Radiation of surface polaritons by a charge circulating inside a dielectric cylindrical waveguide

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Outline

- Radiation processes in media
- Surface plasmon polaritons
- Radiation of SP in cylindrical waveguides: Electric and magnetic fields
- Radiation intensity for SP
- Conclusions

Radiation processes in media

Interaction of charged particles with media gives rise to various types of radiation processes

- Cherenkov radiation
- Transition radiation
- Diffraction and Smith-Purcell radiation
- Parametric X-ray radiation

Previous studies of our group

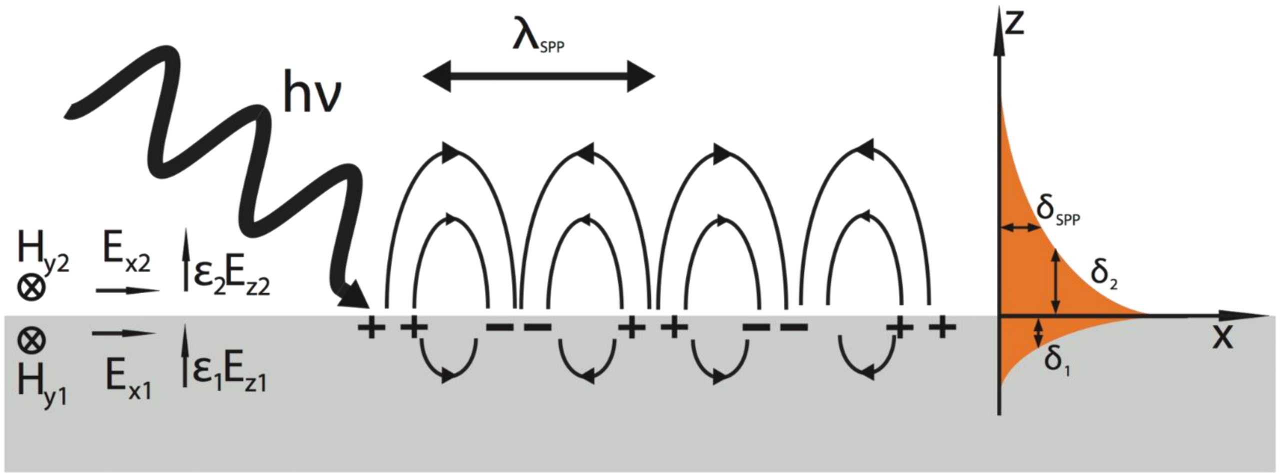
1. A. S. Kotanjyan, A. R. Mkrtchyan, A. A. Saharian, V. Kh. Kotanjyan, *Generation of surface polaritons in dielectric cylindrical waveguides.* Phys. Rev. Accelerators and Beams 22, 040701 (2019).
2. A. A. Saharian, A. S. Kotanjyan, V. Kh. Kotanjyan, *Synchrotron radiation from a charge on eigenmodes of a dielectric cylinder*. J. Contemp. Phys. 54, 111-116 (2019).
3. A.A. Saharian, A.S. Kotanjyan, L.Sh. Grigoryan, H.F. Khachatryan, V.Kh. Kotanjyan, *Synchrotron radiation from a charge circulating around a cylinder with negative permittivity*. Int. J. Mod. Phys. B 34, 2050065 (2020).
4. A.A. Saharian, L.Sh. Grigoryan, A.Kh. Grigorian, H.F. Khachatryan, A.S. Kotanjyan, *Cherenkov radiation and emission of surface polaritons from charges moving paraxially outside a dielectric cylindrical waveguide*. Phys. Rev. A 102, 063517 (2020).
5. A.A. Saharian, L.Sh. Grigoryan, A.S. Kotanjyan, H.F. Khachatryan, *Surface polariton excitation and energy losses by a charged particle in cylindrical waveguides*. Phys. Rev. A 107, 063513 (2023).
6. A. A. Saharian, S. B. Dabagov, H. F. Khachatryan, L. Sh. Grigoryan, *Quasidiscrete spectrum Cherenkov radiation by a charge moving inside a dielectric waveguide*. JINST 19, C06017 (2024).

