Gain coefficient of stimulated radiation in a system of two undulators

Anahit H. Shamamian[1 a 2 b], **Hayk L. Gevorgyan**[1 c d], **and Lekdar A. Gevorgian**[1 a]

[1] Alikhanyan National Laboratory

[2] Military Academy after Vazgen Sargsyan MoD RA

hayk.gevorgyan@aanl.am

[a]Matinyan Center for Theoretical Physics, Alikhanyan National Laboratory (Yerevan Physics Institute), 2 Alikhanyan Brothers St., 0036 Yerevan, Armenia
[b]Department of Exact Subjects, Military Academy after Vazgen Sargsyan MoD RA, 155 Davit Bek St., 0090 Yerevan, Armenia
[c]Experimental Physics Division, Alikhanyan National Laboratory (Yerevan Physics Institute), 2 Alikhanyan Brothers St., 0036 Yerevan, Armenia
[d]Quantum Technologies Division, Alikhanyan National Laboratory (Yerevan Physics Institute), 2 Alikhanyan Brothers St., 0036 Yerevan, Armenia

1. Introduction

In 1977, Vinokurov and Skrinsky proposed a modification of the Free Electron Laser (FEL), specifically the optical klystron (OK), which consists of a system of two undulators separated by a gap [1]. Due to the constructive interference of the radiation fields generated in the individual undulators, the spontaneous radiation is enhanced.
This study investigates a system comprising two identical helical undulators separated by a gap. The trajectories of relativistic particles and the resultant interference field from the two helical undulators are calculated within the system. A formula for the frequency-angular distribution of spontaneous radiation is derived. At a zero angle, the spontaneous radiation line shape is found to be narrower compared to the line shape of the spontaneous radiation spectrum generated in a single undulator without a gap. According to Madey's theory [2], the gain coefficient of stimulated radiation depends on the derivative of the line shape; therefore, an increase in the gain coefficient is observed. An expression for the gain coefficient is obtained.
In the studies [3], X-ray radiation generated by a relativistic charged particle in a crystalline undulator with sections (CUS) was investigated. CUS consists of a system of monocrystals with specified lengths and curvature radii, symmetrically arranged at certain distances from each other. The gain coefficient calculated in the present work corresponds to the values of gain coefficients reported in [3]. For practical applications of the observed effect, it is necessary to determine the optimal values of OK parameter that characterizes the distance between the undulators.

2. Radiation field in a system of two undulators

Consider the motion of a relativistic electron in a system consisting of two identical helical undulators, each with a length L, separated by a gap of sL (where $s > 0$). Within the undulators, the particle follows a helical trajectory with a longitudinal velocity $\beta_{\parallel}(t)c$ along the axis of a helical undulator OZ and a transverse velocity $\beta_{\perp}(t)c$, where c is the speed of light in vacuum. In the gap between the undulators, the particle moves rectilinearly and uniformly.
In the magnetic field of a helical undulator, the particle moves with a velocity given by:

$$\boldsymbol{\beta}(t) = \left\{\beta_{\perp}\sin(\Omega t), -\beta_{\perp}\cos(\Omega t), \beta_{\parallel}(t)\right\}, \tag{1}$$

where $\Omega = \frac{2\pi\beta_{\parallel}c}{l}$ is the angular frequency of rotation, and l is the spatial period of the undulator.
To a good approximation, neglecting small losses due to radiation primarily caused by transverse motion, the particle's energy remains constant. Therefore, we assume that the particle moves with a constant longitudinal velocity, given by $\beta_{\parallel} = \sqrt{\langle\beta_{\parallel}(t)^2\rangle} = \sqrt{\beta^2 - \beta_{\perp}^2}$, where β is the initial velocity of the particle.
The trajectory of the electron follows a helical path with a radius $R = \frac{\beta_{\perp}c}{\Omega}$ given by

$$\boldsymbol{r}(t) = \begin{cases} \left\{-R\cos(\Omega t), -R\sin(\Omega t), \beta_{\parallel}ct\right\} & \text{for } 0 < t < \frac{L}{\beta_{\parallel}c},\ \frac{L(s+1)}{\beta_{\parallel}c} < t < \frac{L(s+2)}{\beta_{\parallel}c}, \\ \left\{0, 0, \beta_{\parallel}ct\right\} & \text{for } \frac{L}{\beta_{\parallel}c} < t < \frac{L(s+1)}{\beta_{\parallel}c}. \end{cases} \tag{2}$$

