

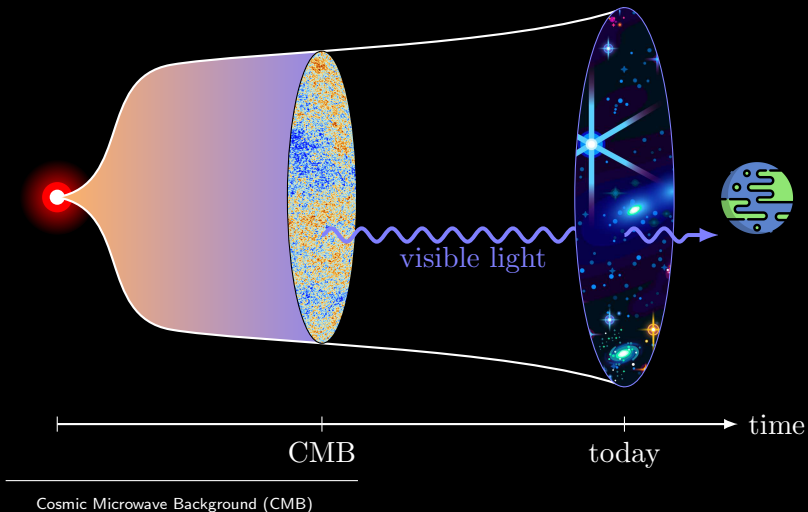
Reliable theoretical predictions for GWs from first-order phase transitions

Philipp Schicho
philipp.schicho@unige.ch
pschicho.github.io

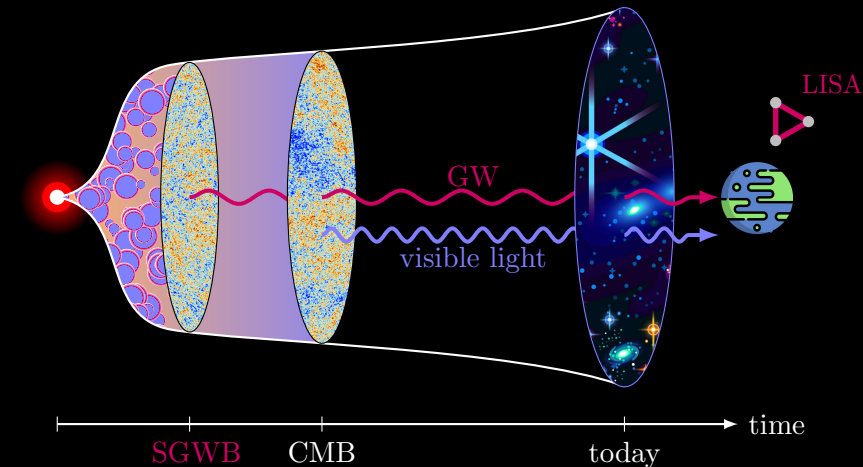
Département de Physique Théorique, Université de Genève

Fundamental physics and gravitational wave detectors
Pollica, 09/2024

Gravitational Waves (GWs) are distortions of spacetime sourced by early-universe **phase transitions** caused by **new physics**.



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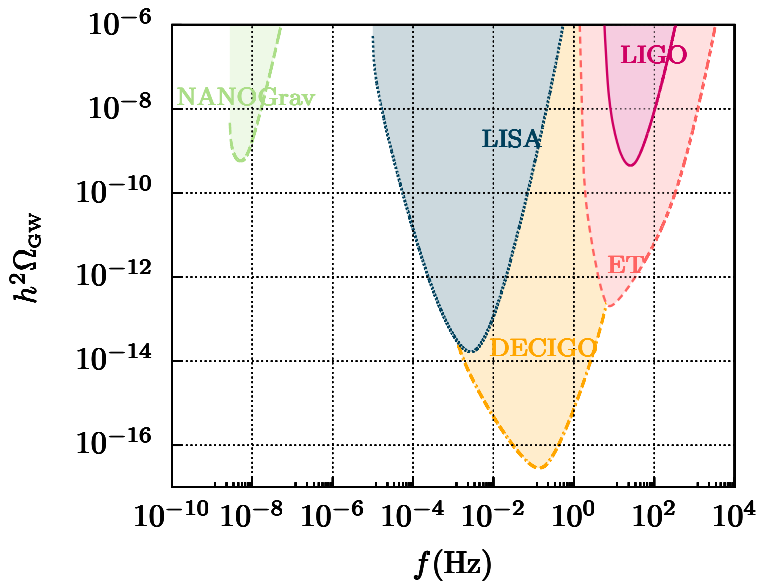


Cosmic Microwave Background (CMB)

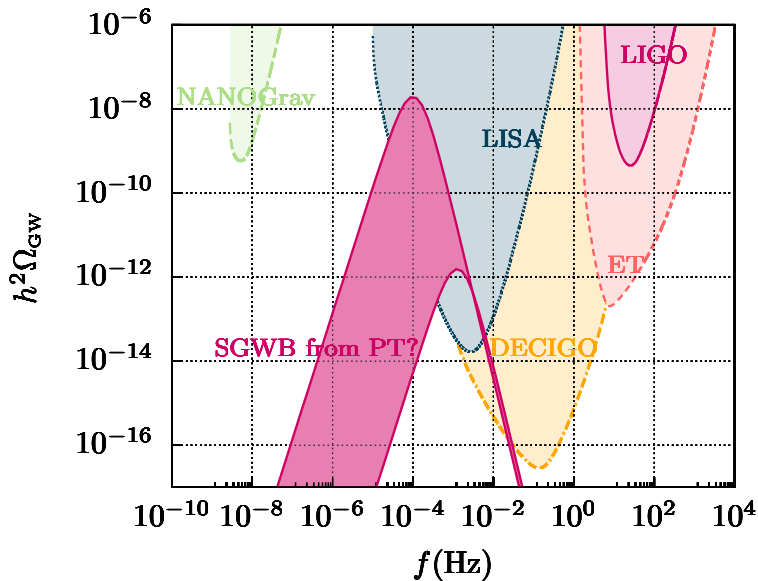
Stochastic GW Background (SGWB)

Laser Interferometer Space Antenna (LISA)

Stochastic gravitational wave background



Stochastic gravitational wave background



The thermal history of electroweak symmetry breaking

Strong first-order cosmic phase transition at EW scale $T_c \sim 100$ GeV:

- ▷ Baryogenesis Baryon asymmetry of the universe
- ▷ Colliding bubbles Gravitational wave (GW) production

In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

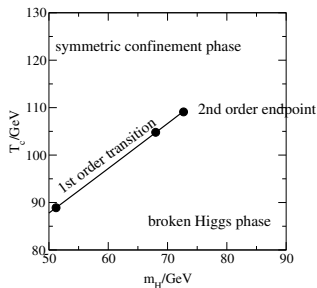


figure by M. Laine, *Electroweak phase transition beyond the standard model*, in *4th International Conference on Strong and Electroweak Matter*, pp. 58–69, 6, 2000 [hep-ph/0010275]

