Institute of Applied Problems of Physics (IAPP)

Few-particle intraband dipole transitions in strongly oblate asymmetric ellipsoid QD

Yevgeni Mamasakhlisov

In collaboration with Aram Nahapetyan, Mher Mkrtchyan, Hayk Sarkisyan

Riccione 2024

Outline:

- Ion – channeling and QDs
- QD and one – particle problem
- Few – particle problem and Moshinsky model
- Kohn’s theorem and QD
- The effect of external electric field
- Results

Channeling through QDs

Available online at www.sciencedirect.com

SCIENCE DIRECT®

NIMB
Beam Interactions with Materials & Atoms

ELSEVIER

Nuclear Instruments and Methods in Physics Research B 241 (2005) 470–474

www.elsevier.com/locate/nimb

Close

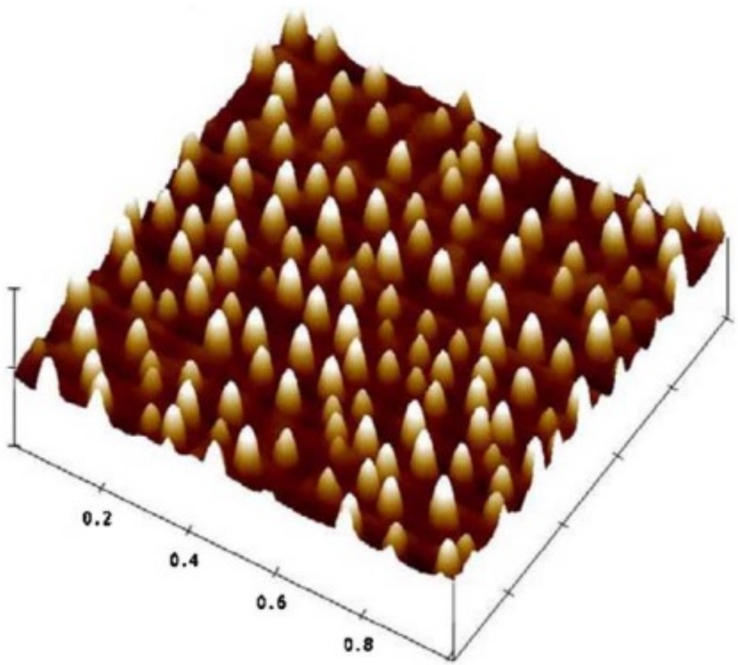
Ion-channeling studies of InAs/GaAs quantum dots

H. Niu [a,*], C.H. Chen [b], H.Y. Wang [c], S.C. Wu [b], C.P. Lee [c]

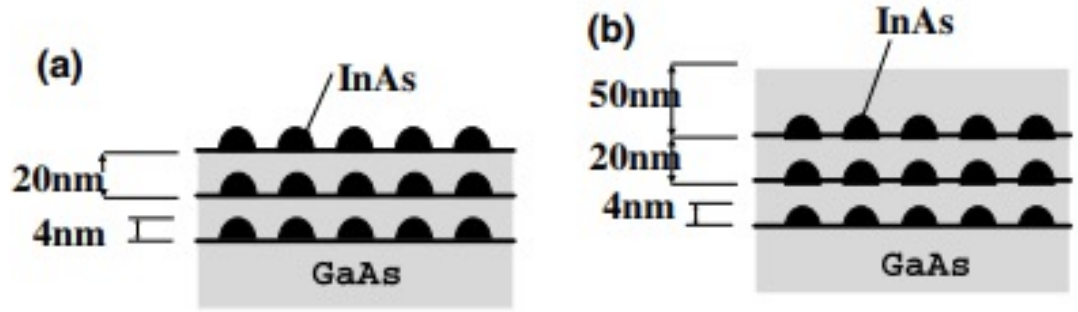
[a] *Nuclear Science and Technology Development Center, National Tsing Hua University, Hsinchu 30043, Taiwan, ROC*
[b] *Department of Physics, National Tsing Hua University, Hsinchu 30043, Taiwan, ROC*
[c] *Department of Electronics Engineering, National Chiao Tung University, Hsinchu 30043, Taiwan, ROC*

Available online 24 August 2005

Channeling through QDs



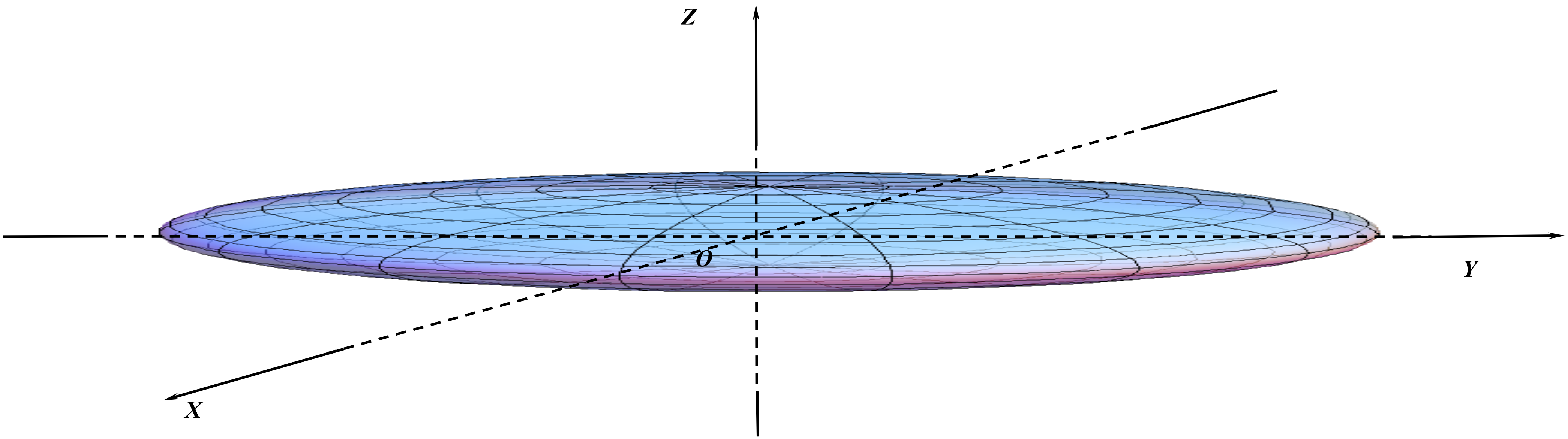
Atomic force microscope (AFM) photograph of the uncapped sample



Schematic view of (a) the uncapped and (b) the capped QDs samples

RBS/w channeling measurement was performed with 4 MeV $^{12}C^{2+}$ beam provided by the 9SDH-2 Tandem accelerator. The obtained results show the existence of strains in and around the synthesized QDs.

