

SEARCHING FOR ULTRALIGHT DARK MATTER WITH GWS

Silvia Gasparotto IFAE

Based on arXiv:2409xxx
In collaboration with Diego
Blas and Rodrigo Vicente



Generalitat de Catalunya
**Departament de Recerca
i Universitats**



Agència
de Gestió
d'Ajuts
Universitaris
i de Recerca

Istanbul, 3rd September 2024

INTRODUCTION TO ULTRALIGHT DARK MATTER (ULDM)

Dark Matter (DM)

Number density: $n_{gal} = \frac{N}{V_{gal}} \sim \frac{M_{gal}}{m} \times \frac{1}{V_{gal}} \sim \frac{1}{m} \times \frac{10^{12} M_{\odot}}{(30 \text{ kpc})^3}$

De Broglie Wavelength: $\lambda_{db} \sim 0.5 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m} \right) \left(\frac{250 \text{ km s}^{-1}}{v} \right)$

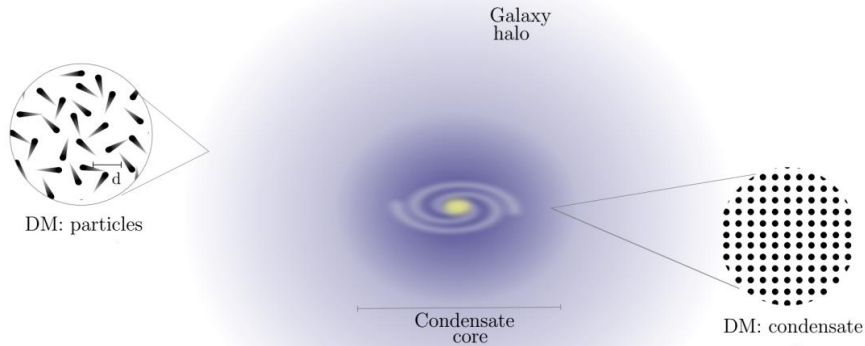
Occupation number : $\mathcal{N} = n \lambda_{db}^3 \sim 10^{92} \times \left(\frac{10^{-22} \text{ eV}}{m} \right)^4$

Given $\mathcal{N} \gg 1$ for $m \ll 0(10) \text{ eV}$ DM can be described by a classical field with

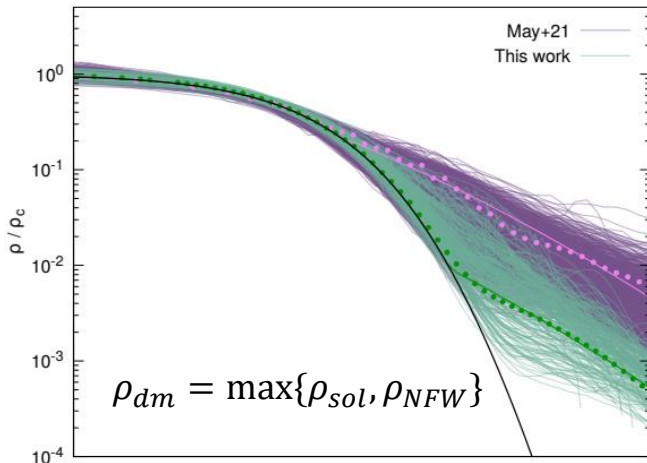
$$\text{EOM: } \square \phi + m^2 \phi = 0$$

Homogeneous solution are given by an oscillating field with frequency $\omega = m$

ULDM FIELD AT GALACTIC SCALE



Ferreira 2005.03254 & Chan 2110.11882



Parametrizing the space dependent part of the field $\phi = \frac{(\psi e^{imt} + \psi^* e^{-imt})}{\sqrt{2m}}$ and the metric

$$g_{\mu\nu} dx^\mu dx^\nu \approx -(1 - 2\Phi) dt^2 + (1 + 2\Psi) \delta_{ij} dx^i dx^j$$