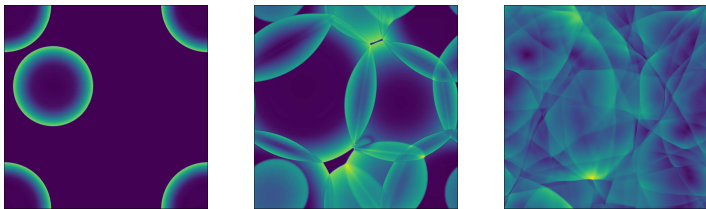
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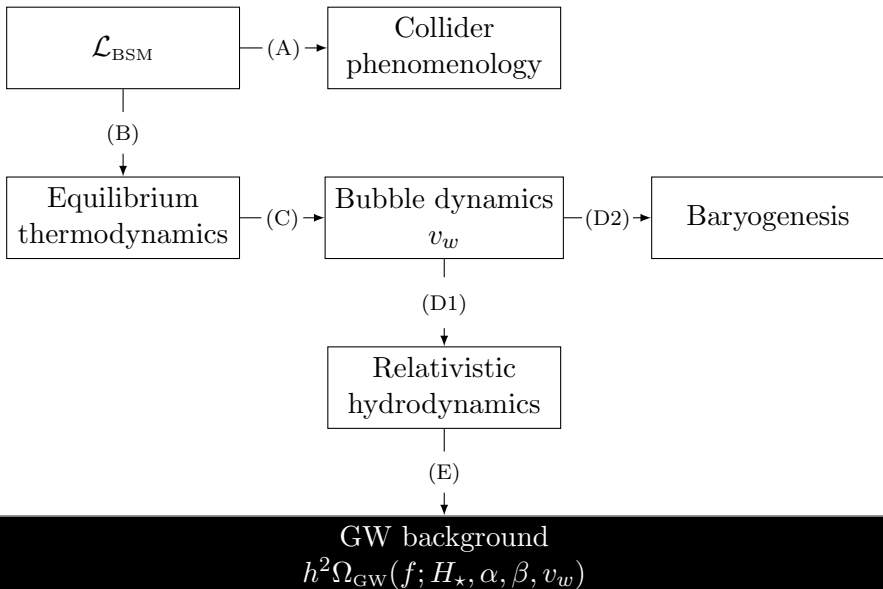
In Standard Model EWSB occurs via a smooth crossover but possible that it is first-order in Beyond the Standard Model (BSM) extensions. Study **BSM physics** near EW scale in context of phase transitions:

- ▷ Light fields strongly coupled to Higgs
- ▷ Collider targets. BSM testing pipeline: Collider phenomenology

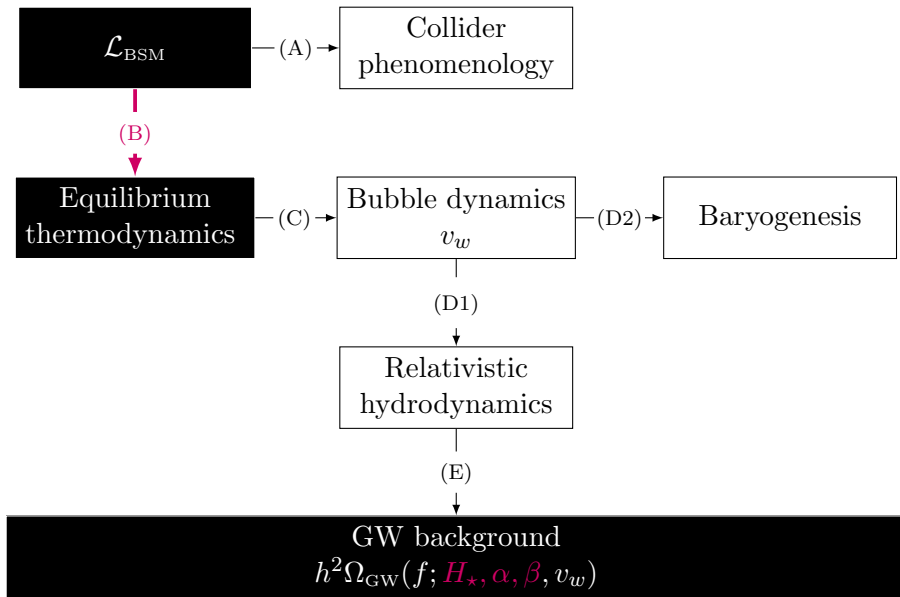


figures by D. Cutting, M. Hindmarsh, and D. J. Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

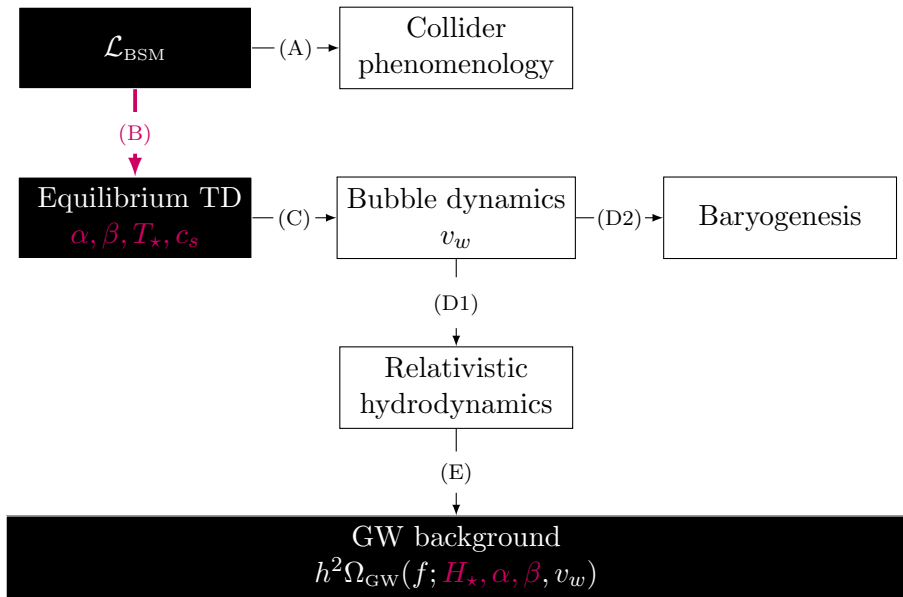
Uncertainties of the gravitational wave pipeline



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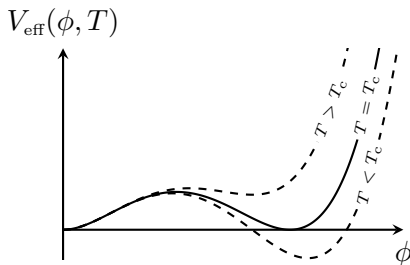
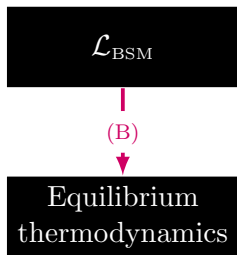
Uncertainties of the gravitational wave pipeline



(B): The effective potential V_{eff} in perturbation theory¹

encodes equilibrium thermodynamics as function of BSM parameters.

Origin of uncertainty.



¹ R. Jackiw, *Functional evaluation of the effective potential*, Phys. Rev. D **9** (1974) 1686

(B): The effective potential V_{eff} in perturbation theory

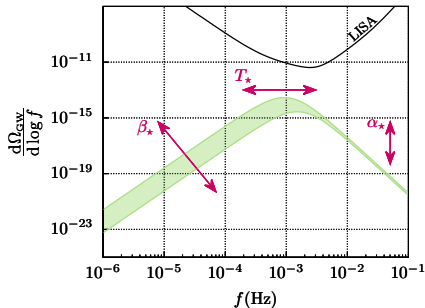
encodes equilibrium thermodynamics as function of BSM parameters.

Origin of uncertainty.

T_* reference temperature of the transition ($T_* = T_n, T_p$),

α phase transition strength,

β/H inverse duration of the transition,



(B): The effective potential V_{eff} in perturbation theory

encodes equilibrium thermodynamics as function of BSM parameters.

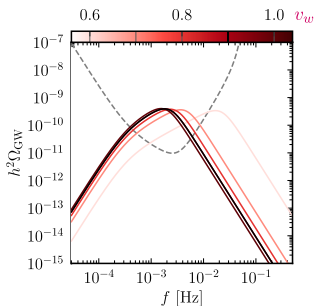
Origin of uncertainty.

