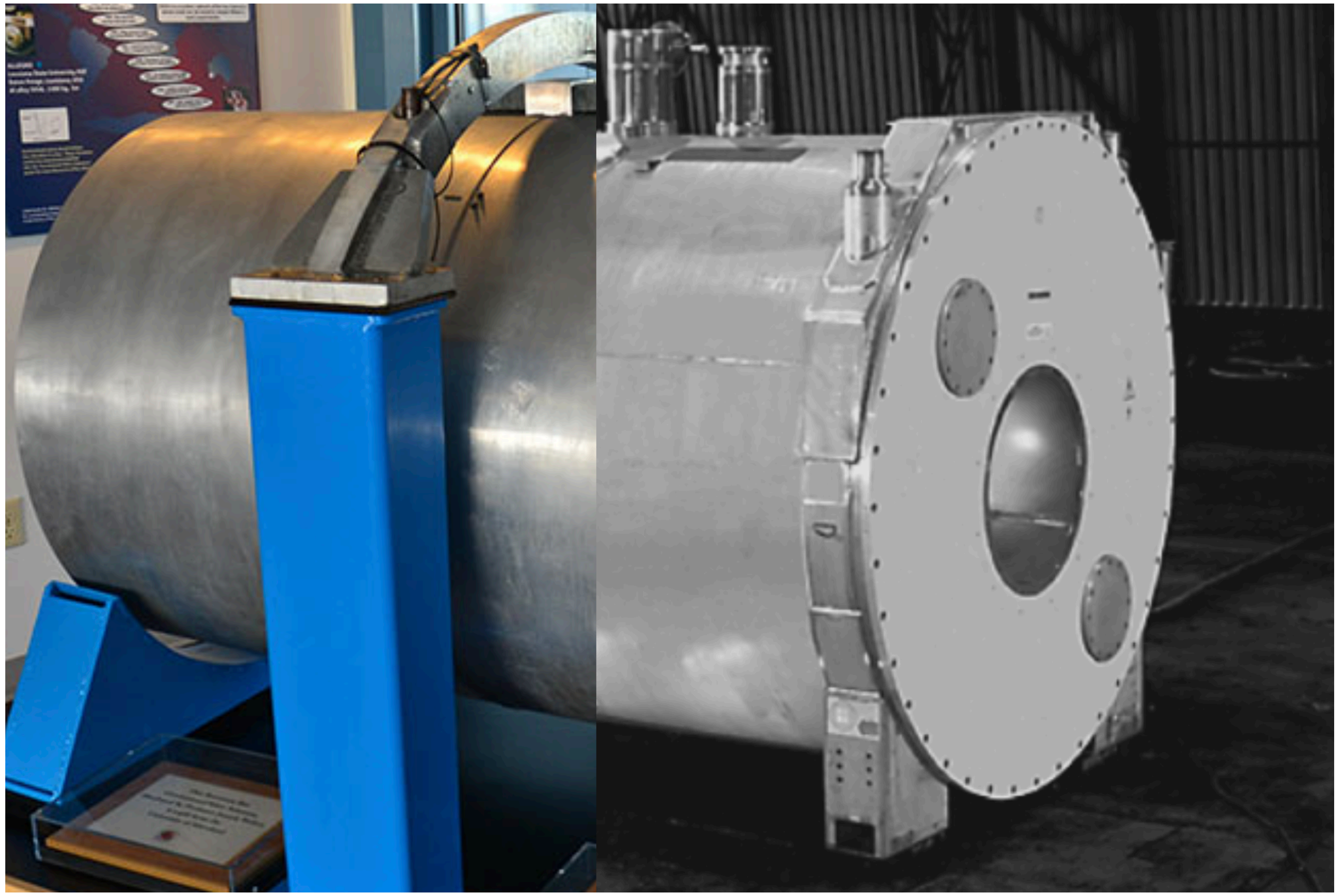


# Magnets are Weber Bars

[2408.01483] w/ Valerie Domcke & Sebastian Ellis

BERKELEY LAB

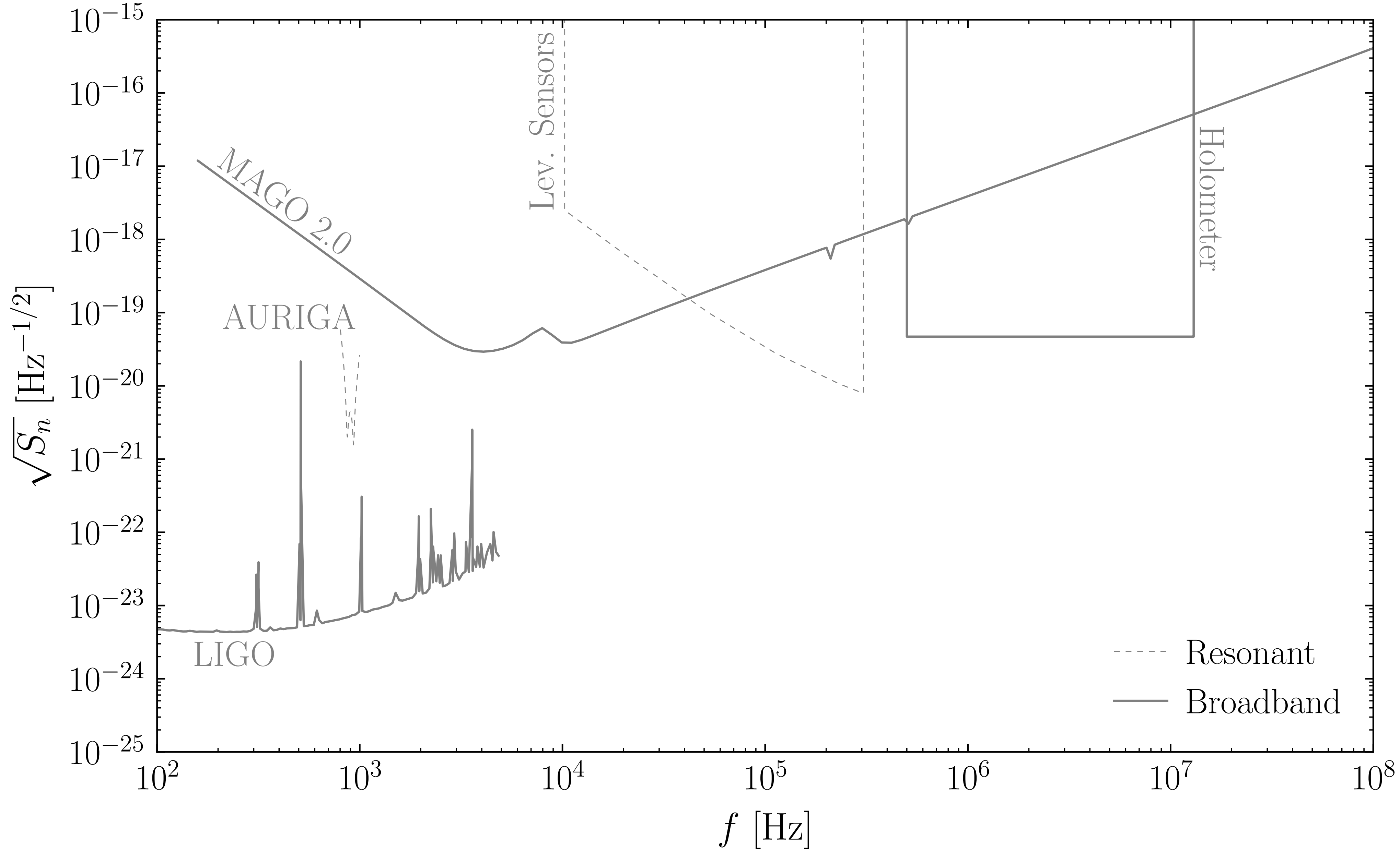
NICK RODD | Pollica | 17 September 2024

# Motivation

How can we best exploit large magnets to search for gravitational waves?

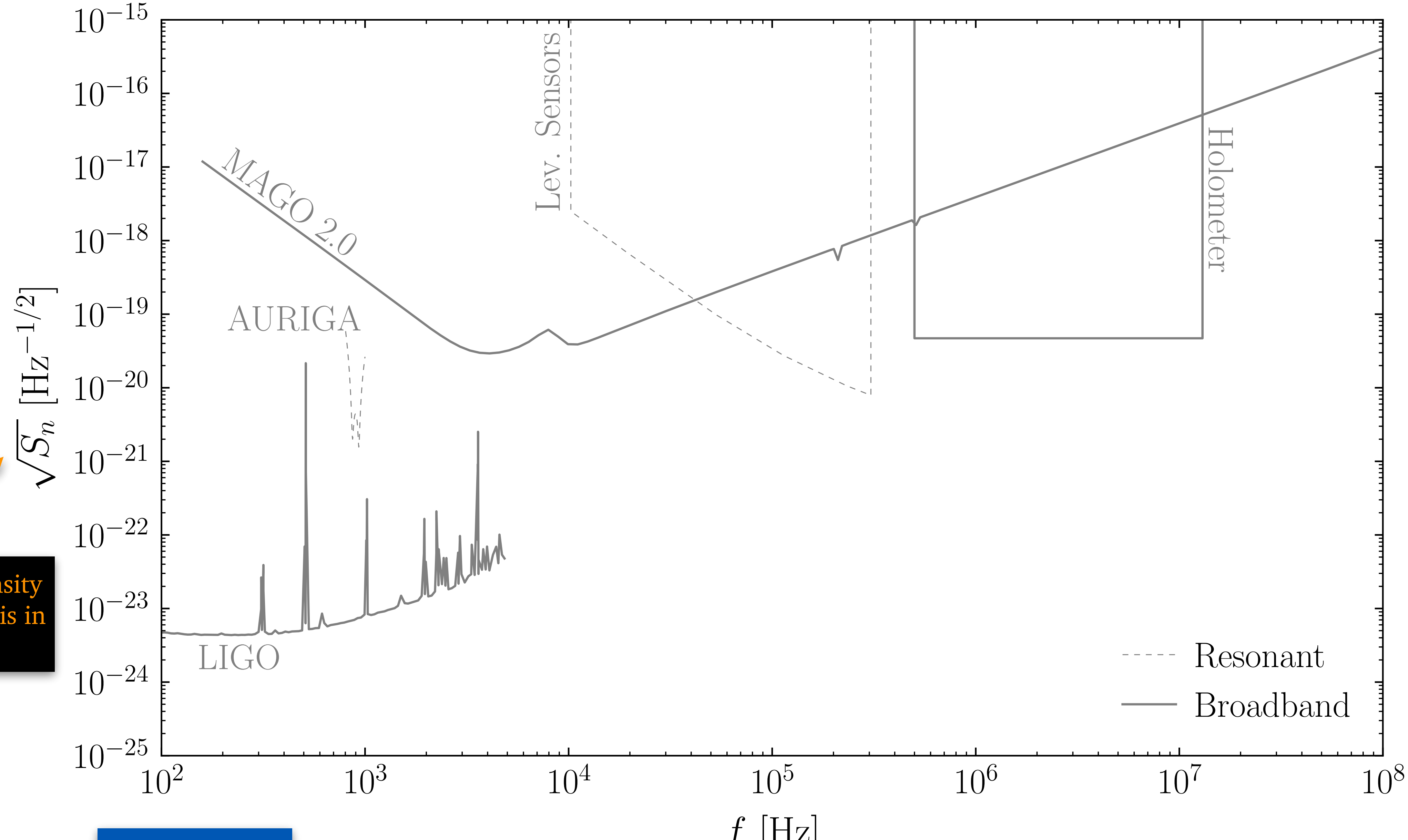
# Challenge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



# Challenge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$




Noise spectral density (How much noise is in the detector?)

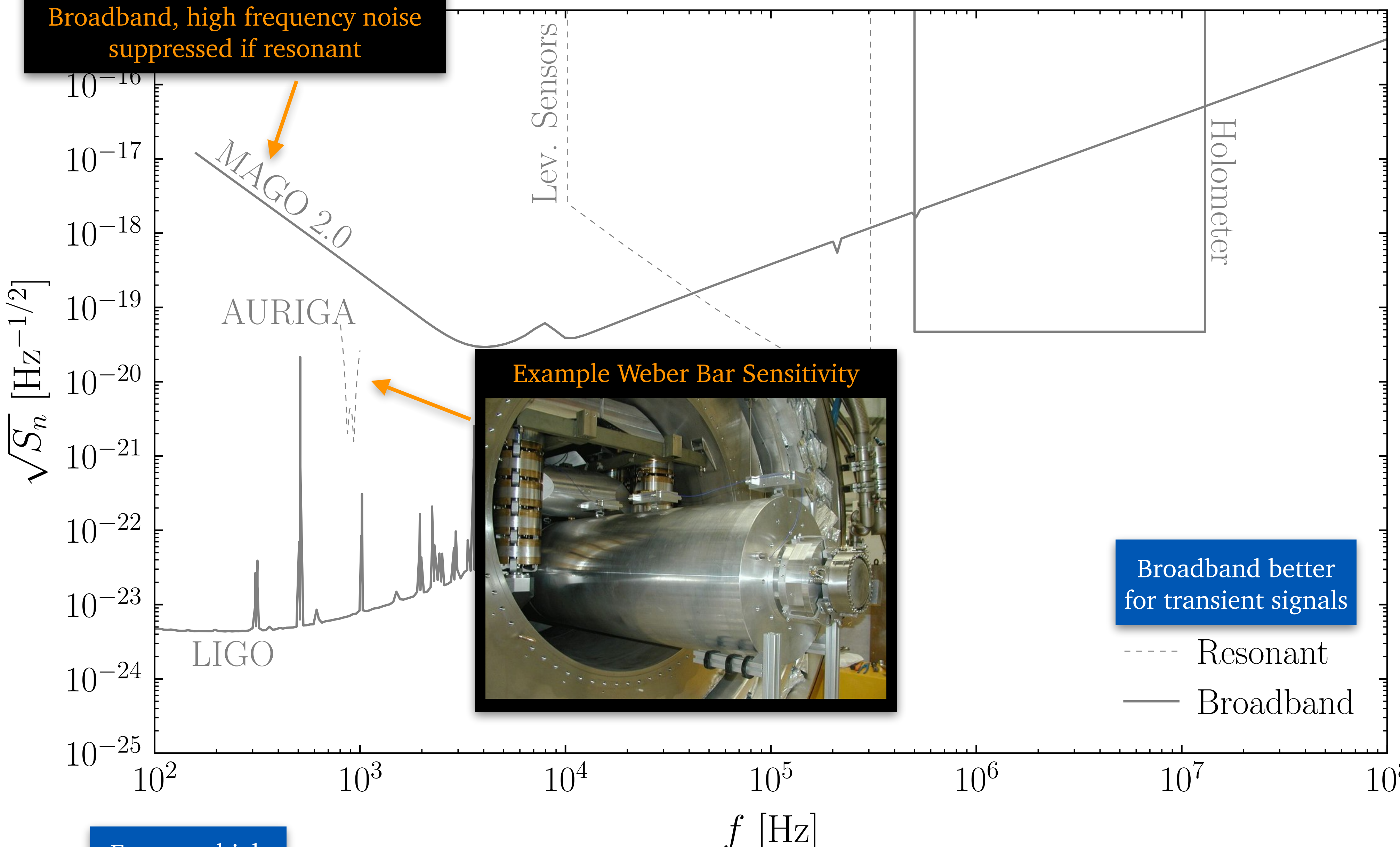
Focus on high frequencies

$h(t) \sim h e^{-2\pi i f t}$

# Challenge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

[Berlin, Schutte-Engel+ 2023]
Broadband, high frequency noise suppressed if resonant



