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Comparative analysis of X-ray CT images and the results of digital modeling of objects printed on a 3D printer

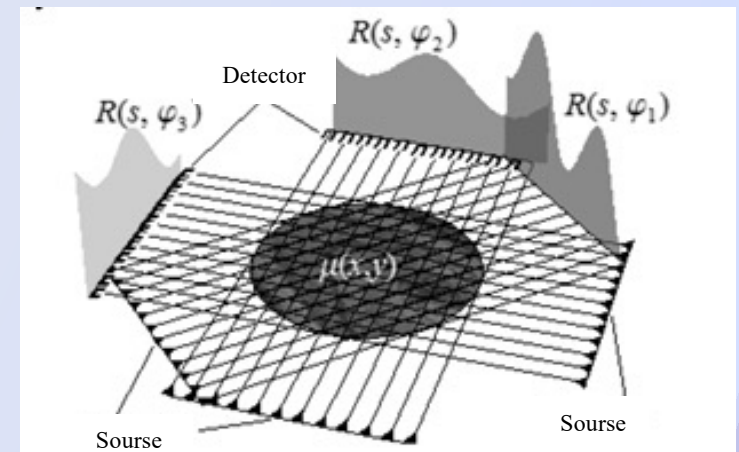
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COMPUTED TOMOGRAPHY (CT) is a non-destructive imaging modality that combines the use of X-rays and computer processing to generate 'slices' of the area scanned.

When examining the internal structure of an object, it is common practice to illuminate it with radiation. Shining a light source on the object from different angles captures multiple shadow images of its three-dimensional shape. This method allows a more detailed analysis of the object's internal composition. By collecting a multitude of projection images taken from various perspectives, it becomes possible to reconstruct a highly accurate representation of the object's internal structure, including details such as its radiation absorption density function.



Theoretical interpretation of CT

During the processing of the received data, it is assumed that the trajectory of the beam is a straight line, and the linear absorption of radiation in the material occurs

$$\frac{dI}{I_0} = -\mu(x, y)dt$$

$\mu(x, y)$ - is the linear attenuation coefficient of the mater

dt - thickness of the layer

I_0 - is the radiation intensity at the entrance of the layer of thickness dt ,

dI - is the intensity change after passing through that layers

Generally the total attenuation \mathbf{R} of a ray at position s , on the projection at angle φ , is given by the line integral:

$$R(s, \varphi) = \ln\left(\frac{I}{I_0}\right) = -\int \mu(x, y)dt$$

The task of CT is to solve the inverse problem and to recover the spatial distribution of the values of the linear absorption coefficient along the direction of propagation of the rays.

The main mathematical model underlying the theoretical basis of reconstruction of tomographic images from two-dimensional projections of an object is the inverse Radon transformation, through which the restoration of the $\mu(x, y)$ function is carried out. The Radon transform is an integral transmission whose inverse is used to reconstruct images from CT scans.

Radon Transform

Radon transform has a simple geometric meaning, it is the integral of a function along a straight line L , perpendicular to the vector $\mathbf{n} = (\cos\varphi, \sin\varphi)$ and passing at a distance s (measured along the vector \mathbf{n} , with the corresponding sign) from the origin of coordinates (Fig).

For fixed φ and s , we can describe the line L by the following equation.

$$x\cos\varphi + y\sin\varphi - s = 0$$

Radon transformation is more commonly defined by

$$R[\mu(x, y)](s, \varphi) = R(s, \varphi) = \iint_{-\infty}^{\infty} \mu(x, y) \delta(x\cos\varphi + y\sin\varphi - s) dx dy$$

Where

$R[\mu(x, y)](s, \varphi)$ is the Radon image of the function $\mu(x, y)$,

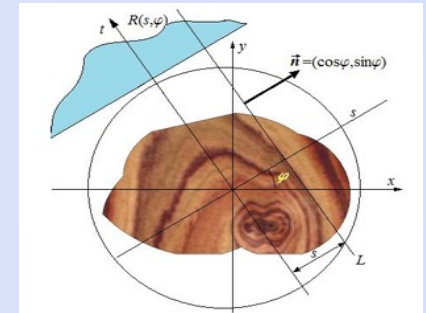
$\delta(x\cos\varphi + y\sin\varphi - s)$ is the Dirac delta function