T_\star reference temperature of the transition ($T_\star = T_n, T_p$),

α phase transition strength,

β/H inverse duration of the transition,

v_w Iteratively solve coupled fluid, scalar field, Boltzmann equations²



²[nu\(oo\)](#) Numerical package for v_w : Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, van de Vis: in progress, B. Laurent and J. M. Cline, *First principles determination of bubble wall velocity*, Phys. Rev. D **106** (2022) 023501 [2204.13120], **LISA Cosmology Working Group** Collaboration, C. Caprini, R. Jinno, M. Lewicki, et al., *Gravitational waves from first-order phase transitions in LISA: reconstruction pipeline and physics interpretation*, [2403.03723]

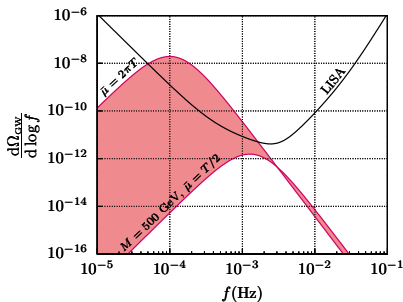
Theoretical predictions are **not robust**

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes³ as Ω_{GW} depends strongly on the transition temperature, T_* , in simulation fits:

$$\Omega_{\text{GW}} \propto \frac{(\Delta V_*)^2}{T_*^8}.$$

Vary RG scale $\bar{\mu}$ in SM extensions:

▷ SMEFT: $\text{SM} + \frac{1}{M^2}(\phi^\dagger\phi)^3$



³ D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080], O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

⁴ S. Biondini, P. Schicho, and T. V. I. Tenkanen, *Strong electroweak phase transition in t-channel simplified dark matter models*, JCAP **10** (2022) 044 [2207.12207]

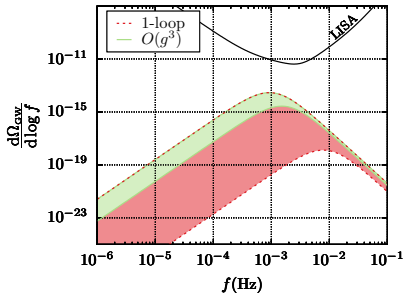
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Vary RG scale $\bar{\mu}$ in SM extensions:

- ▷ SMEFT: SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- ▷ xSM: SM + singlet
- ▷ DM models⁴



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**How to ensure quantitative precision in
finite- T perturbation theory?**

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

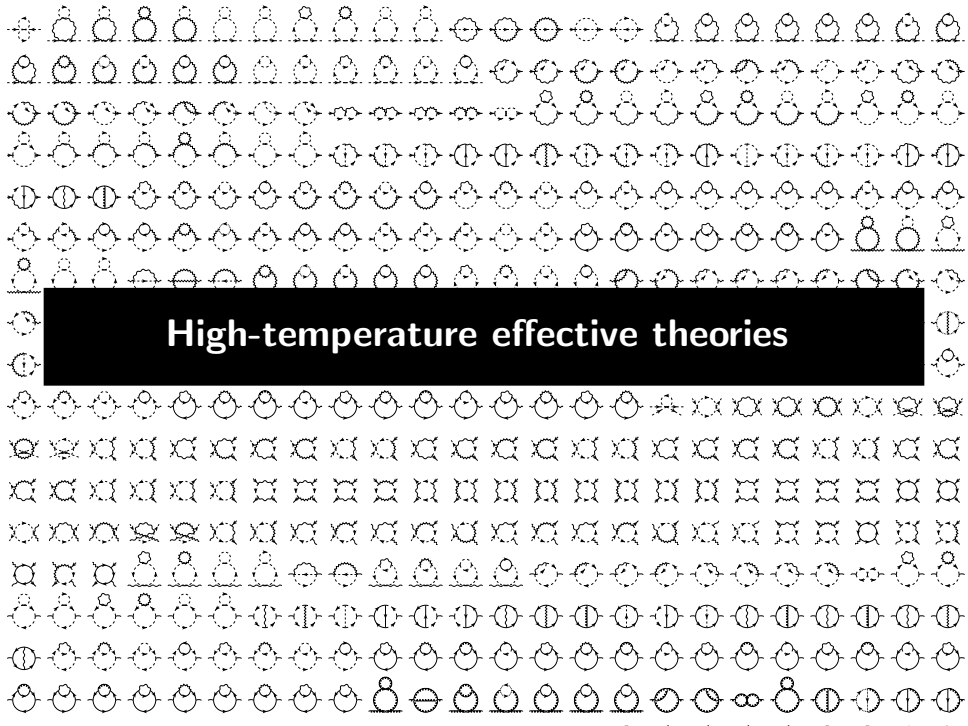
$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}} .$$

Assume particle χ couples to the SM via $g^2 \Phi^\dagger \Phi \chi^\dagger \chi$. If $M_\chi \gg m_\Phi$, integrating out χ introduces Higgs-mass corrections of the form:

$$(\Delta m_\Phi^2) \Phi^\dagger \Phi = \text{---} \sim g^2 M_\chi^2 \Phi^\dagger \Phi, \quad \frac{(\Delta m_\Phi^2)}{m_\Phi^2} = g^2 \left[\frac{M_\chi}{m_\Phi} \right]^2 .$$

Relevant operators ($\sigma > 0$) in the IR get large **UV contributions** and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \right]^\sigma \stackrel{!}{\sim} 1 \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2} \right]^{1/\sigma} \gg 1 \end{cases}$$



High-temperature effective theories

Equilibrium thermodynamics: Imaginary time formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \rightarrow \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \rightarrow -i\tau$,

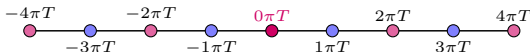
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \exp \left[- \int_0^{\beta=1/T} d\tau \int_{\mathbf{x}} \mathcal{L}_{\text{E}} \right], \quad \phi(0, \mathbf{x}) = \pm \phi(\beta, \mathbf{x}).$$

(Anti-)periodic bosonic(fermionic) fields at boundaries \rightarrow **compact time direction**: $\mathbb{R}^3 \times S^1_{\beta}$.

Finite- τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with **Matsubara frequencies**

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

Absent zero mode $\omega_{n=0}$ for fermions:



Differences to zero temperature

$(d + 1) \rightarrow d$ -dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:

$$\left. \begin{array}{l} \text{-----} \\ A_\mu \\ \text{-----} \\ \psi_i \longrightarrow \end{array} \right\} \propto \frac{1}{P^2 + m^2} = \frac{1}{\mathbf{p}^2 + \omega_n^2 + m^2}, \quad P = (\omega_n, \mathbf{p}).$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{d^{d+1}p}{(2\pi)^{d+1}} f(p) \rightarrow T \sum_n \int \frac{d^d p}{(2\pi)^d} f(\omega_n, \mathbf{p}) = \oint_P f(\omega_n, \mathbf{p}).$$

- ▶ Ultraviolet (UV) contained at $T = 0$
- ▶ Infrared (IR) sensitivity stronger \rightarrow field in reduced spacetime dimension

Multi-scale hierarchy in hot **classicalizing** gauge theories

Evaluated **Matsubara sums** yield Bose(Fermi) distribution. Asymptotically high T and weak $g \ll 1$: **effective expansion parameter**

$$\epsilon_B = g^2 n_B(E) = \frac{g^2}{e^{E/T} - 1} \approx \frac{g^2 T}{E}.$$

Differs from weak coupling g^2 . Fermions are IR-safe $g^2 n_F(E) \sim g^2/2$.

$E \sim$	{	πT	<i>hard scale</i>	quantum theory
		gT	<i>soft scale</i>	
		$g^{3/2}T$	<i>supersoft scale</i>	symmetry breaking
		$g^2 T/\pi$	<i>ultrasoft scale</i>	

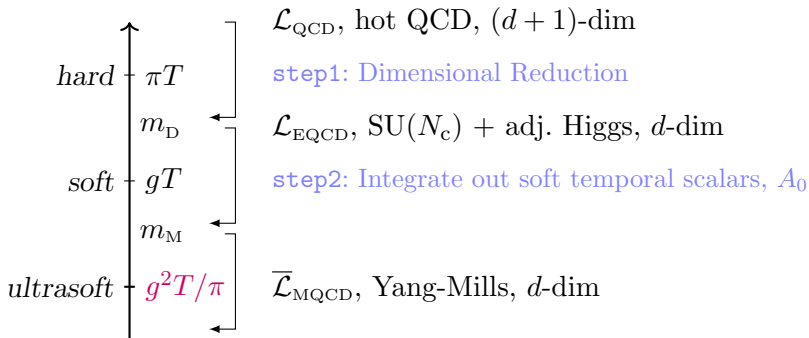
Limit: Confinement-like behavior in ultrasoft sector $g^2 n_B(g^2 T) \sim \mathcal{O}(1)$.
Ultrasoft bosons are non-perturbative at finite T : **Linde IR problem**.⁵

⁵ A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289, O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672]

Effective Field Theory (EFT): Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.

