UNIVERSITÉ DE GENÈVE

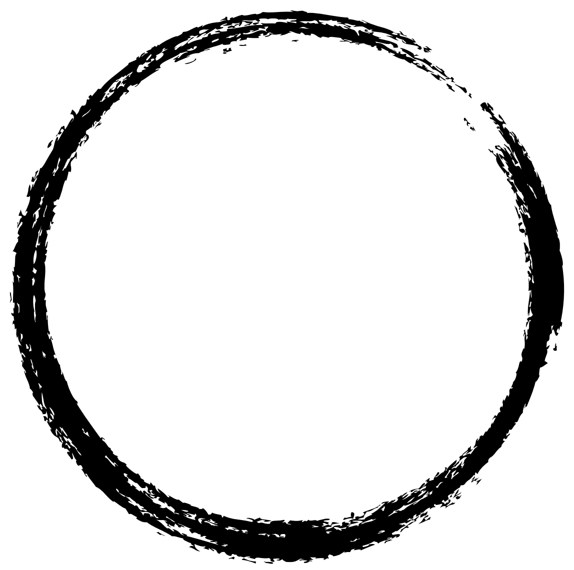
How a black hole rings?

Nonlinearities and Instability of black hole quasi-normal modes

Davide Perrone,
Pollica Physics Workshop,
10/09/2024.

Ringdown, sum of quasi-normal modes

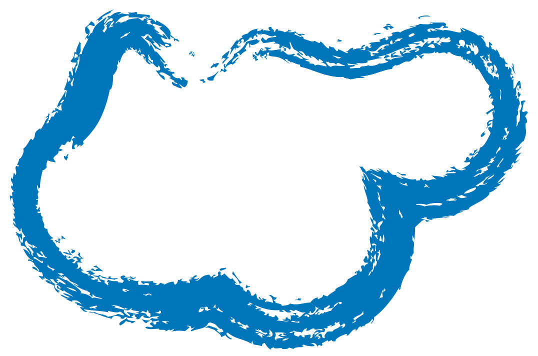
Quasi-normal modes are the proper excitation of black holes



$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \left(\omega^2 - V_{\mathrm{eff}}(x)\right)\right)\psi_{l,m}(x) = 0$$

$$+$$

Angular Equation $= 0$



$x = r + r_+ \log(r - r_+)$ Tortoise Coordinate

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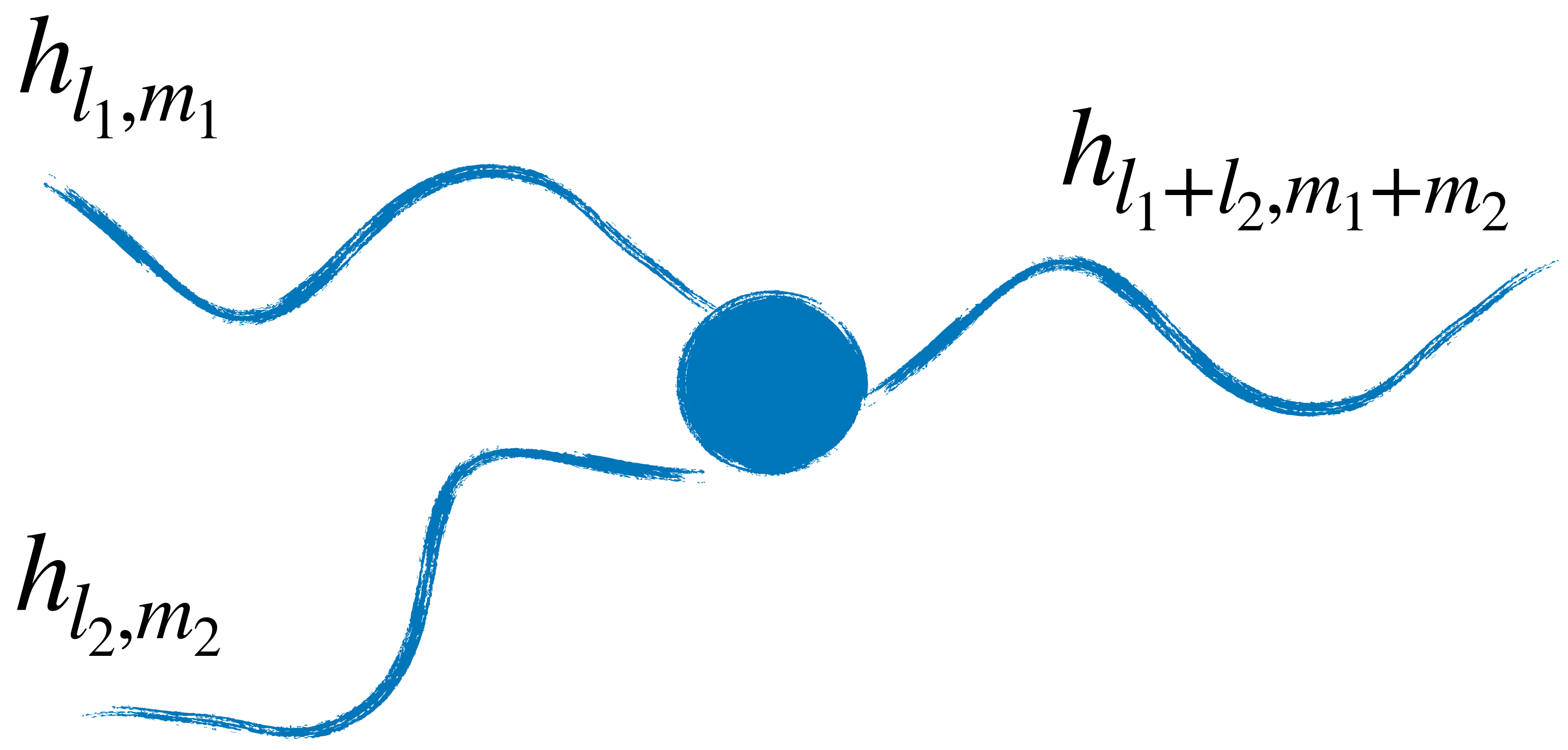
However General Relativity is nonlinear

Linear order solution act as source for nonlinear order



$$\rightarrow \frac{\langle h_{l_1,m_1}\, h_{l_2,m_2}\, h_{l_1+l_2,m_1+m_2}\rangle}{\langle h^2_{l_1,m_1}\rangle\langle h^2_{l_2,m_2}\rangle}$$

