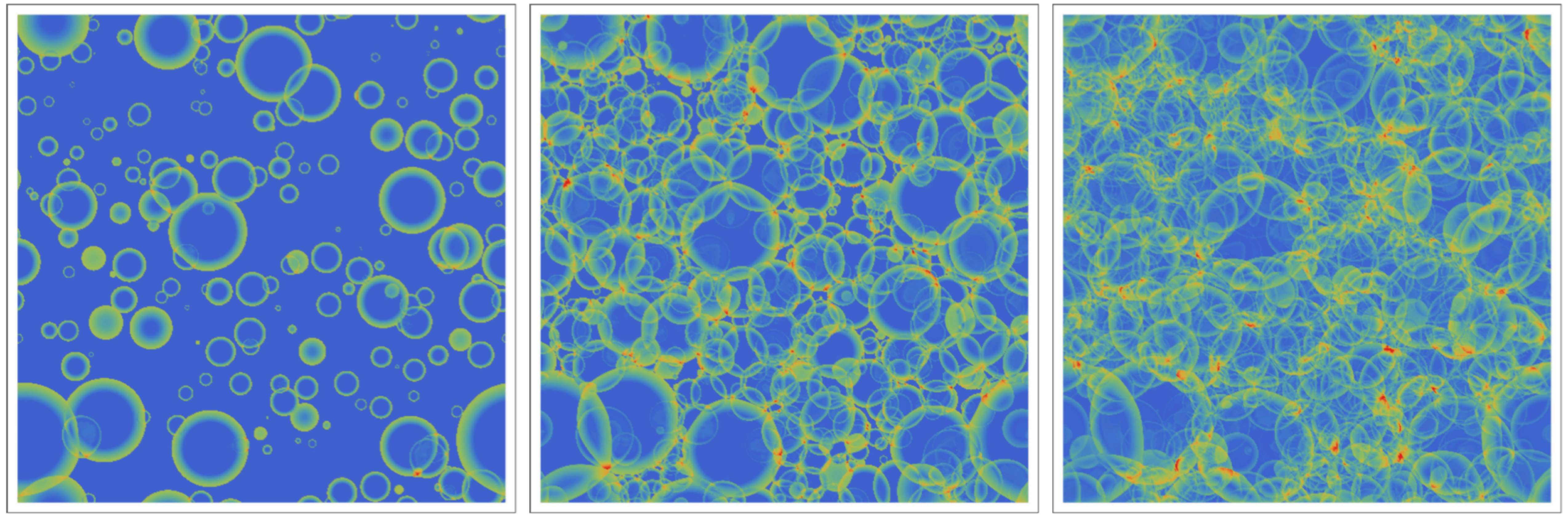
Wall velocity in cosmological phase transitions

Pollica workshop on
Fundamental physics and gravitational wave detectors

Jorinde van de Vis, September 20th 2024

Cosmological phase transitions (see talk by M. Hindmarsh)

- Expanding bubbles of true vacuum
- Isolated bubbles reach a terminal expansion velocity: v_w
- … or they keep accelerating until they collide



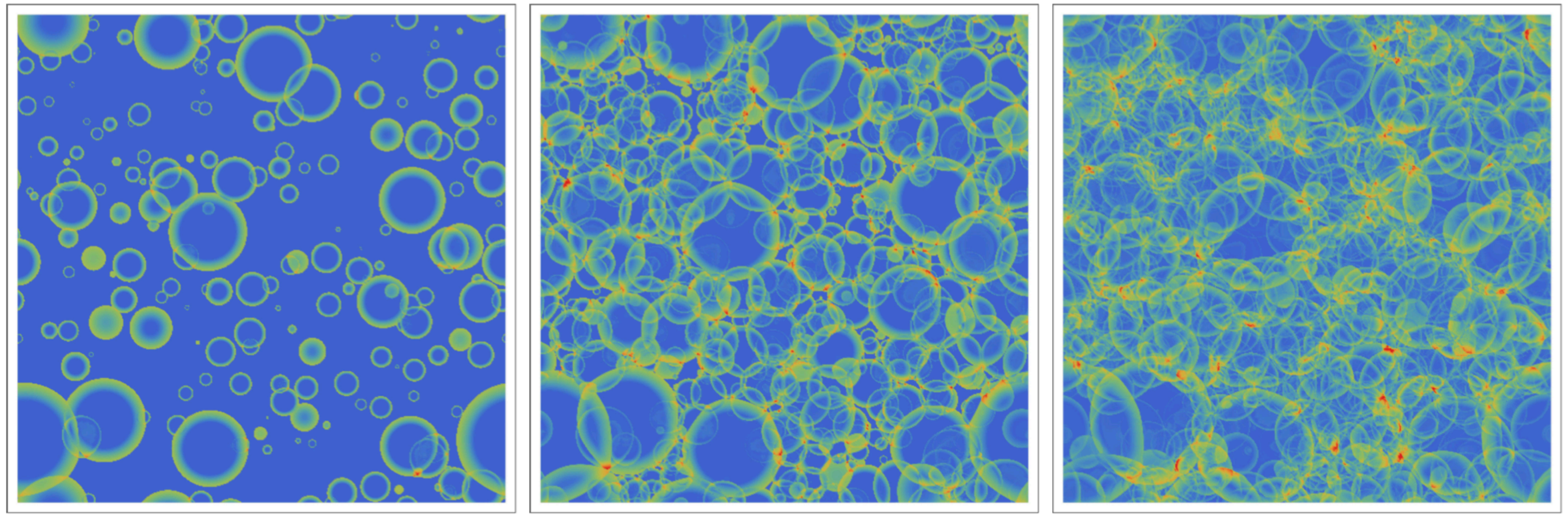
Jinno, Konstandin, Rubira 2020

Cosmological phase transitions (see talk by M. Hindmarsh)

- Expanding bubbles of true vacuum

Topic of this talk

- Isolated bubbles reach a terminal expansion velocity: v_w
- … or they keep accelerating until they collide

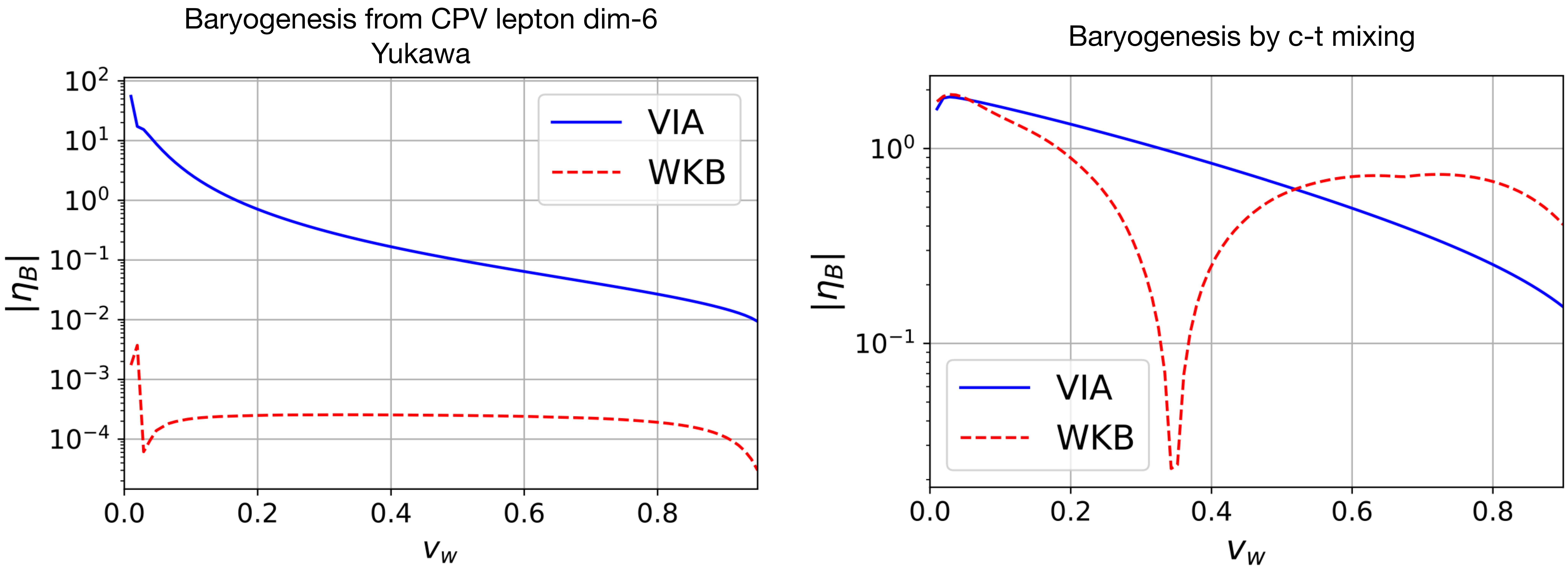


Jinno, Konstandin, Rubira 2020

Dependence on the wall velocity (some examples)

Wall velocity affects the baryon asymmetry

Talk by M. Postma



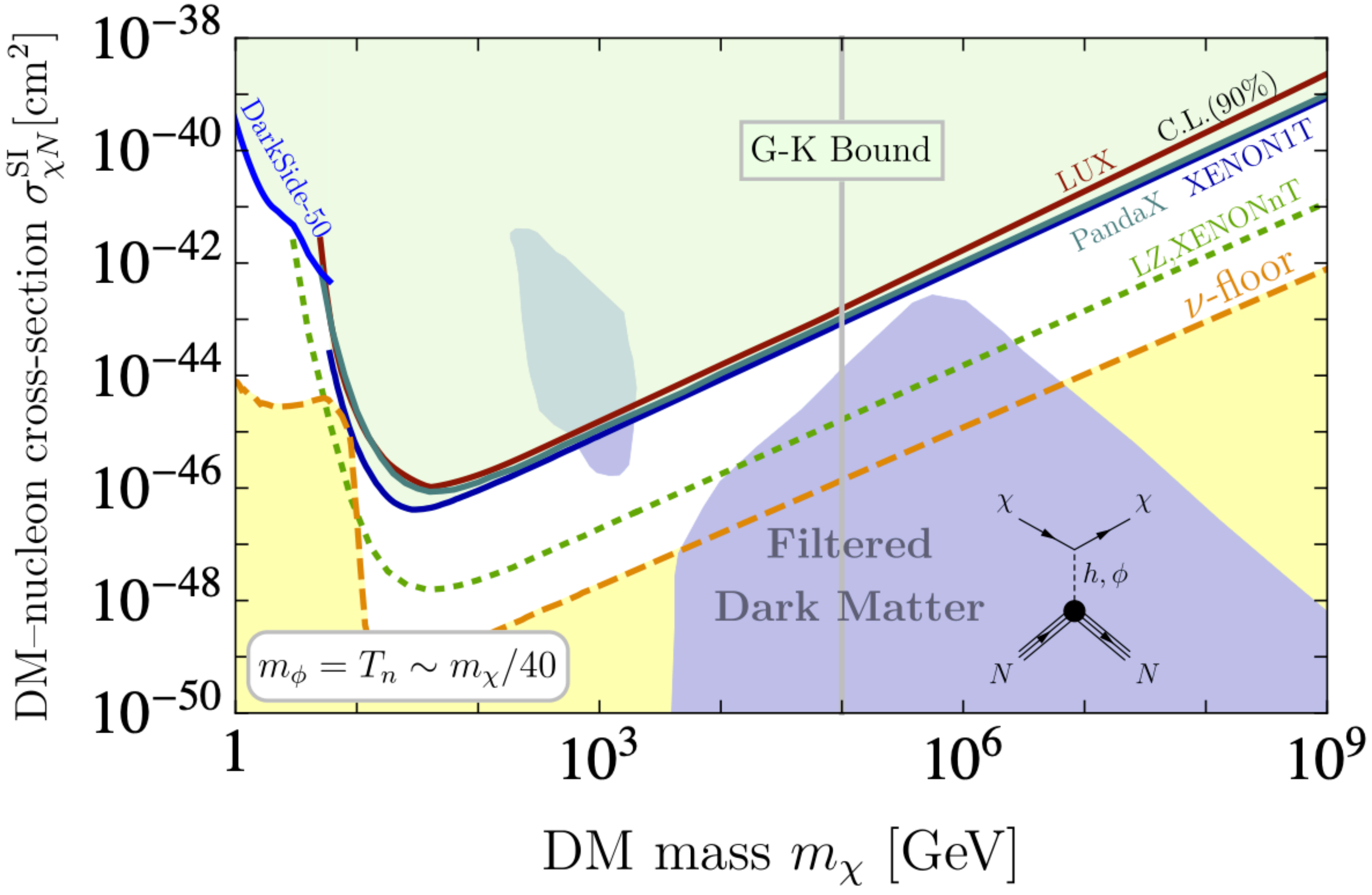
Baryogenesis from CPV lepton dim-6 Yukawa

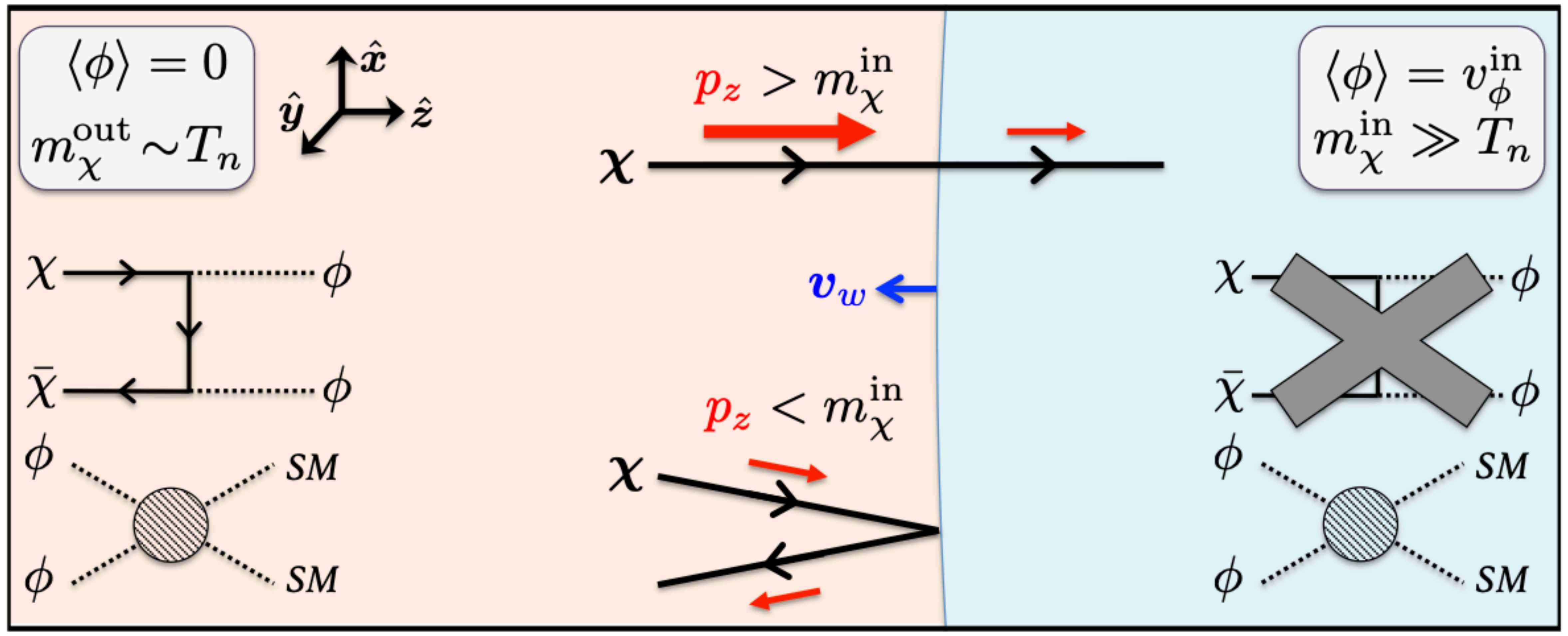
Baryogenesis by c-t mixing

Figures from: Cline, Laurent 2021

Dark matter production

Slow bubbles: Filtered dark matter Baker, Kopp, Long 2019, Chway, Jung, Shin 2019, Marfatia, Tseng 2020

