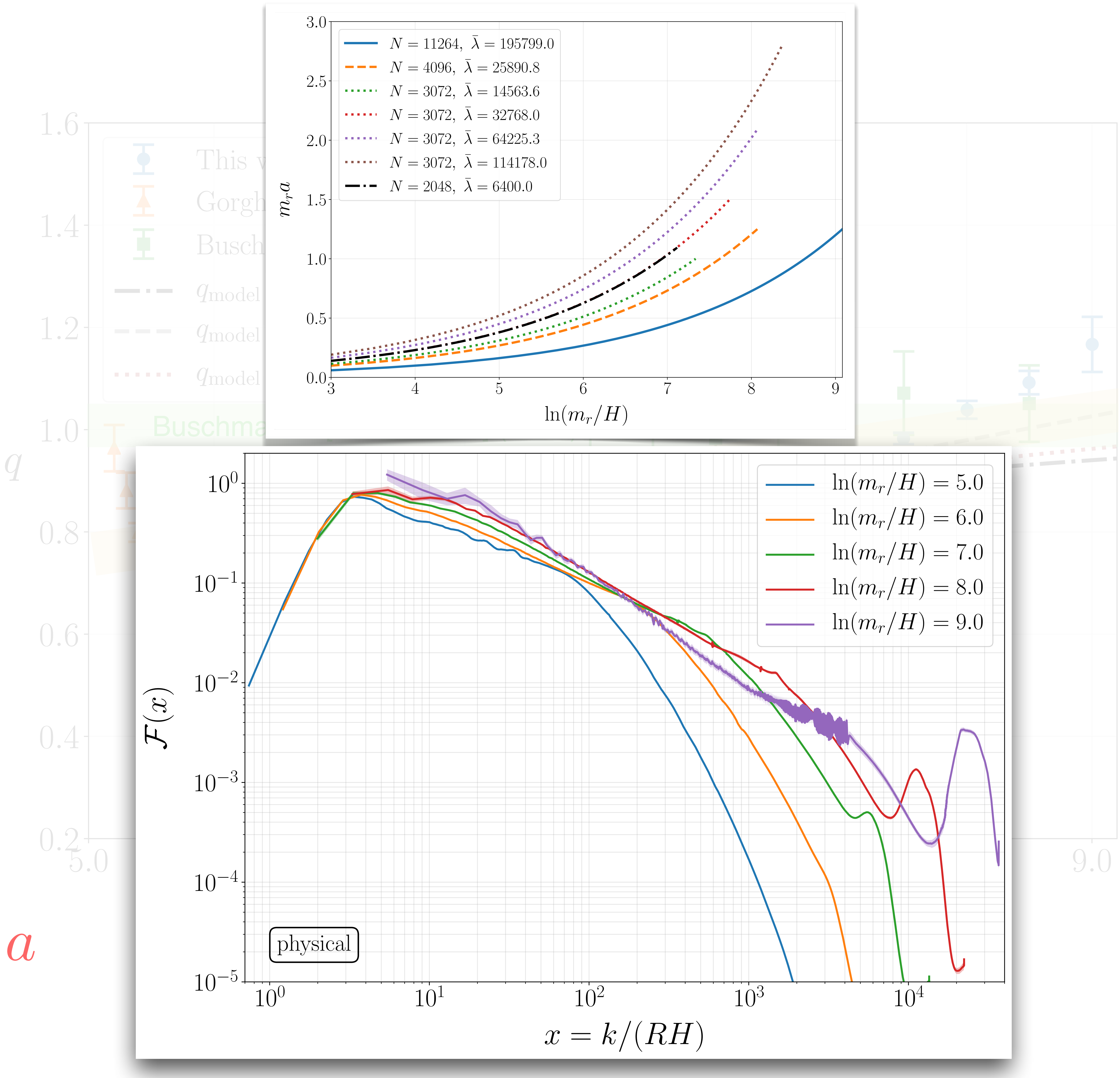
What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
    - **Laplacian**
    - Resolution of the string core $m_r a$

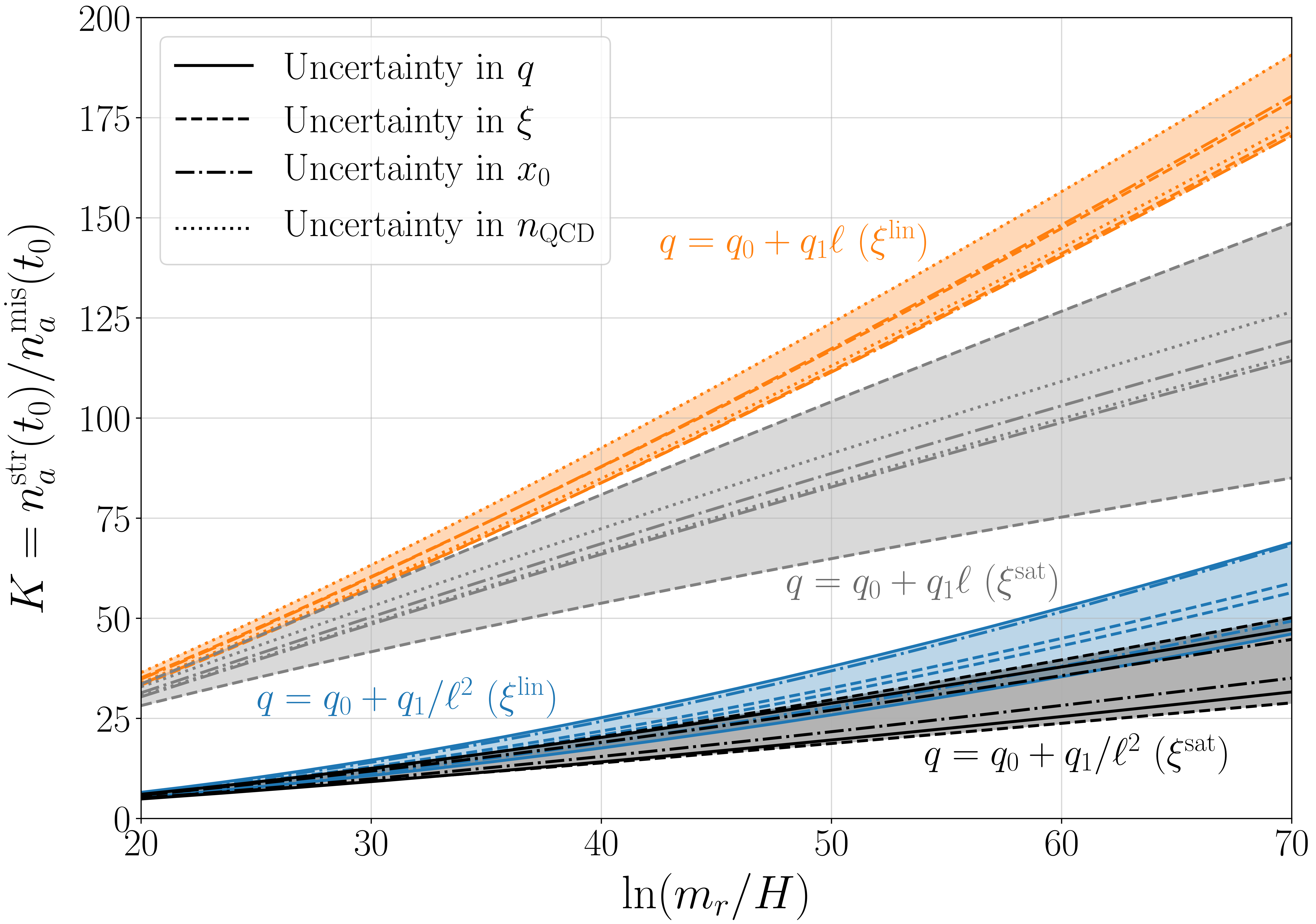


What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
    - Laplacian
    - **Resolution of the string core** $m_r a$



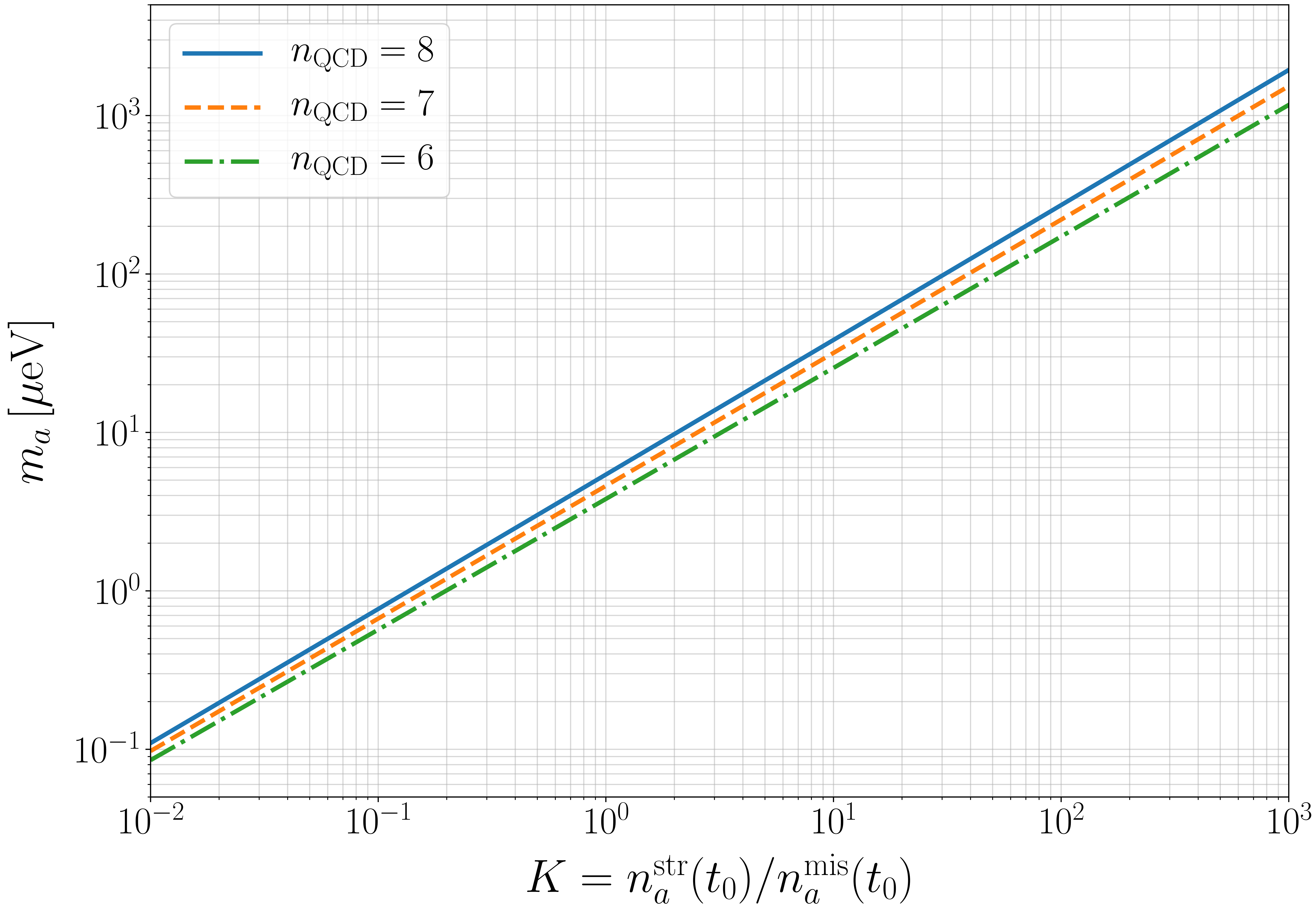
Axion Production: Strings vs. Misalignment



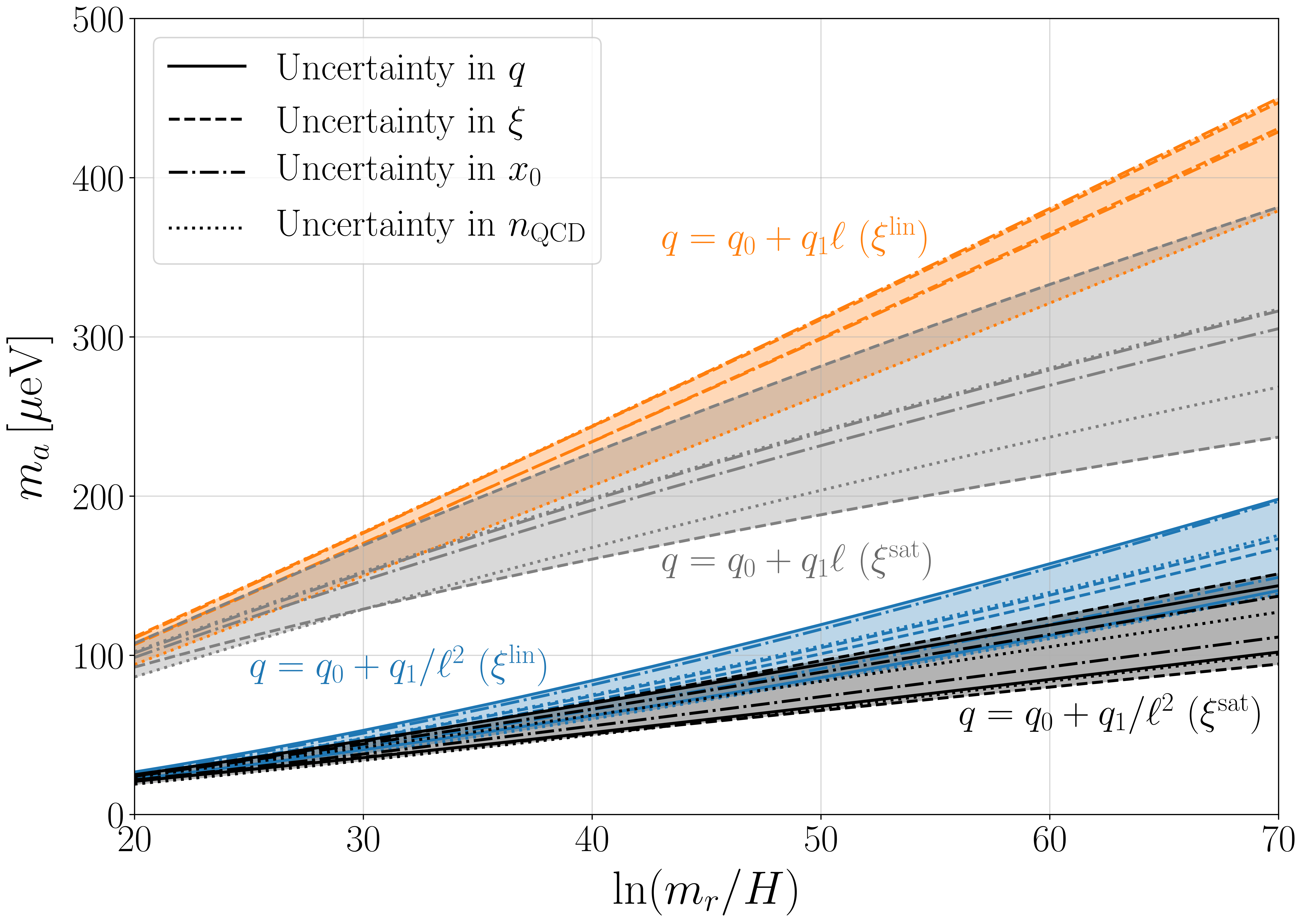
Known for "standard" angle-averaged misalignment:

$$\Omega_a h^2 = K \Omega_a^{\mathrm{mis}} h^2$$

Production Efficiency vs. Axion Mass



Axion Dark Matter Mass Prediction



Challenging ….

MADMAX, ALPHA, ORGAN

Summary

- Understanding of the global string dynamics is very important for a precise prediction of the axion dark matter mass in the post-inflationary scenario.
- Our simulations predict $95\ \mu\text{eV} \lesssim m_a \lesssim 450\ \mu\text{eV}$.
- Fast developments in recent simulations allow us to have a better understanding, albeit serious discrepancies.
- There are several systematic effects that could bias the result, that could explain these discrepancies:
    - Initial conditions
    - Axion field oscillations
    - Discretisation effects
- Further improvement in the dynamical range would be helpful to make the extrapolation trustworthy.

