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# VOLUME REFLECTION UNDULATOR

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# Introduction

- Crystalline undulator based on channeling effect was proposed in [1,2]. Experiments on 10 GeV positron [3] beam and 855 electron beam [4,5] were performed. The idea of the volume reflection undulator was briefly proposed in ref. [6]. The main advantage of the VRU is that it can work with both positrons and electrons
  - Here we consider some properties of the volume reflection undulator. The expression for the undulator parameter as a function of the incident electron energy is derived. The frequencies of radiation emitted from the volume reflection undulator are considered. The threshold character of the radiation is discussed. This project has received funding through the MSCA4Ukraine project, which is funded by the European Union.
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  - [2] V.G. Baryshevsky, I.Ya. Dubovskaya, A.O. Grubich. Generation of gamma-quanta by channeling particles in the presence of a variable external field. Phys. Lett. A 77 (1980) 61-64.
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# Two schemes of the volume reflection undulator

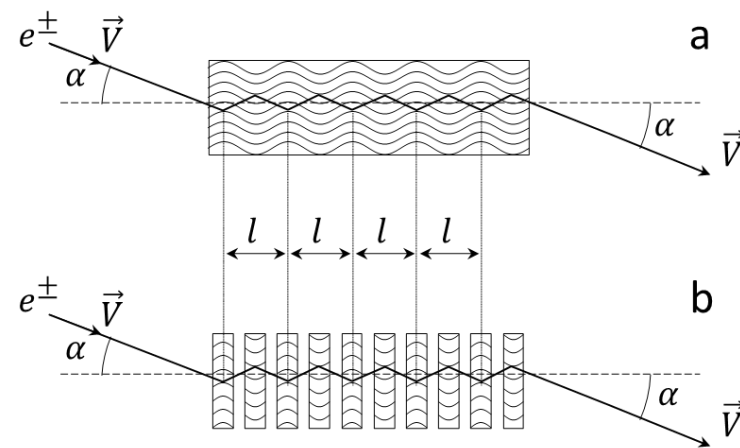


Fig.1. Two schemes of the crystalline undulators based on the volume reflection effect. Bent crystallographic planes are shown by curves. The layered volume reflection undulator is shown in Fig. 1a and the solid state volume reflection undulator is shown in Fig. 1b. The electron  $e^-$  or positron  $e^+$  moves with velocity  $\vec{V}$  in almost straight lines between the reflecting points at angle  $\alpha$  with respect to the undulator axis shown by dashed line.

### Undulator parameter in volume reflection undulator

One of the key characteristics of an undulator device is the undulator parameter  $K$ . The undulator parameter  $K$  can be estimated as the relation of the maximum angle  $\alpha$  between the undulator axis and the electron velocity vector crossing the axis to the value of the inverse relativistic factor of the electron beam  $\gamma^{-1}$

$$K = \alpha\gamma. \quad (1)$$

The undulator device works in the undulator regime and emits coherent quasi-monochromatic radiation if  $K < 1$  and the undulator device works in the wiggler regime and emits incoherent radiation if  $K > 1$ . Below we will believe that the particle moves in almost straight lines between reflecting points and angle  $\alpha = \text{const}$  in the both versions of the volume reflection undulator shown in Fig. 1.

The angle  $\alpha$  in the volume reflection effect is close to the Lindhard angle for both positively and negatively charged particle, i.e. electrons and positrons. The Lindhard angle  $\theta_L$  is

$$\theta_L = \sqrt{\frac{2U}{pV}}, \quad (2)$$

where  $U$  is the depth of the potential well between crystallographic planes,  $p$  and  $V$  are momentum and velocity of the incident particle respectively. The angle  $\alpha$  is

$$\alpha = \xi\theta_L, \quad (3)$$

where the coefficient  $\xi \approx 1$ . Inserting (2,3) to (1), we obtain

$$K = \xi \sqrt{\frac{2U\gamma}{mV^2}}, \quad (4)$$

One can see from Eq. (4) that the undulator parameter monotonically increases on the energy of incident relativistic particle. Equating (1) to unity, we obtain the full particle energy  $E_u$  at which  $K = 1$