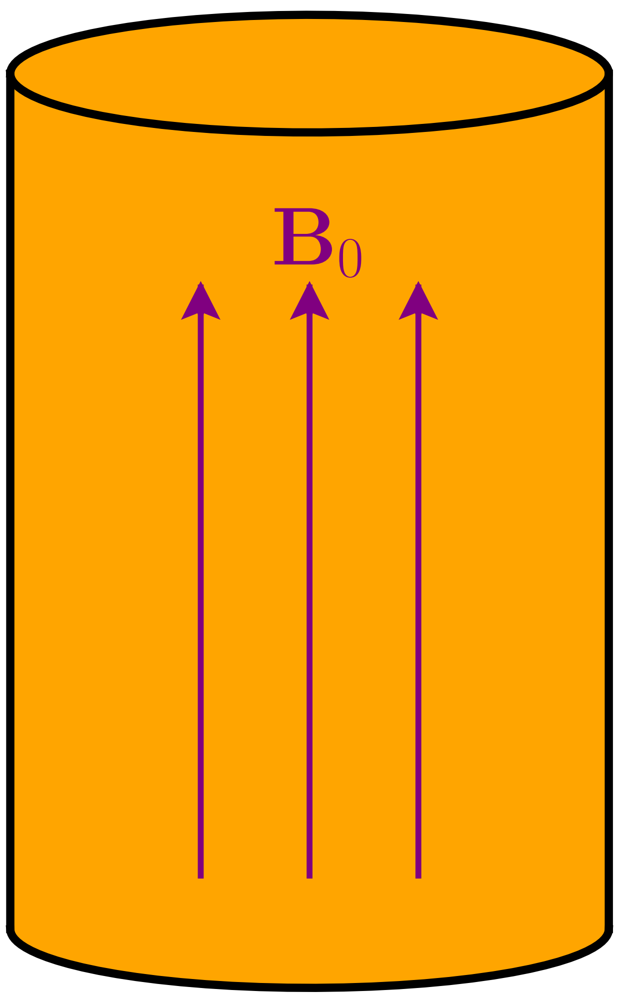

Example Weber Bar Sensitivity

Broadband better for transient signals

Focus on high frequencies

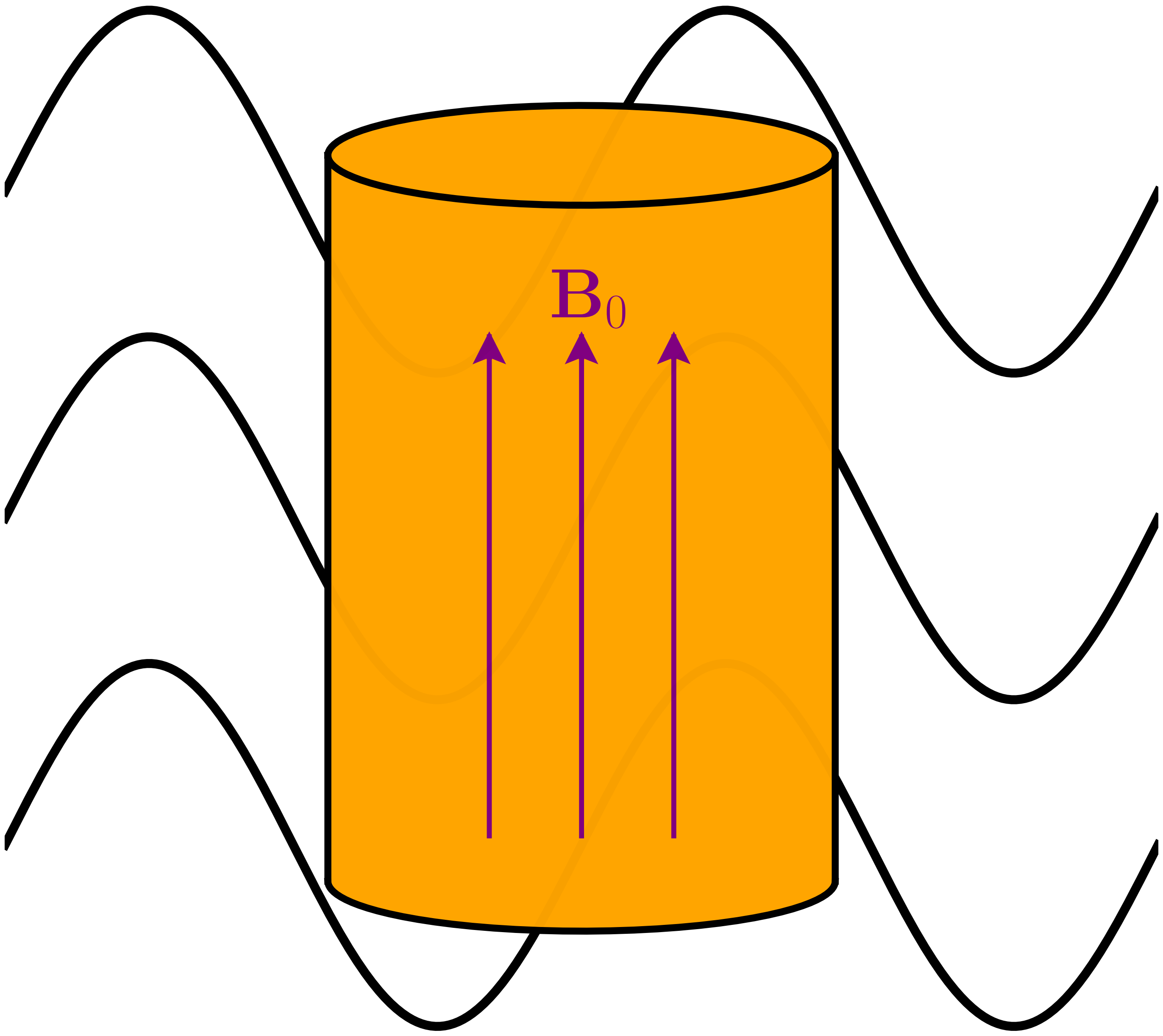
Subset of ideas, see [Aggarwal+ 2020]

# First approach: $hF^2$




Goal: exploit large stored energy in a magnetic field

# First approach: $hF^2$

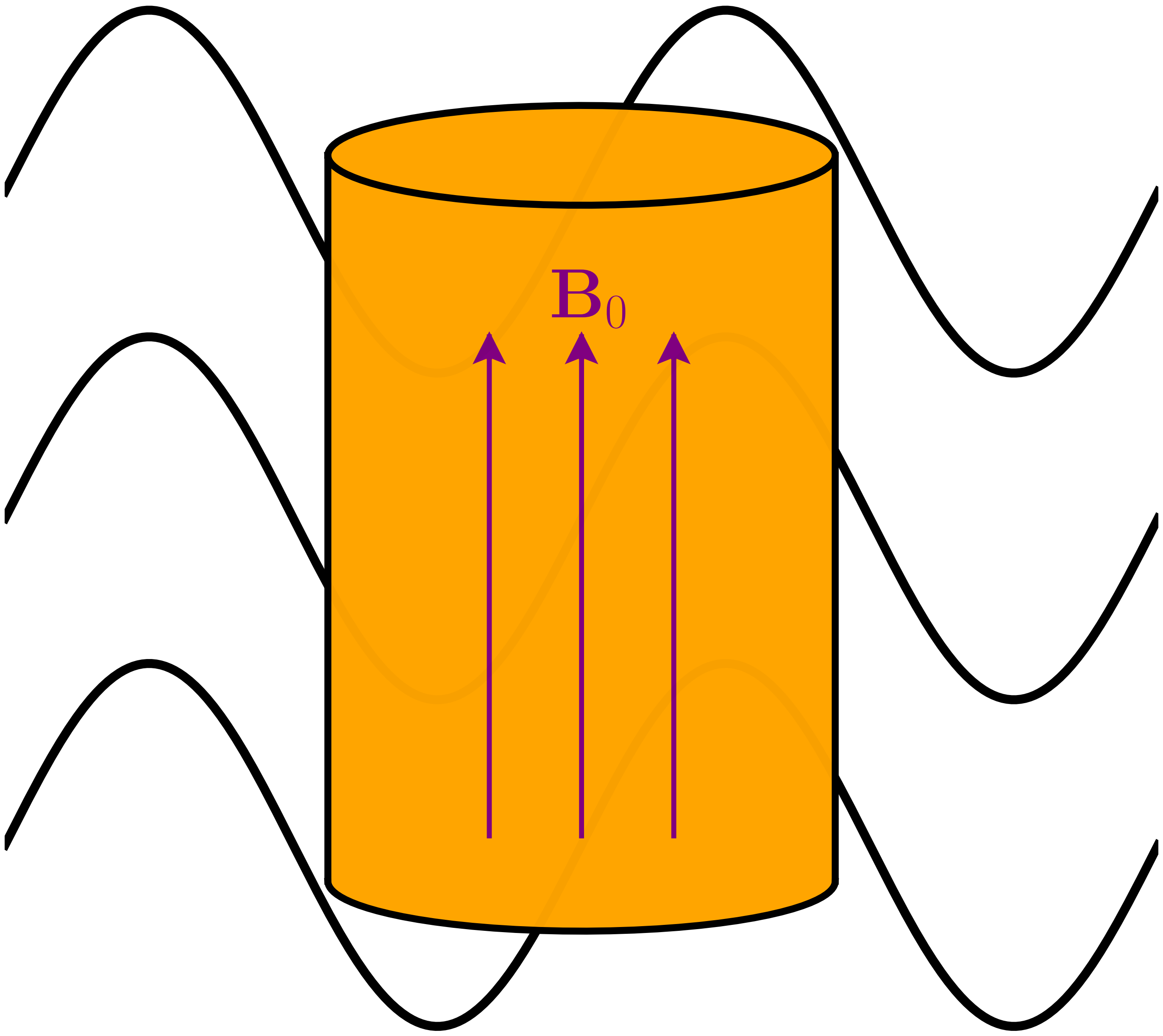



Familiar use

$g_{a\gamma\gamma} a F \tilde{F}$

Axion detection with solenoidal geometry [Sikivie, Sullivan, Tanner 2014]
Earlier suggestion by Romalis & Thomas, see [Svrcek, Witten 2006]

# First approach: $hF^2$




Familiar use

$$g_{a\gamma\gamma} a F \tilde{F}$$

Exploit direct analogy for GW

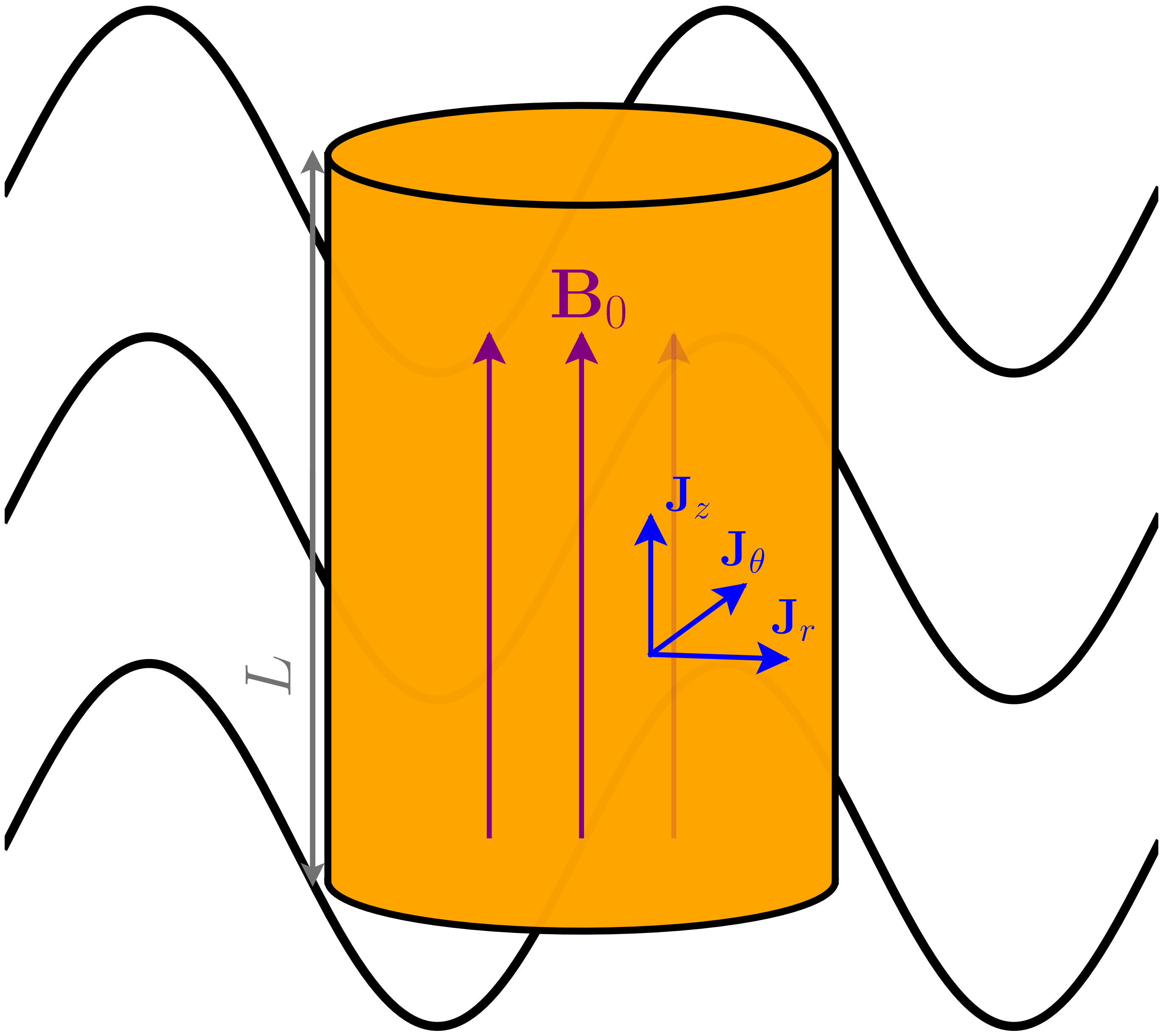
$$hF^2$$

**Axion haloscopes are GW telescopes**

Expand $S \supset \int d^4x \sqrt{-g} \left( -\frac{1}{4} F^2 \right)$ for $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

[Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021]
[Domcke, Garcia-Cely, NLR 2022]