Ellipsoidal Quantum Dot



One particle problem

$$\Psi(x,y,z) = \psi_f(z;(x,y))\ \psi_s(x,y)$$

$$\begin{cases} \psi_f(z;(x,y)) = \sqrt{\dfrac{2}{L(x,y)}}\sin\left(\dfrac{\pi n_z}{L(x,y)}z + \delta_{n_z}\right), \\ \\ E^z_{n_z}(x,y) = \dfrac{\pi^2\hbar^2 n_z^2}{2\mu L^2(x,y)} \end{cases}$$

$$L(x,y) = 2c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

One particle problem

$$\frac{1}{L^2(x,y)} = \frac{1}{4c^2\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)}$$

$$\begin{cases} x \ll a \\ y \ll b \end{cases} \Rightarrow \frac{1}{L^2(x,y)} \approx \frac{1}{4c^2}\left\{1+\frac{x^2}{a^2}+\frac{y^2}{b^2}\right\} \Rightarrow E^z_{n_z}(x,y) = \frac{\pi^2\hbar^2 n_z^2}{8\mu c^2} + \frac{\mu\Omega_a^2(n_z)x^2}{2} + \frac{\mu\Omega_b^2(n_z)y^2}{2}$$

$$\begin{cases} \Omega_a(n_z) = \dfrac{\pi\hbar n_z}{2\mu ac} \\ \Omega_b(n_z) = \dfrac{\pi\hbar n_z}{2\mu bc} \end{cases}$$

One particle problem: 2D Schrodinger equation

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_s(x,y)+\frac{\mu\Omega_a^2(n_z)x^2}{2}\psi_s(x,y)+\frac{\mu\Omega_b^2(n_z)y^2}{2}\psi_s(x,y)=$$

$$=\left(E-\frac{\pi^2\hbar^2 n_z^2}{8\mu c^2}\right)\psi_s(x,y)\equiv E_{n_x,n_y}^{x,y}\psi_s(x,y).$$

One particle Wave function and Energy Spectrum

$$\Psi(x,y,z)=\sqrt{\frac{2}{L(x,y)}}\sin\left(\frac{\pi n_z}{L(x,y)}z+\delta_{n_z}\right)\times\left\{C_{n_x}e^{-\frac{x^2}{2a^2_{\Omega_a(n_z)}}}H_{n_x}\left(\frac{x}{a_{\Omega_a(n_z)}}\right)\right\}\times\left\{C_{n_y}e^{-\frac{y^2}{2a^2_{\Omega_a(n_z)}}}H_{n_y}\left(\frac{y}{a_{\Omega_a(n_z)}}\right)\right\}$$

$$E_{n_x,n_y,n_z}=\hbar\Omega_a(n_z)\left(n_x+\frac{1}{2}\right)+\hbar\Omega_b(n_z)\left(n_y+\frac{1}{2}\right)+\frac{\pi^2\hbar^2n_z^2}{8\mu c^2}$$

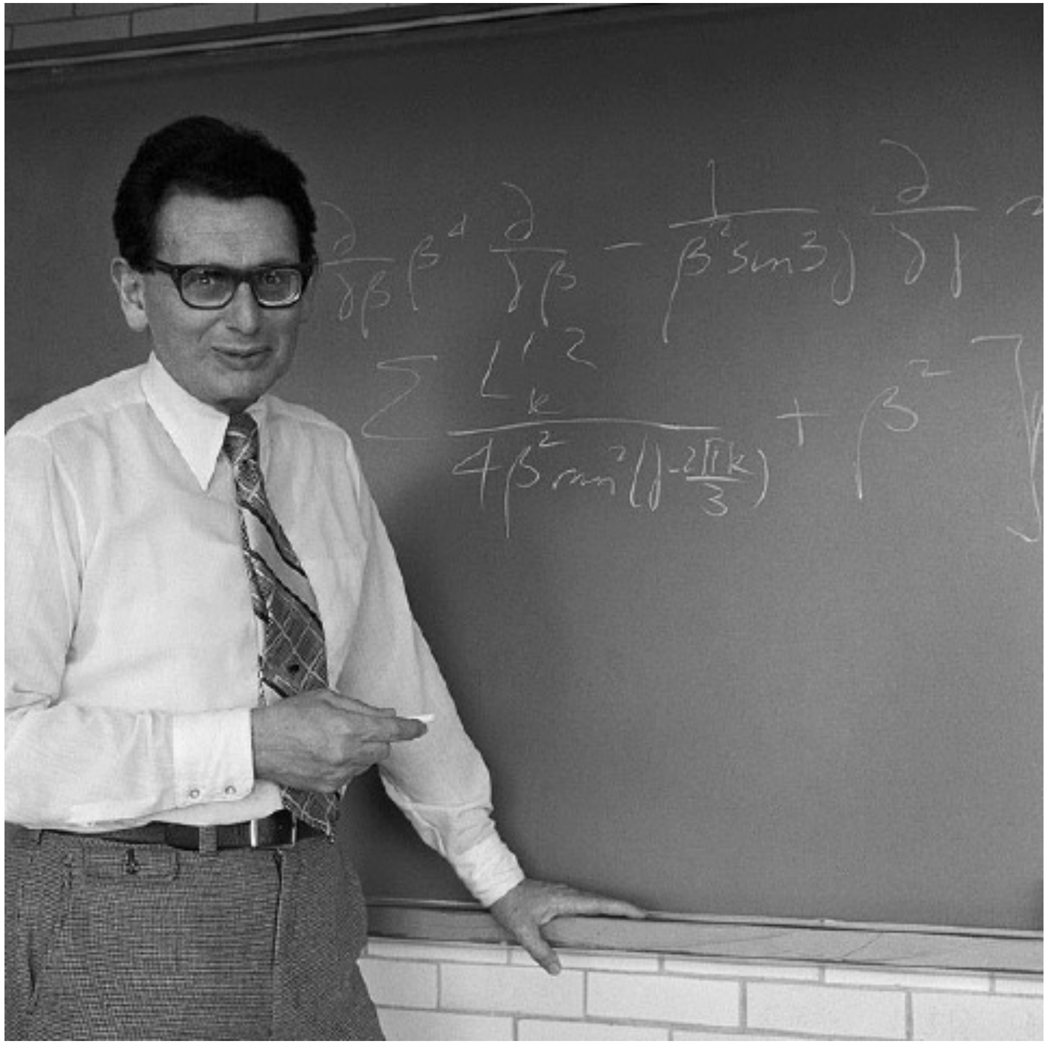
Few particle problem

$$\Psi(\vec{r}_1,\ldots,\vec{r}_N)=\left\{\prod_{i=1}^{N} f(z)\right\}\cdot F(\vec{\rho}_1,\ldots,\vec{\rho}_N)$$