Precision thermodynamics of non-Abelian gauge theories as QCD and (EW) phase transition⁶ using e.g. DRalgo.⁷ Two step procedure:



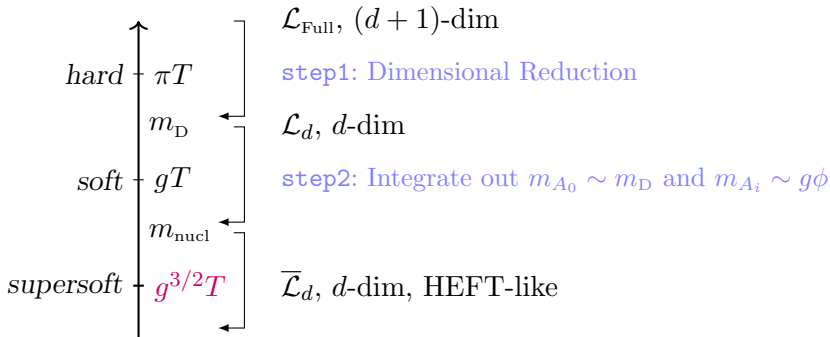
⁶ K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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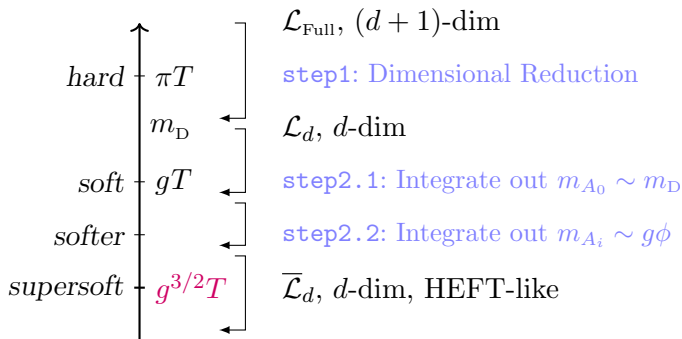
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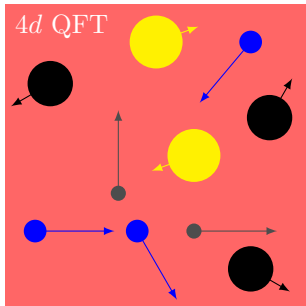
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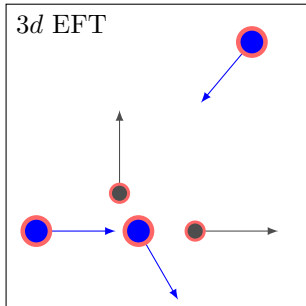
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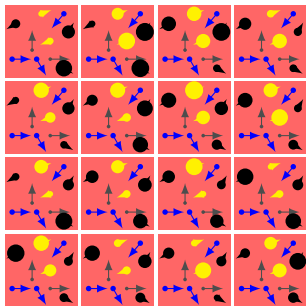
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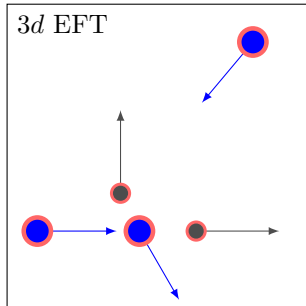
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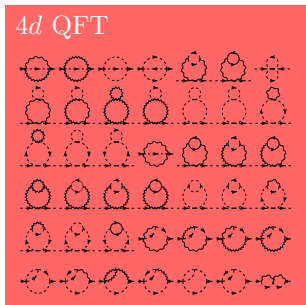
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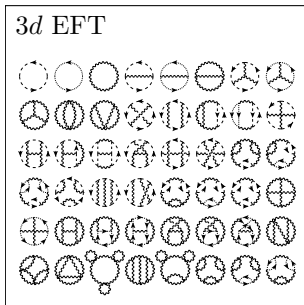
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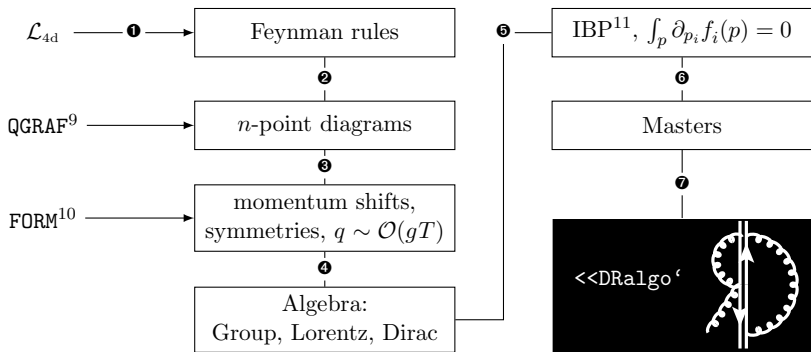


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The Dimensional Reduction algorithm (DRalgo)

State-of-the-art Mathematica package DRalgo.⁸



⁸ github.com/DR-algo/DRalgo, A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo: A package for effective field theory approach for thermal phase transitions, *Comput. Phys. Commun.* **288** (2023) 108725 [2205.08815]

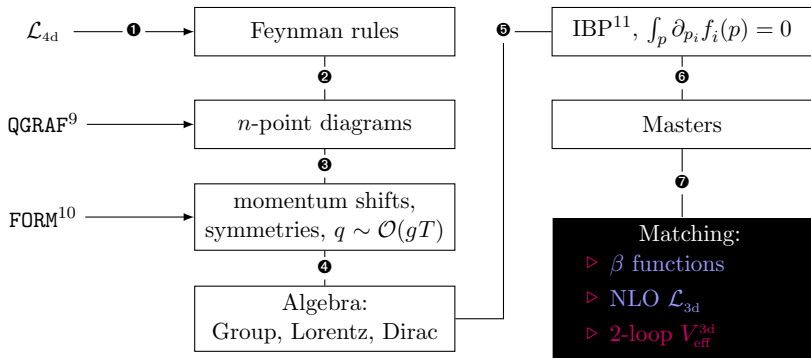
⁹ P. Nogueira, *Automatic Feynman Graph Generation*, *J. Comput. Phys.* **105** (1993) 279

¹⁰ B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [1707.06453]

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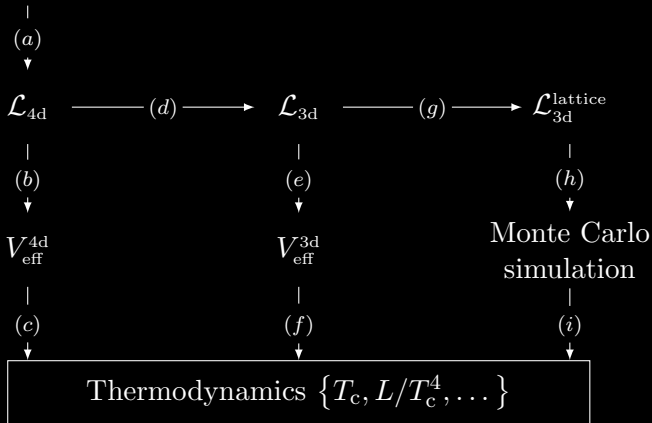
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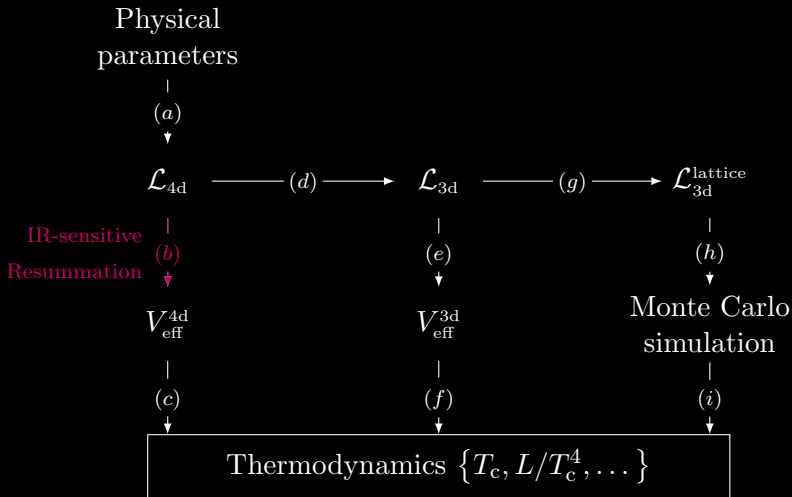
Predictions of GWs sourced by cosmological phase transitions

