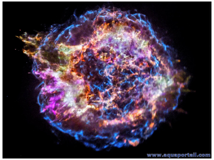


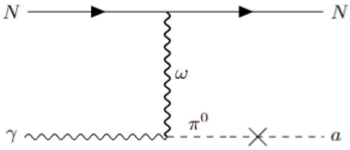
Istanbul CA21106 meeting

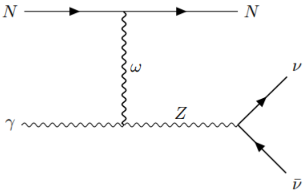
Photo-production in supernovae and neutron stars

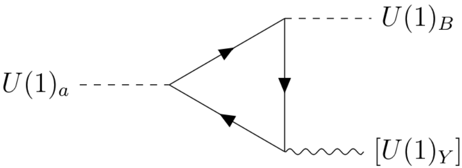
Miguel Vanvlasselaer
miguel.vanvlasselaer@vub.be

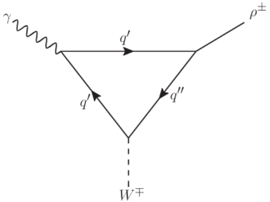
with Aritra Gupta (IFIC) and Sabyasachi Chakraborty (IIT Kanpur): [2403.12169] and JCAP 10 (2023) 030 [2306.15872]

What are the Wess-Zumino-Witten (WZW) interactions ?









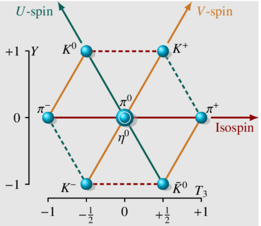
Computing the vertex

$$L \subset \omega \cdot Z \cdot F + \omega \cdot d\pi_0 \cdot F$$

Pion chiral lagrangian and WZW term

- Pion chiral lagrangian for the pseudo scalar mesons

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\text{Tr}\left(D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U\right) \ , \qquad U = e^{2i\vec{\sigma}\cdot\vec{\pi}/f_\pi}$$



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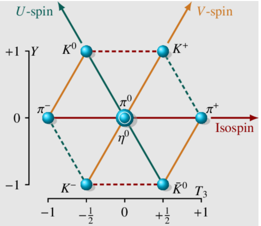

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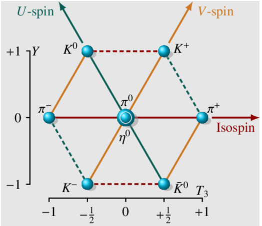

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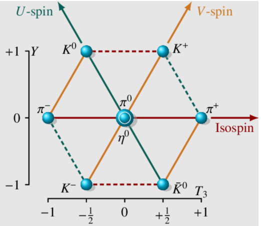

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- Solution: Wess-Zumino-Witten Phys. Lett. B 37 (1971) 95, Nucl. Phys. B 223 (1983) 422.

$$S_{\text{WZW}} = \kappa \int_D d^5x\, \omega \;, \qquad \omega = -\frac{i}{240\pi^2}\, \epsilon^{\mu\nu\rho\sigma\tau}\, \text{Tr}\left(\mathcal{U}_\mu\, \mathcal{U}_\nu\, \mathcal{U}_\rho\, \mathcal{U}_\sigma\, \mathcal{U}_\tau\right) \qquad \mathcal{U}_\mu = U^\dagger \partial_\mu U$$

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$$\text{set } \delta A \qquad \to \ \delta\omega = -e\,\delta A_\mu \hat{J}^\mu \ ,$$

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- Induces a conserved current

$$\hat{J}^\mu = \frac{1}{48\pi^2}\,\epsilon^{\mu\nu\rho\sigma\tau}\,\partial_\nu\,\text{Tr}\left[\{Q, U^\dagger\}\,\partial_\rho U\,U^\dagger\partial_\sigma U\,U^\dagger\partial_\tau U\right]\ .$$

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- Full expression for $S_{\text{WZW}}(U, A_\mu)$

$$\kappa \int_D d^5x\ \omega - \kappa e \int d^4x\ A_\mu J^\mu + \underbrace{\frac{ie^2}{24\pi^2} \int d^4x\ \epsilon^{\mu\rho\sigma\lambda}\ A_\rho\left(\partial_\mu A_\nu\right)\ \left[\text{Tr}\left(\{Q^2, U^\dagger\}\partial_\sigma U\right) - QUQ\partial_\sigma U^\dagger\right]}_{= \frac{ie^2}{48\pi^2} \frac{\pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}}.$$

We found the anomaly !

Repeat the procedure with non-abelian chiral subgroups

Nucl. Phys. B 223 (1983) 422: Witten, PhysRevD.30.594: Kaymakcalan, Rajeev and Schechter

$\alpha \equiv dUU^\dagger \qquad \beta \equiv U^\dagger dU.$

$$\Gamma_{WZW}(U, \mathcal{A}_L, \mathcal{A}_R) = \Gamma_0(U) \; + \mathcal{C} \int \mathrm{Tr} \Big\{ (\mathcal{A}_L \alpha^3 + \mathcal{A}_R \beta^3) - \frac{i}{2}[(\mathcal{A}_L\alpha)^2 - (\mathcal{A}_R\beta)^2]$$
$$+ i(\mathcal{A}_L U \mathcal{A}_R U^\dagger \alpha^2 - \mathcal{A}_R U^\dagger \mathcal{A}_L U \beta^2) + i(d\mathcal{A}_R dU^\dagger \mathcal{A}_L U - d\mathcal{A}_L dU \mathcal{A}_R U^\dagger)$$
$$+ i\big[(d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)\alpha + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)\beta\big] + (\mathcal{A}_L^3 \alpha + \mathcal{A}_R^3 \beta)$$
$$- (d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L) U \mathcal{A}_R U^\dagger + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R) U^\dagger \mathcal{A}_L U$$
$$+ (\mathcal{A}_L U \mathcal{A}_R U^\dagger \mathcal{A}_L \alpha + \mathcal{A}_R U^\dagger \mathcal{A}_L U \mathcal{A}_R \beta) + i \left[\mathcal{A}_L^3 U \mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger \mathcal{A}_L U - \frac{1}{2}(U \mathcal{A}_R U^\dagger \mathcal{A}_L)^2 \right] \Big\}.$$

