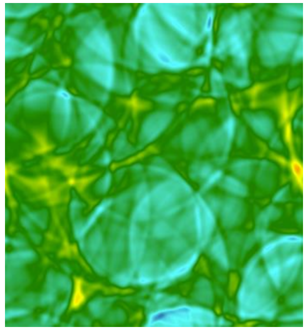

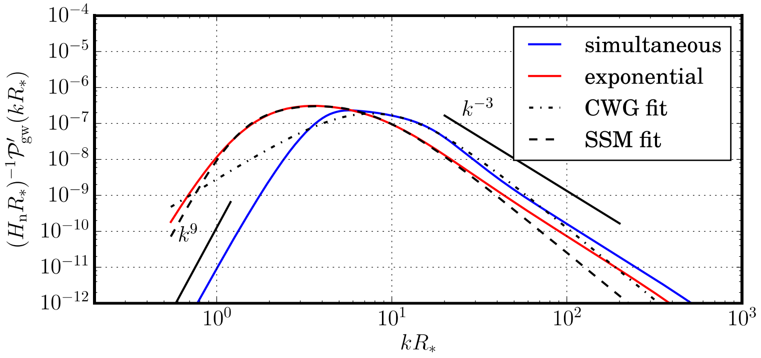

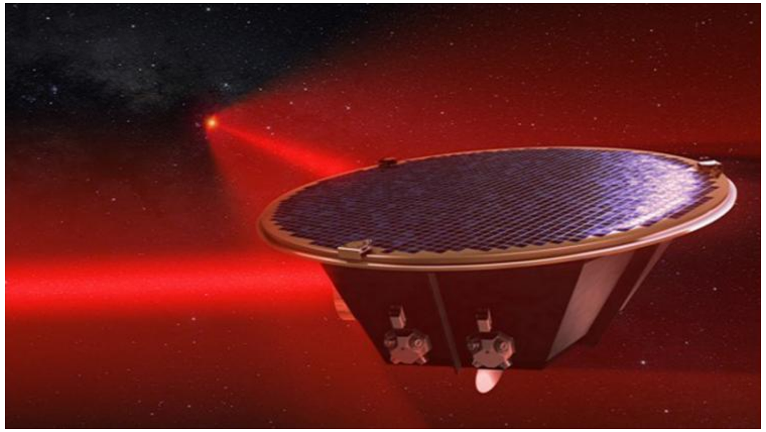

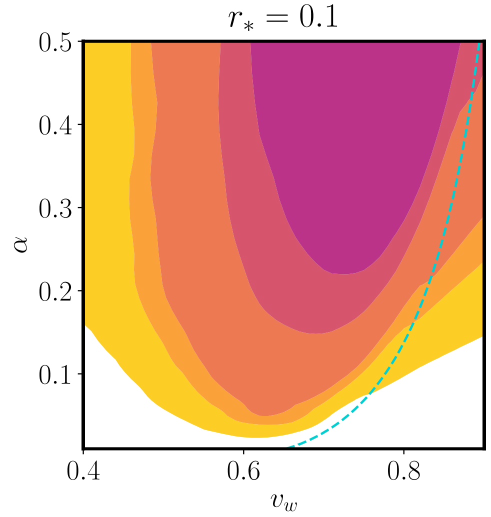

Gravitational waves from sound waves

Mark Hindmarsh

Helsinki Institute of Physics & Dept of Physics,
University of Helsinki

and

Department of Physics & Astronomy,
University of Sussex

Fundamental physics and GW detectors
19. syyskuuta 2024

SUOMEN AKATEMIA
FINLANDS AKADEMI
ACADEMY OF FINLAND

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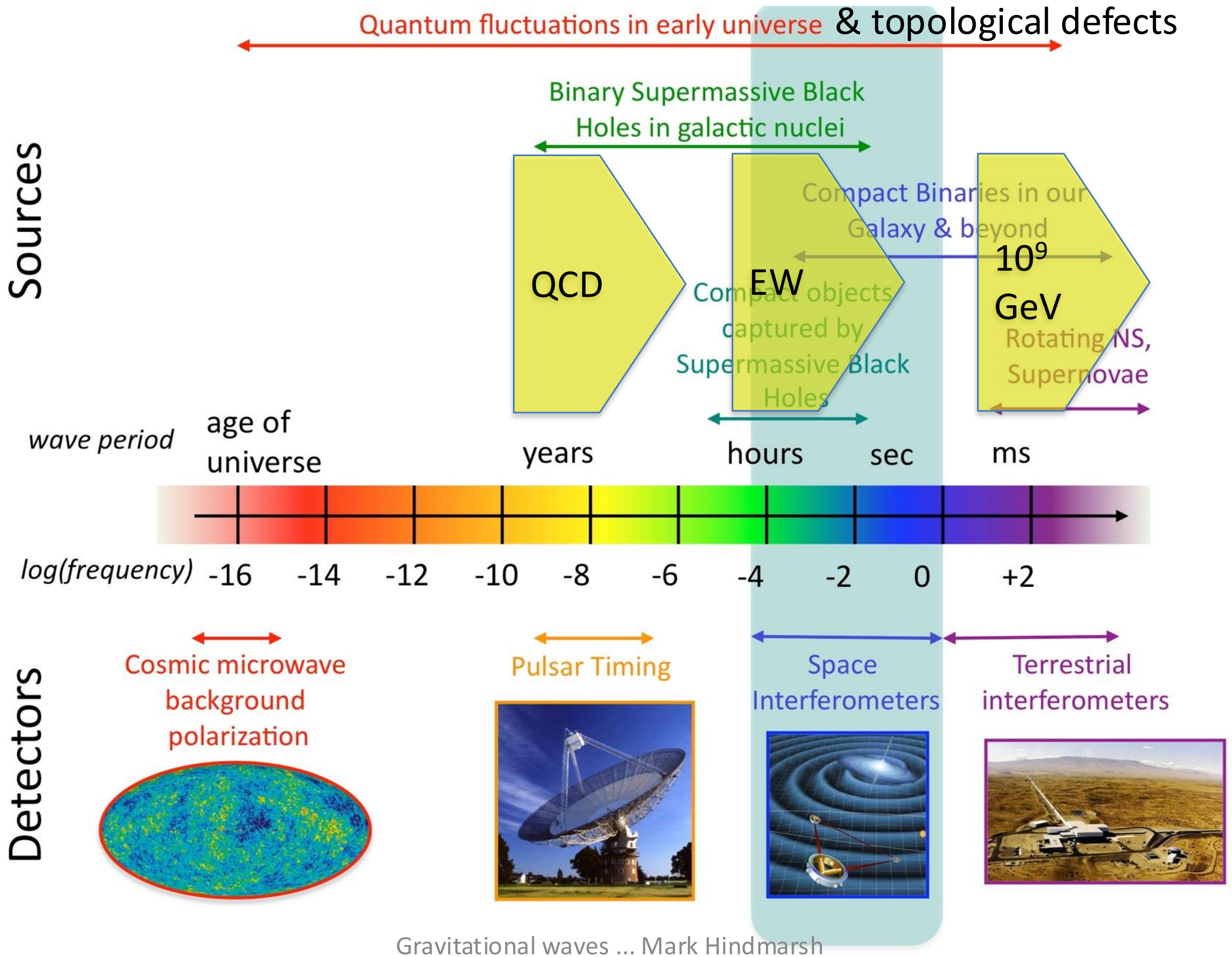
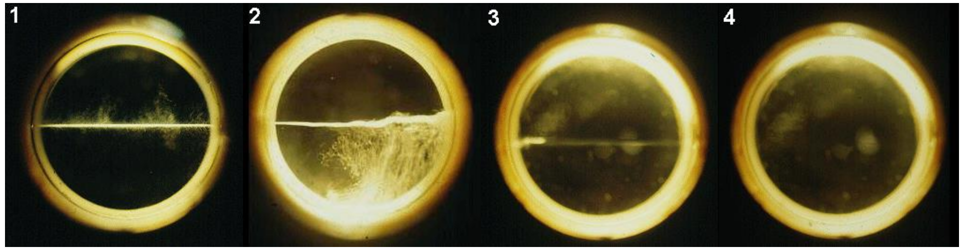
Gravitational wave spectrum