Thermodynamics of electroweak phase transition

Physical
parameters

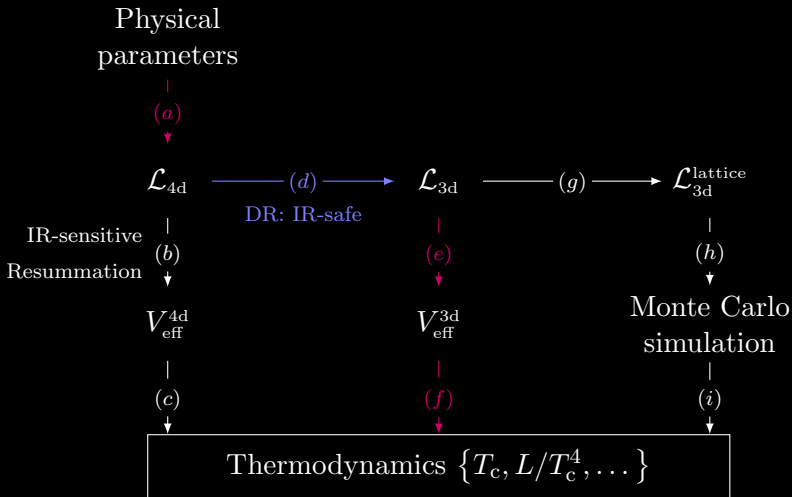


Thermodynamics of electroweak phase transition



▷ 4d approach: Classical predictions within quantum theory

Thermodynamics of electroweak phase transition



- ▷ 4d approach: Classical predictions within quantum theory
- ▷ 3d approach: Classical predictions within classical EFT

Completing the perturbative program for phase transitions

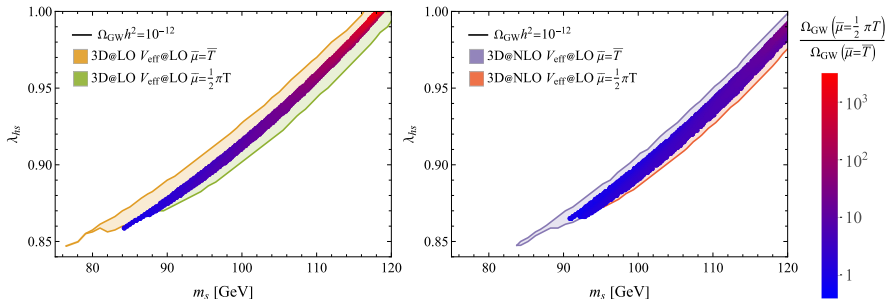
- ① \mathcal{L}^{3d} (hard-to-soft matching)
- ② V_{eff}^{3d} (soft-to-supersoft matching)
- ③ Thermodynamics

① \mathcal{L}_{3d} (hard-to-soft matching)

3d EFT at NLO (gauge-invariant) with DRalgo. **Todo: NNLO.**

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \rightarrow x + s$.
2-loop corrections significantly affect GW signal.¹²

Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$\text{SNR}_{\text{LISA}} > 10$
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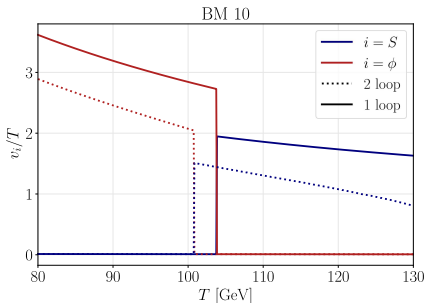
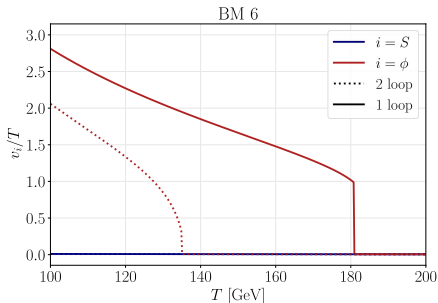


¹² M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Transitions in the xSM (real singlet)

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \rightarrow x + s$.
2-loop corrections are significant.¹³

Benchmark	m_s	λ_{hs}	λ_s
BM6	350 GeV	3.5	0.3
BM10	325 GeV	3.5	0.3



¹³ L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

② V_{eff}^{3d} (soft-to-supersoft matching)

Integrating out vector bosons in two steps up to 2-loops with DRalgo

$$m_{A_0}^2 [\sim (gT)^2] \xrightarrow{\text{step2.1}} \gg m_{A_i}^2 [\sim (g_3\phi)^2] \xrightarrow{\text{step2.2}} \gg m_3^2 [\sim \lambda_3\phi^2] .$$

Focus on **step2.2** and add last perturbative orders N³LO and N⁴LO.¹⁴

WHAT IF WE TRIED
MORE LOOPS ?



¹⁴ A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: the final verdict on perturbation theory*, [2405.18349]

2 V_{eff}^{3d} (soft-to-supersoft matching)

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Focus on **step2.2** and add last perturbative orders N³LO and N⁴LO.¹⁴

$$\begin{aligned}
 V_{\text{eff}}^{\text{sym}} &\sim \underbrace{\text{[diagram]}}_{\text{N}^2\text{LO}} + \underbrace{\text{[diagram]}}_{\text{N}^3\text{LO}} + \underbrace{\text{[diagrams]}}_{\text{N}^4\text{LO}} \dots \\
 V_{\text{eff}}^{\text{bro}} &\sim \underbrace{\text{[diagrams]}}_{\text{LO}} + \underbrace{\text{[diagrams]}}_{\text{NLO}} + \underbrace{\text{[diagram]}}_{\text{N}^2\text{LO}} \\
 &+ \underbrace{\text{[diagrams]}}_{\text{N}^3\text{LO}} + \underbrace{\text{[diagrams]}}_{\text{N}^4\text{LO}}
 \end{aligned}$$

¹⁴ A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: the final verdict on perturbation theory*, [2405.18349]

② V_{eff}^{3d} (soft-to-supersoft matching)

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Focus on **step2.2** and add last perturbative orders N³LO and N⁴LO.¹⁴
 In **strict EFT expansion** this organization can also be understood as:

$$S_{\text{supersoft}}^{\text{tree}} = \int_{\mathbf{x}} \frac{1}{2} (\partial_i s)^2 Z_s + \underbrace{\bullet + \text{blob}}_{\text{LO}} + \underbrace{\text{blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob}}_{\text{NLO}} + \underbrace{\text{blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob} \text{ blob}}_{\text{N}^3\text{LO}},$$

$$S_{\text{supersoft}}^{\text{1-loop}} = \underbrace{\text{blob}}_{\text{N}^2\text{LO}} + \underbrace{\text{blob} \text{ blob}}_{\text{N}^4\text{LO}} .$$