$$\left\{\sum_{i=1}^{N}\left(\frac{\hat{P}_{x_i}^2+\hat{P}_{y_i}^2}{2\mu}\right)+\sum_{i=1}^{N}\frac{\mu\Omega_a^2(n_i^z)x_i^2}{2}+\sum_{i=1}^{N}\frac{\mu\Omega_b^2(n_i^z)y_i^2}{2}+\frac{1}{2}\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N} v(x_i,y_i,x_j,y_j)\right\}\psi_s(x_1,y_1,\ldots,x_N,y_N)=$$

$$=\left[E-\sum_{i=1}^{N}\frac{\pi^2\hbar^2 n_i^{z2}}{8\mu c^2}\right]\psi_s(x_1,y_1,\ldots,x_N,y_N)$$

Marcos Moshinsky (1921-2009)



Moshinsky Atom

How Good is the Hartree-Fock Approximation*

M. Moshinsky
Instituto de Fisica, Universidad de México, México, D. F., México
(Received 12 September 1967)

The problem of two particles in a common harmonic oscillator potential interacting through harmonic oscillator forces is discussed from the standpoint of the Hartree-Fock approximation and compared with the exact solution.

$$H = \tfrac{1}{2}\,[p_1{}^2 + r_1{}^2] + \tfrac{1}{2}\,[p_2{}^2 + r_2{}^2] + \varkappa\,[1/\sqrt{2}\,(\mathbf{r}_1 - \mathbf{r}_2)]^2$$

$$\mathbf{R} = 1/\sqrt{2}\,(\mathbf{r}_1 + \mathbf{r}_2),\;\; \mathbf{r} = 1/\sqrt{2}\,(\mathbf{r}_1 - \mathbf{r}_2)$$

$$H = \tfrac{1}{2}\,[P^2 + R^2] + \tfrac{1}{2}\,[p^2 + (2\varkappa + 1)\,r^2]$$

Moshinsky Model

$$\boxed{\frac{1}{2}\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N} v(x_i,y_i,x_j,y_j)=\frac{1}{2}\left\{\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}\gamma_1(x_i-x_j)^2+\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}\gamma_2(y_i-y_j)^2\right\}}$$

$$\begin{cases}\Omega_a(n_1^z)=\Omega_a(n_2^z)=\ldots=\Omega_a(n_N^z)=\Omega_a(1)=\dfrac{\pi\hbar}{2\mu ac}\\[2ex] \Omega_b(n_1^z)=\Omega_b(n_2^z)=\ldots=\Omega_b(n_N^z)=\Omega_b(1)=\dfrac{\pi\hbar}{2\mu bc}\end{cases}$$

$$\left\{\sum_{i=1}^{N}\left(\frac{\widehat{P}_{x_i}^2+\widehat{P}_{y_i}^2}{2\mu}\right)+\frac{\mu\Omega_a^2(1)}{2}\sum_{i=1}^{N}x_i^2+\frac{\mu\Omega_b^2(1)}{2}\sum_{i=1}^{N}y_i^2+\frac{1}{2}\gamma_1\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}(x_i-x_j)^2+\frac{1}{2}\gamma_2\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}(y_i-y_j)^2\right\}$$

$$\psi_s(x_1,\ldots,x_N,y_1,\ldots,y_N)=\left[E-N\frac{\pi^2\hbar^2}{8\mu c^2}\right]\psi_s(x_1,\ldots,x_N,y_1,\ldots,y_N)\equiv \mathrm{E}(N)\psi_s(x_1,\ldots,x_N,y_1,\ldots,y_N)$$

Two Schrodinger equations along the axes OX and OY

$$\left\{\frac{1}{2\mu}\sum_{i=1}^{N}\hat{P}_{x_i}^{2}+\frac{\mu\Omega_a^2(1)}{2}\sum_{i=1}^{N}x_i^2+\frac{1}{2}\gamma_1\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}(x_i-x_j)^2\right\}\psi_s^x(x_1,\dots,x_N)=E^x\psi_s^x(x_1,\dots,x_N)$$

$$\left\{\frac{1}{2\mu}\sum_{i=1}^{N}\hat{P}_{y_i}^{2}+\frac{\mu\Omega_b^2(1)}{2}\sum_{i=1}^{N}y_i^2+\frac{1}{2}\gamma_2\sum_{i=1,i\neq j}^{N}\sum_{j=1,i\neq j}^{N}(y_i-y_j)^2\right\}\psi_s^y(y_1,\dots,y_N)=E^y\psi_s^y(y_1,\dots,y_N)$$

$$E^x \equiv \mathrm{E}(N)-E^y$$

Transition to new variables and dimensionless Schrodinger equations for X and Y

$$\xi = \frac{x}{\sqrt{\frac{\hbar}{\mu\Omega_a(1)}}}, W_x(N) = \frac{E^x}{\hbar\Omega_a(1)}, g_1 = \frac{\gamma_1}{\mu\Omega_a^2(1)} \qquad \zeta = \frac{y}{\sqrt{\frac{\hbar}{\mu\Omega_b(1)}}}, W_y(N) = \frac{E^y}{\hbar\Omega_b(1)}, g_2 = \frac{\gamma_2}{\mu\Omega_b^2(1)}$$

$$\begin{cases} -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2\psi_s^x}{\partial\xi_i^2} + \left[\frac{1}{2}\sum_{i=1}^{N}\xi_i^2 + \frac{g_1}{2}\sum_{\substack{i=1\\ i\neq j}}^{N}\sum_{j=1}^{N}(\xi_i-\xi_j)^2\right]\psi_s^x = W_x\psi_s^x \\ -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2\psi_s^y}{\partial\zeta_i^2} + \left[\frac{1}{2}\sum_{i=1}^{N}\zeta_i^2 + \frac{g_2}{2}\sum_{\substack{i=1\\ i\neq j}}^{N}\sum_{j=1}^{N}(\zeta_i-\zeta_j)^2\right]\psi_s^y = W_y\psi_s^y \end{cases}$$