Schrodinger-Poisson (SP) system of equations:

$$i \partial_t \psi = - \left\{ \frac{\nabla^2}{2m} + m \left[\langle \Phi \rangle + \frac{\rho_{dm}}{8 m^2 f^2} \right] \right\} \psi$$

$$\nabla^2 \langle \Phi \rangle = - 4 \pi (\rho_{dm} + \rho_m)$$

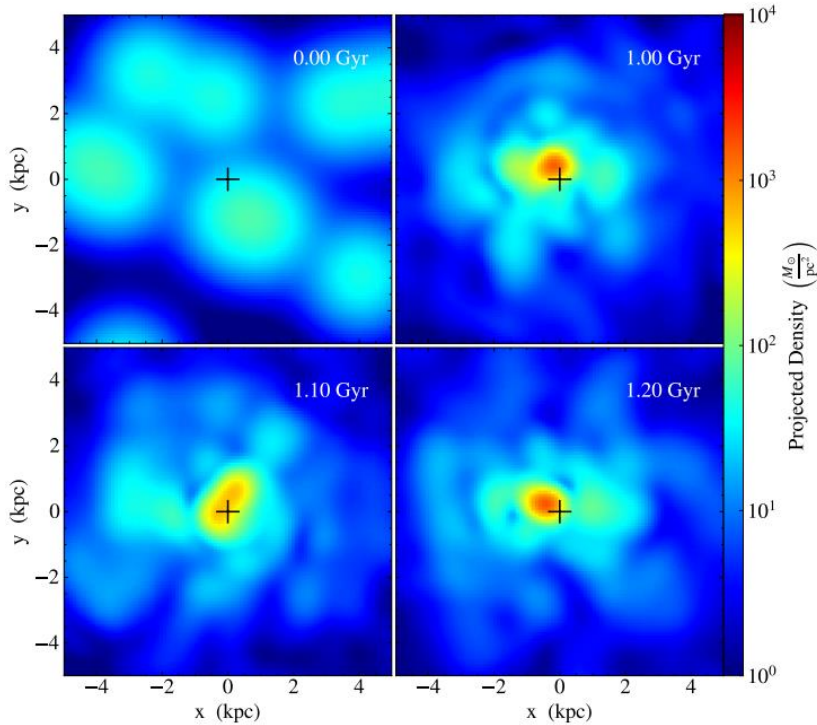
Numerical simulations show **always** the formation of a **dense coherent structure**, i.e. **soliton**, at the centre of the galactic halo.

The soliton is a stationary solution of the SP equations (non-relativistic limit) with density profile

$$\rho_{sol} = \frac{\rho_0}{\left(1 + 0.091 \left(\frac{r}{r_c}\right)^2\right)^8} \text{ with core radius}$$

$$r_c \sim 0.2 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m}\right)^2 \left(\frac{10^9 M_\odot}{M_{sol}}\right) \sim 0.4 \lambda_{db}$$

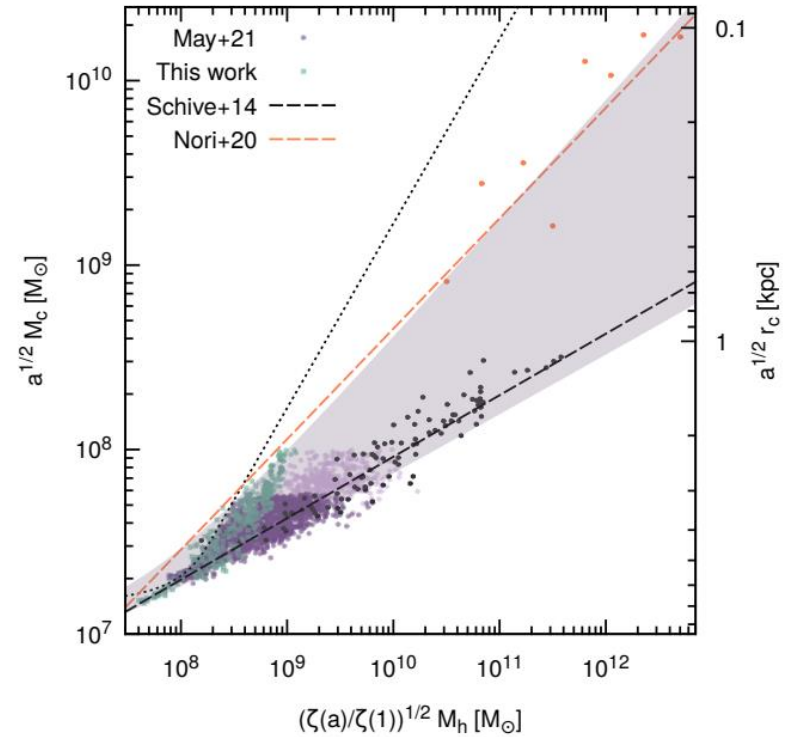
HALO AND SOLITON FORMATION



Hui 2101.11735

Different ideas to test this model \mapsto we focus on the effect of propagation of radiation in this DM environment