¹⁴ A. Ekstedt, O. Gould, and J. Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order*, Phys. Rev. D **106** (2022) 036012 [2205.07241], A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, *Cosmological phase transitions at three loops: the final verdict on perturbation theory*, [2405.18349]

The SM-like 3d EFT (SU(2)+Higgs)

describes the thermodynamics¹⁵ of several parent 4d theories:

$$\mathcal{L}_{3d} = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \Phi)^\dagger (D_i \Phi) + V(\Phi) ,$$
$$V(\Phi) = m_3^2 \Phi^\dagger \Phi + \lambda_3 (\Phi^\dagger \Phi)^2 .$$

If $m_A \sim g_3 \phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) = \bullet + \text{blob} .$$

¹⁵ K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

¹⁶ O. Gould and T. V. I. Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions*, JHEP **01** (2024) 048 [2309.01672],

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$$V_{\text{LO}}(\Phi) = m_3^2 \Phi^\dagger \Phi + \lambda_3 (\Phi^\dagger \Phi)^2 - \frac{g_3^3}{2\pi} \left(\frac{\Phi^\dagger \Phi}{2} \right)^{3/2} .$$

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$$V_{\text{LO}}(\Phi) \rightarrow y \Phi^\dagger \Phi + x (\Phi^\dagger \Phi)^2 - \frac{1}{2\pi} \left(\frac{\Phi^\dagger \Phi}{2} \right)^{3/2} .$$

Since $x \sim \frac{m_3^2}{m_A^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly¹⁶ x -pand the perturbative series using 3D EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2} , \quad y \equiv \frac{m_3^2}{g_3^4} .$$

¹⁵ K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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③ Thermodynamics

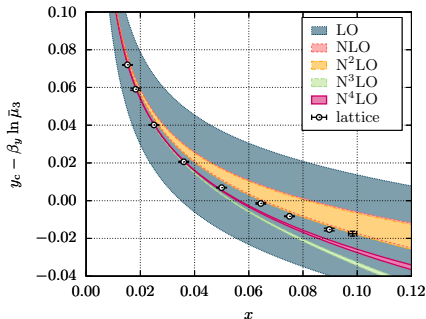
By using $F \sim V_{\text{eff}}(\phi_{\min})$, determine the critical mass y_c (or T_c)

$$\Delta F(y_c(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_c(x), x) = 0,$$

and the scalar condensates¹⁷

$$\Delta\langle\Phi^\dagger\Phi\rangle \equiv \frac{\partial}{\partial y}\Delta F,$$

$$\Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \frac{\partial}{\partial x}\Delta F.$$



¹⁷Lattice data from: K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis*, Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], O. Gould, S. Güyer, and K. Rummukainen, *First-order electroweak phase transitions: a nonperturbative update*, [2205.07238]

③ Thermodynamics

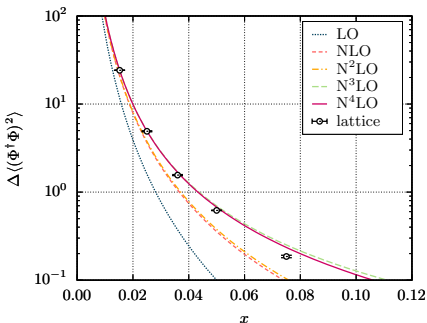
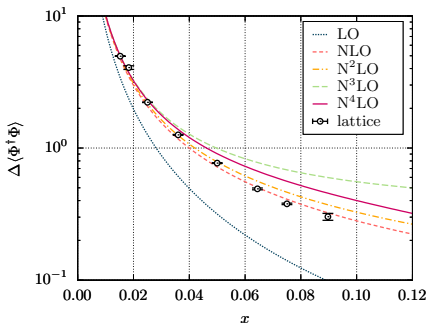
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Predicting Gravitational Waves

Thermodynamics enters the GW spectrum through the strength and inverse duration of the transition, $h^2\Omega_{\text{GW}}(f; H_*, \alpha, \beta, v_w)$:

$$\alpha \sim \frac{d\Delta F(y_c, x)}{d\ln T} = \left(\frac{dy_c}{d\ln T}\right)\Delta\langle\Phi^\dagger\Phi\rangle + \left(\frac{dx}{d\ln T}\right)\Delta\langle(\Phi^\dagger\Phi)^2\rangle,$$

$$\frac{\beta}{H} = -\frac{d\ln\Gamma}{d\ln T}.$$

\implies [UV] \times [IR] factorization.¹⁸

Completed perturbative predictions for: α at N⁴LO,
 β/H at N²LO.

Todo: final perturbative correction for thermal bubble nucleation rate.

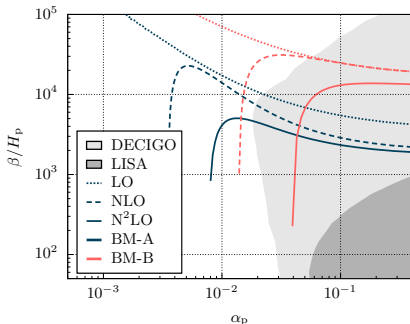
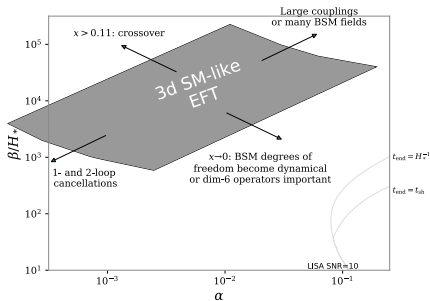
¹⁸ O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

Impact on Gravitational Waves

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.¹⁹
 Better access interesting LISA SNR by increasing loop order:

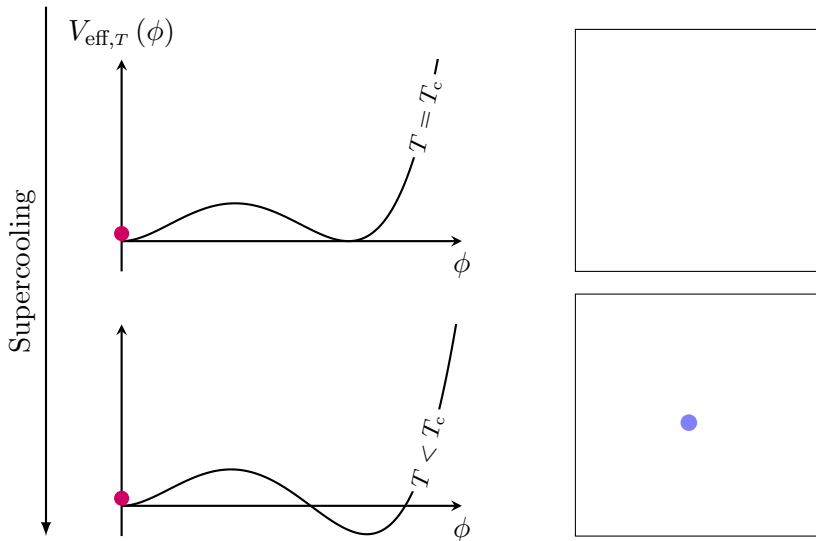
BM-A xSM with weakly portal-coupled singlet (decoupled)

BM-B xSM with strongly portal-coupled singlet



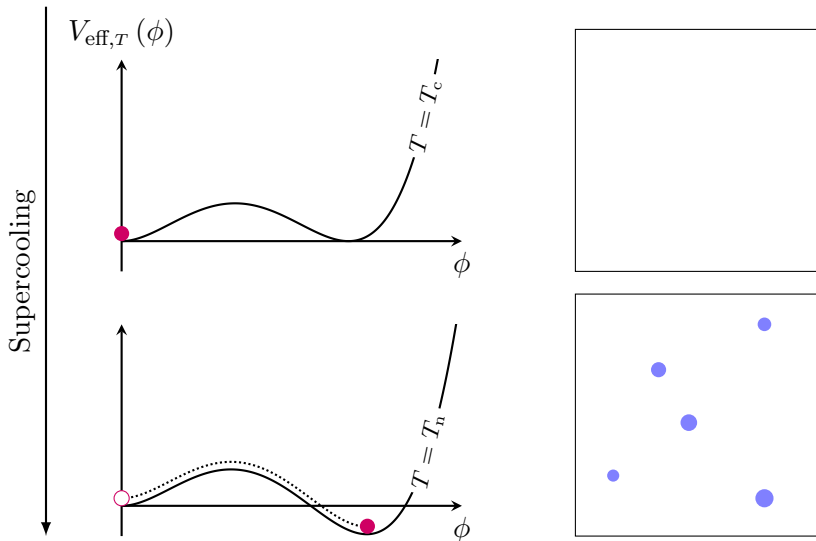
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Nucleation rate and transition reference scale