# First approach: $hF^2$




Familiar use

$$g_{a\gamma\gamma} aF\tilde{F}$$

Exploit direct analogy for GW

$$hF^2$$

Generates AC magnetic field

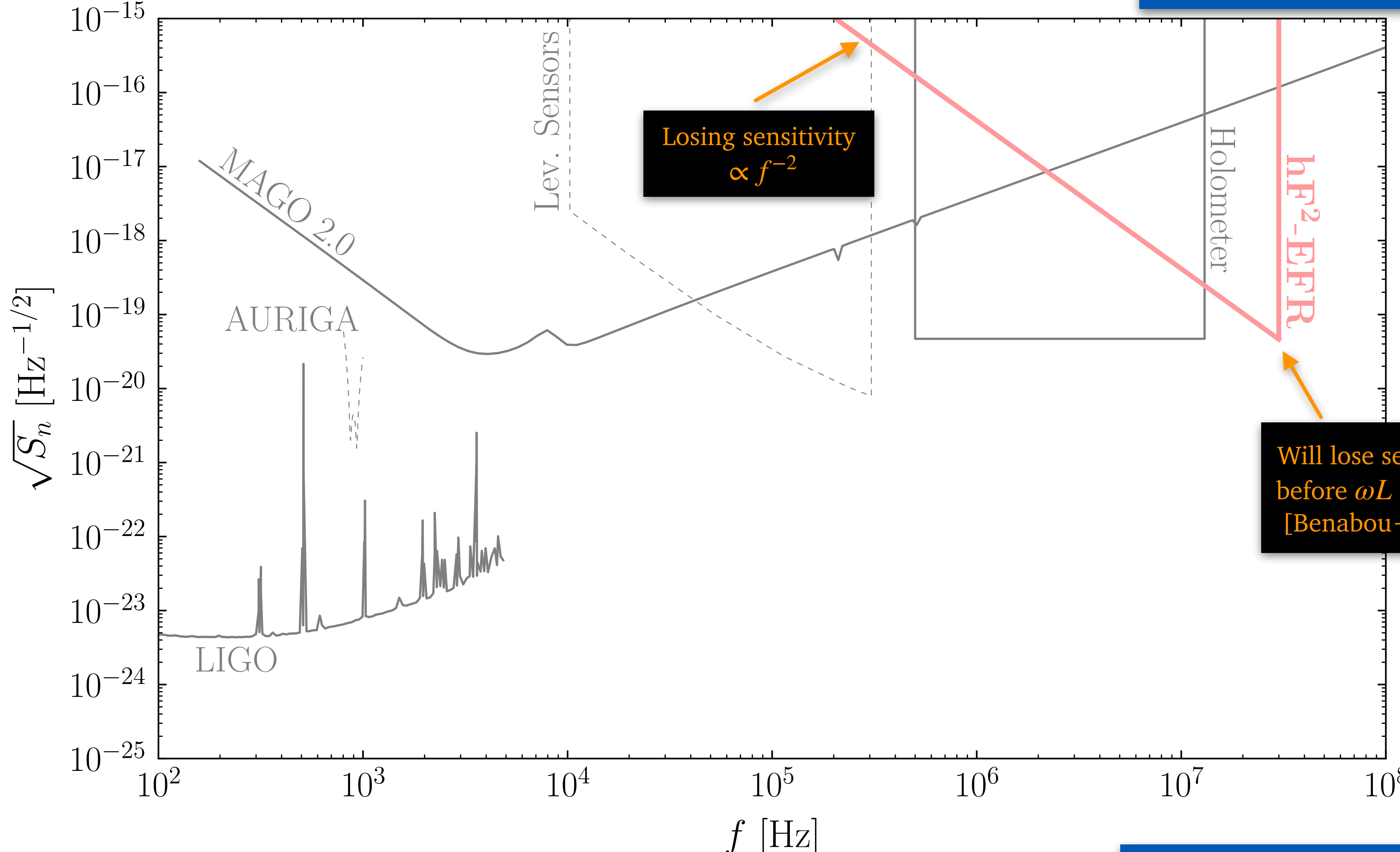
$$B_h^{\rm EM} \sim hB_0(\omega L)^2$$

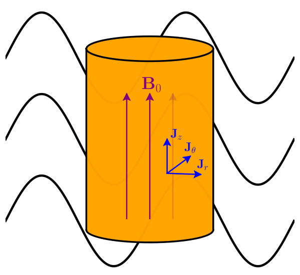
Leading gauge invariant contribution, full solenoidal calculation in [Domcke, Garcia-Cely, Lee, NLR 2024]

# First approach: $hF^2$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Assume ADMX-EFR magnet, SQUID noise limited, broadband readout



As for axion, improves with resonant readout (also true for MAGO 2.0)

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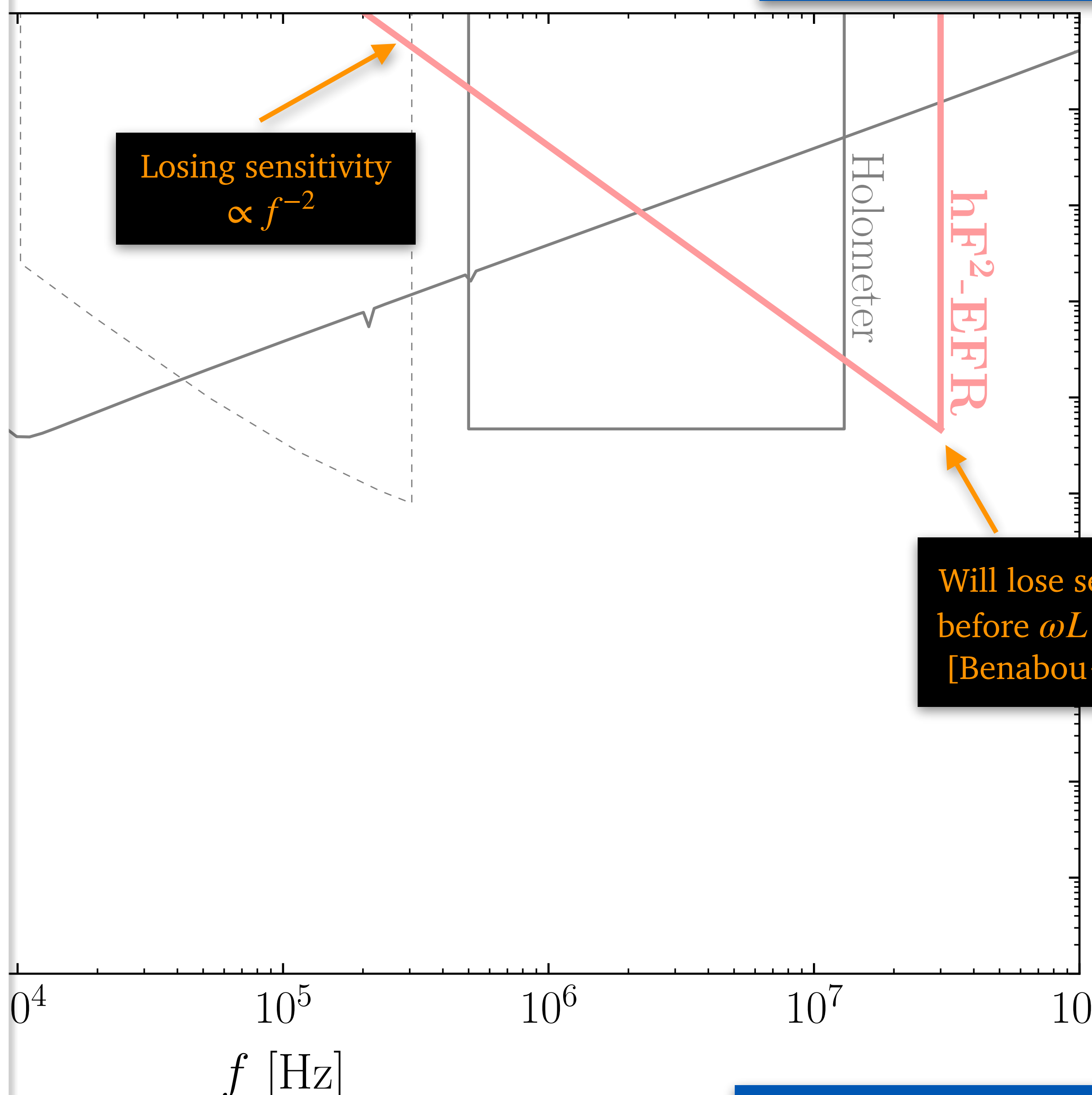
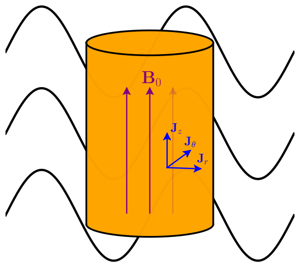



Losing sensitivity $\propto f^{-2}$

Will lose sensitivity before $\omega L \sim 1$, see [Benabou+ 2023]



As for axion, improves with resonant readout (also true for MAGO 2.0)

# Why $B_h \sim (\omega L)^2$ ?

**TT gauge:** $h \sim e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \sim \omega^0$, but $B_0 \sim$ ?

**Proper detector frame:** $B_0 \sim \omega^0$, $h \sim \omega^2$

PDF advocated in [Berlin+ 2021], see also [Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

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**Proper detector frame:** $B_0 \sim \omega^0$, $h \sim \omega^2$

PDF: locally inertial frame along a geodesic $x_0$

$\omega^0$ contribution is $\eta_{\mu\nu}$

Locally flat coordinates $\Rightarrow \mathcal{O}(\omega)$ must vanish

$$0 = \Gamma^{\mu}_{\nu\rho}(x_0) \sim \partial g(x_0)$$

See [Domcke, Garcia-Cely, Lee, NLR 2024]

PDF advocated in [Berlin+ 2021], see also [Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

# Why $B_h \sim (\omega L)^2$ ?

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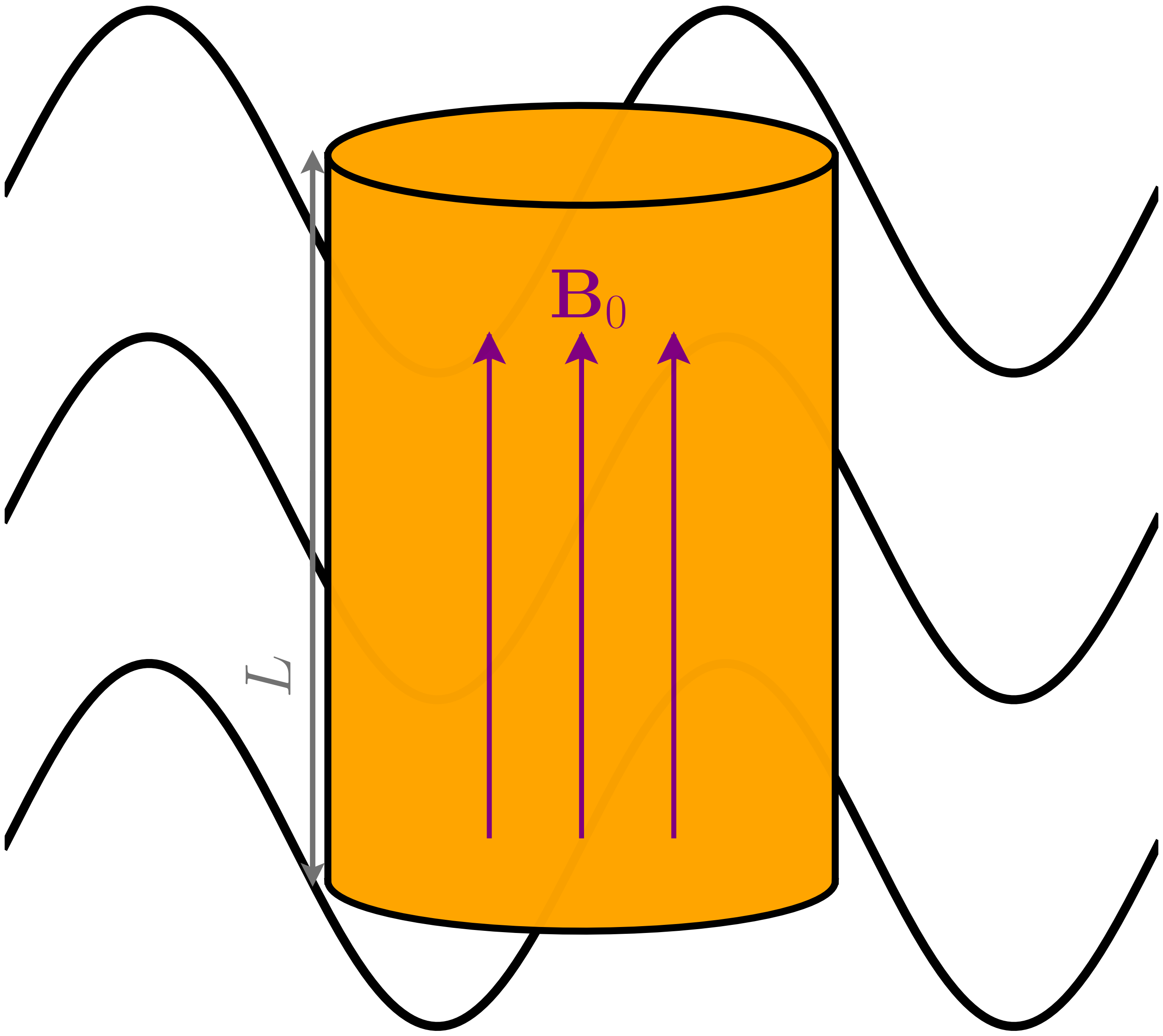
**Proper detector frame:** $B_0 \sim \omega^0$, $h \sim \omega^2$

Couples to an EM modes

$$B_h \sim \frac{\omega^2 h}{\omega^2 - \omega_{\rm EM}^2 + i(\omega\omega_{\rm EM}/Q_{\rm EM})^2} \sim \omega^2 h$$