Ferreira 2005.03254



The mass of the soliton is related to the mass of the DM halo where it is formed. Schive 1407.7762

$$M_{sol} \approx 1.4 \times 10^9 \left(\frac{10^{-22} \text{ eV}}{m_{dm}} \right) \left(\frac{M_{\text{halo}}}{10^{12} M_{\odot}} \right)^{\frac{1}{3}}$$

But some dispersion is observed in the literature

GRAVITATIONAL REDSHIFT



Because of the inhomogeneities of the gravitational background along the line of sight a signal experiences gravitational redshift

$$\frac{\Delta\omega_e}{\omega_e} \simeq \Phi|_e^r + n^i v_i|_e^r - I_{ISW} \text{ where}$$

$$I_{ISW} = (\Phi + \Psi)|_e^r + n^i \int_e^r \partial_i(\Phi + \Psi) d\lambda$$

The DM background oscillates, then the gravitational potentials also oscillate.

Decomposing $\Psi = \langle \Psi \rangle + \delta\Psi \cos(\omega_\delta t)$ as well as for Φ , from Einstein equations one finds

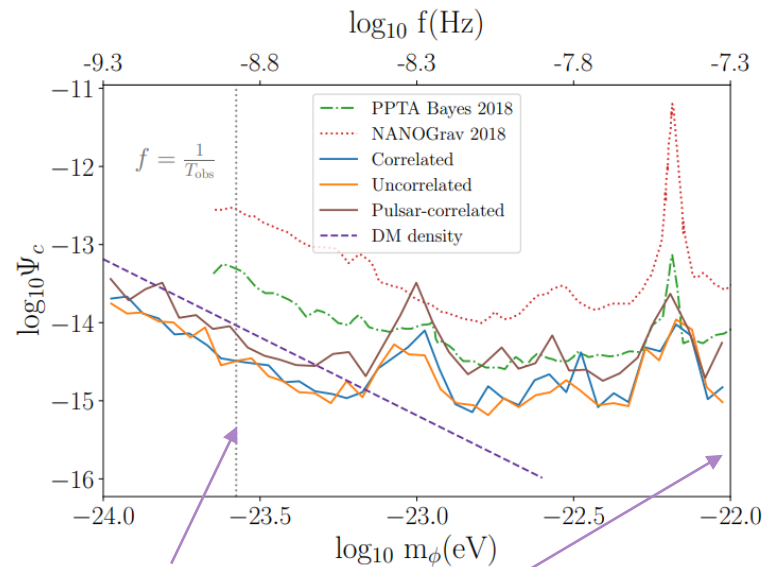
Khmel'nitsky &
Rubakov 1309.5888

$$\delta\Psi = -\frac{\pi\rho}{m^2} \text{ and } \omega_\delta = 2m$$

Periodic modulation in the time of arrival residuals of millisecond Pulsars

$$\Delta t \simeq -\int_0^t \frac{\Delta\omega_e(t')}{\omega_e} dt' \simeq -\int_0^t (\Psi_e - \Psi_r) dt'$$

Clemente et al. 2023, 2306.16228



$$f_{low} = \frac{1}{T_{obs}} \quad f_{high} = \frac{1}{\delta t_{obs}}$$

$$T_{obs} \sim 25 \text{ years} \quad \delta t_{obs} \sim 3 \text{ weeks}$$

CASE OF GRAVITATIONAL WAVES

Look at María José Bustamante-Rosell 2021

The same as for Pulsars will happen for any radiation at a fixed frequency $\omega_e \Rightarrow$ GW will experience frequency modulation. First, let's consider a monochromatic GW:

$$h_{GW} = A \cos(\omega_e u + \varphi) + A \frac{\omega_e}{\omega_\delta} \Upsilon|_e \sin[(\omega_e \pm \omega_\delta) u + \varphi]$$