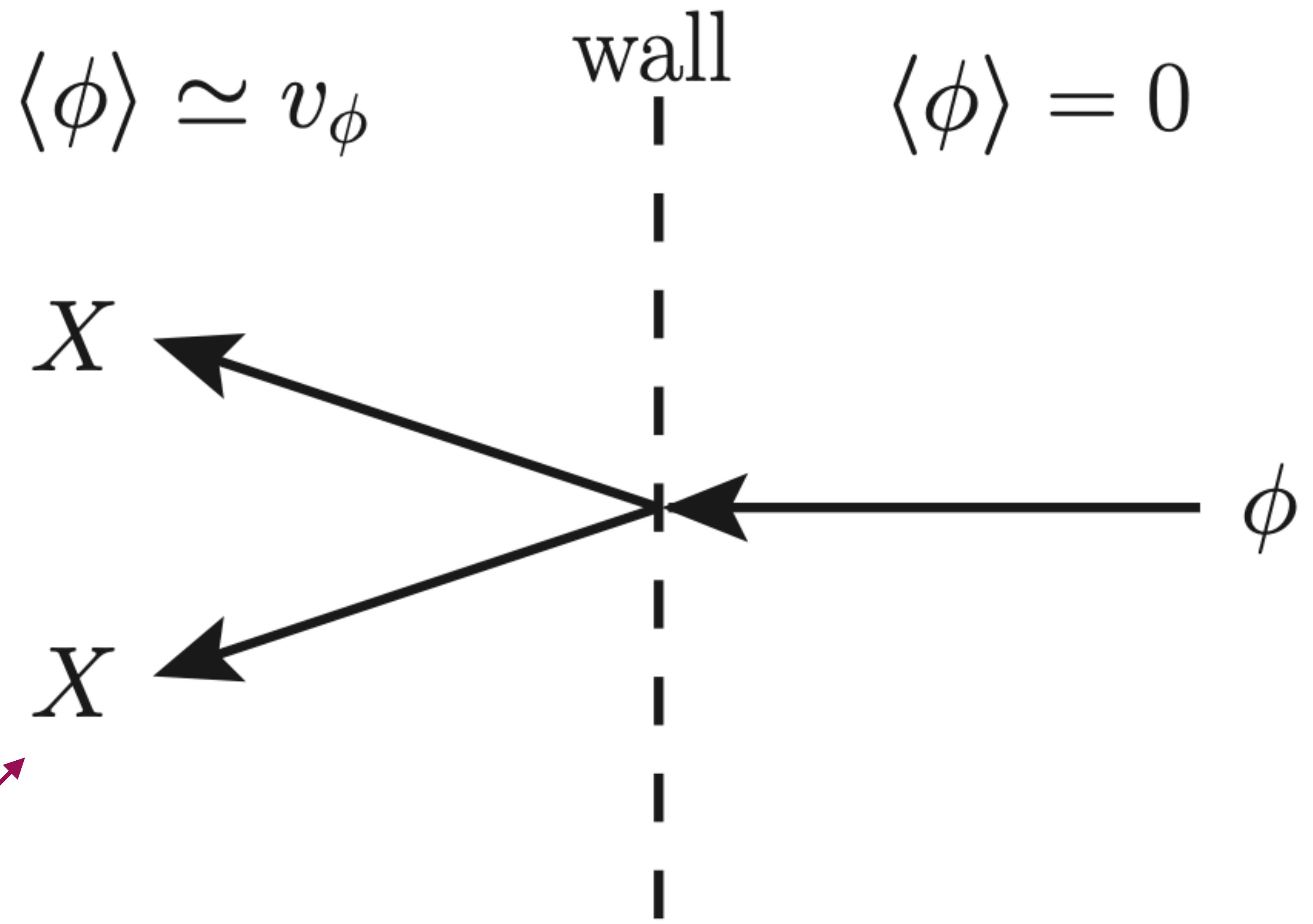




Figures from: Baker, Kopp, Long 2019

Dark matter production

Fast bubbles: Azatov, Vanvlasselaer, Yin 2021; Baldes, Gouttenoire, Sala 2022, …

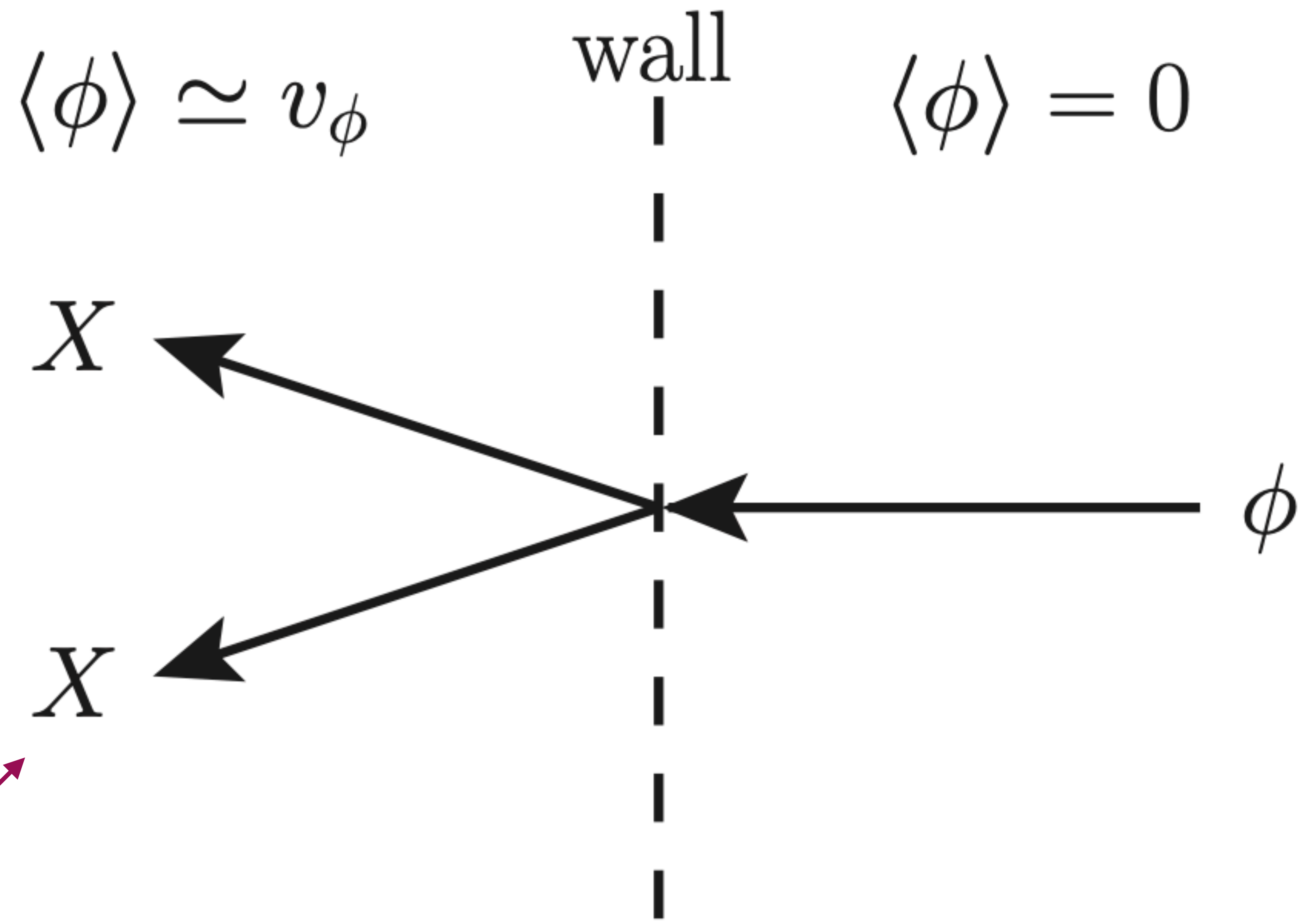


Heavy, out-of-equilibrium dark matter

Figure from: Baldes, Gouttenoire, Sala 2022

Dark matter production

Fast bubbles: Azatov, Vanvlasselaer, Yin 2021; Baldes, Gouttenoire, Sala 2022, …

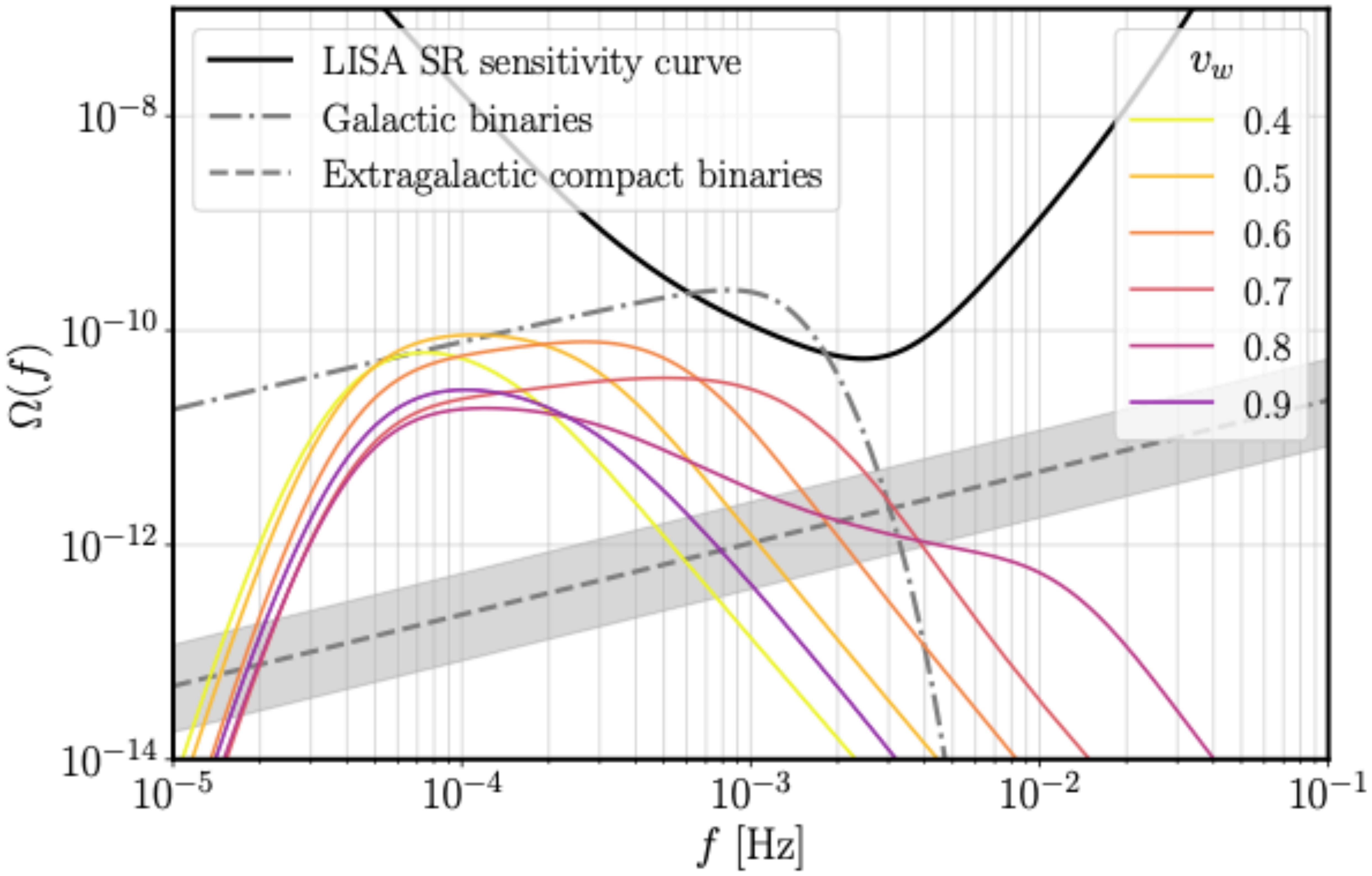


Heavy, out-of-equilibrium dark matter

Figure from: Baldes, Gouttenoire, Sala 2022

- Related mechanism for baryogenesis: Baldes et al. 2021; Azatov, Vanvlasselaer, Yin 2021

The wall velocity affects the GW spectrum



(a) Fixed: $\alpha = 0.2, \quad r_* = 0.1, \quad T_{\rm n} = 100$ GeV.