$$E_u = \frac{(mVc)^2}{2U\xi^2}, \quad (5)$$

where  $m$  is electron mass,  $V \rightarrow c$  for ultra-relativistic particles. At typical values  $U \approx 20$  eV between crystallographic planes (111) and (220) in Si single-crystal [5],  $E_u \approx 6.25$  GeV for  $\xi = 1$ . This means that the undulator device based on (111) or (220) bent crystallographic planes of silicon singlecrystal should work in the undulator regime and emit quasi-monochromatic radiation at the incident electron or positron beam energy below 6.25 GeV or in the wiggler regime at the incident particle beam energy exceeding 6.25 GeV. The behavior of the angles  $\gamma^{-1}$ , and  $\alpha$ , and the undulator parameter  $K$  as functions of the incident electron or positron energy  $E$  at  $\xi = 1$  is shown in Fig. 2.

# Undulator parameter K

$$K = \xi \sqrt{\frac{2U\gamma}{mV^2}}$$

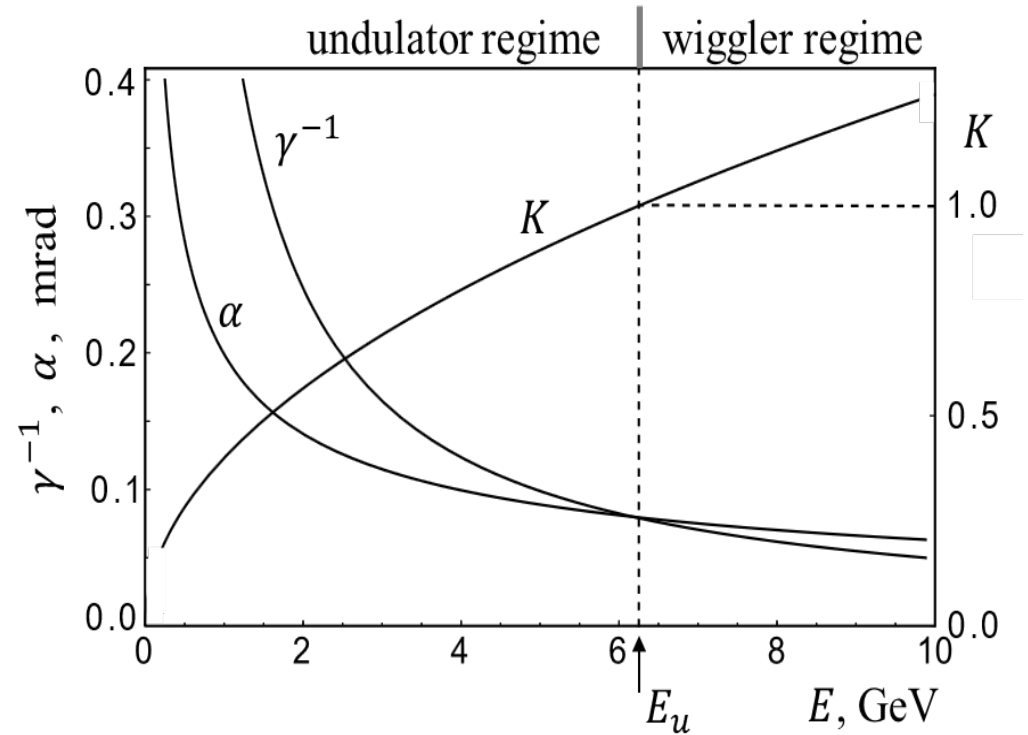
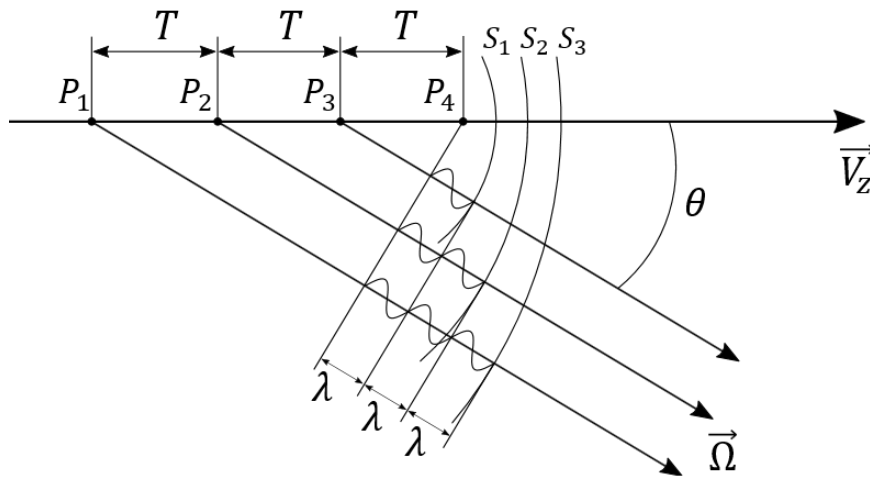