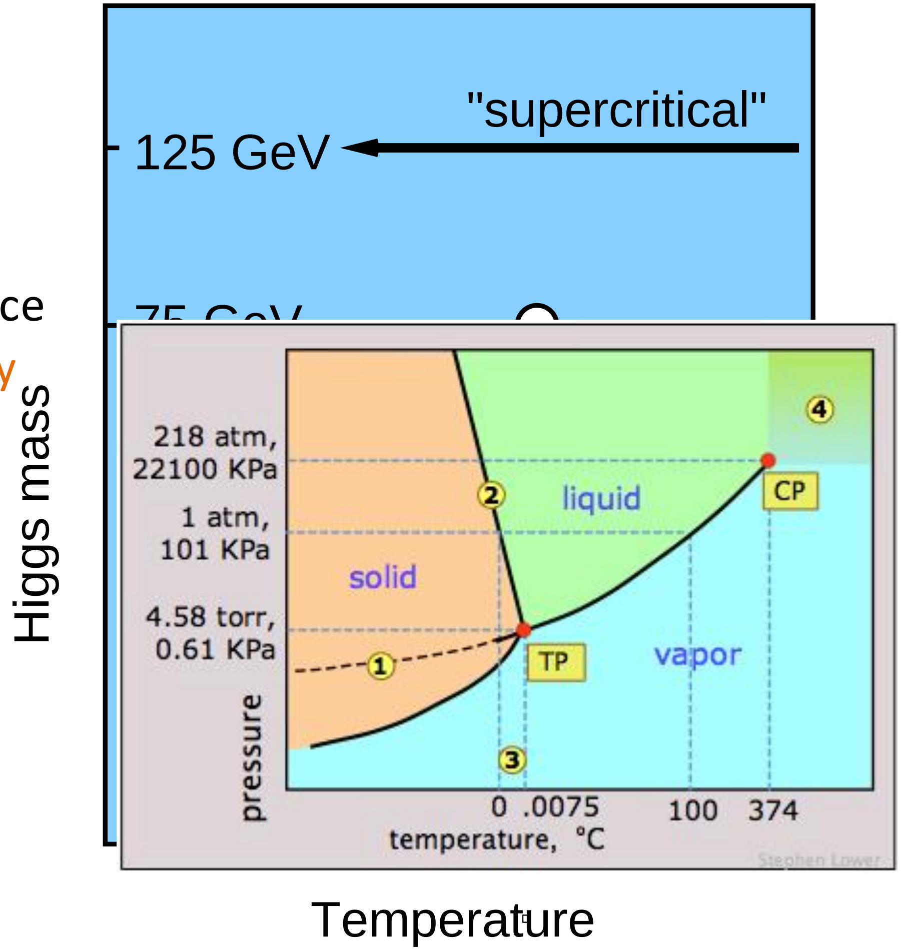


NASA

Electroweak transition: 100 GeV, 10 ps

- Perturbative: weakly first order transition
  Kirzhnitz, Linde (1972,4)
- But: SM is not weakly coupled at high T
  Linde (1980)
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
    P Schicho talk, Friday
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over
  - a supercritical fluid

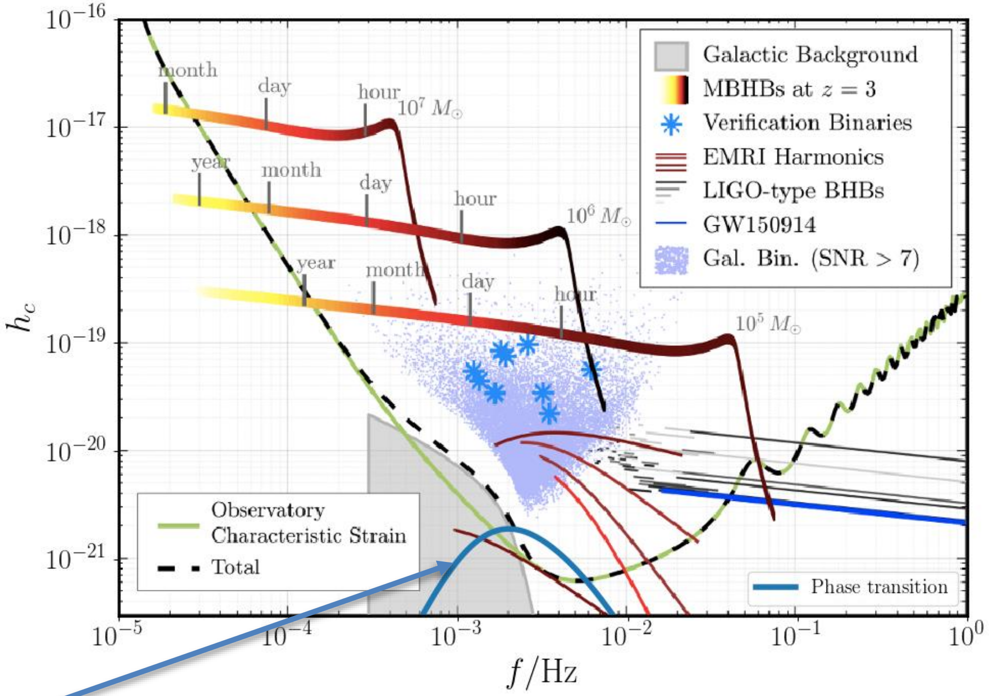




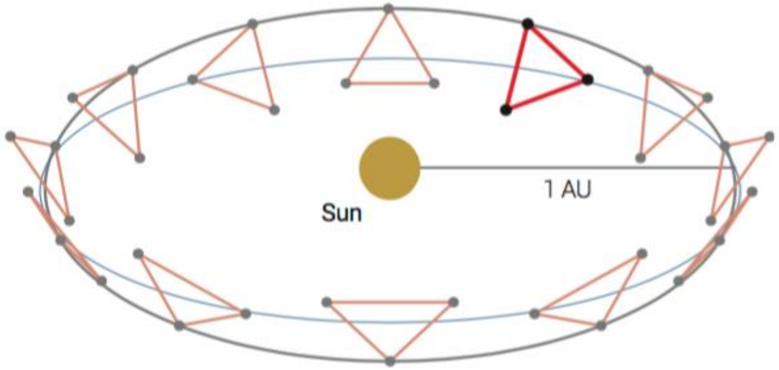
- Search for 1st order transition is a search for physics beyond SM

Laser Interferometer Space Antenna

- Launch mid 2030s
- 4-year mission (up to 10 years)
- 2.5M km arms
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe
- Other missions: Taiji, TianQin
- Proposals: DECIGO, BBO