Figure from: Gowling, Hindmarsh 2021

The wall velocity affects the GW spectrum

- $\frac{d\Omega_{\rm gw}}{d\ln(f)} = 0.687 F_{\rm gw,0} K^2 H_* R_* / c_s \tilde{\Omega}_{\rm gw} C\left(f/f_{p,0}\right)$

  LISA Cosmo-wg 2019, based on Hindmarsh, Huber, Rummukainen, Weir 2015&2017

- With

  $R_* \sim (8\pi)^{1/3}/\beta \max(\mathrm{c_s}, \mathrm{v_w})$

  $K \sim \alpha\kappa/(1+\alpha)$

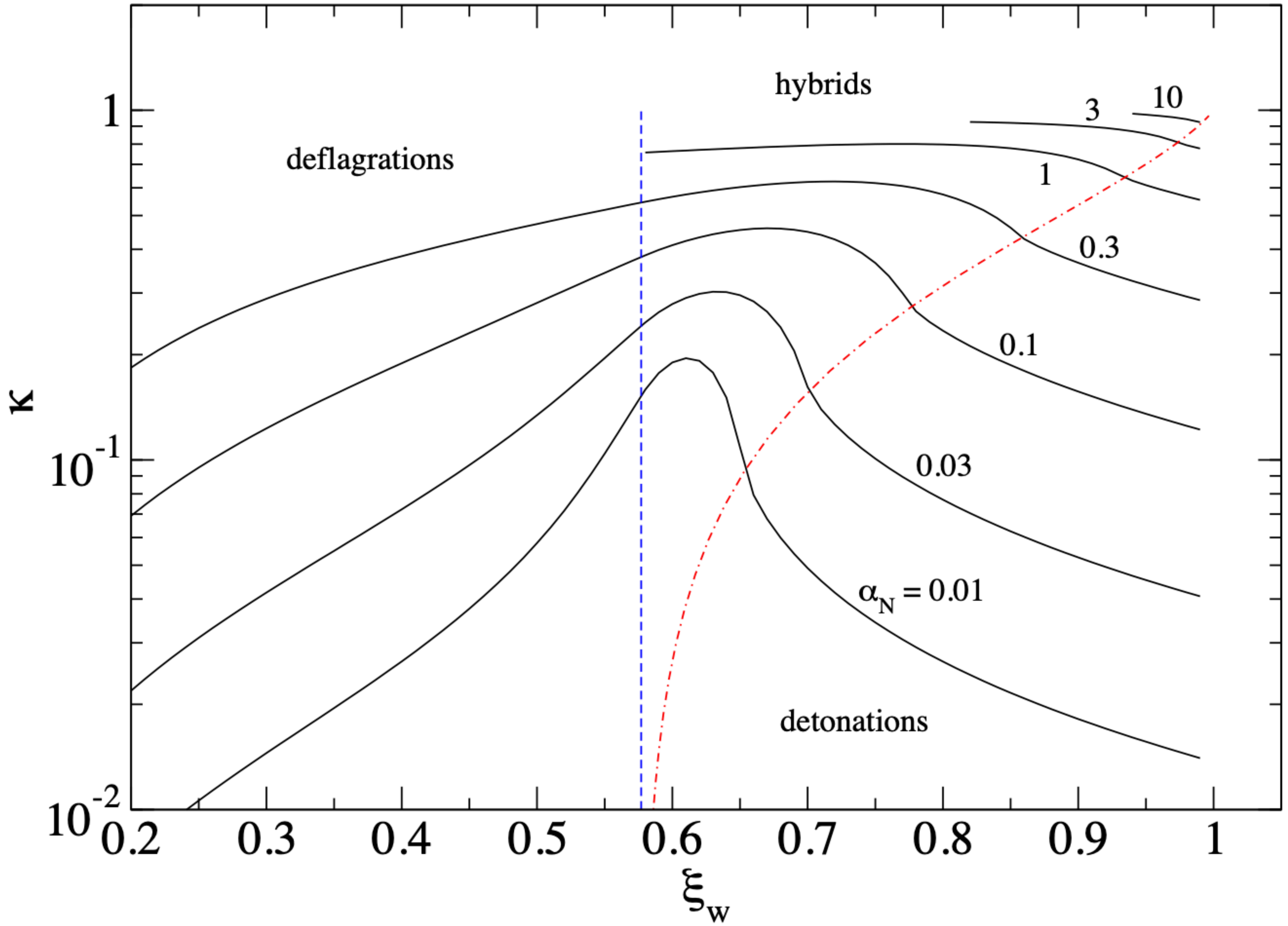


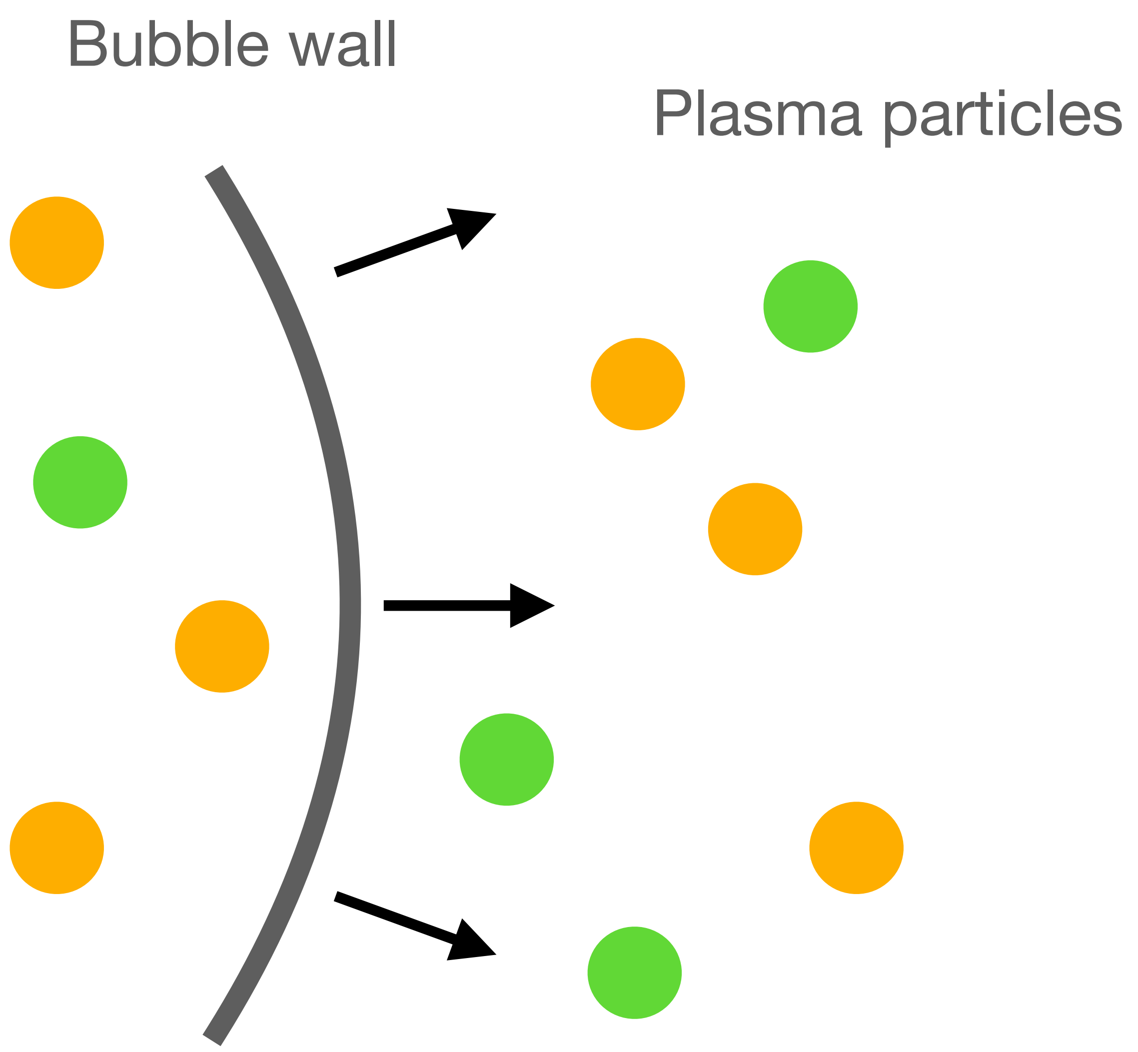
Figure from: Espinosa, Konstandin, No, Servant 2010

Computation of the wall velocity

Coupled bubble wall-plasma system

Assuming weak coupling

- Energy release provides outward pressure
- Plasma particles provide friction by reflections and by gaining mass by entering the bubble
- Hydrodynamic backreaction effects
- Wall velocity follows from $|P_{\text{outward}}| = |P_{\text{inward}}|$



Weakly coupled bubble wall-plasma system

Prokopec, Moore 1995

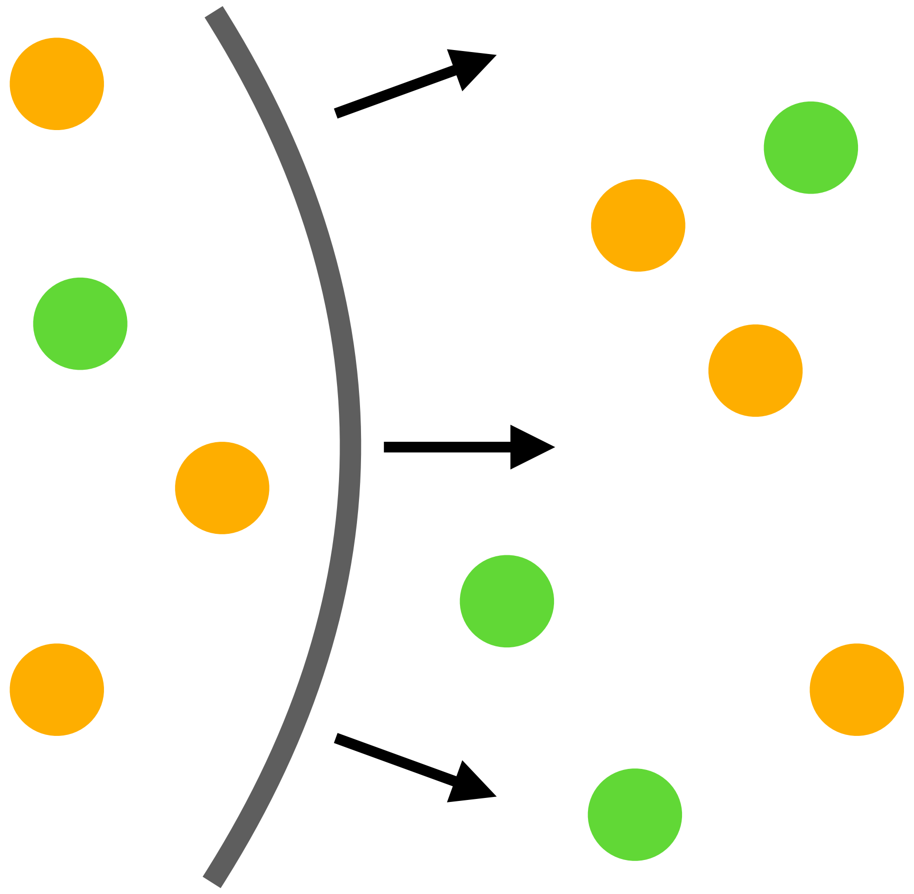
- Scalar field:

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E}\overset{\substack{\text{Out-of-equilibrium particles (top)}\\ \downarrow}}{\delta f}(p,x) = 0$$

- Particles in the plasma (schematically):
$\partial_t f + \dot{\vec{x}} \cdot \partial_{\vec{x}} f + \dot{\vec{p}} \cdot \partial_{\vec{p}} f = -C[f]$

- Temperature and fluid velocity profile from EM conservation

- Vary wall parameters until all equations are satisfied to determine v_w



Challenges

- $$\Box\,\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E}\,\delta f(p,x) = 0$$
- $$\partial_t f_i + \dot{\vec{x}}\cdot\partial_{\vec{x}} f_i + \dot{\vec{p}}\cdot\partial_{\vec{p}} f_i = -\,C_i[f_i]$$

Coupled system of equations

Multiple out-of-equilibrium particles

Matrix elements and collision terms are difficult to compute

- The phase transition could also be strongly coupled

Challenges

-

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi}\int$$

- oupled system of equations

$$\partial_t f_i + \dot{\vec{x}} \cdot \partial_{\vec{x}} f_i + \dot{\vec{p}} \cdot \partial_{\vec{p}} f_i =$$

Multiple out-of-equilibrium particles

C
di

Many papers

- The phase transition could also be strongly coupled

Solutions better than guessing v_w

- Solve the system e.g. Moore, Prokopec 1995, Dorsch, Huber, Konstandin 2021, Laurent, Cline 2022
- Use a (hopefully) reasonable approximation:
    - Local thermal equilibrium Konstandin, No 2011, Barroso Mancha, Prokopec, Swiezewska 2020, Balaji, Spannowski, Tamarit 2020, Ai, Laurent, JvdV 2023, Ai, Nagels, Vanvlasselaer 2024
    - Large jump in degrees of freedom Sanchez-Garitaonandia, JvdV 2023
- Use a numerical package: Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV: in progress

Hydrodynamic-based approximations to the wall velocity

Lightning hydrodynamics recap

- Perfect fluid: $T_{\mu\nu} = wu_\mu u_\nu - pg_{\mu\nu}$
- Fluid equations follow from $\partial_\mu T^{\mu\nu} = 0$
- Matching conditions follow from $\displaystyle\int_{-\delta}^{+\delta} dz \partial_\mu T^{\mu\nu} = 0$

Lightning hydrodynamics recap

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Lightning hydrodynamics recap