What is the pheno in all those terms ? HHH:0705.0697, 0708.1281, Gardner, He: 1101.1128

$$\mathcal{L}^{\pi}_{\mathrm{WZW}} \supset \frac{N_C}{48\pi^2} g_2^2 \tan\theta_W \; \epsilon^{\mu\nu\rho\sigma} \; F_{\mu\nu} \mathcal{A}_\rho Z_\sigma - 2\frac{N_C}{48\pi^2} \frac{\partial_\alpha \pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \mathcal{A}_\beta$$

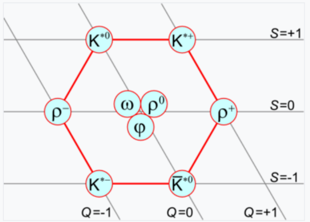
How to introduce vector mesons ?

$$\mathcal{L} \subset \omega \cdot F \cdot \ldots \; ??$$

Vector mesons not part of the U matrix ! How to introduce them ?

$\Downarrow$ HHH: Phys. Rev. D 77 (2008) 085017

Introduce a background field B with QN of ω, ρ : $\Gamma(U, \mathcal{A}_\mu + B_\mu)$



- Introduce $B_\mu j^\mu_B$ $\Rightarrow \partial_\mu j^\mu \propto \epsilon \cdot F_A F_B$ (mixed anomaly in the vector current)

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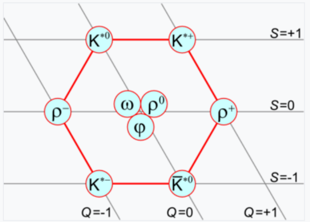

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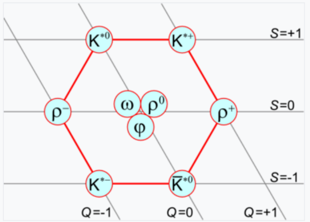

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- Non-vectorlike: Recipe is

$$S_{\text{full}} = S_{WZW}(U, A_L + B_L, A_R + B_R) - \Gamma_c$$

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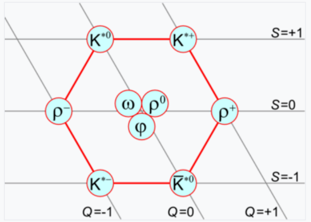
Background field method dictates HHH: Phys. Rev. D 77 (2008) 085017



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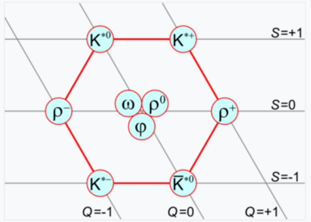

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$$\frac{\pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} = -2 \frac{\partial_\alpha \pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \mathcal{A}_\beta \to -2 g_\omega \frac{\partial_\alpha \pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \omega_\beta$$

Pion-axion mixing: $a - \pi_0$

Introduce the axion

- The KSVZ or KSVZ-like axion

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + c_g \frac{\alpha_s}{8\pi}\frac{a}{f_a} G\widetilde{G} + \frac{1}{2}\left(\partial_\mu a\right)^2 - \frac{1}{2} m_{a_0}^2 a^2$$

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- The mixing angle for heavy ALP Aloni, Fanelli, Soreq, Williams 19' PRL 123, 071801: Traditional recipe

$$\boxed{\mathcal{M}_{i\to fa} = \theta^{\mathrm{ALP}}_{\pi^0-a}\mathcal{M}_{i\to f\pi_0}}$$

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-

$$\theta_{\pi^0-a} \simeq \frac{1}{2}\frac{f_\pi}{f_a}\left(2c_{GG}\frac{(\kappa_u-\kappa_d)m_a^2}{m_\pi^2 - m_a^2}\delta - \frac{m_\pi^2}{m_\pi^2-m_a^2}\Delta_\kappa\right) \equiv \frac{C_A^{\mathrm{ALP}} f_\pi}{f_a}, \qquad \Delta_\kappa = 0 \quad (\mathrm{Tr}[\vec{\kappa}] = 1)$$

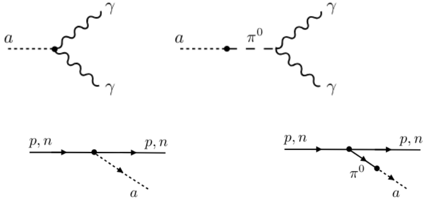
The mixing angle for heavy ALP

- Build axion-pion lagrangian Bauer,Neubert, Renner,Schnubel,Thamm 20' :

κ_q unphysical but disappears in the sum!

$$C_{a\gamma\gamma} = -1.92 c_\gamma - \frac{m_a^2}{m_\pi^2 - m_a^2}\left(c_g \delta + c_u - c_d\right)$$

$$g_{ap} = g_0(c_u + c_d + 2c_g) + g_A \frac{m_\pi^2}{m_\pi^2 - m_a^2}(c_u - c_d + 2c_g\delta)$$



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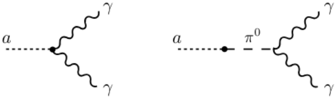

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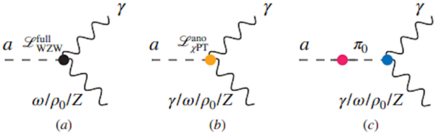
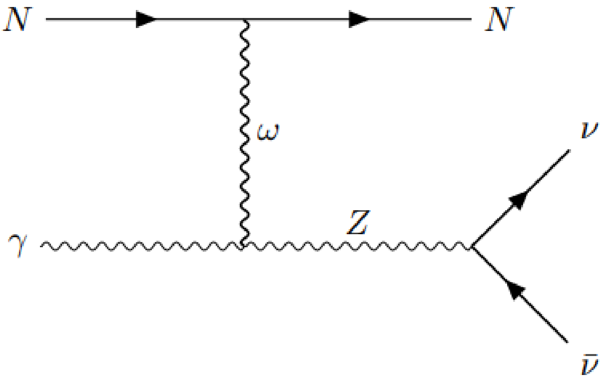
- Consistent method: incorporate the axion as a background field Bai, Chen, Liu, Ma, arxiv:[2406.11948]



$$C_{\omega\gamma a}^{\text{anomalous}} = \frac{3}{8\pi^2}\left(\frac{m_\pi^2}{m_\pi^2 - m_a^2} c_g \delta - \frac{m_a^2}{m_\pi^2 - m_a^2}\frac{c_u - c_d}{2}\right)$$