Fig. 2. Angles  $\gamma^{-1}$ ,  $\alpha$ , and the undulator parameter  $K$  as functions of the full electron or positron energy  $E$  at  $U = 20$  eV and  $\xi = 1$ . Calculations were performed by formulae (2-5).

# What frequencies of radiation are possible from a periodic structure?

The equation for radiation frequency was obtained from the Huygens construction. This cinematic consideration of coherent radiation is independent of the mechanism of the radiation. The mechanism may be, e.g. coherent transition radiation or coherent bremsstrahlung.



$$\hbar\omega \ll E \quad \varepsilon = 1 - \left( \frac{\omega_p}{\omega} \right)^2$$

$$\omega = \frac{2\pi}{T} \frac{V_z}{1 - \frac{\sqrt{\varepsilon}}{c} V_z \cos \theta}$$

Equation for frequency  
by Ter-Mikaelian

This is quadratic equation

The solution of the quadratic equation (9,10) is

$$\omega = \frac{\omega_0}{2} \pm \sqrt{\left(\frac{\omega_0}{2}\right)^2 - \frac{(\gamma\omega_p)^2}{1+K^2+(\gamma\theta)^2}}. \quad (11)$$

where

$$\omega_0 = \frac{4\pi nc}{l} \gamma^2 \frac{1}{[1+K^2+(\gamma\theta)^2]} \quad (12)$$

is the solution of the of Eq. (10) if the term  $(\gamma\omega_p)^2$  is neglected.

$$\gamma_{thr} = \frac{\frac{\xi^2 U}{2mc^2} + \sqrt{\left(\frac{\xi^2 U}{2mc^2}\right)^2 + 4 \left[\frac{2\pi nc}{l\omega_p}\right]^2 - \theta^2}}{2 \left[\left(\frac{2\pi nc}{l\omega_p}\right)^2 - \theta^2\right]}$$

The asymptote of the low-frequency decision  $\omega_1^{as}$  can be found from (11)

$$\omega_1^{as} = \frac{l\omega_p^2}{4\pi nc}. \quad (16)$$

The asymptote (16) is independent of the relativistic factor and harmonics frequencies are inversely proportional to the harmonics number. The asymptote of the high-frequency decision  $\omega_2^{as}$  is

$$\omega_2^{as} = \omega_0 - \omega_1^{as} = \frac{4\pi nc}{l} \gamma^2 \frac{1}{[1+K^2+(\gamma\theta)^2]} - \frac{l\omega_p^2}{4\pi nc} \quad (17)$$