If time allows …

Towards the Continuum Limit of Global Loop Decays

ongoing project(s) with J. Redondo, I. Y. Rybak and A. Drew

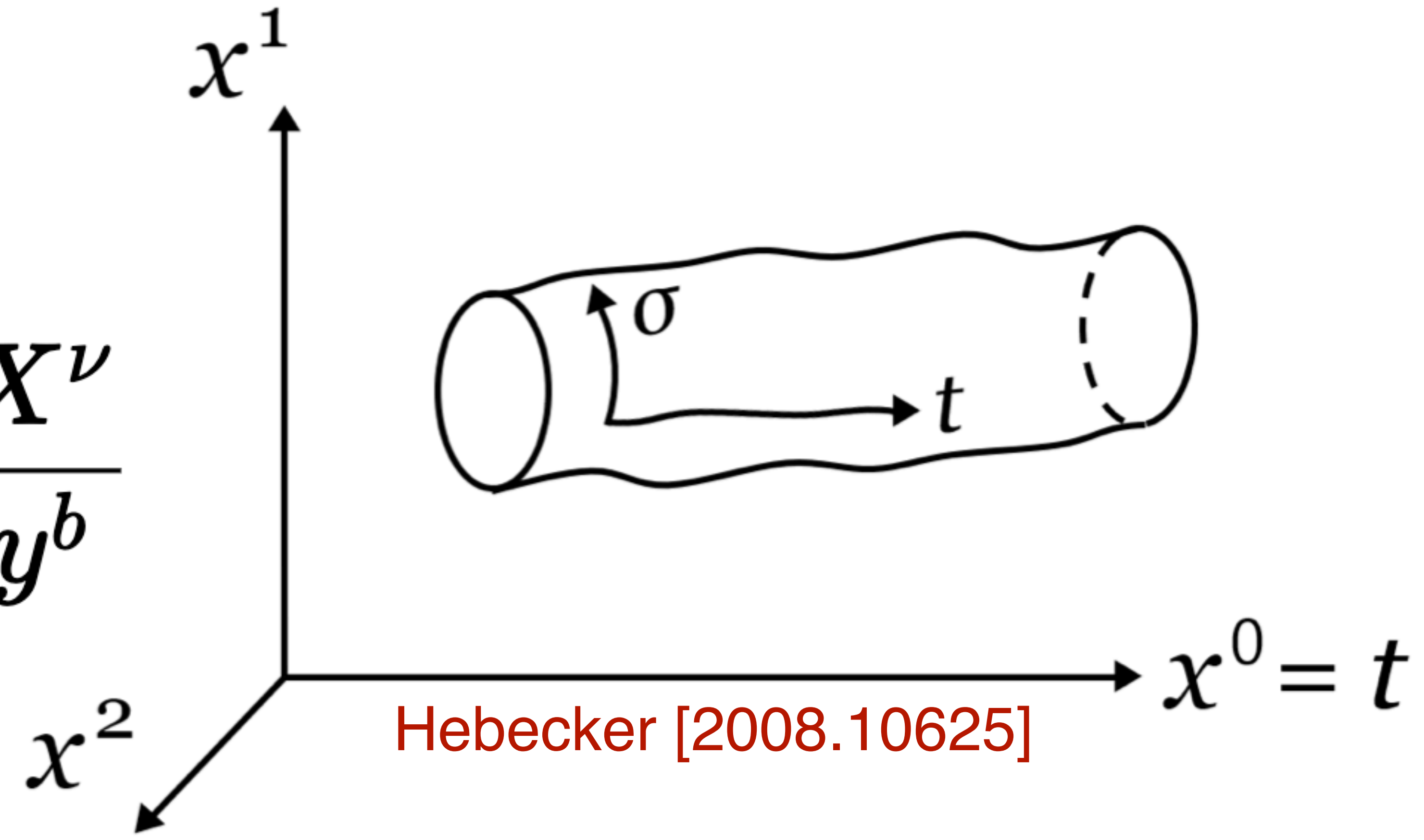
Departamento de Física Teórica
Universidad Zaragoza
Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza
GOBIERNO DE ARAGON
COSMIC WISPERS
cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

String Dynamics

- (Local) String dynamics governed by the **Nambu-Goto** action:

$$S_{\mathrm{NG}} = \mu_0 \int \mathrm{d}^2\sigma \sqrt{-h} \quad \text{with } h = \det\left(h_{ab}\right),\ h_{ab} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}$$

Nambu (1970), Goto (1971)



- For **Global Strings,** couple to antisymmetric **Kalb-Ramond** field:

$$S_{\mathrm{KR}} = S_{\mathrm{NG}} - \frac{1}{6} \int \mathrm{d}^4x F^{\mu\alpha\beta} F_{\mu\alpha\beta} + 2\pi f_a \int \mathrm{d}^2\sigma B_{\alpha\beta} \partial_a X^\alpha \partial_b X^\beta \epsilon^{ab}$$

Kalb & Ramond (1974), Dabholkhar & Quashnock (1990)

- Can be used to compute the axion field around strings (in close analogy to EM):

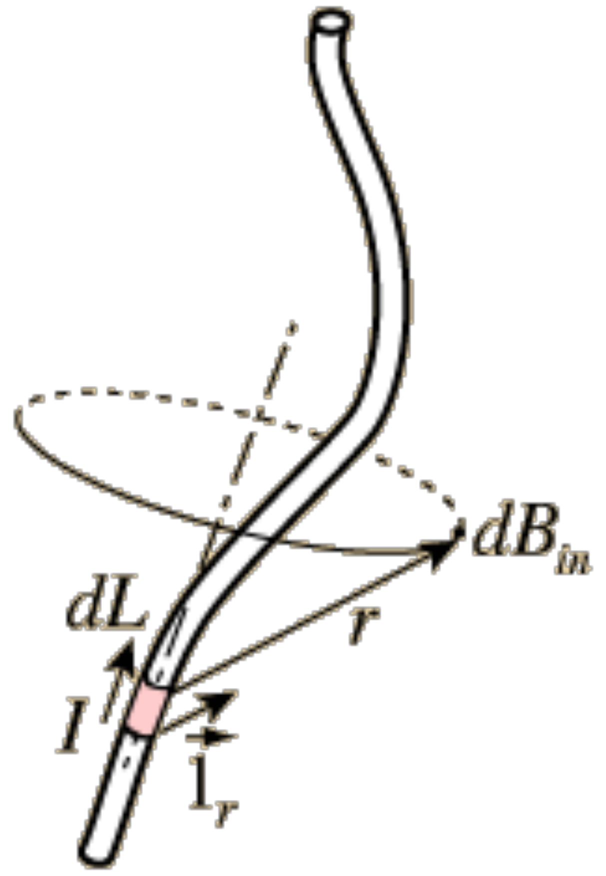
$$f_A \partial_\mu \theta = \frac{1}{6} \epsilon_{\mu\nu\alpha\beta} F^{\nu\alpha\beta}$$

e.g. Fleury & Moore [1509.00026]

Constructing the Axion Field around Strings

- Contribution of a short, straight section of the string to the **axionic *B*-field**:

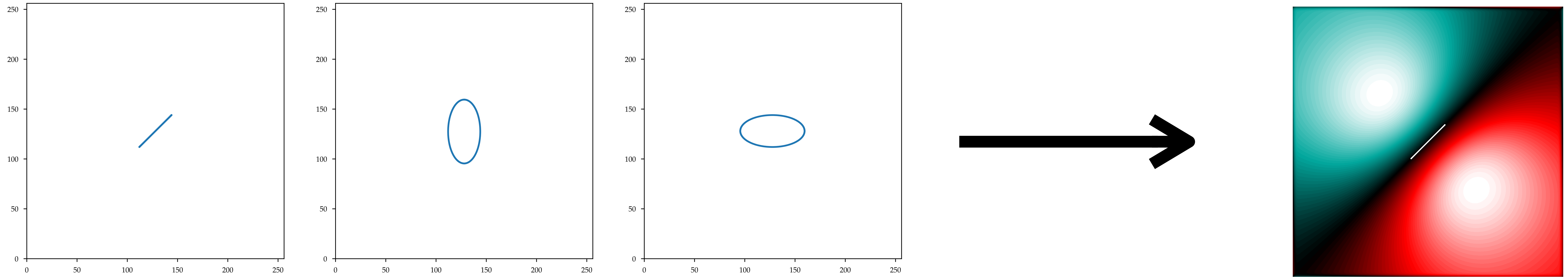
$$\nabla\theta = K \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



Biot-Savard law (Link)

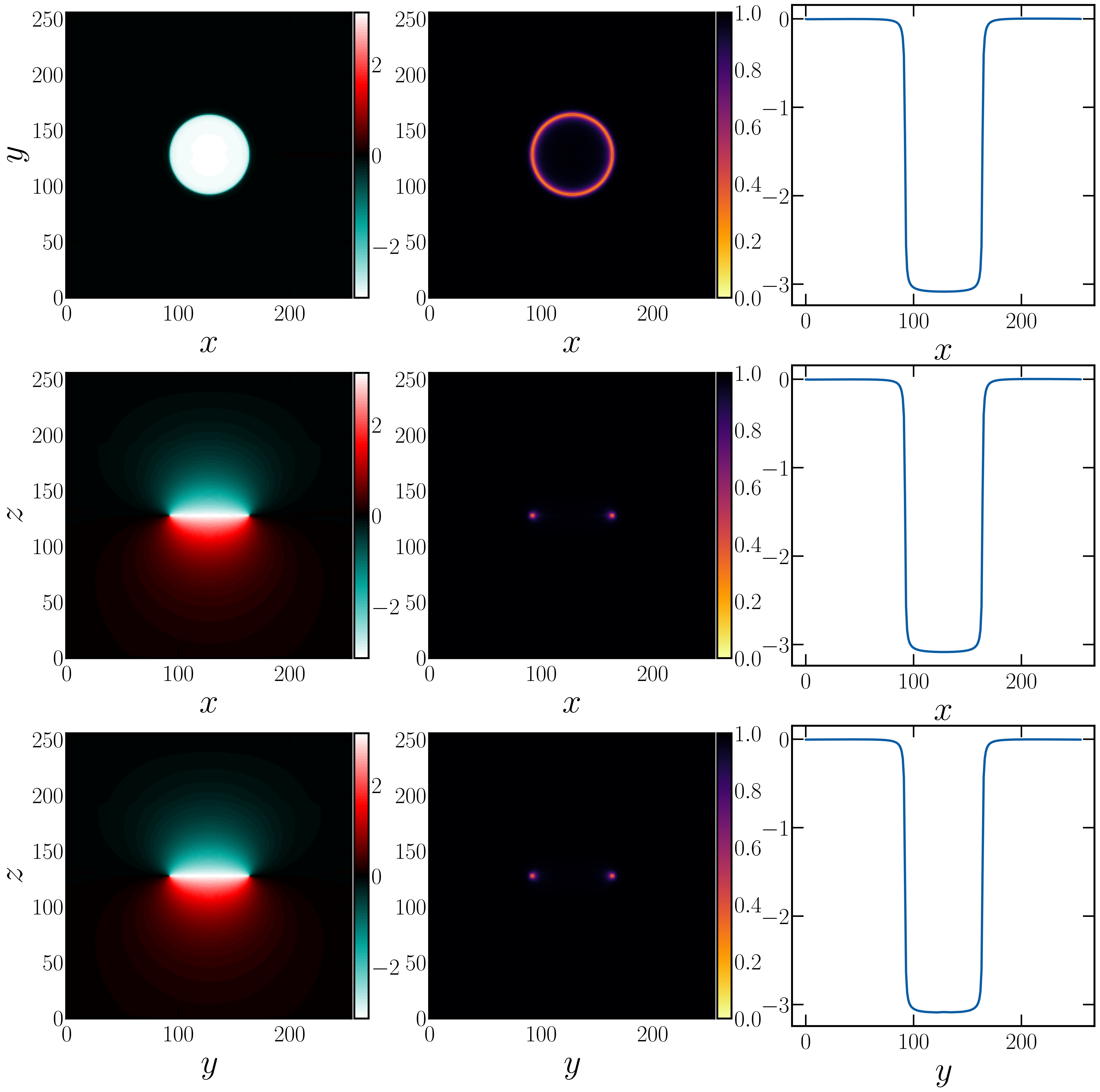
- Calculate links to construct the axion field in the full plane:

$$\theta_{\mathbf{x}+\mathbf{d}x} - \theta_{\mathbf{x}} = \int_x^{x+dx} d^3\mathbf{x} \cdot \nabla\theta = -\frac{1}{2}\int_x^{x+dx} d^3\mathbf{x} \cdot \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



Constructing the Axion Field around Strings

- Contribution of a s … B-field:

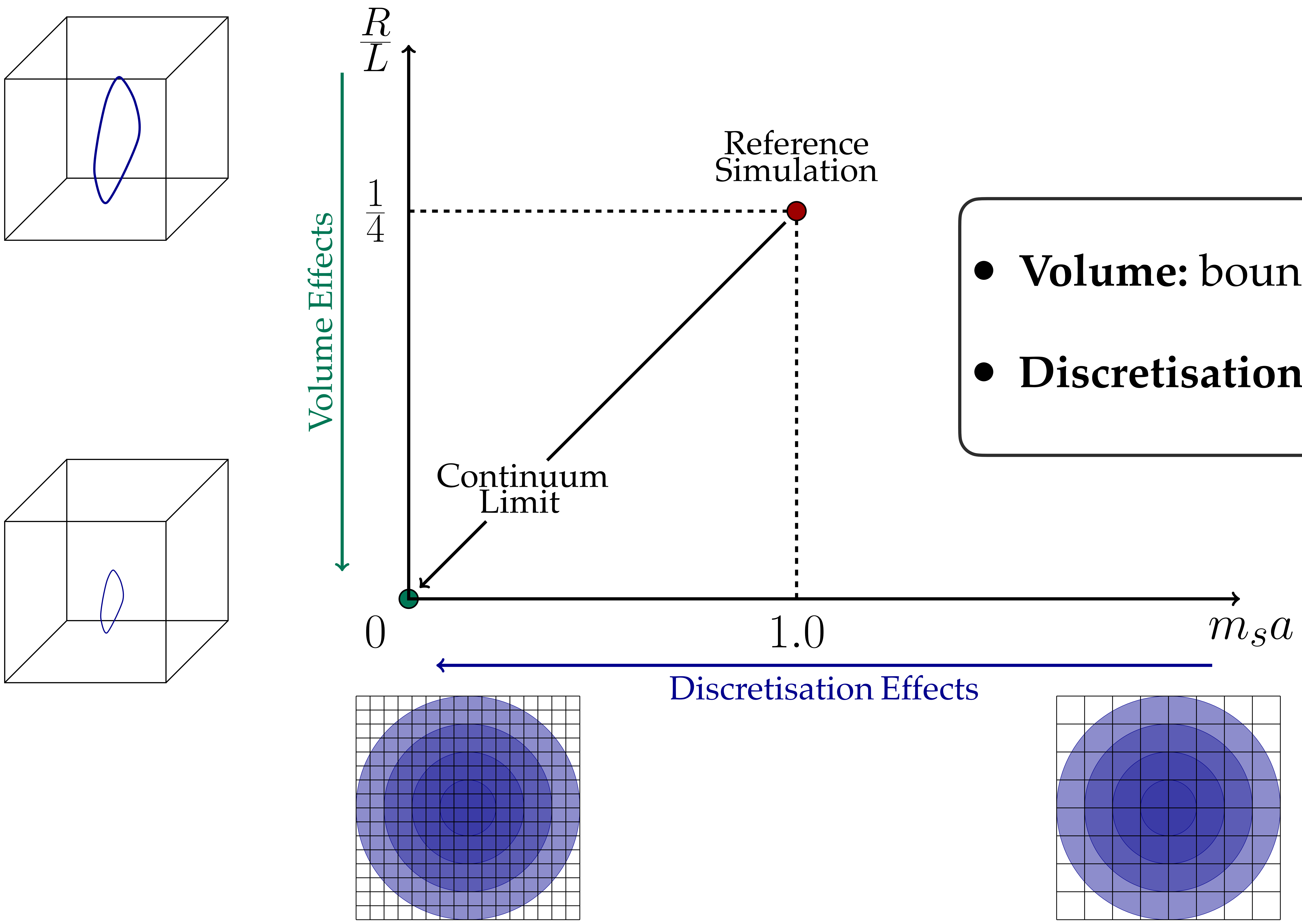


Biot-Savard law (Link)

- Calculate links to c …

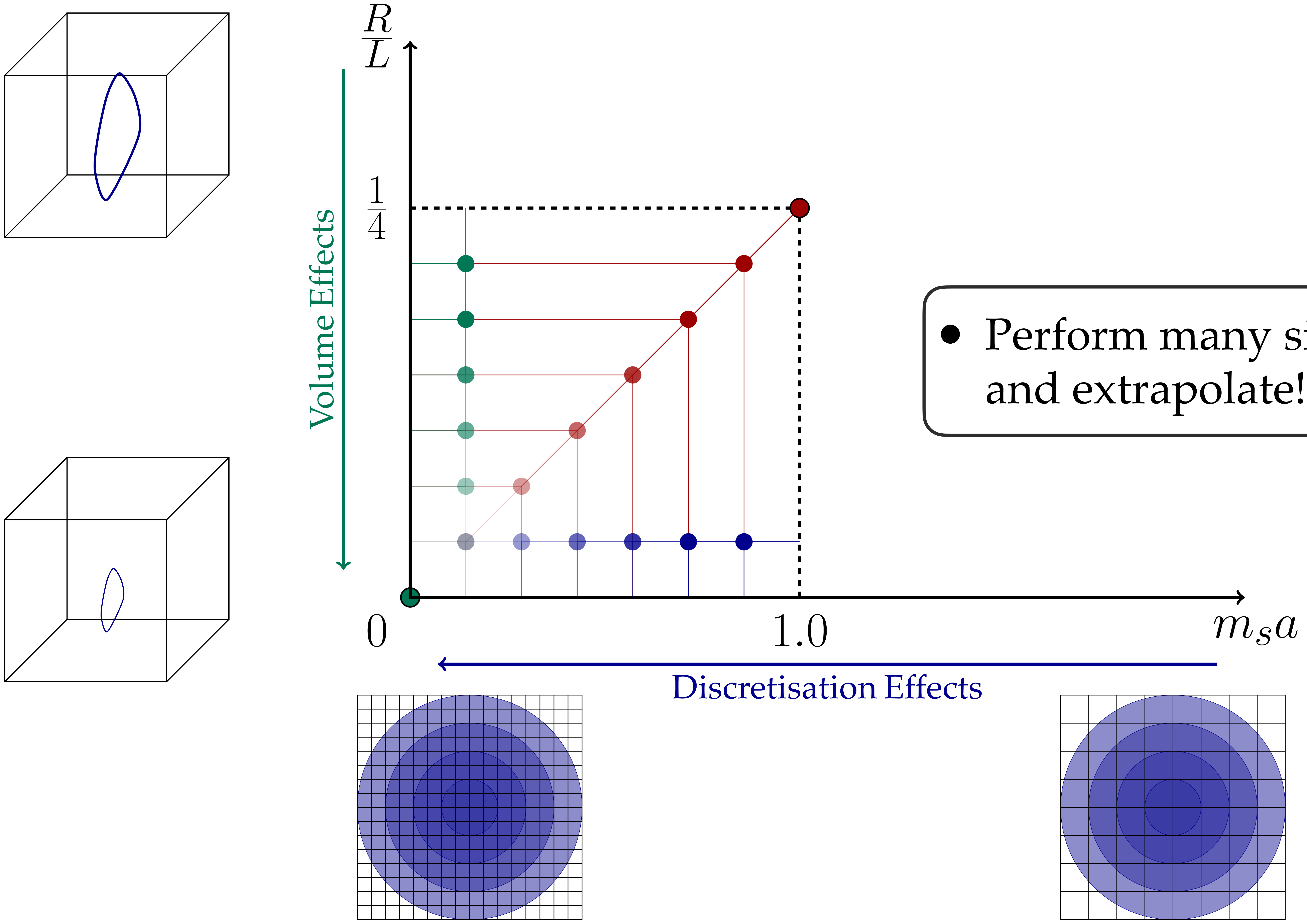
$\theta_{\mathbf{x}+\mathbf{d}x} - \theta_{\mathbf{x}} = \; \cdots \; - \mathbf{X}(\sigma)) \times \mathbf{X}' \; / \; \cdots \mathbf{x} - \mathbf{X}(\sigma)|^3$

Towards the Continuum Limit



- **Volume:** boundary effects, periodic copies?
- **Discretisation:** resolution of the string cores

