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# DE BROGLIE WAVE AND LONGITUDINAL DENSITY EFFECT

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# Abstract

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- The de Broglie wave characterizing a relativistic particle moving in a medium and X-ray radiation emitted in the medium by the same relativistic charged particle are considered. Phase and group velocities of both, de Broglie and electromagnetic waves are compared. The criterion for the appearance of the Ter-Mikaelian longitudinal density effect (dielectric suppression effect) is formulated in terms of phase and group velocities.

## Longitudinal Ter-Mikaelian density effect

When a relativistic charged particle moves through a medium, it emits bremsstrahlung radiation. At lower photon energies, the intensity of this radiation is suppressed because of Ter-Mikaelian longitudinal density effect [1], that sometimes is called as dielectric suppression [3]. The suppression occurs at photon energies  $\hbar\omega$  below the critical or cut-off energy  $E_{\text{eff}} = \hbar\omega_{\text{eff}}$ , at

$$\hbar\omega < E_{\text{eff}} = \hbar\omega_{\text{eff}},$$

where

$$E_{\text{eff}} = \hbar\omega_{\text{eff}} = \gamma\hbar\omega_p,$$

where  $\gamma$  is the relativistic Lorentz factor,  $\hbar$  is the Plank constant divided by  $2\pi$ ,  $\omega_p$  is the plasma frequency in the medium, was confirmed experimentally in refs. [3]. In the present paper we will consider the connection between the critical of cutoff energy and de Broglie wave of the incident particle and wavelength of emitted electromagnetic radiation.

## Phase and group velocities of X-ray radiation in a medium

The module of the wave vector of radiation in a medium is  $k = \frac{\omega\sqrt{\varepsilon}}{c}$ , where the dielectric constant

in the X-ray range is  $\varepsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2$ . The phase light velocity is

$$c_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon}} = \frac{c}{\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}}.$$

The group light velocity is

$$c_{gr} = \frac{d\omega}{dk} = c\sqrt{\varepsilon} = c\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}.$$

One can see that

$$c_{ph}c_{gr} = \frac{c}{\sqrt{\varepsilon}}c\sqrt{\varepsilon} = c^2.$$

The phase light velocity  $c_{ph}$  always exceeds the common light velocity  $c$ . The group light velocity  $c_{gr}$  always is below the common light velocity  $c$ . But both phase and group velocities are close to the common light velocity  $c$  in X-ray waves because of  $\omega \gg \omega_p$  in the X-ray range.

## Phase and group velocities in de Broglie wave of relativistic particle

The wave function of moving particle is the de Broglie wave [2].

The group velocity of the de Broglie wave of relativistic charged particle moving with velocity  $V$  is

$$v_{gr} = \frac{d\omega}{dk} = V .$$

The phase velocity of the de Broglie wave of relativistic charged particle is

$$c_{ph} = \frac{\omega}{k} = c \frac{c}{V} .$$

One can see that for de Broglie velocities

$$v_{ph} v_{gr} = c \frac{c}{V} V = c^2$$

similar to relation between the phase and group velocities for X-ray waves.

## De Broglie and electromagnetic waves

Let us consider the case, when the group velocity of the de Broglie wave of relativistic particle is equal to the group velocity of X-ray radiation in a medium and also when the phase velocity of the de Broglie wave of relativistic particle is equal to the phase velocities of X-ray radiation in a medium. These two conditions are described by two equations

$$v_{ph} = c_{ph} \Rightarrow \frac{c}{\sqrt{1-\gamma^{-2}}} = \frac{c}{\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}}$$
$$v_{gr} = c_{gr} \Rightarrow c\sqrt{1-\gamma^{-2}} = c\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}$$

Both equations have only one solution, that exactly correspond to the critical or cutoff frequency for Ter-Mikaelian density effect

$$\omega = \omega_{\text{eff}} = \gamma\omega_p.$$

## Results and discussion

The Ter-Mikaelian longitudinal density effect is insignificant, when the the group velocity of the de Broglie wave of relativistic particle ( $V$ ) is less than the group velocity of X-ray wave radiated by the particle in a medium and simultaneously the the phase velocity of the de Broglie wave of relativistic particle exceeds the phase velocity of X-ray wave radiated by the particle in a medium.

The Ter-Mikaelian longitudinal density effect is sufficient and leads to suppression of bremsstrahlung radiation, when the the group velocity of the de Broglie wave of relativistic particle exceeds the group velocity of X-ray wave radiated by the particle in a medium and simultaneously the the phase velocity of the de Broglie wave of relativistic particle is less than the phase velocity of X-ray wave radiated by the particle in a medium.

The critical condition for Ter-Mikaelian longitudinal density effect is

$$\omega_{eff} = \gamma\omega_p$$

occurs when both the group and phase velocities of de Broglie wave of relativistic particle and X-ray wave radiated by the particle in a medium are equal one to another.

Thus, we found the connection between the de Broglie wave and longitudinal density effect

## References

1. M.L. Ter-Mikaelian., High-Energy Electromagnetic Processes in Condensed Media, New York: Wiley-Interscience, 1972; Translated from Russian: Vliyanie Sredy Na Elektromagnitnye Protsessy Pri Vysokikh Energiyakh, Erevan: Izd. AN Arm. SSR, (1969).
2. Don M. Sparlin, Wave Properties of Particles, <https://web.mst.edu/~sparlin/phys107/lecture/chap03.pdf>
3. P. L. Anthony, R. Becker-Szendy, P. E. Bosted, M. Cavalli-Sforza, L. P. Keller, L. A. Kelley, S. R. Klein, G. Niemi, M. L. Perl, L. S. Rochester, and J. L. White, Bremsstrahlung suppression due to the Landau-Pomeranchuk-Migdal and dielectric effects in a variety of materials, Physical Review D 56 (1997) 1373 – 1390.

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# Two schemes of the volume reflection undulator

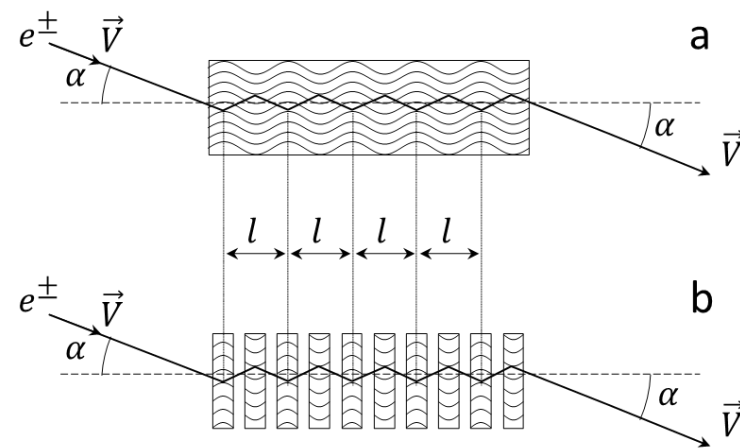


Fig.1. Two schemes of the crystalline undulators based on the volume reflection effect. Bent crystallographic planes are shown by curves. The layered volume reflection undulator is shown in Fig. 1a and the solid state volume reflection undulator is shown in Fig. 1b. The electron  $e^-$  or positron  $e^+$  moves with velocity  $\vec{V}$  in almost straight lines between the reflecting points at angle  $\alpha$  with respect to the undulator axis shown by dashed line.