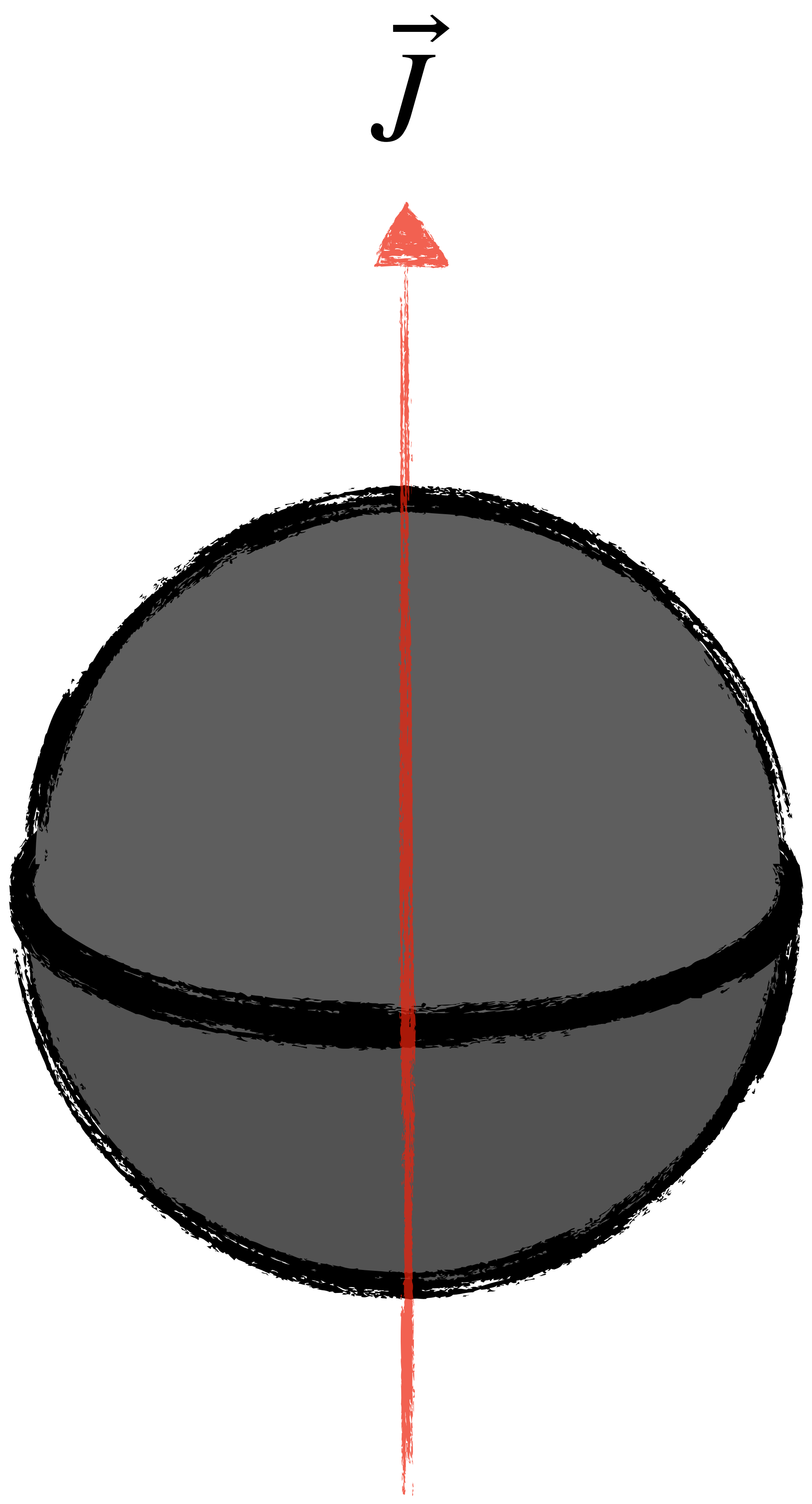
$$\frac{\left|A^{(2,2,0)\times(2,2,0)}_{(4,4)}\right|}{\left|A_{(2,2,0)}\right|\,\left|A_{(2,2,0)}\right|} = 0.1637 \pm 0.0018$$

$$\frac{\left|A^{(2,2,0)\times(3,3,0)}_{(5,5)}\right|}{\left|A_{(2,2,0)}\right|\,\left|A_{(3,3,0)}\right|} = 0.4735 \pm 0.0062$$

K. Mitman et al. (2022), M. Cheung et al. (2022)

Extremal black hole scenario

Additional symmetries simplify the problem



J. M. Bardeen et al. (1999), M. Guica et al. (2008)

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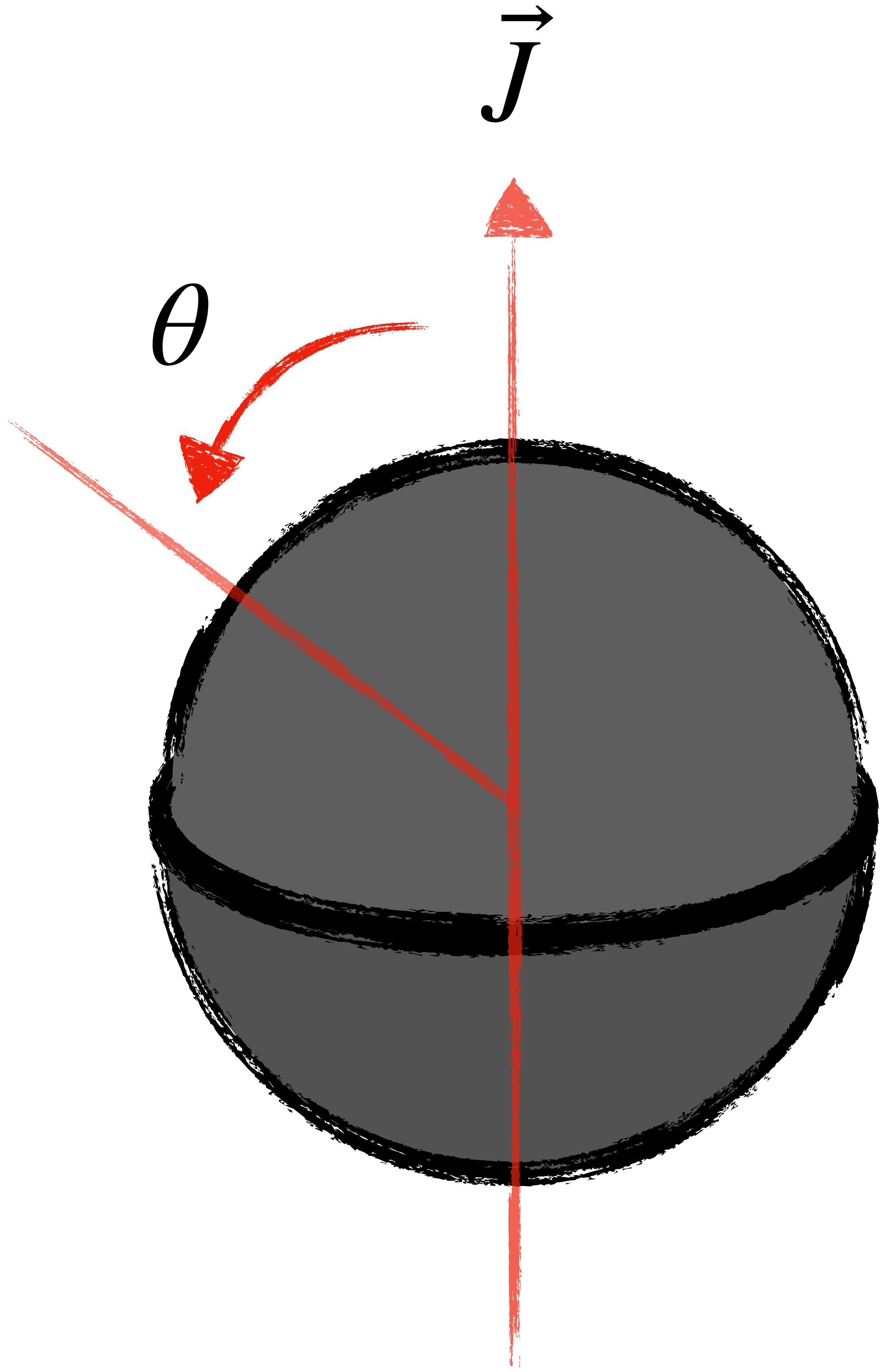

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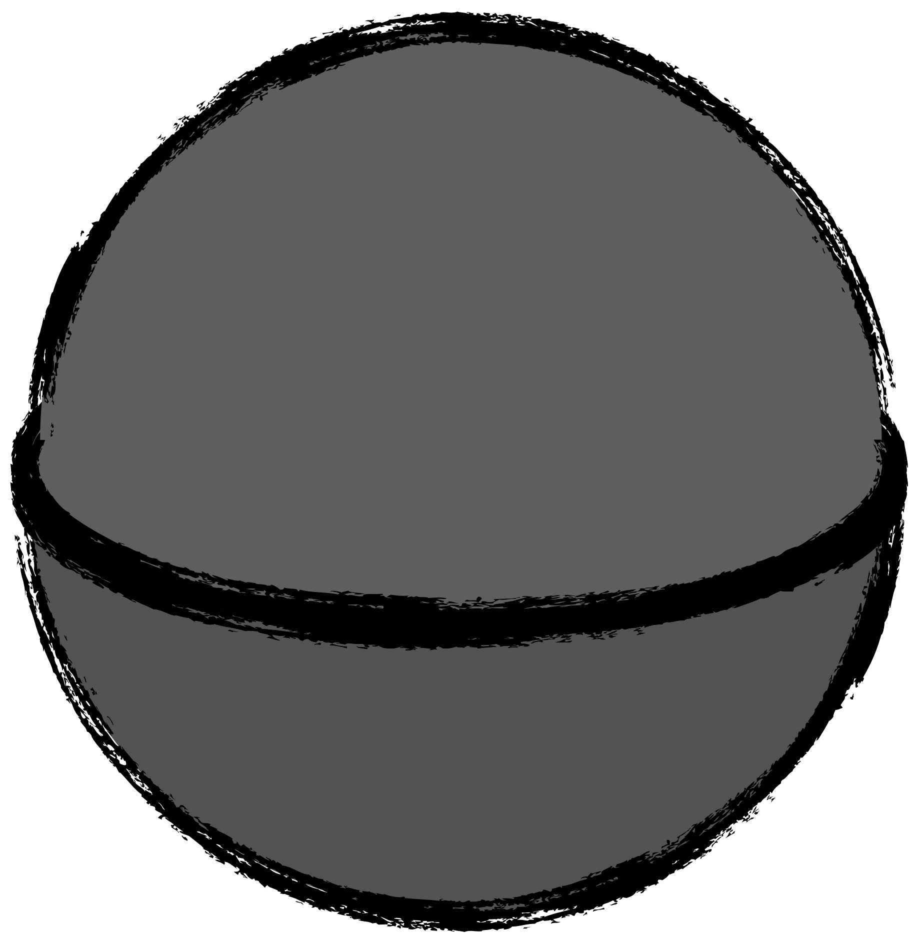