Towards the Continuum Limit

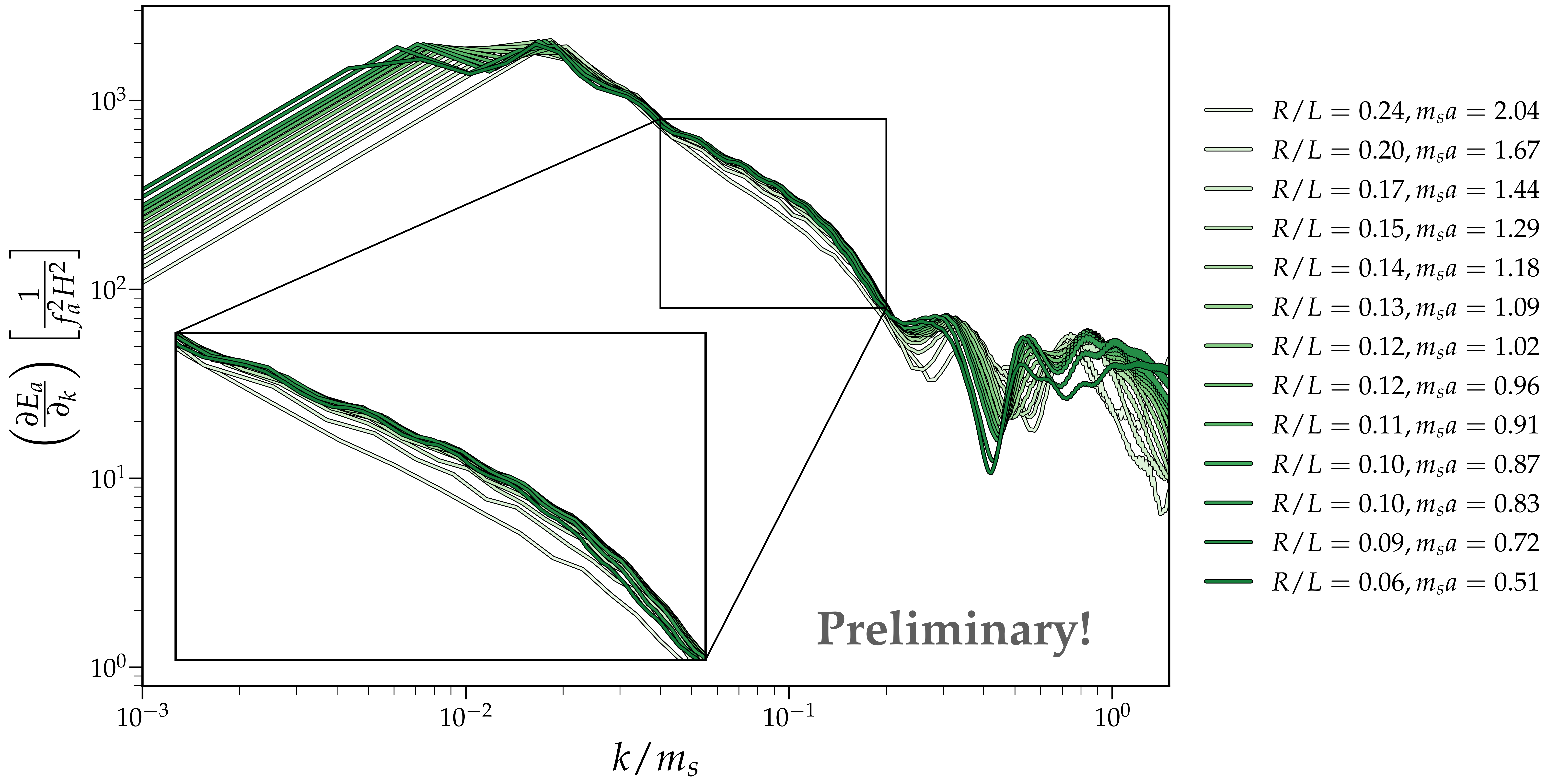


- Perform many simulations, test convergence and extrapolate!

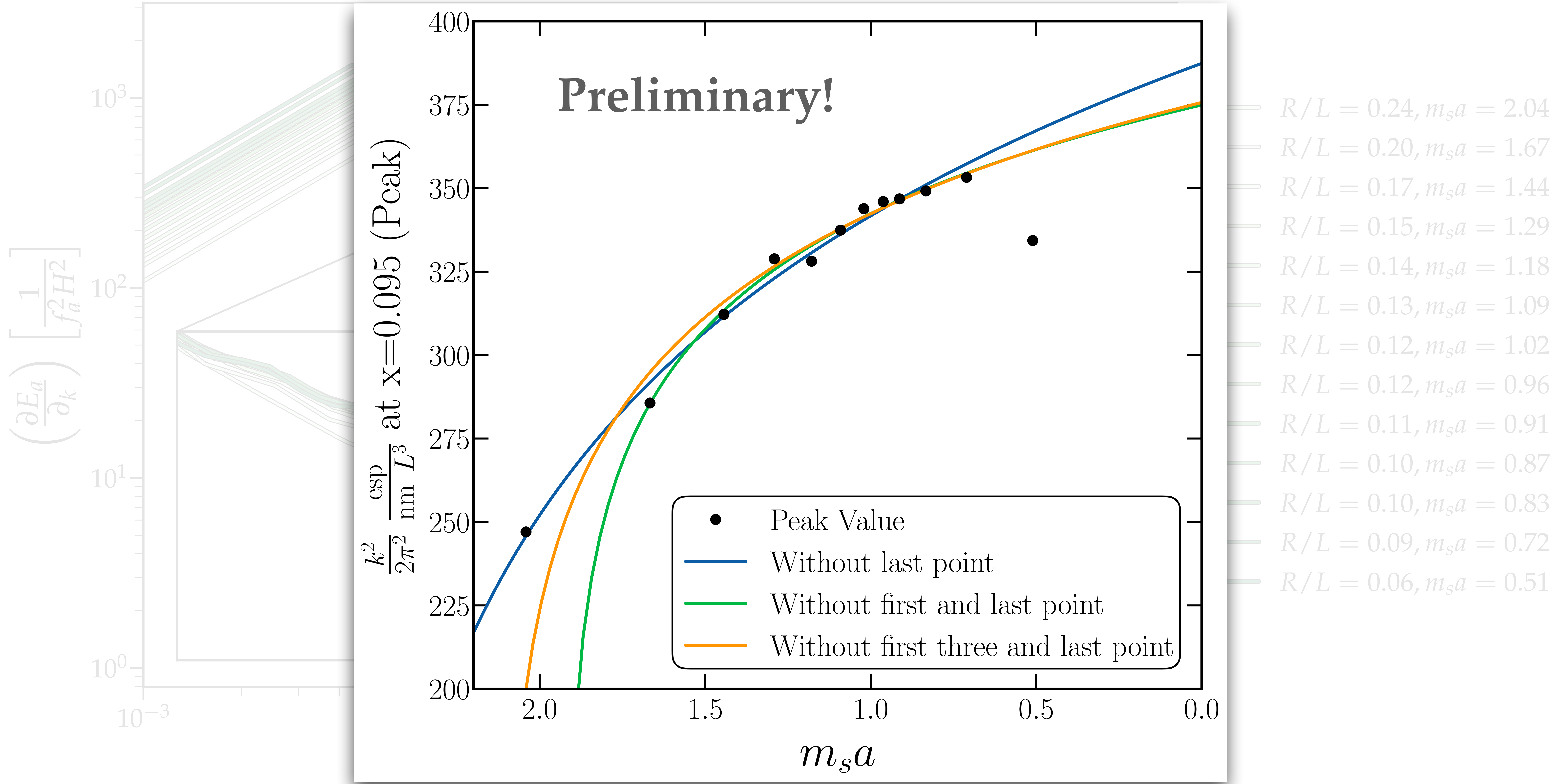
Open Problems

- $\mathcal{F}$ is a useful quantity when there are many strings but is it also good for single string configurations?
- How can we treat the **self-field** and the **radiation** separately? Does this bias the spectrum?
- Is there a **characteristic time scale/wavelength** that limits our ability to correctly track the string dynamics?

Extrapolation of the Radiation Spectrum

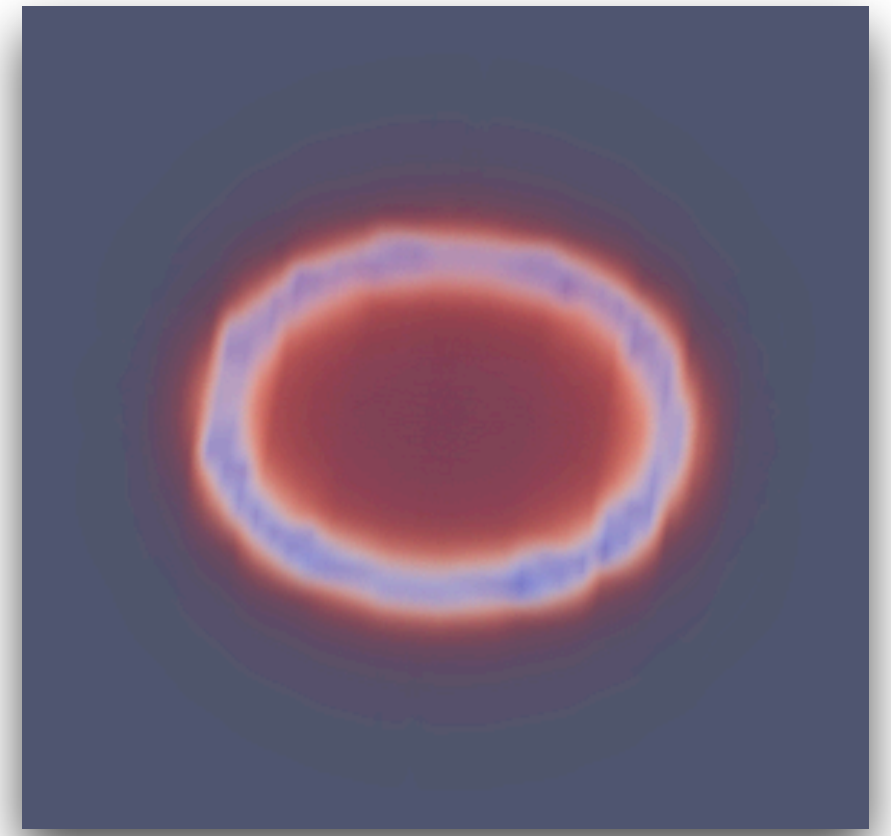


Extrapolation of the Radiation Spectrum

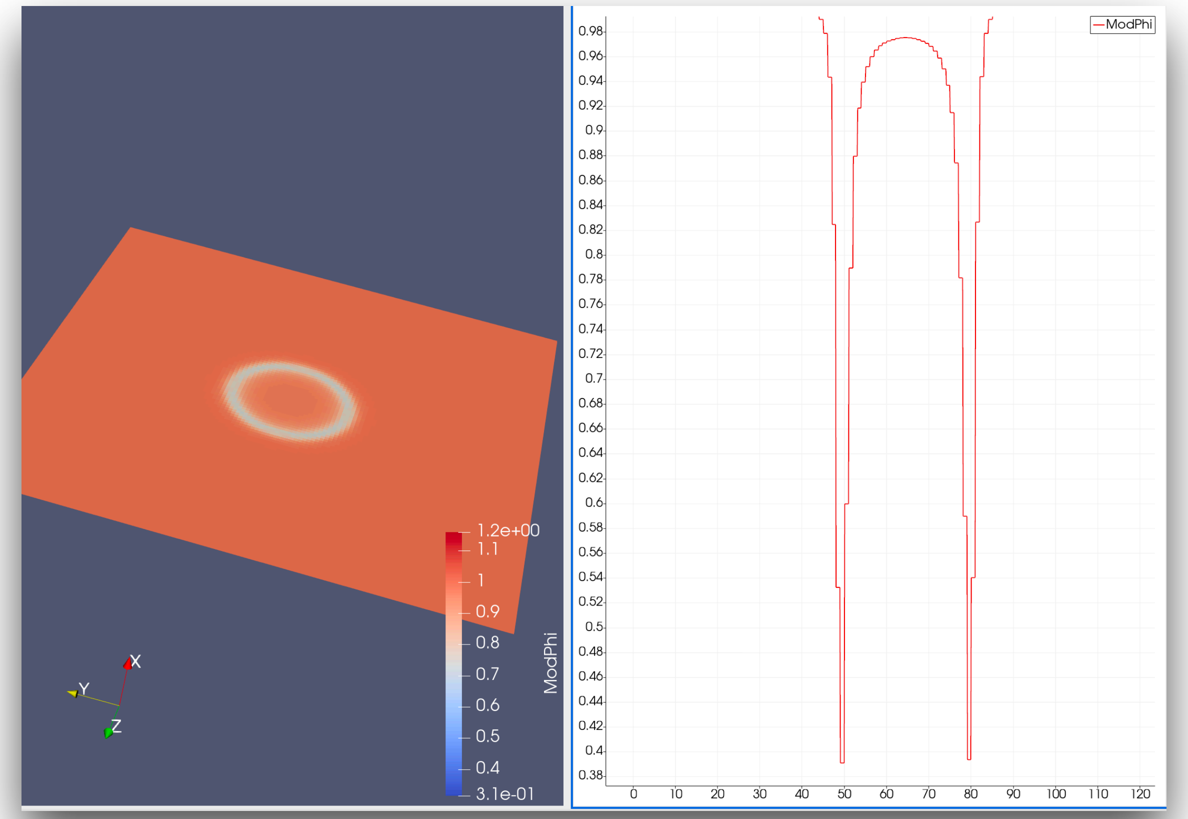


New Insights into the Network Spectrum?

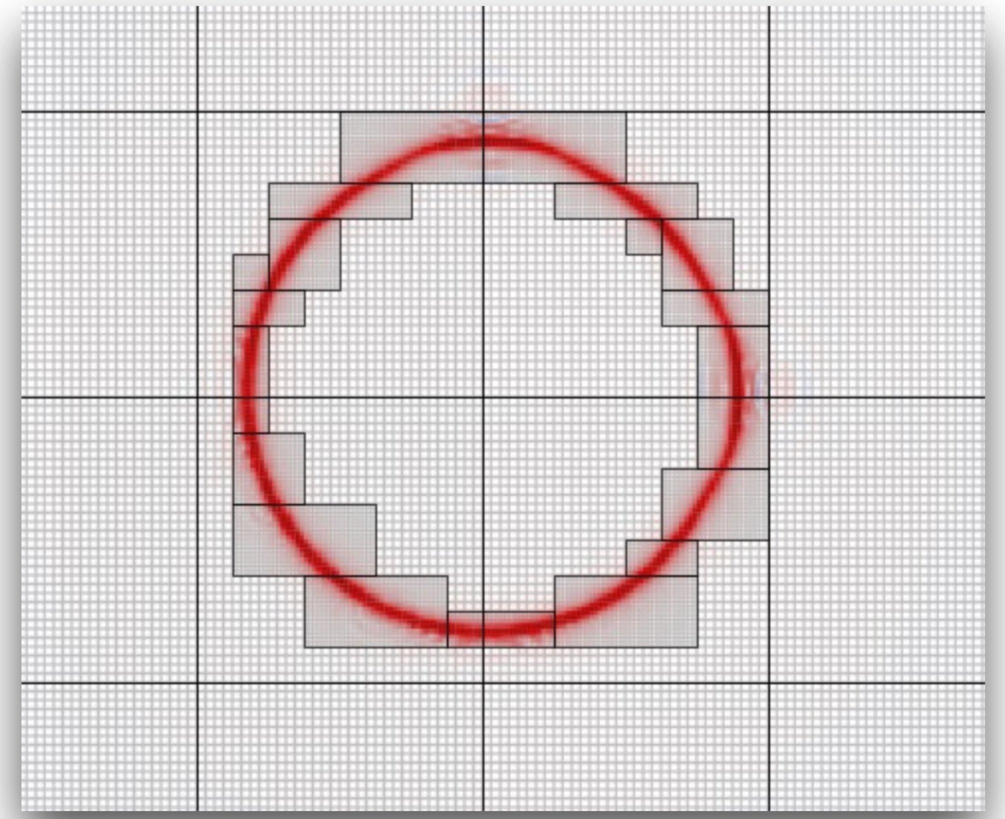
- We are running lots of simulations with different individual string configurations and try to obtain their radiation spectrum.
- We use both static-grid and **Adaptive Mesh Refinement** simulations and try to directly sync and compare results from different codes to better understand the discrepancies. (STSM with Amelia Drew in Cambridge)
- The goal is to understand the contributions from the constituents of the string network in more detail.



Jaxions ICs in Chombo



Visualisation with ParaView



axioNyx AMR Code

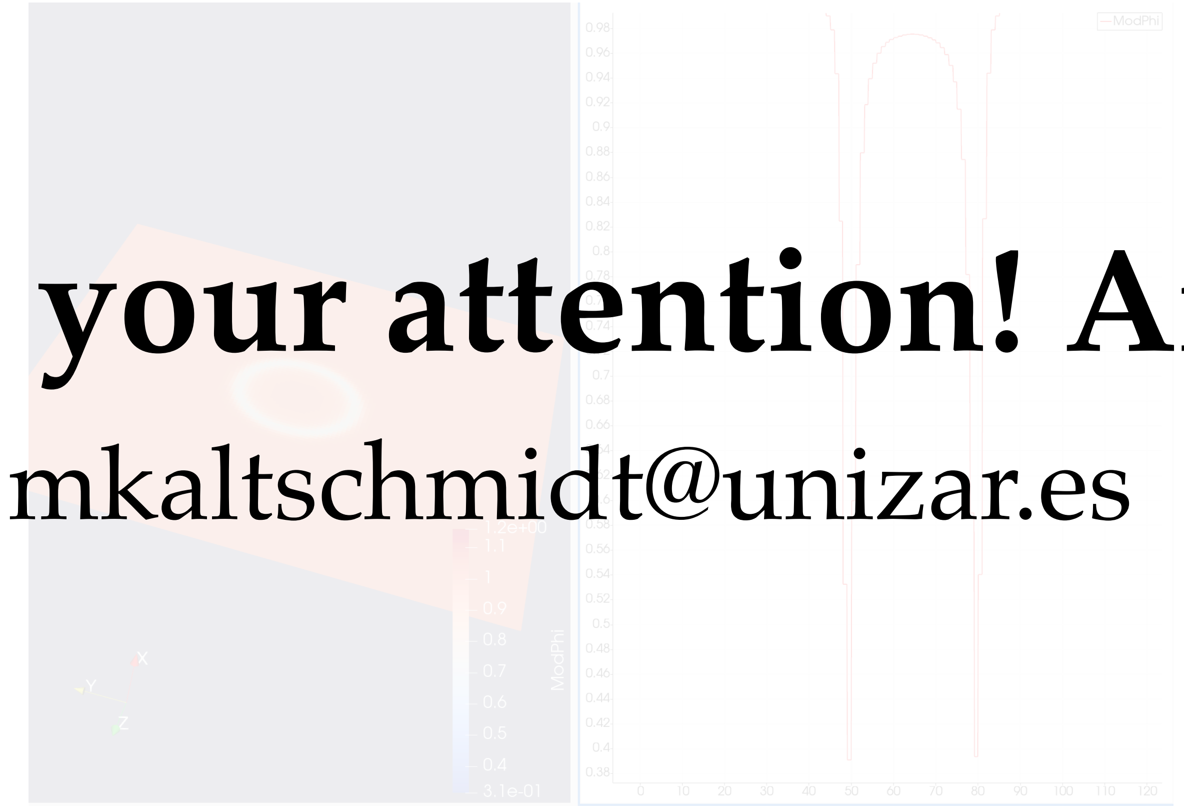
New Insights into the Network Spectrum?