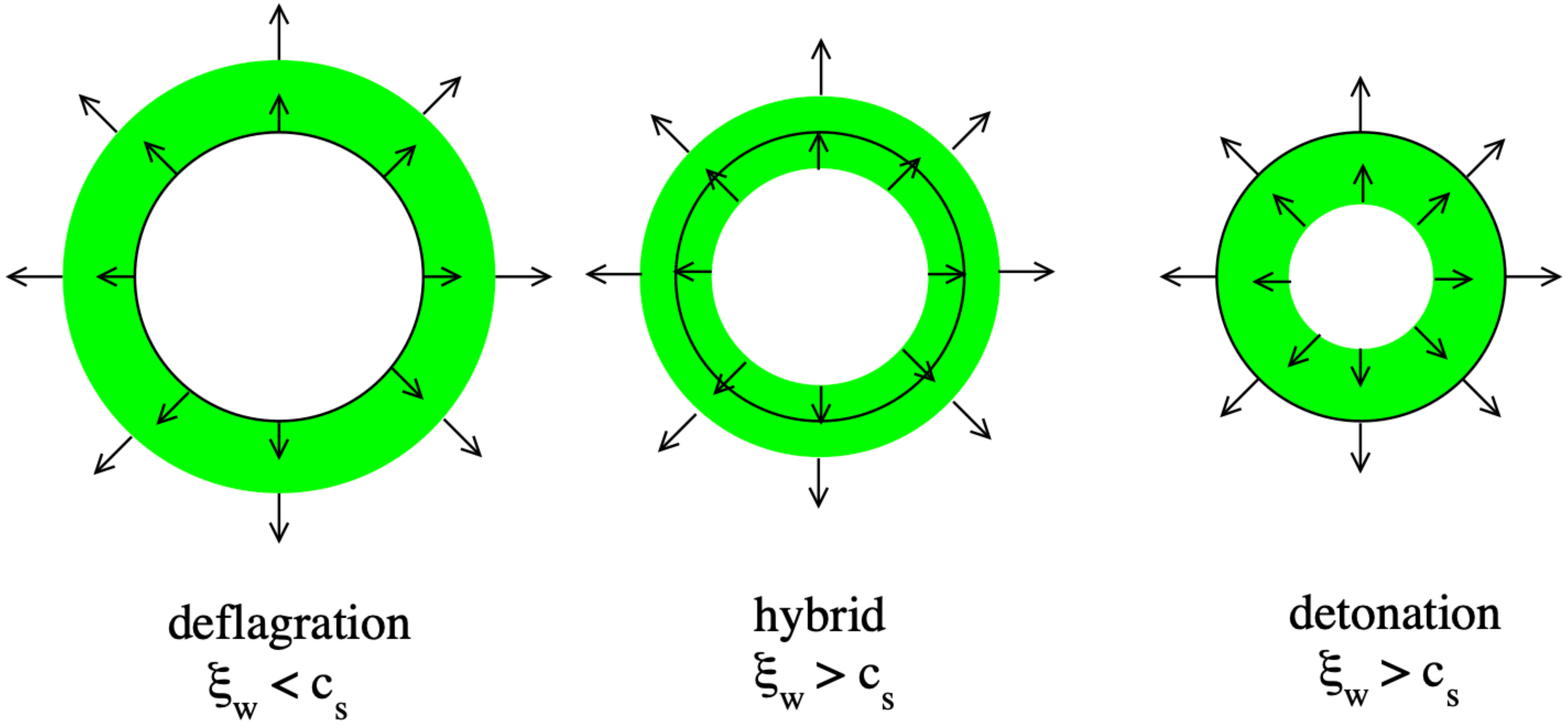
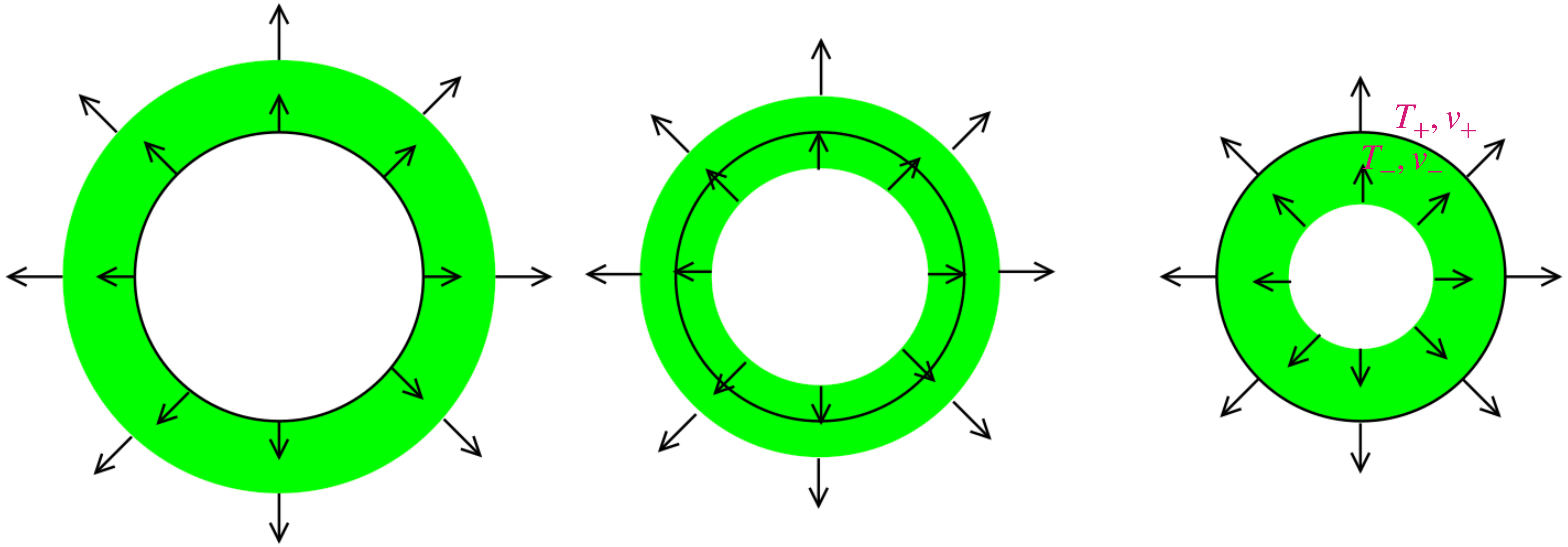


Figure from: Espinosa, Konstandin, No, Servant 2010

Lightning hydrodynamics recap



deflagration $\xi_w < c_s$

hybrid $\xi_w > c_s$

detonation $\xi_w > c_s$

Figure from: Espinosa, Konstandin, No, Servant 2010

Local thermal equilibrium

- Even in local thermal equilibrium, hydrodynamic effects provide friction (backreaction) on the wall
  Ignatius, Kajantie, Kurki-Suonio, Laine 1994; Konstandin, No 2011; Barroso Mancha, Prokopec, Swiezewska 2020; Balaji, Spannowski, Tamarit 2020
- Equilibrium-only friction is a reasonable approximation for deflagrations and hybrids in SM+singlet
  Laurent, Cline 2022

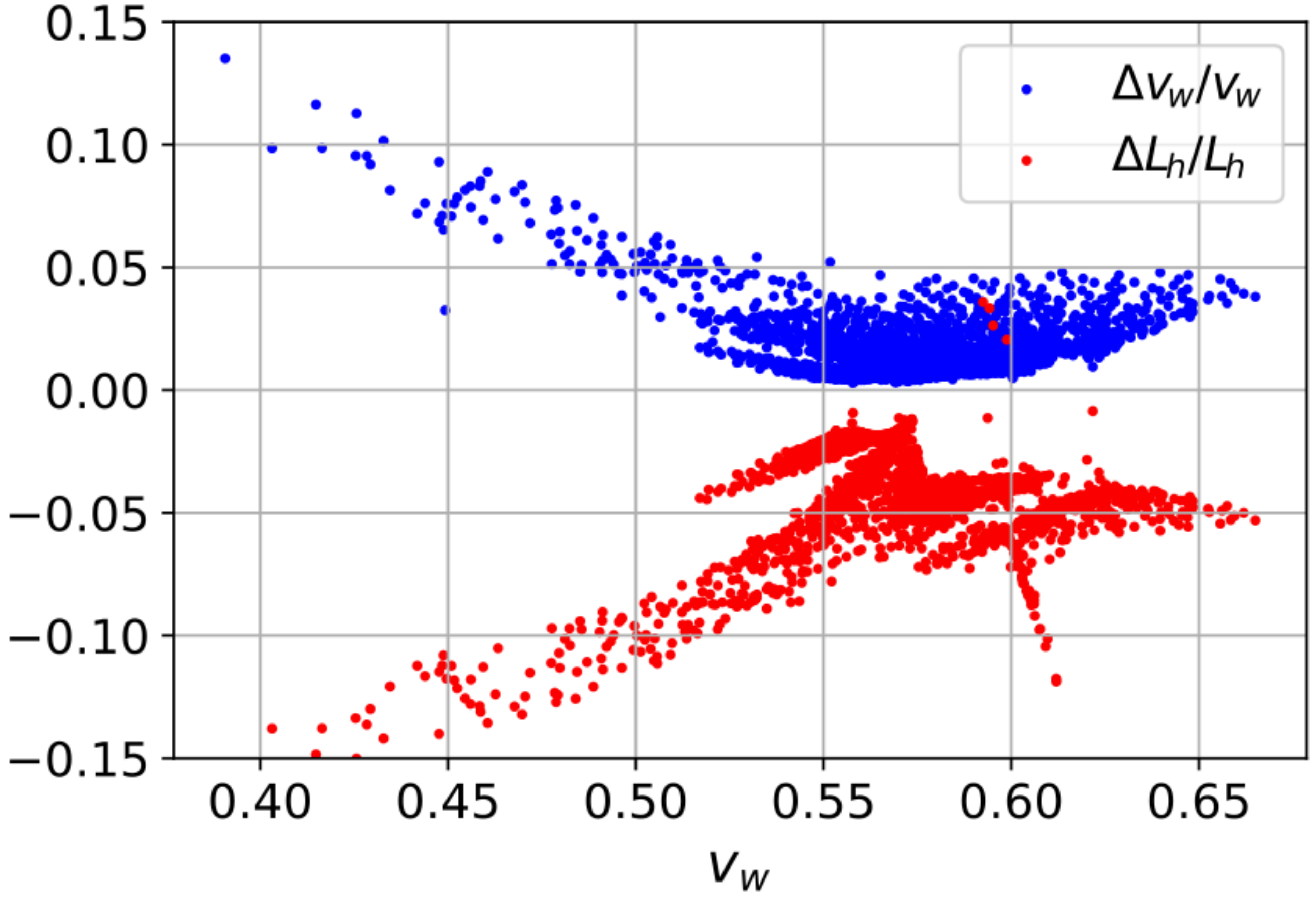


Figure 5. Scatter plot of relative errors of the wall velocity (blue points) and thickness (red points) due to neglecting the out-of-equilibrium pressure contribution, as a function of v_w.

Figure from: Laurent, Cline 2022

Model-independent computation of the wall velocity in LTE

- LTE can be understood as additional matching condition: $s_+\gamma_+v_+ = s_-\gamma_-v_-$, the wall velocity can be determined without solving the scalar field equation of motion Ai, Garbrecht, Tamarit 2021
- We use the template model to find v_w model-independently Ai, Laurent, JvdV 2023
- Determined by $\alpha, c_b, c_s, \Psi_n \equiv w_b(T_n)/w_s(T_n)$

Template model

A generalization of the bag equation of state

- $p_s = \frac{1}{3} a_+ T^\mu - \epsilon$ $\quad$ $p_b = \frac{1}{3} a_- T^\nu$

  $\mu = 1 + \frac{1}{c_{s,\text{sym}}^2}$ $\quad$ $\nu = 1 + \frac{1}{c_{s,\text{brok}}^2}$

Leitao, Megevand, 2015

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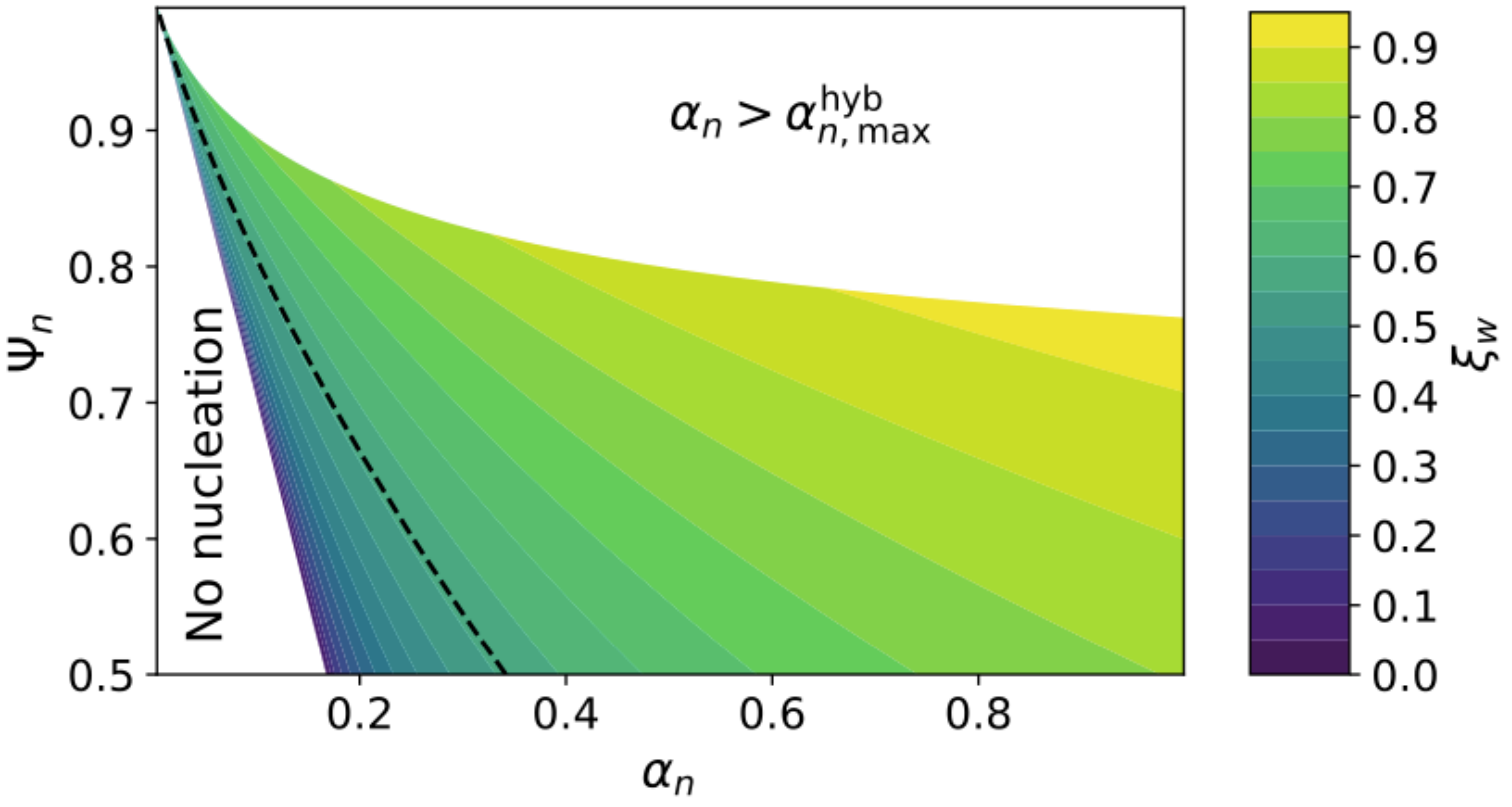
Model-independent computation of the wall velocity in LTE