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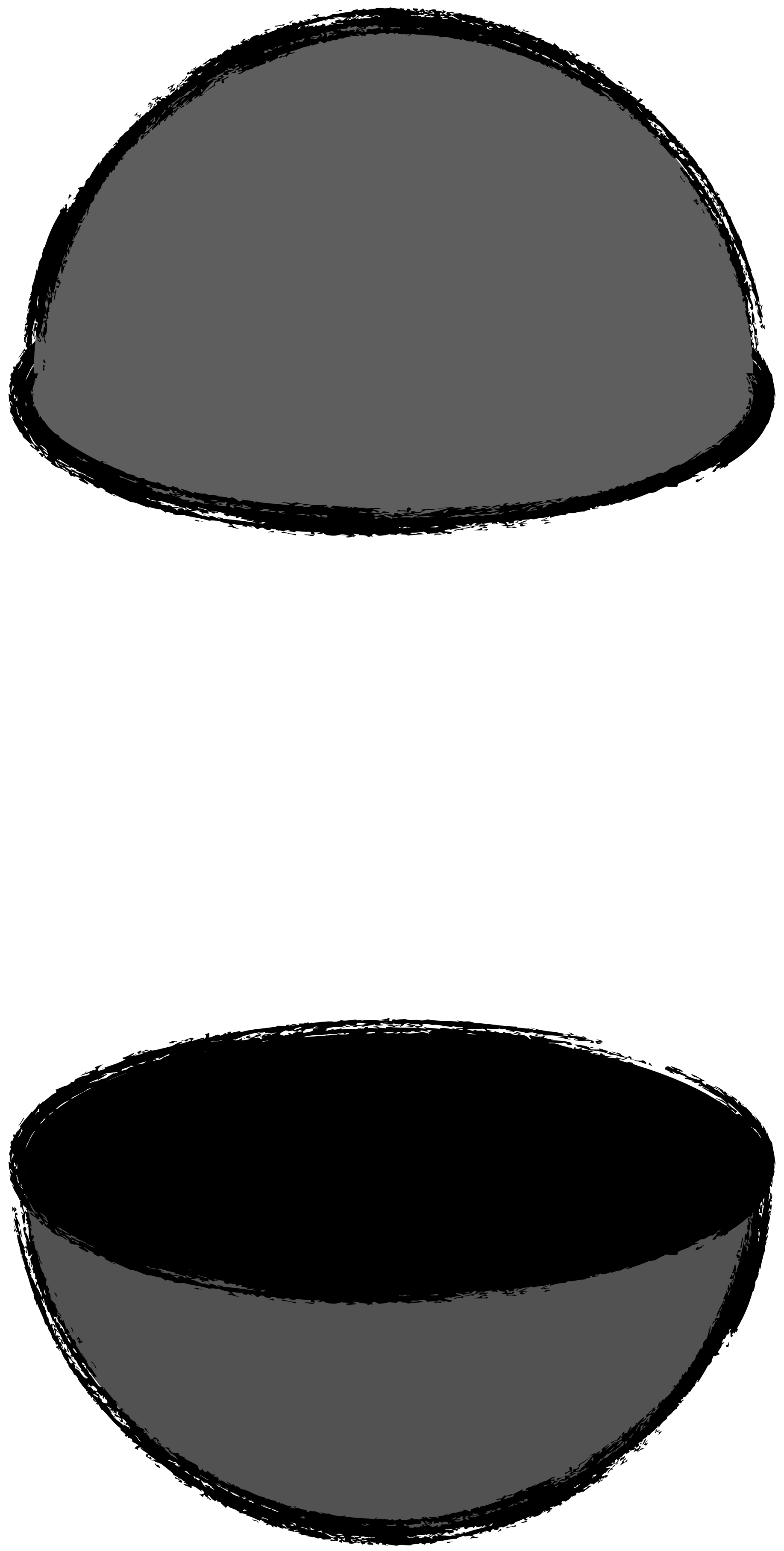

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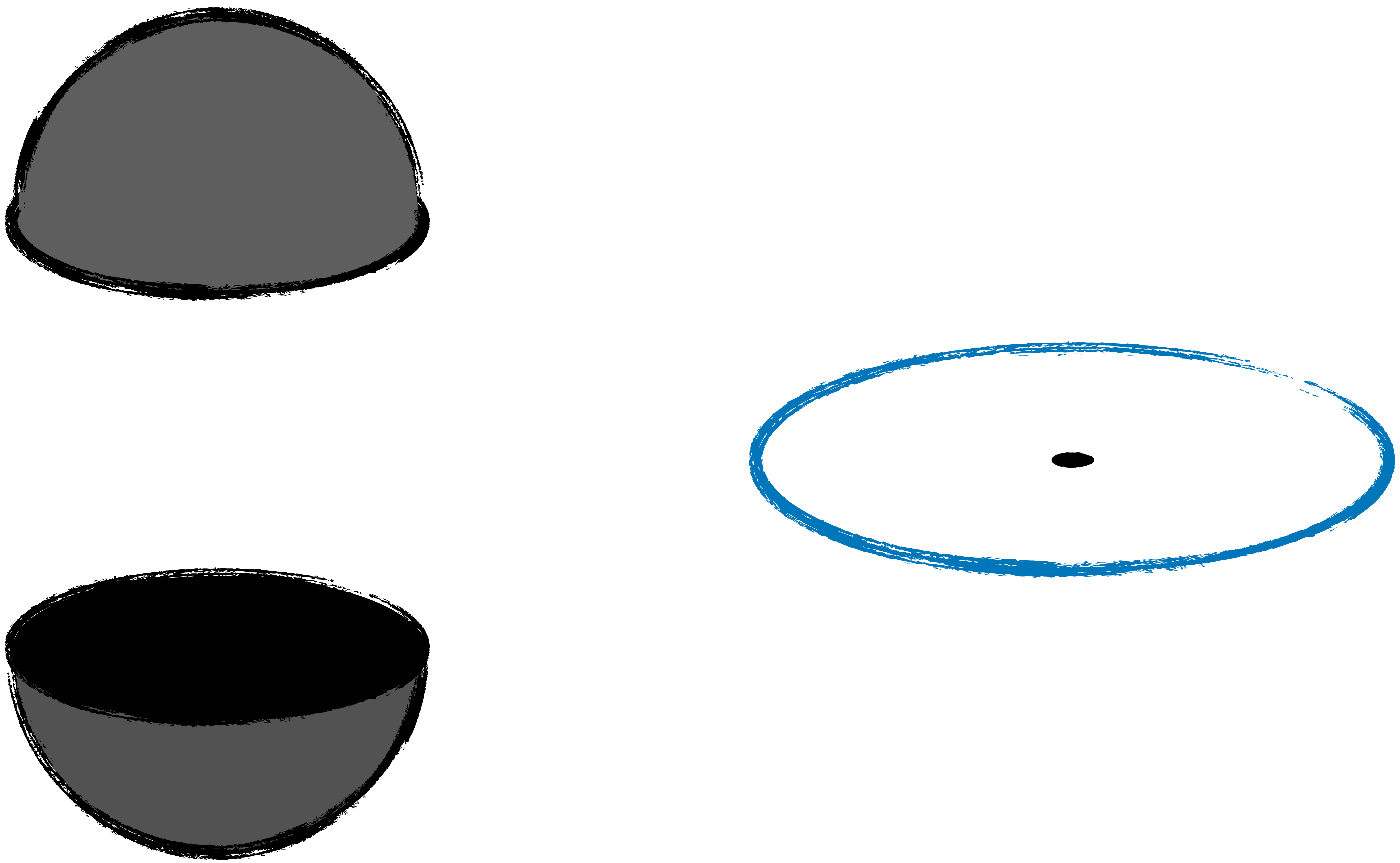