Jacobi coordinates and Schrodinger equations

$$X = \frac{\xi_1 + \xi_2 \ldots + \xi_N}{\sqrt{N}}, \qquad X_i = \sqrt{\frac{i-1}{i}}\left(\xi_i - \frac{1}{i-1}\sum_{k=1}^{i-1}\xi_k\right), \quad i = 2,3,\ldots,N$$

$$Y = \frac{\zeta_1 + \zeta_2 \ldots + \zeta_N}{\sqrt{N}}, \qquad Y_i = \sqrt{\frac{i-1}{i}}\left(\zeta_i - \frac{1}{i-1}\sum_{k=1}^{i-1}\zeta_k\right), \quad i = 2,3,\ldots,N$$

$$\begin{cases} \left[\left(-\frac{1}{2}\frac{\partial^2}{\partial X^2} + \frac{X^2}{2}\right) + \sum_{i=2}^{N}\left(-\frac{1}{2}\frac{\partial^2}{\partial X_i^2} + \frac{1}{2}\tilde{\Omega}_a^2 X_i^2\right)\right]\psi_s^x = W_x \psi_s^x, \\ \left[\left(-\frac{1}{2}\frac{\partial^2}{\partial Y^2} + \frac{Y^2}{2}\right) + \sum_{i=2}^{N}\left(-\frac{1}{2}\frac{\partial^2}{\partial Y_i^2} + \frac{1}{2}\tilde{\Omega}_b^2 Y_i^2\right)\right]\psi_s^y = W_y \psi_s^y \end{cases}$$

$$\tilde{\Omega}_a = \sqrt{1 + 2Ng_1}$$

$$\tilde{\Omega}_b = \sqrt{1 + 2Ng_2}$$

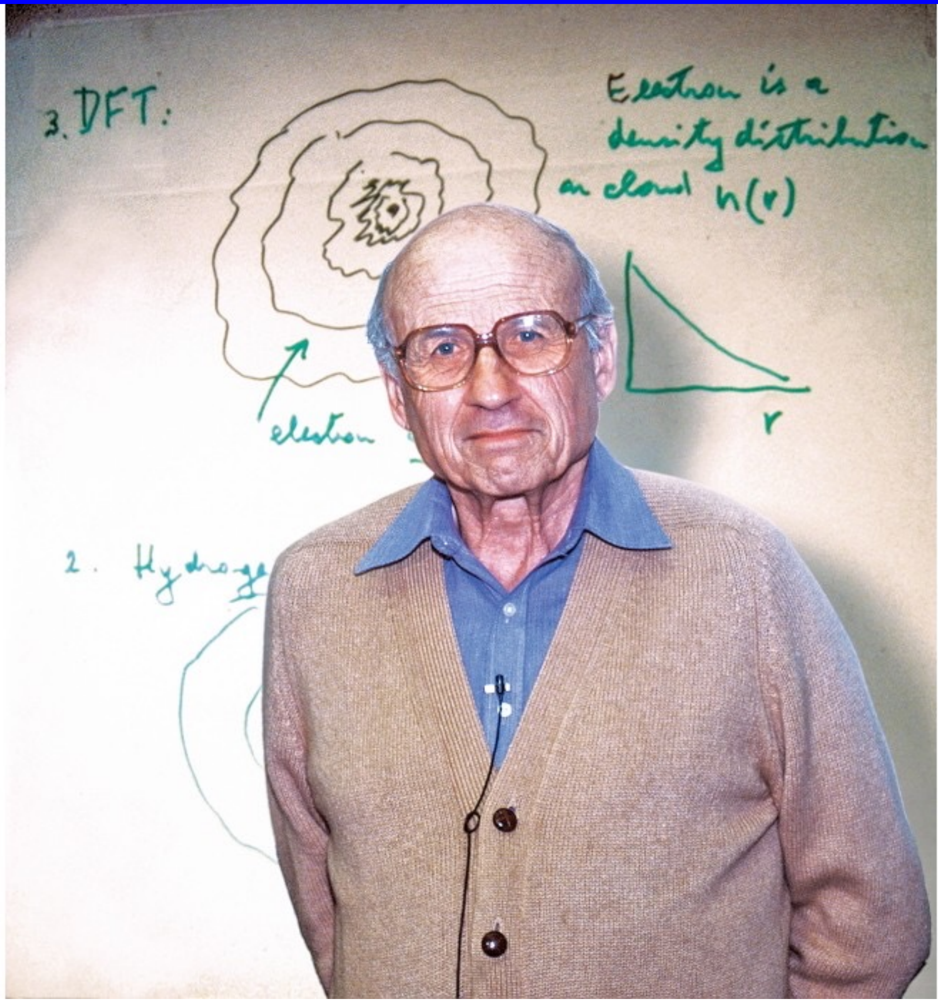
System energy spectrum and wave functions

$$E_{state} = N\frac{\pi^2\hbar^2}{8\mu c^2} + \hbar\Omega_a(1)\left(n^x_{c.m.}+\frac{1}{2}\right) + \hbar\Omega_a(1)\cdot\varpi_a\sum_{i=2}^{N}\left(n^x_{rel.i}+\frac{1}{2}\right) + \hbar\Omega_b(1)\left(n^y_{c.m.}+\frac{1}{2}\right) + \hbar\Omega_b(1)\cdot\varpi_b\sum_{i=2}^{N}\left(n^y_{rel.i}+\frac{1}{2}\right)$$

$$state \equiv \left\{n^x_{c.m.}, n^x_{rel}, n^y_{c.m.}, n^y_{rel}\right\}$$

$$\begin{cases} \psi(X, X_i) = \dfrac{1}{\sqrt{2^{n^x_{c.m.}}\, n^x_{c.m.}!}}\left(\dfrac{1}{\pi}\right)^{1/4} e^{-X^2/2} H_{n^x_{c.m.}}(X) \displaystyle\prod_{i=2}^{N} \dfrac{1}{\sqrt{2^{n^x_{rel_i}}\, n^x_{rel_i}!}}\left(\dfrac{\varpi_a}{\pi}\right)^{1/4} e^{-\frac{\varpi_a}{2}X_i^2} H_{n^x_{rel_i}}\left(\sqrt{\varpi_a}X_i\right), \\ \psi(Y, Y_i) = \dfrac{1}{\sqrt{2^{n^y_{c.m.}}\, n^y_{c.m.}!}}\left(\dfrac{1}{\pi}\right)^{1/4} e^{-Y^2/2} H_{n^y_{c.m.}}(Y) \displaystyle\prod_{i=2}^{N} \dfrac{1}{\sqrt{2^{n^y_{rel_i}}\, n^y_{rel_i}!}}\left(\dfrac{\varpi_b}{\pi}\right)^{1/4} e^{-\frac{\varpi_b}{2}Y_i^2} H_{n^y_{rel_i}}\left(\sqrt{\varpi_b}Y_i\right) \end{cases}$$

Walter Kohn (1923-2016)



Generalized Kohn theorem: The frequency of resonant absorption of long-wave radiation from a pair-interacting electron gas localized in a parabolic QD doesn't depend on the number of particles. In other words, single-particle transitions are realized in a multiparticle system.