Using Fourier slice theorem, by capturing an infinite number of projections at an infinite range of angles, we can recreate the original object with precision.

$$\mathcal{F}[R(\omega, \varphi)] = \mathcal{F}[\mu](\omega\cos\varphi, \omega\sin\varphi)$$

And inverse Radon transform will be

$$\mu(x, y) = \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} |\omega| \mathcal{F}[R(\omega, \varphi)] e^{i\omega s} d\omega d\varphi.$$



Filtered back-projection

There are numerous reconstruction algorithms, **Filtered back-projection** reconstruction is most widely used in CT scanners. This is an image restoration method based on the radon transform, which is mainly used in computed tomography. FBP restores the original 2D image from a set of one-dimensional projections in different directions. To do this, the projections are first filtered and then moved across the image surface in the appropriate direction. The advantage of this method is that it is fast, since it requires little computing power.

With a sampled discrete system, the inverse Radon transform is

$$\mu(x, y) = \frac{1}{2\pi} \int_0^\pi I(s, \varphi) d\varphi$$

Where $I(s, \varphi)$ is the convolution of $h(s)$ and $R(s, \varphi)$

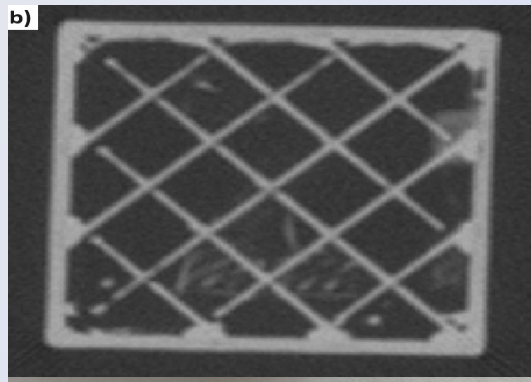
$$I(s, \varphi) = h(s) * R(s, \varphi)$$

the function $h(s)$ is called filter (Ramp filter).

Computer simulation in the CT

Computer simulation has become an invaluable tool in the field of computed tomography for predicting and analysing experimental results.

Fast prototyping technologies have made it easy to create complex-shaped samples quickly for imaging tasks. Fused deposition modeling with polymer materials is commonly used to create imaging phantoms for the human body and small animals. Plastic materials can imitate biological tissues for imaging studies, showing promise for 3D printing in imaging applications. However, inaccuracies can occur during the 3D printing process, leading to differences in the internal structure of printed samples. This study explores analyzing these inaccuracies using computer simulation of CT images.



Appearance of 3D printed cube sample (a) and image of slice obtained with micro-CT (b)

We used a 3D-printed cubic sample as an origin for computer model and experiment. In our simulation, we used the image of one slice of the object corresponding to the slice of the reconstructed CT image. The slice was obtained for the plane parallel to the top layer of the sample.

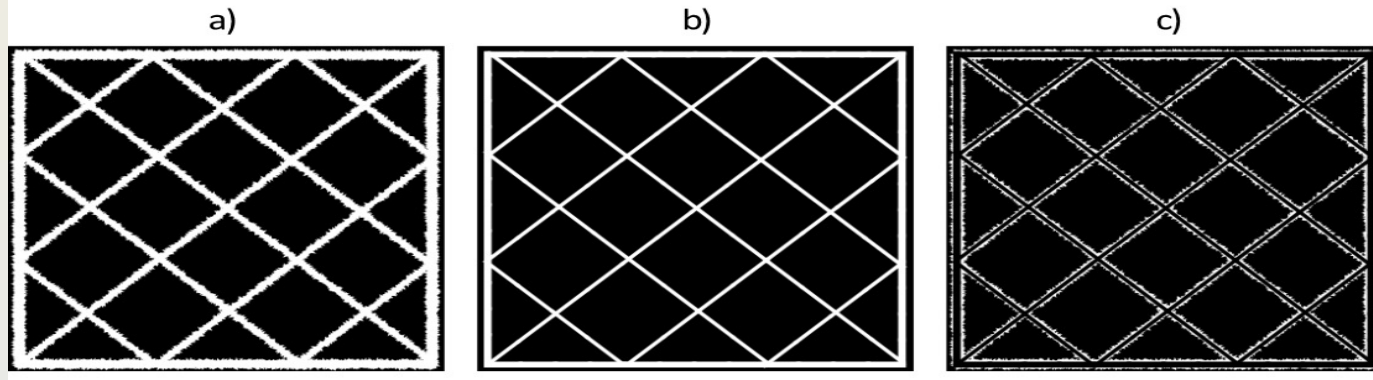


Figure 2. Models simulating slice of printed sample (a) and ideal rectangular fill (b); differential image of two models (c).
Differential image is pixel-by-pixel difference of a) and b) images.