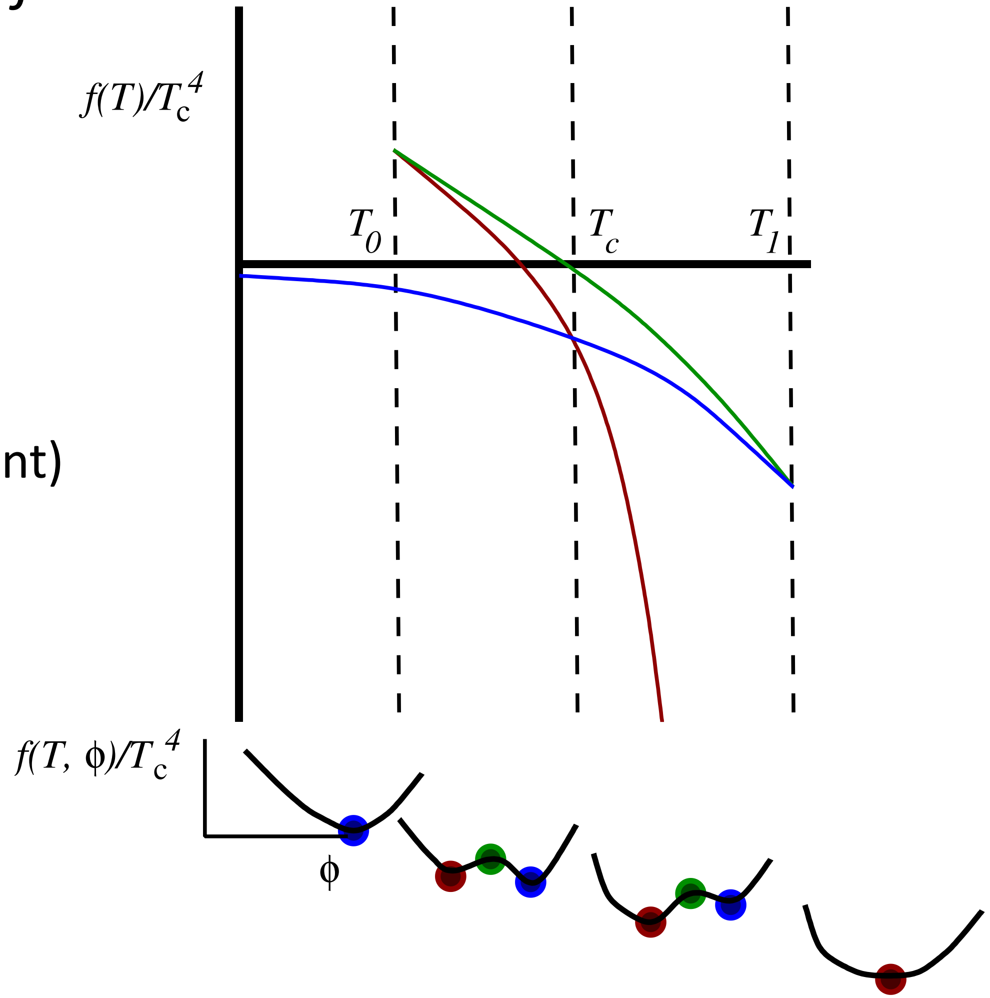
LISA sensitivity



LISA

First order phase transitions

- Phase: a local minimum of the free energy density f
- Behaviour of free energy around 1st order PT:
  - $T > T_2$: one equilibrium phase
  - $T_0 < T < T_1$: two equilibrium phases, one unstable
  - $T = T_c$: equal free energy, critical temperature
  - $T_0 < T < T_c$: high temperature phase is metastable
  - $T = T_0$: high temperature phase is unstable (spinodal point)
- Metastable phase can persist to $T = 0$
  - Example: superfluid ^{3}He, A phase
- Keep track of phase with order parameter ϕ
- In equilibrium: $\partial_\phi f(T, \phi_{\rm eq}) = 0$
- Equilibrium free energy: $f(T) \equiv f(T, \phi_{\rm eq})$



Little bangs in the Big Bang

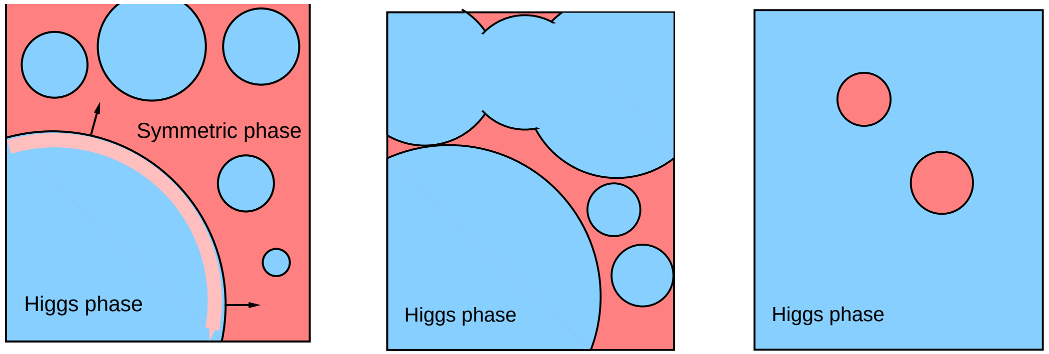
- 1st order transition by nucleation of bubbles of low-T phase

  Langer 1969, Coleman 1974, Linde 1983

- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Gravitational wave (GW) production
- GW spectrum has information about phase transition
- Departure from equilibrium: needed for baryogenesis

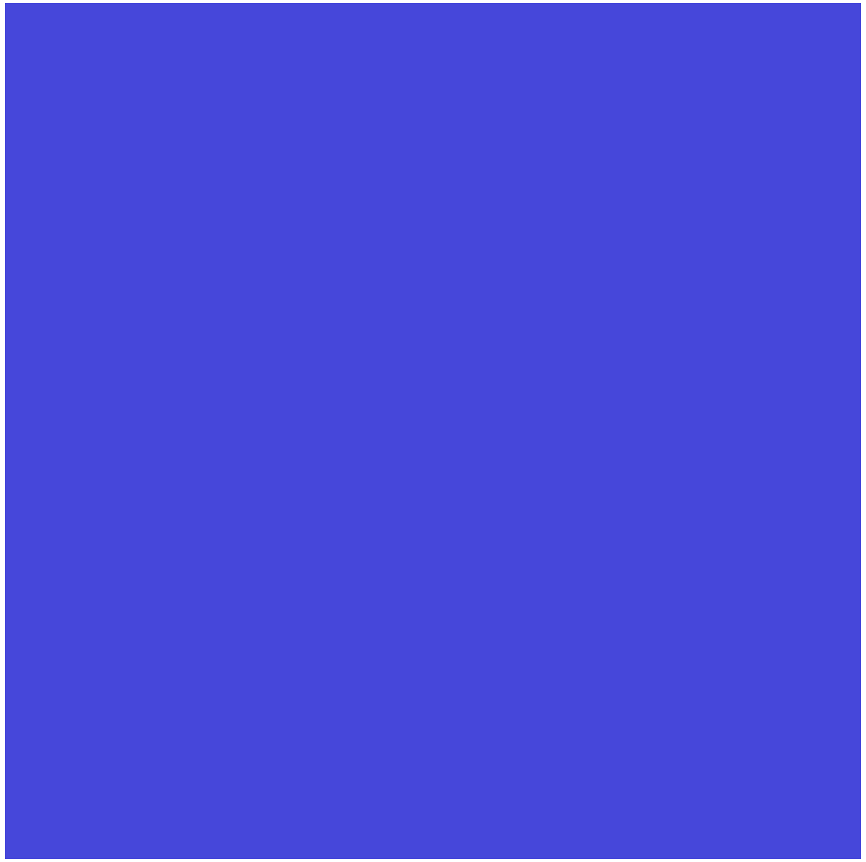
  Sakharov (1967)
  Kuzmin, Rubakov, Shaposhnikov (1985)

M. Postma, Friday



Steinhardt (1982); Hogan (1983,86);
Gyulassy et al (1984); Witten (1984)

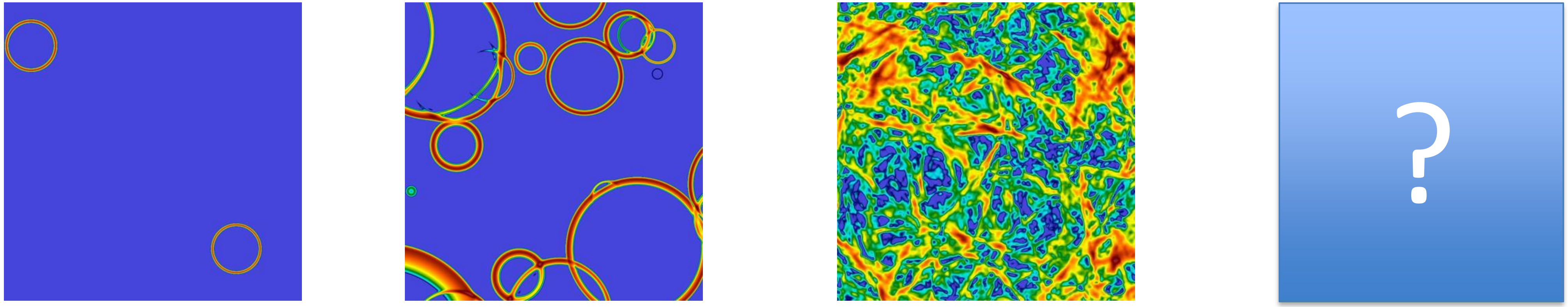
Fluid kinetic energy



David Weir

MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

Phases of a phase transition



1 2 3 4

1. Bubble nucleation and expansion
2. Collision
3. Acoustic waves (vorticity)
4. Non-linear (shocks, turbulence)

$$\tau_{\text{nl}} \sim L_f / \bar{U}_f$$

L_{f} – fluid flow length scale
U_{f} – RMS fluid velocity

'exponential' nucleation rate/volume p(t)

$$p(t) = p_n e^{\beta(t - t_n)}$$

$$\tau_{\text{co}} = \beta^{-1}$$

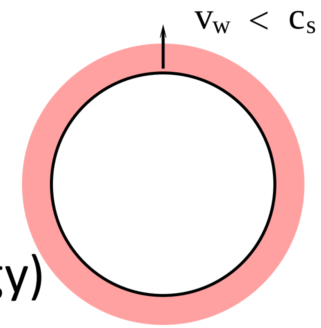
β – transition rate parameter
$\beta > H$ for successful transition