The process of photon radiation in undulators is described with sufficient accuracy by classical theory. The radiation fields in the first and second undulators are determined using the following integrals

$$\boldsymbol{I}_1 = \int_0^{\tau} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\left[\boldsymbol{n}\times\boldsymbol{\beta}(t)\right]dt, \tag{3a}$$

$$\boldsymbol{I}_2 = \int_{(s+1)\tau}^{(s+2)\tau} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\left[\boldsymbol{n}\times\boldsymbol{\beta}(t)\right]dt, \tag{3b}$$

where $\boldsymbol{\beta}(t) = \frac{\boldsymbol{v}(t)}{c}$, $\boldsymbol{v}(t) = \frac{d\boldsymbol{r}(t)}{t}$ is the velocity of electrons and $\tau = \frac{L}{\beta_{\parallel}c}$ is their time of flight through the undulator.
Thus, considering expressions, the vector $\boldsymbol{I}$ that defines the total radiation field produced by an electron in a system of undulators separated by a gap is given by

$$\begin{aligned}\boldsymbol{I} = \boldsymbol{I}_1 + \boldsymbol{I}_2 = \frac{2L}{\beta_{\parallel}c}\sum_{p=-\infty}^{\infty}\Bigg(J_p(\xi)\left(\beta_{\parallel}\sin\theta - \frac{pR\Omega}{c\xi}\cos\theta\right)(\mathbf{i}\sin\varphi - \boldsymbol{j}\cos\varphi) - \\ -i\frac{dJ_p(\xi)}{d\xi}\frac{R\Omega}{c}\left(\boldsymbol{i}\cos\varphi\cos\theta + \boldsymbol{j}\sin\varphi\cos\theta + \boldsymbol{k}\sin\theta\right)\Bigg)\cdot e^{i(z(s+2)+p(\phi+\pi/2))}\cdot\frac{\sin z}{z}.\end{aligned} \tag{4}$$

Note that the expression (4) accounts for the absence of radiation in the time intervals

$$-\infty < t < 0, \quad \tau < t < (s+1)\tau, \quad (s+2)\tau < t < +\infty. \tag{5}$$

3. Frequency-angular distribution of radiated photons

For the frequency-angular distribution of the number of radiated photons in a system of two undulators separated by a gap, we have

$$\frac{d^2N_{\text{ph}}}{d\omega d\Omega} = \frac{\alpha\omega}{4\pi^2}|\boldsymbol{I}|^2, \quad \boldsymbol{I} = \boldsymbol{I}_1 + \boldsymbol{I}_2, \tag{6}$$

where $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$ is the fine-structure constant, $dO = \sin\theta d\theta d\varphi$ is the solid angle of radiation at the frequency ω, e is the electron charge, and $\hbar$ is the reduced Planck constant.

4. Gain coefficient of stimulated radiation generated in the undulator with a gap

The gain coefficient of radiation is given by

$$g(x) = \frac{\delta N_{\text{st}}}{N_{\text{sp}}} = 4\pi\lambda^2\rho_e\delta N_{\text{sp}}(x,u) = 32\pi^3\frac{\alpha\lambda_c Q\rho_e\lambda n^3 q^2 x^2}{\gamma}\left(-\frac{dF(z)}{dz}\right). \tag{7}$$

The maximum value of the derivative of the spontaneous radiation line shape takes place at $z_{max} \approx 1$ and is approximately 0.6 (more precisely 0.54).
For the gain coefficient of stimulated radiation in an SASE FEL (Self-Amplified Spontaneous Emission Free Electron Laser) at a wavelength $\lambda = 1.5\times10^{-8}\text{cm}$ ($\hbar\omega = 8.3\text{keV}$) taking into account the parameters of the LCLS electron bunch (Linac Coherent Light Source) — $N_e = 1.56\times10^9$, $\sigma_{\parallel} = 9\times10^{-4}\text{cm}$, $\gamma = 2.66\times10^4$ ($E = 13.6\text{GeV}$), and undulator parameters $q = 3.5$ and $l = 3$, we have

$$g_{\text{max}} = 6.04\times10^{-7}L^3. \tag{8}$$

Note that for the undulator with length $L \gtrsim 100\text{cm}$, the spontaneous undulator radiation increases ($g_{\text{max}} \gtrsim 1$). For SASE FEL, radiation generation starts if the interaction length exceeds 1m. The gain coefficient for stimulated radiation (7), defined as the relative increase in radiation intensity due to one passage of the electron through the undulator system per unit length, depends on the derivative of the spontaneous radiation line shape $F_s(z_p)$ (or $f_s(Z_p)$,

$$\begin{aligned} F_s(z_p) &= \cos^2\left((s+1)z_p\right)F(z_p), \quad F(z_p) = \frac{\sin^2 z_p}{z_p^2}, \\ f_s(Z_p) &= \frac{\cos^2\left((s+1)Z_p/2\right)}{\cos^2(Z_p/2)}f(Z_p), \quad f(Z_p) = \frac{\sin^2 Z_p}{Z_p^2} \end{aligned} \tag{9}$$

specifically, the function $\frac{dF_s(z_p)}{dz_p}$.
The gain coefficient for stimulated radiation for a system of undulators is transformed as follows

$$g = 8\pi^2\frac{r_0 q^2 n^2 L\rho_e}{\gamma^3}g(Z_p), \quad g(Z_p) = \left(-\frac{df_s(Z_p)}{dZ_p}\right) = G(z_p) = \left(-\frac{dF_s(z_p)}{dz_p}\right). \tag{10}$$