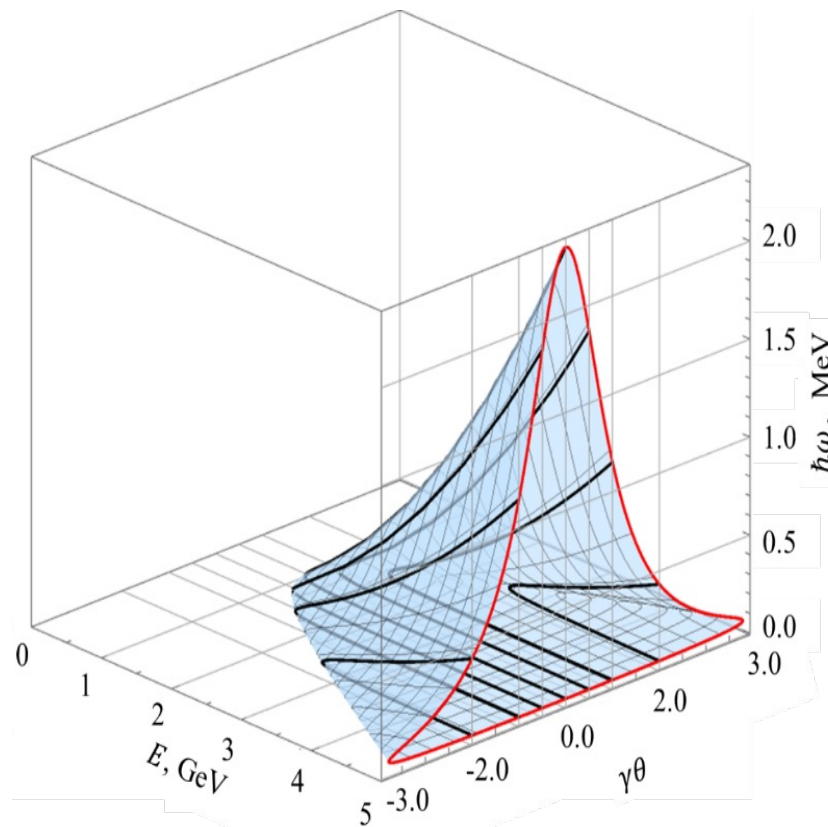
The asymptote (17) approaches to the solution (12) without account of the term  $(\gamma\omega_p)^2$  in Eq. (10) at increasing of the harmonic number.

Similar asymptotes were obtained by Baier and Katkov in [i], where they considered properties of transition radiation. Their results were developed by Potylitsyn in his book [ii].

i V.N Baier, V.M Katkov. Transition radiation as a source of quasi-monochromatic X-rays. Nuclear Instruments an Methods A 439 (2000) 189-198.

ii A.P. Potylitsyn. Electromagnetic Radiation of Electrons in Periodic structures. Springer - Verlag Berlin Heidelberg, 2011

**Point-like detector** The undulator radiation photon energy  $\hbar\omega$  as a function of the positron or electron energy  $E$  and emission angle  $\gamma\theta$  in units. Calculations are performed by Eq. (11) for Si solid state volume reflection undulator shown in Fig. 1a for harmonic number 1 and period 100 micrometers. Photon energies of radiation emitted at 0, 0.5, 1.0, 2.0 are shown by black thick curves. Photon energies emitted at positron or electron energy 5 GeV are shown by red curve.