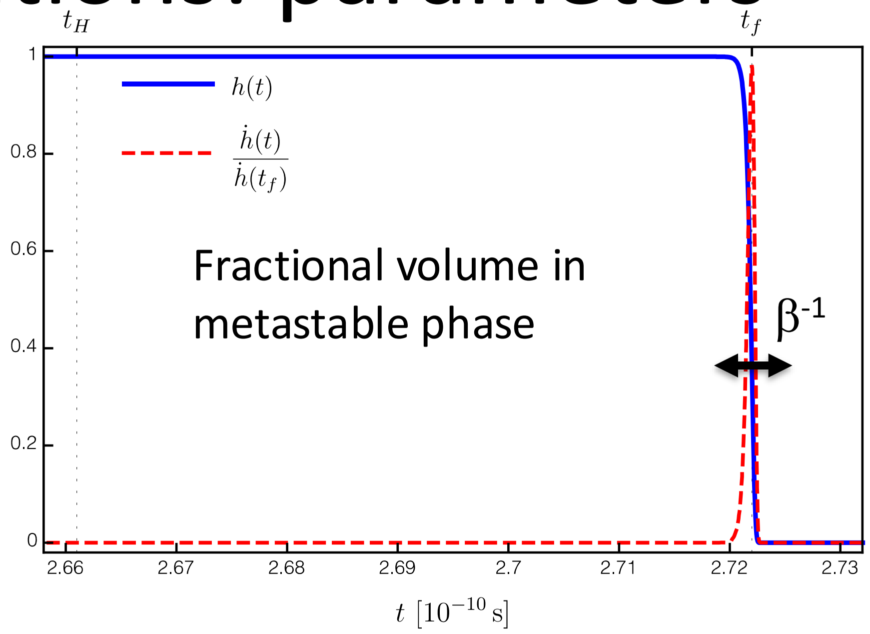
Guth, Weinberg 1983; Enqvist et al 1992;
Turner, Weinberg, Widrow 1992;

Review: MH, Lüben, Lumma, Pauly 2021

GWs from first order phase transitions: parameters

- Parameters of transition:
  - T_n = Temperature at nucleation
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - α = (potential energy change*)/(heat energy)
  - c_s = sound speed



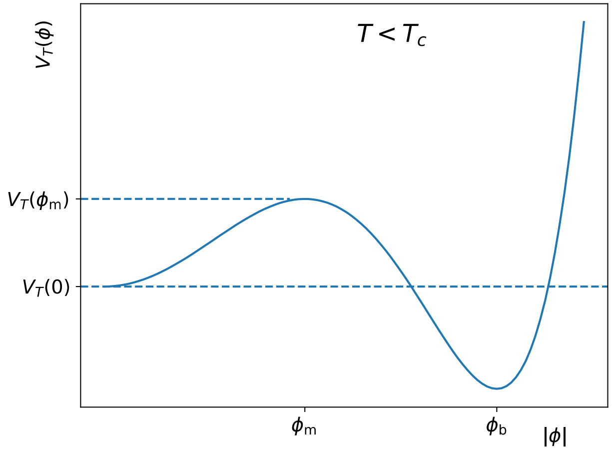


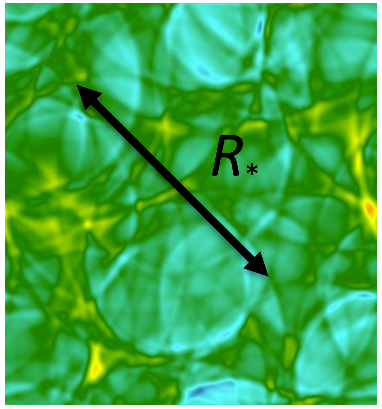
Giese et al 2020

- Useful derived parameters:
  - r_* = (bubble centre spacing R_*)/Hubble length
  - K = fluid kinetic energy fraction

Steinhardt ’84
Espinosa et al 2010

- Fluid kinetic energy makes GWs
  - Energy release via self-similar bubble solutions





$$* \quad \frac{1}{4}\Delta(e - 3p)$$

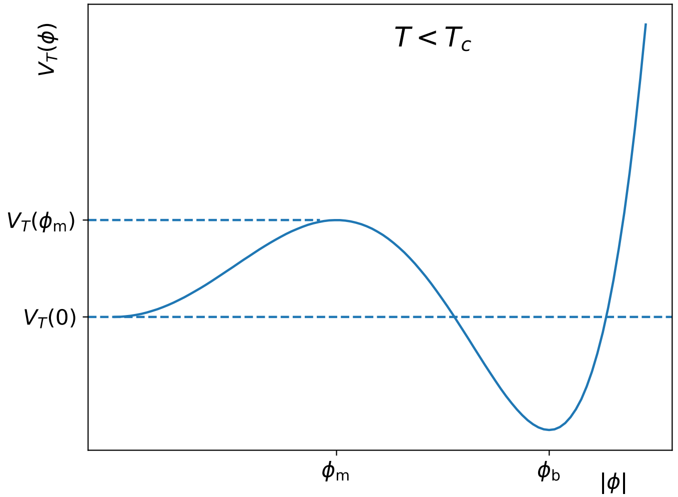
GWs from an early universe phase transition

Assume rapid transition, $\beta >> H$, neglect expansion of universe

- Effective theory: Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - **Higgs field** $\Box\phi - V_T'(\phi) = \eta_T(\phi) U \cdot \partial\phi$
    - $V_T(\phi)$ equation of state
    - $\eta_T(\phi)$ field-fluid coupling (models friction)
  - **Relativistic fluid (ideal limit)**

$$T_{\mathrm{f}}^{\mu\nu} = (e+P)U^\mu U^\nu + P g^{\mu\nu}$$

$$\partial_\mu T_{\mathrm{f}}^{\mu\nu} + \partial^\nu \phi V_T'(\phi) = \eta_T(\phi)(U \cdot \partial\phi)\partial^\nu\phi$$



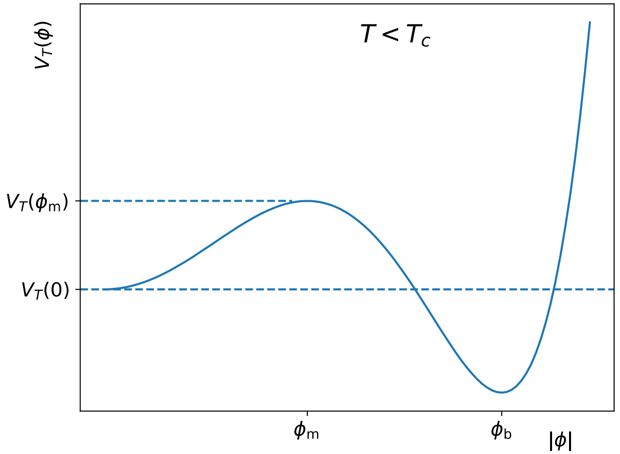
  - **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \Rightarrow \tilde{h}_{ij}(\mathbf{k}) = \Lambda_{ij,kl}^{TT} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

GWs from an early universe phase transition

Assume rapid transition, $\beta >> H$, neglect expansion of universe



- Ingredients for theory: Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - **Higgs field** $-\ddot{\phi} + \nabla^2\phi - V_T'(\phi) = \eta_T(\phi) W(\dot{\phi} + V^i \partial_i \phi)$
    - $V_T(\phi)$ equation of state
    - $\eta_T(\phi)$ field-fluid coupling ("friction", high T: $\eta_T(\phi) \propto \phi^2/T$)
  - **Relativistic fluid**

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi} W(\dot{\phi} + V^i \partial_i \phi) = \eta W^2 (\dot{\phi} + V^i \partial_i \phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_i V^j) + \partial_i P + \frac{\partial V}{\partial \phi} \partial_i \phi = -\eta W (\dot{\phi} + V^j \partial_j \phi) \partial_i \phi.$$

    - E = W*(energy density), Z_i = momentum density, V_i = 3-velocity, W = Lorentz factor
  - Discretisation * Wilson & Matthews (2003)

Also: Brandenburg, Enqvist, Olesen (1996); Giblin, Mertens (2013); Jinno, Konstandin, Rubira (2020)

  - **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda^{TT}_{ij,kl} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

Connection to fundamental theory

- Scalar hydrodynamics $-\ddot{\phi} + \nabla^2\phi - V_T'(\phi) = \eta_T(\phi)W(\dot{\phi} + V^i\partial_i\phi)$