Computing the full interaction



Road to our Lagrangian

- $$\mathcal{L}_{\text{WZW}} \supset \left(\frac{N_c}{48\pi^2} g_2^2 g_\omega \, \tan\theta_W\right) \times \, \epsilon^{\mu\nu\rho\sigma} \, \overbrace{F_{\mu\nu}}^{\text{light}} \overbrace{\omega_\rho Z_\sigma}^{\text{heavy}} + \ldots$$

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- Integrating out ω_ν and Z_μ : vertex $\gamma N N \nu\nu$

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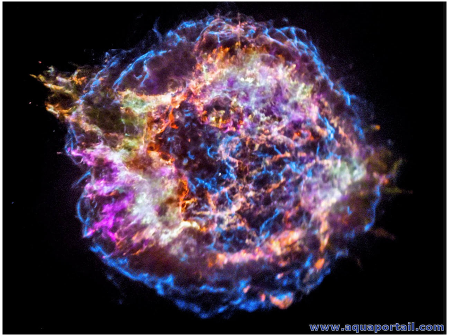
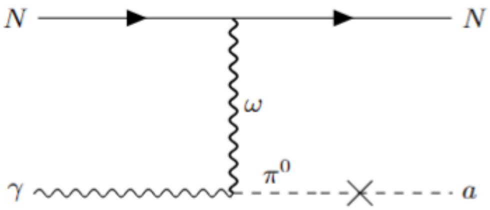
- vertex γNNa

$$\mathcal{L}_{\text{int}} = \left(C_A \frac{eN_c}{24\pi^2} \frac{g_\omega^2}{m_\omega^2}\right) \times \epsilon_{\mu\nu\alpha\beta} \frac{\partial_\mu a}{f_a} F^{\nu\alpha} \bar{N}\gamma^\beta N$$

Emission of axions from SN

Photo-production of axions in Supernovae





With Sabyasachi Chakraborty (IIT-Kanpur) and Aritra Gupta (IFIC Valencia): 2403.12169

The cooling argument in supernovae and SN1987A

- Neutrino received during SN1987A:

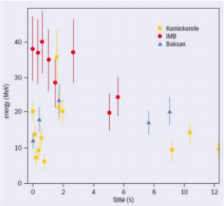


Figure: Credit:NirCam JWST

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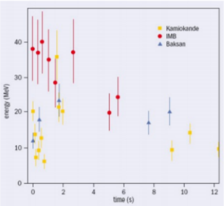

- Raffelt bound:

$$\frac{Q_{\text{additional}}}{\rho} \lesssim 10^{19} \text{ erg } s^{-1} g^{-1}$$

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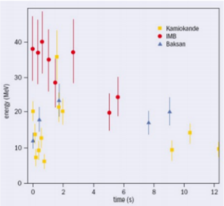

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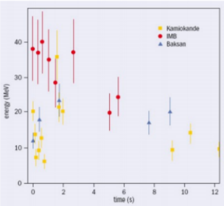


- Raffelt bound:

$$\frac{Q_{\text{additional}}}{\rho} \lesssim 10^{19}\ \text{erg}\ s^{-1} g^{-1}$$

- This is a coupling constraint !
- We need: Emissivity $Q =$, energy per unit of time and cc emitted

Figure: Credit:NirCam JWST

How to carry energy away from the SN?

Supernovae extreme medium:

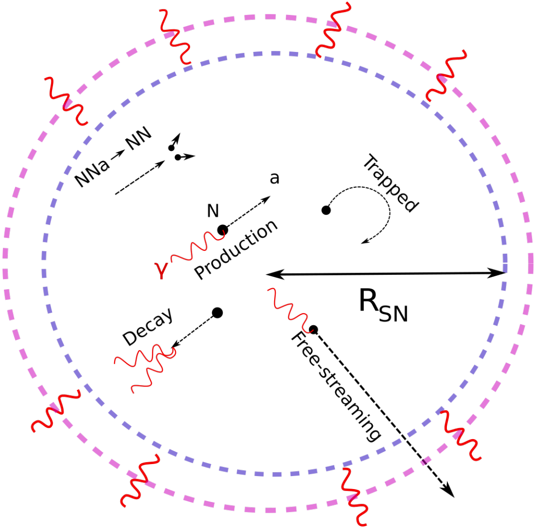
$\rho \sim \rho_0, \qquad T \sim 30 - 50$ MeV

bremsstrahlung and pion conversion

$$NN \to NNa \qquad N\pi \to Na$$

What about the photo-production ?

$$\boxed{N\gamma \to Na}$$

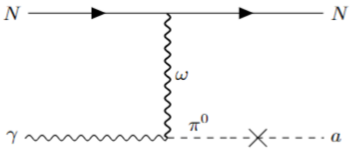


Emissivities of the $\gamma N \to Na$

$$Q_{N\gamma\to Na} \approx \int \frac{dE_\gamma}{\pi^2} E_\gamma^2 f_\gamma(p_\gamma) \underbrace{n_B}_{\text{targets}} \underbrace{\sigma_{\gamma N\to Na}(E_\gamma)}_{\text{x-section}} \times \underbrace{E_\gamma}_{\text{energy escaping}}$$

- WZW Non-degenerate regime

$$\frac{Q^{\rm WZW,ND}_{N\gamma\to Na}}{10^{34}\ \text{erg/s/cm}^3} \approx 5.6\ C_A^2\ g_{40}^4\ T_{40}^8\ \rho_{15} \left(\frac{10^9\ \text{GeV}}{f_a}\right)^2$$