$\omega \ll \omega_{\rm EM} \sim 500$ MHz

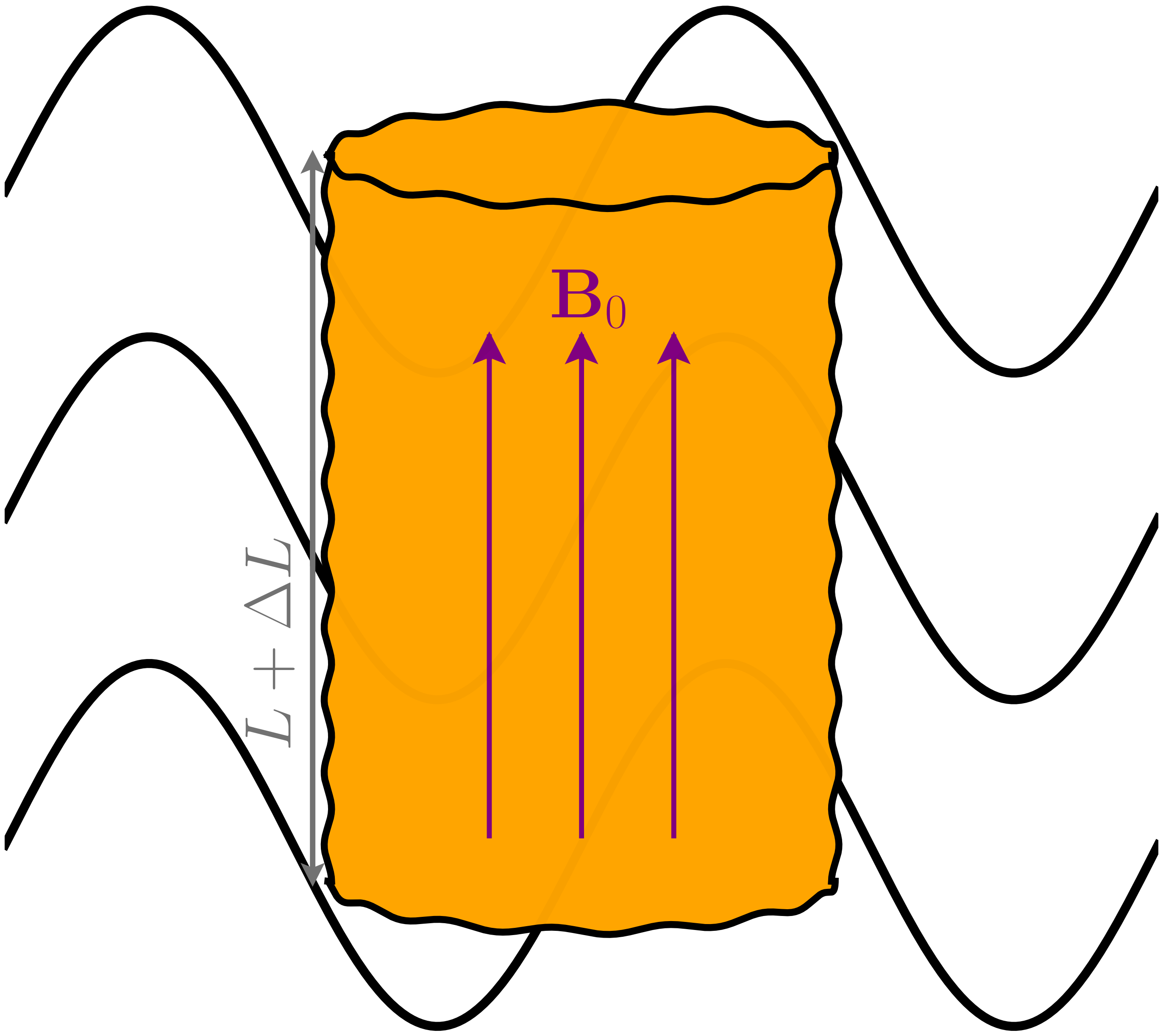
# Mechanical Coupling




Solenoidal Magnet:

$$B_0 = \frac{NI}{L}$$

# Mechanical Coupling




Solenoidal Magnet:

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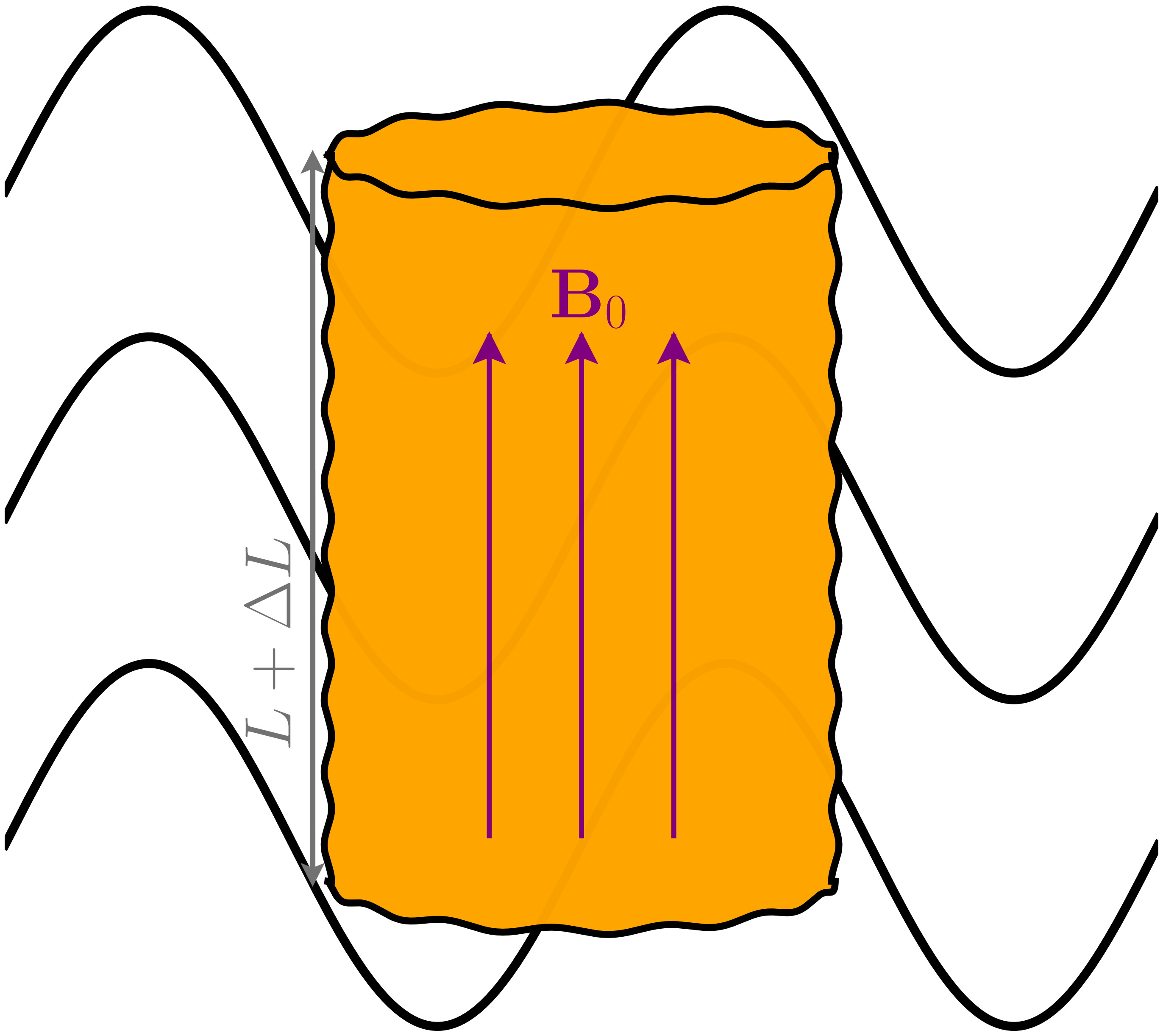
GW oscillates the magnet length

$$L \to L + hL$$

# Mechanical Coupling

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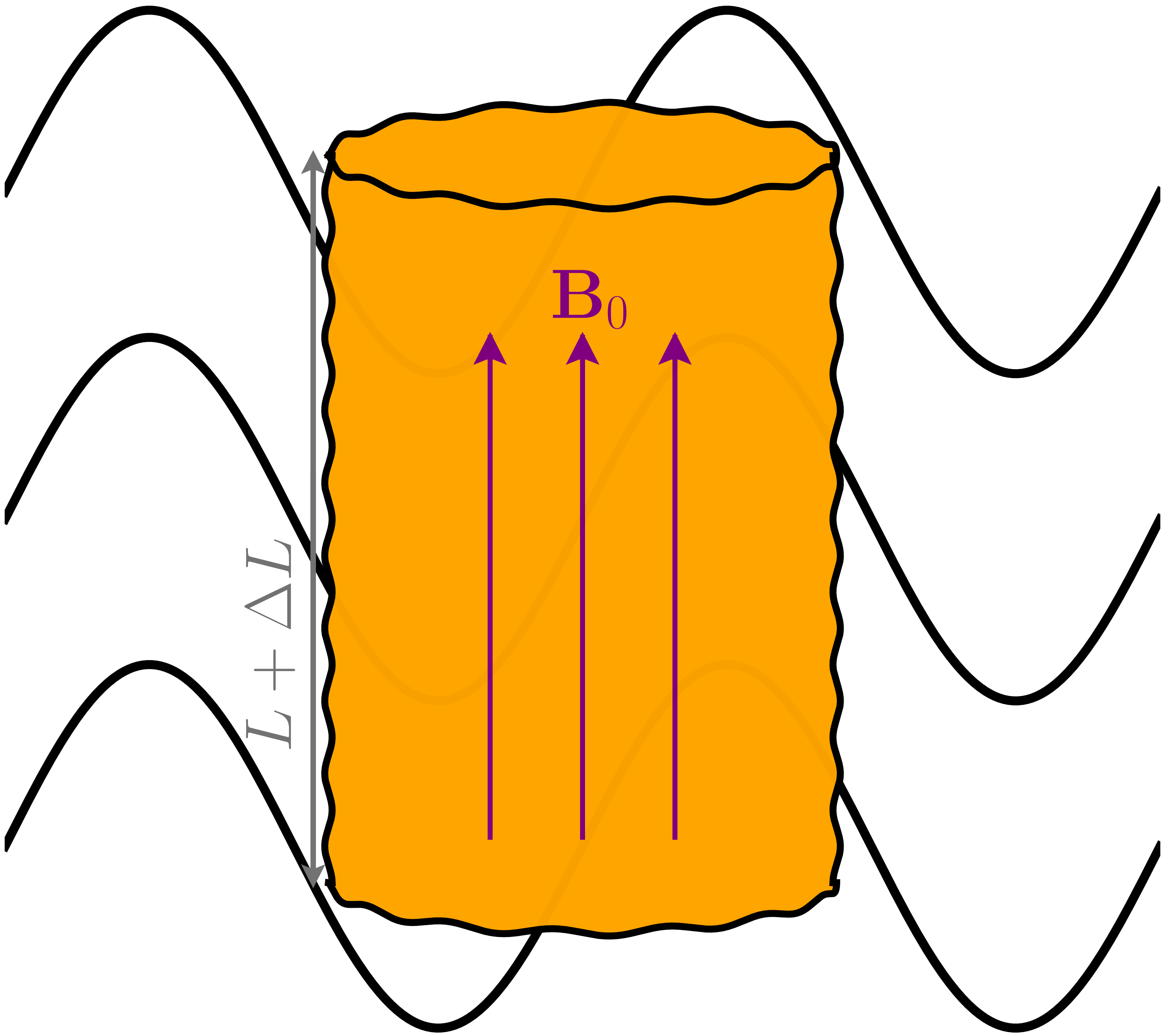
$$B_h^{\rm mech} \sim hB_0 \gg B_h^{\rm EM}$$

# Mechanical Coupling

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GW oscillates the magnet length

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Generates AC magnetic field

$$B_h^{\rm mech} \sim hB_0 \gg B_h^{\rm EM}$$

**A magnet is a Weber bar!**

# Where is the $\omega^2$

**PDF:** GW generates a force density

$$[\mathbf{f}_g]_i = \frac{1}{2}\rho \ddot{h}^{\rm TT}_{ij} x^j$$

$\ddot{h}^{\rm TT} \sim \omega^2 h$

# Where is the $\omega^2$

**PDF:** GW generates a force density

$$[\mathbf{f}_g]_i = \frac{1}{2}\rho \ddot{h}^{\mathrm{TT}}_{ij} x^j$$

**Key:** GW is coupling to mechanical not EM modes

$$B_h \sim \frac{\omega^2 h}{\omega^2 - \omega^2_{\mathrm{Mech}} + i(\omega\omega_{\mathrm{Mech}}/Q_{\mathrm{Mech}})^2} \sim h$$

$\omega \gg \omega_{\mathrm{Mech}} \sim 5\ \mathrm{kHz} \sim c_s \omega_{\mathrm{EM}}$

# Estimated Sensitivity

GW Flux: $\Phi \sim hB_0\pi R_p^2$

Couple to a SQUID with coupling $\kappa \sim 10^{-2}$

SQUID noise: $\Phi_{\mathrm{SQ}} \sim 10^{-21}\,\mathrm{Wb}/\sqrt{\mathrm{Hz}}$

# Estimated Sensitivity

GW Flux: $\Phi \sim hB_0\pi R_p^2$

Couple to a SQUID with coupling $\kappa \sim 10^{-2}$

SQUID noise: $\Phi_{\rm SQ} \sim 10^{-21}\,{\rm Wb}/\sqrt{\rm Hz}$

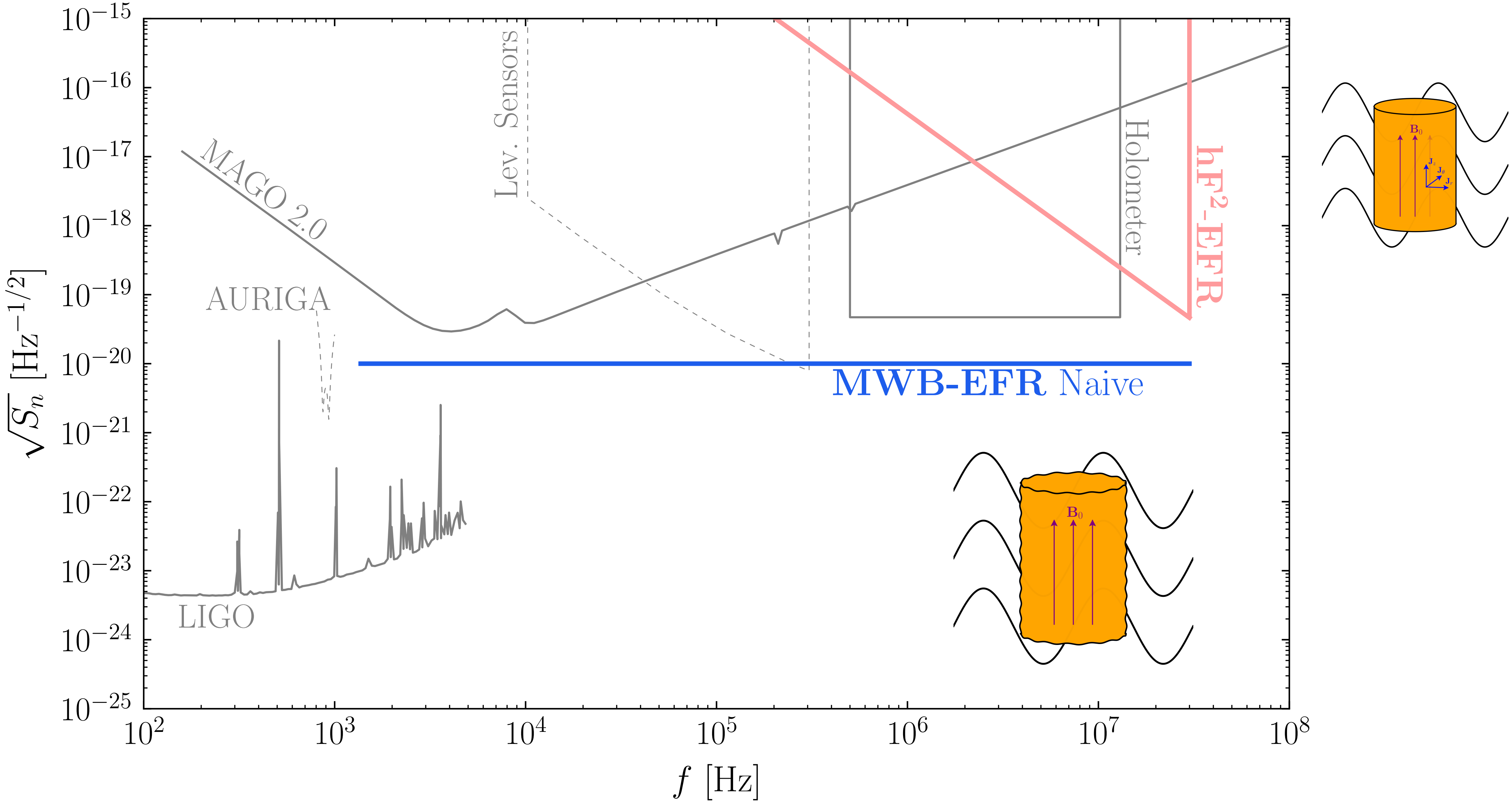
Approximate noise spectral density

$$\sqrt{S_n} \sim \Phi_{\rm SQ}/(\kappa B_0 \pi R_p^2) \sim 10^{-20}/\sqrt{\rm Hz}$$