- Scalar effective potential $V_T(\phi)$ → equilibrium, quasi-eqm. (T_n, α, β, c_s, g_{eff}) P. Schicho, Friday
- Scalar-fluid coupling $\eta_T(\phi)$ → non-equilibrium (v_w) J. van de Vis, Friday

$$\Omega_{gw}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

Phase transition parameters :
T_n = nucleation temperature
g_{eff} = effective d.o.f. in plasma
α ~ (latent heat)/(thermal energy)
c_s = sound speed(s)
β = transition rate
v_w = bubble wall speed

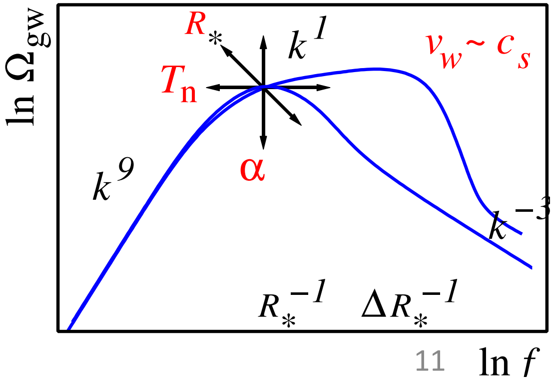
Simulations, Modelling →

$H_n(T_n, g_{eff})$ (Hubble rate)

$K(v_w, \alpha, c_s)$ (kinetic energy fraction)

$R_*(\beta, v_w)$ (mean bubble separation)

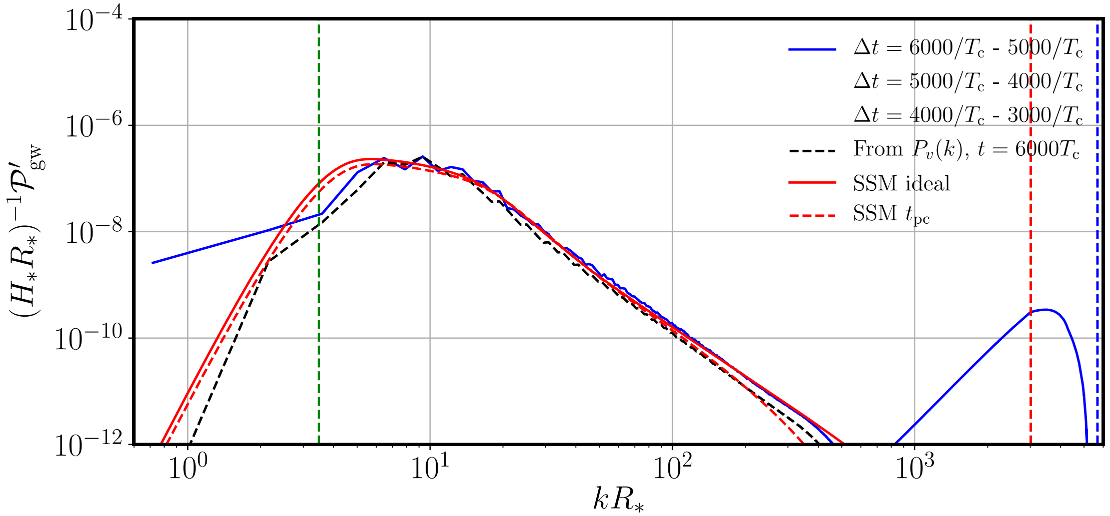
GW spectrum
Ω_p = peak amplitude
f_p = peak frequency
σ_i = shape parameters



3D hydrodynamic simulations of phase transitions

Hindmarsh et al 2013, 2015,2017, 2019; Jinno et al 2023

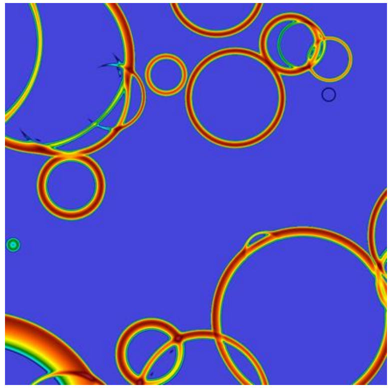
- Relativistic fluid + scalar order parameter ("Higgs")
- Linearised gravitational wave production
- Discretise on 4200^3 lattice (run on 24k CPUs)
  Wilson & Matthews (2003)
- Key output: GW power spectrum
  (fractional GW energy density per log wavenumber)

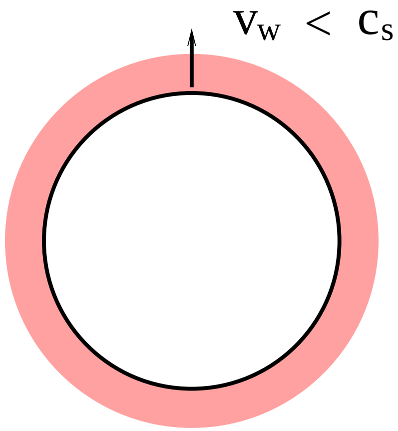


- While sound waves persist, GW power spectrum grows
- Plot: GW power spectrum **growth rate** (scaled)

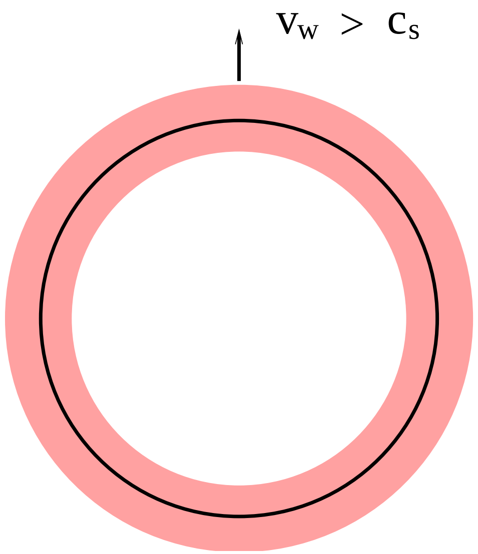
C S C

Towards a model: relativistic combustion

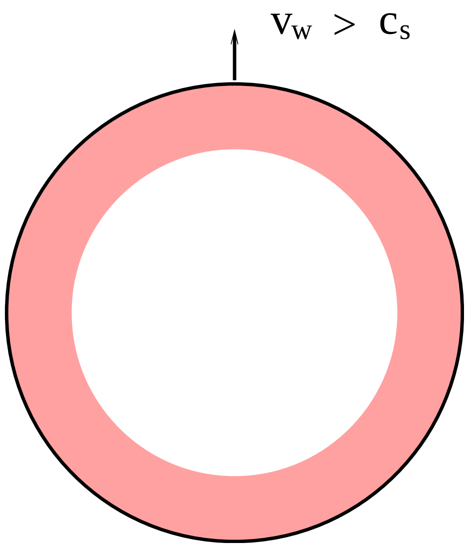




Deflagration



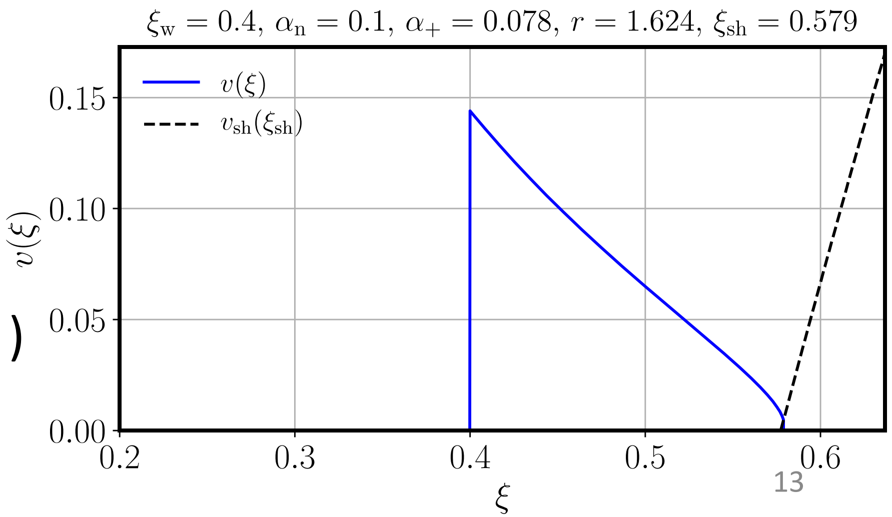
Supersonic deflagration ("hybrid")