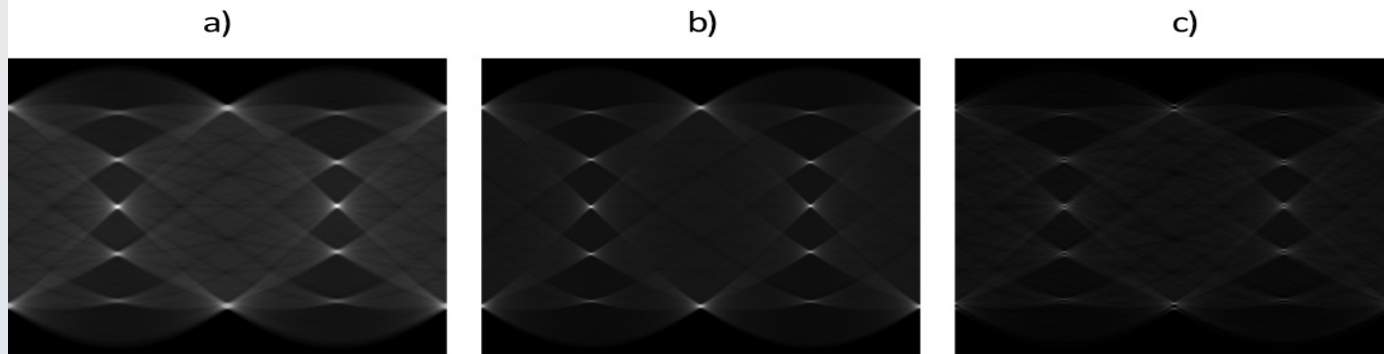
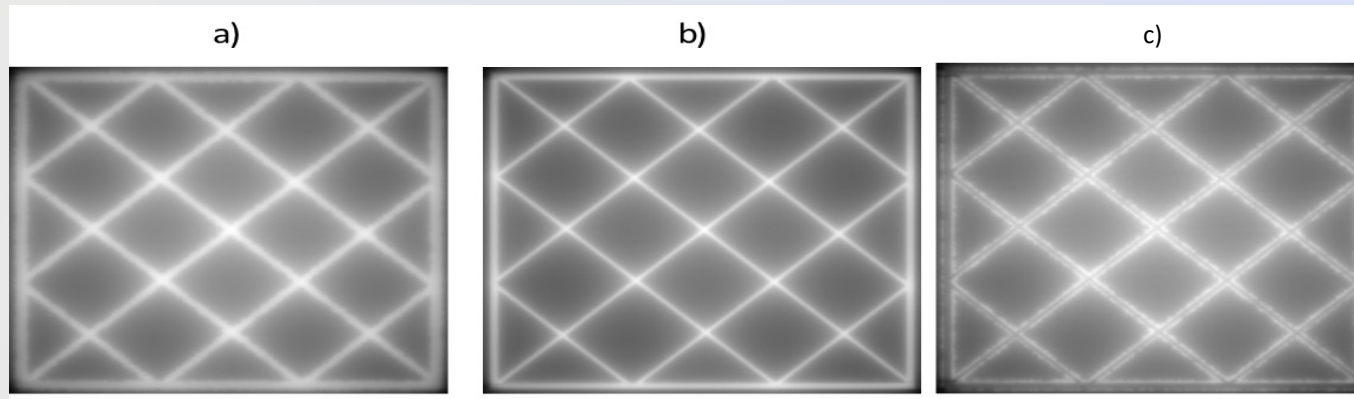


Figure 3. Sinogram obtained for slice simulating printed sample (a) and ideal rectangular fill (b); differential image of two sinograms (c).
Differential image is pixel-by-pixel difference of a) and b) images.