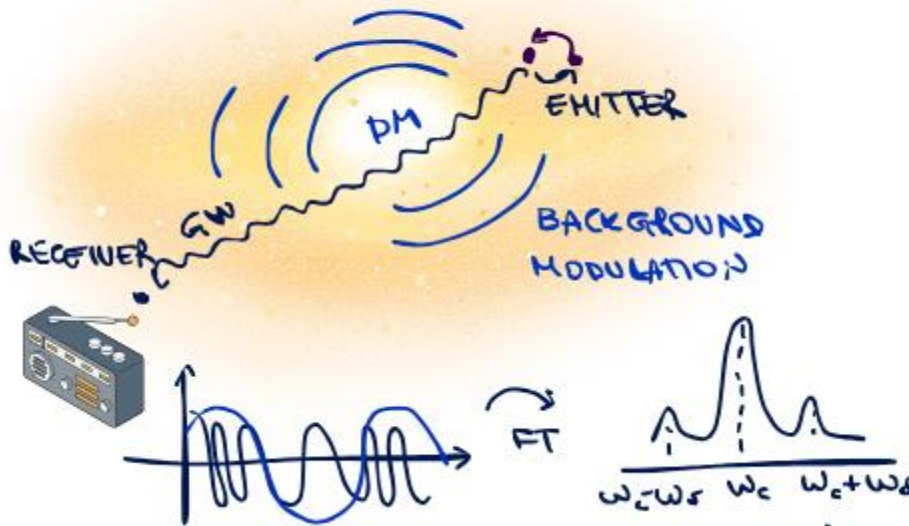
carrier frequency
modulation frequency

- GW emitters could come from inside the soliton (not contaminated by dust in the GC)
- Could be more abundant than Pulsars in PTA
- No limitation on observation time (higher frequency could be reached)
- Signal from other Galaxies

Signal-to-Noise-Ratio (SNR) of sidebands:

$$SNR_\delta = \frac{1}{\sqrt{2}} \frac{\omega_e}{\omega_\delta} \Upsilon(\rho_0, m, x_e) SNR_h$$

Amplitude of the modulation
of the carrier



AMPLITUDE OF THE MODULATION Υ

- Minimal coupling: pure gravitational interaction

With $\delta\Phi = \Phi_2 \cos(2mt)$ and $V = \frac{1}{2} m^2 \phi^2 (1 - \frac{1}{12} \frac{\phi^4}{f^4})$

Solutions are given by 2nd order Einstein equations

$$\nabla^2 \left[\Psi_2 + \frac{\pi\rho}{m^2} \right] = -\frac{\pi}{6f^2 m^2} \rho^2$$

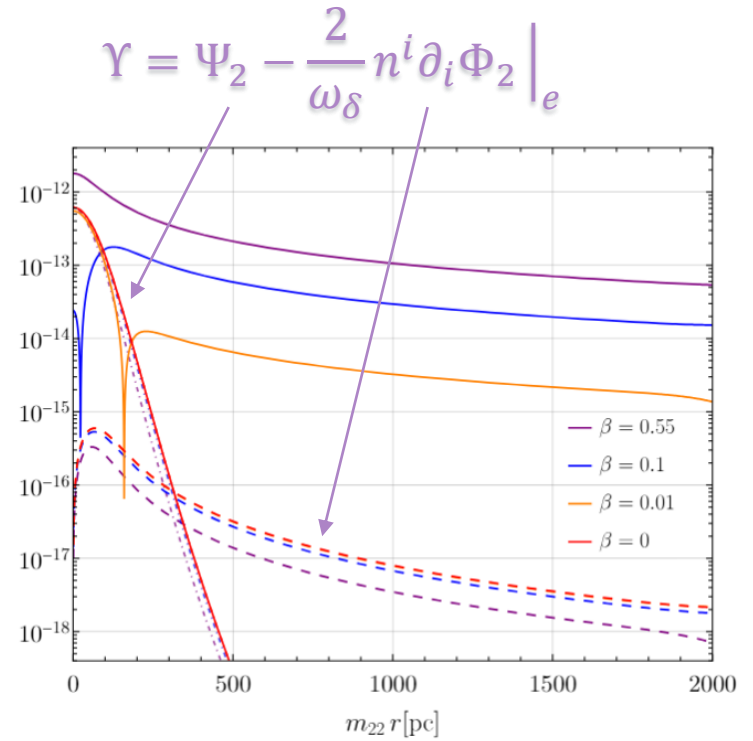
$$\nabla^2 \Phi_2 = 8\pi \left[5\langle\Phi\rangle + \gamma - \frac{\rho}{12f^2 m^2} \right] \rho$$

- Direct coupling: ULDM directly coupled to SM

(e.g. $m_\chi \frac{\phi}{\Lambda_1} \bar{\chi}\chi$ or $m_\chi \frac{\phi^2}{\Lambda_2^2} \bar{\chi}\chi$), under a conformal transformation to the Jordan-Fierz metric:

$$\widetilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}, \text{ with } A \simeq 1 + \frac{\phi}{\Lambda_1} \text{ or } A \simeq 1 + \frac{\phi^2}{\Lambda_2}$$

$$\Rightarrow \Upsilon = \frac{\sqrt{2}}{\Lambda_1} \left(\frac{\rho}{m^2} \right)^{1/2} |_e \text{ and } \Upsilon = \frac{1}{\Lambda_2^2} \left(\frac{\rho}{m^2} \right) |_e$$



$$\beta \simeq \frac{0.024}{m_{22}} \left(\frac{10^{16} \text{Gev}}{f_\phi} \right)^2 \sqrt{\frac{\rho_0}{10^3 M_\odot \text{pc}^{-3}}}$$