Surface Plasmon Polaritons

- Interfaces between two media with different electromagnetic characteristics give arise to the existence of new types of electromagnetic modes
- Surface modes depend on the geometry of the separating boundary and carry an important information on the electromagnetic properties of the contacting media
- Among the various types of surface waves, the surface plasmon polaritons have been a powerful tool in the wide range of investigations including
  - Surface imaging
  - Surface-enhanced Raman spectroscopy
  - Data storage and Biosensors
  - Plasmonic waveguides
  - Light-emitting devices
  - Plasmonic solar cells…

Surface Plasmon Polaritons

- SPPs are evanescent electromagnetic waves propagating along a metal-dielectric interface as a result of collective oscillations of electron subsystem coupled to electromagnetic field
- SPPs exist in frequency ranges where the real part of the permittivity undergoes a change of the sign at the interface
- Perpendicular to the interface SPPs have subwavelength-scale confinement



- Remarkable properties of SPPs include
  - Possibility of concentrating electromagnetic fields beyond the diffraction limit of light waves
  - Enhancing the local field strengths by orders of magnitude

Surface Polaritons

- Although SPPs are the most thoroughly investigated type of surface polaritons, depending on the dielectric properties of the active medium other forms of surface polaritons may exist

- In particular, other materials besides metals, such as semiconductors, organic and inorganic dielectrics, ionic crystals, can support surface polariton type waves

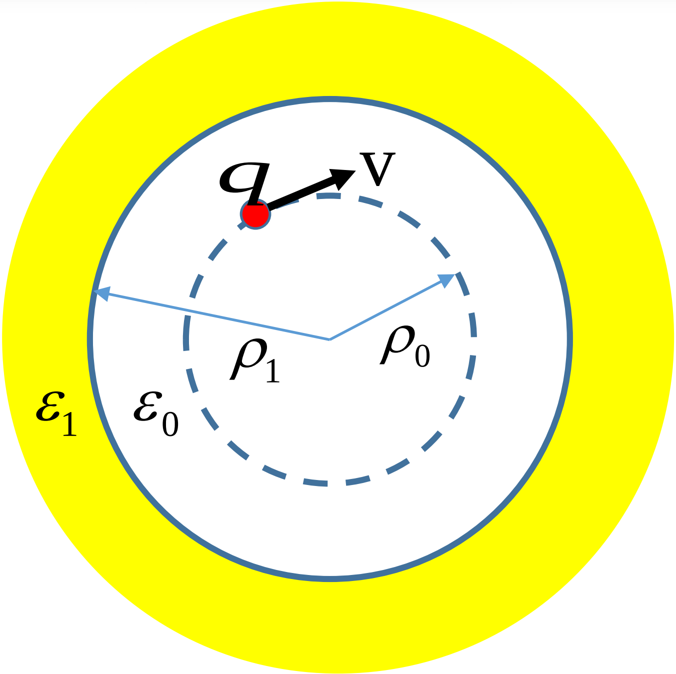
- An important direction of recent developments is the extension of plasmonics to the infrared and terahertz ranges of frequencies.

- This can be done by a suitable choice of the active medium such as doped semiconductors and artificially constructed materials (metamaterials)

Generation of Surface Polaritons

- Techniques used to excite surface polariton modes include
  - Prism and grating coupling
  - Strongly focused optical beams
  - Guided photonic modes from another waveguide
  - Electron beams (in particular, in scanning electron microscopes)
- The first experimental observation of surface plasmon polaritons was based on measurements of the electron energy loss spectra

Problem setup



- Cylindrical waveguide immersed into homogeneous medium with dielectric permittivity ε_1
- Charge rotates along a circular trajectory coaxial with the cylinder
- Types of the radiation present:

- Radiation at large distances from the cylinder (Synchrotron radiation in a medium influenced by the cylinder, Cherenkov radiation)
- Radiation of guided modes
- Radiation of surface polaritons