Assume $B_0 \sim 10\,{\rm T}$, $R_p \sim 0.4\,{\rm m}$

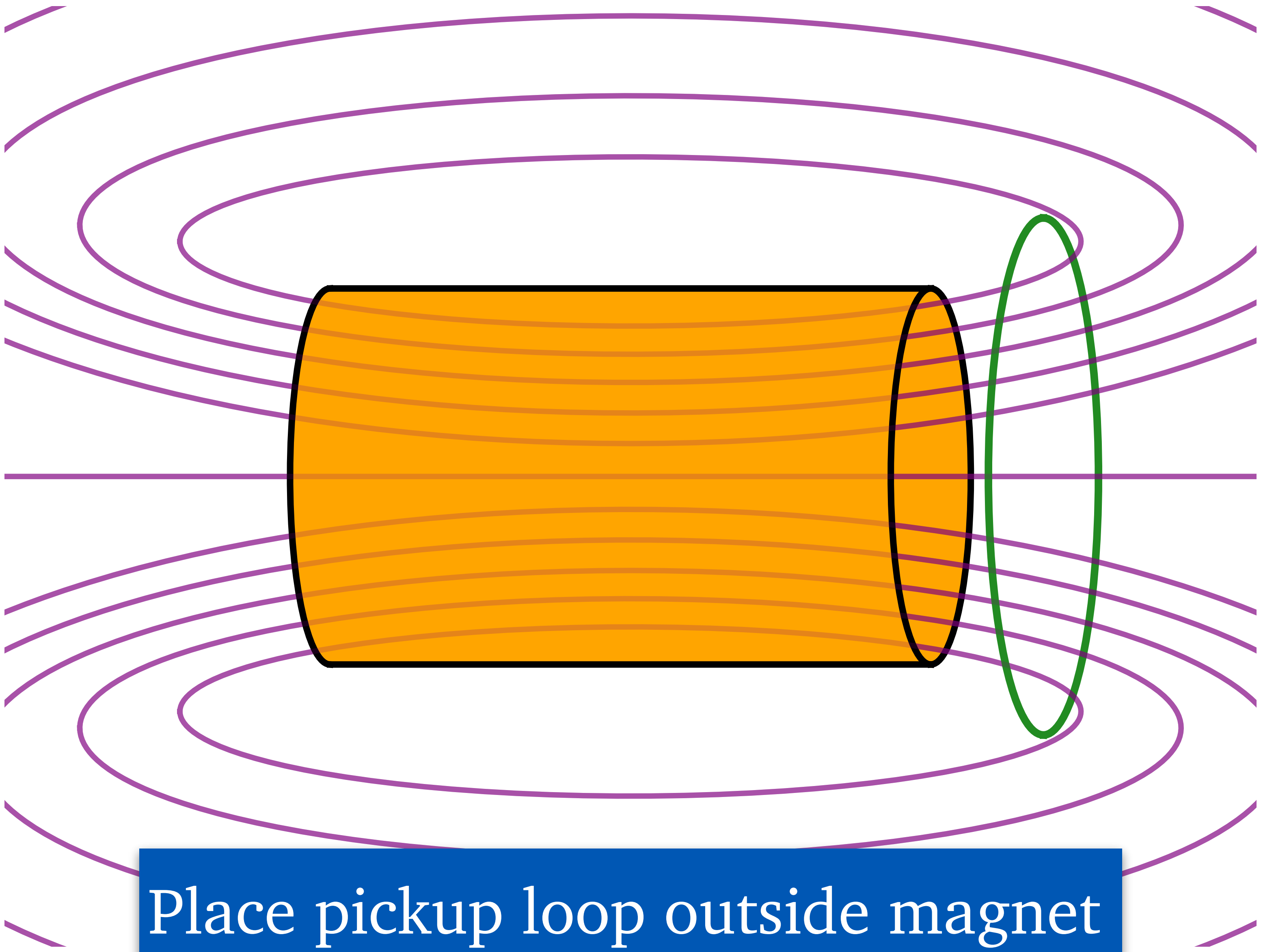
# Estimated Sensitivity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



# Detailed Sensitivity

Consider sensitivity in three regimes



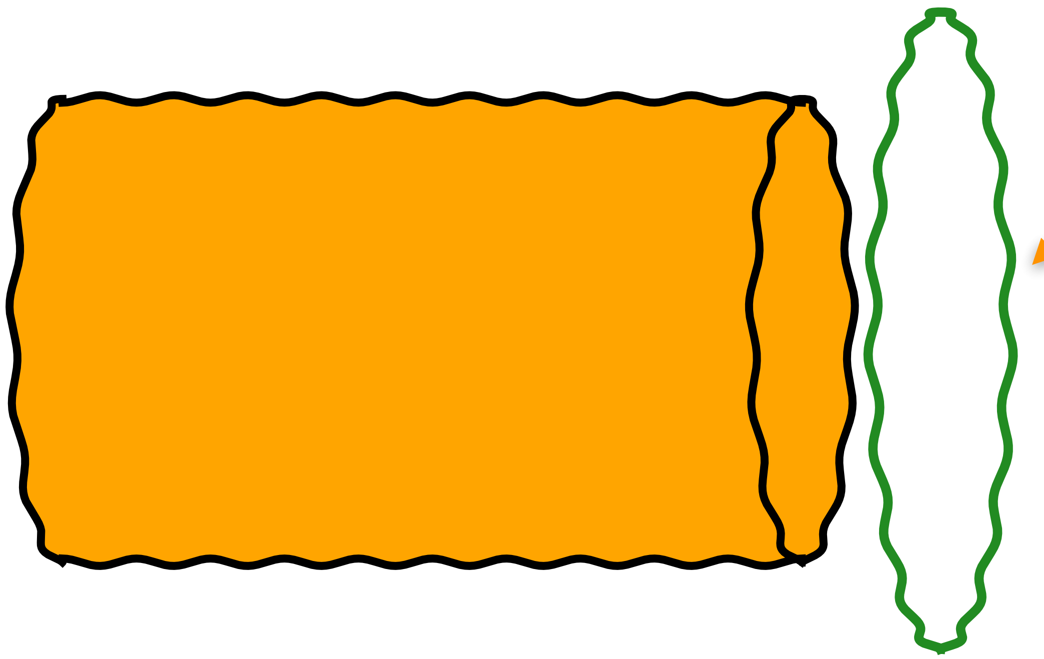
Place pickup loop outside magnet

- Run in parallel to axion experiments
- Loop can be larger than magnet bore

# Detailed Sensitivity

Regime 1: $\omega_{\text{mech}} \ll \omega \ll \omega_{\text{EM}}$

Treat magnet and loop as freely falling



Loop fluctuations enhanced when in a region of large $B_0$ gradient

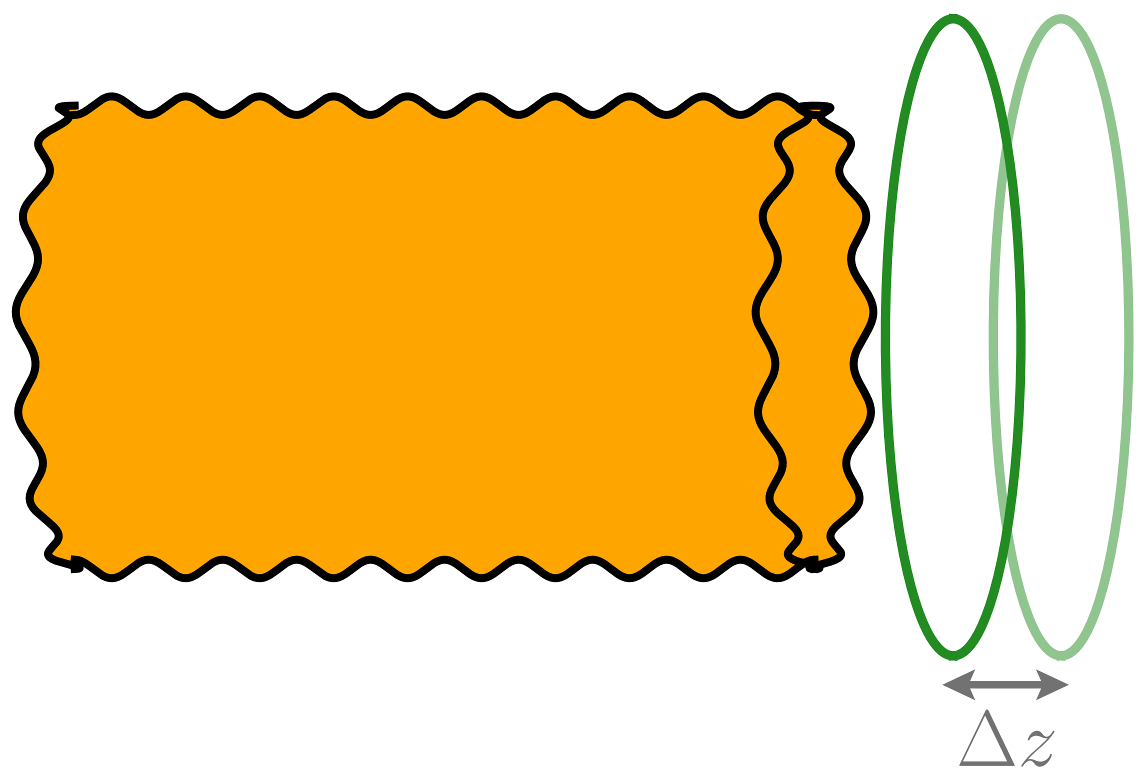
$$S_n \simeq \frac{8L_p}{LB_0^2A_p^2}S_{\text{SQ}}$$

Valid up to $\mathcal{O}(1)$ factor due to exact loop position and GW incident direction ($L$, $L_p$ SQUID, pickup inductances)

# Detailed Sensitivity

Regime 2: $\omega_{\rm mech} \sim \omega$

**Resonantly excite the magnet**



# Detailed Sensitivity

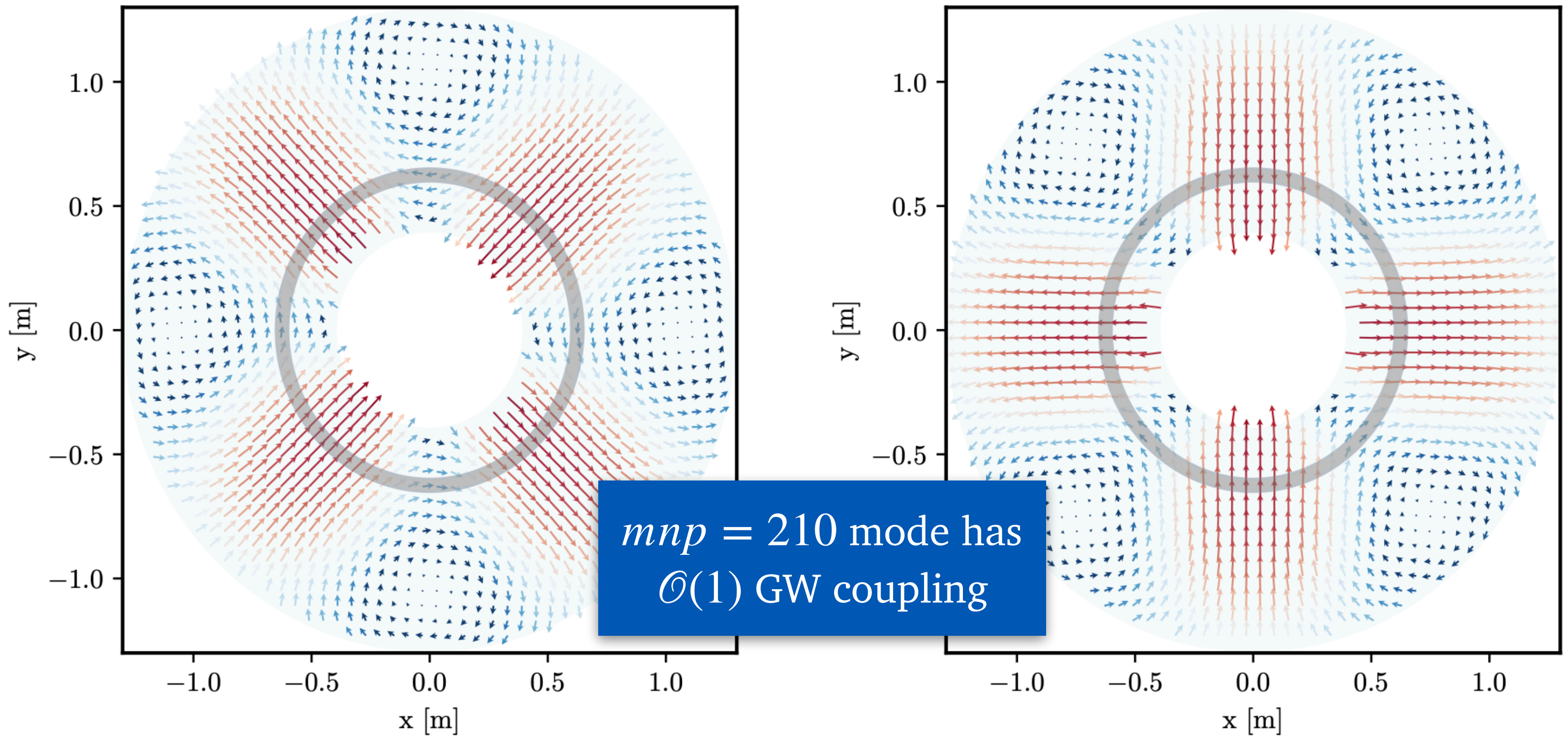
Regime 2: $\omega_{\text{mech}} \sim \omega$