Self-interaction increases the effect at larger radius

RESULTS FOR THE MILKY WAY

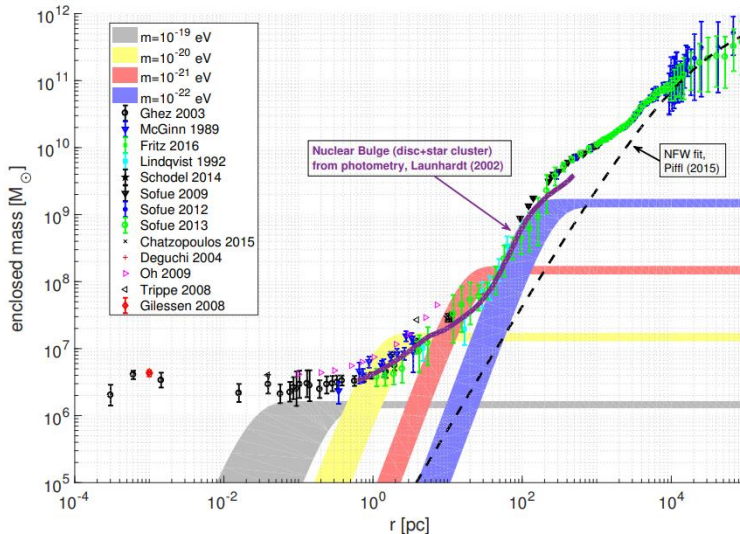
- Observational constraints:

kinematic data show a preference of a for an inner cored profile in the bulge with mass

$$M_{core} \leq 6 \times 10^9 M_{\odot} \text{ within } r_c \sim 0.25 \text{ kpc},$$

$$\text{average density of } \rho_0 \sim 10^3 M_{\odot} \text{ pc}^{-3}$$

compatible with a **ULDM soliton**



- Astrophysical populations of galactic monochromatic GW sources:

- **White Dwarfs/ X-MRIS** $\rightarrow f_e \sim \text{mHz}$
(detectable by LISA, TianQin, Taiji)

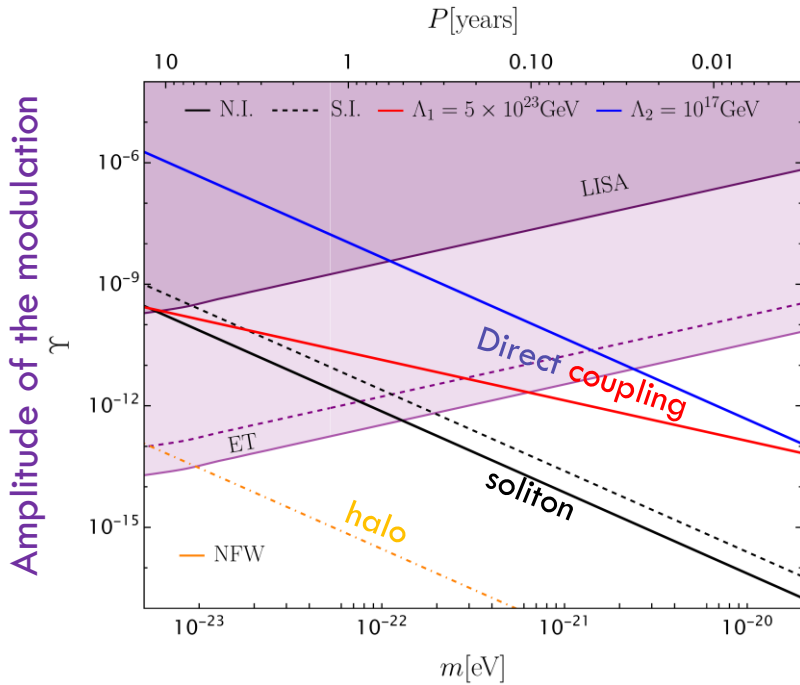
- **Deformed Spinning Neutron Stars** $\rightarrow f_e \sim \text{kHz}$
(detectable ET/CE)

Expected/Simulated values for the calculation of the sensitivity from different astrophysical populations (N carriers)

$$SNR_{\delta} = \frac{1}{\sqrt{2}} \frac{\omega_e}{\omega_{\delta}} \Upsilon(\rho_0, m, x_e) \sqrt{N} SNR_h$$