Radiation at large distances

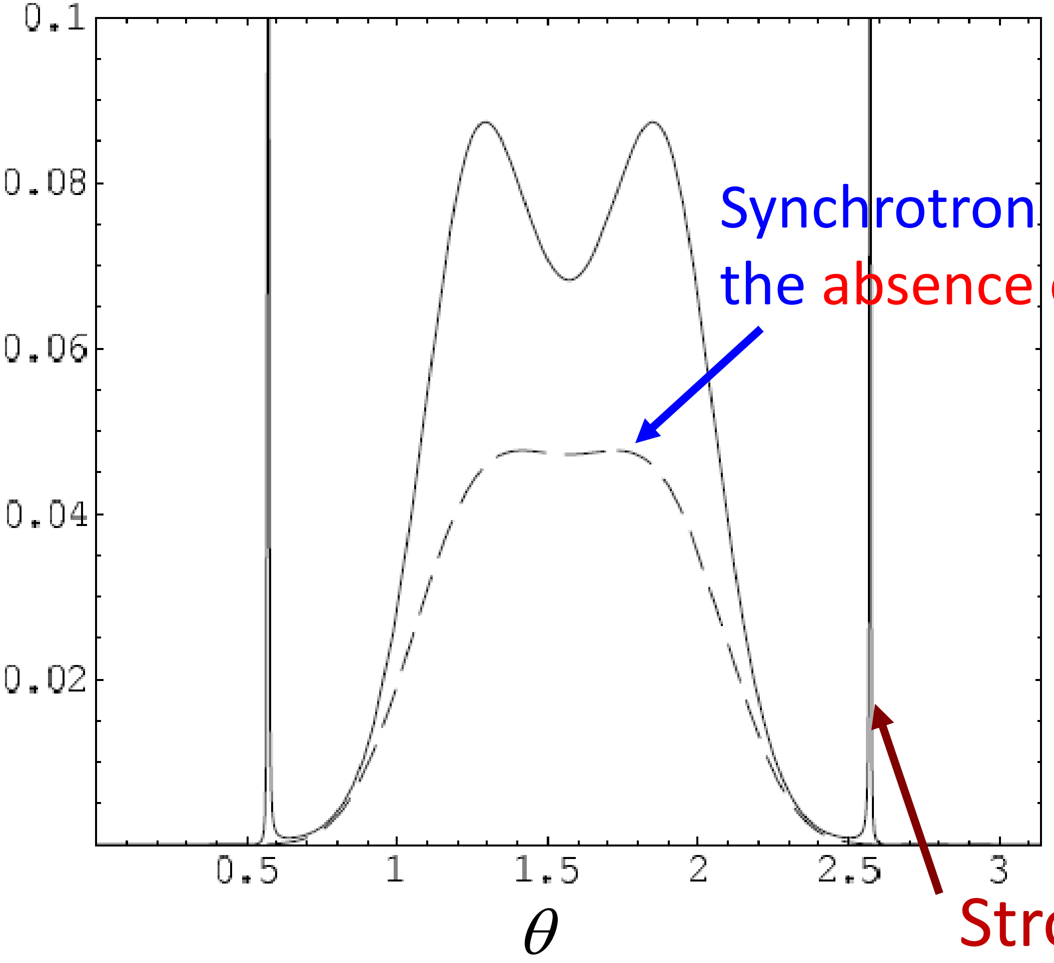
- Under the Cherenkov condition for the material of the cylinder and the velocity of the particle image on the cylinder surface, strong narrow peaks appear in the angular distribution of the radiation intensity
- At the peaks the radiated energy exceeds the corresponding quantity in the case of a homogeneous medium by orders of magnitude
- Necessary condition for the appearance: $\varepsilon_0 > \varepsilon_1, \quad \tilde{v}\sqrt{\varepsilon_0}/c > 1$ (Velocity of particle image on the cylinder surface)
- Angular range: $\cos^2\vartheta > \varepsilon_0/\varepsilon_1$ (ϑ – Radiation direction with respect to the cylinder axis)
- Equation determining the angular locations of the peaks is obtained from the equation for eigenmodes of cylinder by the replacement