Kohn's theorem in the case of asymmetric parabolic QD

RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 42, NUMBER 2 15 JULY 1990-I

Magneto-optics in parabolic quantum dots

F. M. Peeters*

Bellcore, 331 Newman Springs Road, Red Bank, New Jersey 07701-7020

(Received 3 April 1990)

We show that the position of the resonance lines in the magneto-optical absorption spectrum of a quantum dot with a (asymmetric) parabolic confinement potential is independent of the electron-electron interaction and the number of electrons in the quantum dot.

Hamiltonian for N noninteracting electrons in an asymmetric QD, its diagonalization and energy spectrum

$$H_0 = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \frac{1}{2} m \sum_{j=1}^{N} \left(\omega_x^2 x_j^2 + \omega_y^2 y_j^2 \right) \Rightarrow H_0 = \hbar\omega_x \left(C_x^+ C_x^- + \frac{1}{2} \right) + \hbar\omega_y \left(C_y^+ C_y^- + \frac{1}{2} \right)$$

$$C_{x(y)}^+ = \sum_{j=1}^{N} c_{j,x(y)}^+ \qquad c_{j,x(y)}^+ = \left(\frac{m\omega_{x(y)}}{2\hbar} \right)^{1/2} \left(x_j(y_j) - i \frac{p_{j,x(y)}}{m\omega_{x(y)}} \right) \qquad c_{j,x(y)}^- = \left(c_{j,x(y)}^+ \right)^+$$

$$E_{n_x,n_y} = \hbar\omega_x \left(n_x + \frac{1}{2} \right) + \hbar\omega_y \left(n_y + \frac{1}{2} \right)$$

Hamiltonian of the interacting system, electron-electron interaction and commutation relations

$$H = H_0 + U \qquad U(1,\dots,N) = \sum_{i<j=1}^{N} u(\vec{r}_i - \vec{r}_j) \qquad u(\vec{r}) = \frac{e^2}{\varepsilon\left|\vec{r}\right|}$$

$$\left[H_0, C_x^+\right]\psi^0_{n_x,n_y} = H_0 C_x^+ \psi^0_{n_x,n_y} - C_x^+ H_0 \psi^0_{n_x,n_y} = \hbar\Omega_a(1) C_x^+ \psi^0_{n_x,n_y}$$

$$\psi^0_{n_x,n_y} \to E^0_{n_x,n_y} \quad \Rightarrow \quad C_x^+ \psi^0_{n_x,n_y} \to \hbar\Omega_{a,b}(1) + E^0_{n_x,n_y}$$

$$\left[U(1,\dots,N), C^{\pm}_{x,y}\right] = 0 \quad \Rightarrow \quad \left[H, C^{\pm}_{x,y}\right] = \pm\hbar\Omega_{a,b}(1) \times C^{\pm}_{x,y}$$

System in an electric field, commutation relation

$$H = H_0 + H'$$

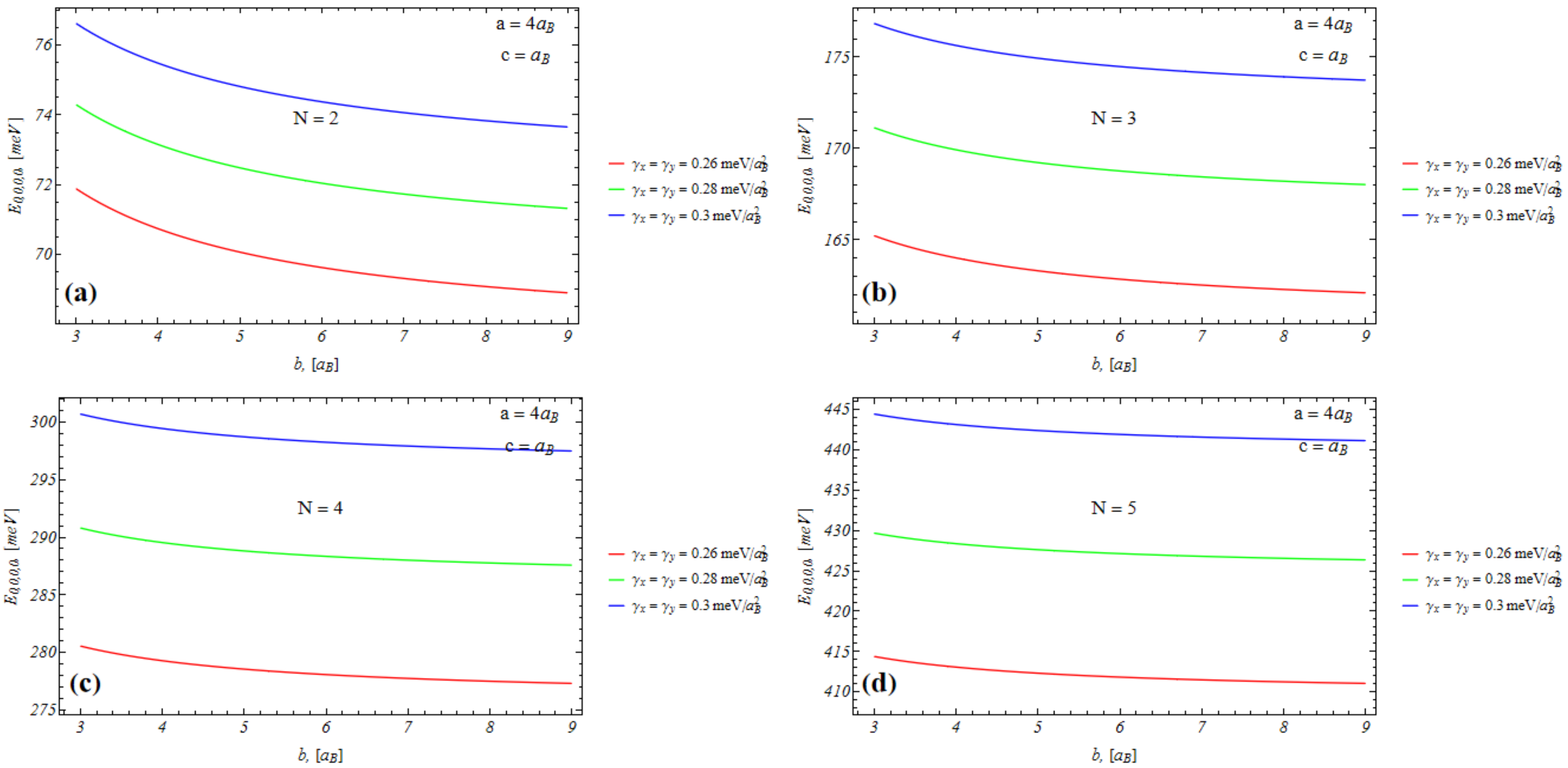
$$E(t) = e^{-i\omega t} E_0(\cos\theta, \sin\theta) \quad \Rightarrow \quad H' = -e\sum_j \vec{r}_j \cdot \vec{E}(t)$$