The detector of angular radius  $\gamma^{-1}$

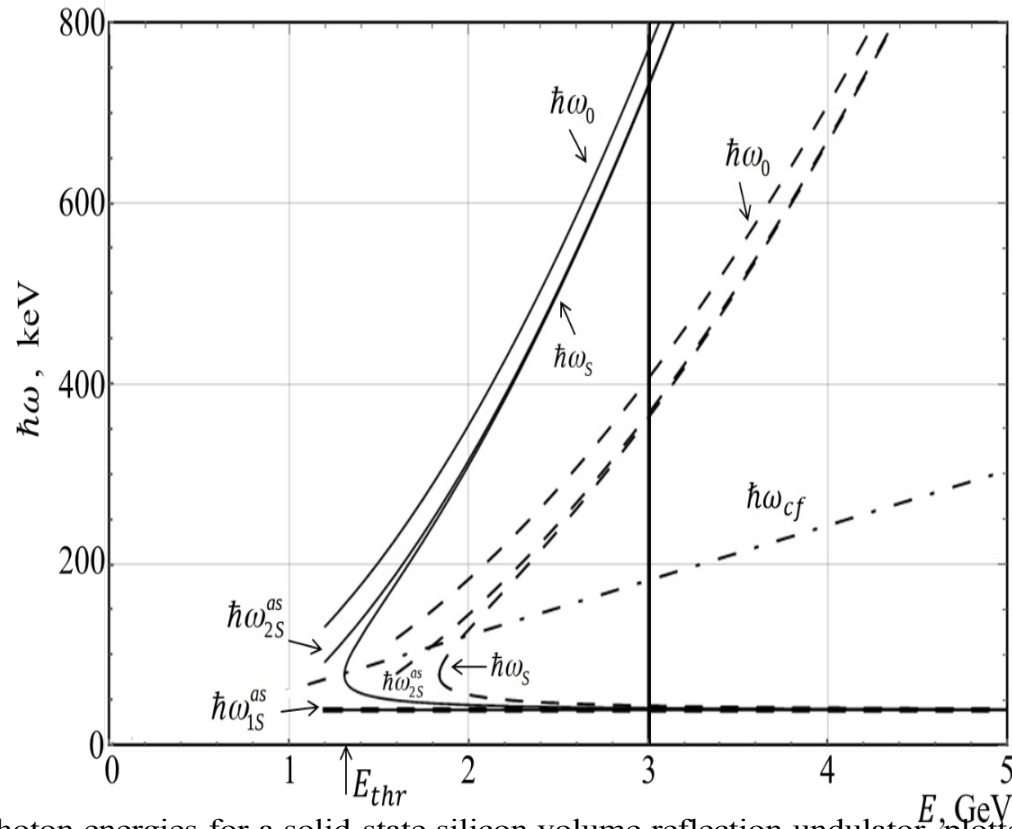


Fig. 5. Photon energies for a solid-state silicon volume reflection undulator, plotted as function of the electron or positron energy  $E$ . The undulator has a period of  $l = 100 \mu\text{m}$  and a potential well depth of  $U = 20 \text{ eV}$ . Further parameters are  $\xi = 1$ ,  $n = 1$ ,  $\hbar\omega_p = 31.1 \text{ eV}$ . The calculations of  $\hbar\omega_s$ ,  $\hbar\omega_0$ ,  $\hbar\omega_{1s}^{as}$ ,  $\hbar\omega_{2s}^{as}$  according to Eqs. (11, 12, 24, 25) for  $\theta = 0$  are indicated by solid lines. The same calculations at observation angle  $\theta = \gamma^{-1}$  are represented by dashed lines. The energy threshold  $E_{thr} = mc^2\gamma_{thr}$ , calculated using Equation (18), is indicated by an arrow. Additionally, the cutoff energy  $\hbar\omega_{cf} = \gamma\hbar\omega_p$ , applicable to volume reflection undulator radiation from any crystalline undulator, transition radiation, and bremsstrahlung radiation, is shown by a dash-dotted line and discussed in the last section.



# Radiation frequency in layered undulator

The wavefront passes the period during the time

$$\frac{2l_1\sqrt{\varepsilon_1}}{c} + \frac{2l_2\sqrt{\varepsilon_2}}{c} = \frac{l\sqrt{\varepsilon_{eff}}}{c}, \quad (18)$$

where  $\varepsilon_{eff} = 1 - \left(\frac{\omega_{peff}}{\omega_l}\right)^2$  and  $\omega_{peff}$  are the effective values of the permittivity and plasma frequency in the layered undulator. The effective value of the plasma frequency can be found from Eq. (18)

$$\omega_{peff}^2 = \frac{2l_1\omega_{p1}^2 + 2l_2\omega_{p2}^2}{l} \quad (19)$$

In practically important case of vacuum gap at  $\omega_{p2} = 0$  or air gap at  $\omega_{p2} \approx 0$  we obtain

$$\omega_{peff}^2 = \frac{2l_1}{l} \omega_{p1}^2 \quad (20)$$

and corresponding expression for effective permittivity

$$\varepsilon_{eff} = 1 - \frac{2l_1}{l} \left(\frac{\omega_{p1}}{\omega_l}\right)^2 \quad (21)$$

Below we will use these expressions (20,21) for calculations of the frequencies of undulator radiation from a layered undulator installed in vacuum or air.

