



# Cosmology with binary black holes

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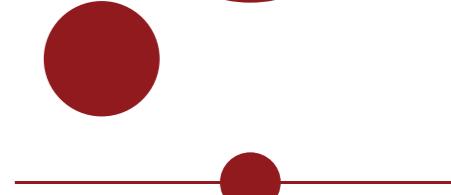
[jose.ezquiaga@nbi.ku.dk](mailto:jose.ezquiaga@nbi.ku.dk)

[ezquiaga.github.io](https://ezquiaga.github.io)

VILLUM FONDEN

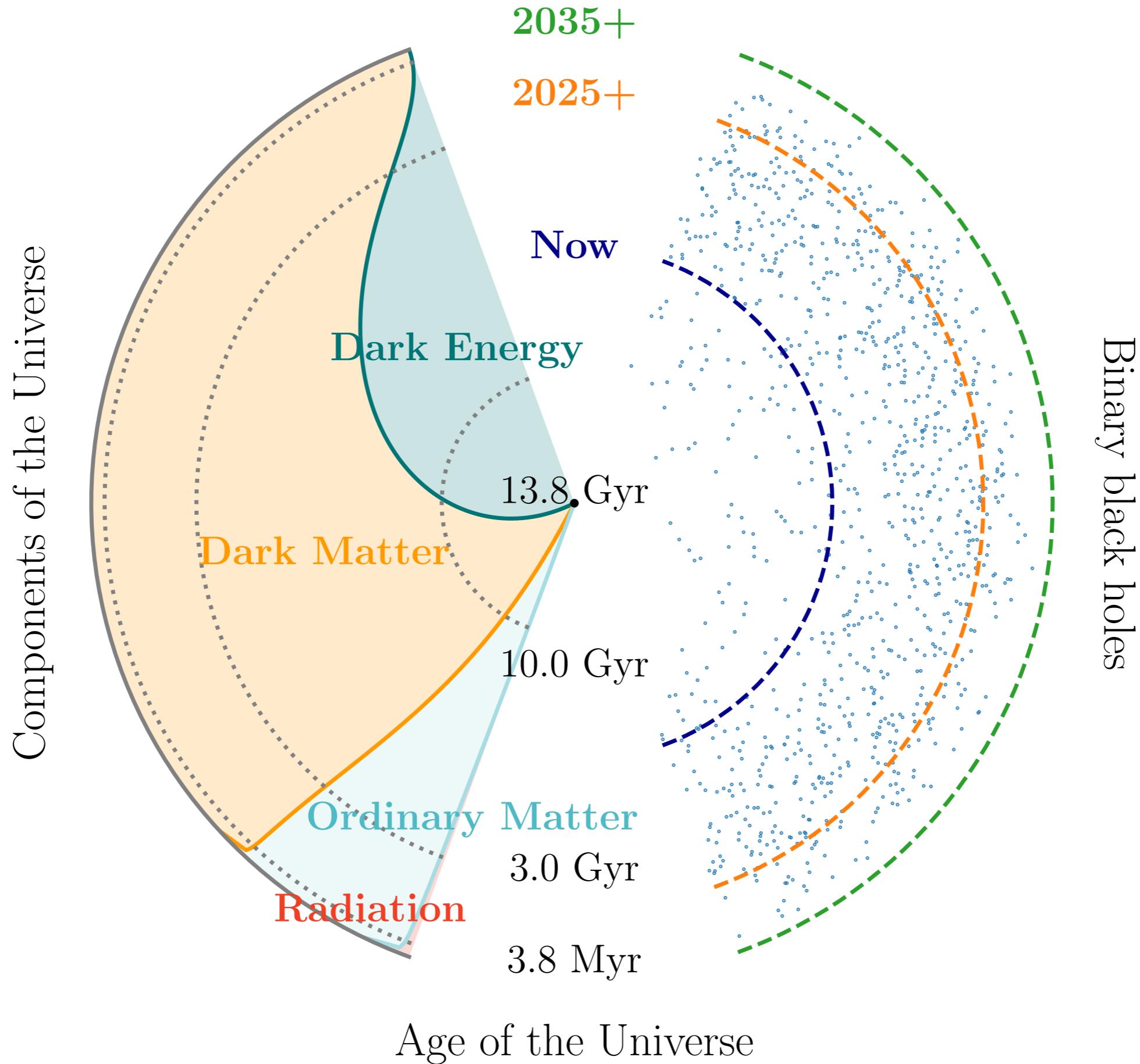


[Pompeii]