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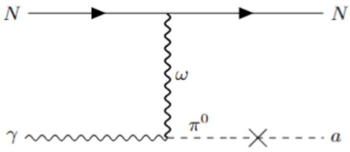
$$\frac{Q^{\rm WZW,D}_{N\gamma\to Na}}{10^{34}\text{erg/s/cm}^3} \approx 1.7\ C_A^2\ g_{40}^4 T_{40}^9\left(\frac{10^9\ \text{GeV}}{f_a}\right)^2$$



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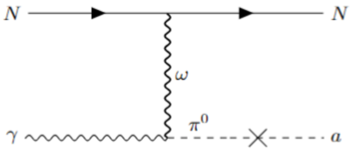
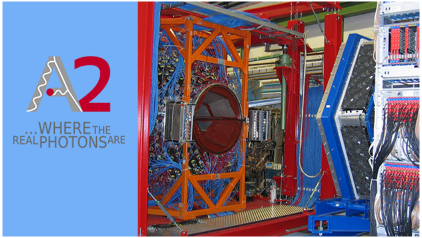
- ND computation holds for $T \gtrsim 30$ MeV.

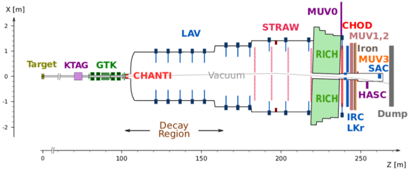


What is the value of g_ω ?

- Large range of theoretical prediction: $g_\omega \in 8 - 60$!

MAMI and NA60 collabs





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- Large range of theoretical prediction: $g_\omega \in 8 - 60!$
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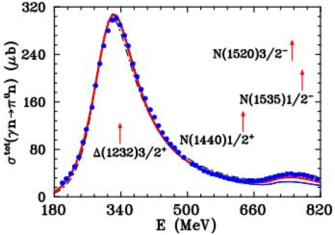

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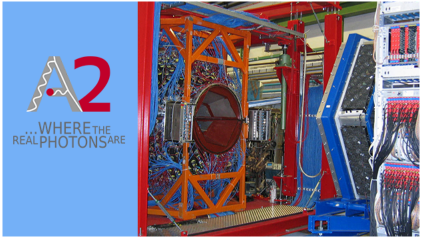

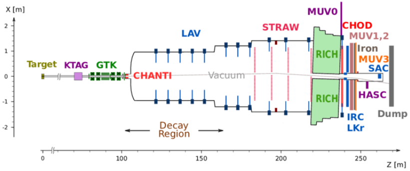

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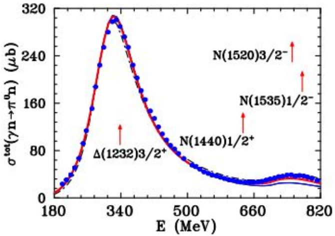

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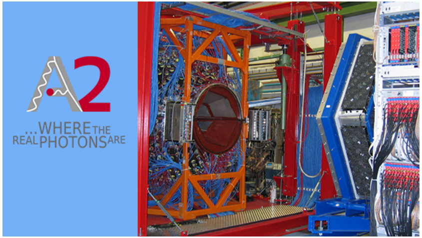

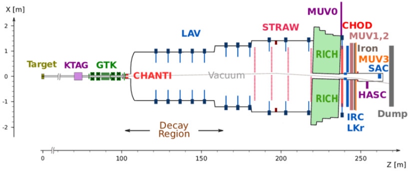

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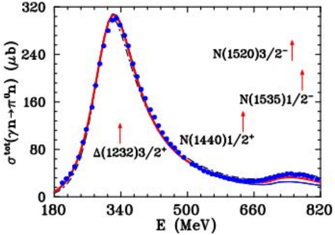
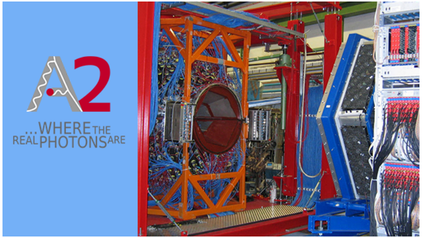
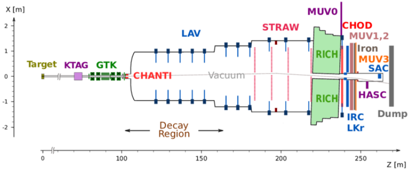


- Consistent with $g_\omega \sim 8 - 12$
- Data driven method available: GlueX Aloni, Fanelli, Soreq, Williams: PRL 123, 071801

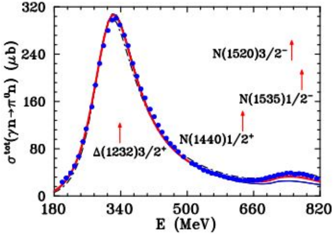
MAMI and NA60 collabs

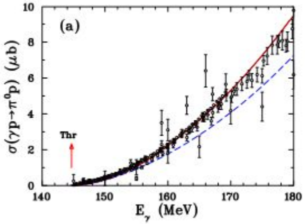




Data-driven photo production

- $\sigma_{\gamma N \to N a} \approx \frac{f_\pi C_A}{f_a} \sigma_{\gamma N \to N \pi_0}, \quad E > 145$ MeV





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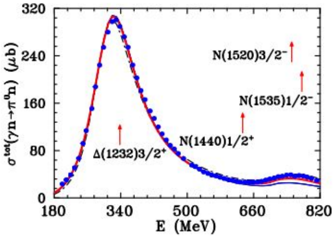