$$\begin{cases} \sum_j x_j = (\hbar / m\omega_x)^{1/2}(C_x^- + C_x^+) \\ \sum_j y_j = (\hbar / m\omega_y)^{1/2}(C_y^- + C_y^+) \end{cases}$$

$$\left[H, C_{x,y}^{\pm}\right] = \left[H_0 + H', C_{x,y}^{\pm}\right] = \left[H_0, C_{x,y}^{\pm}\right] + \left[H', C_{x,y}^{\pm}\right]$$

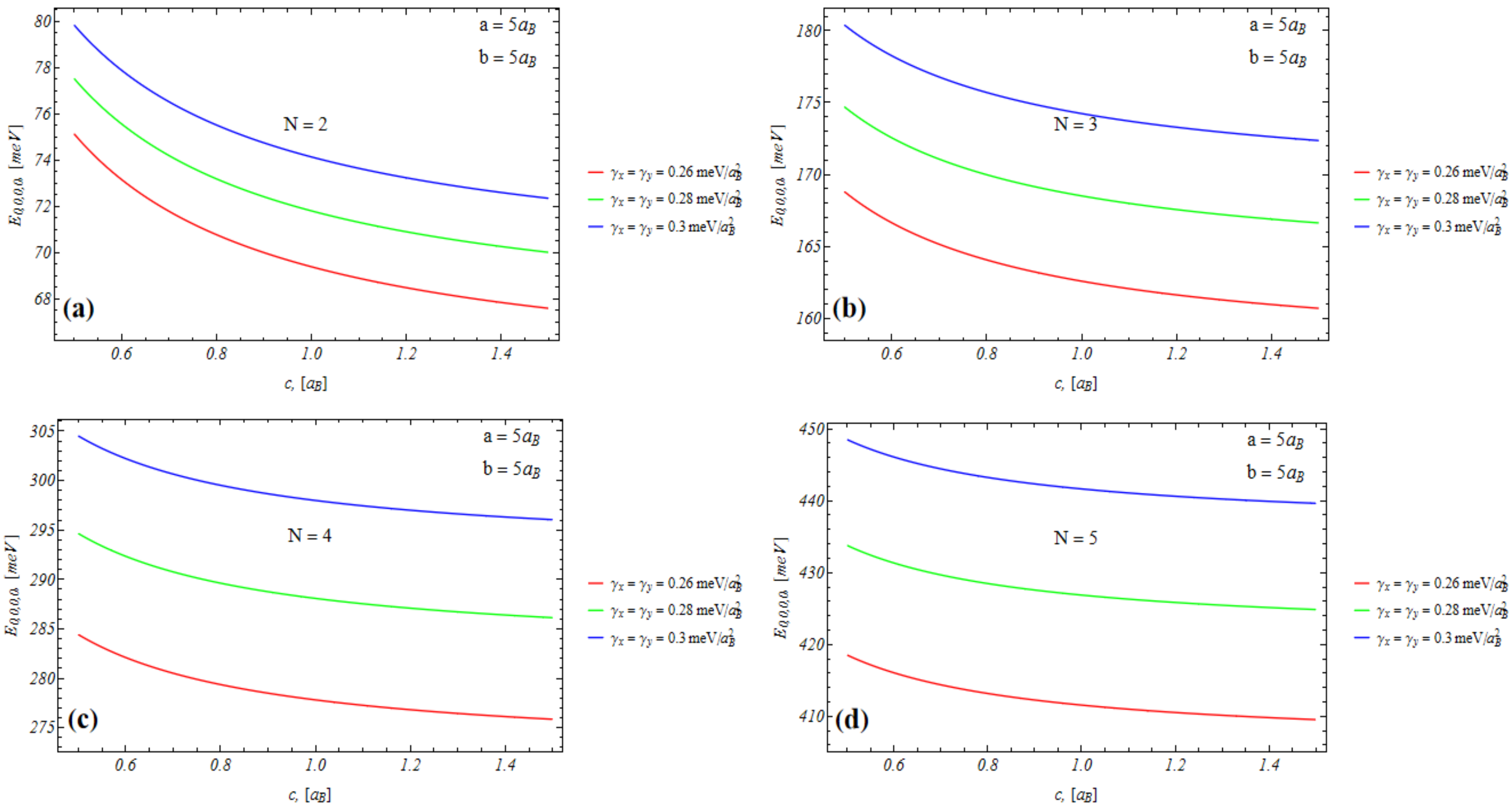
$$\left[H', C_{x,y}^{\pm}\right] = 0 \quad \Rightarrow \quad \left[H, C_{x,y}^{\pm}\right] = \left[H_0, C_{x,y}^{\pm}\right] = \pm\hbar\Omega_{a,b}(1) \times C_{x,y}^{\pm}$$