Detonation

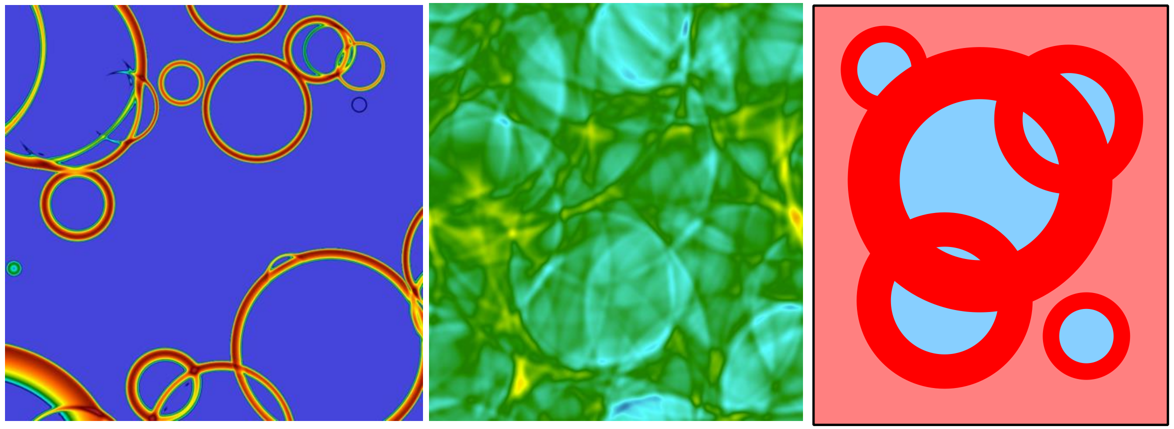
Landau & Lifshitz; Steinhardt (1984)
Kurki-Suonio, Laine (1991), Espinosa et al (2010)

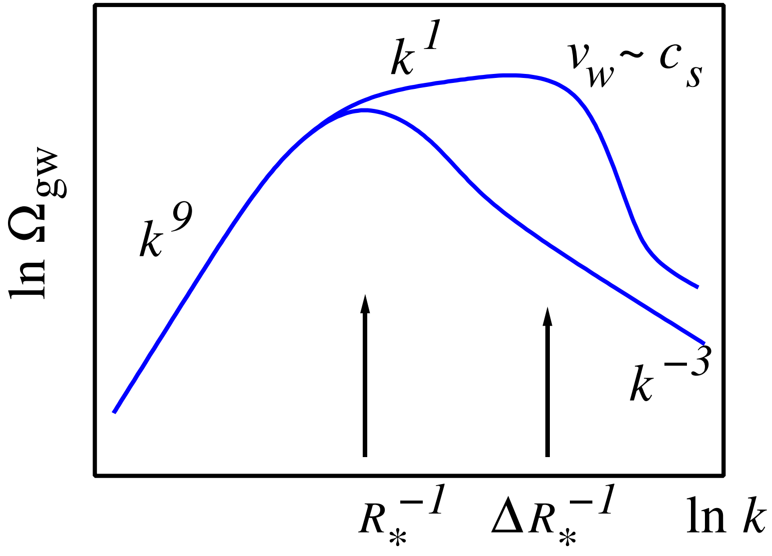
- Large scales: ideal relativistic hydrodynamics
- Microphysics: $e(T)$, $p(T)$, v_w
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
  - **Similarity solution $v(x)$, $w(x)$: $\xi = r/t$**
- Low-friction or ultra-strong transition: can be "runaway" ($v_w \to 1$)
  - (not considered here) Bodeker Moore 2010, 2017



GWs from phase transitions: Sound shell model

- GWs from Gaussian velocity field
  Caprini, Durrer, Servant (2007,2009)
- Velocity field: weighted addition of self-similar sound "shells" $\mathbf{v}_{\mathbf{q}}(t_i)$ from bubbles
  MH 2016, MH, Hijazi (2019)
- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_*|v_w - c_s|/c_s$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude proportional to:
  - Bubble spacing
  - Shear stress lifetime
  - (Kinetic energy)^2
- Similar: bulk flow model (real space)
  Jinno, Konstandin, Rubira 2020





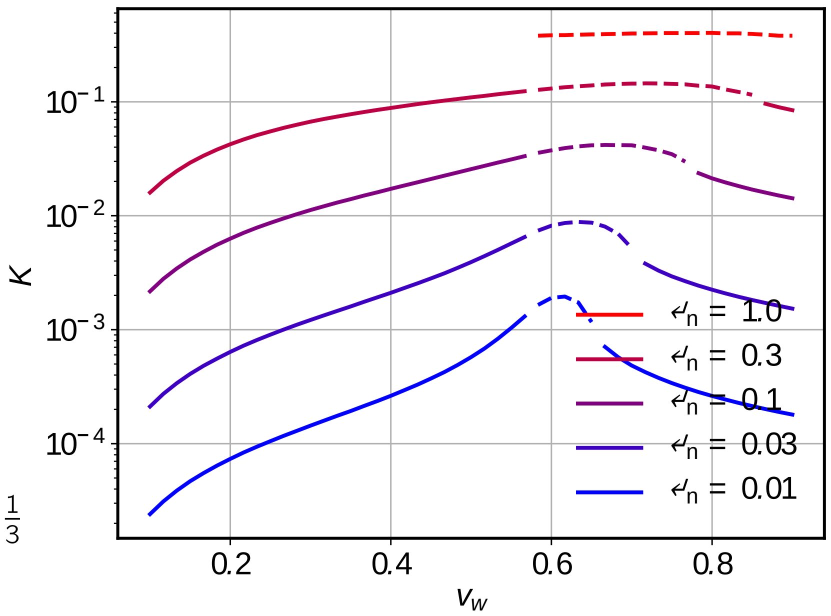
Estimating GW power

$$\Box h \sim T \longrightarrow P_{\dot{h}}(t,k) \sim \int^t dt_1 \int^t dt_2 \cos[k(t-t_1)]\cos[k(t-t_1)]\langle T_k(t_1) T_k^*(t_2)\rangle$$

- GW energy fraction: $\longrightarrow$ $$\Omega_{\rm gw} \sim (H_{\rm n}\tau_{\rm v})(H_{\rm n}R_*)K^2$$
  - H_n Hubble rate at nucleation
  - τ_v duration of stresses
  - τ_c coherence time
- Coherence time:
  - $\tau_c \sim R_*$ (bubble spacing)
- Shear stress lifetime (shocks):
  - $\tau_v = H_n^{-1}/(1 + U_f/R_* H_n)$
  - U_f (~ RMS velocity) ~ $\sqrt{K}$
  - or $K = (4/3)U_f^2$

$R_*(\beta, v_w$, also $\alpha_n)$ (mean bubble spacing) from bubble growth dynamics

$K(v_w, \alpha_n)$ (kinetic energy fraction) from self-similar hydro solution



$$\Omega_{\rm gw,0} \simeq F_{\rm gw,0}\frac{r_*}{1+\sqrt{K/r_*}}K^2\tilde{\Omega}_{\rm gw}$$

Numerical simulations:

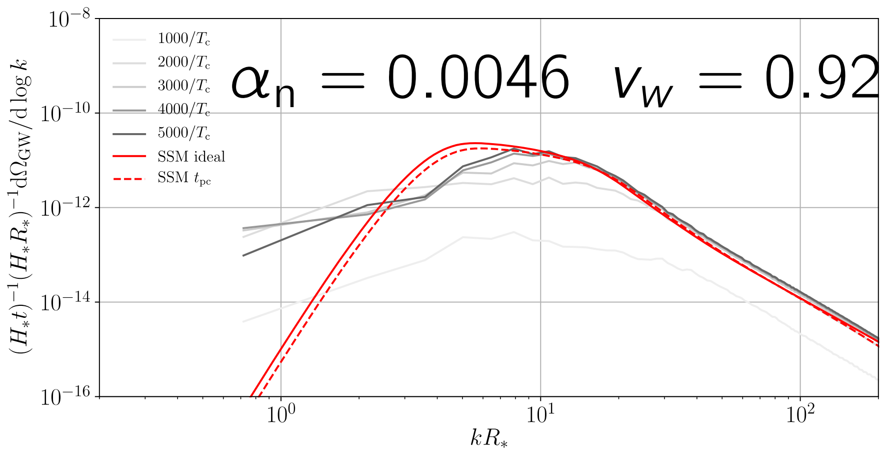
$$\tilde{\Omega}_{\rm gw} = O(10^{-2})$$