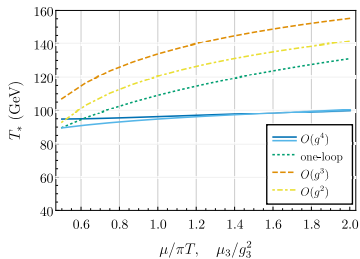
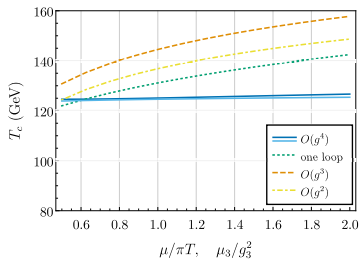
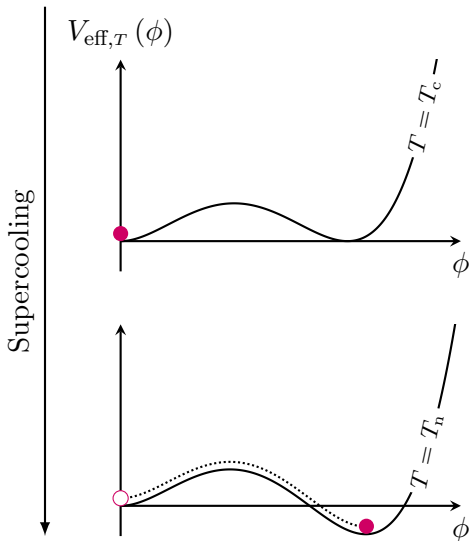
O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

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The semiclassical bubble nucleation rate

$$\Gamma \sim Ae^{-B},$$

is known to N²LO and factors into:

B Internal energy of critical bubble. Solve one non-linear ODE

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r} \frac{d\phi_b}{dr} - V'(\phi_b) = 0$$

A log entropy of fluctuations about bubble. Solve infinite number of linear ODEs

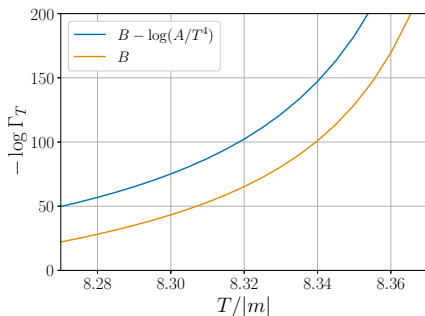
$$\left[\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} + V''(\phi_b) \right] \psi_l^b(r) = 0$$

Need both *A* and *B* for **reliable** rate computation

$$\Gamma \sim [m^4 e^S] \times e^{-E_b/T} \sim m^4 e^{-(E_b - ST)/T}.$$

Determining the Bubble determinant

BubbleDet²⁰ computes term A . Here: Yukawa model.



Nucleation not just energy and entropy \rightarrow out-of-equilibrium effects²¹

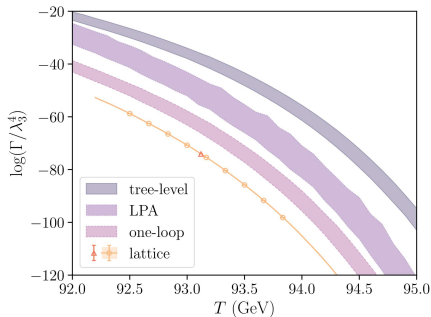
$$\Gamma \sim \frac{\kappa_{\text{dyn}}}{2\pi} m^3 e^{-(E_b - ST)/T}.$$

²⁰ A. Ekstedt, O. Gould, and J. Hirvonen, *BubbleDet: A Python package to compute functional determinants for bubble nucleation*, [2308.15652]

²¹ J. Langer, *Statistical theory of the decay of metastable states*, Ann. Phys. (N. Y). **54** (1969) 258, J. Hirvonen, *Nucleation Rate in a High-Temperature Quantum Field Theory with Hard Particles*, [2403.07987], O. Gould, A. Kormu, and D. J. Weir, *A nonperturbative test of nucleation calculations for strong phase transitions*, [2404.01876]

Determining the Bubble determinant

BubbleDet²⁰ computes term A . Here: ϕ^4 model.



Nucleation not just **energy** and **entropy** \rightarrow out-of-equilibrium effects²¹

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Conclusions

Precision thermodynamics of BSM theories:

- ▶ reliably describe cosmological FOPT and GW production,
- ▶ practical approach: **Effective Theories**.

Precision cosmology with dimensionally reduced 3d EFT:

- ▶ multi-loop sport – automatic all-order high- T resummation,
- ▶ systematic higher-loop/operator improvement,
- ▶ **universality**.

Final perturbative steps:

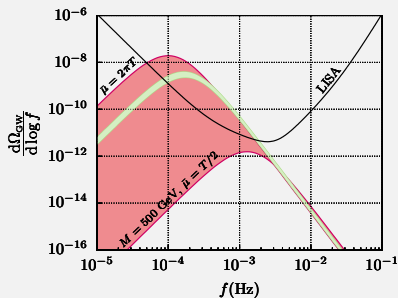
- ✱ higher dimensional operators at $\mathcal{O}(g^6)$ in the 3d EFT,
- ⊕ purely perturbative $\mathcal{O}(g^6)$ contributions from hard scale πT ,
- final perturbative corrections for bubble nucleation rate.

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Precision thermodynamics of BSM theories:

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Precision cosmology with dimensionally reduced 3d EFT:

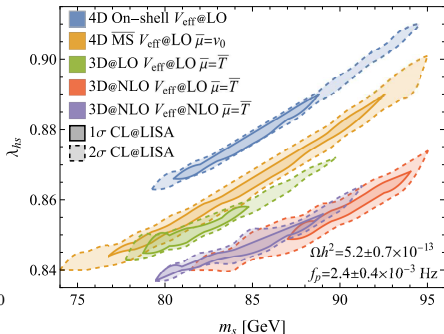
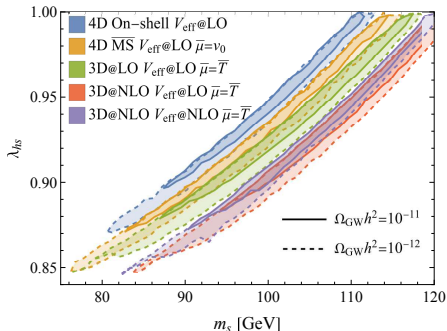


Backup

Computational diligence at $\mathcal{O}(g^4)$

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \rightarrow x + s$.
 2-loop corrections **move “Bananas”**: significant effect on GW signal.²²

Parameters m_s λ_{hs} $\lambda_s = 1$ $\alpha > 1$ $\text{SNR}_{\text{LISA}} > 10$

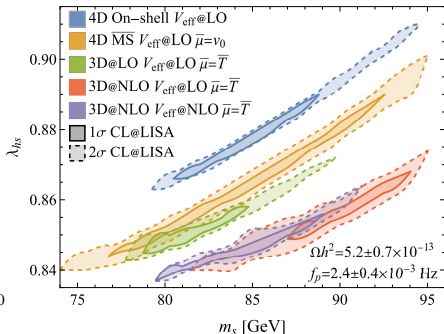
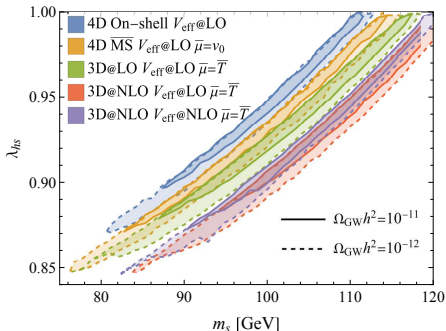


²² M. Lewicki, M. Merchand, L. Sagunski, P. Schicho, and D. Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], L. Niemi, P. Schicho, and T. V. I. Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]

Does the **banana** move?

