DOPPLER EFFECT IN A MEDIUM IN THE X-RAY RANGE

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Abstract

- Ter-Mikaelian noted that PXR frequency can be considered as a manifestation of the Doppler effect [2].

- The Doppler effect in X-ray range emitted by a relativistic oscillator in a medium or by a relativistic charged particle moving in a periodical medium is considered.
- The radiation can be emitted, for instance, due to mechanisms of parametric X-ray radiation, coherent bremsstrahlung, undulator radiation in a crystalline undulator, transition radiation from a stack of foils.
- The illustration of the Doppler low as an ellipsoid in the momentum space is shown.

Doppler effect in vacuum as an ellipsoid in the momentum space

First of all, remind the radiation frequency of an oscillator with own angular frequency ω_0 which moves rectilinearly and uniformly with velocity V in vacuum. The observer can see the radiation with single frequency ω_V which is described by the Doppler low

$$\omega_V = \frac{\omega_0 \gamma^{-1}}{1 - \frac{V}{c}\cos\theta}, \tag{1}$$

where $\gamma = \left[1 - \left(\frac{V}{c}\right)^2\right]^{-\frac{1}{2}}$ is the relativistic Lorentz factor of the oscillator and θ is the observation angle between the oscillator velocity vector and the observation direction. In the case of the longitudinal Doppler effect, the radiation frequency observed in forward direction at $\theta = 0$ is

$$\omega_V(\theta = 0) = \omega_0 \gamma \left(1 + \sqrt{1 - \gamma^{-2}}\right) \sim 2\omega_0 \gamma, \tag{2}$$

and the radiation frequency observed in backward direction at $\theta = \pi$ is

$$\omega_V(\theta = \pi) = \omega_0 \gamma \left(1 - \sqrt{1 - \gamma^{-2}}\right) \sim \frac{\omega_0}{2\gamma}. \tag{3}$$

In the case of transversal Doppler effect, the radiation frequency observed in transversal direction at $\theta = \frac{\pi}{2}$ is

$$\omega_V\left(\theta = \frac{\pi}{2}\right) = \omega_0 \gamma^{-1}. \qquad (4).$$

Radiation of frequency (1) in vacuum is possible at any own oscillator frequency ω_0 , and velocity V, and value of the Lorentz factor γ .

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Doppler low

$$\omega_V = \frac{\omega_0 \gamma^{-1}}{1 - \frac{V}{c}\cos\theta}, \quad (5)$$

Ellipsoid in polar coordinates

$$\rho = \frac{d}{1 - \xi\cos\theta}, \quad (6)$$

where ξ is the eccentricity of the ellipsoid.

Rewrite the Doppler low as a function of gamma-factor

$$\left|\vec{k}\right| = \frac{\hbar\omega_V}{c} = \frac{\hbar\omega_0\gamma^{-1}}{c\left(1 - \sqrt{1-\gamma^{-2}}\cos\theta\right)}, \quad (7)$$

and the ellipsoid formula as a function of long a and short b ellipsoid semi-diameters

$$\rho = \frac{b^2 / a}{1 - \sqrt{1 - \left(\frac{b}{a}\right)^2}\cos\theta}, \quad (8)$$

Comparing Eq. (7) and Eq. (8), one can see that

Eccentricity $\xi = \frac{V}{c}$, gamma-factor $\gamma = \frac{a}{b}$, wave vector module $\left|\vec{k}\right| = \rho$, $\frac{\hbar\omega_0}{c} = b$