- We are running lots of simulations with different individual string configurations and try to obtain their radiation spectrum.
- We use both static-grid and **Adaptive Mesh Refinement** simulations and try to directly sync and compare results from different codes to better understand the discrepancies. (STSM with Amelia Drew in Cambridge)
- The goal is to understand the contributions from the constituents of the string network in more detail.

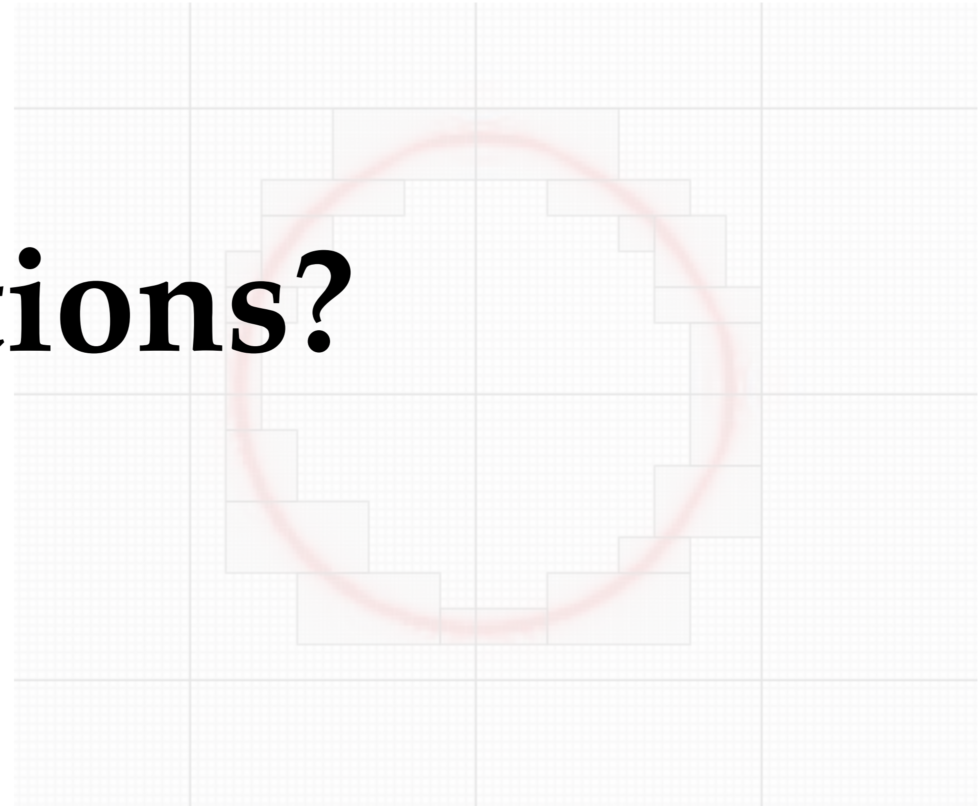
Thank you for your attention! Any questions?

mkaltschmidt@unizar.es

Jaxions ICs in Chombo



Visualisation with ParaView



axioNyx AMR Code

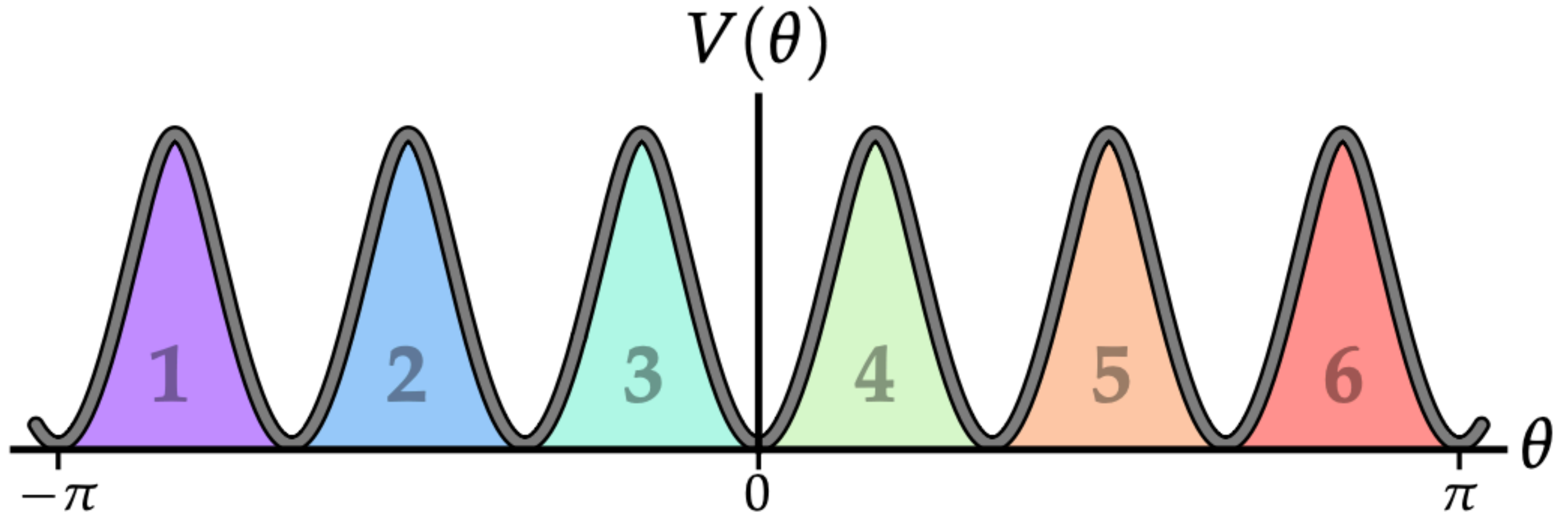
Backup Slides

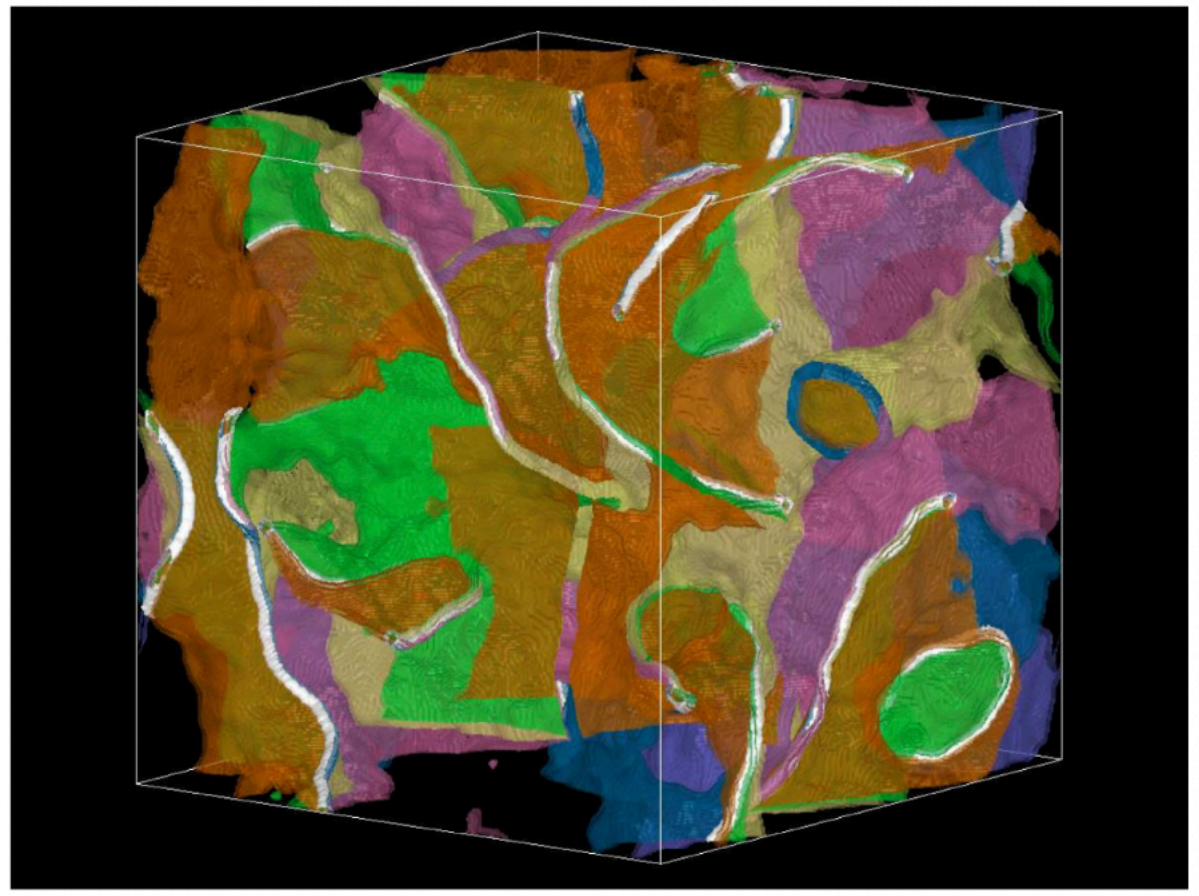
General

$N_{\rm DW} > 1$: Axion Domain Wall Problem

- Axion cycles around $N_{\rm DW}$ times between $(-\pi, \pi)$
- In general we get more axions from wall decay, so preferred m_a is higher.
- Phenomenologically difficult. Domain wall network gets stuck and overwhelms the cosmic energy density.
- Must have some preferred minimum!

$$V(\theta) \approx -\chi(T)\cos(N_{\rm DW}\theta)$$





(e) $N_{\rm DW} = 6$

Hiramatsu+ [1207.3166]

More details on the recent Paper

Departamento de Física Teórica
Universidad Zaragoza
CAPA Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza
GOBIERNO DE ARAGON
COSMIC WISPERS
cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Simulation Overview

- More than 1500 simulations performed at
  - RAVEN and COBRA supercomputers at Max Planck Computing and Data Facility (MPCDF)
  - SQUID supercomputer at Cybermedia Center, Osaka University
- Box sizes of up to 11.264^3 (256 CPU nodes)

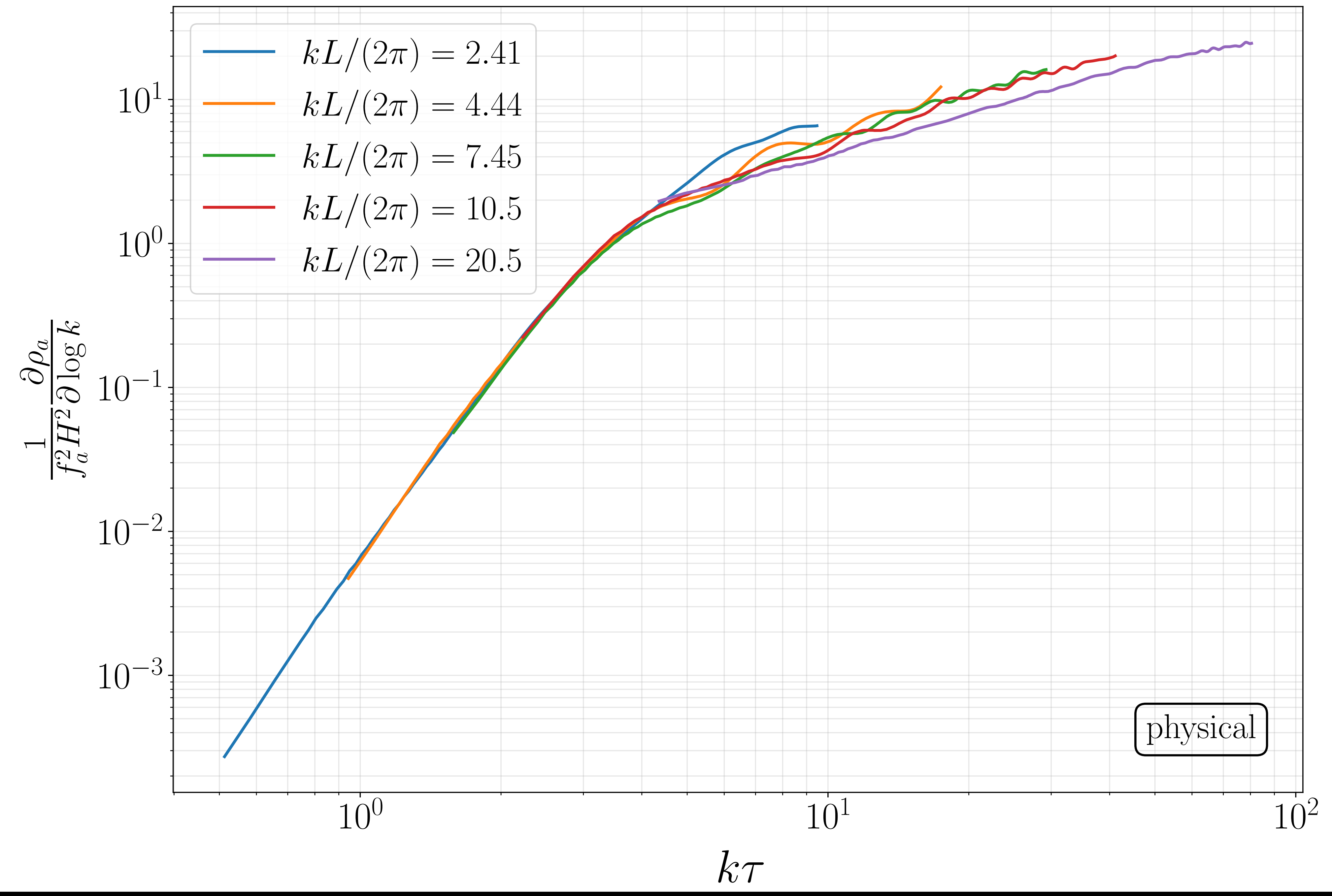
Type[a]	Grid size (N^3)	Laplacian	Final time (τ_f/L)	$\ln(m_r/H)$ at τ_f	Parameter	Number of simulations
Physical	11264^3	4-neighbours	0.625	9.08	$\bar{\lambda} = 195799$	20
Physical	4096^3	1-neighbour	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	2-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	3-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	4-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	3072^3	4-neighbours	0.5	7.34	$\bar{\lambda} = 14563.6$	30
Physical	3072^3	4-neighbours	0.5	7.74	$\bar{\lambda} = 32768$	30
Physical	3072^3	4-neighbours	0.5	8.08	$\bar{\lambda} = 64225.3$	30
Physical	3072^3	4-neighbours	0.5	8.37	$\bar{\lambda} = 114178$	30
Physical	2048^3	4-neighbours	0.55	7.12	$\bar{\lambda} = 6400$	30×30[b]
Physical	1024^3	4-neighbours	0.5	6.23	$\bar{\lambda} = 1600$	30×4[c]
Physical	3072^3	4-neighbours	0.458367	7.5	$\bar{\lambda} = 28571.2$	30
Physical	2560^3	4-neighbours	0.550042	7.5	$\bar{\lambda} = 13778.5$	30
Physical	2048^3	4-neighbours	0.687552	7.5	$\bar{\lambda} = 5643.68$	30
Physical	1536^3	4-neighbours	0.916735	7.5	$\bar{\lambda} = 1785.69$	30
Physical	1024^3	4-neighbours	1.3751	7.5	$\bar{\lambda} = 352.73$	30
PRS	8192^3	4-neighbours	0.55	6.80	$m_r a = 0.2$	20
PRS	8192^3	4-neighbours	0.55	7.21	$m_r a = 0.3$	20
PRS	8192^3	4-neighbours	0.55	7.72	$m_r a = 0.5$	20
PRS	8192^3	4-neighbours	0.55	8.06	$m_r a = 0.7$	20
PRS	8192^3	4-neighbours	0.55	8.41	$m_r a = 1.0$	20
PRS	8192^3	4-neighbours	0.55	8.82	$m_r a = 1.5$	20
PRS	4096^3	4-neighbours	0.55	7.72	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.55	7.03	$m_r a = 1.0$	30
PRS	1024^3	4-neighbours	0.55	6.33	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.5	6.93	$m_r a = 1.0$	1