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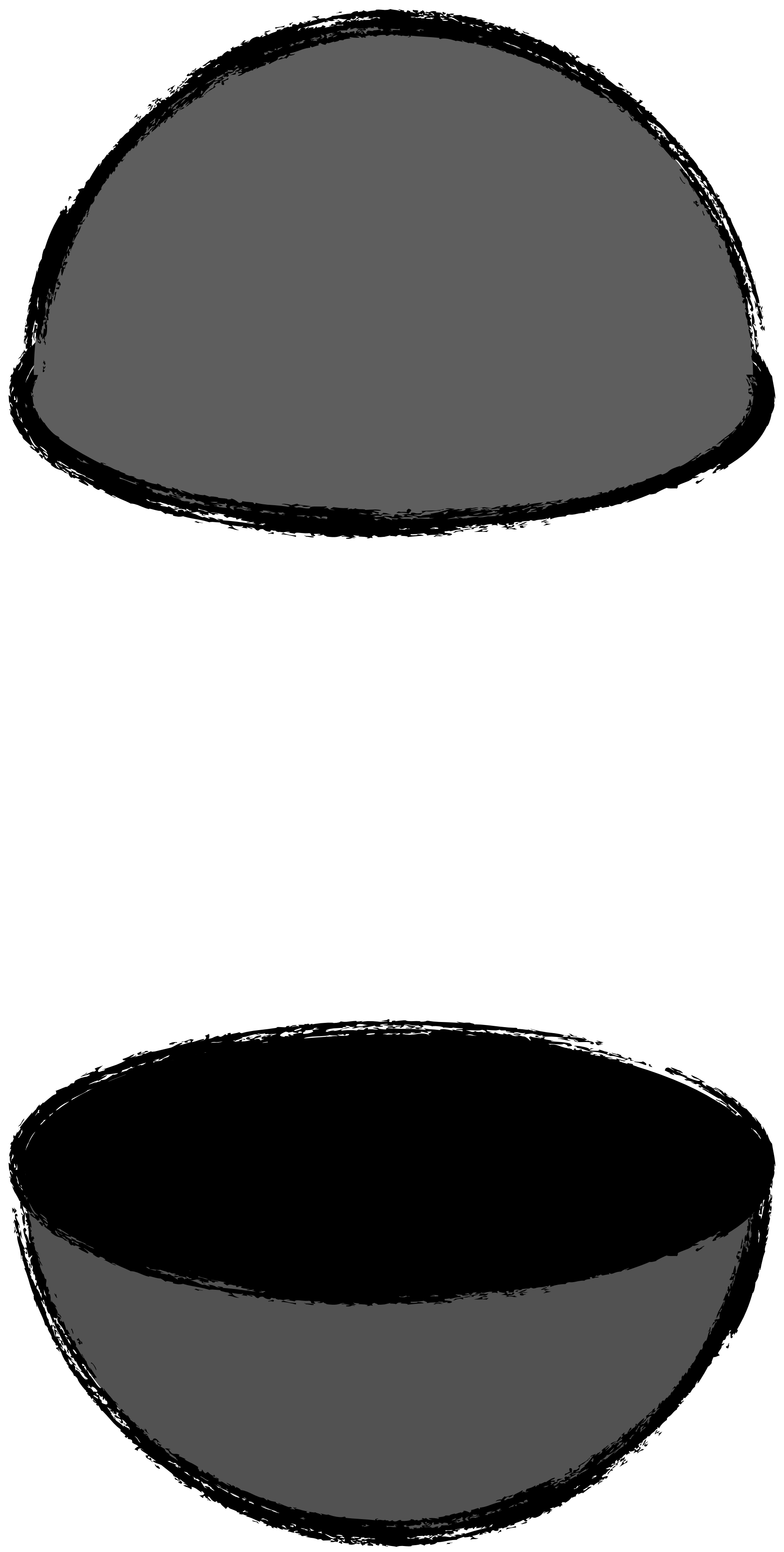