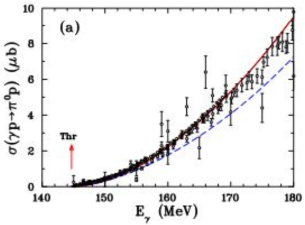

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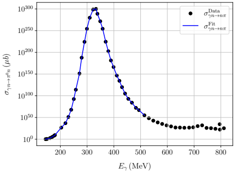
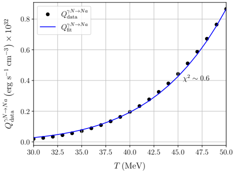
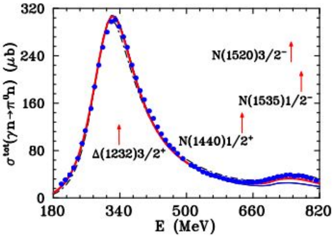
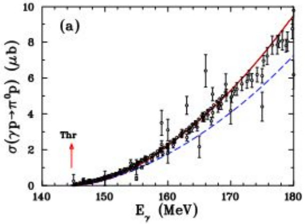
- Axion emissivities:

$$\frac{Q_{\text{data}}^{\gamma N \to N a}}{1.6 \times 10^{33}\ \text{cm}^{-3} s^{-1} \text{erg}} \approx \left(\frac{C_A 10^9}{f_a/\text{GeV}} \right)^2 \times \rho_{15} T_{40}^{6.73}$$









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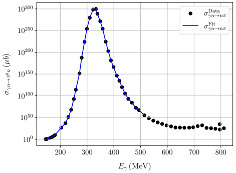
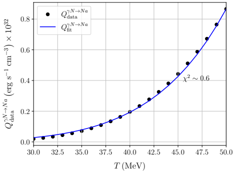
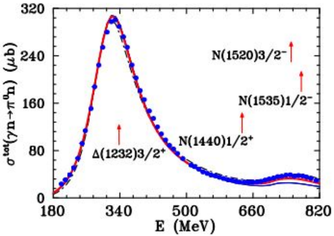
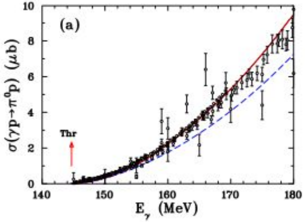








- The data-driven piece dominates if $g_\omega < 20$:

$$g_\omega \sim 12: \qquad Q_{N\gamma \to Na}^{\text{data,ND}} \sim 10 Q_{N\gamma \to Na}^{\text{WZW}}$$

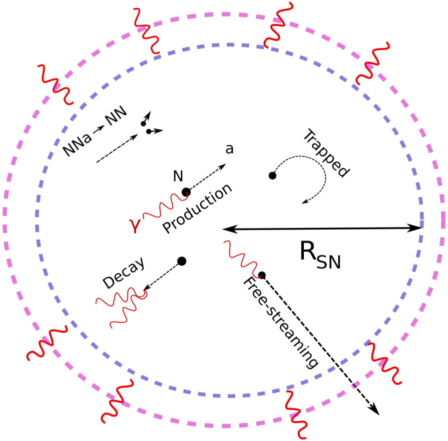
Emissivities of the $\gamma N \to Na$: further subtleties

- Absorption via $aN \to \gamma N$ or $NNa \to NN$:

$$L_a \approx \frac{1}{\Gamma_{aN\to\gamma N}} + \frac{1}{\Gamma_{NNa\to NN}}$$

$$\frac{L_a^{apn\to pn}}{10\text{ Km}} \approx \frac{10^2}{X_p C_{ap}^2}\left(\frac{f_a}{10^9\text{GeV}}\right)^2 \frac{E_a}{1\text{ GeV}}$$

We cut if $L_a < R_{SM} \sim 10$ km



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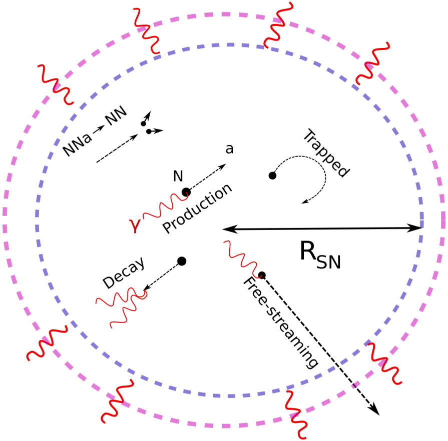
- Lapse effects: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

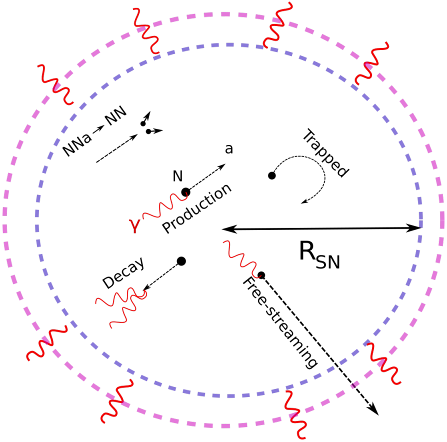
$$E_\infty = E_{\text{emission}} \times \alpha \qquad n_a^\infty = n_{\text{emission}} \times \alpha$$

$$\implies Q_\infty = \alpha^2 Q_{\text{emission}}$$

$$\alpha(M,r) \approx \sqrt{1-2M/r}$$



Emissivities of the $\gamma N \to Na$: further subtleties when axions are massive



Massive axions regime

- Redshift effect: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

$$Q \approx \int_{m_a/\alpha} dE \ldots$$

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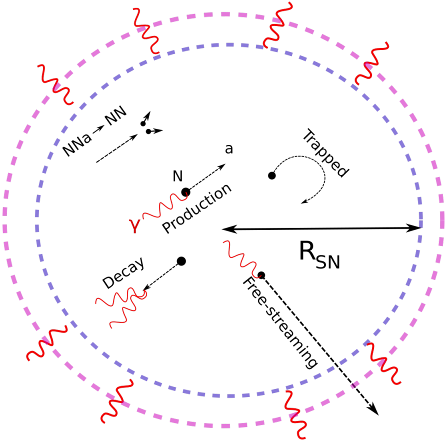