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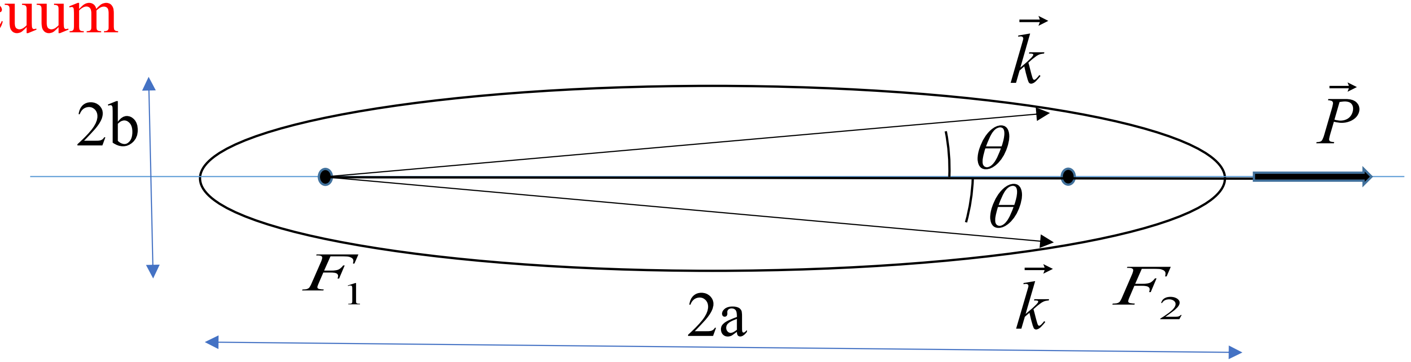
Doppler low as an ellipsoid in momentum space

Doppler: $\left|\vec{k}\right| = \frac{\hbar\omega_V}{c} = \frac{\hbar\omega_0\gamma^{-1}}{c\left(1-\sqrt{1-\gamma^{-2}}\cos\theta\right)} \quad = \quad \rho = \frac{b^2/a}{1-\sqrt{1-\left(\frac{b}{a}\right)^2}\cos\theta}$ **- Ellipsoid**

Eccentricity $\xi = \frac{V}{c}$, gamma-factor $\gamma = \frac{a}{b}$, wave vector module $\left|\vec{k}\right| = \rho$, $\frac{\hbar\omega_0}{c} = b$

At ellipsoid is strongly elongated along the velocity of momentum vector $\vec{P}$ at -factor $\gamma = \frac{a}{b} >> 1$

in vacuum



Cross section of the ellipsoid in vacuum in momentum space

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The Doppler effect in a medium in X-ray range

Let us consider radiation frequency of the same relativistic oscillator with own frequency ω_0 moving in a medium. If the particle moves in the medium with the permittivity ε , we have to rewrite the equation (1) as

$$\omega = \frac{\omega_0 \gamma^{-1}}{1 - \frac{V\sqrt{\varepsilon}}{c}\cos\theta}. \tag{6}$$

Below we will consider radiation in X-ray and gamma-ray ranges, when

$$\varepsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2, \tag{7}$$

where ω_p is the plasma frequency in the medium, and emitted frequency ω is out of the resonant atomic or nuclear frequencies, and also should be

$$\omega > \omega_p\,, \tag{8}$$

because of radiation with $\omega < \omega_p$ is absorbed in the medium.

The Cherenkov radiation is impossible in this case because the phase light velocity $\frac{c}{\sqrt{\varepsilon}}$ exceeds the common light velocity c and, naturally, the oscillator or charged particle of velocity V . Inserting Eq. (7) to Eq. (6), we obtain the quadratic equation for emitted by the oscillator frequency (see Appendix 1)

$$\omega^2\left[1 - \left(\frac{V\cos\theta}{c}\right)^2\right] - \omega 2\omega_0\gamma^{-1} + \omega_0^2\gamma^{-2} + \omega_p^2\left(\frac{V\cos\theta}{c}\right)^2 = 0. \tag{9}$$

Eq. (9) has two solutions (see Appendix 2)

$$\omega_{+/-} = \frac{\omega_0\gamma^{-1}\left[1 \pm \frac{V\cos\theta}{c}\sqrt{1 - \left(\frac{\omega_p\gamma}{\omega_0}\right)^2\left[1 - \left(\frac{V\cos\theta}{c}\right)^2\right]}\right]}{\left[1 - \left(\frac{V\cos\theta}{c}\right)^2\right]} = \omega_V \frac{1 \pm \frac{V\cos\theta}{c}\sqrt{1 - \left(\frac{\omega_p\gamma}{\omega_0}\right)^2\left[1 - \left(\frac{V\cos\theta}{c}\right)^2\right]}}{1 + \frac{V\cos\theta}{c}} \text{(10}$$

But the solution (10) with negative sign does not satisfy Eq. (6) at $\omega_p = 0$. In following, we will consider solution (10) with positive sign.