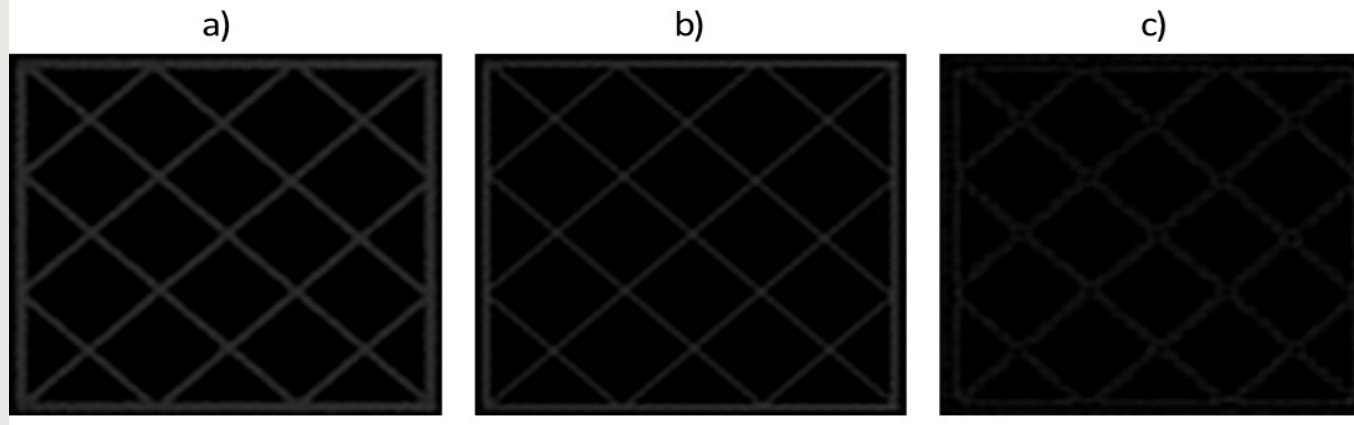
We used for computer simulation two models of the slice shown in previous slide. One of them corresponds to the ideal model used for printing the sample, which contains only solid lines. Another model was designed to imitate possible inaccuracy during the printing of the sample. For this purpose, we used random noises which could fill pixels near the lines of the first model. In both these models we used only maximal (white) and minimal (black) brightness of the pixels without intermediate (grey) values. Figure 2c shows the differential image of two models which was the pixel-by-pixel difference of two 'models' images and was also used for computer processing. Actually, this differential image was the image of random noises used to create the second model. Wolfram Mathematica software was used for images simulation and processing. In our computer simulation we used standard functions of Wolfram Mathematica dealing with Radon and Inverse Radon transforms. Figure 3 shows sinograms for both our models and the differential image of these two sinograms.



Reconstructed images of the slice of simulating printed object (a) and ideal rectangular fill (b).

c) differential image of two reconstructed slice images

After that, we reconstruct the slice image from the corresponding sinogram. The reconstructed images are shown in the Figures. As one can see from the figure a), the noise in the image printed object is still present. However, it seems smooth in comparison with the initial image. Figure c) displays a differential image of two reconstructed slice images, showing dark lines that correspond to rectangular filling lines. The border is not as sharp as in the initial models, due to the tomography reconstruction approach. This results in some loss of information, causing the reconstructed images to not fully match the original sample. However, the main features of the internal structure are still visible. Additionally, figures reveal a non-uniform gray background in the reconstructed images, a result of the reconstruction process. To accurately analyze the results of a real experiment, it is crucial to use an image processed with direct and inverse Radon transforms to distinguish between imaging artifacts and printing inaccuracies. This approach may allow distinguishing details caused by images processing (imaging “artefacts”) and by printing inaccuracies.