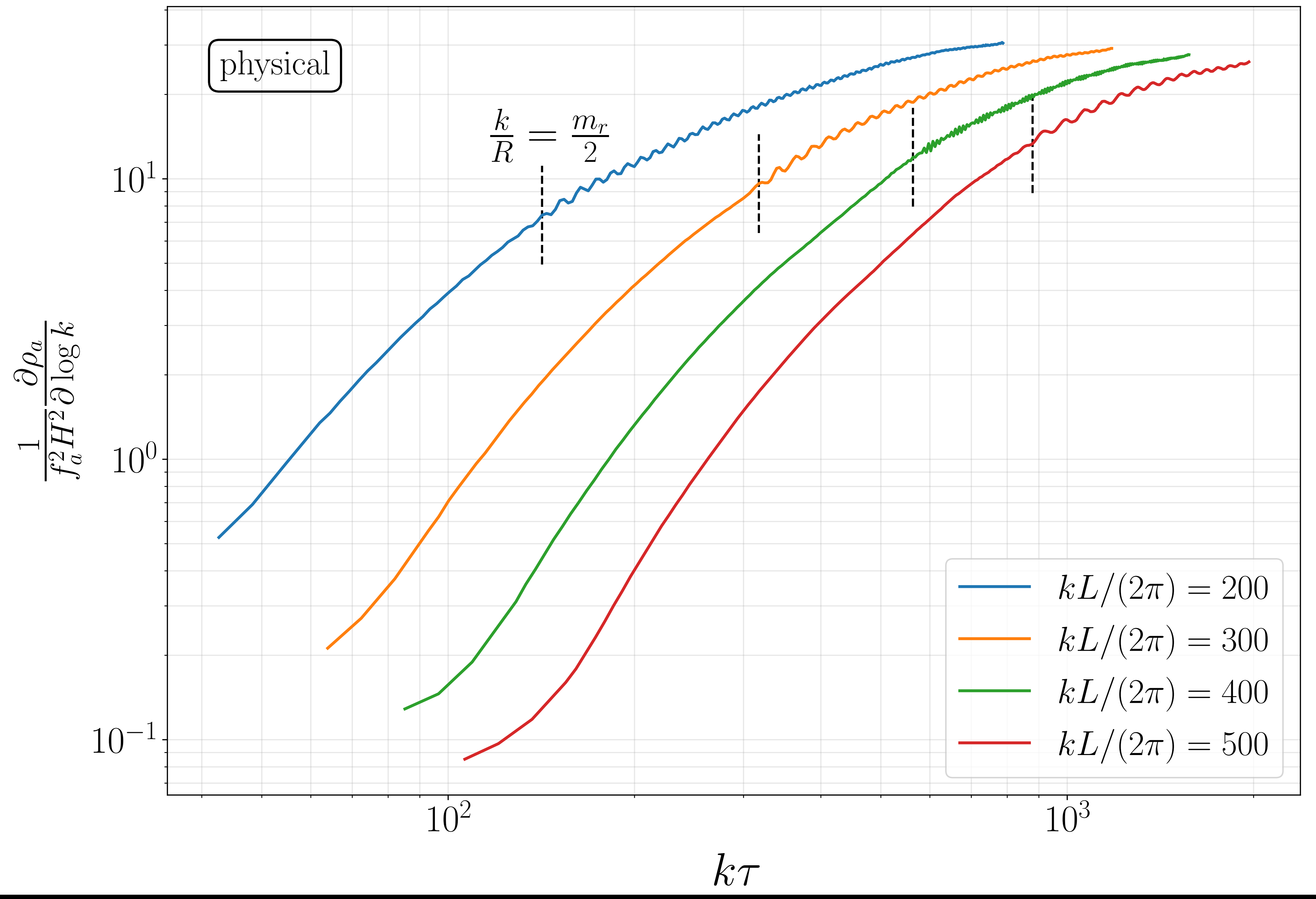
Axion Mode Evolution

- To calculate the differential spectrum, we need to know the time evolution of one mode:

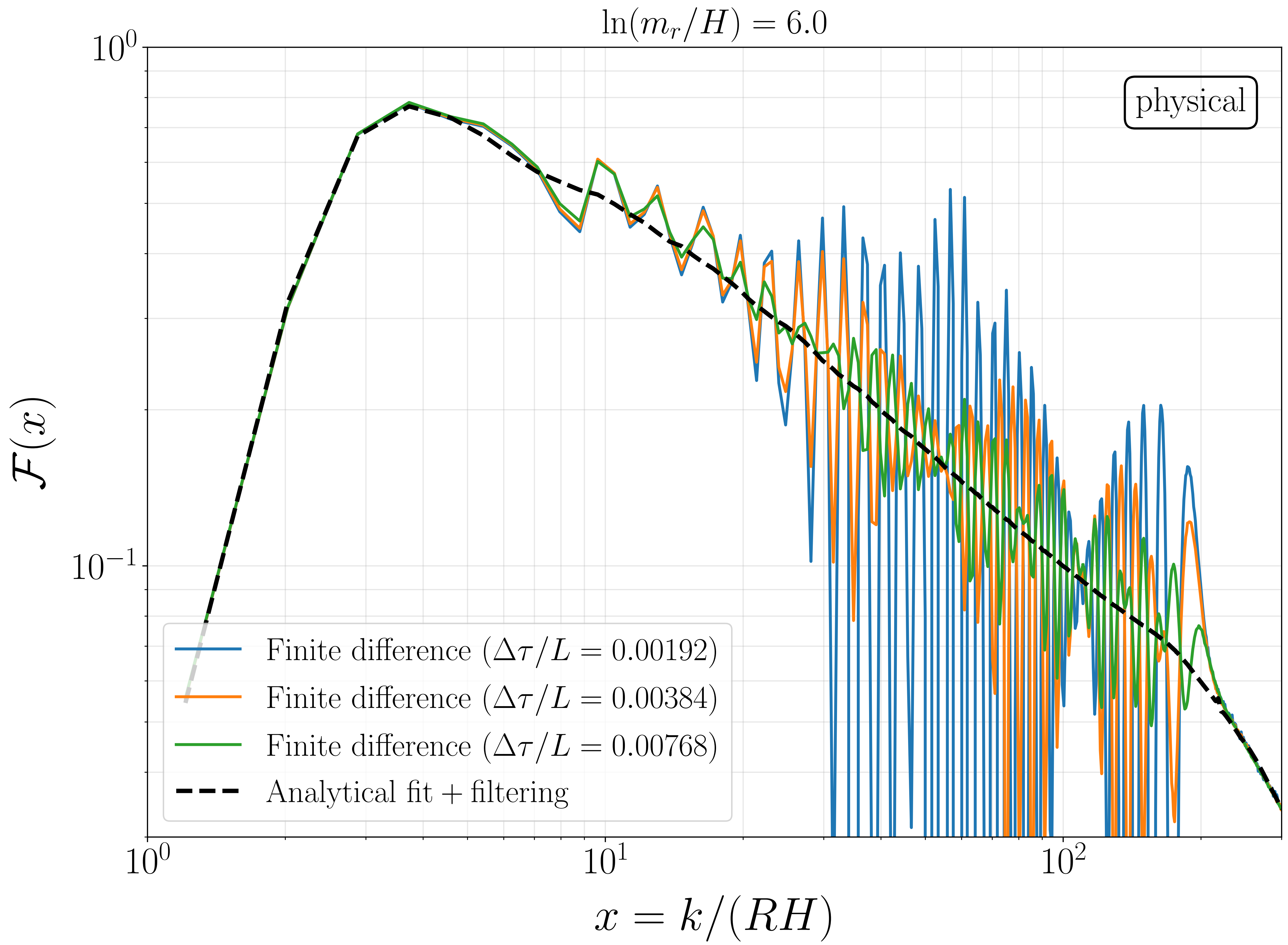
$$\mathcal{F} = \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t} \left(R^4 \frac{\partial \rho_a}{\partial k} \right)$$

- Contains oscillating components with frequency $\sim 2k$, interpreted as axion field oscillations after the horizon entry or production from the radial field.





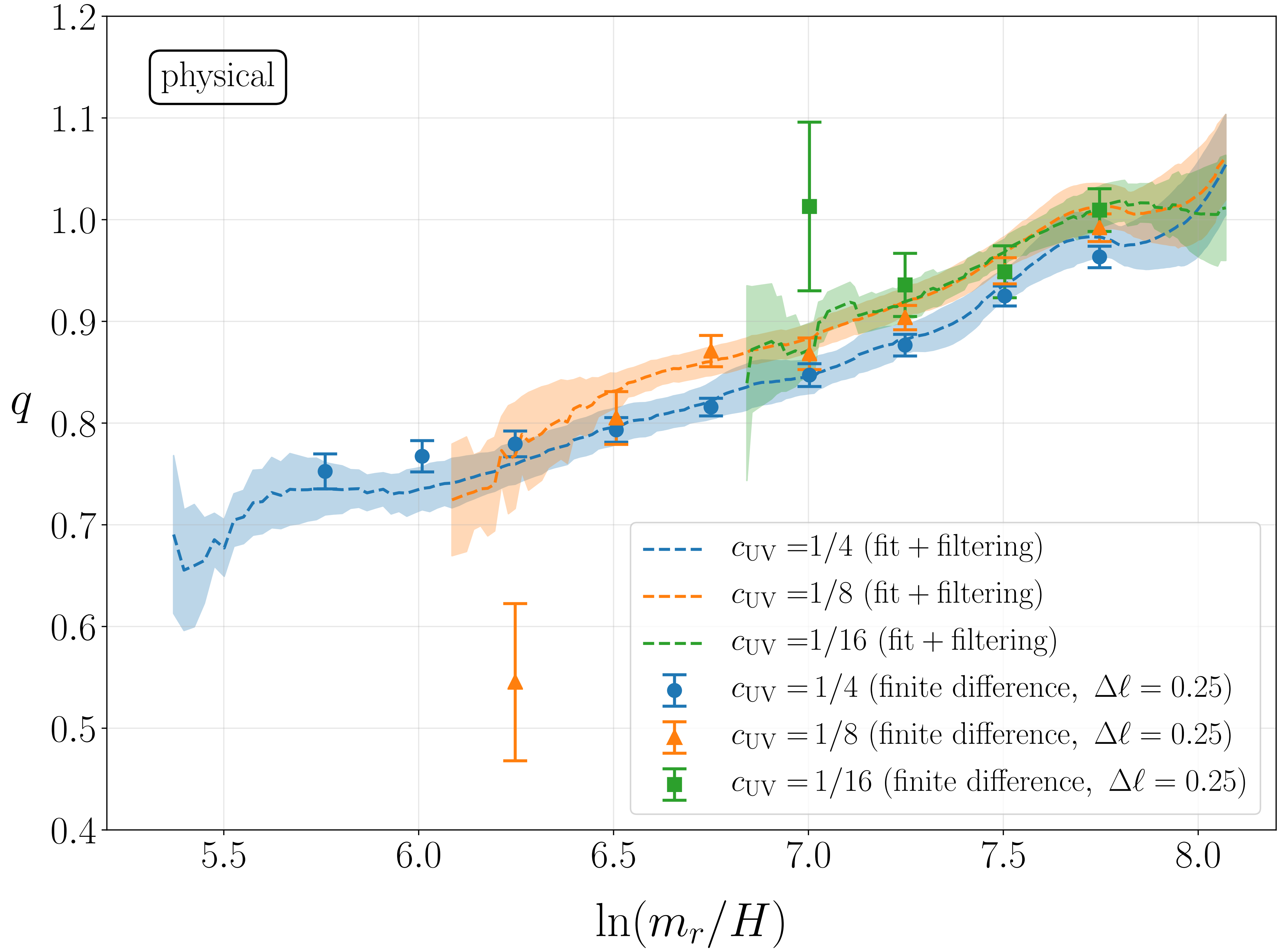
Calculation of the Instantaneous Spectrum



- Simple finite difference leads to a lot of contaminations from axion field oscillations.
- One can reduce them by applying a filter to remove high frequency components in the mode evolution data.

Axion Field Oscillations

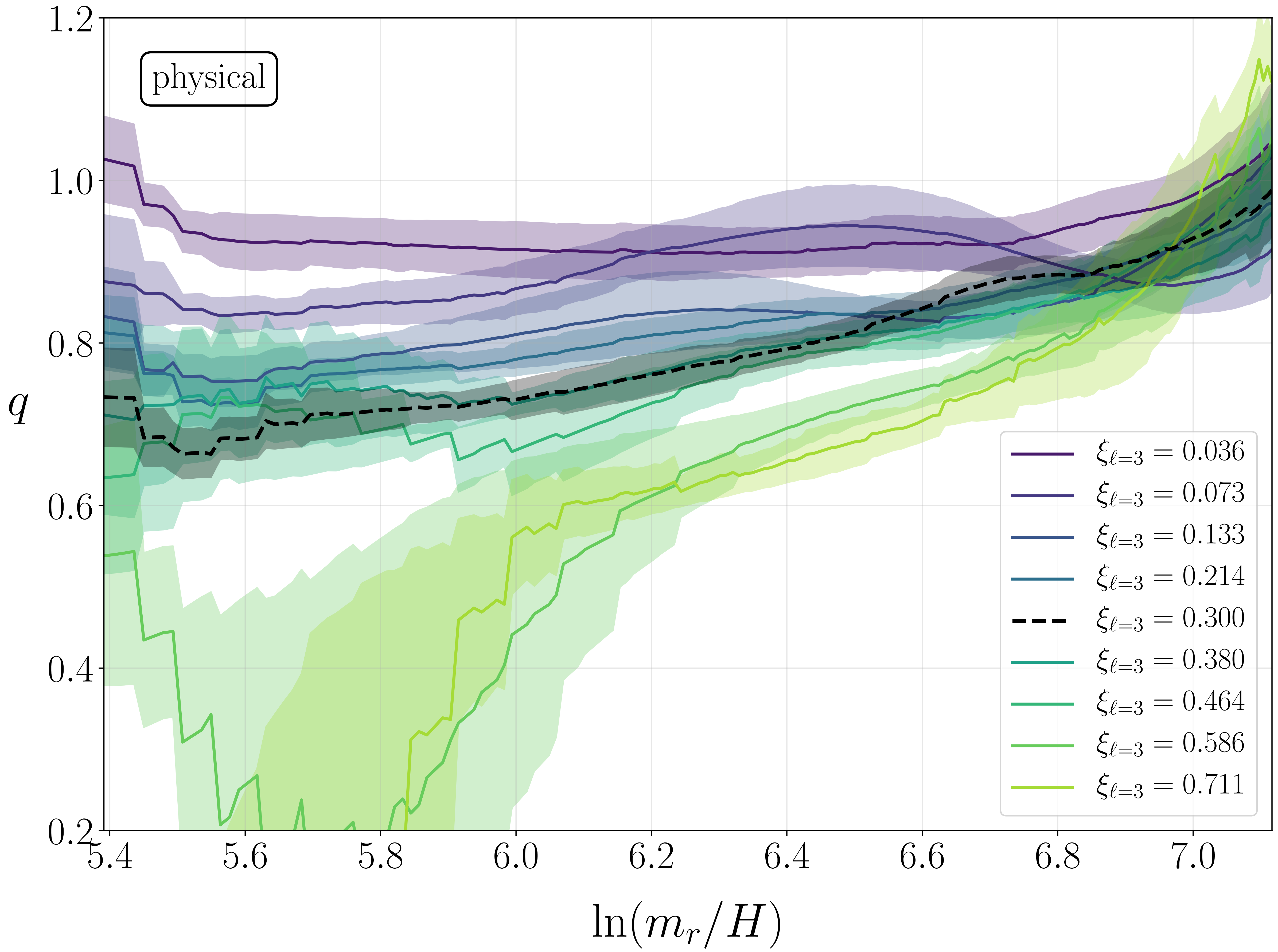
- Fit power law $\mathcal{F} \sim k^{-q}$ to the data in the range $c_{\rm IR} H < k/R < c_{\rm UV} m_r$



- The oscillations in the IR modes have an impact on the measurement of q.

- The effect can be alleviated by taking a broader range for the fit.

Initial Conditions

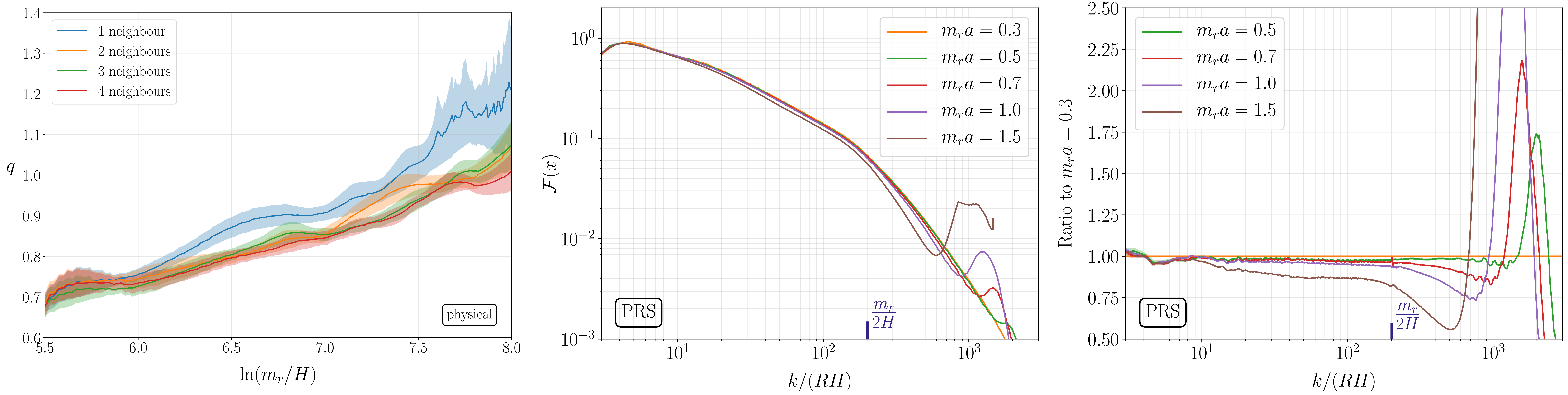


- Differences in the initial string density affect the slope of the radiation spectrum.
- Overdense (underdense) initial conditions could bias the estimation of q towards lower (higher) values.