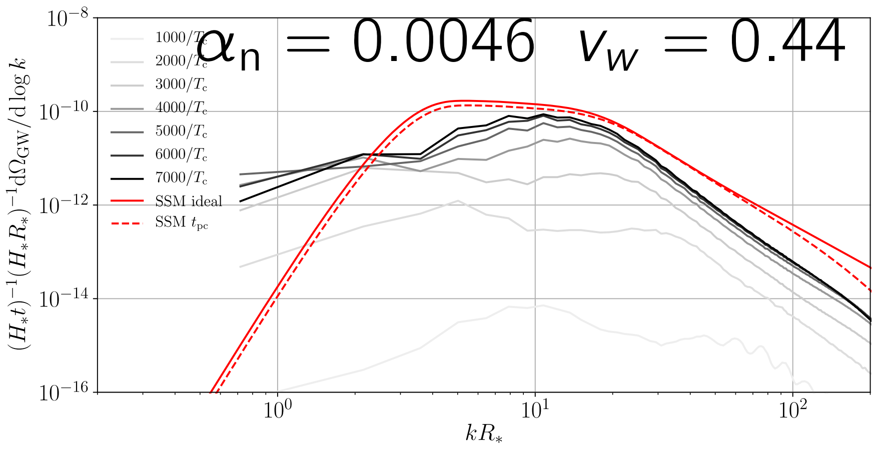
Standard cosmology:

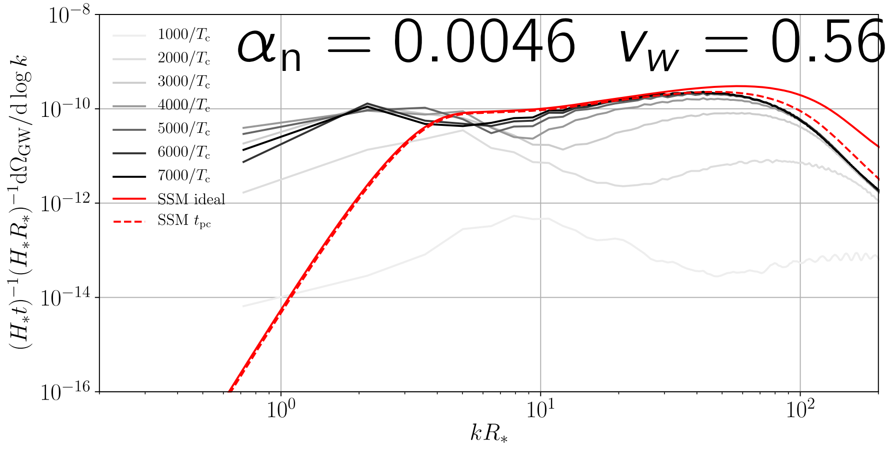
$$F_{\rm gw,0} = 3.6 \times 10^{-5}\left(\frac{100}{g_{\rm eff}}\right)^{\frac{1}{3}}$$

Sound shell model vs. simulations P_{gw}

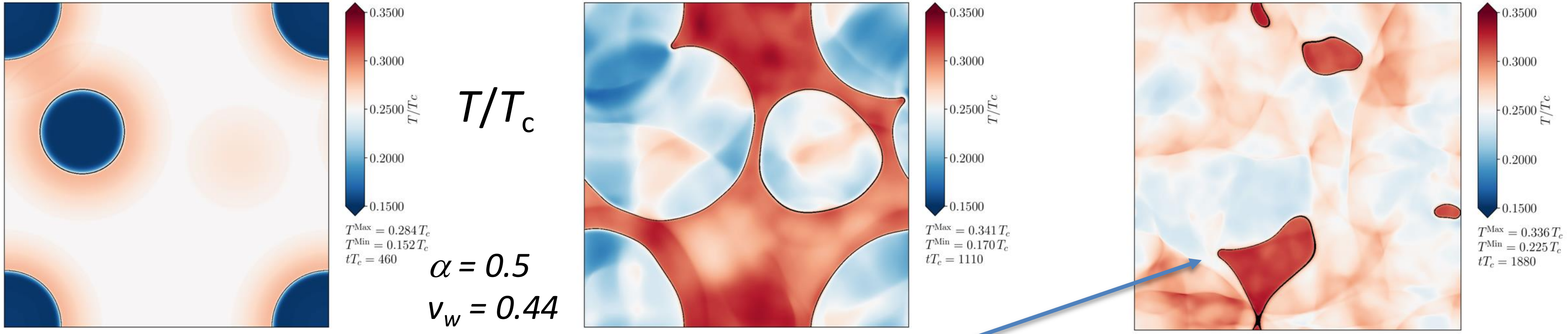
- Solid: ideal self-similar sound shell
- Dash: evolving sound shell at peak collision time in 1+1D scalar hydro
- Grey: simulations: MH et al 2017
  - simultaneous nucleation of bubbles







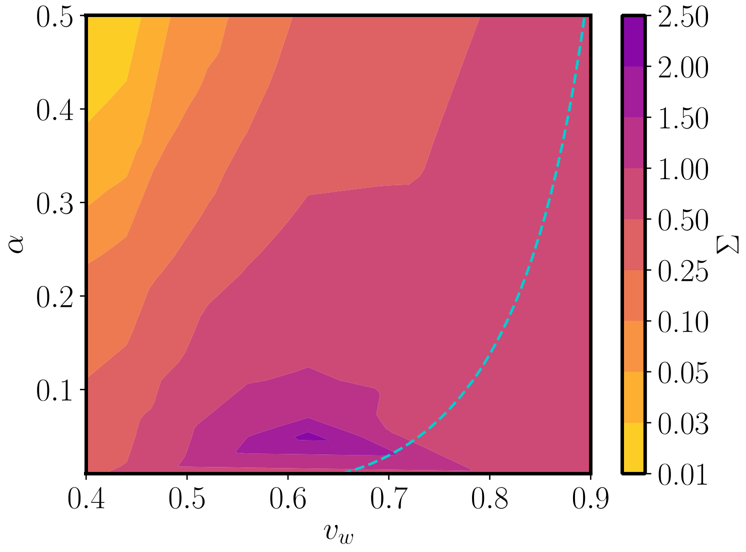
Nonlinearities 1: Kinetic energy & GW suppression



Cutting, MH, Weir, 2020

- **Deflagrations**: heat up fluid in front
- Pressure in front of wall increases, walls slow down
- Formation of hot droplets
- Less transfer into kinetic energy, more into heat.
- Include **GW suppression factor** as a numerical parameter (**right**)
- Also: nucleation suppression
  - bigger bubbles, boosts signal

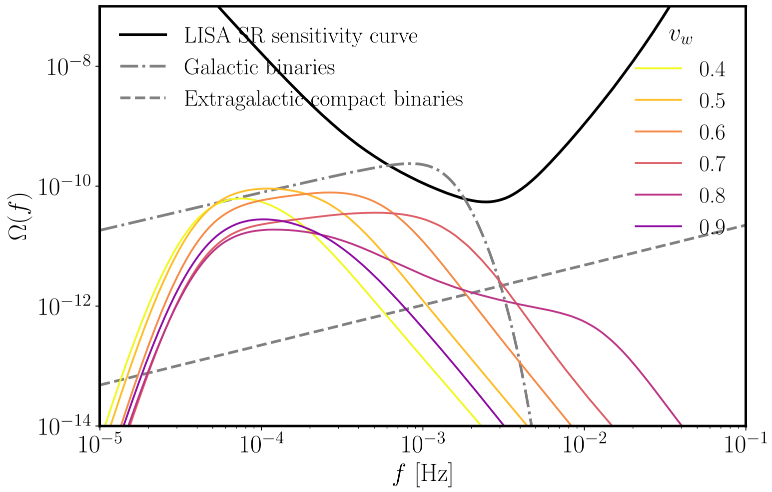
Al-Ajmi, MH (2023)