Resonantly excite the magnet

Compute the mechanical modes of the magnet with Navier-Cauchy eq.

$$\rho \ddot{\mathbf{U}} = (\lambda + \mu)\nabla(\nabla \cdot \mathbf{U}) + \mu \nabla^2 \mathbf{U} + \mathbf{f}_g$$

**U**: displacement field



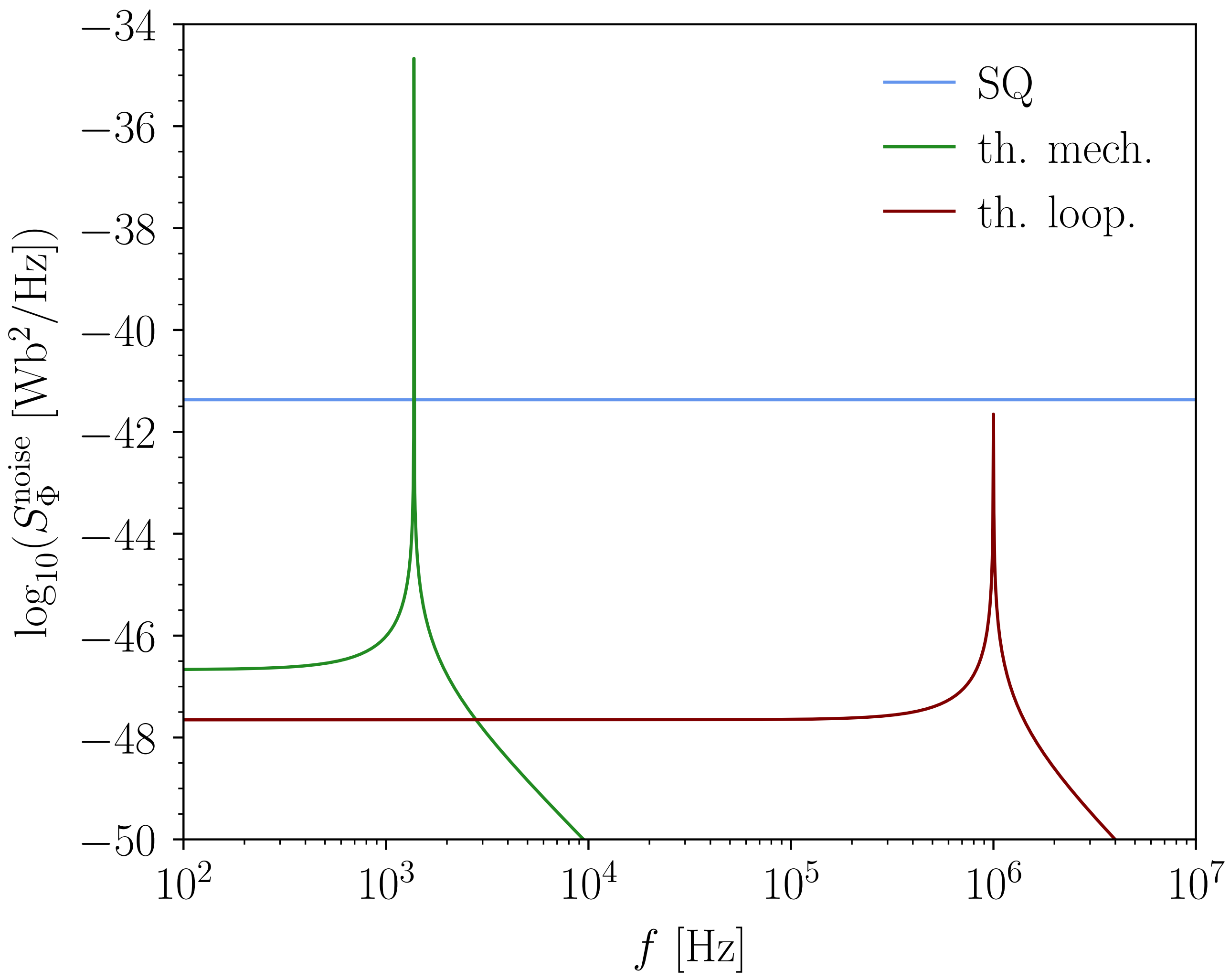

$mnp = 210$ mode has $\mathcal{O}(1)$ GW coupling

# Detailed Sensitivity

Regime 2: $\omega_{\text{mech}} \sim \omega$

**Resonantly excite the magnet**

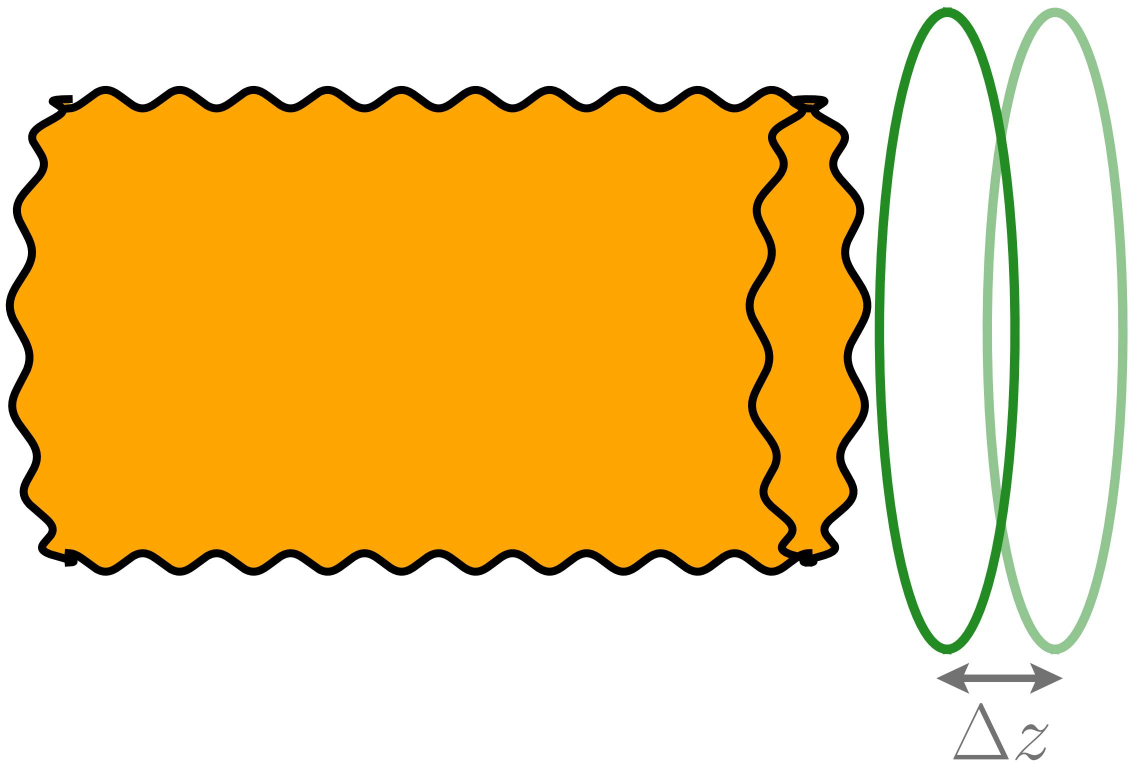
Thermal vibrations of the magnet also resonantly enhanced



# Detailed Sensitivity

Regime 2: $\omega_{\rm mech} \sim \omega$

**Resonantly excite the magnet**



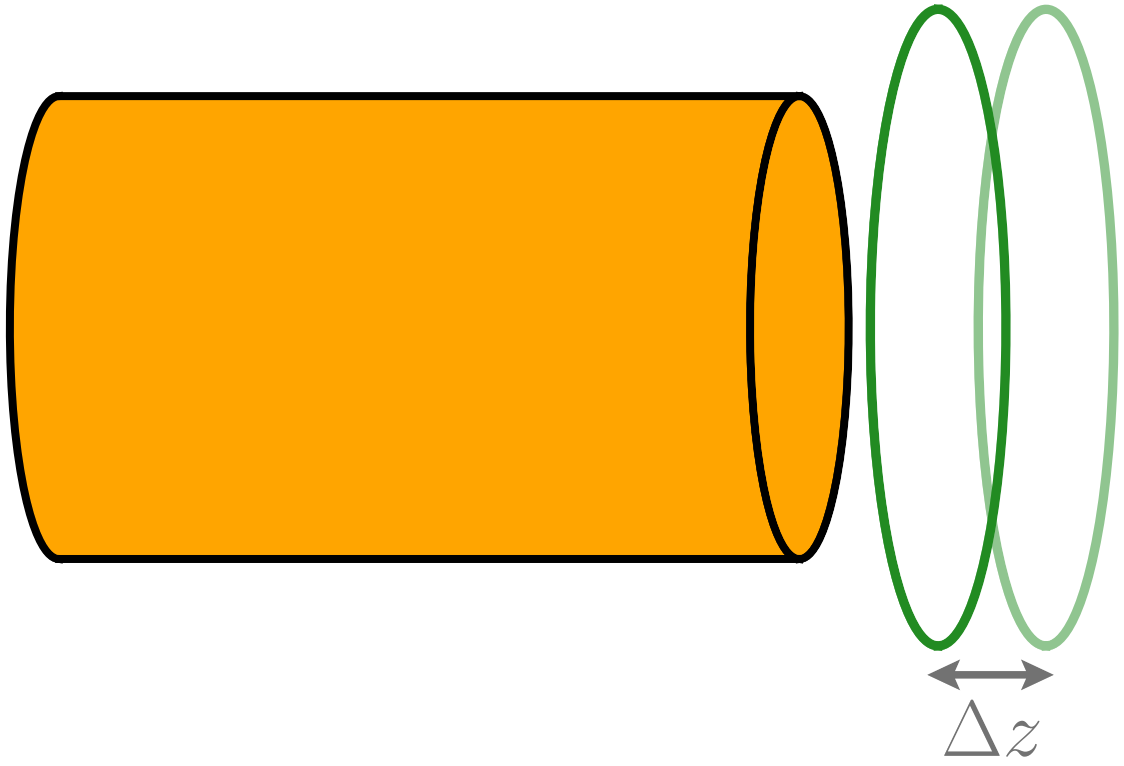

$$S_n \simeq \frac{4T}{M(\eta_{210}L)^2\omega_{\rm mech}^3 Q_{\rm mech}}$$

Sig. & bkg. enhanced by $B_0$, dependence vanishes ($\eta$ overlap factor)