Discretisation Effects

Effects that can bias q towards larger values:

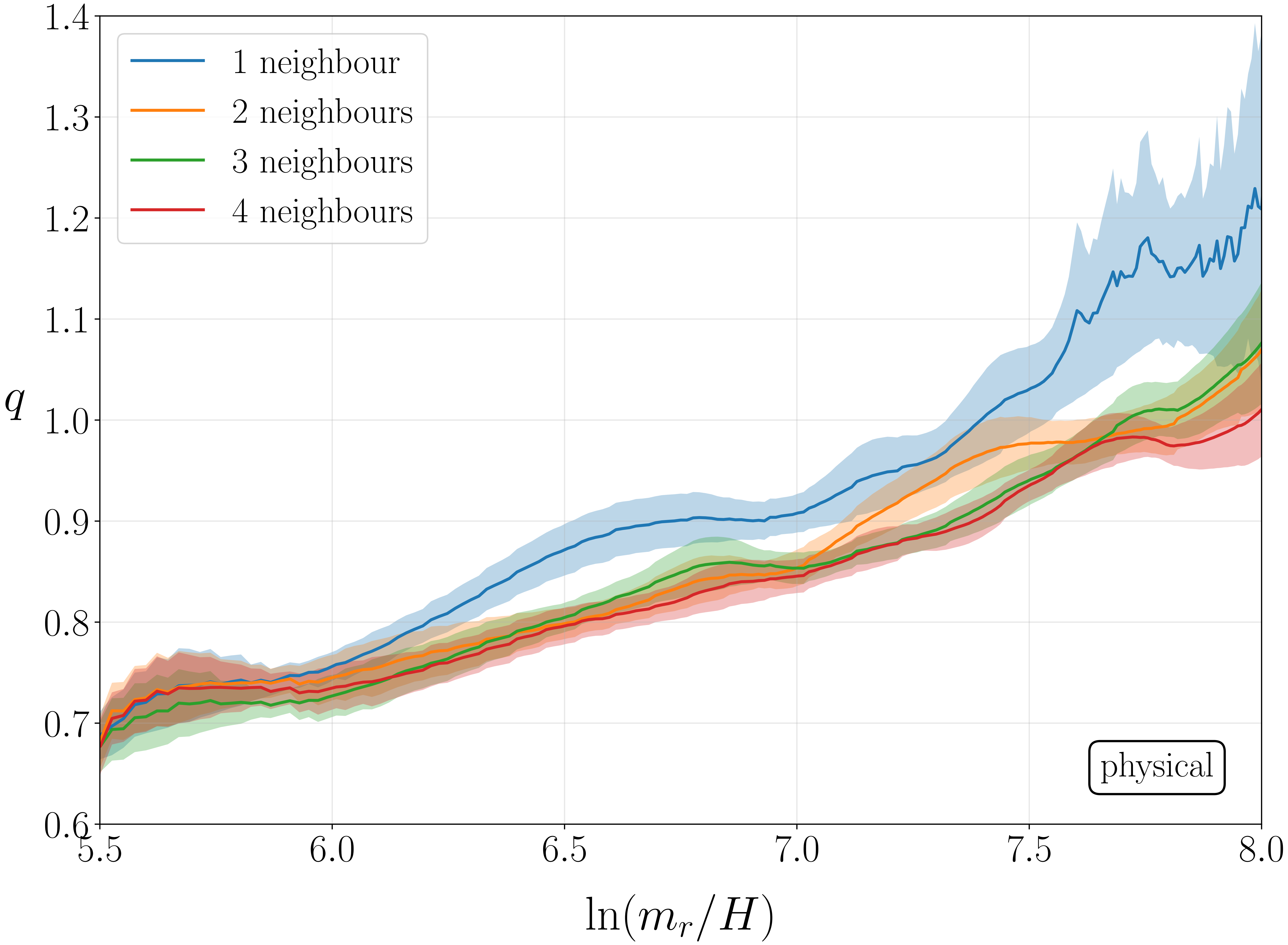
- Discretisation scheme of the Laplacian
- Resolution of the string core (parametrised by $m_r a$)



Discretisation Effects

Effects that can b

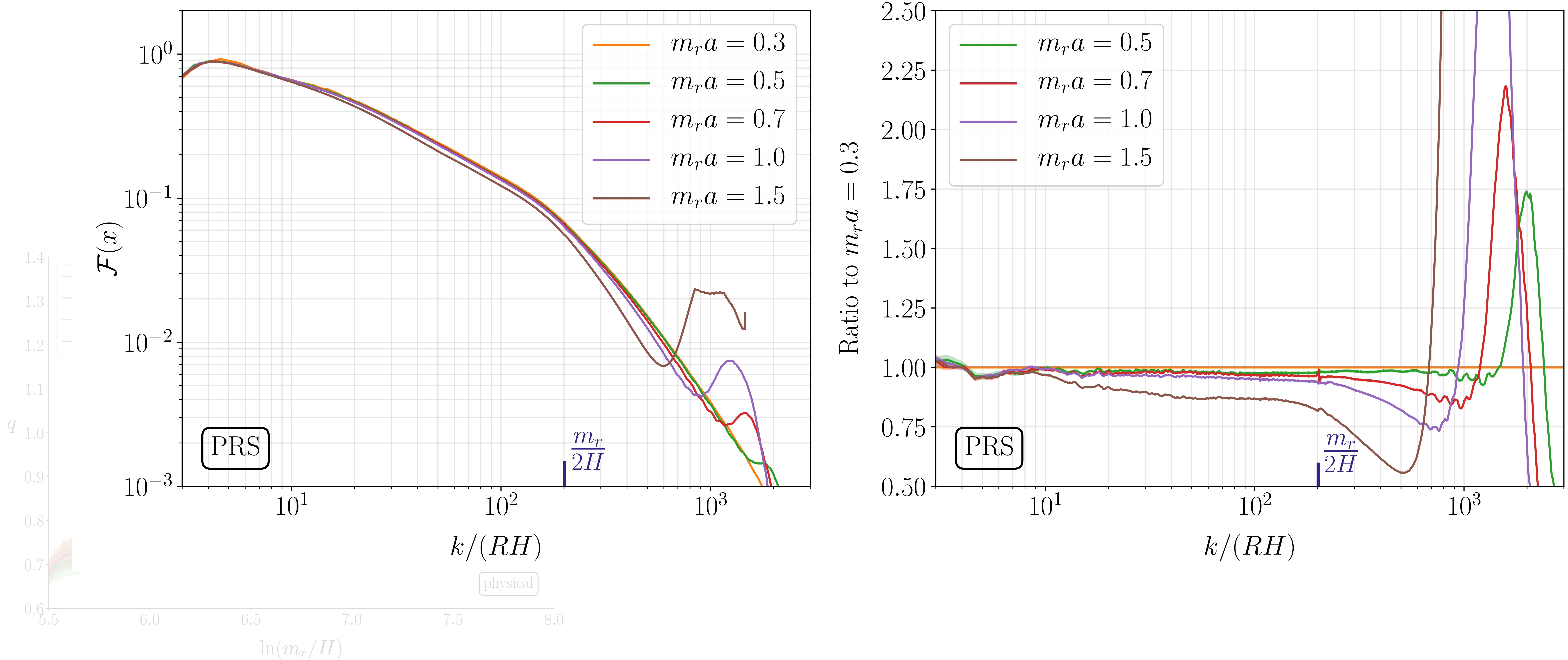
- Discretisati
- Resolution



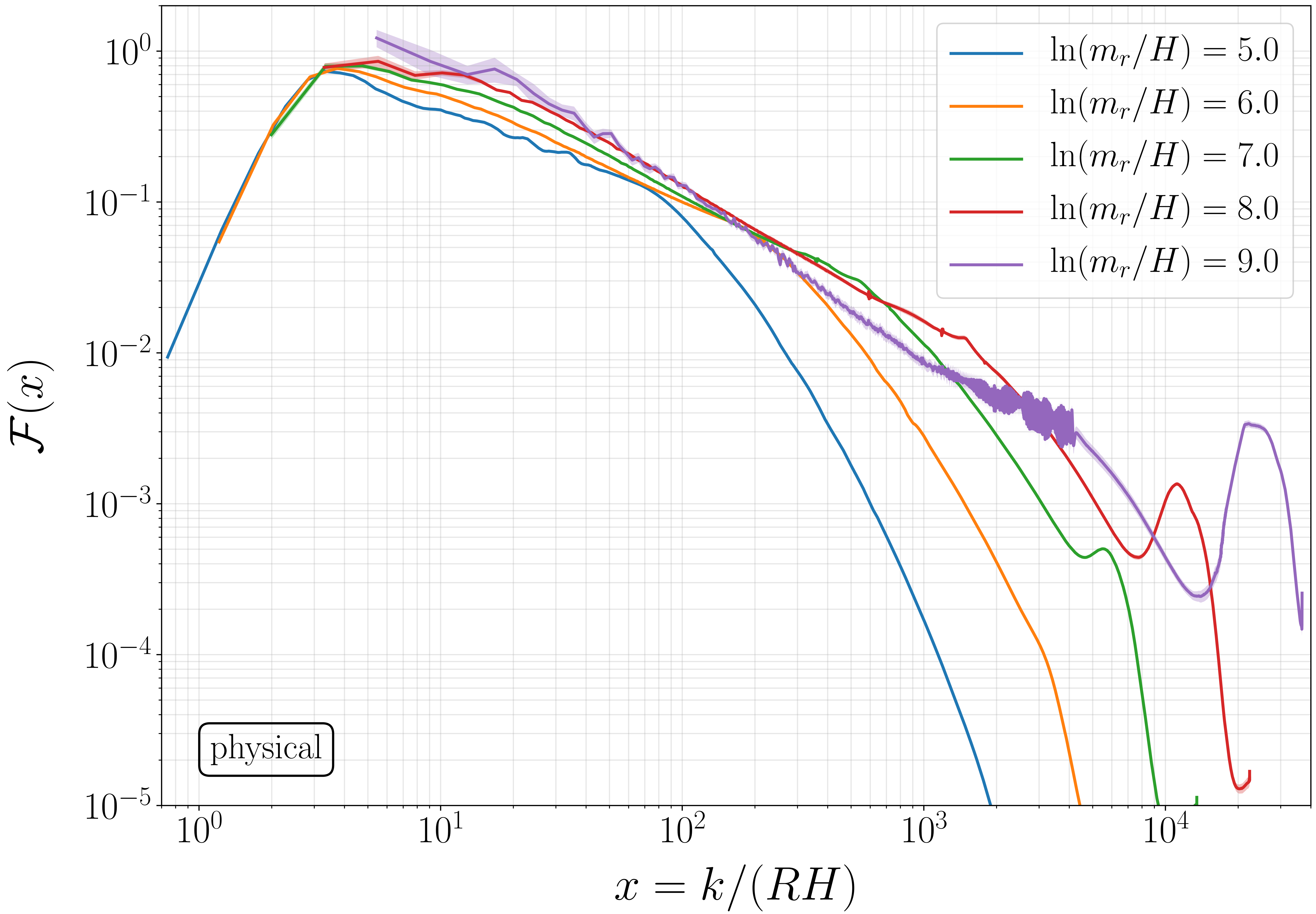
We observe that the value of q can be **overestimated** for smaller N_g!

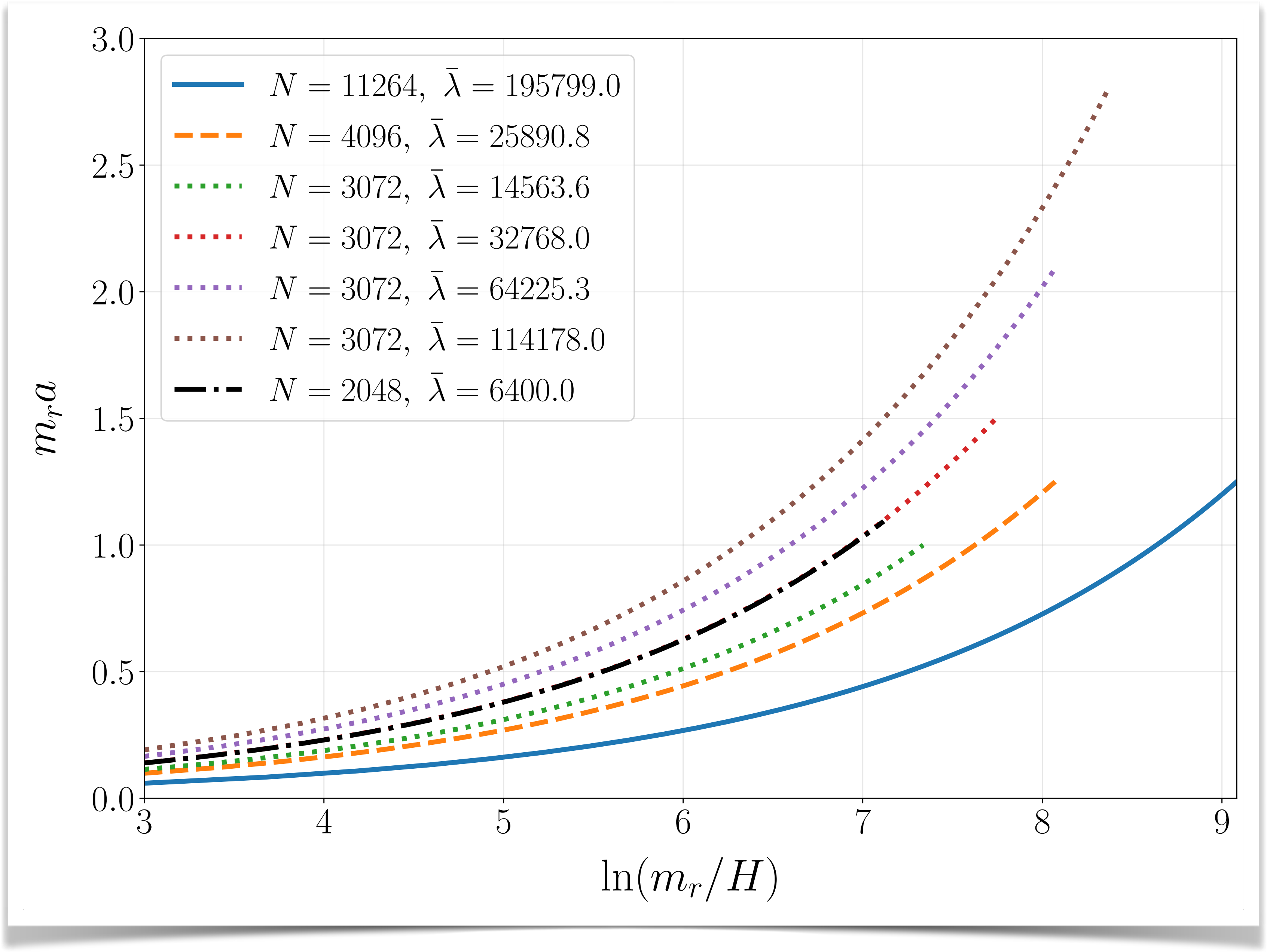
Discretisation Effects

Effects that can bias q towards larger values:



Discretisation Effects



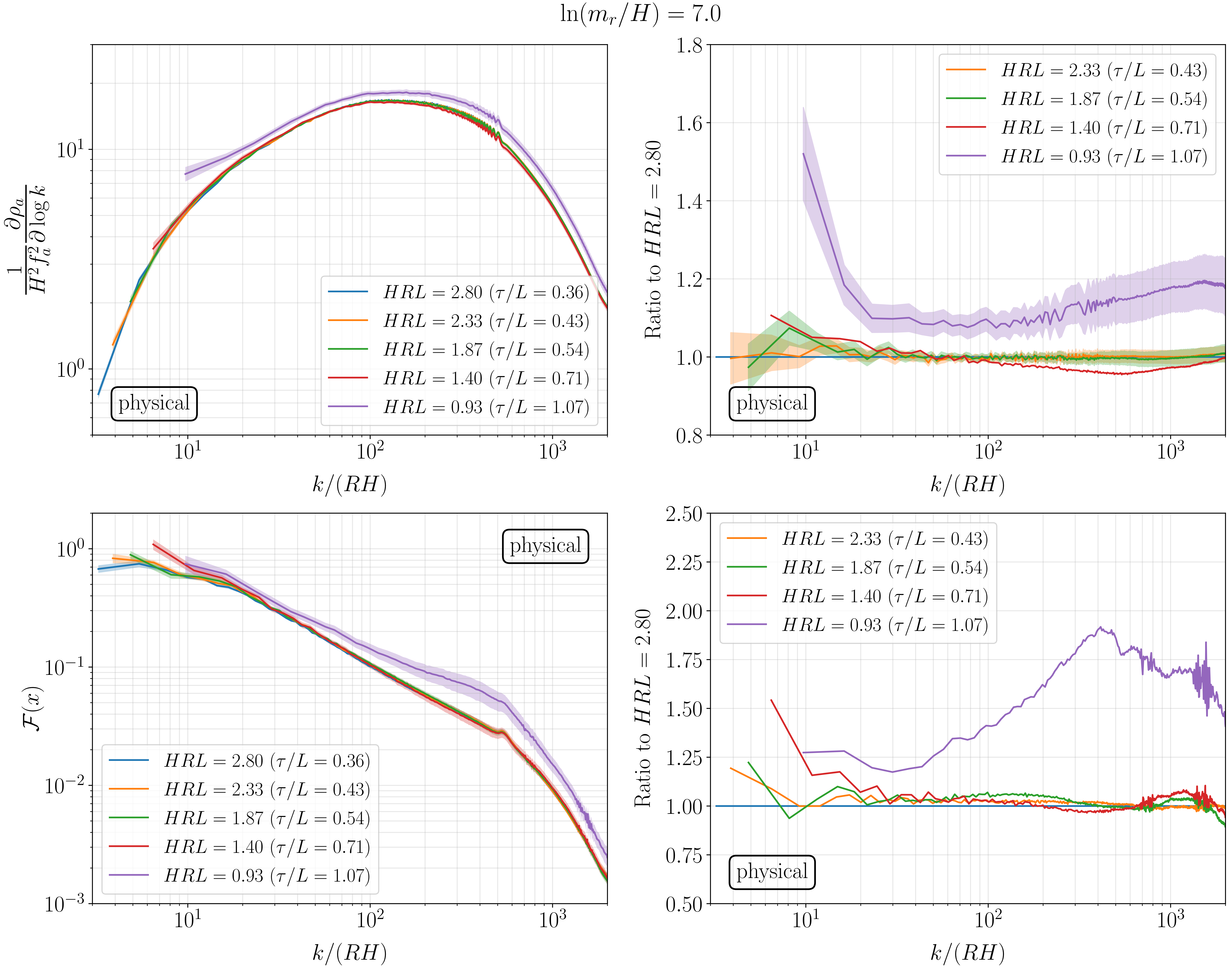


Effects increases drastically at larger $\ln(m_r/H)$, leading to a significant distortion of the spectrum.

Finite Volume Effects

- Fix $m_r a = 1.0$ and vary ratio of phys. box size RL to Hubble radius H^{-1} at $\ln(m_r/H) = 7$
- Results converge for $HRL \gtrsim 1.4$ (or $\tau/L \lesssim 0.7$)
- We terminate the simulations at $\tau/L \leq 0.625$

Should **not** be a problem!