Ai, Laurent, JvdV 2023



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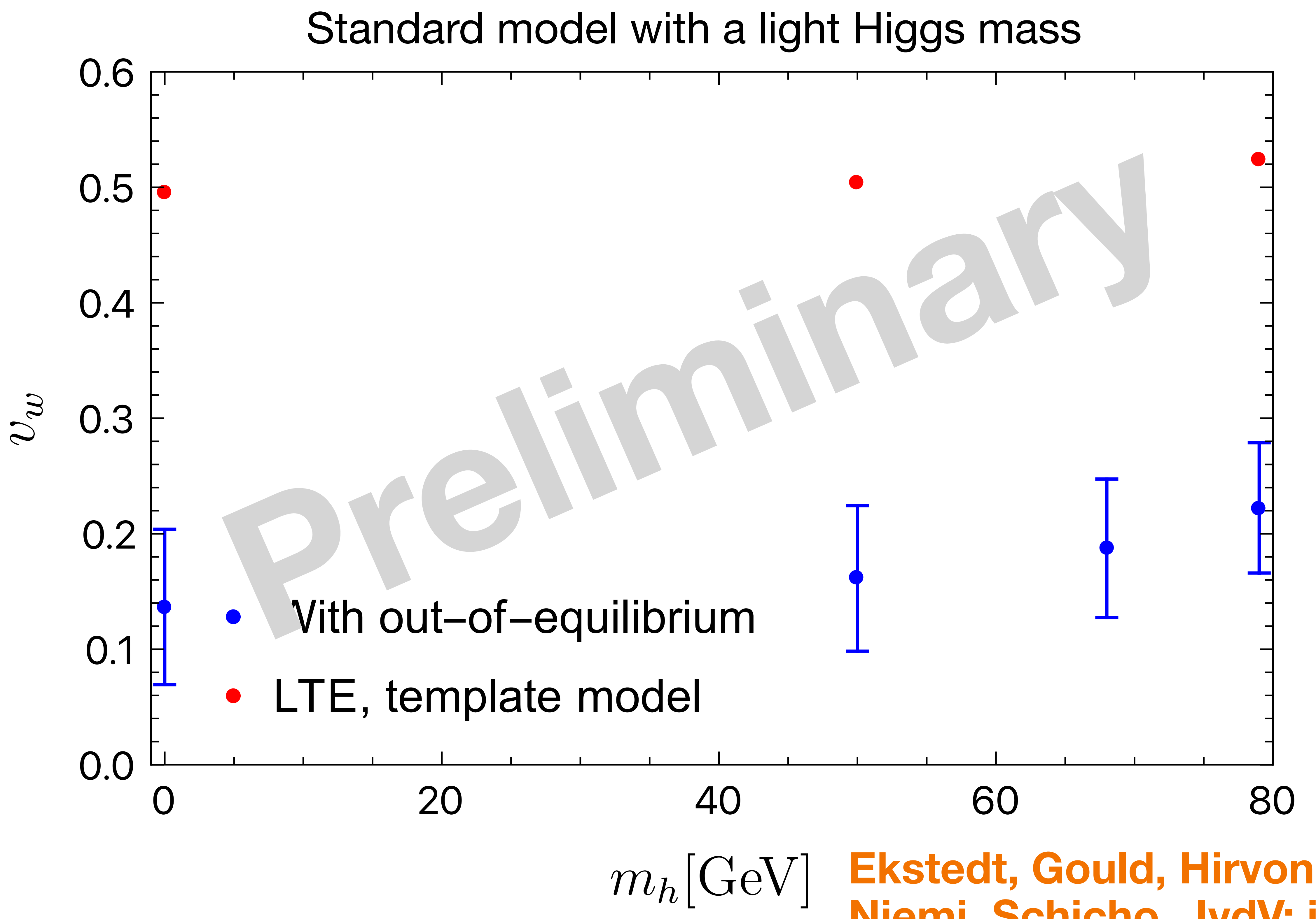
- Determined by $\alpha, c_b, c_s, \Psi_n \equiv w_b(T_n)/w_s(T_n)$
- Provides an *upper bound* on the wall velocity

Discussion of local thermal equilibrium approximation

- How well does the LTE approximation work in other models?



Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV: in progress

Discussion of local thermal equilibrium approximation

- How well does the LTE approximation work in other models?
- Is the LTE solution reached dynamically? See Krajwski, Lewicki, Zych 2024

v_w in phase transitions in strongly coupled sectors

New strongly coupled sectors (e.g. SU(N))

- Can provide stable dark matter candidate, solution to hierarchy problem
- Cosmological strongly coupled phase transitions and GWs?

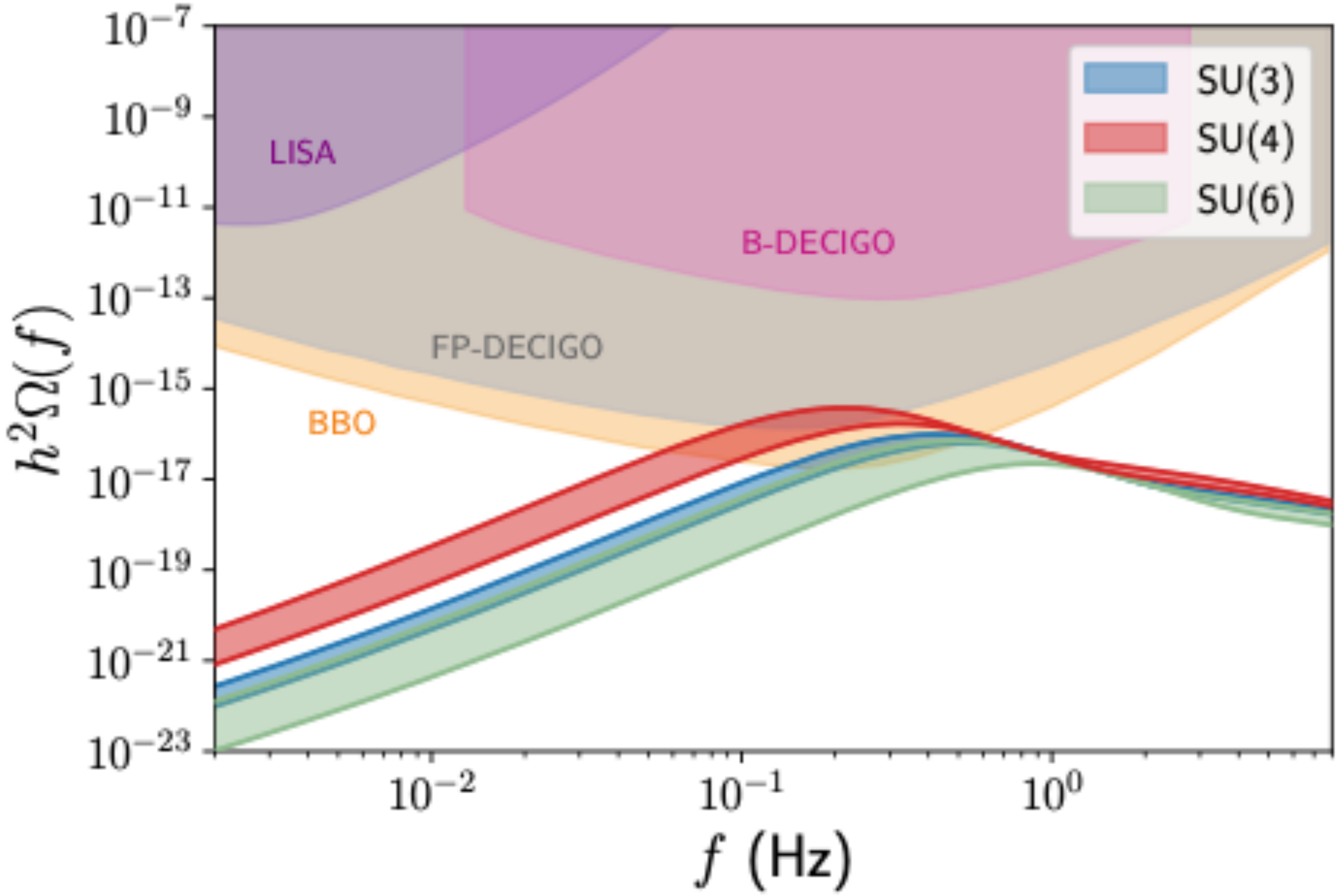
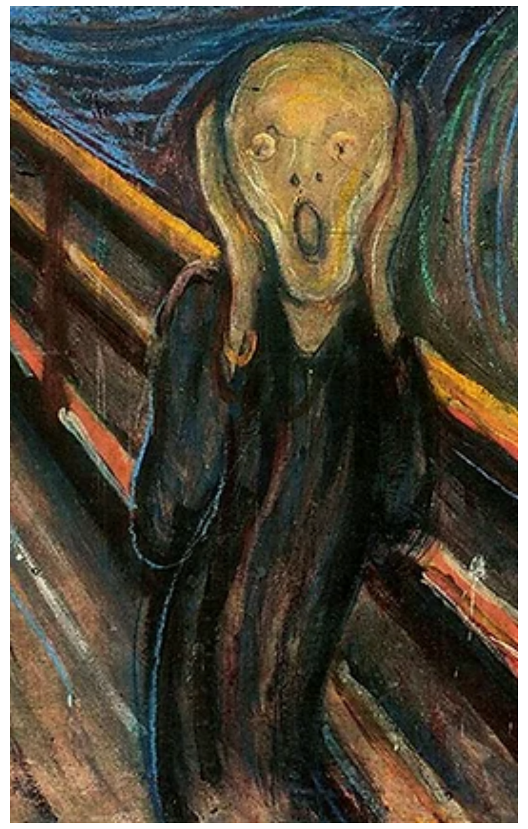


Fig. Halverson, Long, Haiti, Nelson, Salinas 2020

New strongly coupled sectors (e.g. SU(N))

- Can provide stable dark matter candidate, solution to hierarchy problem
- Cosmological strongly coupled phase transitions and GWs?
- Non-perturbative computation of v_w?



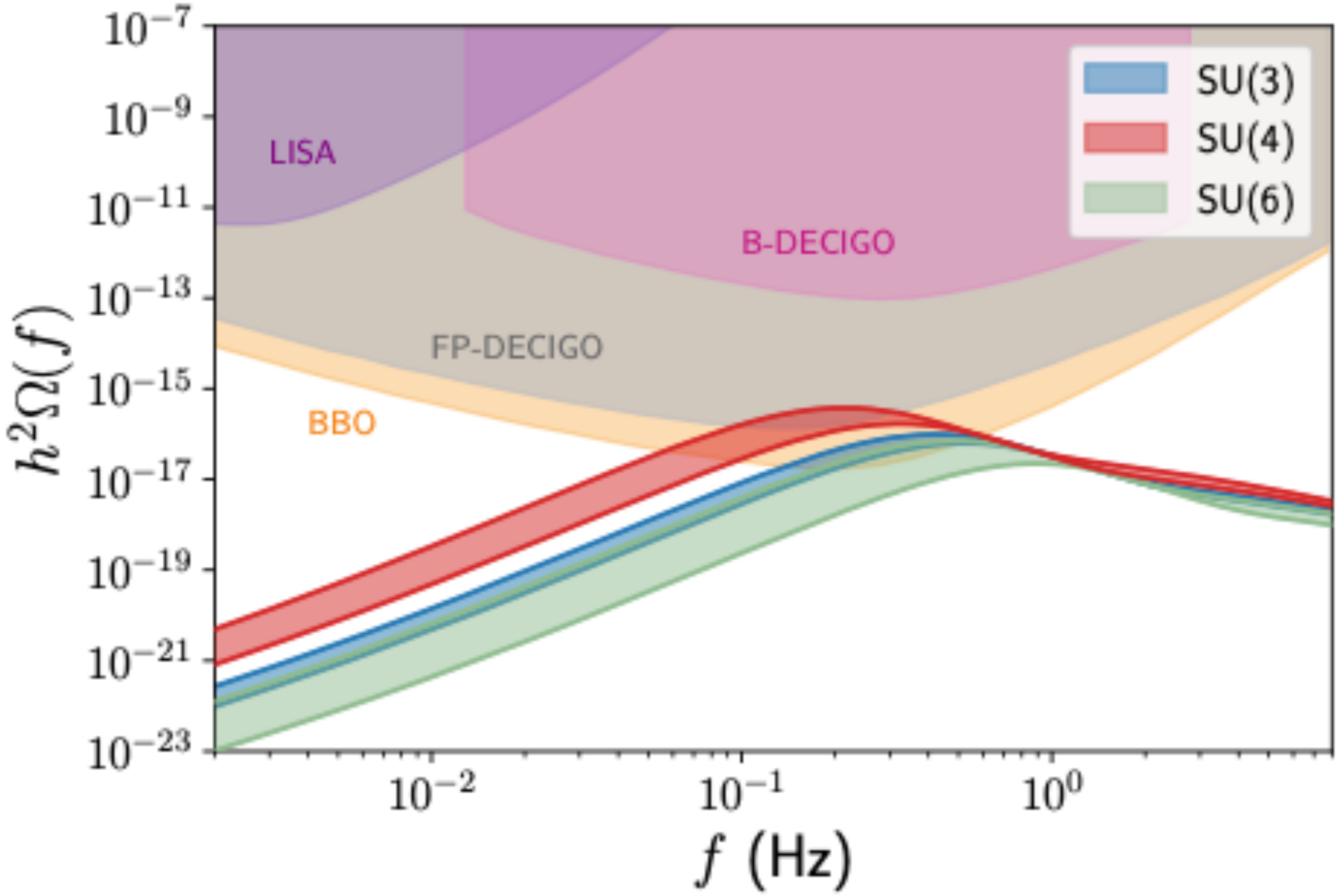


Fig. Halverson, Long, Haiti, Nelson, Salinas 2020

Large jump in degrees of freedom

- Strongly coupled PTs typically feature a large enthalpy jump (large jump in dof $\propto N^2$)

