Coherent radiation of modulated positron bunch formed in crystalline undulator

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1. Theory of crystalline undulator radiation

The observed periodic *sinusoidal*-like bending of the crystallographic planes enables us to find an analytic formula for the crystalline undulator radiation (CUR) using Jacobi–Anger expansion. Hereby, under the assumption of preserved channeling, an average trajectory and, hence, an average motion of a charged particle along the bent crystallographic planes can be considered as

$$\boldsymbol{r}(t) = y(t)\boldsymbol{j} + z(t)\boldsymbol{k} = y_0 \cos\Omega t \boldsymbol{j} + c\beta_\parallel t\boldsymbol{k}, \tag{1a}$$

$$\boldsymbol{\beta}(t) = \beta_\perp(t)\boldsymbol{j} + \beta_\parallel(t)\boldsymbol{k} = -\beta_\perp \sin\Omega t\boldsymbol{j} + \beta_\parallel\boldsymbol{k}, \tag{1b}$$

where $\beta_\perp = \frac{x_0\Omega}{c}$ is the maximum value of the transverse velocity of a charged particle, $\beta_\parallel c$ is a longitudinal velocity of positrons, $\Omega = \frac{2\pi\beta_\parallel c}{l}$, where l is a spatial period of CU, y_0 is a CU bending amplitude, c is a velocity of light, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors along the transverse y and longitudinal z directions, respectively. There are two considerations in (1):

1. Radiation energy losses are negligible and are much less than the initial energy $\gamma m_e c^2$ of a charged particle, where γ is a Lorentz factor, m_e is a rest mass of the same.
2. Constancy of longitudinal velocity $\left\langle \beta_\parallel(t) \right\rangle = \beta_\parallel$. This leads to the expression $\gamma_\parallel = \frac{1}{\sqrt{1-\beta_\parallel^2}} = \frac{\gamma}{\sqrt{1+q^2/2}}$ for the longitudinal Lorentz factor, where the following formulas are used: $\beta^2 = \left\langle \beta_\parallel^2(t) \right\rangle + \left\langle \beta_\perp^2(t) \right\rangle$ and $\left\langle \beta_\perp^2(t) \right\rangle = \left\langle \beta_\perp^2 \sin^2\Omega t \right\rangle = \beta_\perp^2/2$. Thus, the longitudinal energy is less than the total energy by $\sqrt{1+q^2/2}$ times, where $q = \beta_\perp\gamma$ is called the undulator parameter.

The energy radiated per unit solid angle per unit frequency interval is

$$\frac{d^2W}{dOd\omega} = \frac{e^2\sqrt{\varepsilon}\omega^2}{4\pi^2 c}\left|\boldsymbol{A}(\omega)\right|^2, \quad \boldsymbol{A}(\omega) = \int_{-\infty}^{\infty} (\boldsymbol{n}\times\boldsymbol{\beta})\, e^{i(\omega\tau - \boldsymbol{k}\cdot\boldsymbol{r}(\tau))} d\tau, \tag{2a}$$

where $\boldsymbol{A}(\omega)$ characterizes the radiation field, $\boldsymbol{n}$ is a unit vector directed from the position of the charge to the observation point, representing direction of the instantaneous energy flux (Poynting's vector), $\boldsymbol{k} = \sqrt{\varepsilon}\frac{\omega}{c}\boldsymbol{n}$ is the wave vector of radiated photon with the frequency ω in the medium with dielectric permittivity ε, e is an elementary charge, c is a light velocity and τ is the retarded time.

In (2) in the sum $|\boldsymbol{A}|^2 = \sum_{m,m'=1}^{\infty} \boldsymbol{A}_m\boldsymbol{A}_{m'}^*$ remain only equiharmonic terms, since the interference terms with factors of different parities are imaginary and nullify $\boldsymbol{A}_m\boldsymbol{A}_{p+2k+1}^* + \boldsymbol{A}_m^*\boldsymbol{A}_{m+2k+1} = 0$, when the interference terms with factors of similar parities $\boldsymbol{A}_m\boldsymbol{A}_{m+2k}^* + \boldsymbol{A}_m^*\boldsymbol{A}_{m+2k}$ are negligible with respect to the equiharmonic terms at $x_m \approxeq 0$ and $n \gg 1$ according to $\left|\frac{\sin nx_{m+2k}}{x_{m+2k}}\right| \leq \frac{1}{2\pi k} \ll \frac{\sin nx_m}{x_m}$. Thus, formula (2) becomes

$$\frac{d^2W}{dOd\omega} = \frac{e^2\sqrt{\varepsilon}\omega^2}{c\Omega^2}\sum_{m=1}^{\infty}\left(\beta_\parallel^2\sin^2\vartheta - \frac{m}{\alpha_2}\beta_\parallel\beta_\perp\sin 2\vartheta\sin\varphi + \frac{m^2}{\alpha_2^2}\beta_\perp^2\left(1-\sin^2\vartheta\sin^2\varphi\right)\right)J_m^2(\alpha_2)\frac{\sin^2 nx_m}{x_m^2}. \tag{3}$$