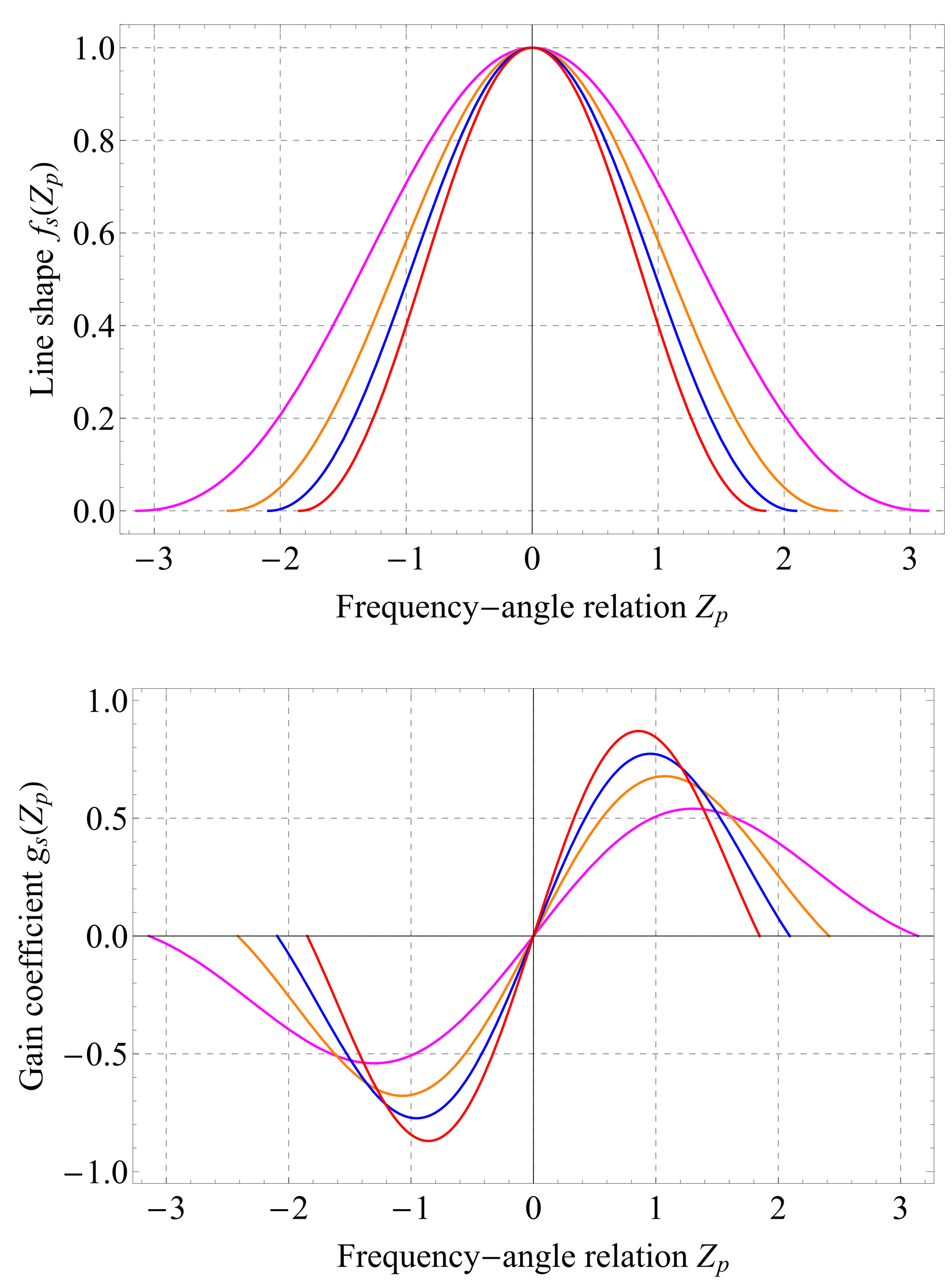

Figure 1: Line shape (top) and gain coefficient (bottom) versus Z_p representing the frequency angular relation for the gaps $s = 0$ (Magenta), $s = 0.3$ (Orange), $s = 0.5$ (Blue), $s = 0.7$ (Red) between two helical undulators.

References

[1] N.A. Vinokurov and A.N. Skrinsky, Budker Institute for Nuclear Physics Report No. BINP 77–59, 1977; Budker Institute for Nuclear Physics Report No. BINP 77–67, 1977; N.A. Vinokurov, *Moshchnye lazery na svobodnykh élektronakh na osnove opticheskogo klistrona*, Doctoral dissertation, Budker Institute for Nuclear Physics, 1995.

[2] J.M. Madey, Nuovo Cim., B; (Italy), **50**, 64 (1979).

[3] L.A. Gevorgian, R.O. Avakian, K.A. Ispirian, and A.H. Shamamian. Nucl. Instr. and Meth. B **227**, 104 (2005); H.L. Gevorgian, L.A. Gevorgian, and A.H. Shamamian, J. Contemp. Phys. **56**, 159 (2021).

The 10th International Conference "Charged & Neutral Particles Channeling Phenomena", Riccione (RN) (Italy), 8-13 September 2024