Obtained results



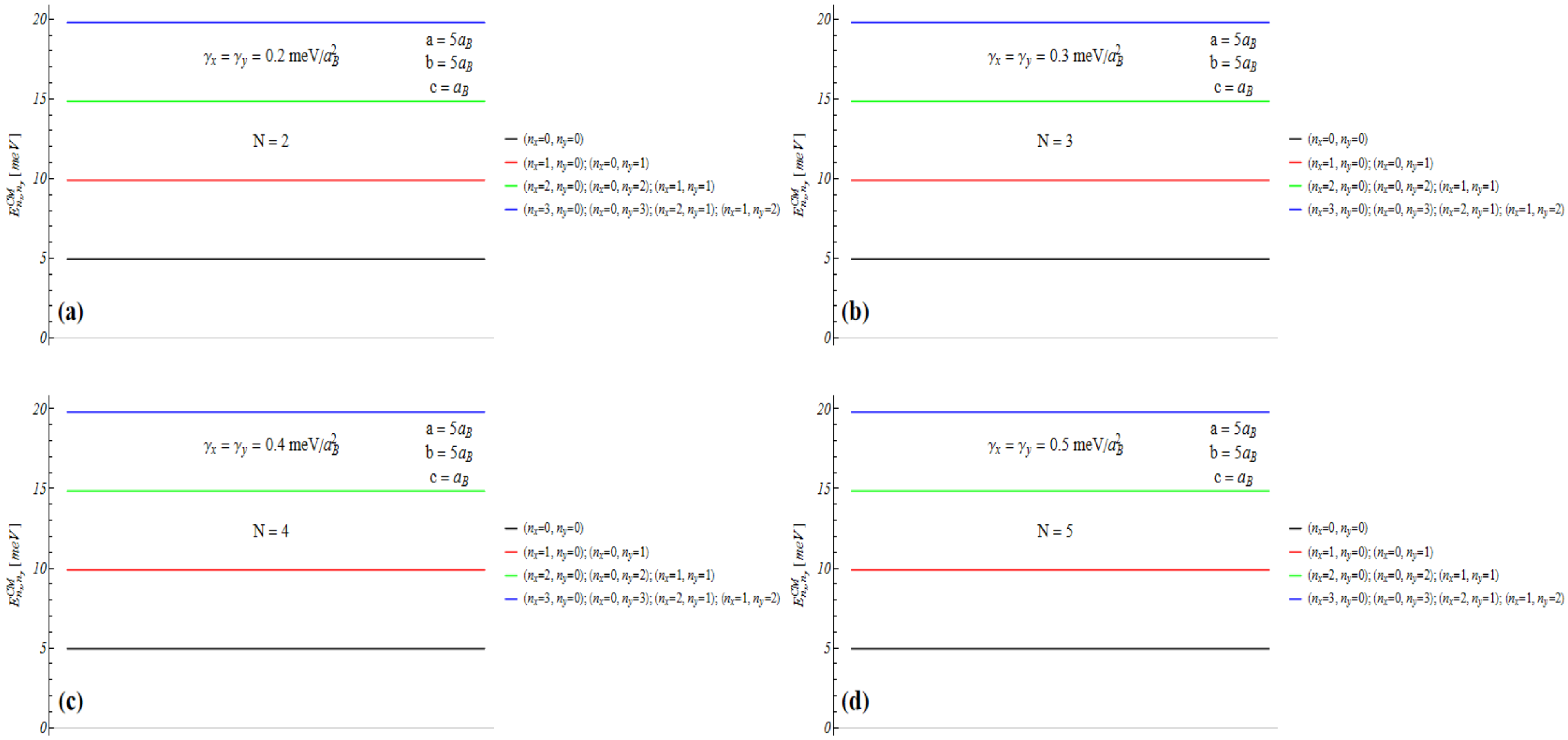
Electron gas total energy dependency from QD semiaxis b for different interaction parameter $\gamma_{1(2)}$

Obtained results



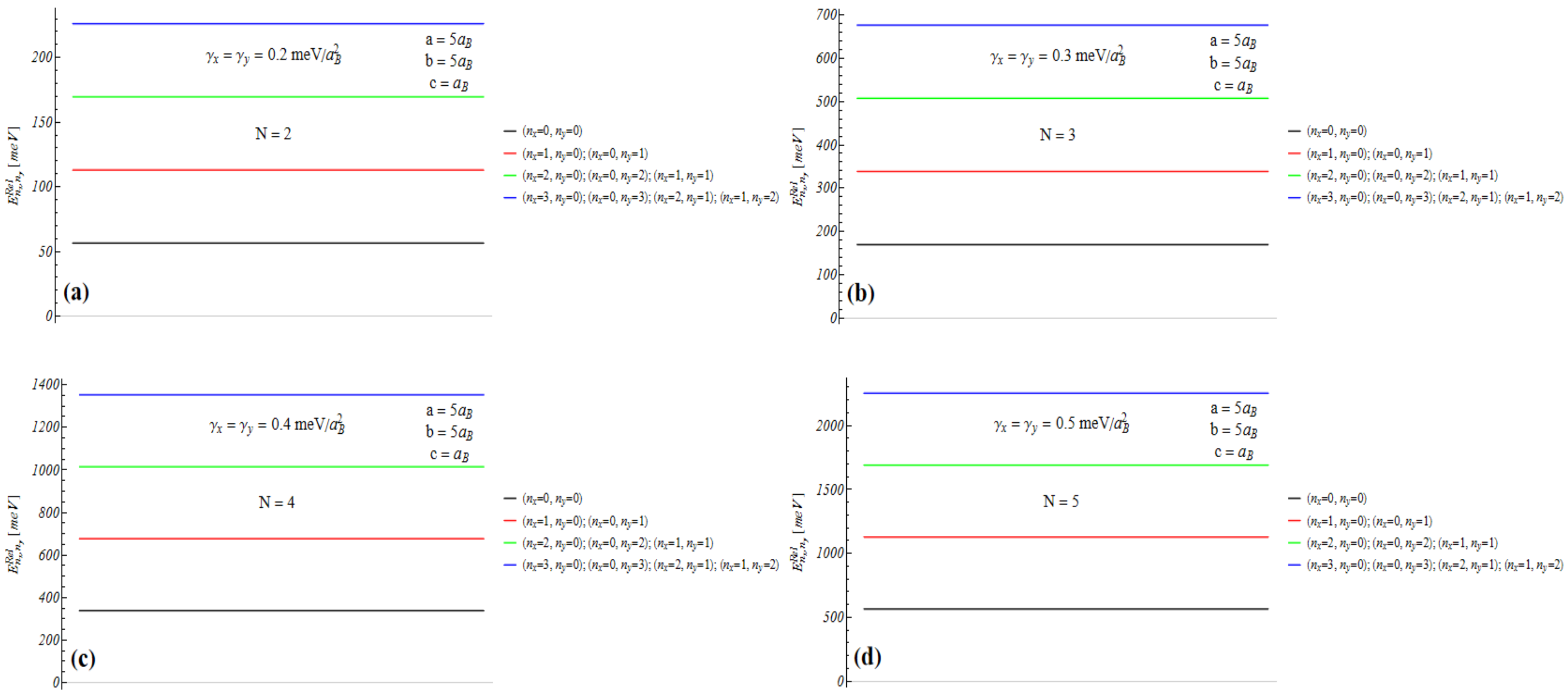
Electron gas total energy dependency from QD semiaxis *c* for different interaction parameter $\gamma_{1(2)}$