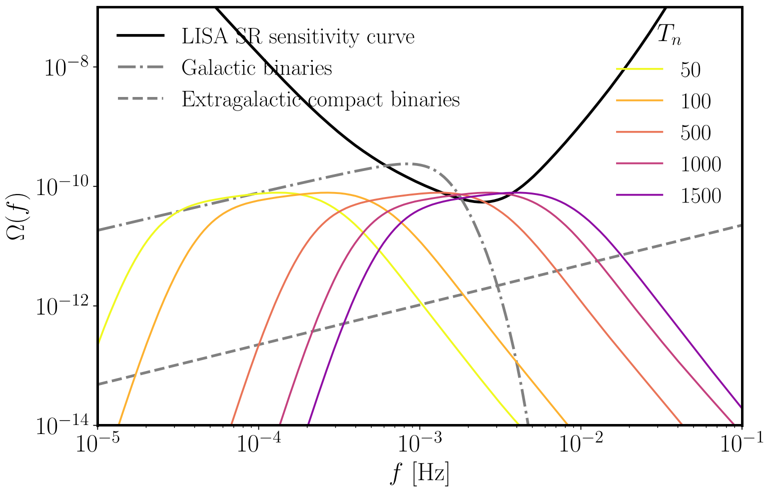
Gowling, MH (2021)

GW power spectra in the SSM

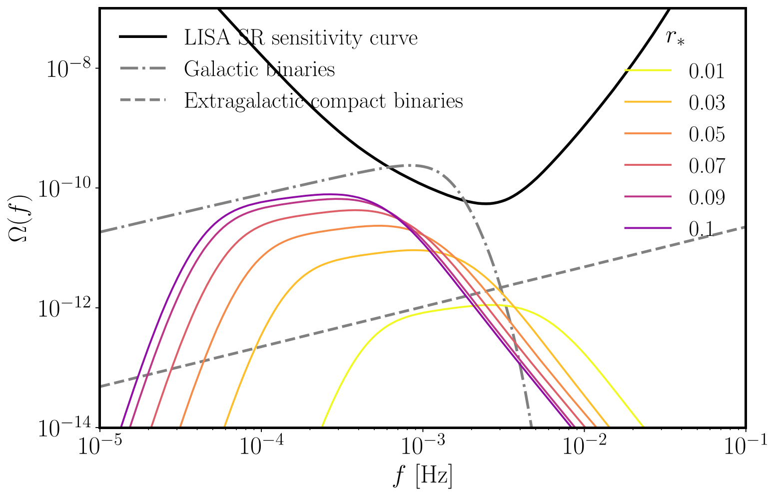
- **Sound shell model predictions, acceptable accuracy for** Gowling, MH (2021)
  - near-linear flows ($\alpha \leq 0.3$); fast walls: $v_w > 0.4$; sub-Hubble bubble separations ($r_* << 1$)



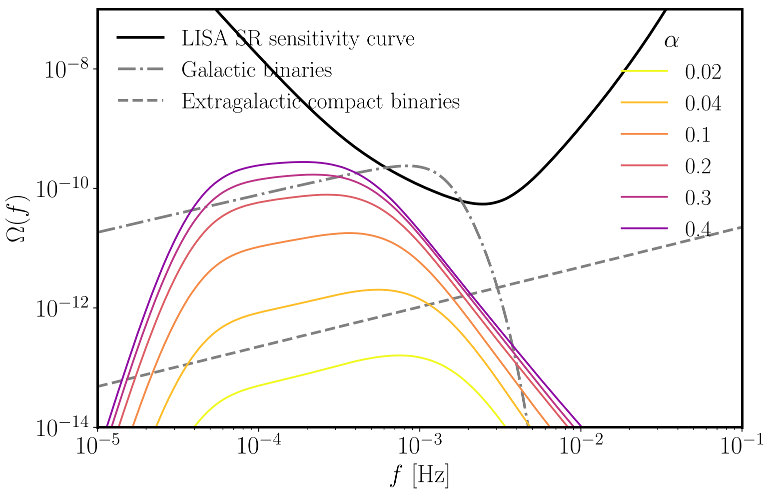
Vary wall speed



Vary transition temp



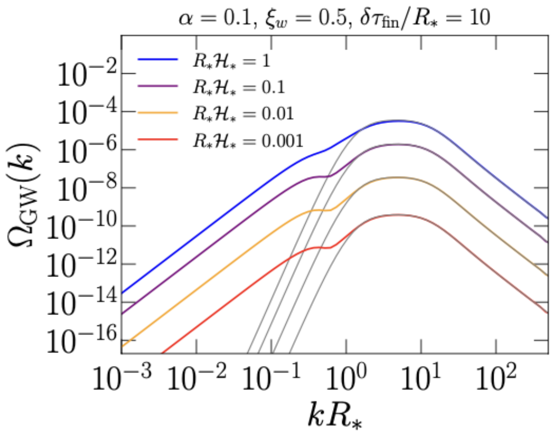
Vary bubble separation



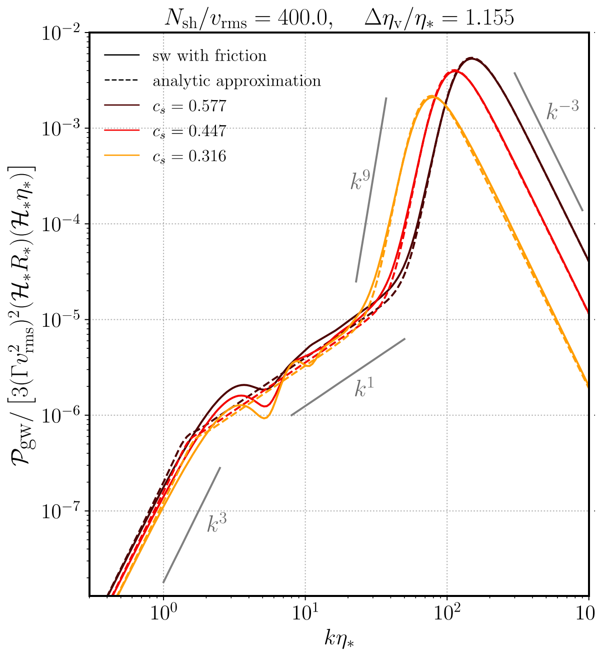
Vary transition strength

Recent advances in hydrodynamics

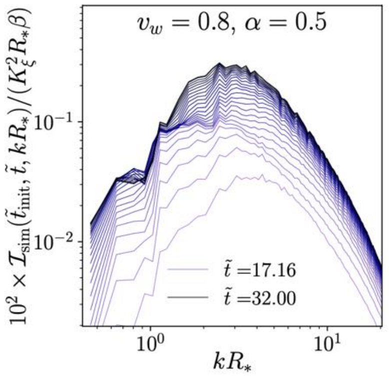
- Shape of GW spectrum at low *k*
  Sharma, Dahl, Brandenburg, MH 2023; Roper-Pol, Procacci, Caprini 2023
- Effect of sound speed on GW spectrum
  Giombi, Dahl, MH 2024
  – post-transition equation of state Racco, Poletti 2022
- GWs from strong transitions
  – v_{rms} ~ 0.1 – 0.3, decaying flow (shock dissipation)
  – convergence of GW spectrum, peak shape change



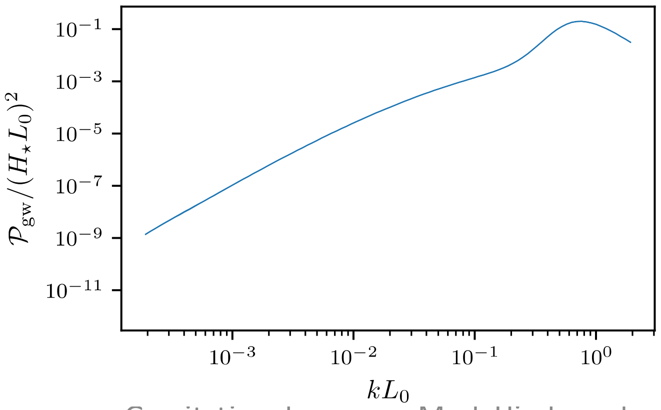
Roper Pol, Procacci, Caprini 2023



Giombi, Dahl, MH 2024



Caprini et al 2024



Dahl et al 2024

Conclusions

- LISA and other missions will probe physics of electroweak-scale transitions from mid-2030s
  - Measure/constrain phase transition parameters
    - Wall speed likely to be best determined (if signal seen) Gowling, MH (2022)
  - Parameters from underlying particle physics models
    - Recent progress with equilibrium parameters
    - Wall speed the hardest (non-equilibrium)
- Towards accurate calculations of GW power spectrum from parameters
  - Non-linear evolution not well understood yet
  - Strongly supercooled transitions?
    - e.g. "nearly conformal dynamics" Konstandin, Servant (2011)
- Ambition: make GWs as good a probe of the electroweak era as CMB is for the decoupling era