Massive axions regime

- Redshift effect: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

$$Q \approx \int_{m_a/\alpha} dE \ldots$$

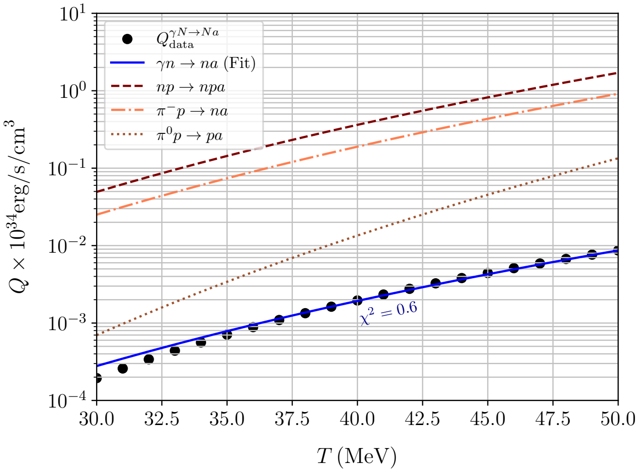
- Decay of the heavy axions:

$$L_{a\to\gamma\gamma} \approx \frac{4\times 10^4\ \text{km}}{(G_{a\gamma\gamma}/10^{-9}\text{GeV}^{-1})^2} \frac{E_a/100\ \text{MeV}}{(m_a/100\ \text{MeV})^4}$$

Cooling if $\quad L_{a\to\gamma\gamma} > R_{\text{SN}}$

Impact of the photo-production on axion constraints for KSVZ model

Contribution from photo-production to the emissivity for QCD axion



Impact of the photo-production on axion constraints for KSVZ model

Contribution from photo-production to the emissivity for massive ALPs

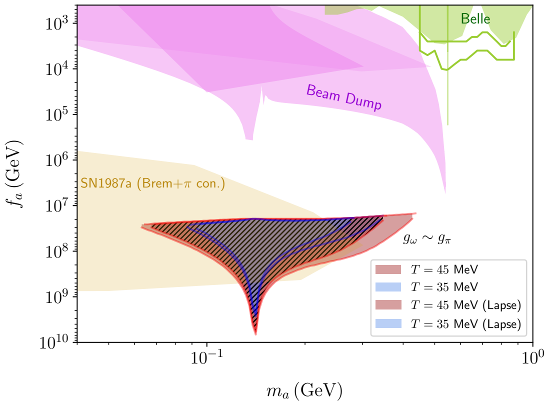
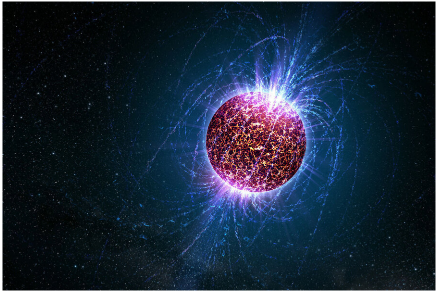
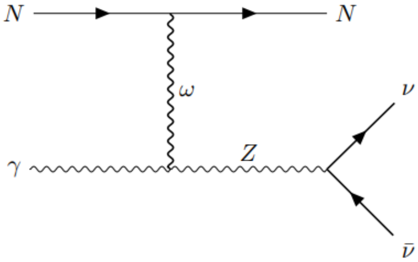


Figure: Lella, Carenza, Co', Lucente, Giannotti, Mirizzi, Rauscher

Emission of neutrino from NS

Photo-production of neutrino in NS

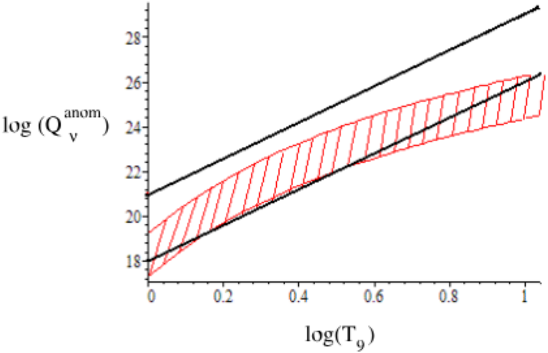




With Sabyasachi Chakraborty and Aritra Gupta: 2306.15872

WZW could contribution to NS cooling !

First computation of $N\gamma \to N\nu\bar{\nu}$ in [Harvey, Hill and Hill, arXiv:0708.1281]



But neglect the degeneracy effect ...

How do we compute the emissivity from a star ?

- Emissivity computation for $\gamma N \to N X$

$$Q^{N\gamma\to NX}_{\text{cooling}} \equiv \int \frac{d^3p_\gamma g_\gamma f_\gamma(p_\gamma)}{(2\pi)^3 2E_\gamma} \int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1} (2\pi)^3 2E_{N_2}} \prod_{X_i} \int \frac{(2\pi)^4 d^3p_{X_i}}{(2\pi)^3 2E_{X_i}} \langle|\mathcal{M}|^2\rangle \left(\sum_i E_{X_i}\right)$$
$$f_N(E_{N_1})\,(1 - f_N(E_{N_2}))\delta(E_1 - E_2 - Q_0)\,\delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q})\,. \tag{1}$$

How do we compute the emissivity from a star ?

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$$f_N(E_{N_1})\,(1 - f_N(E_{N_2}))\delta(E_1 - E_2 - Q_0)\,\delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q})\,. \quad (1)$$

- How to compute:

$$\boxed{\int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1} (2\pi)^3 2E_{N_2}} f_N(E_{N_1})\,(1 - f_N(E_{N_2}))}$$

How do we compute the emissivity from a star ?

- Emissivity computation for $\gamma N \to NX$

$$Q^{N\gamma\to NX}_{\text{cooling}} \equiv \int \frac{d^3p_\gamma g_\gamma f_\gamma(p_\gamma)}{(2\pi)^3 2E_\gamma} \int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1}(2\pi)^3 2E_{N_2}} \prod_{X_i} \int \frac{(2\pi)^4 d^3p_{X_i}}{(2\pi)^3 2E_{X_i}} \langle|\mathcal{M}|^2\rangle \left(\sum_i E_{X_i}\right)$$
$$f_N(E_{N_1})\,(1 - f_N(E_{N_2}))\delta(E_1 - E_2 - Q_0)\,\delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q})\,. \quad (1)$$

- How to compute:

$$\boxed{\int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1}(2\pi)^3 2E_{N_2}} f_N(E_{N_1})\,(1 - f_N(E_{N_2}))}$$