$$\omega = \frac{\omega_0 \gamma^{-1}\left[1+\frac{V\cos\theta}{c}\sqrt{1-\left(\frac{\omega_p\gamma}{\omega_0}\right)^2\left[1-\left(\frac{V\cos\theta}{c}\right)^2\right]}\right]}{\left[1-\left(\frac{V\cos\theta}{c}\right)^2\right]} = \omega_V \frac{1+\frac{V\cos\theta}{c}\sqrt{1-\left(\frac{\omega_p\gamma}{\omega_0}\right)^2\left[1-\left(\frac{V\cos\theta}{c}\right)^2\right]}}{1+\frac{V\cos\theta}{c}} \quad (10a)$$

The radiation with frequency (10) is possible, if the radicand in (10a) is not negative

$$\left(\frac{\omega_p\gamma}{\omega_0}\right)^2\left(1-\left(\frac{V\cos\theta}{c}\right)^2\right) < 1 \quad (11)$$

One can find conditions for existence of the radiation from inequality (11) (see Appendix 2a)

$$|\sin\theta| < \frac{c}{V\gamma}\sqrt{\left(\frac{\omega_0}{\omega_p}\right)^2 - 1} \quad (12)$$

and also one condition for existence of the radiation (in addition to condition (8))

$$\omega_0 > \omega_p . \quad (13)$$

At $\gamma >> 1$ and $\omega_0 >> \omega_p$ the condition (12) looks as

$$\theta < \frac{\omega_0}{\gamma\omega_p} \quad (14)$$

Thus, the relativistic oscillator in a medium can emit X-ray radiation only in the forward direction within the angles around of γ^{-1} similarly to bremsstrahlung radiation emitted by a relativistic charged particle in a medium.

The frequency of radiation emitted in forward direction at $\theta = 0$ is (see Appendix 5)

$$\omega_+(\theta=0) = \omega_0\gamma\left[1+\sqrt{\left(1-\gamma^{-2}\right)\left(1-\left(\frac{\omega_p}{\omega_0}\right)^2\right)}\right]. \quad (15)$$

This frequency is some reduced in comparison to vacuum frequency (2).

Examples of Doppler effect

Usually, an oscillator is considered as a particle without charge. But we can consider radiation of an elementary particle moving in a periodical medium and emitting radiation periodically.
Ter-Mikaelian first turned attention that PXR frequency emitted in a crystal can be considered as Doppler effect. We believe that all types of radiation emitted in a periodical medium can be considered as manifestation of the Doppler effect. In this case we have to insert into Doppler formula expression $\frac{2\pi V}{l}$ instead of $\omega_0\gamma^{-1}$. Consider examples:

Parametric X-ray radiation.

Mechanism of radiation – polarization type. Particle moves rectilinearly.

$$\omega_{PXR} = \frac{2\pi V}{l}\frac{1}{1-\frac{V\sqrt{\varepsilon}}{c}\cos\theta}$$

Coherent bremsstrahlung (at $\hbar\omega << E_e$).

Mechanism of radiation – bremsstrahlung on nuclei. Particle moves rectilinearly.

$$\omega_{BS} = \frac{2\pi V}{l}\frac{1}{1-\frac{V\sqrt{\varepsilon}}{c}\cos\theta}$$

Transition radiation on a stack of foils (Bayer, Karkov)

Mechanism of radiation – polarization type. Particle moves rectilinearly.

$$\omega_{TR} = \frac{2\pi V}{l}\frac{1}{1-\frac{V\sqrt{\varepsilon}}{c}\cos\theta}, \text{ where } \varepsilon = 1-\left(\frac{\omega_{peff}}{\omega_L}\right)^2, \quad \omega_{peff}^2 = \frac{2l_1\omega_{p1}^2+2l_2\omega_{p2}^2}{l}$$

Undulator radiation from crystalline undulator

Mechanism of radiation – undulator type. Particle moves with transverse velocity.

$$\omega_{CU} = \frac{2\pi n}{l}\frac{V_z}{1-\frac{\sqrt{\varepsilon}}{c}V_z\cos\theta}. \qquad (8)$$

where longitudinal velocity in sinusoidal undulator is $V_z = c\left(1-\frac{1+\frac{K^2}{2}}{2\gamma^2}\right)$, $K = \frac{2\pi A}{l}\gamma$

and longitudinal velocity in volume reflection undulator is $V_z = V\cos\alpha$

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