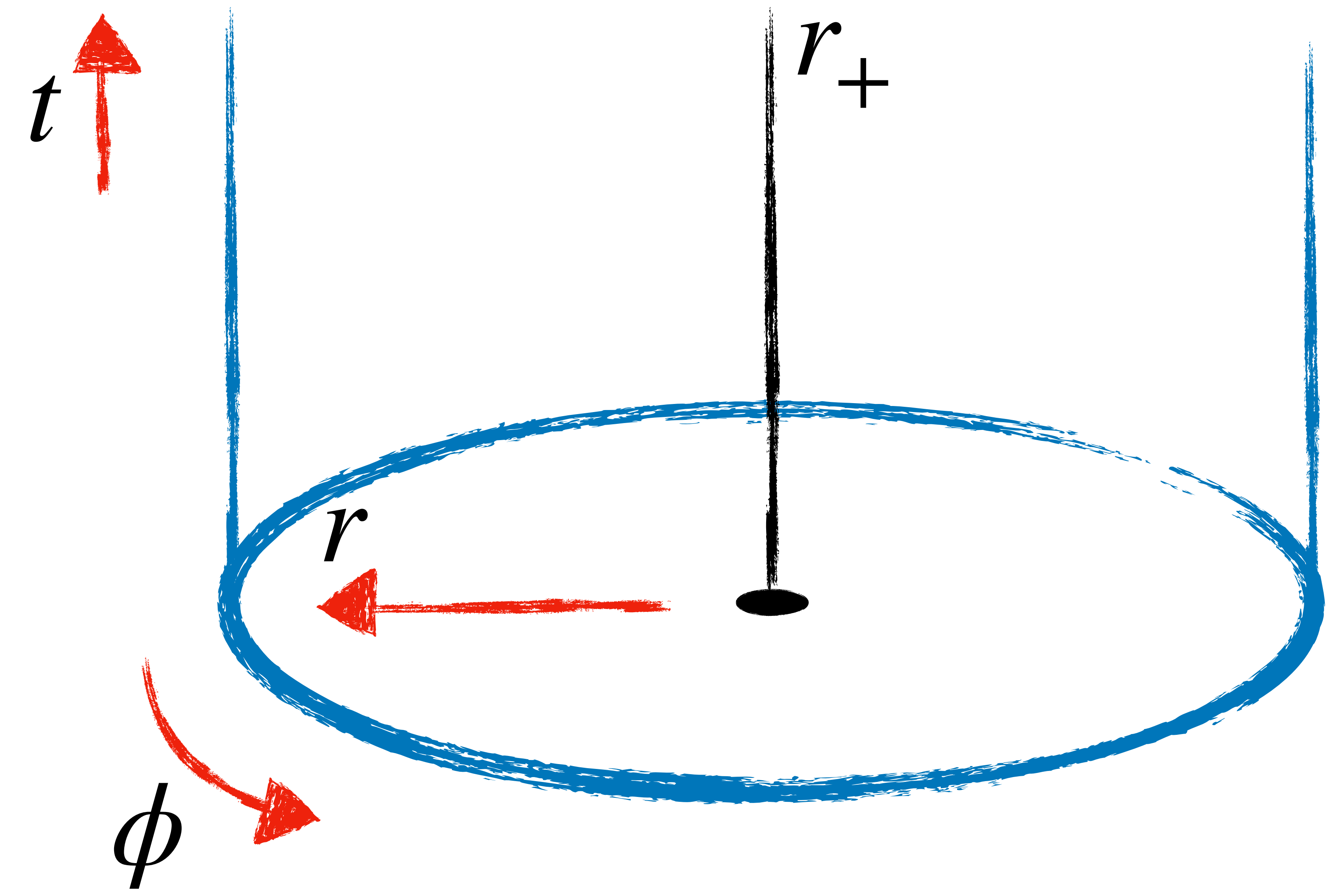

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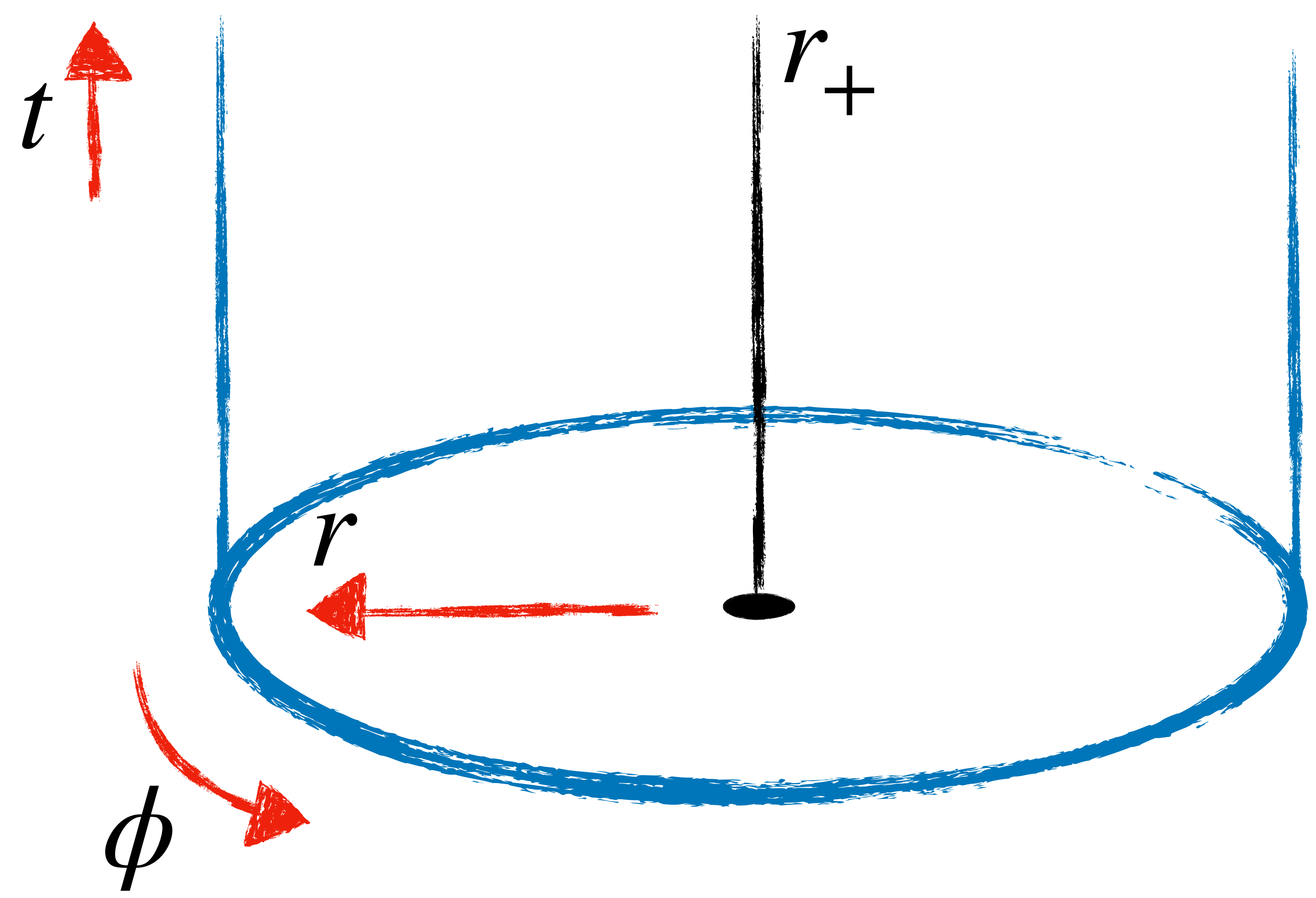

AdS_3 metric

$\mathrm{SL}(2,\mathbb{R}) \otimes U(1)$

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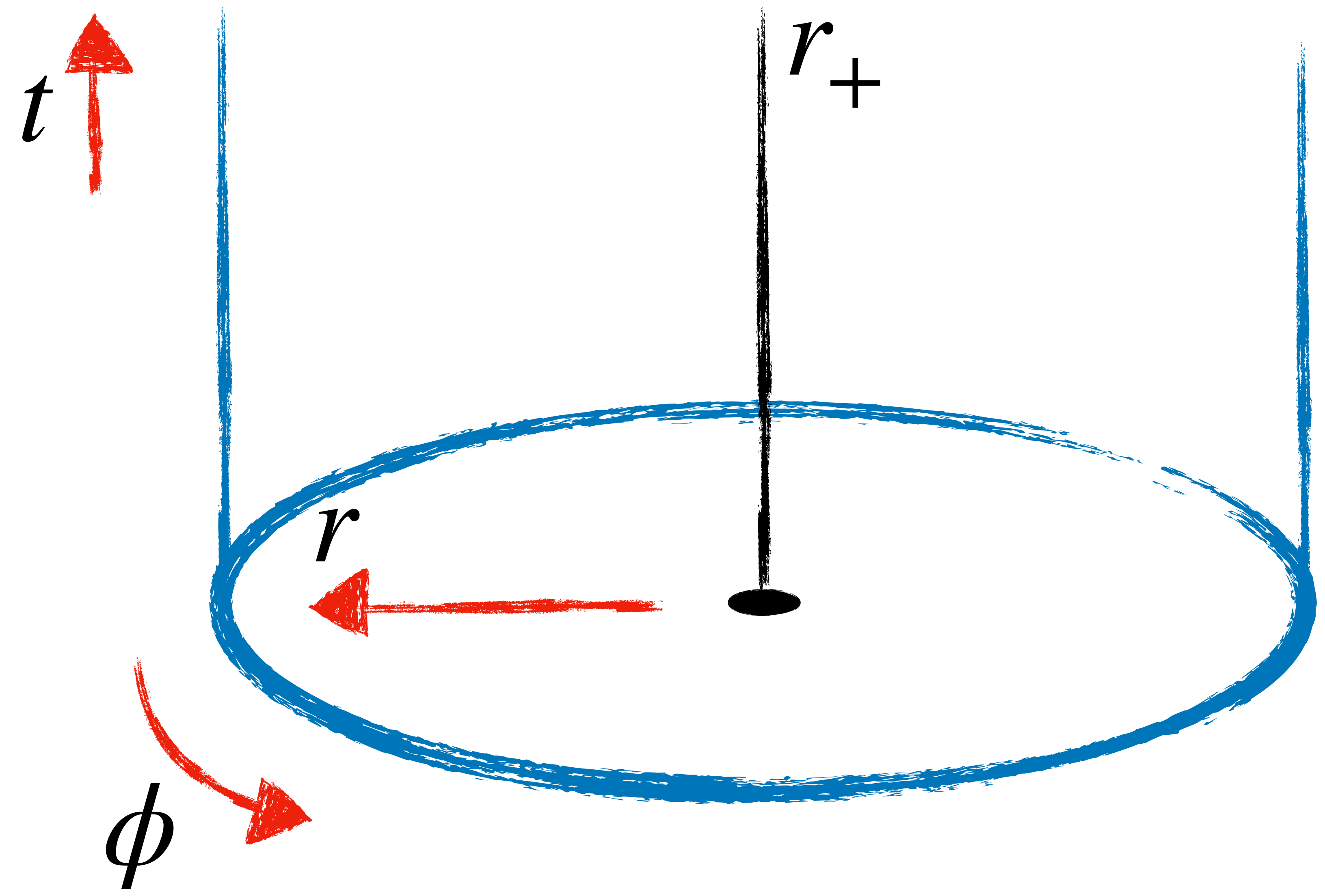


AdS_3 metric

$$\mathrm{SL}(2,\mathbb{R}) \otimes U(1)$$

Described with a CFT

$$\langle T_{m_1} T_{m_2} \rangle, \quad \langle T_{m_1} T_{m_2} T_{m_3} \rangle$$