- Degenerate case: $(1 - f_N(E_{N_2})) \ll 1$: $\quad \xi \equiv \frac{p_F^2}{2\pi m_N T} \gg 1$

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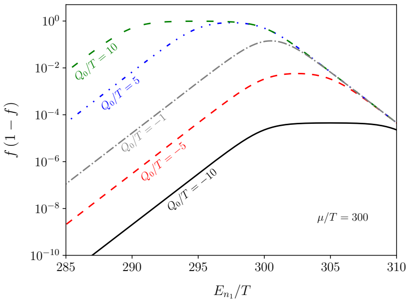
- Non-degenerate case: $(1-f_N(E_{N_2})) \to 1$: $\qquad \xi \ll 1$

Degenerate computation



$$Q^{2\to3} = \frac{64\, n_F}{4}\frac{g_\gamma}{3}\kappa^2 \int \frac{d^3p_\gamma}{(2\pi)^3}\frac{f_\gamma}{2E_\gamma}|\vec{p}_\gamma|^2 \int \frac{d^3p_1\, d^3p_2}{(2\pi)^6 2E_1\, 2E_2} E_1\, E_2\, (E_1+E_2)\, S(q^\mu)\ ,$$

$$S(Q_0, q) = \frac{M_N^2\, T}{\pi q}\frac{z}{1-e^{-z}}\Theta(\mu - E_-)\ .$$

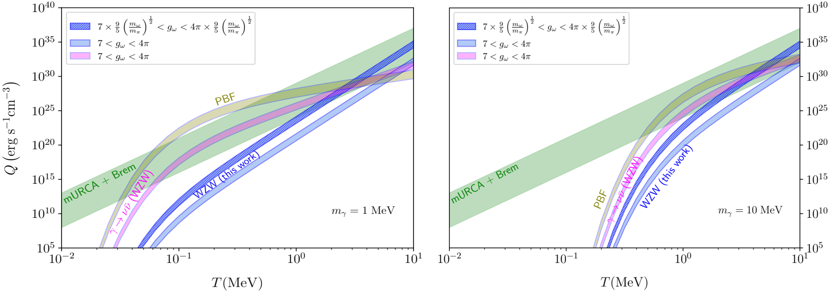
Standard cooling NS paradigm

- mURCA: $n\,n \longrightarrow n\,p\,e\,\bar{\nu}_e\ , \quad n\,p\,e \longrightarrow n\,n\,\nu_e$.:

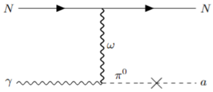
$$Q^{\mathrm{mURCA}} \simeq 10^{26-29}\left(\frac{T}{1\,\mathrm{MeV}}\right)^{8}\ \mathrm{erg\,s^{-1}\,cm^{-3}}$$

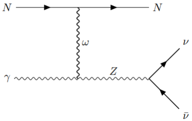
- Bremsstrahlung

$$Q^{\,\nu-\mathrm{Brem}} \simeq 10^{24-28}\left(\frac{T}{1\,\mathrm{MeV}}\right)^{8}\ \mathrm{erg\,s^{-1}\,cm^{-3}}$$



Conclusions and outlook





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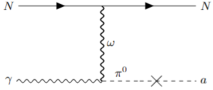

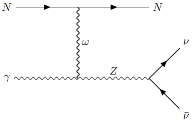

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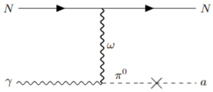

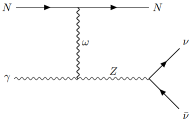

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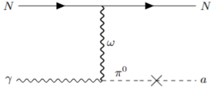

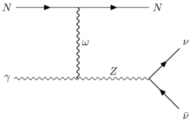

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- pheno of WZW: "Circulez, Y'a rien à voir"?

Better keep exploring: Harvey Hill Hill arxiv:0712.1230

$$\Gamma_{AAB} = \mathcal{C}\int dZZ\left[\frac{s_W^2}{c_W^2}\rho^0 + \left(\frac{3}{2c_W^2}-3\right)\omega - \frac{1}{2c_W^2}f\right] + dAZ\left[-\frac{s_W}{c_W}\rho^0 - \frac{3s_W}{c_W}\omega\right] + dZ\left[W^-\rho^+ + W^+\rho^-\right]\frac{s_W^2}{c_W}$$
$$-s_W dA\left[W^-\rho^+ + W^+\rho^-\right] + (DW^+W^- + DW^-W^+)\left[-\frac{3}{2}\omega - \frac{1}{2}f\right],$$

$$\Gamma_{ABB} = \mathcal{C}\int Z\Big\{d\rho^0\Big[-\frac{3}{2c_W}\omega - \frac{s_W^2}{c_W}a^0 + \left(-\frac{3}{2c_W}+3c_W\right)f\Big] + d\omega\Big[-\frac{3}{2c_W}\rho^0 + \left(-\frac{3}{2c_W}+3c_W\right)a^0 - \frac{s_W^2}{c_W}f\Big]$$
$$+da^0\left[\frac{s_W^2}{c_W}\rho^0 + \left(\frac{3}{2c_W}-3c_W\right)\omega - \frac{1}{2c_W}f\right] + df\Big[\left(\frac{3}{2c_W}-3c_W\right)\rho^0 + \frac{s_W^2}{c_W}\omega - \frac{1}{2c_W}a^0\Big]\Big\}$$
$$+s_W dA\left(\rho^0a^0 + 3\rho^0 f + 3\omega a^0 + \omega f + \rho^+a^- + \rho^-a^+\right) - \frac{s_W^2}{c_W}dZ\left(\rho^+a^- + \rho^-a^+\right)$$
$$+\frac{3}{2}\left[W^+D\rho^- + W^-D\rho^+)\right](-\omega+f) + \frac{3}{2}\left[W^+(-\rho^- + a^-) + W^-(-\rho^+ + a^+)\right]d\omega$$
$$+\frac{1}{2}\left[W^+Da^- + W^-Da^+\right](-3\omega - f) + \frac{1}{2}\left[W^+(-3\rho^- - a^-) + W^-(-3\rho^+ - a^+)\right]df,$$