	N	$\langle SNR_h \rangle$	$\sqrt{N} \langle SNR_h \rangle \langle f_c \rangle [\text{Hz}]$	
<i>Double White Dwarfs</i>				
LISA	$5.5(1.6) \times 10^3$	37(38)	7.8(4.3)	
TianQin	$2.5(0.7) \times 10^3$	37(37)	5.1(2.9)	
Taiji	$5.8(1.7) \times 10^3$	59(60)	13(6.8)	
μAres	$504(148) \times 10^3$	49(48)	97(52)	
<i>X-MRIs</i>				
LISA	$\mathcal{O}(5)$	$\sim 10^3$	~ 10	1903.10871
<i>Spinning NSs</i>				
ET/CE	$\mathcal{O}(200)$	~ 30	$\sim 10^5$	2303.04714

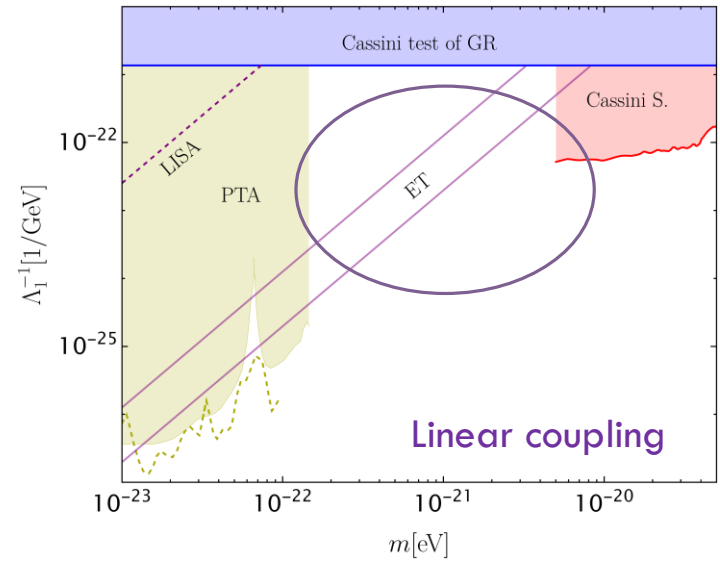
SENSITIVITIES TO ULDM IN THE MW



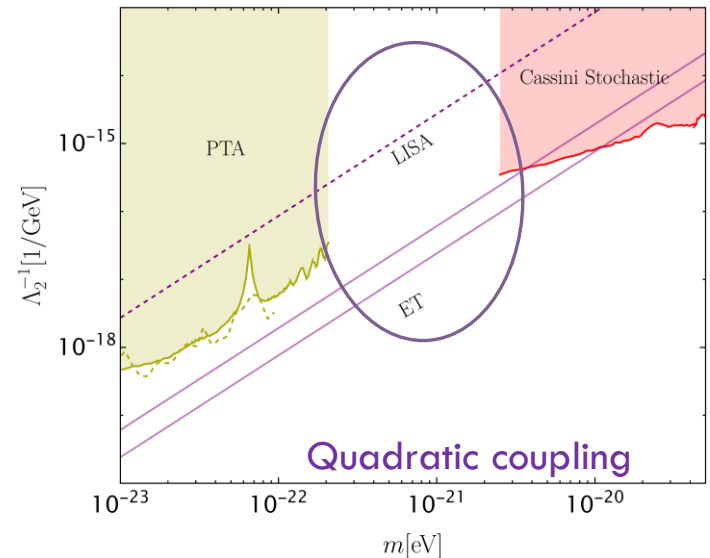
Spinning NS in ET outperform compared to LISA binaries

Not possible to probe NFW profile for $m > 10^{-23}$ eV

ET could probe the ULDM soliton for the minimal coupling for masses $m \leq 2 \times 10^{-22}$ eV



Great prospects for direct coupling: ET and LISA can probe the uncover window at masses $2 \times 10^{-22} \text{ eV} \leq m \leq 3 \times 10^{-21} \text{ eV}$