J. M. Bardeen et al. (1999), M. Guica et al. (2008)

Kerr Black Hole Non-Linearity

And its estimate in Extremal limit

$$\frac{\langle h_{(\ell_1,m_1)} h_{(\ell_2,m_2)} h_{(\ell_1+\ell_2,m_1+m_2)}\rangle}{\langle h^2_{(\ell_1,m_1)}\rangle\langle h^2_{(\ell_2,m_2)}\rangle} = \frac{6\sqrt{2}}{2\pi}\ {}_{-2}C^{m_1,m_2,m_1+m_2}_{\ell_1,\ell_2,\ell_1+\ell_2}\ \frac{G^{3,3}_{3,3}\left(\begin{matrix} -im_1, & 0, & im_2 \\ 1-im_1, & 1, & 1+im_2 \end{matrix}\ \middle|\ e^{i\pi}\right)}{\left|\Gamma\left(2-i(m_1+m_2)\right)\right|^2}$$

$$\frac{\langle h_{(2,2)} h_{(2,2)} h_{(4,4)}\rangle}{\langle h^2_{(2,2)}\rangle^2} \simeq 0.62 \cdot \frac{5}{24}\sqrt{\frac{7}{\pi}} \simeq 0.19 \quad \text{vs} \quad \frac{\left|A^{(2,2,0)\times(2,2,0)}_{(4,4)}\right|}{\left|A_{(2,2,0)}\right|^2} = 0.1637 \pm 0.0018$$

$$\frac{\langle h_{(2,2)} h_{(3,3)} h_{(5,5)}\rangle}{\langle h^2_{(2,2)}\rangle\langle h^2_{(3,3)}\rangle} \simeq 1.57 \cdot \frac{2}{3}\sqrt{\frac{7}{11\pi}} \simeq 0.47 \quad \text{vs} \quad \frac{\left|A^{(2,2,0)\times(3,3,0)}_{(5,5)}\right|}{\left|A_{(2,2,0)}\right|\left|A_{(3,3,0)}\right|} = 0.4735 \pm 0.0062$$

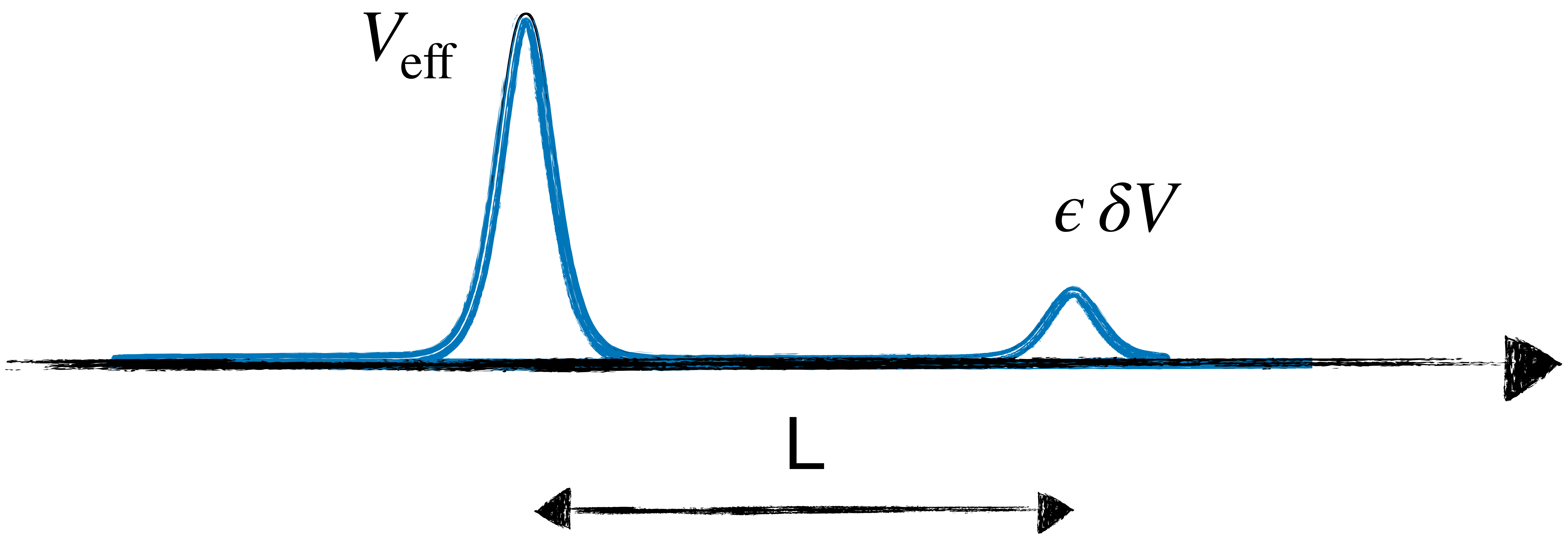
A. Kehagias, D.P., F. Riva, A. Riotto (2023)

Linear level frequency change

A bump in the potential modifies the proper response of the black hole

$$\left(\frac{\mathrm{d}^2}{\mathrm{dx}^2} + \left(\omega^2 - V_{\mathrm{eff}}(x) + \epsilon\, \delta V(x)\right)\right)\psi_{l,m}(x) = 0$$

→ Transfer matrix Formalism



A. Ianniccari, A.J. Iovino, A. Kehagias, P. Pani, G. Perna, D.P., A. Riotto, (2024)

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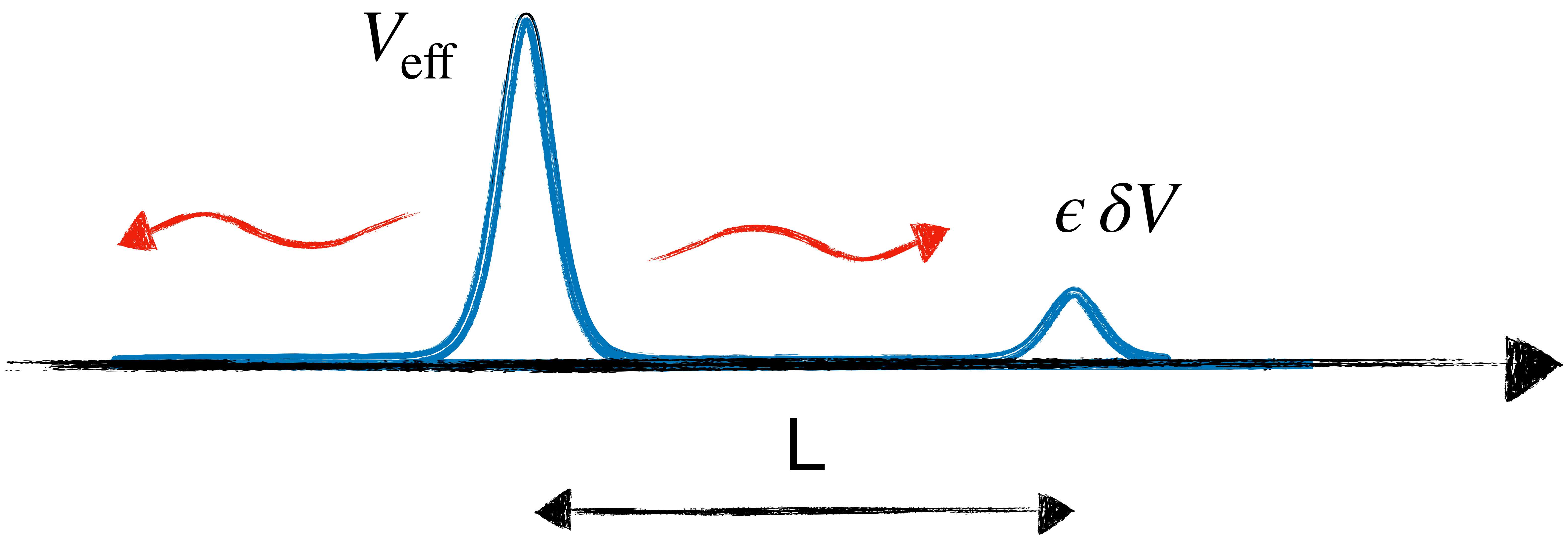