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Resummation

Dynamically generated masses through collective plasma effects

$$m_T = g^n T + m .$$

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high- T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_B(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2 . Fermions are IR-safe $g^2 n_F |p| \sim g^2/2$.

Cure IR-sensitive contributions at $m_T \sim gT$ by thermal resummation:

$$V_{\text{eff}} \supset \text{[diagram of a loop with } N \text{ vertices]} \propto g^{2N} \left[m_T^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_T} \right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T , light bosons are non-perturbative.

The Nielsen identities²³

(A useful tool for showing gauge invariance.) Vary effective potential with gauge parameter

$$\xi \frac{\partial S_{\text{eff}}}{\partial \xi} = - \int_{\mathbf{x}} \frac{\delta S_{\text{eff}}}{\delta \phi(x)} \mathcal{C}(x) ,$$

use derivative expansion of functional

$$\mathcal{C}(x) = C(\phi) + D(\phi)(\partial_{\mu}\phi)^2 - \partial_{\mu}(\tilde{D}(\phi)\partial_{\mu}\phi) + \mathcal{O}(\partial^4)$$

results in Nielsen identities

$$\xi \frac{\partial}{\partial \xi} V_{\text{eff}} = -C \frac{\partial}{\partial \phi} V_{\text{eff}} , \quad (1)$$

$$\xi \frac{\partial}{\partial \xi} Z = -C \frac{\partial}{\partial \phi} Z - 2Z \frac{\partial}{\partial \phi} C - D \frac{\partial}{\partial \phi} V_{\text{eff}} - \tilde{D} \frac{\partial^2}{\partial \phi^2} V_{\text{eff}} . \quad (2)$$

Identity (1): **EWSB is gauge invariant.**

²³ N. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, Nucl. Phys. B **101** (1975) 173

A minimal extension of the EW sector

Real singlet (S) extension of the SM (xSM)

$$\mathcal{L}_{\text{xSM}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_S - V(\phi, S) ,$$

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu S)^2 ,$$

$$V(\phi, S) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \\ + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{4}\lambda_s S^4 + \frac{1}{2}\lambda_{hs} S^2 \phi^\dagger \phi ,$$

with portal coupling λ_{hs} . Phases at finite T :

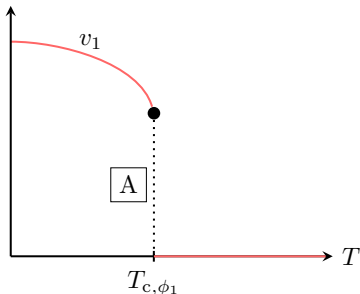
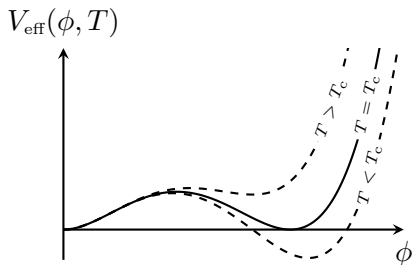
- ▷ ϕ condenses, SM-like Higgs regime
- ▷ S condenses
- ▷ No direct Dark Matter candidate. See complex singlet with parameterization $S \rightarrow v_S + S + iA$.

The effective potential in perturbation theory

receives thermal corrections $\Pi_T \sim \gamma T^2$ with $\gamma \sim g^n$. Dynamical generation of scales close to critical temperature T_c :

$$V_{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \Pi_T)\phi^2 + \frac{1}{2}\lambda\phi^4 - \frac{g^3 T}{16\pi}\phi^3 + \dots$$

$$(-\mu^2 + g^n T^2) \sim \boxed{0 \times (gT)^2}_{\text{soft}} + \boxed{0 \times (g^{3/2}T)^2}_{\text{supersoft}} + \boxed{\#(g^2 T)^2}_{\text{ultrasoft}}.$$

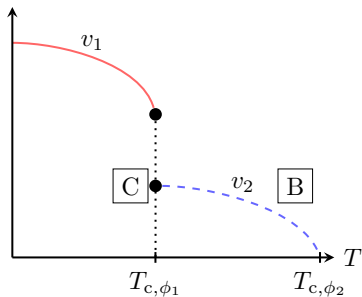
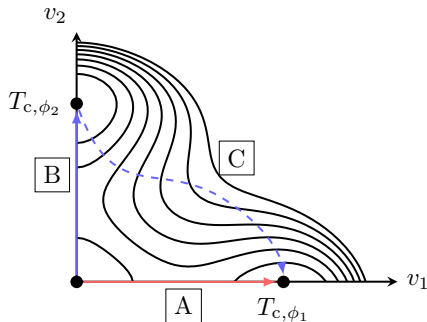


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The thermal effective potential at LO

$$V_{\text{eff}} = V_{\text{eff}}^{\text{tree}} + V_{\text{eff}}^{1\ell}.$$

At 1-loop sum over n -point functions at $Q_i = 0$ external momenta

$$\begin{aligned}
 V_{\text{eff}}^{1\ell} &= \text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---} + \frac{1}{3} \text{---}\text{---}\text{---} + \dots \Big|_{Q_i=0} \\
 &= \frac{1}{2} \oint_P \ln(P^2 + m^2) \\
 V_{\text{eff}}^{1\ell} &= \underbrace{\frac{1}{2} \int_P \ln(P^2 + m^2)}_{\equiv V_{\text{CW}}(m)} - T \underbrace{\int_p \ln(1 \mp n_{\text{B/F}}(E_p, T))}_{\equiv V_{T,b/f}\left(\frac{m^2}{T^2}\right)} \\
 &= \underbrace{\frac{T}{2} \int_p \ln(p^2 + m^2)}_{\equiv TV_{\text{soft}}(m)} + \underbrace{\frac{1}{2} \oint'_{P/\{P\}} \ln(P^2 + m^2)}_{\equiv V_{\text{hard}}(m)}.
 \end{aligned}$$

Renormalization scale (in)dependence at finite T

At zero temperature

$$V_{\text{eff}}(\phi, \bar{\mu}) = \boxed{\frac{V_{\text{eff}}^{\text{tree}}}{\mathcal{O}(g^2)}} + \boxed{\frac{V_{\text{CW},1\ell}}{\mathcal{O}(g^4)}}, \quad \mu \frac{d}{d\mu} \left(V_{\text{eff}}^{\text{tree}} + V_{\text{CW},1\ell} \right) = 0.$$

At finite temperature²⁴

$$V_{\text{eff}}^{\text{res.}}(\phi, T, \bar{\mu}) = \boxed{\frac{V_{\text{eff}}^{\text{tree}}}{\mathcal{O}(g^2)}} + \boxed{\frac{V_{\text{res.,soft}}}{\mathcal{O}(g^3)}} + \boxed{\frac{V_{\text{hard}}}{\mathcal{O}(g^2 T^2) + \mathcal{O}(g^4)}},$$

running of 1-loop thermal masses is of the same order as 2-loop thermal-mass logarithms.

Automatically included in dimensionally reduced 3d EFT:

$$\mu \frac{d}{d\mu} \text{---}\bullet\text{---} \sim \mu \frac{d}{d\mu} \text{---}\bigcirc\text{---} \sim \text{---}\bigoplus\text{---} \sim \text{---}\bigcirc\text{---} \sim \mathcal{O}(g^4 T^2)$$

²⁴ O. Gould and T. V. I. Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