Technicalities

Masking the Spectrum

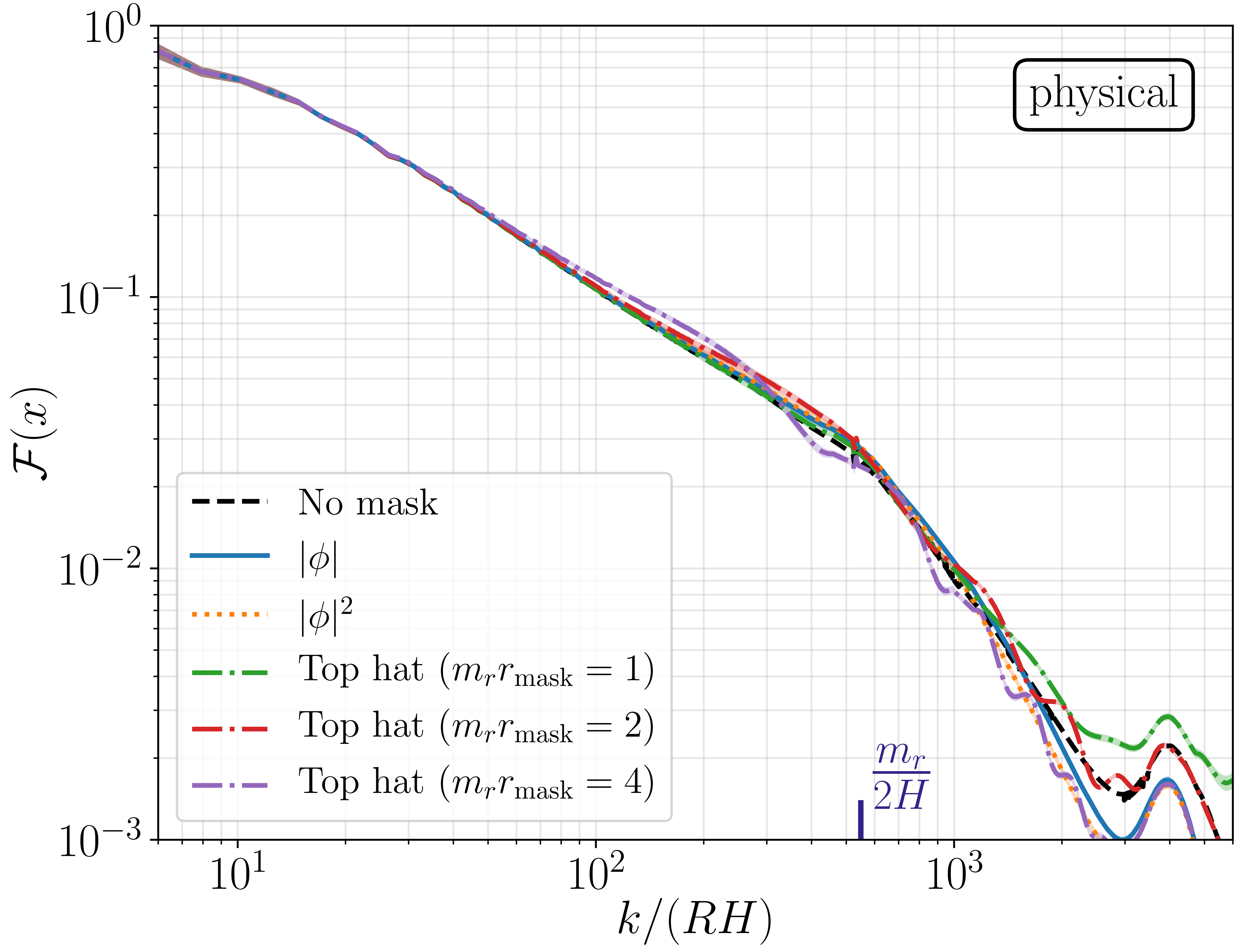
- To try to mitigate the contamination from the string core, we can introduce masks to to compute derivatives:

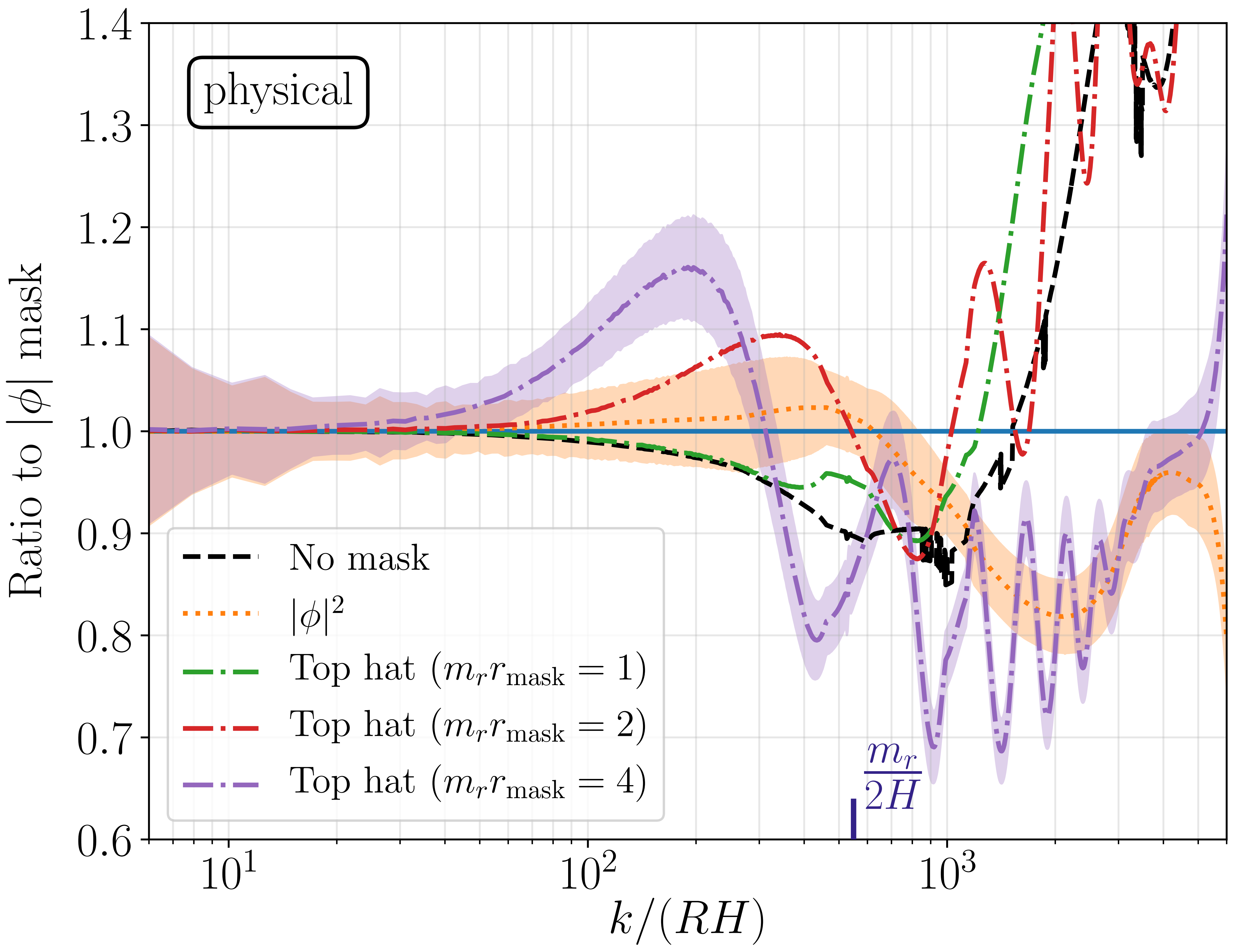
$$\dot{X}^{\text{mask}}(\boldsymbol{x}) = M(\boldsymbol{x})\dot{X}(\boldsymbol{x})$$

- Simple choice is to use the fact that the value of the radial field $|\phi|$ is zero inside the core.

$$M(x) = \left(\frac{|\phi(x)|}{f_a}\right)^k$$

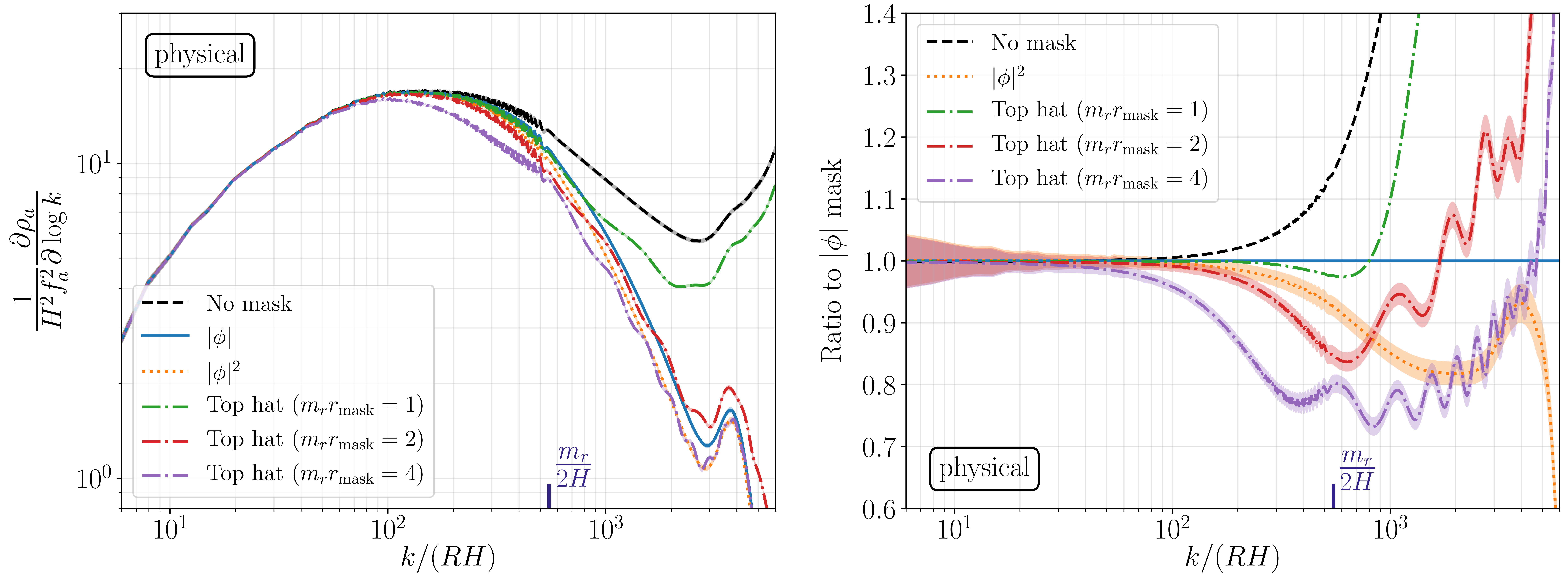
Masking the Spectrum



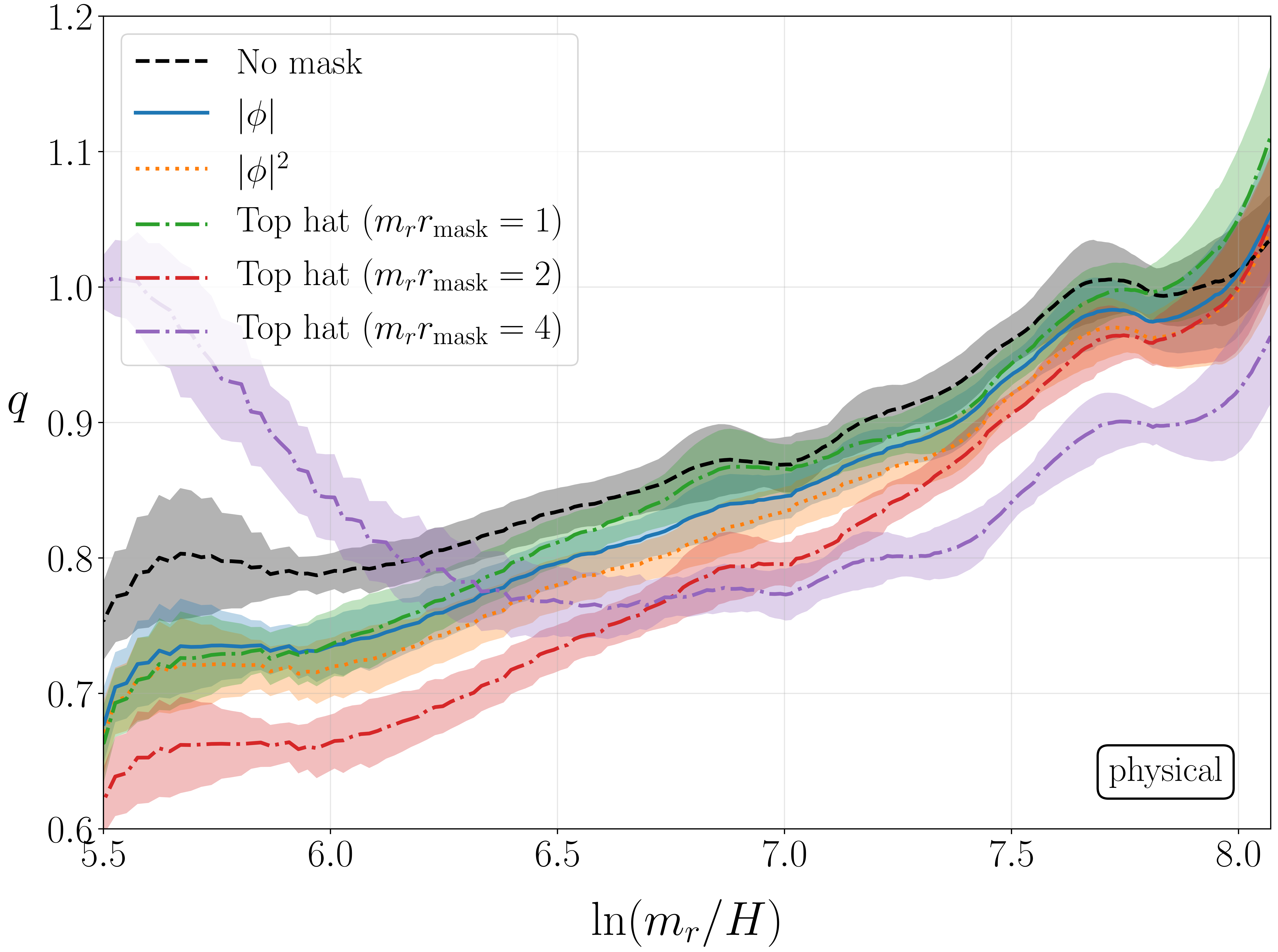


Masking the Spectrum

$\ln(m_r/H) = 7.0$



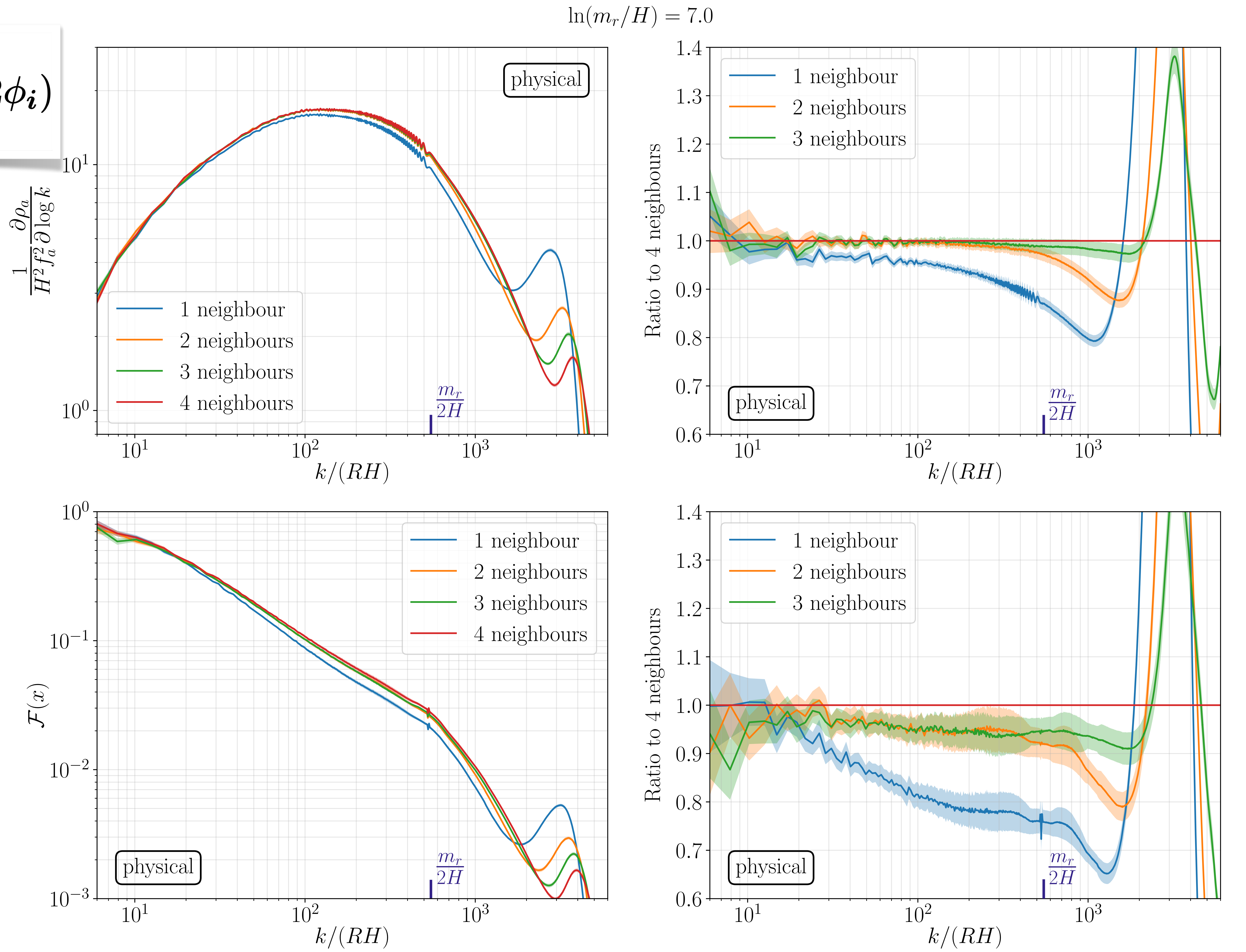
Masking the Spectrum



Discretisation of the Laplacian

$$(\nabla^2\phi)_{\boldsymbol{i}} = \frac{1}{\delta^2}\sum_{u=x,y,z}\sum_{n=1}^{N_g} C_n(\phi_{\boldsymbol{i}+n\boldsymbol{n}_u} + \phi_{\boldsymbol{i}-n\boldsymbol{n}_u} - 2\phi_{\boldsymbol{i}})$$