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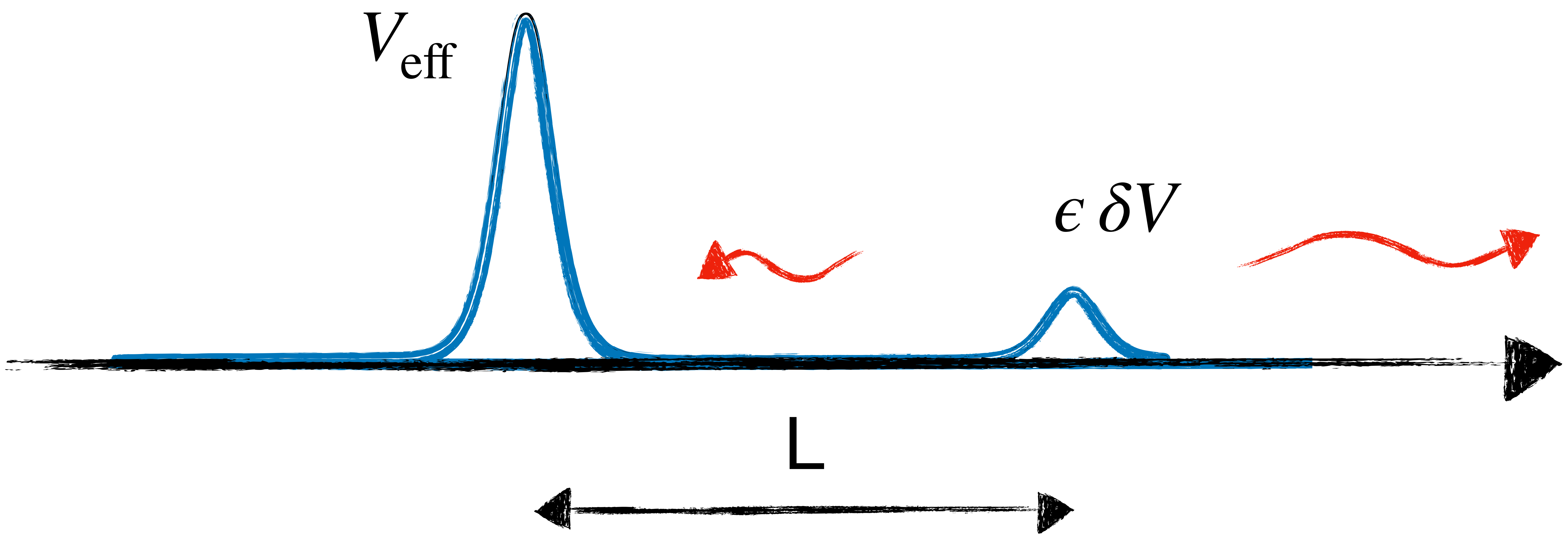

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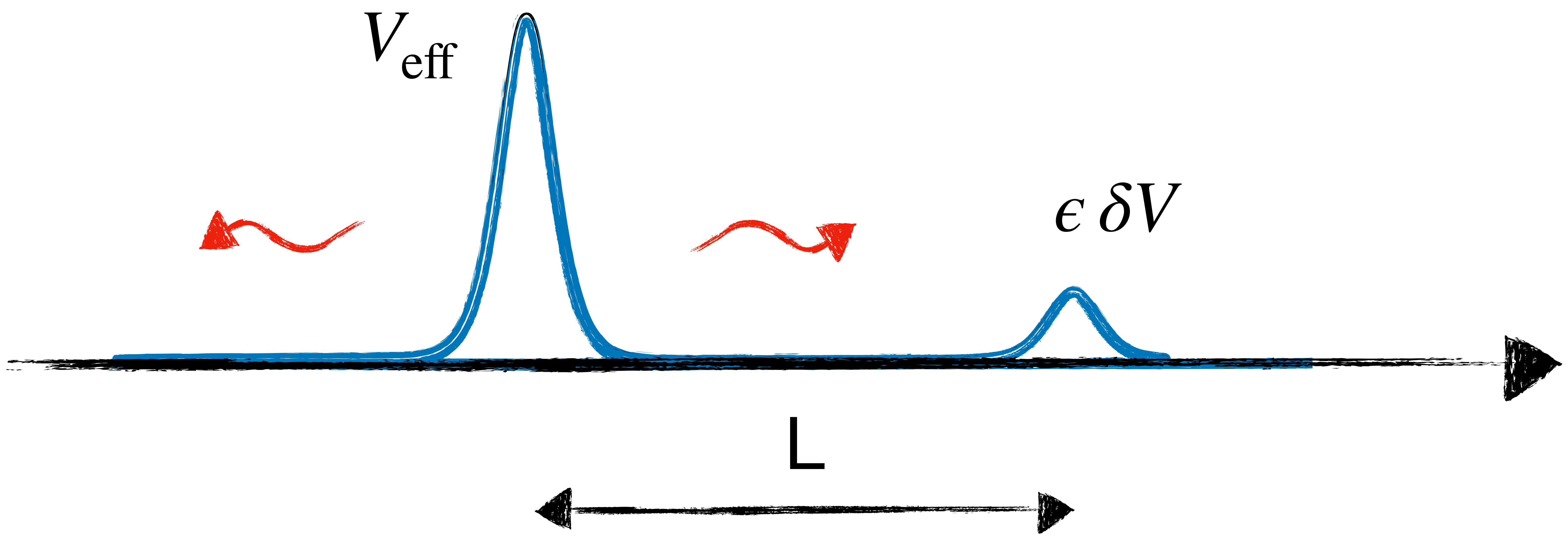

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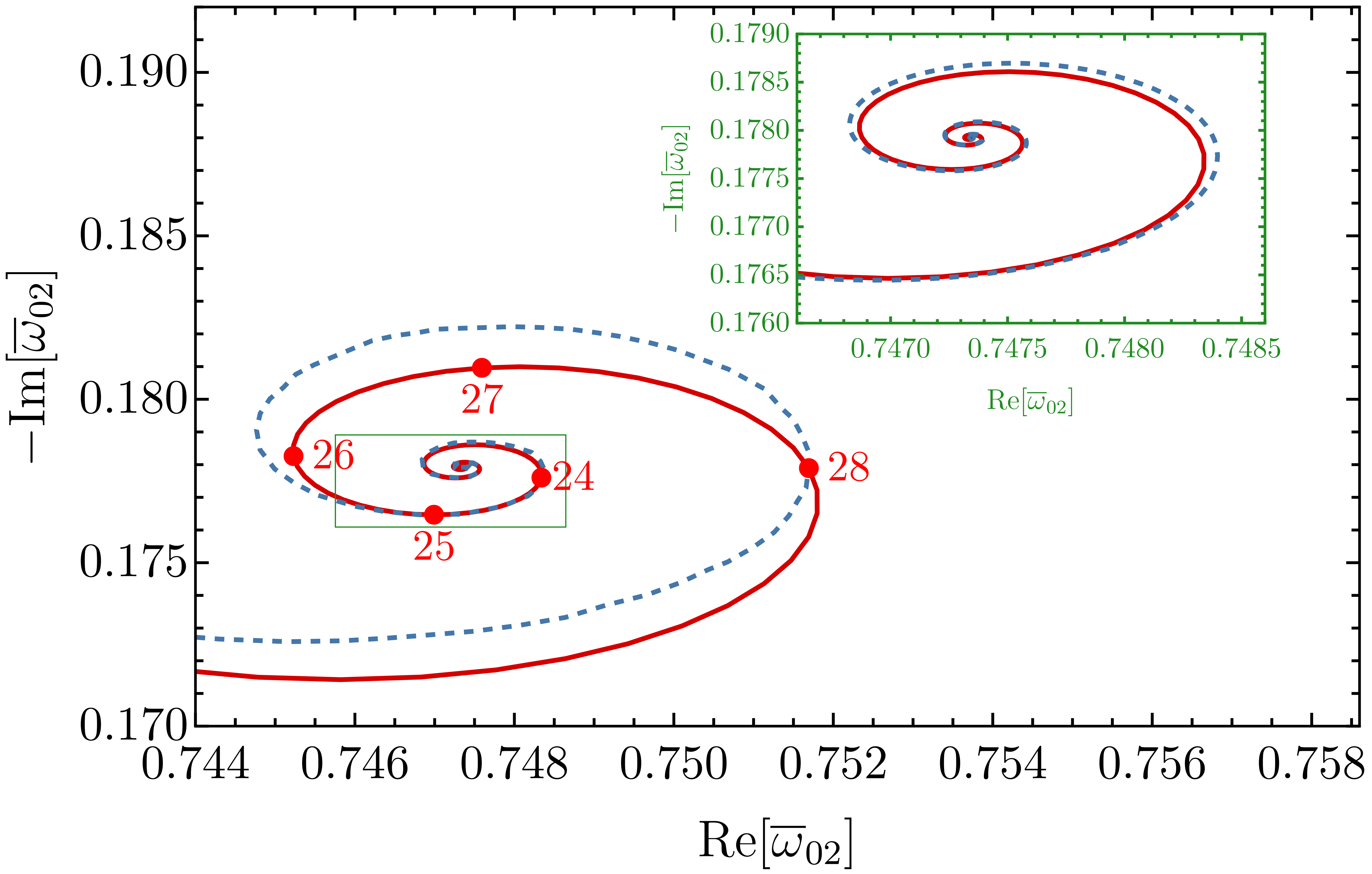
Analytical prediction on frequency migration

Using the transfer matrix formalism



$$\delta\omega_{02} \propto e^{2i\omega_{02}L} R_1(\omega_{02})\,\mathrm{Res}(R_0)\,|_{\omega_{02}}$$