2. Coherent radiation of modulated positron bunch

The formula for frequency-angular average distribution of the number of photons of any nature radiation, emitted by freeform positron bunch, has the following form

$$\begin{aligned} N_{tot}(\omega,\vartheta) &= N_{incoh}(\omega,\vartheta) + N_{coh}(\omega,\vartheta), \\ N_{incoh}(\omega,\vartheta) &= N_b(1-F)N_{ph}(\omega,\vartheta), \\ N_{coh}(\omega,\vartheta) &= N_b^2 F N_{ph}(\omega,\vartheta), \end{aligned} \tag{4}$$

where $N_{ph}(\omega,\vartheta)$ is the frequency-angular distribution of photon number for single positron radiation, N_b is the positron number of the bunch.

The bunch form-factor $F = F_Z(\omega,\vartheta)\times F_R(\omega,\vartheta)$ is determined with longitudinal $F_Z(\omega,\vartheta)$ and transverse $F_R(\omega,\vartheta)$ form-factors

$$F_Z(\omega,\vartheta) = \left|\int f(Z)e^{-i\frac{\omega}{c}cos\vartheta}dZ\right|^2, \quad F_R(\omega,\vartheta) = \left|\int f(R)e^{-i\frac{\omega}{c}sin\vartheta}dR\right|^2, \tag{5}$$

where $f(Z)$ and $f(R)$ are the positron bunch density distributions in longitudinal Z and transverse R directions accordingly.

2.1 Symmetric bunch

For the symmetric bunch, when $f(Z)$ and $f(R)$ are Gaussian distribution functions with dispersions σ_Z^2 and σ_R^2 accordingly, we have

$$F_Z(\omega,\vartheta) = \exp\left\{\left[-\left(\frac{\omega\sigma_Z cos\vartheta}{c}\right)^2\right]\right\}, \quad F_R(\omega,\vartheta) = \exp\left\{\left[-\left(\frac{\omega\sigma_R sin\vartheta}{c}\right)^2\right]\right\}. \tag{6}$$

Therefore form-factors at a zero angle are the following

$$F_Z(\omega) = \exp\left\{\left[-\left(\frac{\omega\sigma_Z}{c}\right)^2\right]\right\}, \qquad F_R = 1. \tag{7}$$

If we assume that the bunch density is modulated by law $b\times cos(\frac{\omega_r}{c}Z)$, where b is the modulation depth and ω_r is the modulation frequency. The longitudinal form-factor of modulated bunch has the following form

$$F_Z(\omega) = \frac{b^2}{4}\exp\left\{\left[-\left(\frac{\omega_r\sigma_Z}{c}\right)^2\left(\frac{\omega-\omega_r}{\omega_r}\right)^2\right]\right\}, \tag{8}$$

where ω_r is the resonant frequency.

The relative linewidth of $F_Z(\omega)$ is equal to $2\Delta\omega/\omega_r = \lambda_r/(\pi\sigma_Z)$, where λ_r is the resonant wavelength. In the case $\lambda_r \ll \pi\sigma_Z/n$, the linewidth of coherent radiation coincides to the linewidth of $F_Z(\omega)$ due to the linewidth of diffraction sine much more. For the number frequency distribution of radiated photons formed in CU is obtained the following formula

$$N_{coh}(\omega) = N_{ph}(\omega_r)\frac{b^2}{4}N_b^2 e^{-(\frac{\sigma_Z}{c})^2(\omega-\omega_r)^2}. \tag{9}$$

For the total number of the coherently radiated photons, formed over the length $L = n\times l$, we have

$$N_{coh} = \frac{\sqrt{\pi}\alpha}{2\sigma_Z\lambda_r}\left(\frac{qLN_b}{4\gamma}\right)^2 b^2. \tag{10}$$

The photon beam energy spread is equal to $\delta_{coh} = \lambda_r/(2\pi\sigma_Z)$.

The number of incoherent radiated photons with the same energy spread is

$$N_{incoh} = \frac{\alpha N_b}{\sigma_Z\lambda_r}\left(\frac{qL}{2\gamma}\right)^2. \tag{11}$$

The ratio between the coherent and incoherent parts of the number of radiated photons is

$$K = \frac{\sqrt{\pi}}{8}N_b b^2. \tag{12}$$

When the condition $b \gg \frac{2}{\sqrt{N_b}}$ takes place we have $N_{coh} \gg N_{incoh}$.

3. Numerical results and discussion

Numerical results for the case when the modulated positron bunch falls perpendicular to the CU-section are shown below. Numerical calculations are performed for diamond and the modulated positron bunch, for which the parameters of LCLS bunch modulated in SASE FEL (self-amplified spontaneous emission) process are used.