Obtained results



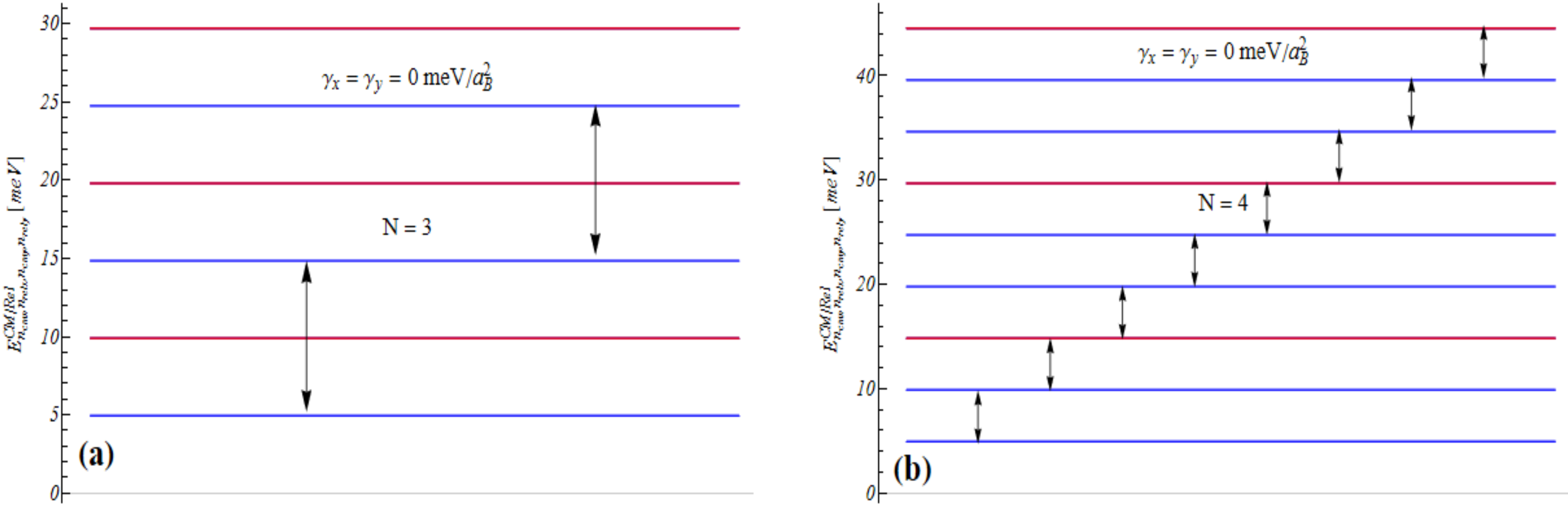
Electron gas center of mass energy diagram

Obtained results



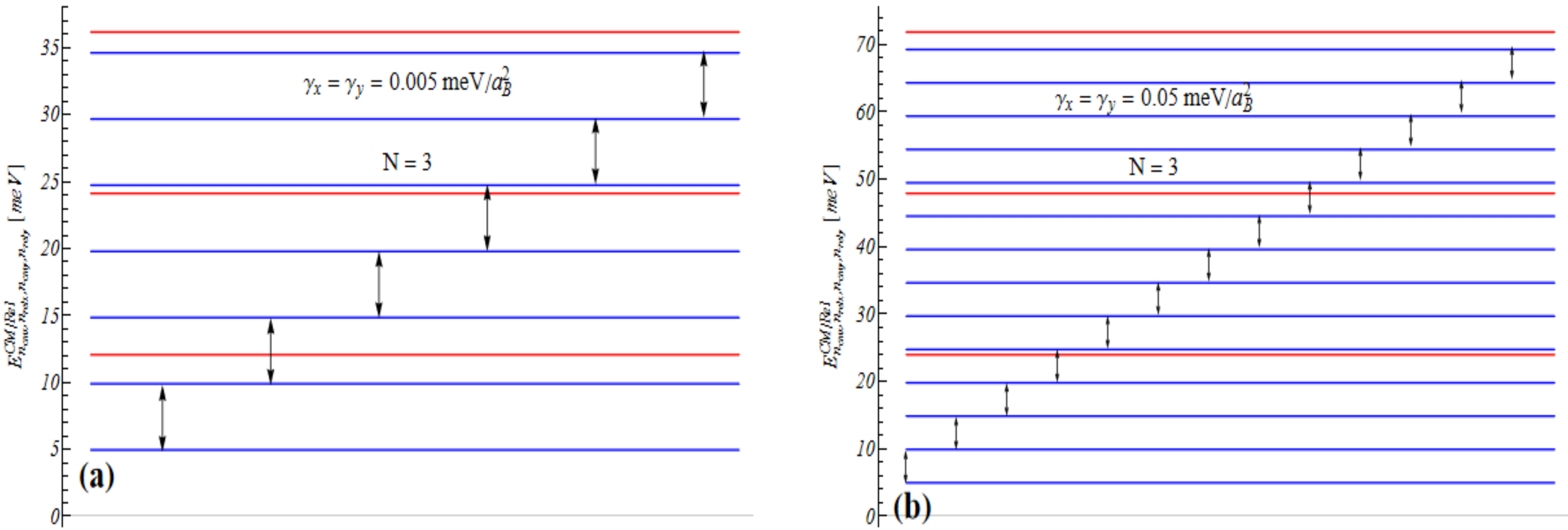
Electron gas relative energy diagram

Obtained results



Electron gas total energy diagram

Obtained results



Electron gas total energy diagram

Main Results:

1. In the case of a strongly oblate asymmetric ellipsoidal quantum dot, the confining potential of the slow subsystem is described within the framework of a two-dimensional asymmetric parabolic well.

2. In the quantum dot under consideration, for the case of a pair-interacting electron gas, the conditions for fulfilling the generalized Kohn theorem are realized.

3. The resonance frequencies of the transitions described by the generalized Kohn theorem depend on the geometric parameters of the ellipsoidal quantum dot and can be controlled by changing these parameters.

Thank You!