Hankel function → $H_m \to Y_m$ ← Neumann function

Synchrotron radiation in the exterior medium

Angular density for the number of radiated quanta per period

$(T\sqrt{\varepsilon_1}\hbar c/q^2)\,dN_m/d\Omega$



Strong narrow peaks appear

Radiation spectrum is discrete

$\omega = m\omega_0$ $\quad T = 2\pi/\omega_0$

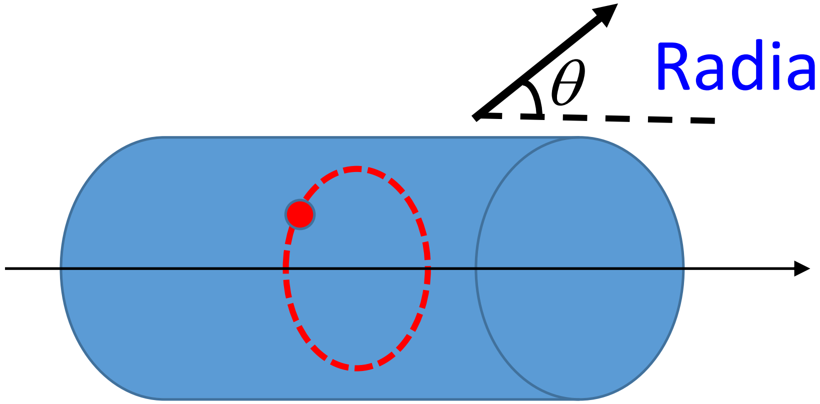
$v/c = 0.9$

$\varepsilon_1 = 1,\ \varepsilon_0 = 3,$

$\rho_1/\rho_0 = 1.05,$

$m = 10$ ⇦ Radiation harmonic

Radiation direction



Radiation fields inside the cylinder

- Waves propagating inside the cylinder are radiated on the eigenmodes of the cylinder with the frequency $n\omega_0$ (ω_0 ⟸ angular velocity of the charged particle)
- For the corresponding modes $\lambda_1^2 < 0$, and the radial dependence is in the form of the function $K_{n+p}(|\lambda_1|\rho)$ $p = 0, \pm 1$ $\lambda_j^2 = n^2\omega_0^2\varepsilon_j/c^2 - k_z^2, \quad j = 0, 1$ ⟸ $\rho > \rho_1$
- Dependence on the radial coordinate for a given mode is described by the function $J_{n+p}(\lambda_0\rho)$ ⟸ $\rho < \rho_1$
- Guiding modes (oscillating modes): $\lambda_0^2 > 0$
- Surface-type modes: $\lambda_0^2 < 0$ (radial dependence is in the form $I_{n+p}(|\lambda_0|\rho)$)
- Surface-type modes are present if the permittivities (real parts) have opposite signs

$$\varepsilon_0' \cdot \varepsilon_1' < 0$$

- In the limit $\rho, \rho_1, \rho_0 \to \infty$ with $\rho - \rho_0, \rho - \rho_1$ fixed, surface polaritons are obtained in the geometry of planar boundary

Electromagnetic fields inside the cylinder

- Fourier expansion of the fields

$$F_l(\mathbf{r},t) = \sum_{m=-\infty}^{\infty} e^{im(\phi-\omega_0 t)} \int_{-\infty}^{\infty} dk_z e^{ik_z z} F_{ml}(k_z,\rho),\ F = E, H$$

For electric and magnetic fields

- Cylindrical components of the magnetic field strength ($l = \rho, \phi, z$)

$$H_{ml} = \frac{qvk_z}{4ci^{\sigma_l - 1}} \sum_{p=\pm 1} p^{\sigma_l - 1} \left[J_{m+p}(\lambda_0\rho_0) H_{m+p}(\lambda_0\rho) + B_{1,m}^{(p)} J_{m+p}(\lambda_0\rho) \right],\ l = \rho, \phi,$$

Hankel (1) function — Bessel function

$$\sigma_\rho = 1,\ \sigma_\phi = 2$$

$$H_{mz} = \frac{iqv\lambda_0}{4c} \sum_{p=\pm 1} p \left[J_{m+p}(\lambda_0\rho_0) H_m(\lambda_0\rho) + B_{1,m}^{(p)} J_m(\lambda_0\rho) \right],$$

Parts corresponding to the fields in a homogeneous medium with $\varepsilon = \varepsilon_0$

Electric field is found from

$$\mathbf{E} = ic(\omega\varepsilon_0)^{-1} \nabla \times \mathbf{H}$$

- Notations: $B_{1,m}^{(p)} = -J_{m+p}(\lambda_0\rho_0) \frac{W_{m+p}^H}{W_{m+p}^J} + \frac{ip\lambda_1 H_{m+p}(\lambda_1\rho_1)}{\pi\rho_1\alpha_m W_{m+p}^J} H_m(\lambda_1\rho_1) \sum_{l=\pm 1} \frac{J_{m+l}(\lambda_0\rho_0)}{W_{m+l}^J},$

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{1}{2}\lambda_0 J_m(\lambda_0\rho_1) \sum_{l=\pm 1} l \frac{H_{m+l}(\lambda_1\rho_1)}{W_{m+l}^J},\quad W_m^F = \lambda_1 F_m(\lambda_0\rho_1) H_m'(\lambda_1\rho_1) - \lambda_0 H_m(\lambda_1\rho_1) F_m'(\lambda_0\rho_1)$$