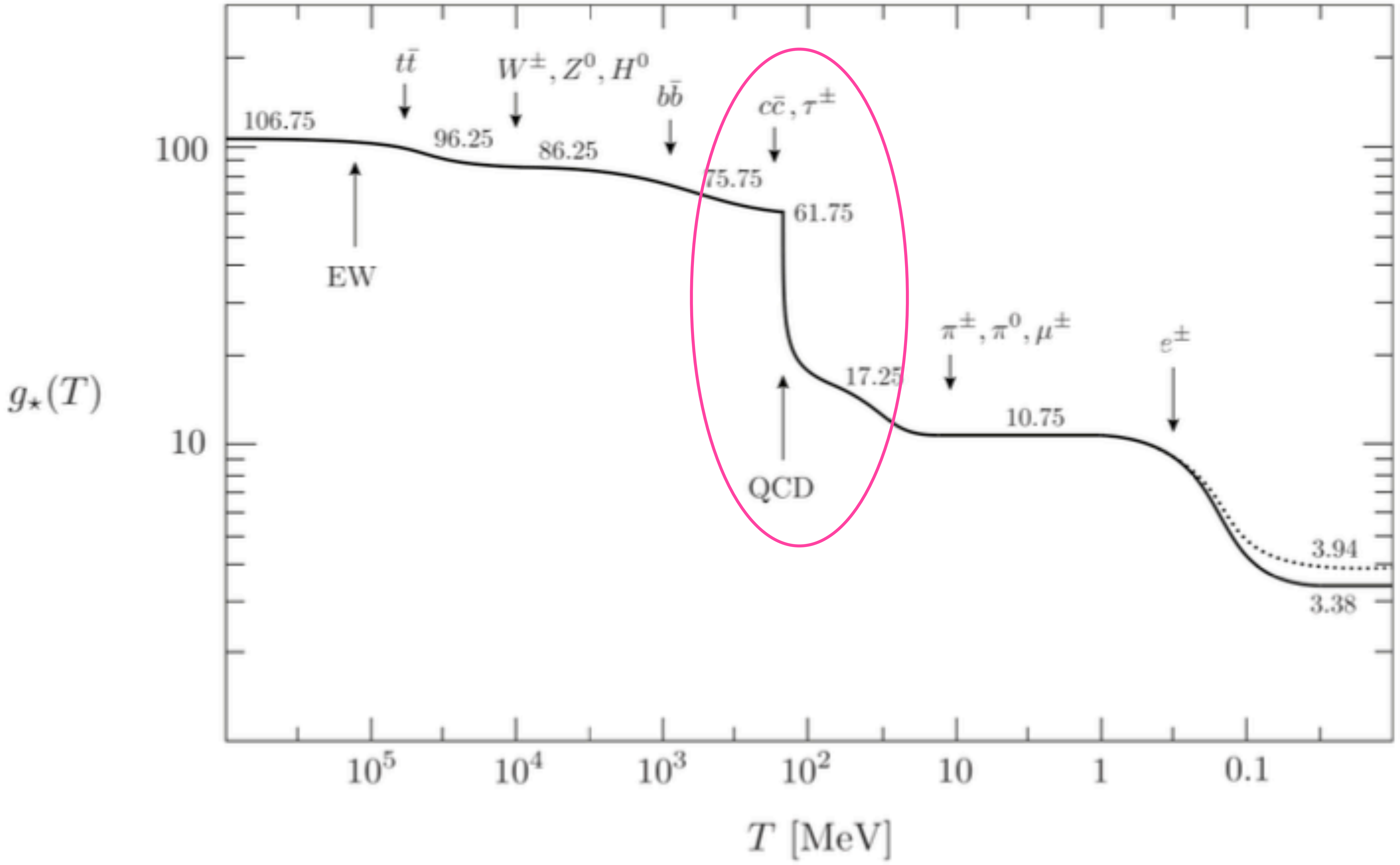


Fig. Daniel Baumann 2013

Using the large enthalpy jump to predict v_w

Sanchez-Garitaonandia, JvdV, 2023

- Strongly coupled PTs typically feature a large enthalpy jump (large jump in dof $\propto N^2$)
- We estimate the wall velocity from hydrodynamics in the large-N limit
- We make no further assumptions related to strong coupling (so result would also apply to weakly coupled theories)

Equation of state with a large enthalpy jump

- Low-enthalpy (confined) phase suppressed by $\frac{1}{N^2}$ compared to high-enthalpy (de-confined) phase*

$$p_L(T) \sim \frac{\bar{p}}{N^2}, \quad w_L(T) \sim \frac{\bar{w}}{N^2}, \quad e_L(T) \sim \frac{\bar{e}}{N^2}$$

* $\bar{p}, \bar{w}, \bar{e}$ are $\mathcal{O}(1)$ numbers in the appropriate units

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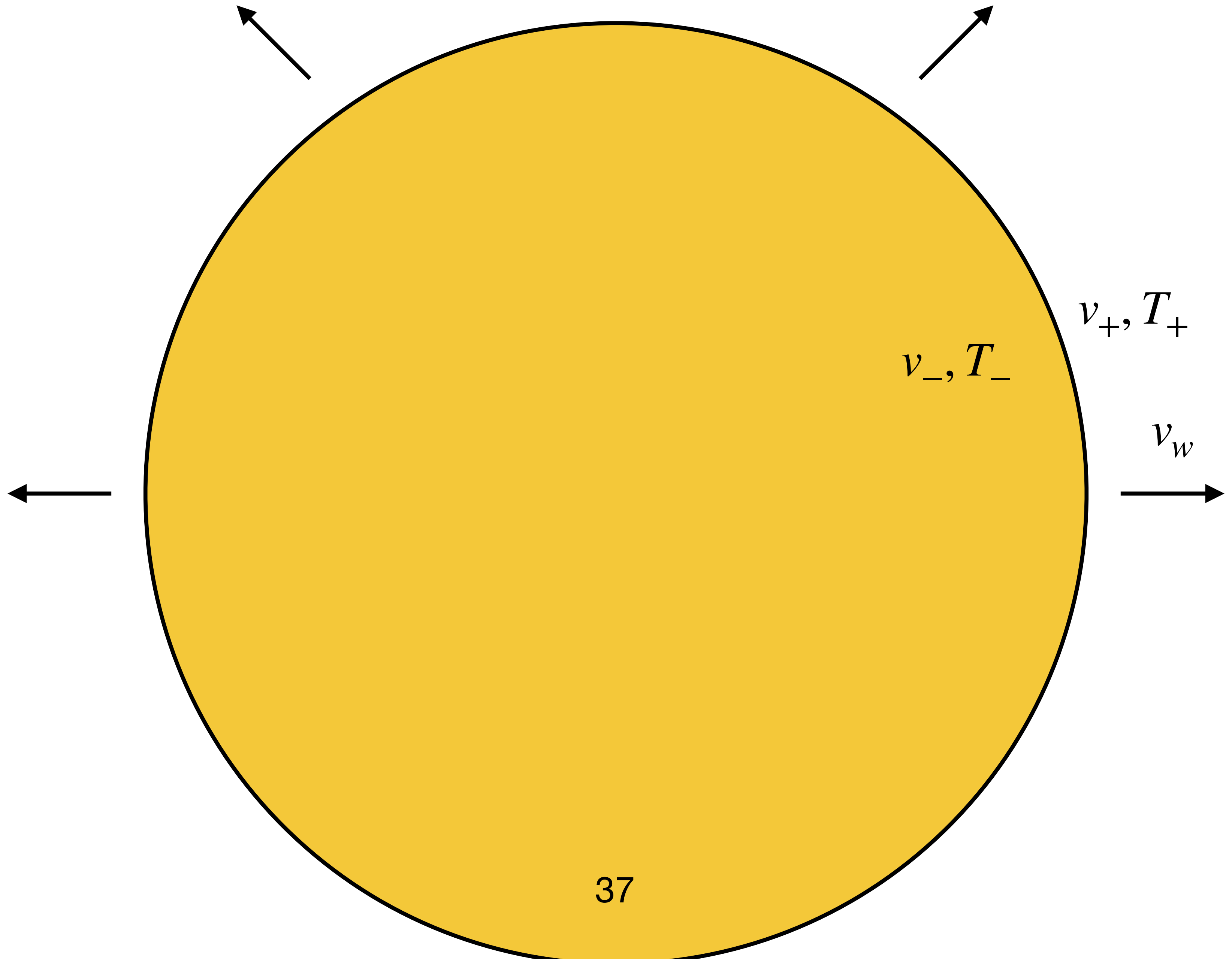
E.g. (bag EoS) $p_H = \dfrac{a_H}{3}T^4 - \epsilon, \quad p_L = \dfrac{a_H}{3N^2}T^4$

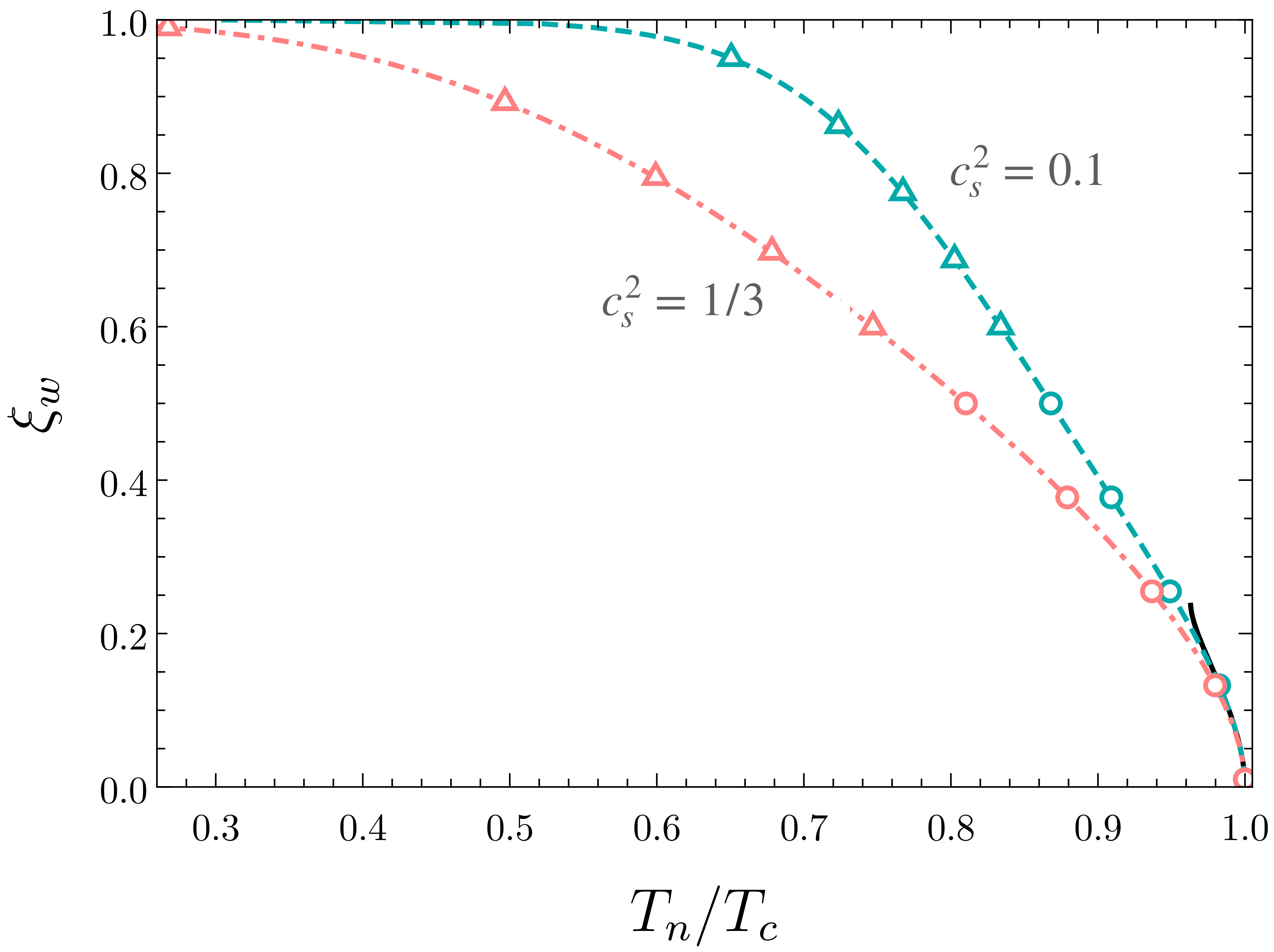
Large-N limit dictates v_+, T_+

- Matching relations: $v_+ v_- = \dfrac{p_+ - p_-}{e_+ - e_-}, \qquad \dfrac{v_+}{v_-} = \dfrac{e_- + p_+}{e_+ + p_-}$
- Only when $T_+ = T_c$, and $v_+ = 0$, matching equations can be fulfilled



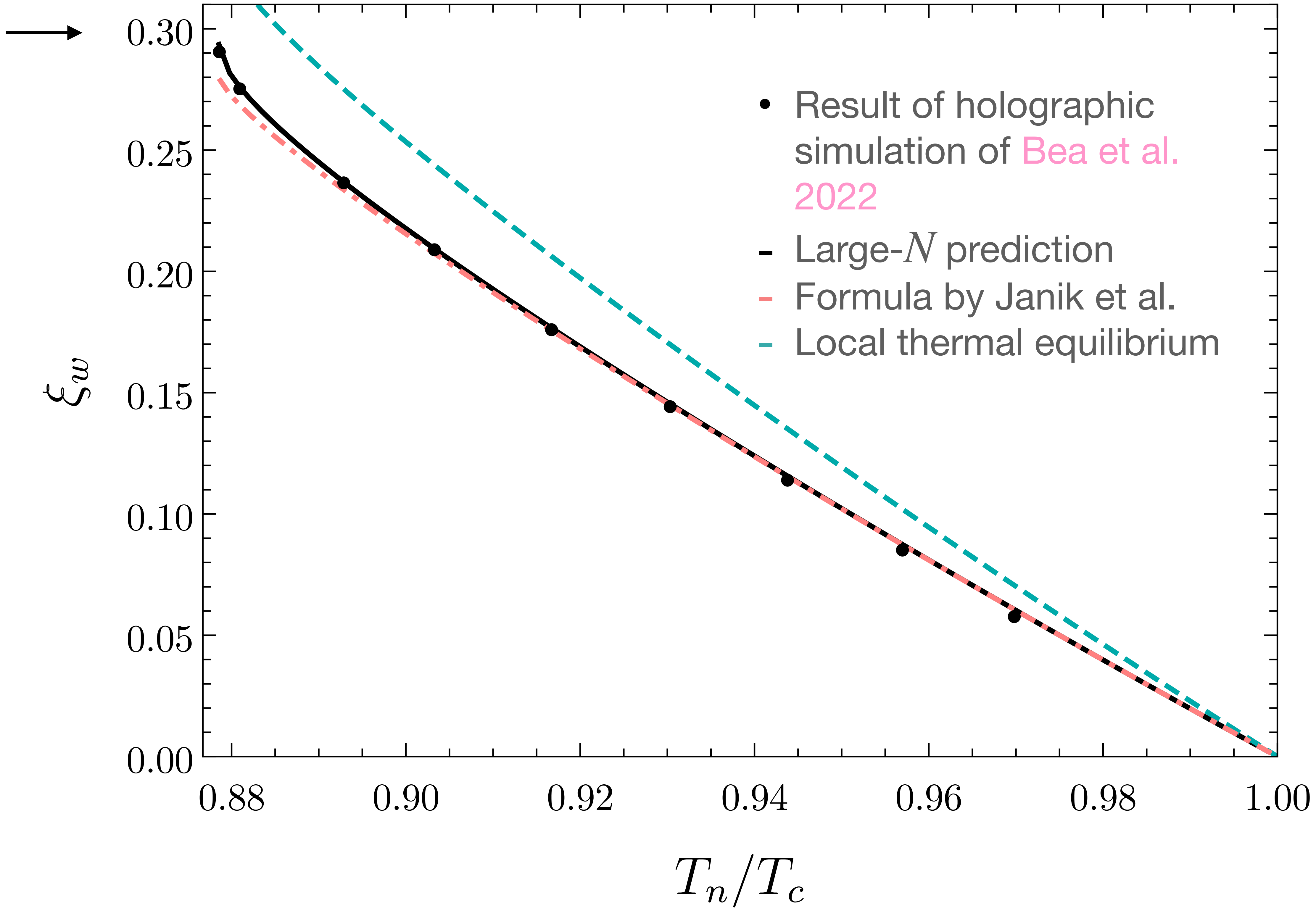
Solve the fluid profile

- Knowing T_+, v_+, v_w and an EOS we can solve the fluid profile: this determines T_n
- **Unique relation between v_w and T_n**