Executing the calculations similar to ones in previous section, we obtain the frequency

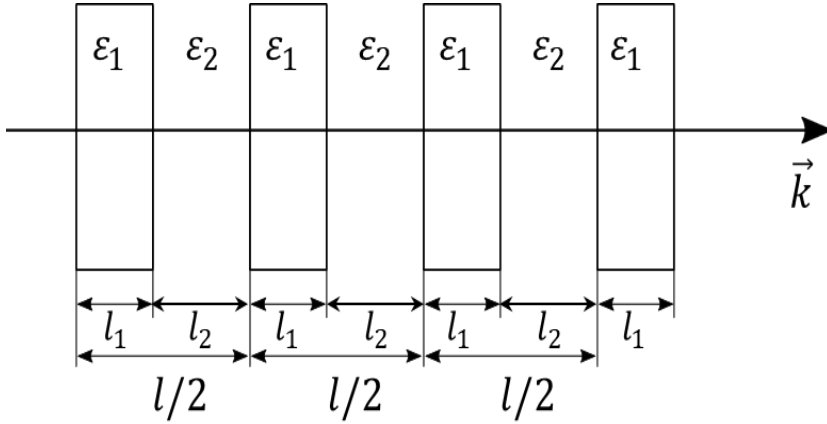
$$\omega_l = \frac{\omega_0}{2} \pm \sqrt{\left(\frac{\omega_0}{2}\right)^2 - \frac{2l_1(\gamma\omega_{p1})^2}{l[1+K^2+(\gamma\theta)^2]}}. \quad (22)$$

and asymptotes

$$\omega_{l1} = \frac{2l_1\omega_p^2}{4\pi nc}. \quad (25)$$

and

$$\omega_{l2} = \omega_0 - \omega_{l1} = \frac{4\pi nc}{l} \gamma^2 \frac{1}{[1+K^2+(\gamma\theta)^2]} - \frac{2l_1\omega_p^2}{4\pi nc} \quad (26)$$