- Eigenmodes of the cylinder for real dielectric permittivities are roots of the equation

$$\alpha_m = 0$$

Surface-type modes

- We consider the case $\varepsilon_0 > 0$, Surface-type modes are present under the condition

$$|k_z| > m\omega_0\sqrt{\varepsilon_0}/c$$

- Equation determining the eigenvalues for the projection of wave vector on the cylinder axis for a given radiation harmonic: $k_z = k_{m,s}$

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} + \frac{1}{2}\sum_{l=\pm 1}\left[1 + \frac{\gamma_1 I_{m+l}(\gamma_0\rho_1)K_m(\gamma_1\rho_1)}{\gamma_0 I_m(\gamma_0\rho_1)K_{m+l}(\gamma_1\rho_1)}\right]^{-1} = 0$$

- Energy losses per unit time $\frac{dW}{dt} = q\mathbf{v}\cdot\mathbf{E} = qvE_2|_{\phi=\omega_0 t, z=0}$

- Energy losses per unit time on a given harmonic

$$\frac{dW_m}{dt} = 2qv\mathrm{Re}\left[\int_0^\infty dk_z\, E_{m2}(m, k_z, \rho_0)\right]$$

Energy losses

- Energy losses per unit time in the range $(k_z, k_z + dk_z)$ ⇨ dW_m

$$\frac{dW_m}{dk_z} = \frac{dW_{0m}}{dk_z} - \frac{q^2 v^2}{2\pi m \omega_0} \mathrm{Im}\left[\frac{1}{\varepsilon_0}\sum_{p,j=\pm 1}\left(\frac{m^2\omega_0^2\varepsilon_0}{c^2} + pjk_z^2\right) I_{m+j}(\gamma_0\rho_0) B_m^{(p)}\right]$$

Energy losses in a homogeneous medium with permittivity ε_0

$$\gamma_j = \sqrt{k_z^2 - m^2\omega_0^2\varepsilon_j/c^2},\ j = 0, 1$$

$$\frac{dW_{0m}}{dk_z} = \frac{q^2 v^2}{2\pi m \omega_0} \mathrm{Im}\left[\frac{1}{\varepsilon_0}\sum_{p,j=\pm 1}\left(\frac{m^2\omega_0^2\varepsilon_0}{c^2} + pjk_z^2\right) I_{m+j}(\gamma_0\rho_0) K_{m+p}(\gamma_0\rho_0)\right]$$

$$B_m^{(p)} = I_{m+p}(\gamma_0\rho_0)\frac{W_{m+p}^K}{W_{m+p}^I} + p\frac{\gamma_1 K_{m+p}(\gamma_1\rho_1)}{2\rho_1\alpha_m W_{m+p}^I} K_m(\gamma_1\rho_1)\sum_{l=\pm 1} l\frac{I_{m+l}(\gamma_0\rho_0)}{W_{m+l}^I}$$

- We will assume that the imaginary part of the permittivity ε_0 is small and it will be taken real and positive, $\varepsilon_0 > 0$ $\beta\sqrt{\varepsilon_0} < 1$ ⇨ $dW_{0m}/dk_z = 0$

$$\frac{dW_m}{dk_z} = -\frac{q^2 v^2}{2\pi m \omega_0}\frac{1}{\varepsilon_0}\sum_{p,j=\pm 1}\left(\frac{m^2\omega_0^2\varepsilon_0}{c^2} + pjk_z^2\right) I_{m+j}(\gamma_0\rho_0)\,\mathrm{Im}\left[B_m^{(p)}\right]$$

Wavelength of the radiated SP

- Dispersion relation for SP on a given harmonic is determined solving

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1(\omega)-\varepsilon_0} + \frac{1}{2}\sum_{l=1,2}\left[1+\frac{\gamma_1 I_{m+l}(\gamma_0\rho_1)K_m(\gamma_1\rho_1)}{\gamma_0 I_m(\gamma_0\rho_1)K_{m+l}(\gamma_1\rho_1)}\right]^{-1} = 0, \quad \gamma_j = \sqrt{k_z^2 - \omega^2\varepsilon_j / c^2}$$

$$\omega = \omega(k_z)$$

- In the problem under consideration the wavelength of the radiated SP is determined by the intersection points of the curve $\omega = \omega(k_z)$ and $\omega = m\omega_0$