Comparison with simulation result in holographic model

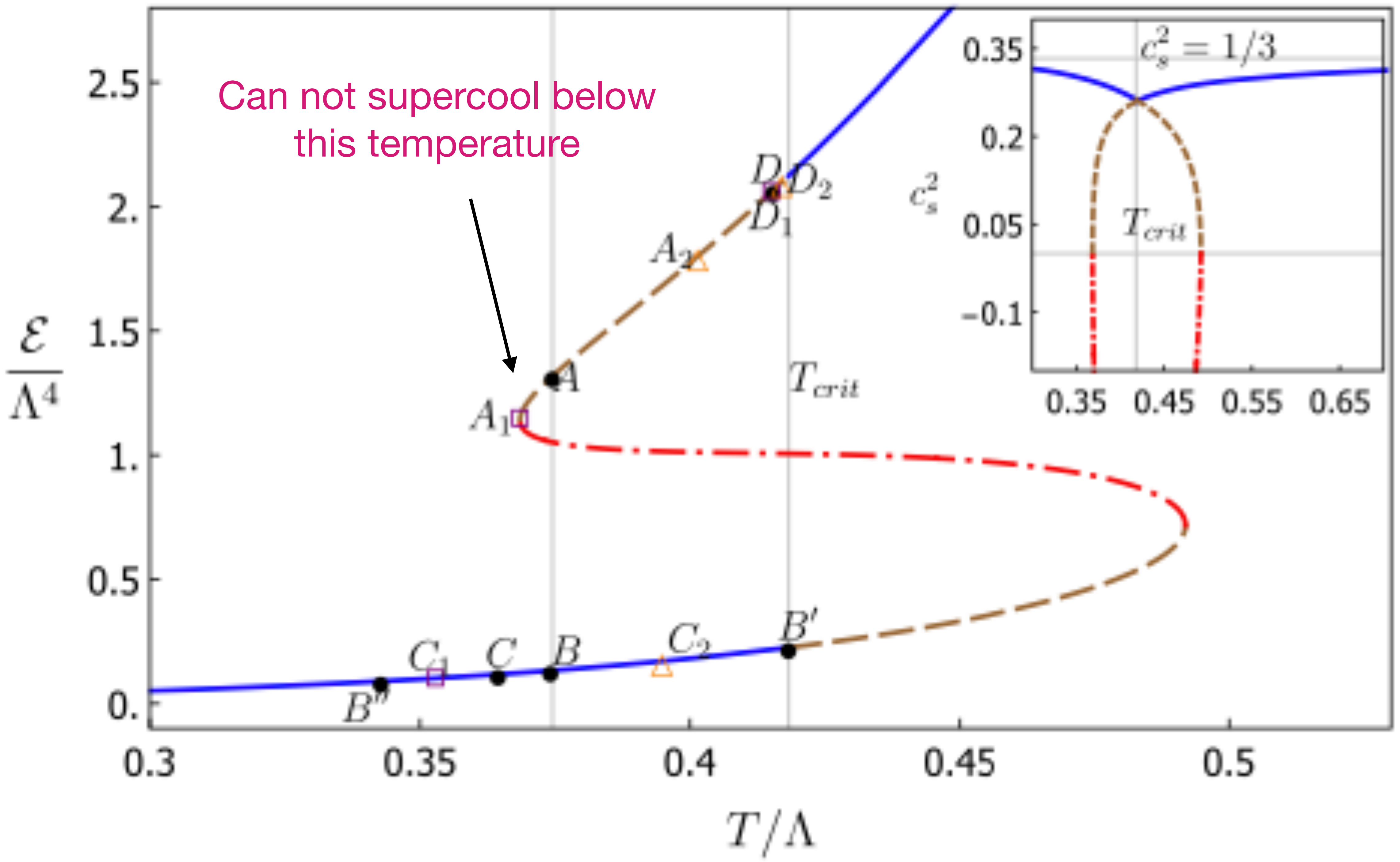
- Large-N reproduces simulations really well (even though $N \sim 3$)



Not really fast ⟶

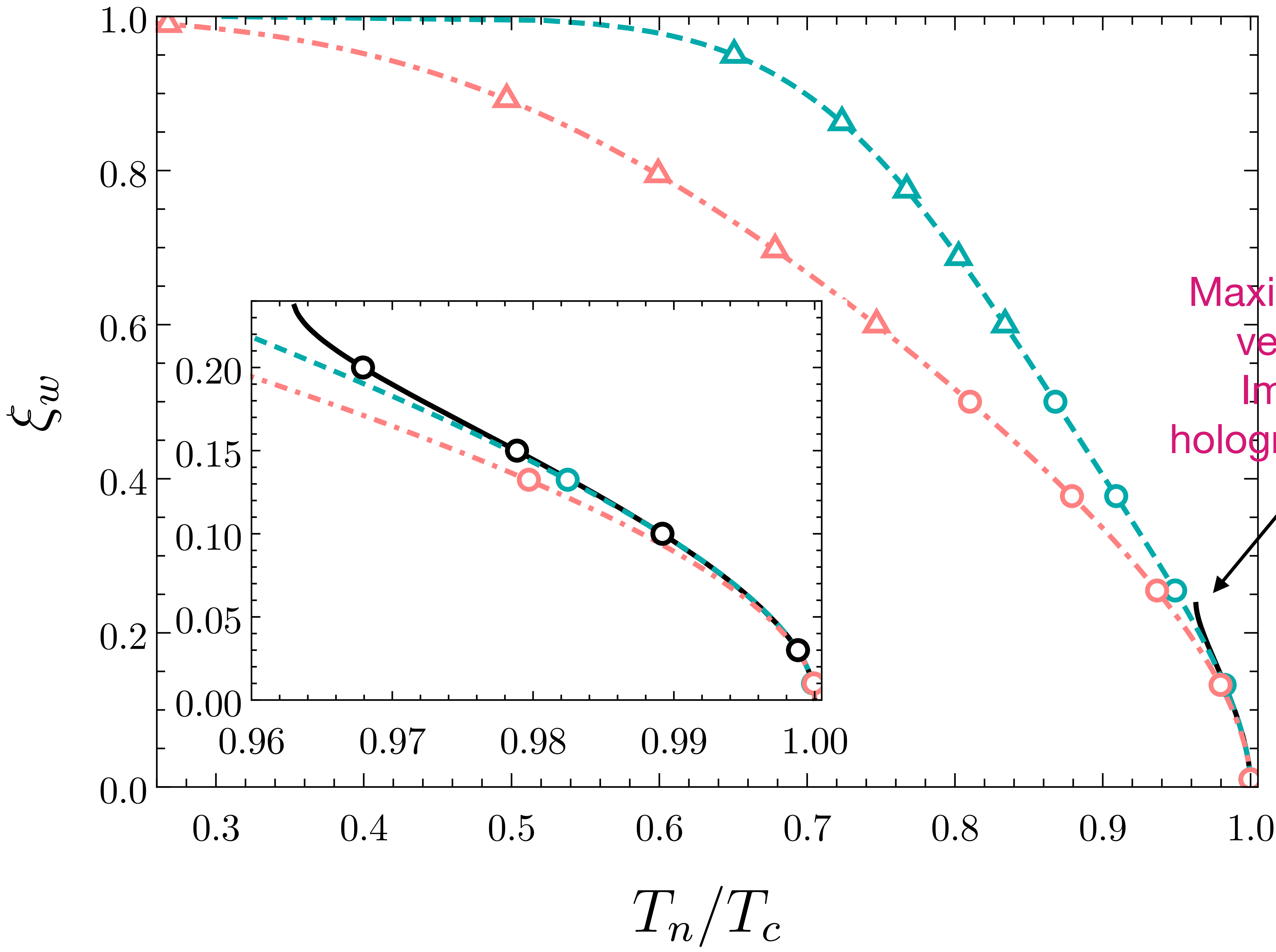
Why was the wall velocity so small in the simulation?

See talk by R. Mishra



Can not supercool below this temperature

Fig. Bea, Caselderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia, Zilhão 2022



Maximum wall-velocity in Improved holographic QCD

Computing v_w with out-of-equilibrium contributions*

*For weakly coupled theories

Ekstedt, Gould, Hirvonen, Laurent, Niemi, Schicho, JvdV: 2410.xxxx

Publicly availabe code for the computation of the wall velocity

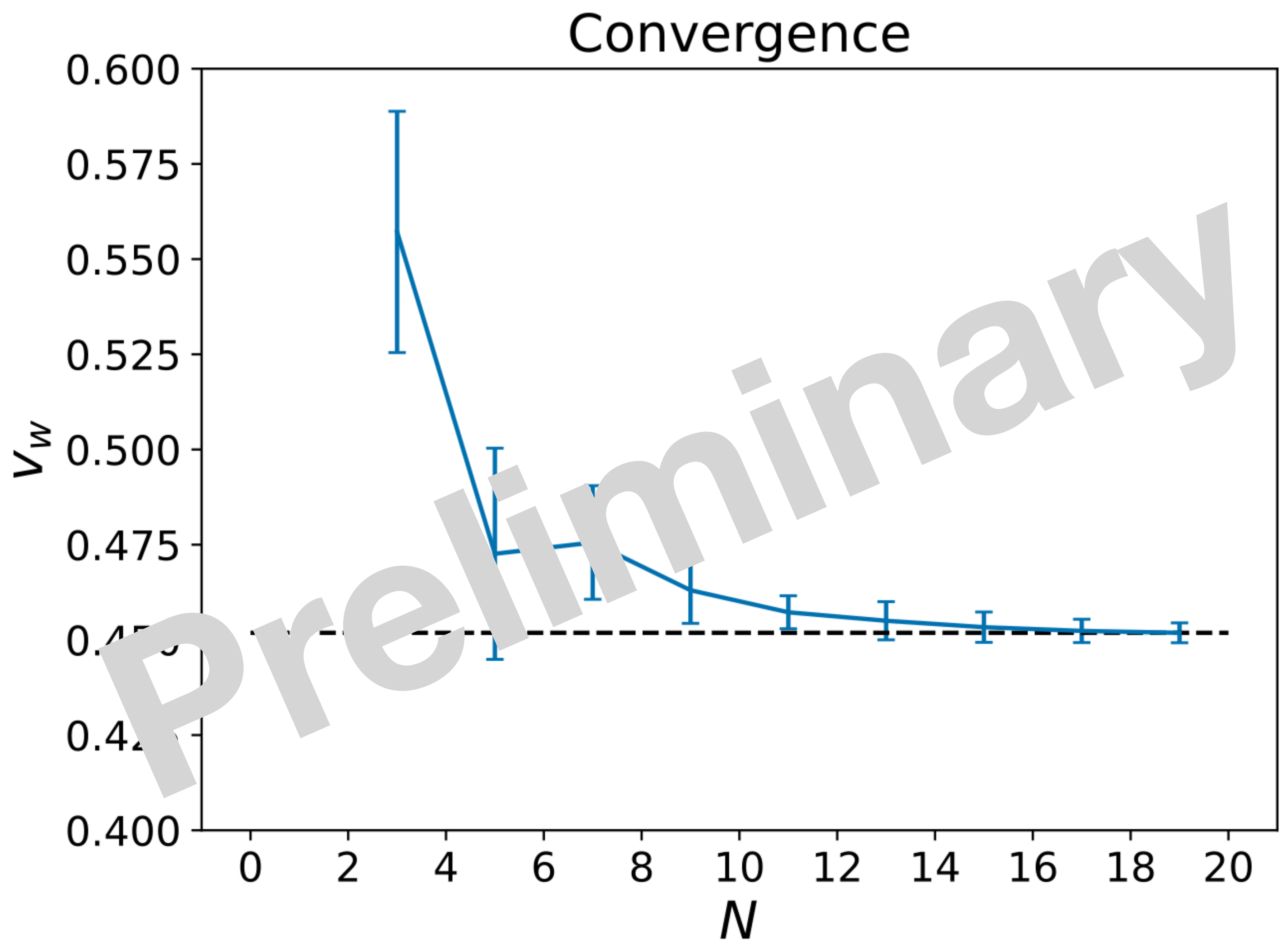
with out-of-equilibrium contributions

What does it do?

- Computes matrix elements for out-of-equilibrium particles, based on `DRalgo` `(Mathematica)` Ekstedt, Schicho, Tenkanen 2022
- Computes the corresponding matrix elements in `C++`
- Solves the equation of motion for the scalar field(s), fluid equations and Boltzmann equations for out-of-equilibrium particles in `Python`
- The model and the set of out-of-equilibrium particles are user-defined

Some details on the implementation

- Spectral method of Laurent, Cline 2022 : expansion of $\delta f(z, p_z, p_\parallel)$ in Chebyshev polynomials: $\delta f_a(z, p_z, p_\parallel) = \sum_{ijk} \delta f_a^{ijk} T_i(z) T_j(p_z) T_k(p_\parallel)$