$$\Gamma_{BBB} = \mathcal{C}\int 2\Big[\left(\rho^- f + \omega a^-\right)D\rho^+ + \left(\omega a^+ + \rho^+ f\right)D\rho^- + \left(\omega a^0 + \rho^0 f\right)d\rho^0 + \left(\rho^+a^- + \rho^-a^+ + \omega f + \rho^0a^0\right)d\omega\Big],$$

$$\Gamma_{AAAB} = \mathcal{C}\int iW^+W^-Z\left[3c_W\omega + \left(c_W + \frac{1}{2c_W}\right)f\right],$$

$$\Gamma_{AABB} = \mathcal{C}\int i\Big\{W^+W^-\left[\frac{3}{2}(\rho^0+a^0)\omega - \frac{1}{2}(\rho^0-a^0)f\right]$$
$$+W^+Z\Big[\left(\frac{3c_W}{2}-\frac{1}{c_W}\right)\rho^- f - \frac{3c_W}{2}\rho^-\omega - \frac{c_W}{2}a^- f + \frac{3c_W}{2}\omega a^-\Big]$$

Back up

In medium effects

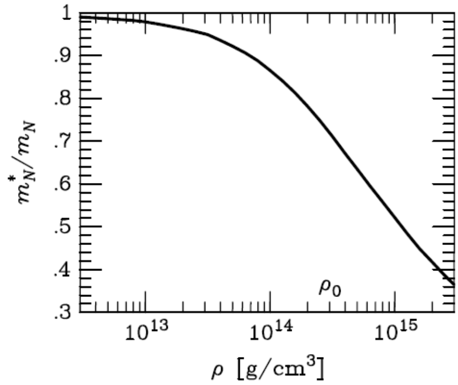

Figure: Credit: Raffelt

$\rho_{\rm SN} \sim$ few ρ_0

- The Brown-Rho scaling Brown, Rho 91'

$$\frac{f_\pi^\star}{f_\pi} \approx \frac{m_\omega^\star}{m_\omega} \approx \frac{m_N^\star}{m_N} \Rightarrow \frac{g_\omega^\star}{g_\omega} \approx \frac{g_\pi^\star}{g_\pi} \approx \frac{g_A^\star}{g_A}$$

g_A dependence: Voskresensky 01', Raffelt 01'

$$g_A^\star/g_A \approx 1/(1+(1/3)(m_N^\star/m_N)(\rho/\rho_0)^{1/3})$$

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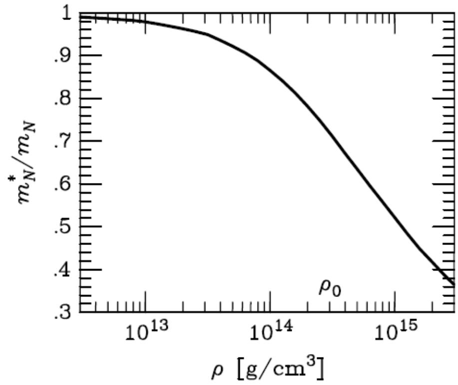

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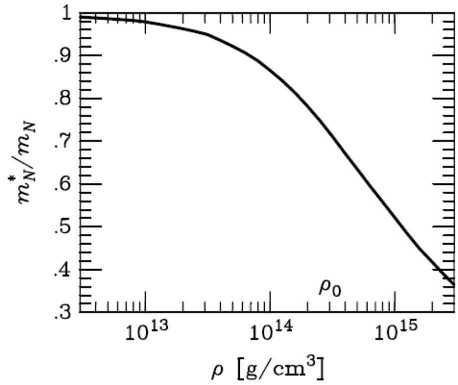

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$$\frac{Q^\star_{N\gamma\to Na}}{Q_{N\gamma\to Na}} \approx \frac{m_\omega^4}{g_\omega^4}\frac{g_{\omega\star}^4}{m_{\omega\star}^4} \in [0.8, 1.5] \quad \rho \in [0.5, 2]\rho_0$$

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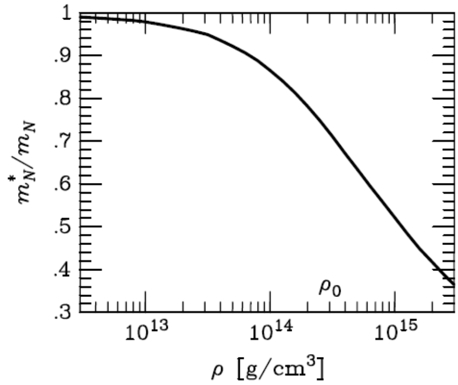

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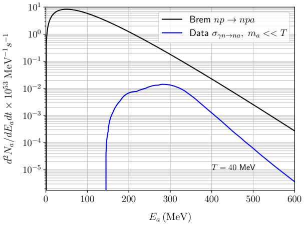
- For data-driven ??

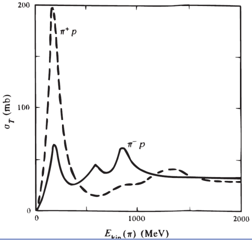
Can we observe the axions emitted from the supernovae

Bremsstrahlung peak: $E_a \sim 1.25T \sim 50 - 60$ MeV
Photo-production peak: $E_a \sim 6T \sim 250 - 300$ MeV

$$\frac{d^2 N_a}{dE_a dt} \approx C_f \rho_{15} \left(\frac{C_A 10^9}{f_a/\text{GeV}}\right)^2 g_{40}^4 \left(\frac{E_a}{\text{MeV}}\right)^6 e^{-E_a/T} ,$$

$C_f = 4.6 \times 10^{42}$ MeV^{-1}s^{-1}





$$\sigma_{ap^+ \to N\pi^+} \approx 10^{-25} C_A^2 (f_\pi / f_a)^2 cm^{-2}$$

$$\frac{dN_\pi}{dt} \approx 6\rho_{15} C_A^4 \left(\frac{10^9}{f_a/\text{GeV}}\right)^4 g_{40}^4 T_{40}^7 \times \left(\frac{K \ldots ol}{\ldots {}_2O} \right)$$