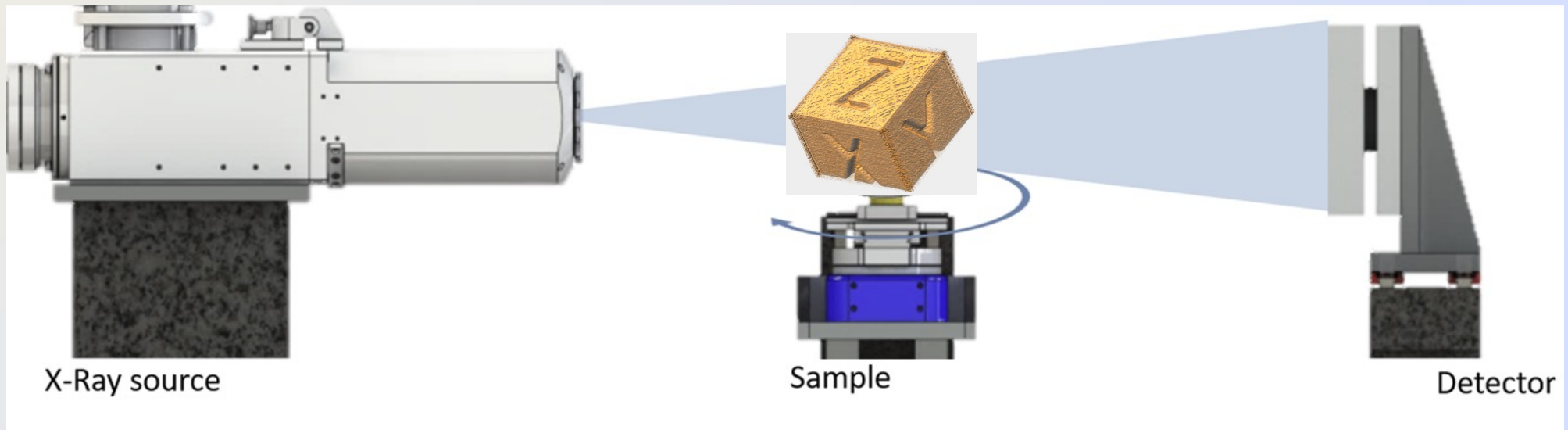


Reconstructed images of the slice of simulating printed object (a) and ideal rectangular fill (b) using ramp filter and their differential image (c).

In the final stage, we obtained images from sinograms shown in Figure 2a, 2b using inverse radon transformation with ramp filter which is commonly used to improve the quality of CT images, and analyzed their differential image. A ramp filter is usually applied to enhance the high-frequency components and reduce the low-frequency noise that causes blurring in the image. Figures demonstrates these images.

The differential image in Figure c) demonstrates that after reconstruction data with a ramp filter, we cannot exclude rectangular filling from the differential image. It should be noted that most filters used in imaging tasks are developed to improve the quality of an object investigated, while for our task we need a filter that will allow better distinguishing of differences between experimental and model images. The selection or design of a mathematical filter that may allow the solution of this task is a separate issue and will be considered in future works. However, it is important to note that standard filters may simultaneously improve the quality of a particular reconstructed image and lead to losing details for comparing it with model one.