# Detailed Sensitivity

Regime 3: $\omega_{\text{mech}} \gg \omega$

**Magnet and loop rigid & freely falling**

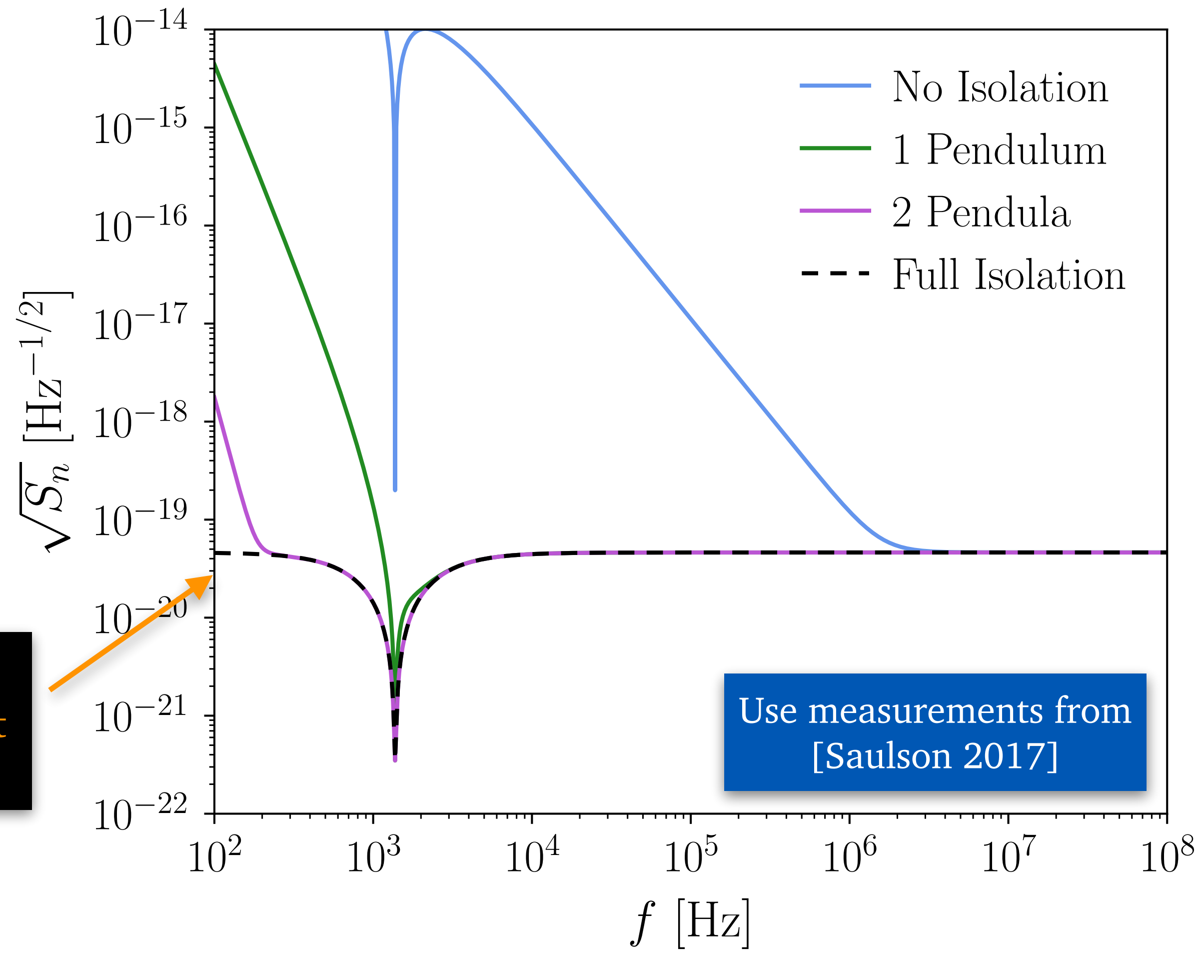


# Detailed Sensitivity

Regime 3: $\omega_{\text{mech}} \gg \omega$

**Magnet and loop rigid & freely falling**

Sensitivity as in regime 1 up to seismic noise

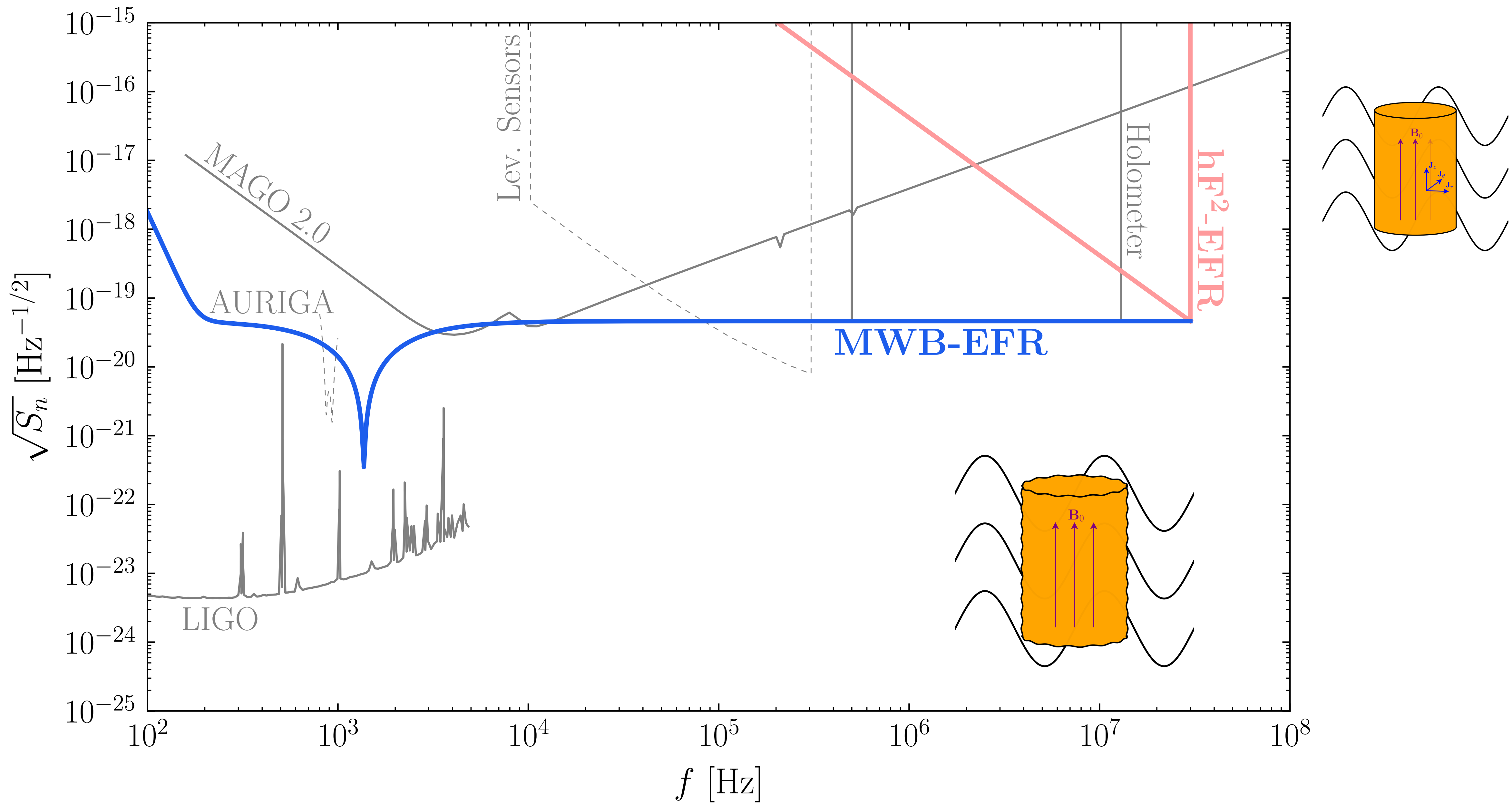



Can't win forever, eventually gravity gradient noise [Saulson 1984]

Suggests also important for e.g. DMRadio

# Detailed Sensitivity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



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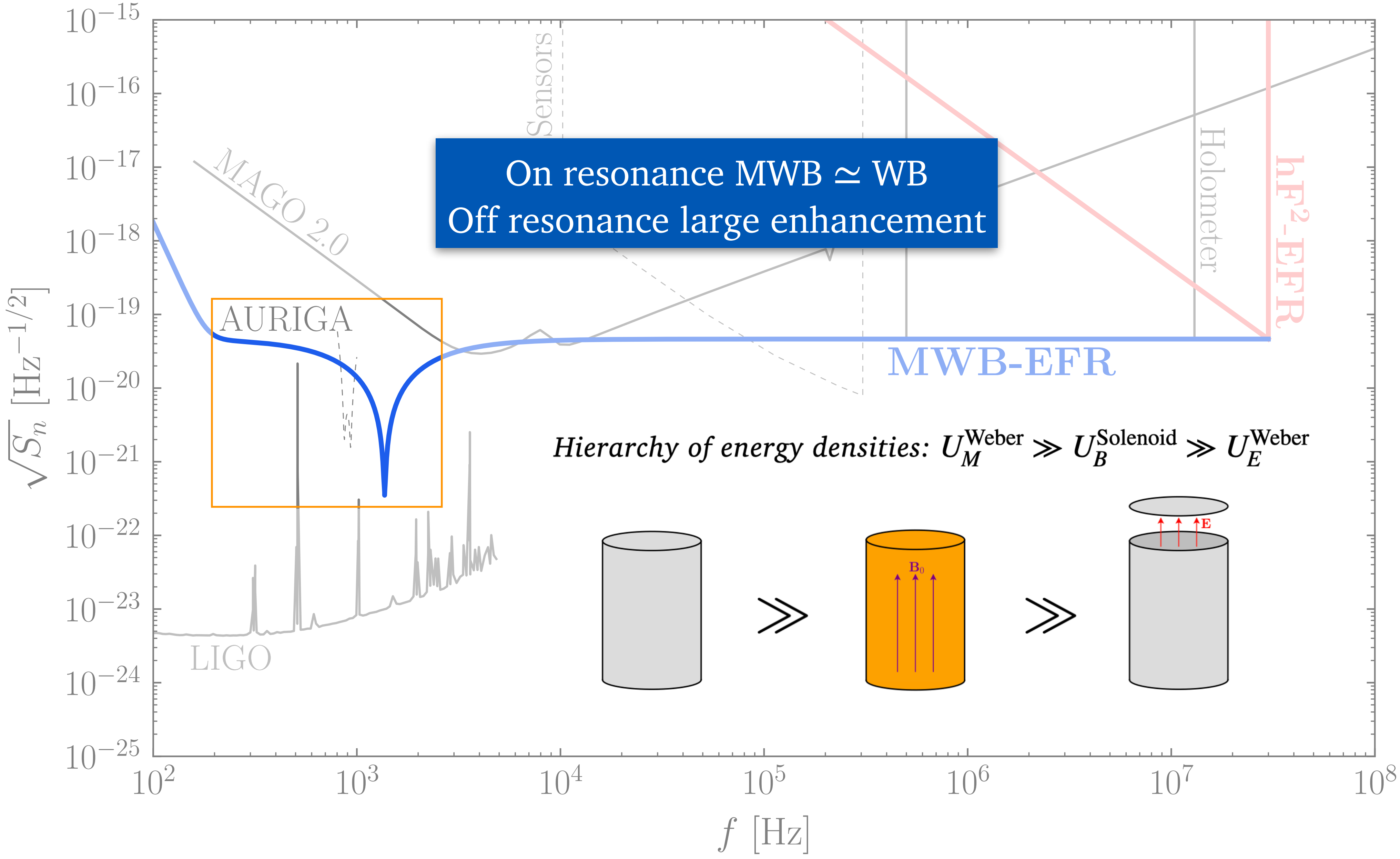

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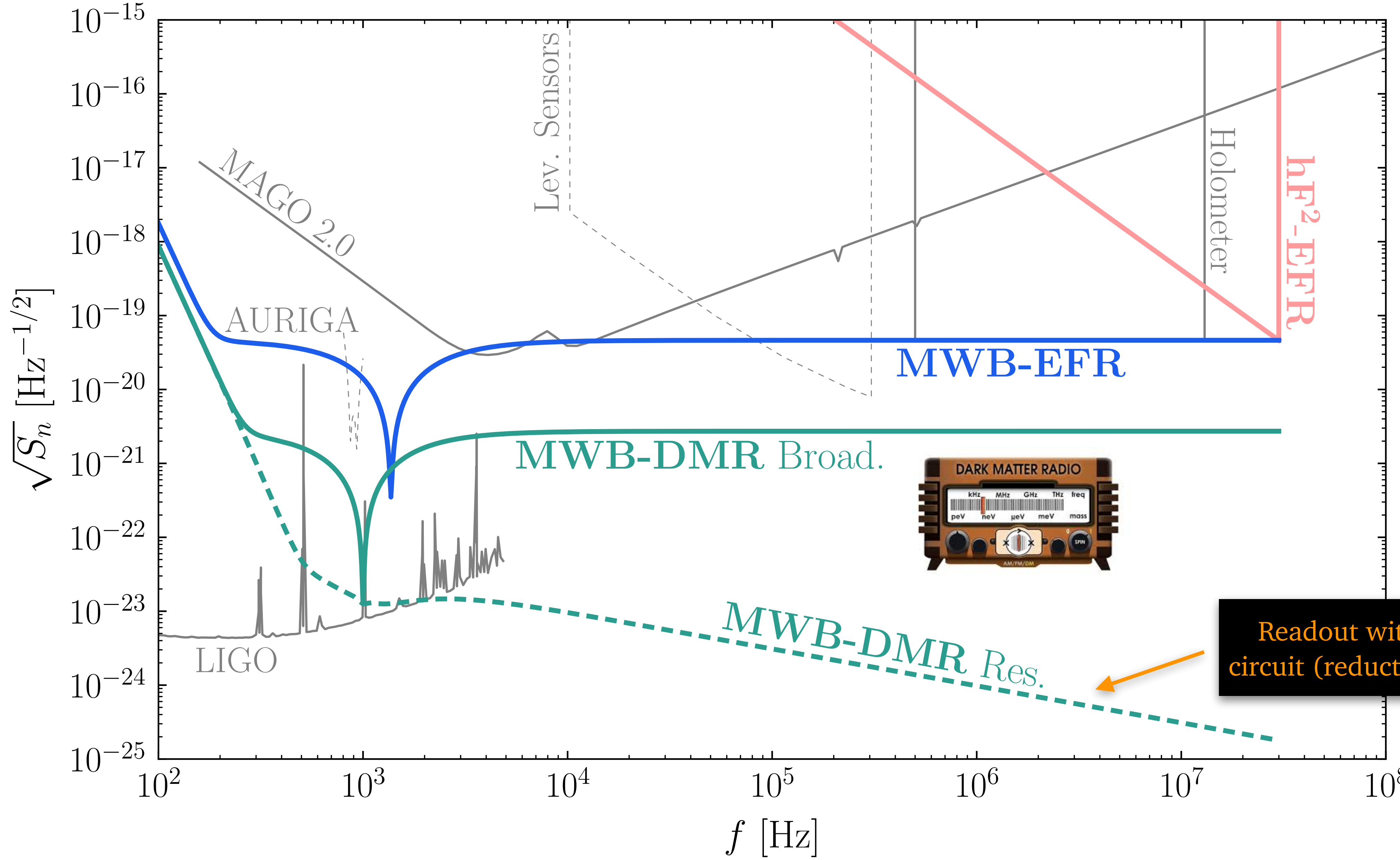
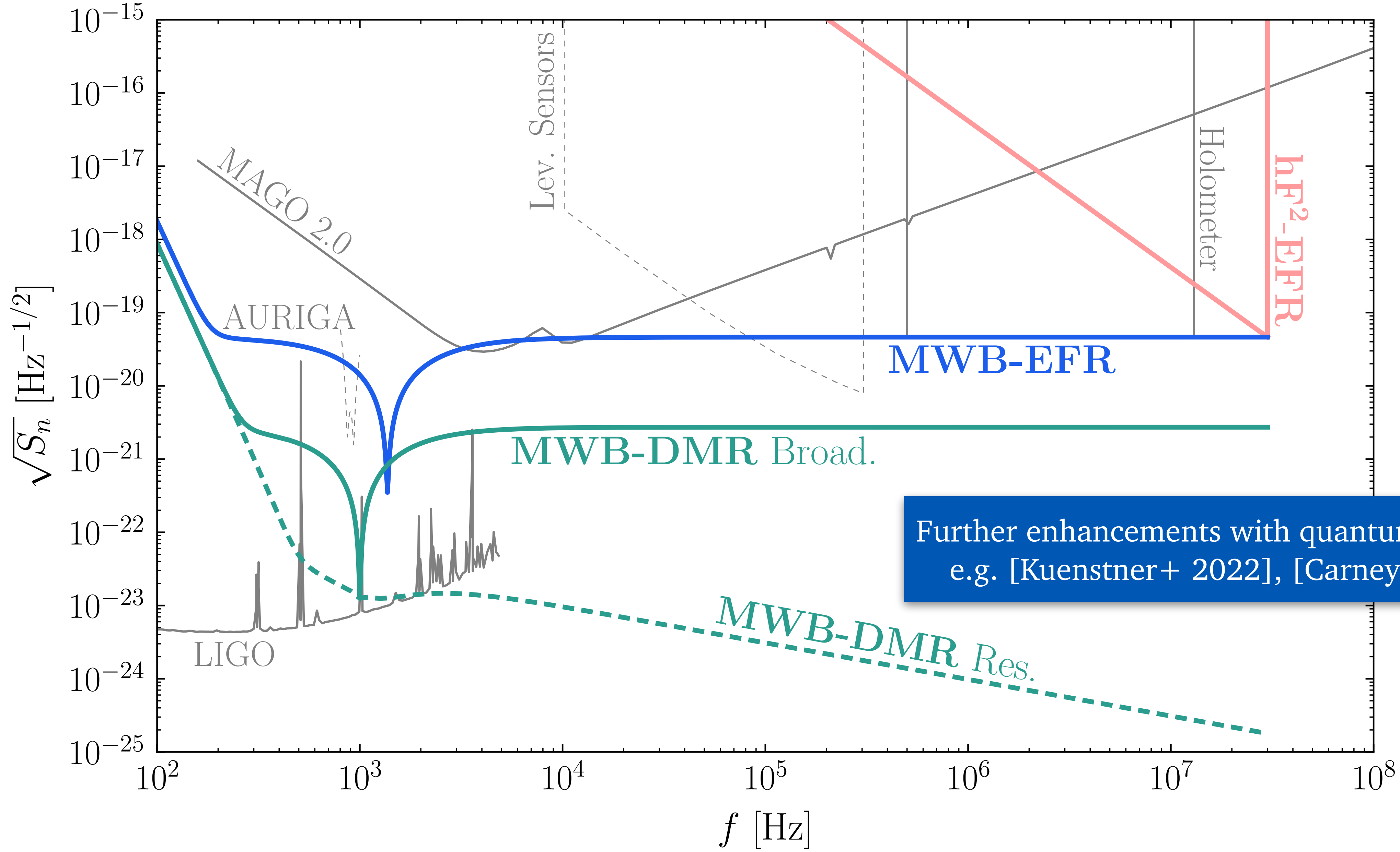



Readout with resonant RLC circuit (reduction in bandwidth)

**EFR:** $B_0 = 10\,\mathrm{T}, T = 4\,\mathrm{K}, R_p = 0.4\,\mathrm{m}, Q_{\mathrm{mech}} = 10^6$
**DMR:** $B_0 = 16\,\mathrm{T}, T = 0.02\,\mathrm{K}, R_p = 3\,\mathrm{m}, Q_{\mathrm{mech}} = 10^7$