- Numerical results will be considered for the Drude model of exterior dielectric permittivity

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

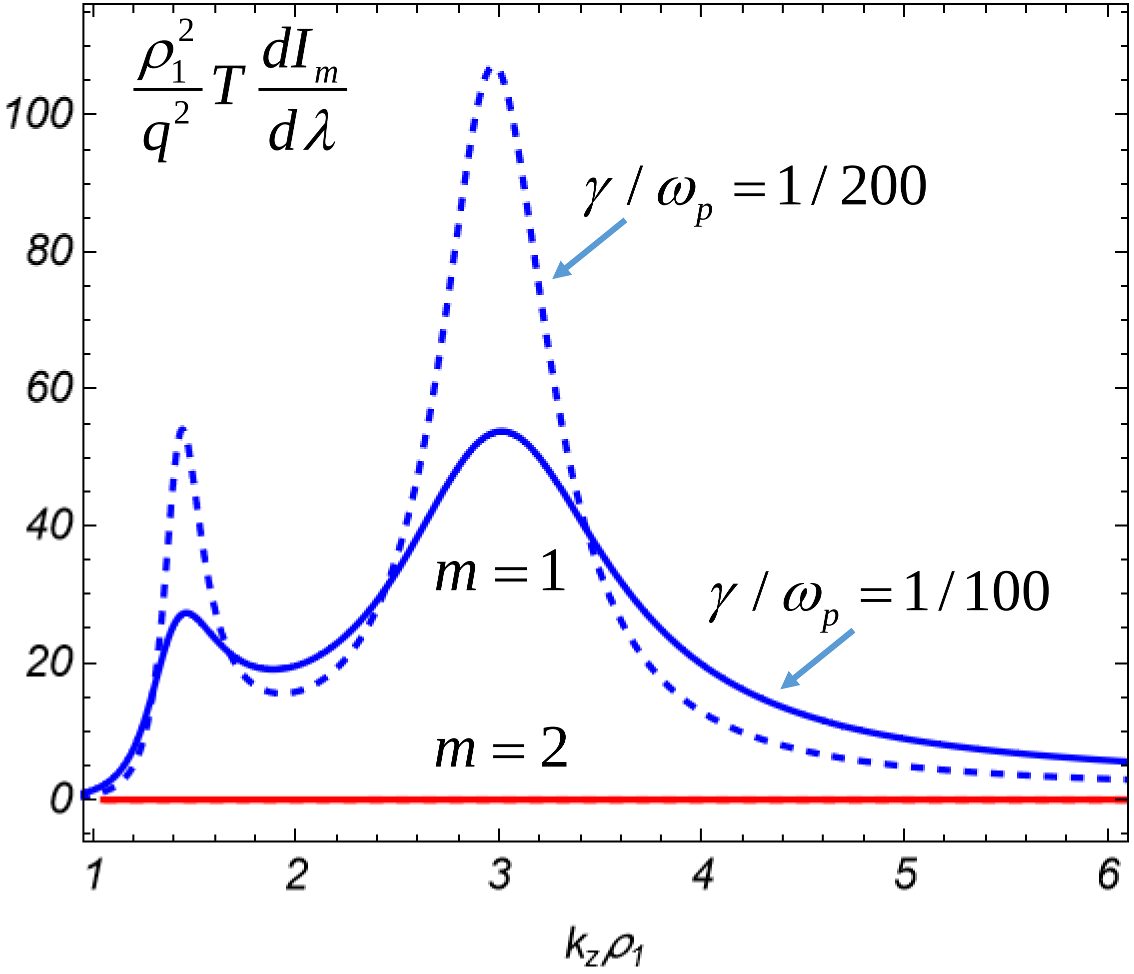
Numerical example 1

Energy losses per rotation period T

$\lambda = 2\pi / k_z$ ⇐ wavelength

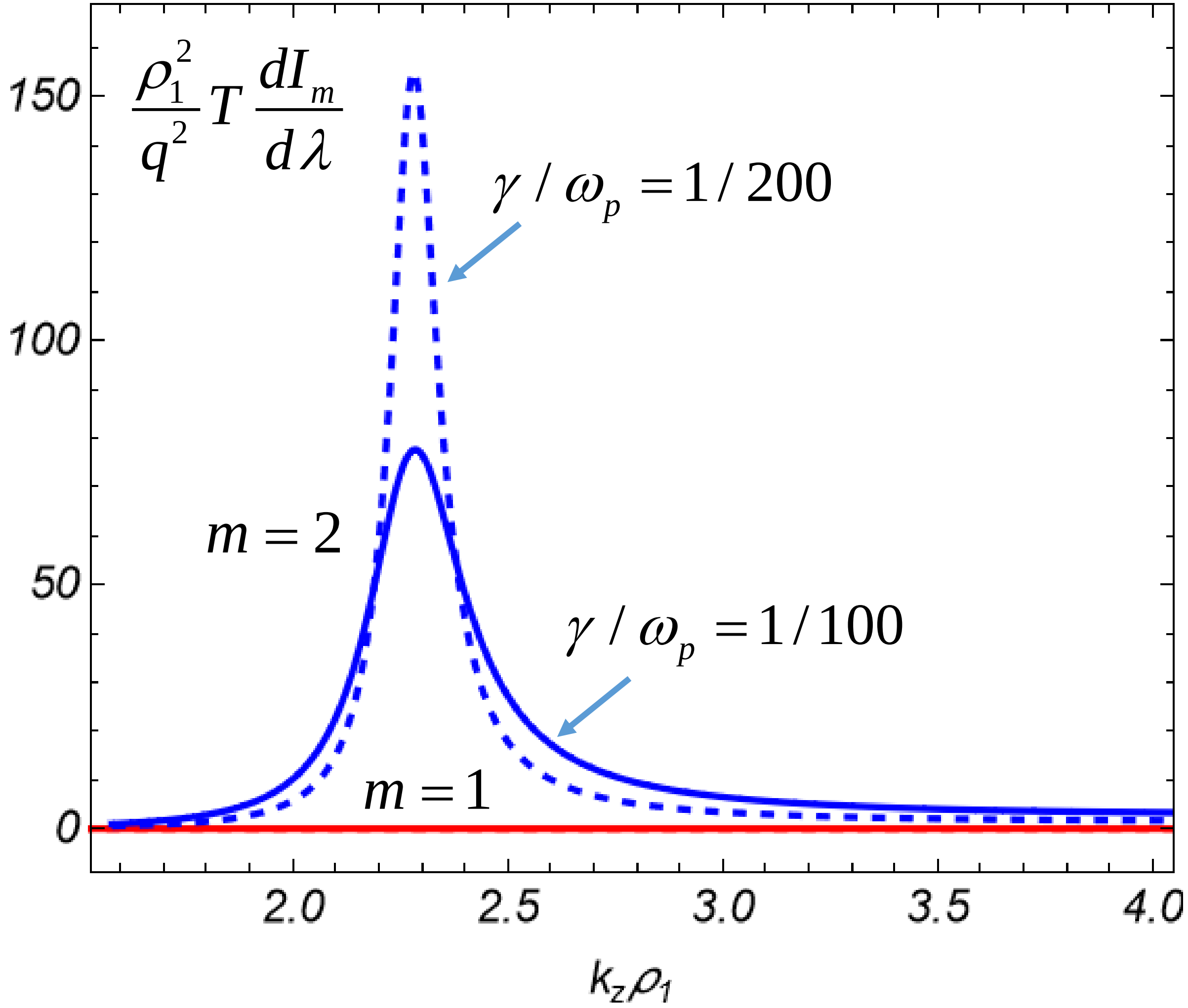
$\beta = v / c = 0.5,\ \rho_0 / \rho_1 = 0.95,\ \omega_p \rho_1 / c = 0.7$

SP mode is present for *m=1* only



Numerical example 2

Energy losses per rotation period T



$\lambda = 2\pi / k_z$ ⇐ wavelength

$\beta = \mathrm{v}/c = 0.75,\ \rho_0/\rho_1 = 0.95,\ \omega_p \rho_1 / c = 2.3$

SP mode is present for *m=2* only

Conclusions

- Presence of cylindrical waveguide may essentially change the spectral-angular distribution of the Synchrotron and Cherenkov radiations in the exterior medium
- Two types of modes are radiated propagating inside the cylinder with an exponential damping in the exterior region: Guided modes and Surface-type modes (Surface polaritons)
- Electromagnetic fields and the radiation intensity for SP are evaluated
- Depending on the waveguide radius one can have radiation in the spectral range from wicrowaves to optics

Thank You!