The modulated positron bunch parameters are: $N_b = 1.56\times10^9$, $E = 13.6$ GeV $(\gamma = 2.66\times10^4)$, $\hbar\omega_r = 8.3$ KeV ($\lambda_r = 1.5$ Å), $\sigma_Z = 9\times10^{-4}$ cm.

Positrons are channeled between the (1 1 0) crystallographic planes of a diamond crystal with interplanar distance $d = 1.26$ Å. Then the depth of potential well is $U_0 = 24.9$ eV ($\nu = U_0/mc^2 = 4.87\times10^{-5}$), and the energy of plasma oscillations of the diamond medium is $\hbar\omega_p = 38$ eV ($\lambda_p = 3.26\times10^{-6}$ cm). CU represents the periodically bent monocrystal of diamond with the space period $l = 1.4\times10^{-3}$ cm ($\lambda_r = \lambda_+ = 2\lambda_p^2/l = 1.5$ Å) and with the amplitude $A = 1.27$ Å $(q = 1.5, Q = 2.125)$. The condition $q < q_{ch}$ is satisfied since $q_{ch} = \sqrt{2\nu\gamma} = 1.62$.

Then on the interacted length $L = 0.14$ cm ($n = 10^2$, $L \leq L_D \approx 0.14$ cm) will be generated $4.6\times10^{15}b^2$ photons with the energy 8.3 KeV. The relative linewidth of coherent radiated photons' beam is 5.3×10^{-6} (energy spread is equal to 0.022 eV). The incoherent radiated photon number with the same linewidth is 1.3×10^7.

The total number of incoherent radiated photons with the natural linewidth 2×10^{-4} (with 41.5 eV energy spread) is equal to 2.5×10^{10}. The coherent part of radiation is predominant if the condition $b \gg 10^{-4}$ takes place and one can determine the important parameter b (modulation depth).

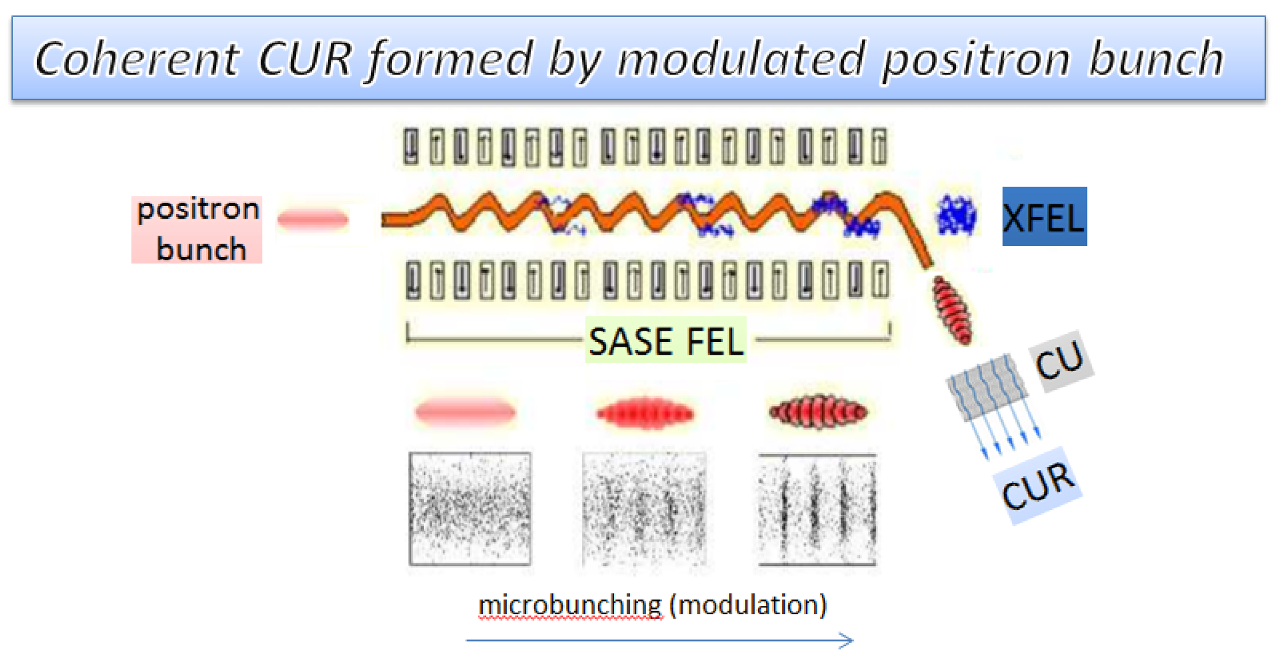

Figure 1. Scheme of the experiment to obtain the intense, super-monochromatic, directed X-ray photon beam

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