RESULTS FOR OTHER GALAXIES

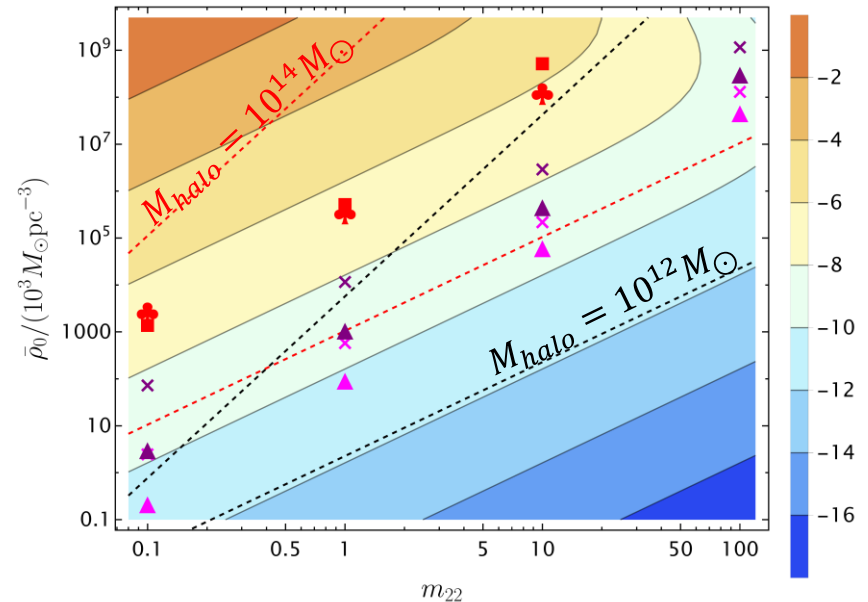
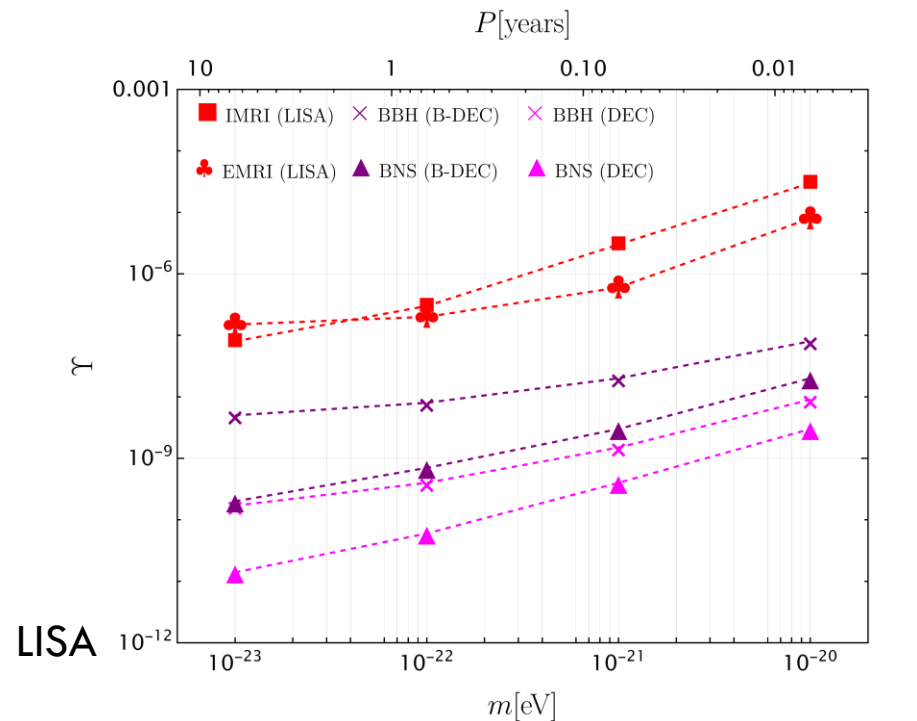
With GWs we can probe the ULDM environment of other galaxies (more massive soliton).

We consider four case-examples of binaries:

- EMRI: $(m_1, m_2) = (10^6 M_\odot, 60 M_\odot)$ at Gpc
 - IMRI: $(m_1, m_2) = (10^4 M_\odot, 10 M_\odot)$ at Gpc
 - BBH: GW170608-like event
 - BNS: GW170817-like event
- } (B-)DECIGO

These sources are chirping, the correction to the phase of the GW is more complicated and we cannot easily isolate its SNR \rightarrow we compute the uncertainty on Υ through a Fisher matrix and define the threshold when $\sigma_\Upsilon \leq \Upsilon$

Also look at J. Stegmann et al. 2023 2311.06335
and P. Brax et al 2024 2402.04819