Experimental scheme of micro CT

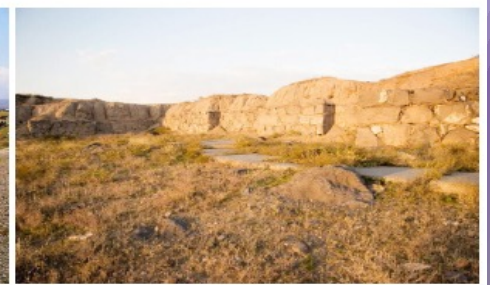


The sample was examined by the CT method

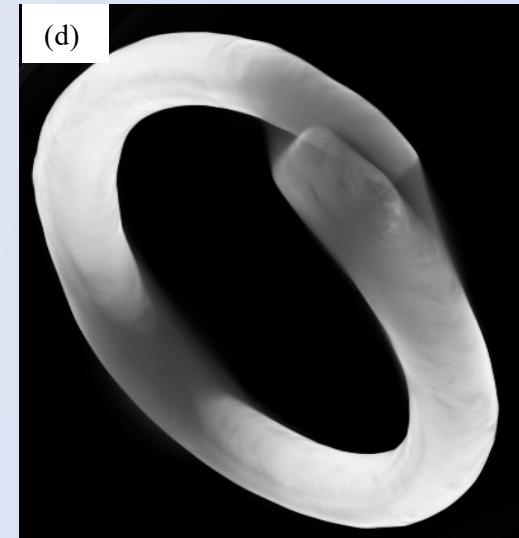
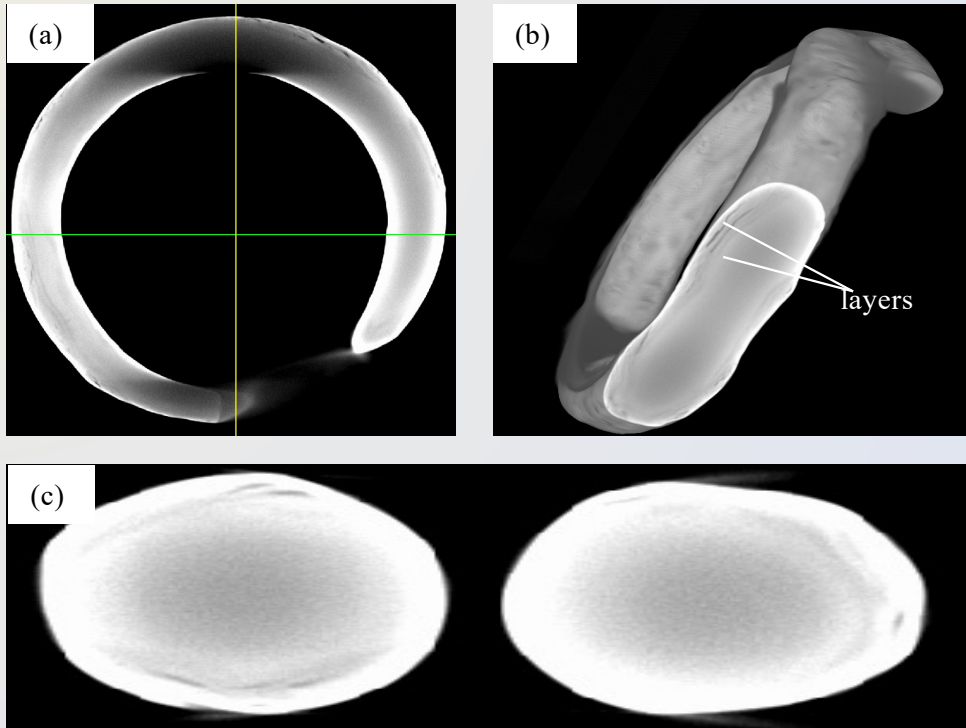
An X-ray micro-CT with a corresponding software package was designed and created at our institute (IAPP NAS RA), which enables 3D scanning of a sample with a diameter up to 30 cm and a height of 22 cm with a resolution of 60 micron.

The tomograph operates on the principle of absorption contrast. The implemented software package uses standard filters and options (such as Beam hardening correction, Ring artifacts removal, etc.), and the reconstruction is carried out using Radon transformations.

I would like to present one of our recent studies using the above described micro-CT. The study was conducted to obtain the internal structure of an archaeological sample provided by the “Research Center for Historical and Cultural Heritage of Yerevan”. The sample was discovered in the southwestern suburb of Yerevan, from the Karmir Blur archaeological site on the left bank of the Hrazdan River, in the area of which BC 7th century in the 1st quarter, during the reign of the Urartian king Rusa II (685-645), the fortress-fortress of Teishebaini was founded. It was a large administrative and economic center, with a two-story citadel, where pottery, weaponry, and metalworking workshops, wine and beer cellars, grain stores, cattle sheds (first floor), colonnaded halls and living rooms (second floor) were located.



Karmir Blur (Red Hill) ancient site (Teishebaini)



To determine the peculiarities and defects of the sample internal structure we conducted microtomographic studies. As a source, we used a tungsten X-ray tube. Slices were obtained from μ CT study with 150 kV copper filtered X-ray beam.

From the obtained images, a conclusion can be made about the sample preparation technology. It can be seen from the pictures that it has a layered structure. We assume that it was made in an open mold, after which it was worked and hammered into a bracelet.

Here is shown an example of slices obtained in the frontal (a), sagittal (b) and horizontal planes (c). Also shown is the 3D model (d) obtained from reconstructed tomographic slices.

Conclusion

- Simulated CT images for models of 3D-printed sample filling with and without random noises imitating inaccuracies occurred during printing process.
- It is shown possibility to development of an algorithm for distinguishing inaccuracies of internal structure of 3D printed samples.
- This approach potentially can be used for analysis of inaccuracies of real 3D printed sample by comparing of CT experimental results and filling model of this sample processed with direct and inverse Radon transforms.

Thank you for your attention!