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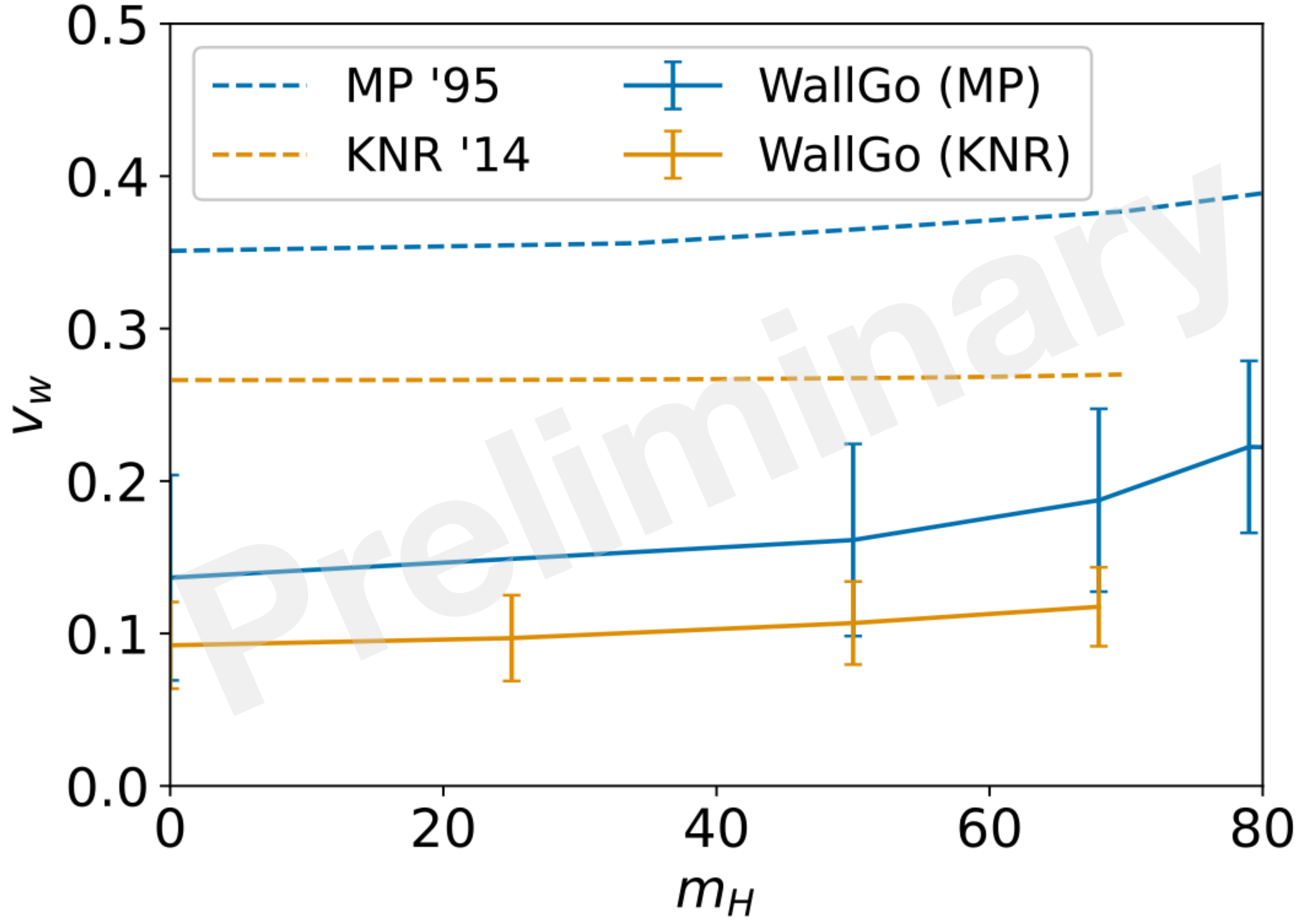
- Tanh-Ansatz for the scalar field(s): solve for width(s and offsets)
- All tree-level $2 \rightarrow 2$ scattering processes in the matrix elements

Comparison with earlier computation for SM with light Higgs

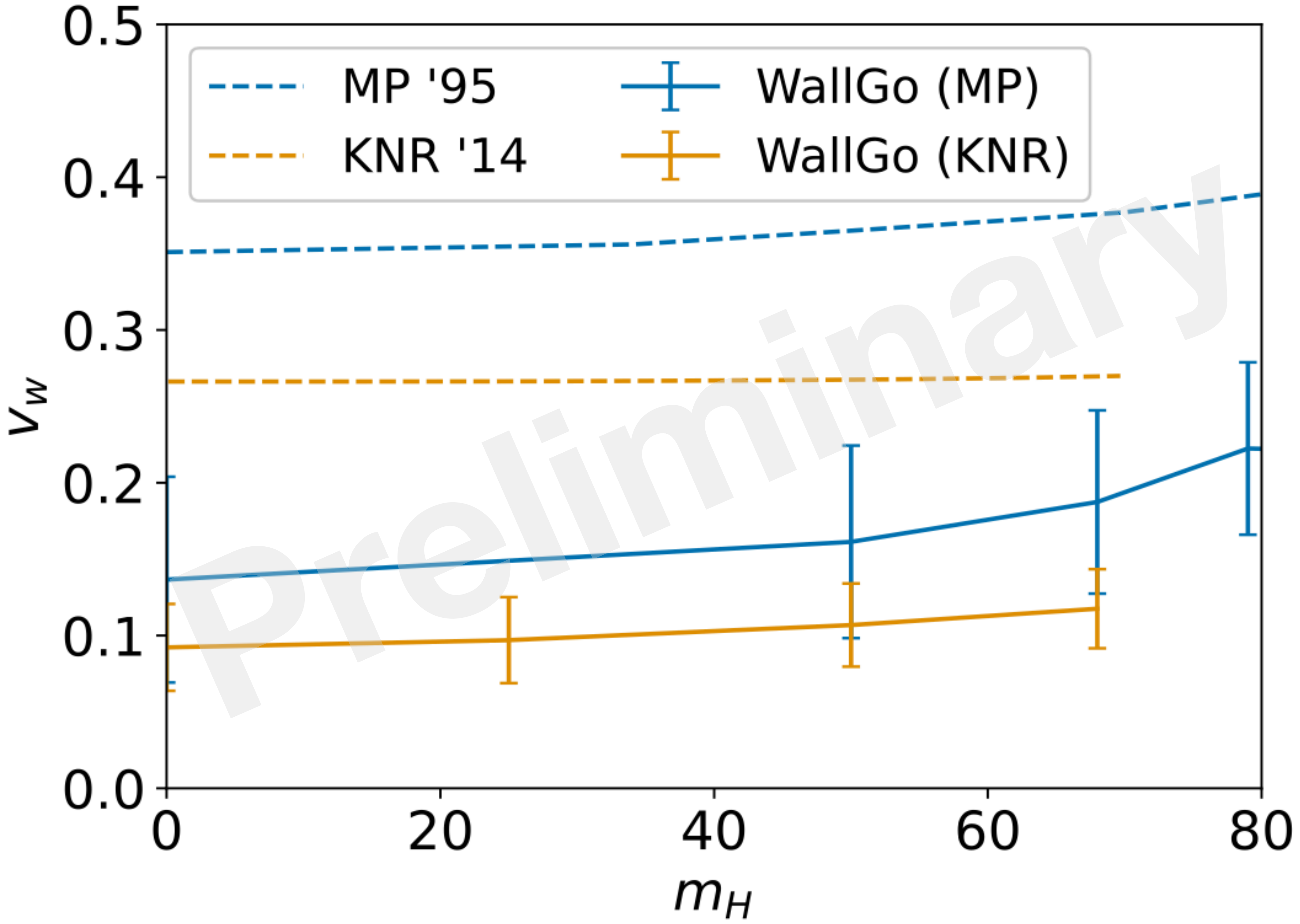
Moore, Prokopec 1995; Konstandin, Nardini, Rues 2014



Comparison with earlier computation for SM with light Higgs

Moore, Prokopec 1995; Konstandin, Nardini, Rues 2014

- Spectral method ($N = 11$) versus three moments
- Some differences in matrix elements
- Mixing in the Boltzmann equations (e.g. eq. for δf_{top} depends on δf_W)
- Different treatment of hydrodynamics to MP



What can we learn from WallGo ?

- A better estimate of v_w (and thus η_B, η_{DM}, Ω_{GW}, …) for many models

What can we learn from WallGo ?

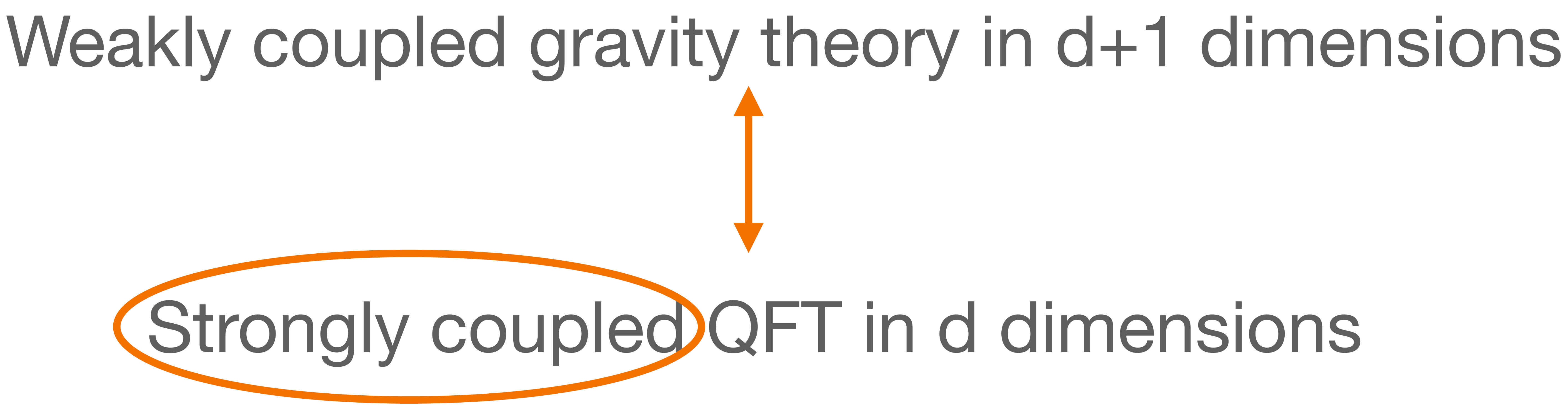
- A better estimate of v_w (and thus η_B, η_{DM}, Ω_{GW}, …) for many models
- What are the largest sources of uncertainty in the computation of v_w ?
  - The effective potential See talk by P. Schicho
  - (Leading log) collisions
  - Tanh Ansatz (for future versions)
  - …
- When does the linearization in δf break down?
- …

Summary

- The wall velocity is an important parameter in particle and GW production in first order phase transitions, but difficult to compute
- Hydrodynamics-based approximations:
  - Local thermal equilibrium. Code snippet available for model-independent computation
  - Large jump in the number of degrees of freedom. Applicable for a large jump in the degrees of freedom
- WallGo: publicly available code for the computation of v_w with out-of-equilibrium effects. To be released very soon!

Back-up

Alternative approach: holography



- Weakly coupled gravity theory in d+1 dimensions

  Strongly coupled QFT in d dimensions

- Originally: correspondence between type IIB string theory on $\mathrm{AdS}_5 \times S^5$ to $N = 4$ supersymmetric Yang-Mills theory Maldacena 1998, Gubser, Klebanov, Polyakov 1998, Witten 1998

- Different gravity descriptions can be used to correspond to different QFTs

Improved Holographic QCD

Gursoy, Kiritsis 2008

- 5D gravity theory $(g_{\mu\nu}, \Phi, a)$ (metric, dilaton, axion) with two solutions:

Thermal gass ↔ Confined phase

AdS Black hole ↔ Deconfined phase

- Dual: large-N Yang-Mills
- Reproduces e.g. linear confinement, asymptotic freedom in the UV, …

Wall velocity from holography

- Numerical simulations of a bubble in a holographic model Bea, Caselderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia, Zilhão 2022
- Gravity 5D Einstein-scalar model. $N \sim 3$. Modification of Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Oliván, Sopuerta, Triana, Zilhão 2016
- We will use these results as a test of our prediction

Not really fast

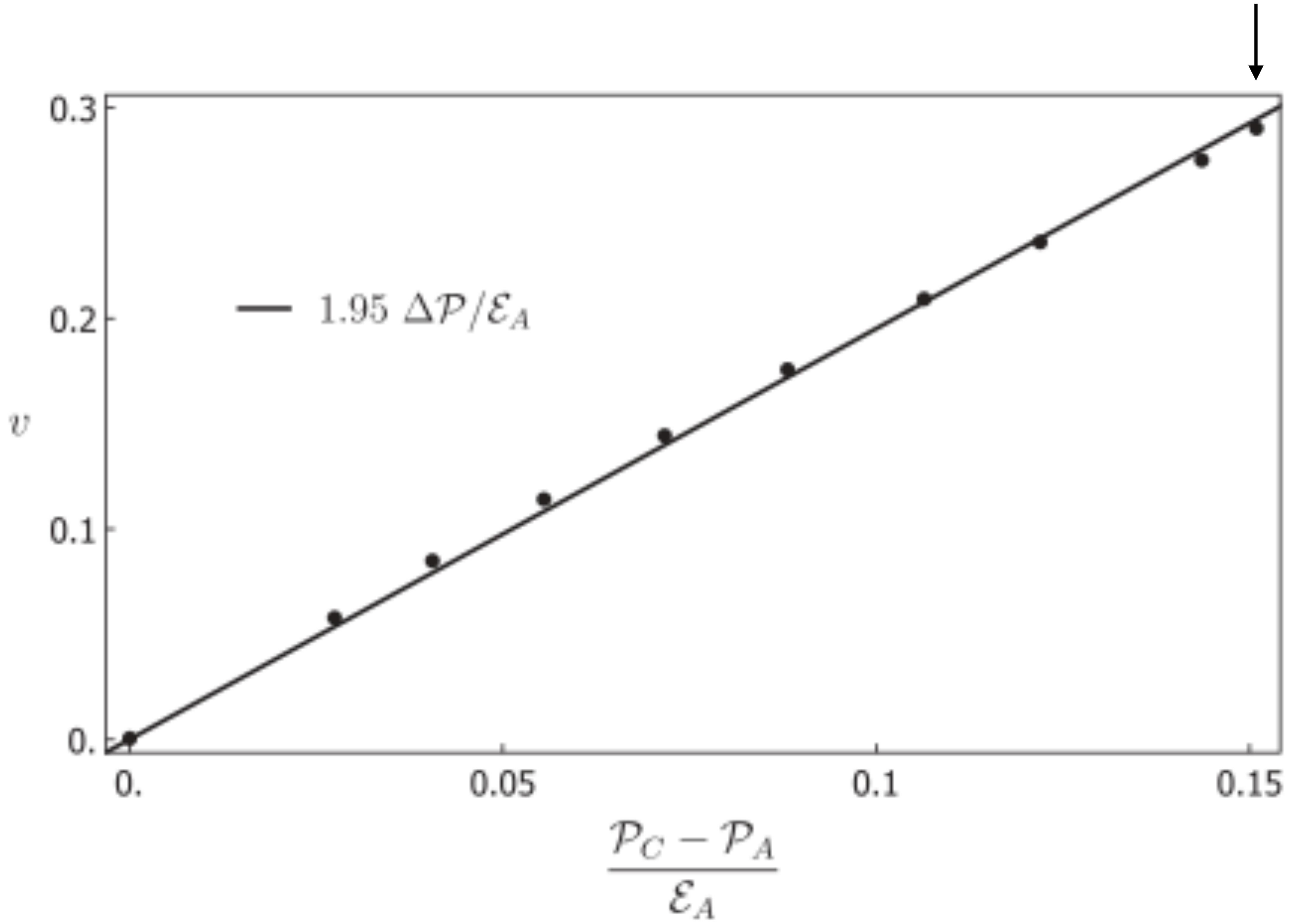


Fig. Bea, Caselderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia, Zilhão 2022

Hydrodynamic equations and matching conditions

- Hydrodynamic equations

$$2\frac{v}{\xi} = \underset{\substack{\uparrow \\ \text{Lorentz factor}}}{\gamma^2}(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v, \qquad \frac{\partial_v w}{w} = \left(\frac{1}{c_s^2} + 1\right)\gamma^2 \overset{\substack{\text{Velocity boost} \\ \downarrow}}{\mu}$$

- Boundary conditions

$$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \qquad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$