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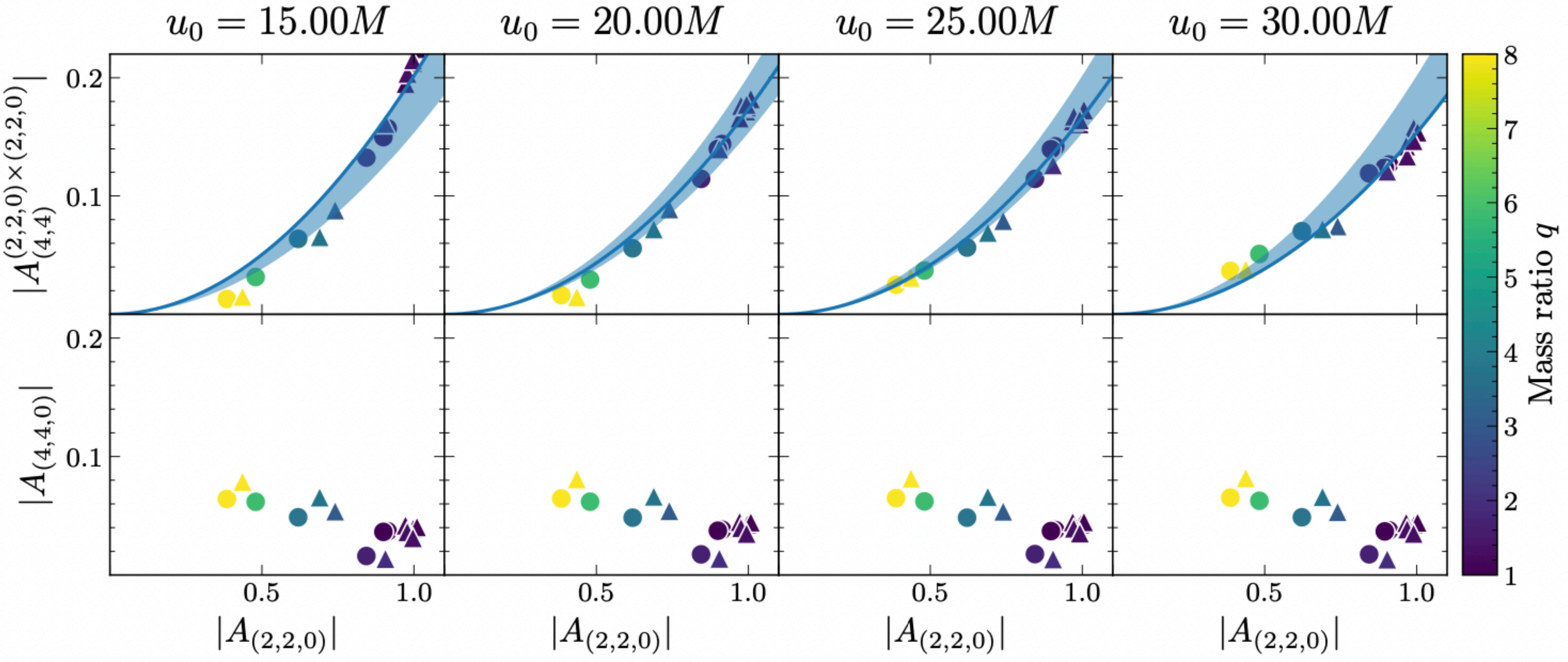
Summary and outlook

- Ringdown is crucial for the future of Gravitational Wave analysis
- A lot of techniques are being developed to evaluate it precisely
- Some features could be captured by symmetry

Second-order amplitudes are sizeable



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2D CFT correlators

Exploiting the duality

$$Z_{\mathrm{AdS,eff}}[h] = e^{iS[h]} = \langle\, e^{\int_{\partial \mathrm{AdS}} T^{\mu\nu} h_{\mu\nu}} \rangle_{\mathrm{CFT}}$$

Gravitational strain in bulk
=
Stress-Energy tensor on boundary

↓

$$\langle T(w_1) T(w_2) \rangle, \quad \langle T(w_1) T(w_2) T(w_3) \rangle$$

Known correlators in 2D CFT

↓

$$w_i = \phi_i \to m_i$$

Fourier transform of correlators

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Kerr/CFT correspondence

Evaluating the Asymptotic Symmetry Group

$$\mathrm{AdS}_3 \ \text{metric}$$

$$\mathrm{SL}(2,\mathbb{R}) \otimes U(1)$$

Boundary CFT

M. Guica et al. (2008)

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Boundary CFT

Not extended

Half Virasoro Algebra, $c = 12J$

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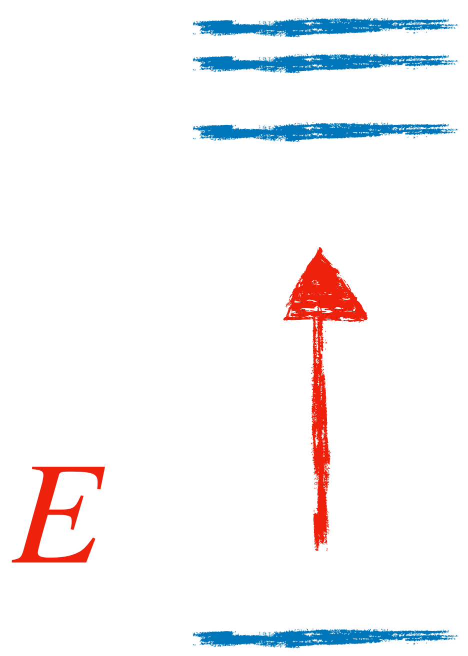

Infinite Gap
in extremal limit

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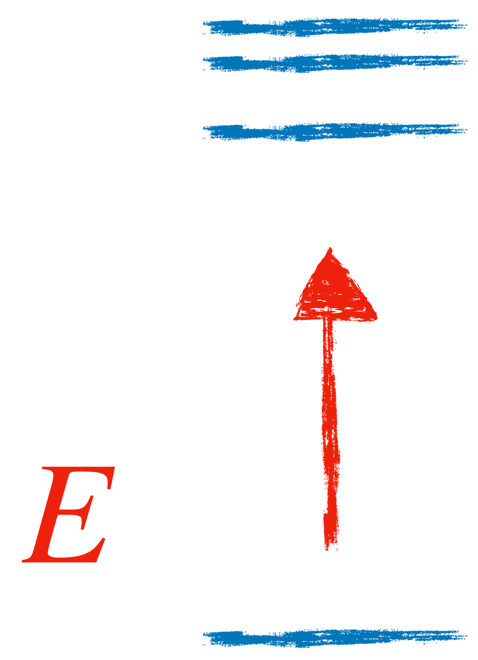

Infinite Gap
in extremal limit

Left movers excited
with $T_L = \dfrac{1}{2\pi}$

M. Guica et al. (2008)

Prescription

- Calculate correlators of the 2D energy-momentum tensor at finite temperature

$$\langle T_{m_1} T_{m_2} \rangle, \quad \langle T_{m_1} T_{m_2} T_{m_3} \rangle$$

- Find the gravitational strain correlators

$$\langle h_m, h_{-m} \rangle' = -\frac{1}{\mathrm{Re}\langle \mathrm{T_m T_{-m}} \rangle'}, \quad \langle h_{m_1} h_{m_2} h_{m_3} \rangle' = \frac{2\mathrm{Re}\langle \mathrm{T_{m_1} T_{m_2} T_{m_3}} \rangle'}{\prod_i^3 \left(-2\mathrm{Re}\langle \mathrm{T_{m_i} T_{-m_i}} \rangle' \right)}$$

- Integrate the spin-weighted spherical harmonics over the polar angle θ

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Near Horizon Extremal Kerr geometry

Zooming close to the Horizon

$$\mathrm{ds}^2 = 2\mathrm{M}^2\Gamma(\theta)\left[-\mathrm{r}^2\mathrm{dt}^2 + \frac{\mathrm{dr}^2}{\mathrm{r}^2} + \mathrm{d}\theta^2 + \Lambda^2(\theta)(\mathrm{d}\phi + \mathrm{rdt})^2\right]$$

$$\Gamma(\theta) = \frac{1+\cos^2\theta}{2}, \quad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$$

NHEK is a warped AdS_3 geometry

Isometry group $\mathrm{SL}(2,\mathbb{R}) \otimes U(1)$

J. M. Bardeen et al. (1999), M. Guica et al. (2008)