- Spectrum **underestimated** at intermediate momenta for smaller N_g
- Observation of peak-like structure in the UV, height related to N_g



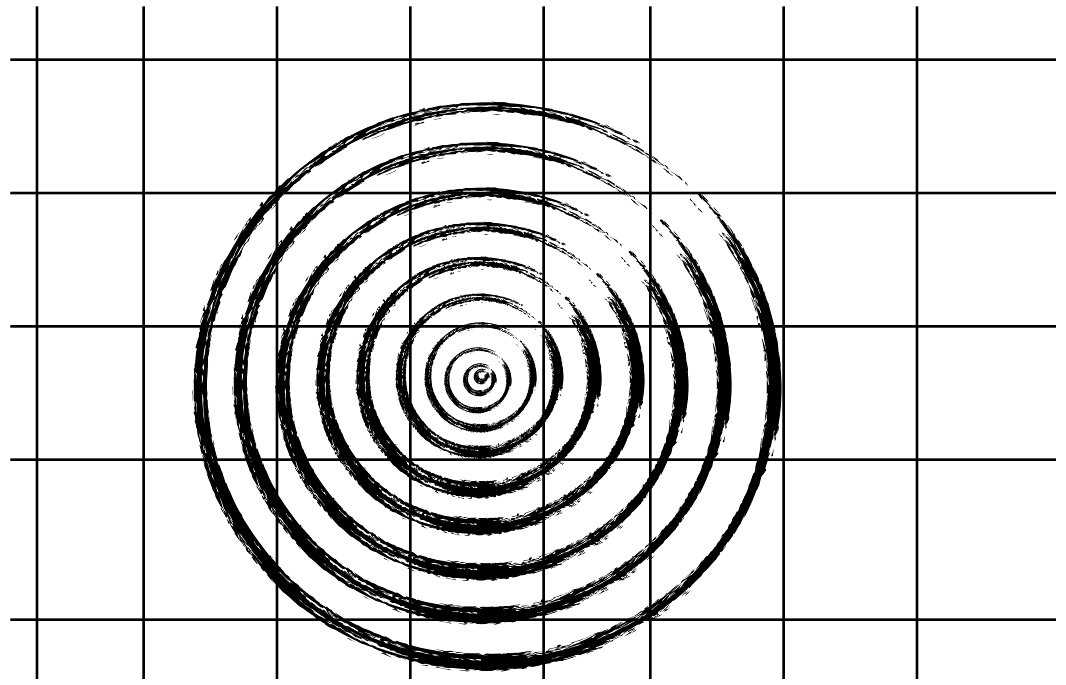
Dynamical Range (AMR)

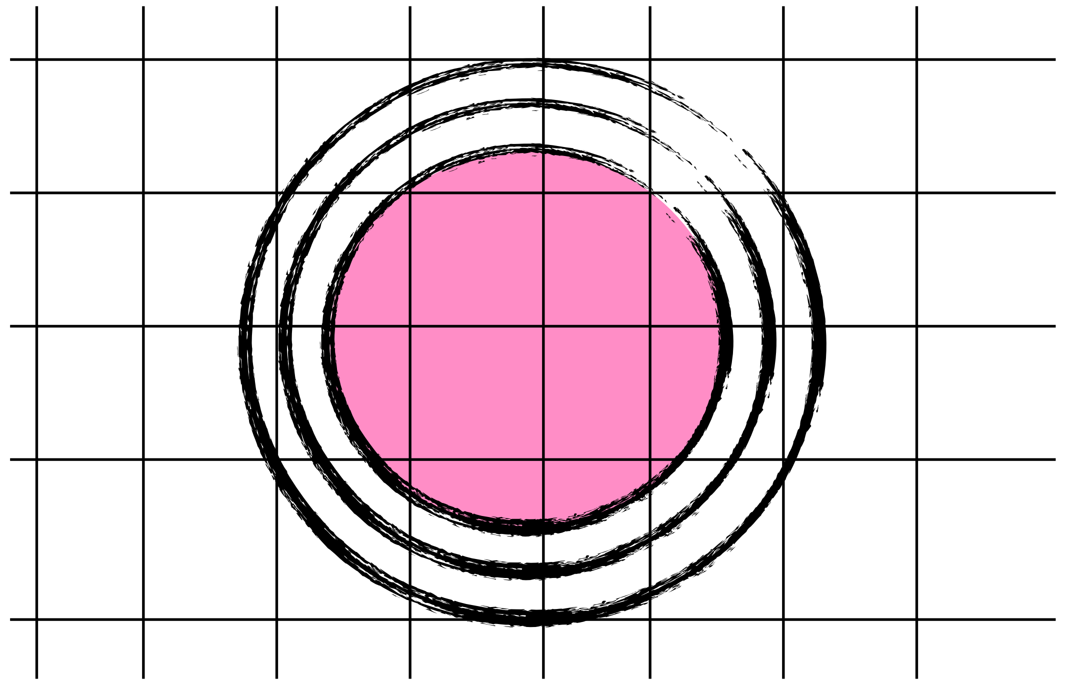
Departamento de Física Teórica
Universidad Zaragoza
1542
CAPA Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza
GOBIERNO DE ARAGON
COSMIC WISPERS
CA21106
cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

How can we reach a larger dynamical range?

- **Brute Force**: Larger simulations on more powerful supercomputers
- **Better:** Use the given computational power more efficiently: **AMR!**
- **In addition:** Study effective models that allow us to study the network dynamics at high tension (**Moore strings**) with 2+3 extra degrees of freedom (two additional complex scalars + one vector field)

Klaer, Moore [1707.05566, 1708.07521, 1912.08058]

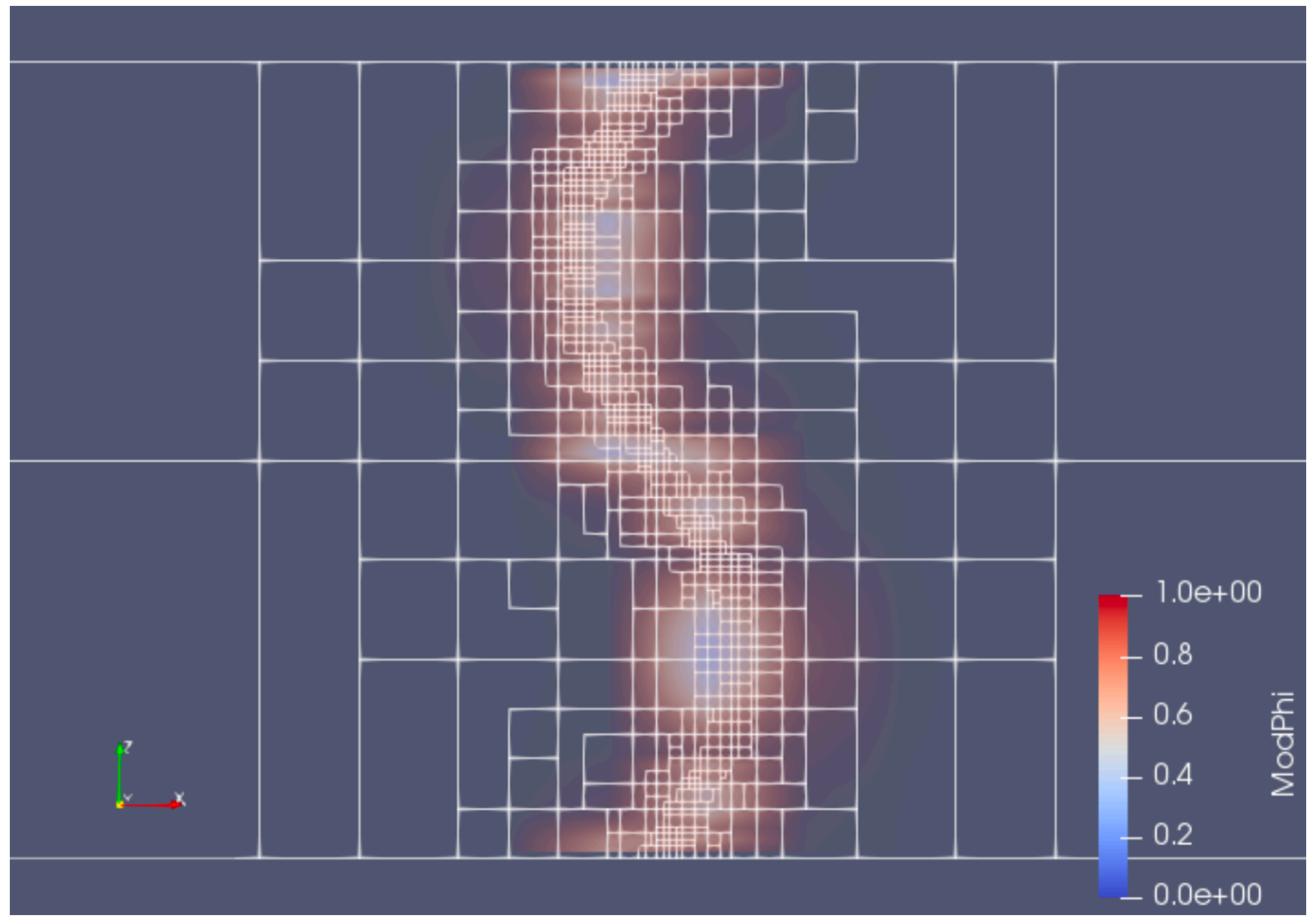




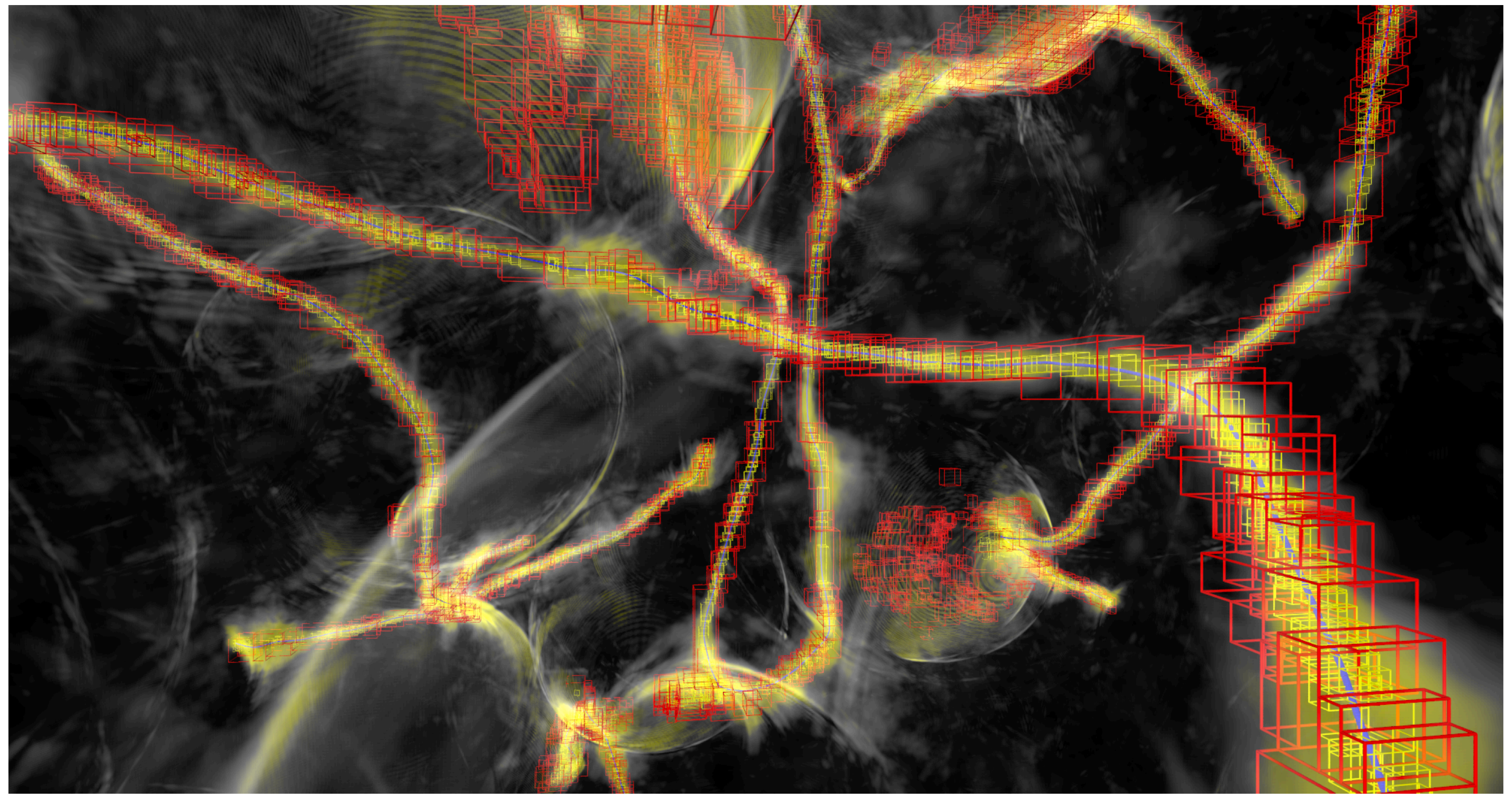
$$\log \sim 2(q_1^2 + q_2^2)$$

Adaptive Mesh Refinement (AMR)

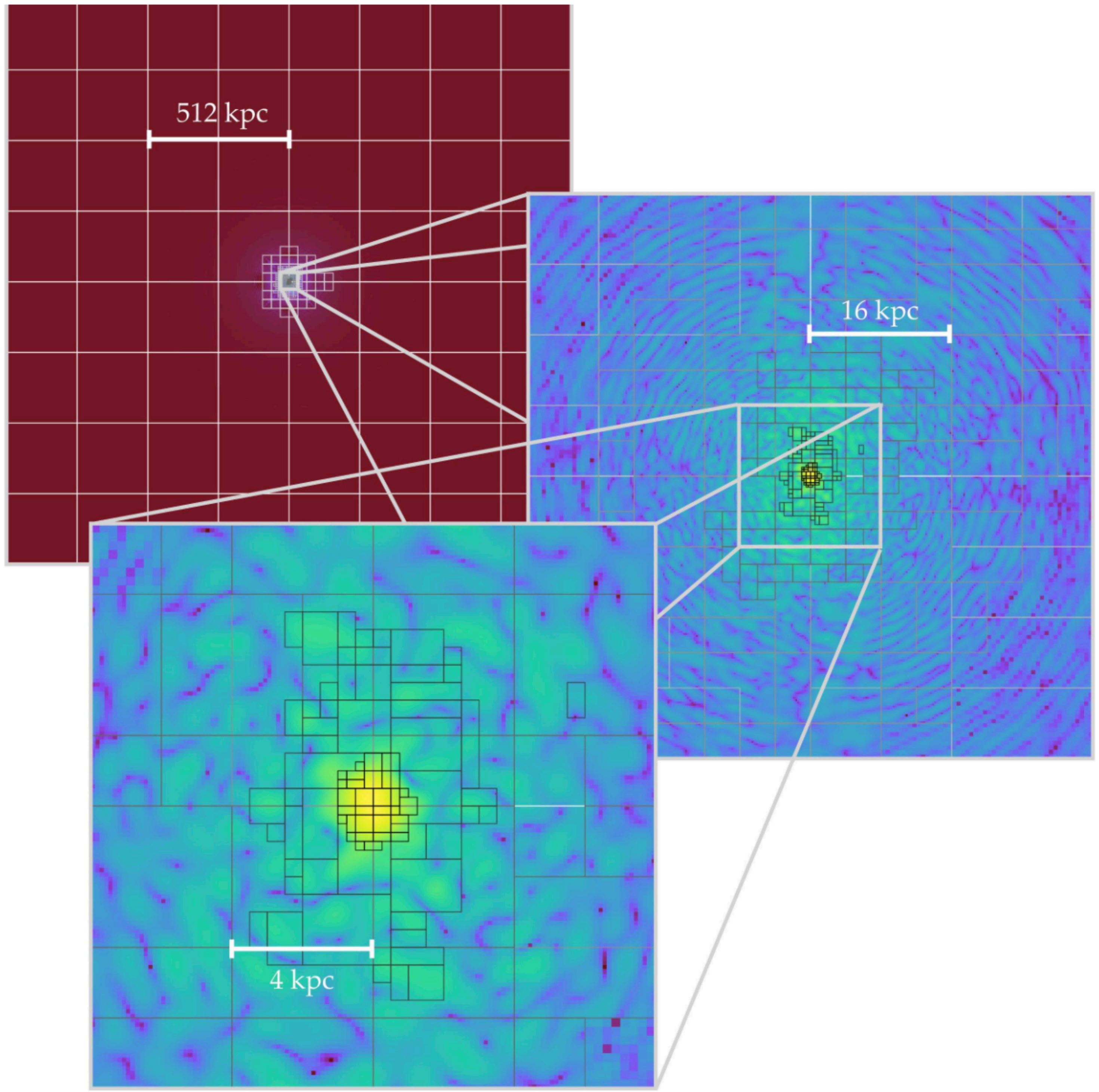
- **Idea:** Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on AMReX: https://amrex-codes.github.io/amrex/



Drew & Shellard [1910.01718]
"GRChombo"



Benabou+ [2308.01334]
"sledgehamr"



Schwabe+ [2007.08256]
"axioNyx"

Potential Improvement with AMR

- We can estimate the RAM needed to perform an AMR complex scalar simulation:

$$\mathrm{RAM} = 2 \times 2 \times 4\ \mathrm{bytes}\ \times \left(N_0^3 + \frac{\pi n_c n_r^2}{4} \frac{r^\ell - 1}{r - 1} N_p \right)$$

$N_p = \xi \times 6(L/(N_0\tau))^2 \times N_0^3$
Fleury & Moore [1509.00026]

- This takes into account, that we refine only around the strings and that we want to balance the RAM between the root and the refined grids

- Suggests time-dependent number of refinement levels:.

$$\ell + 7 \simeq \log_2\left(N_0^3/(\pi N_p)\right) = \log_2\left(N_0^2 \tau^2/(\pi 6 \xi L^2)\right)$$

- Results in log ~ 13,16,18 for base grids of $N_0 = 2048,\ 4096,\ 8192$ with $\ell = 9,\ 11,\ 13$. In practice not so trivial ..

Potential Improvement with AMR

• W… …nulation:

$\times 6(L/(N_0\tau))^2 \times N_0^3$

& Moore [1509.00026]

• Th… …ve want

to…

• Su…

• Re…

ℓ