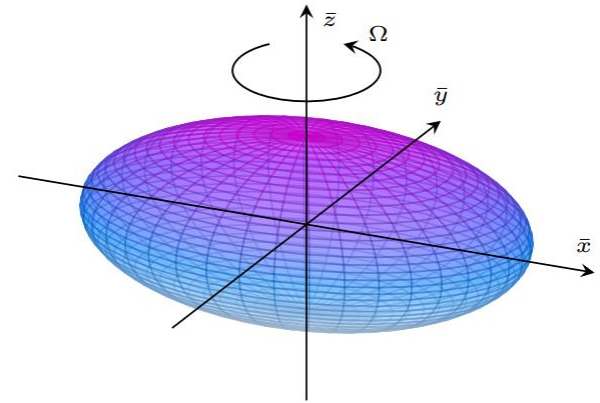


CONCLUSION AND FUTURE DIRECTIONS

- Imprint the **ULDM oscillations in the phase of GWs** (minimal and direct coupling)
- Extend previous computations to **solitons** and with **self-interaction**
- The SNR of the effect in terms of the carrier SNR, the amplitude Υ and the ratio ω_e/ω_δ
- Sensitivity of different astrophysical population for the MW \Rightarrow **Spinning NS detectable with ET/CE have optimal prospects**
- LISA sources could fill the gap in the $2 \times 10^{-22} \text{ eV} \leq m \leq 3 \times 10^{-21} \text{ eV}$ mass window
- Extragalactic (chirping) sources could probe **ULDM over densities in other Galaxies**

GWS FROM SPINNING NS

Reviews e.g Gittins 2401.01670,
Piccinni 2202.01088



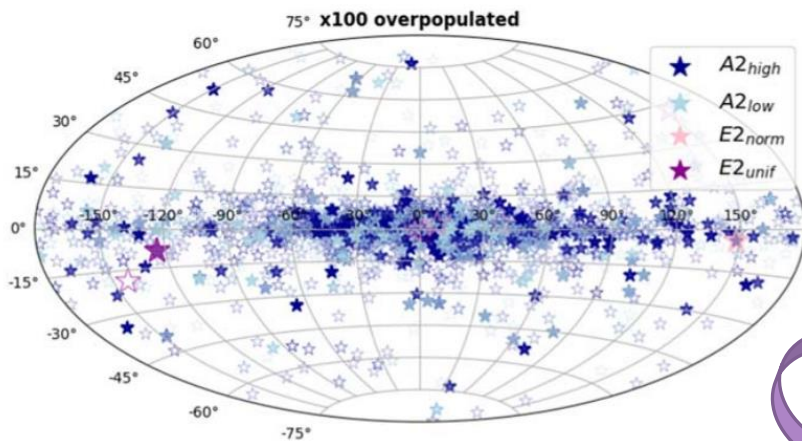
Rotating NS can support long-lived, non-axisymmetric deformations known as mountains \Rightarrow potential sources of continuous GW

$$h_0 = \frac{4G}{c^4} \frac{\epsilon I_3 \Omega^2}{d} \approx 10^{-25} \left(\frac{10 \text{ kpc}}{d} \right) \left(\frac{\epsilon}{10^{-6}} \right) \left(\frac{I_3}{10^{45} \text{ g cm}^2} \right) \left(\frac{\nu}{500 \text{ Hz}} \right)^2$$

Ellipticity parameter $\epsilon = (I_2 - I_1)/I_3$

Average number of detectable sources from 2303.04714

Model	\bar{n}	
	ET	CE
A2 _{low}	231.9 ± 14.6	338.1 ± 16.8
A2 _{high}	387.2 ± 19.4	524.3 ± 22.6
E2 _{norm}	0.5 ± 0.6	2.0 ± 1.4
E2 _{unif}	1.7 ± 1.3	5.2 ± 2.2



Great uncertainty on the detection prospects

CHIRPING CASE

- Gravitational redshift $\chi = \Phi|_e^r + n^i v_i|_e^r - I_{iSW}$
- Relative phase correction $\eta = \frac{\int \omega_e \chi}{\int \omega_e}$
- Quadrupolar result for the GW frequency

$$f_e = \frac{1}{\pi} \left(\frac{2GM}{c^3} \right)^{-\frac{5}{8}} \left(\frac{5}{256\tau} \right)^{3/8}$$

$$\eta_r(\tau_r) = -\frac{|\Upsilon|}{13} \left(13 {}_1F_2 \left(\frac{5}{16}; \frac{1}{2}, \frac{21}{16}; -\frac{1}{4} \tau^2 \omega_\delta^2 \right) \cos \Theta \right. \\ \left. + 5 \tau \omega_\delta {}_1F_2 \left(\frac{13}{16}; \frac{3}{2}, \frac{29}{16}; -\frac{1}{4} \tau^2 \omega_\delta^2 \right) \sin \Theta \right) + \Theta_c$$