# Discussion

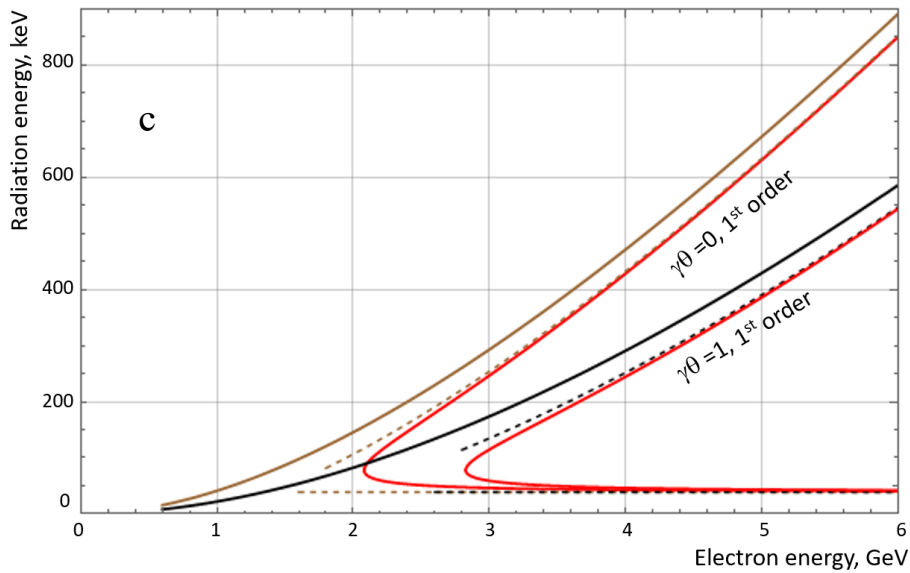
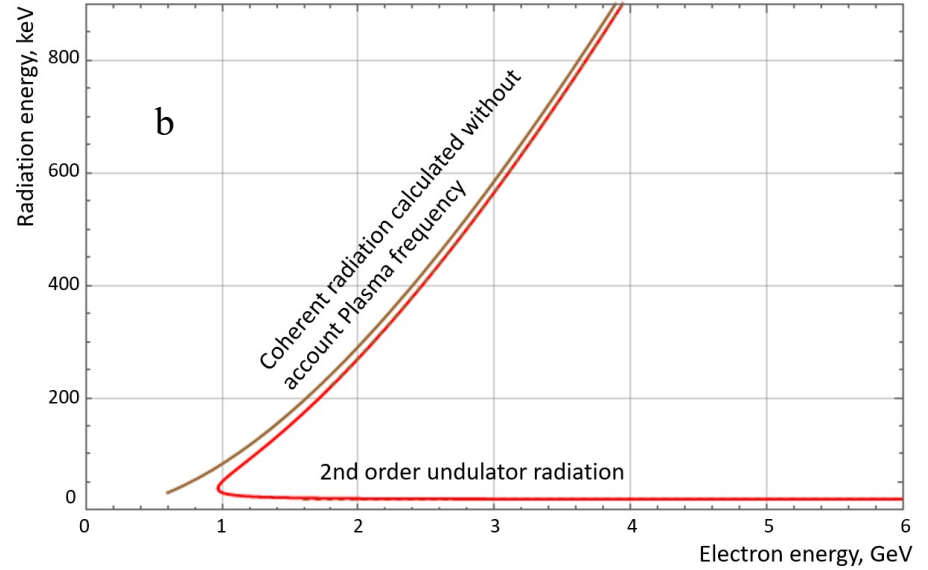
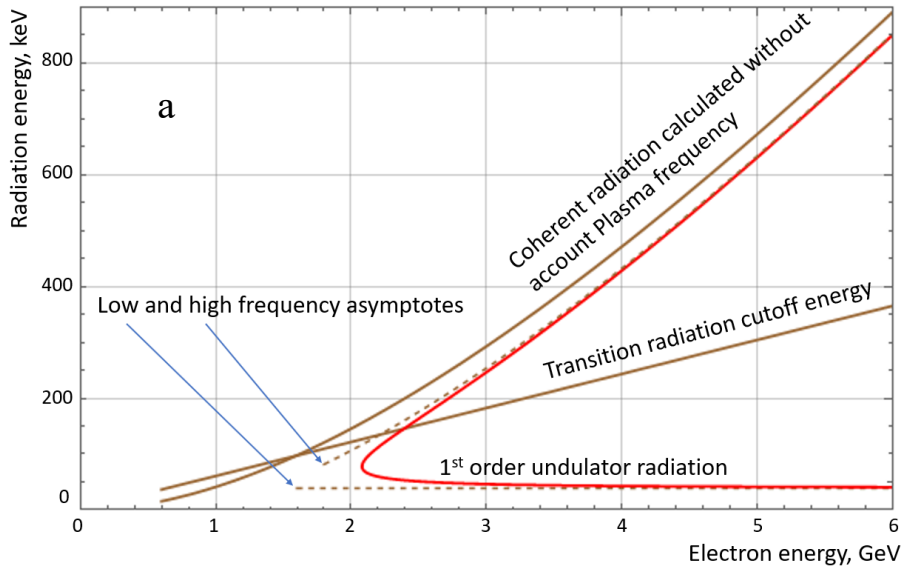
Here, we considered only the undulator parameter and frequencies of radiation from the volume reflection undulator. The important questions remain about stability of the motion of particles in volume reflection undulator. The experimental research can be performed at existing [i] or planned [ii] extracted electron/positron beams of energies up to 6 GeV in DESY. Undulator radiation with photons of energy about 10-20 MeV is in the range of giant nuclear resonance. It can be used **for nuclear transmutation of radioactive wastes**.

i R. Diener et al, The DESY II test beam facility. Nucl. Instrum. Methods Phys. Res. A 922 (2019) 265–286

ii A. Sytov. G. Kube, L. Bandiera et al., First design of a crystal-based extraction of 6 GeV electrons for the DESY II Booster Synchrotron. The European Physical Journal C volume 82, Article number: 197 (2022).

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- Thank you for attention.



Energies of radiation from the layered volume reflection undulator as functions of incident electron energy. Reflections from Si (110) or (111) planes in bent crystalline plates. Undulator period is 200um, bent crystalline plates are of thickness 50um. Undulator radiation is shown by red curves.

- a) 1<sup>st</sup> order  $\gamma\theta = 0$
- b) 2<sup>nd</sup> order  $\gamma\theta = 0$
- c) 1<sup>st</sup> order  $\gamma\theta = 0$  ,  $\gamma\theta = 1$

The threshold of radiation is at electron energies about 1-2 GeV

$$\gamma_r = \frac{\omega_p \sqrt{1 + K^2}}{\sqrt{\frac{(2\pi n c)^2}{2l|l} - (\theta\omega_p)^2}}$$