# Detailed Sensitivity

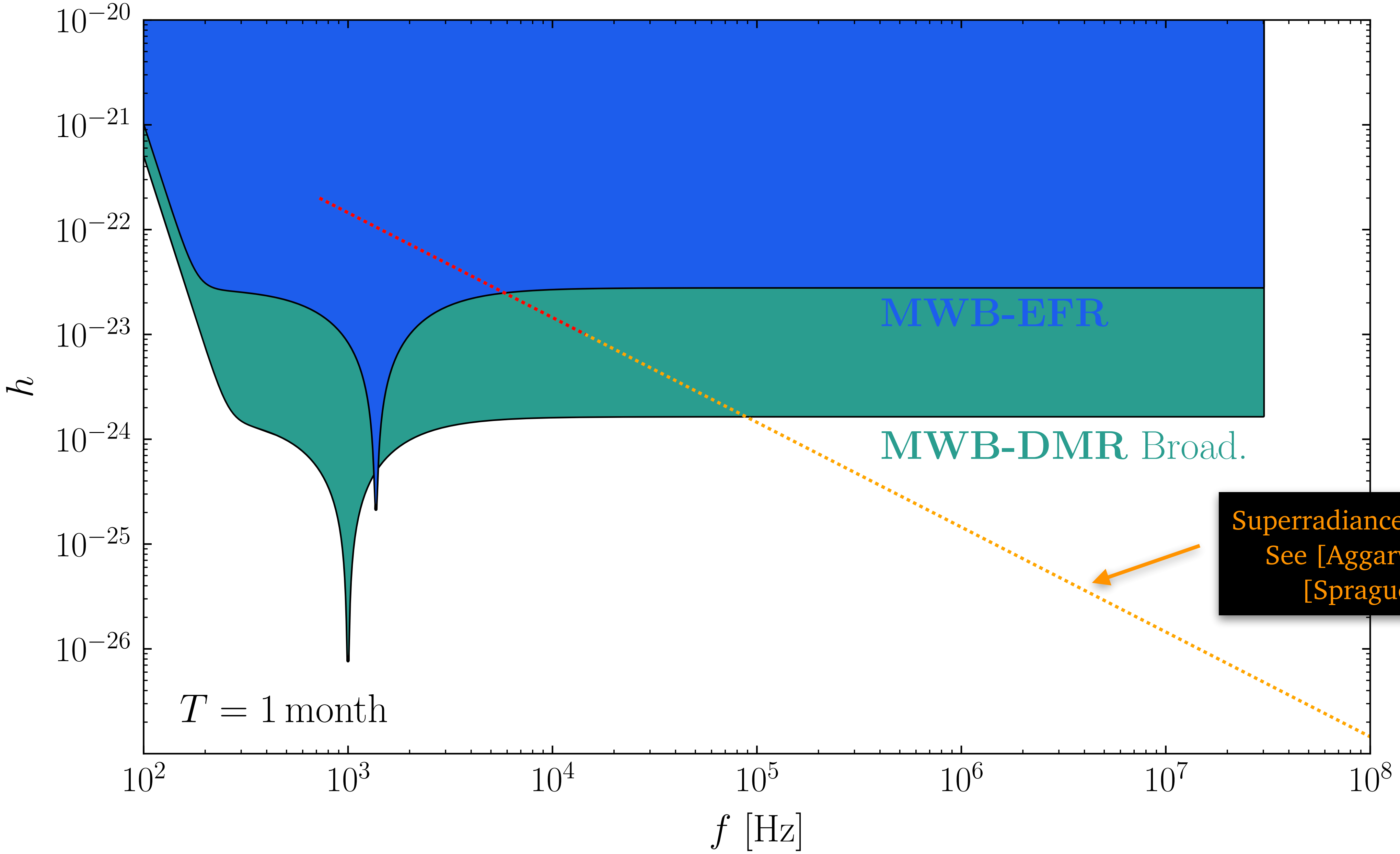
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$




Further enhancements with quantum readout?
e.g. [Kuenstner+ 2022], [Carney+ 2024]

# Detailed Sensitivity

Reach for persistent monochromatic signal $h \sim \sqrt{S_n/T}$

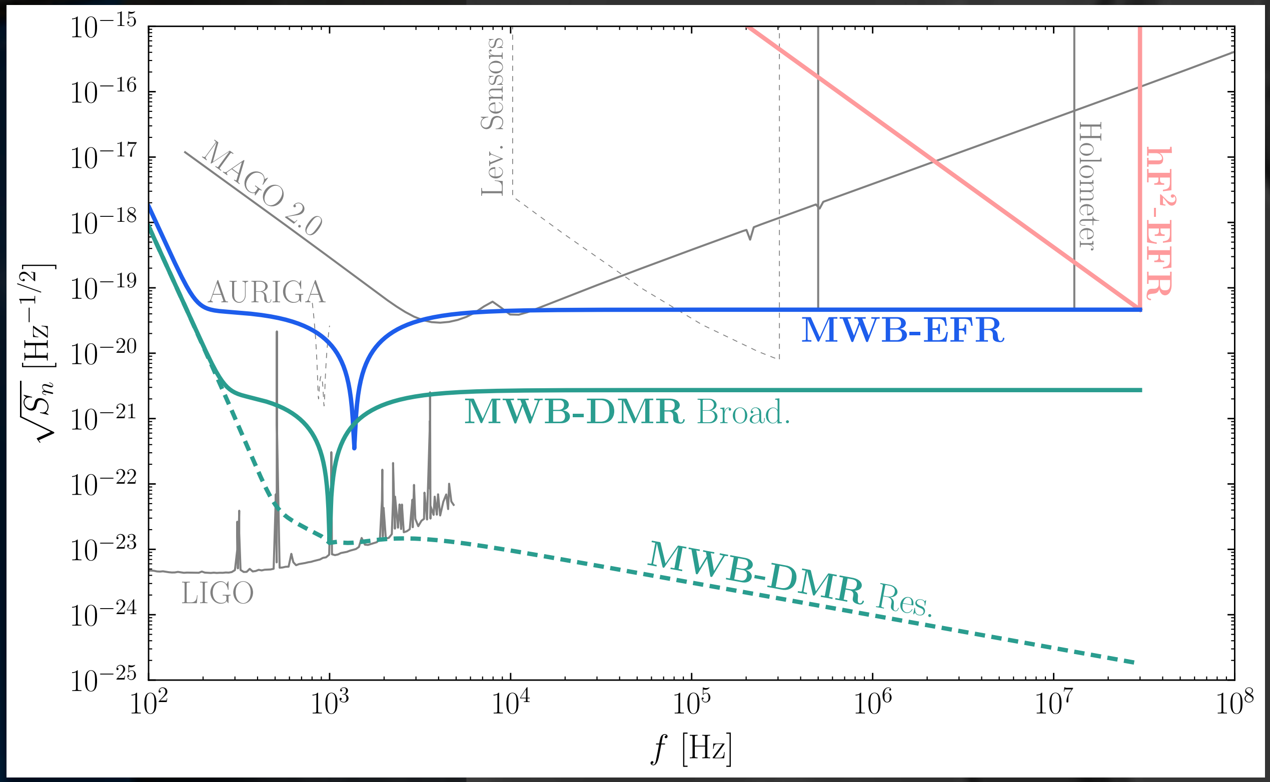



Superradiance for a 1 kpc BH
See [Aggarwal+ 2022],
[Sprague+ 2024]

Currently studying the reach to various signals w/ [Domcke, Ellis, Ning, Schutte-Engel]

# Conclusion

Magnets can have leading sensitivity to high-frequency gravitational waves




**[2408.01483 Domcke, Ellis, NLR]**

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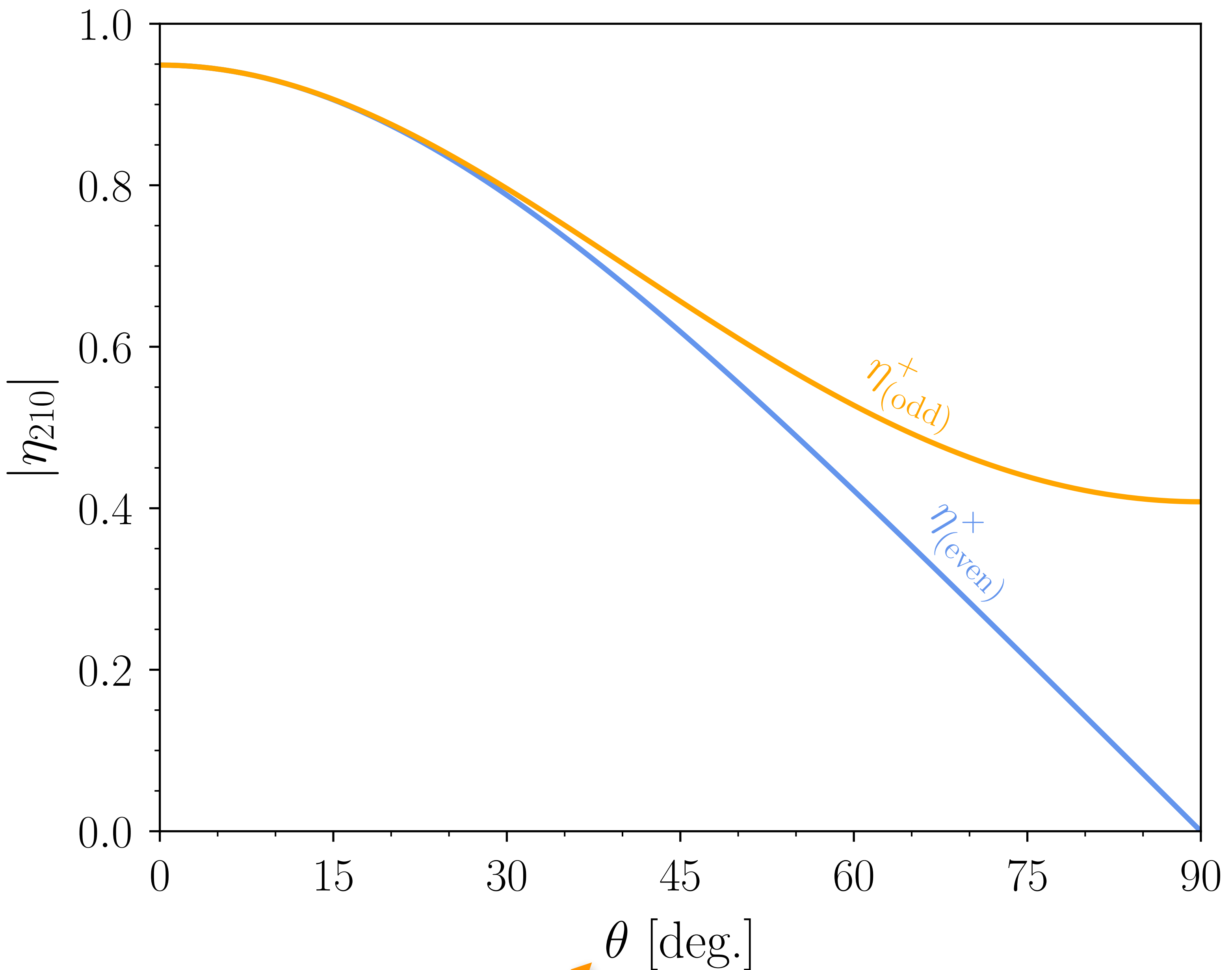
# Backup Slides

# Overlap Factor




Angle from the z-axis

# Proper Detector Frame

TT gauge: GW is a plane wave $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$h_{00} = \omega^2 F(\mathbf{k}\cdot\mathbf{r})\,\mathbf{b}\cdot\mathbf{r}, \qquad b_j \equiv r_i h_{ij}^{\mathrm{TT}}\Big|_{\mathbf{r}=0},$$

$$h_{0i} = \frac{1}{2}\omega^2\left[F(\mathbf{k}\cdot\mathbf{r}) - iF'(\mathbf{k}\cdot\mathbf{r})\right]\left(\hat{\mathbf{k}}\cdot\mathbf{r}\;b_i - \mathbf{b}\cdot\mathbf{r}\;\hat{k}_i\right),$$

$$h_{ij} = -i\omega^2 F'(\mathbf{k}\cdot\mathbf{r})\left(|\mathbf{r}|^2\,h_{ij}^{\mathrm{TT}}\Big|_{\mathbf{r}=0} + \mathbf{b}\cdot\mathbf{r}\,\delta_{ij} - b_i r_j - b_j r_i\right),$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$$

See [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021], [Domcke, Garcia-Cely, NLR 2022], [Domcke, Garcia-Cely, Lee, NLR 2024]

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TT gauge: GW is a plane wave $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Proper Detector Frame: more involved

$$h_{00} = \omega^2 F(\mathbf{k}\cdot\mathbf{r})\,\mathbf{b}\cdot\mathbf{r} \qquad b_i = x_j h^{\rm TT}_{ij}\big|$$

$$h_{0i} = \frac{1}{2}\omega^2 \left[F(\ldots\right.$$

$$h_{ij} = -i\omega^2 F'(\ldots \; x_i),$$

$$F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2 = -1/2 + \mathcal{O}(\xi)$$

$$g_{\mu\nu}(x) = \underbrace{g_{\mu\nu}(x_0)}_{=\eta_{\mu\nu}} + \underbrace{(x-x_0)^\alpha \partial_\alpha g_{\mu\nu}(x_0)}_{=0\ (\because\ \Gamma^\mu_{\nu\rho}(x_0)=0)} + \underbrace{(x-x_0)^\alpha (x-x_0)^\beta \partial_\alpha \partial_\beta g_{\mu\nu}(x_0)}_{\mathcal{O}(\omega^2 R^2)} + \ldots$$

See [Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel 2021], [Domcke, Garcia-Cely, NLR 2022], [Domcke, Garcia-Cely, Lee, NLR 2024]

# Proper Detector Frame

Use Fermi normal coordinates

Locally inertial coordinates along a geodesic **[Fermi 1922]**

$$h_{ij} = -2 \sum_{n=0}^{\infty} \frac{n+1}{(n+3)!} \hat{R}_{ikjl,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n},$$

$$h_{0i} = -2 \sum_{n=0}^{\infty} \frac{n+2}{(n+3)!} \hat{R}_{0kil,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n},$$

$\hat{R}$ is evaluated at the coordinate origin

$$h_{00} = -2 \sum_{n=0}^{\infty} \frac{n+3}{(n+3)!} \hat{R}_{0k0l,m_1 \ldots m_n} r_k r_l r_{m_1} \ldots r_{m_n}$$

[Fortini and Gualdi 1982], [Marzlin 1994], [Rakhmanov 2014]

# Proper Detector Frame

Proper detector frame:
Fermi normal coordinates transformed to the non-inertial reference frame of the detector

[Ni, Zimmermann 1978]

Non-inertial corrections (Earth's gravity, Coriolis effect, etc) are irrelevant at higher frequencies - effectively can just use Fermi normal coordinates