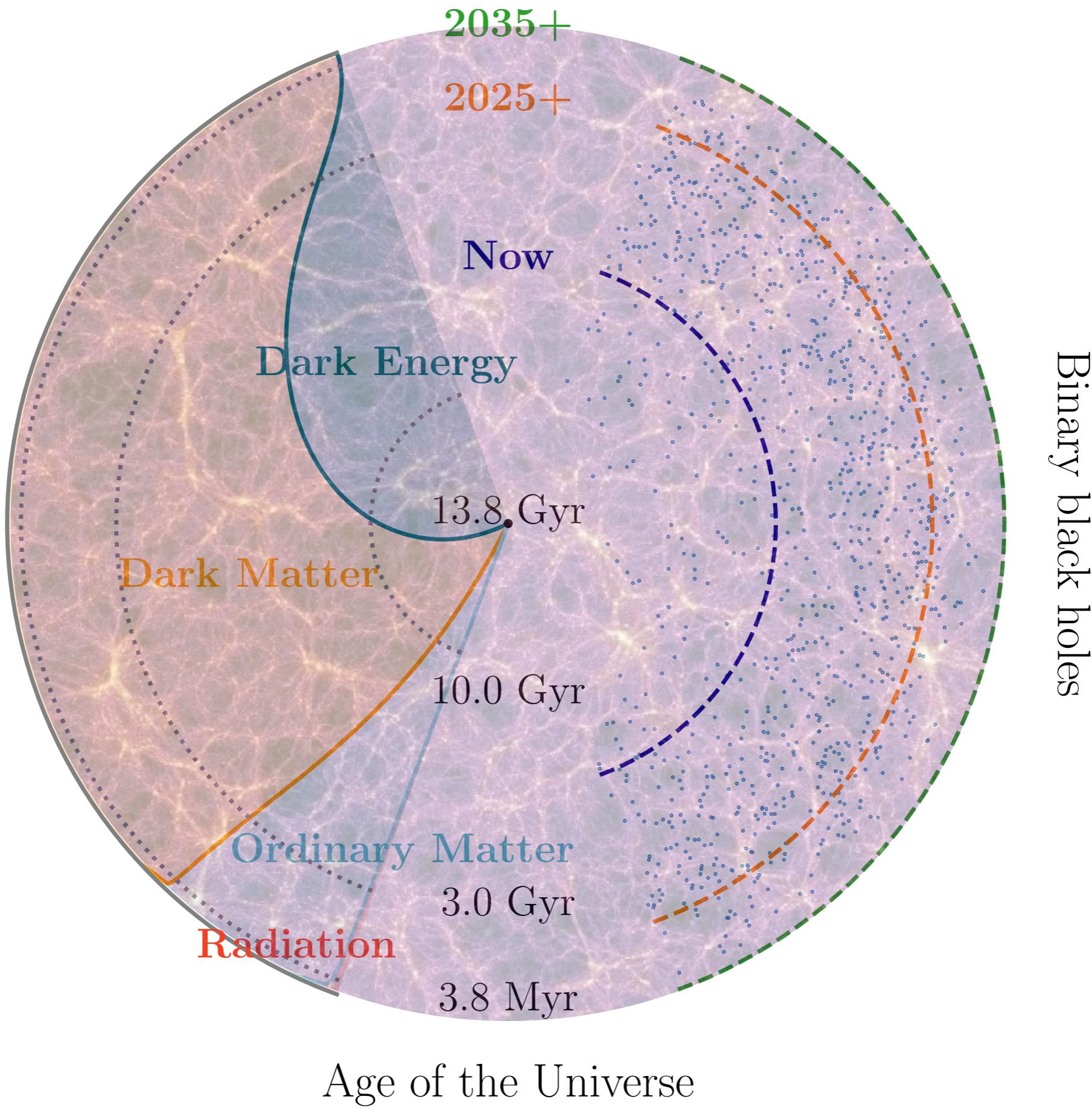
KØBENHAVNS  
UNIVERSITET

# Gravitational Wave horizons



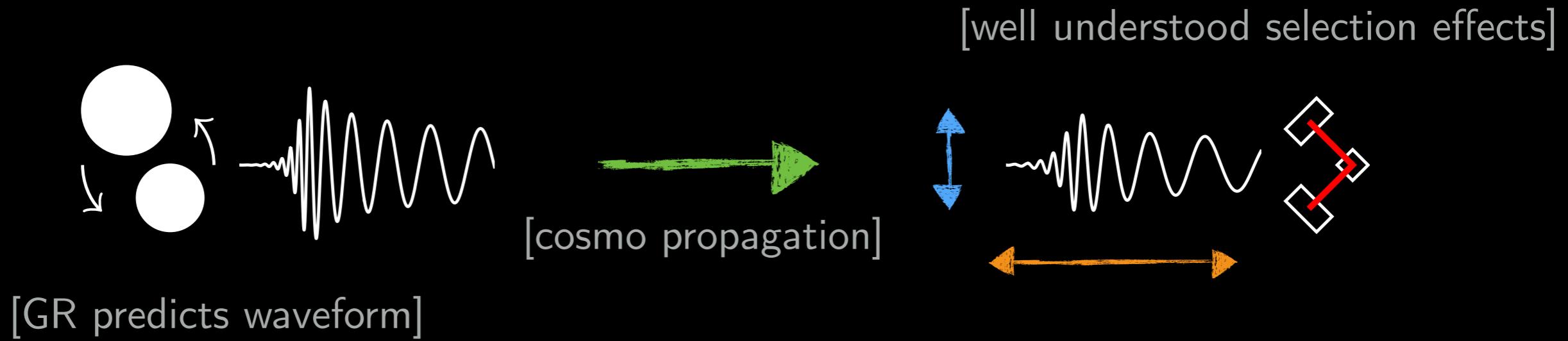
# Gravitational Wave horizons

Components of the Universe



Age of the Universe

# GWs are standard sirens

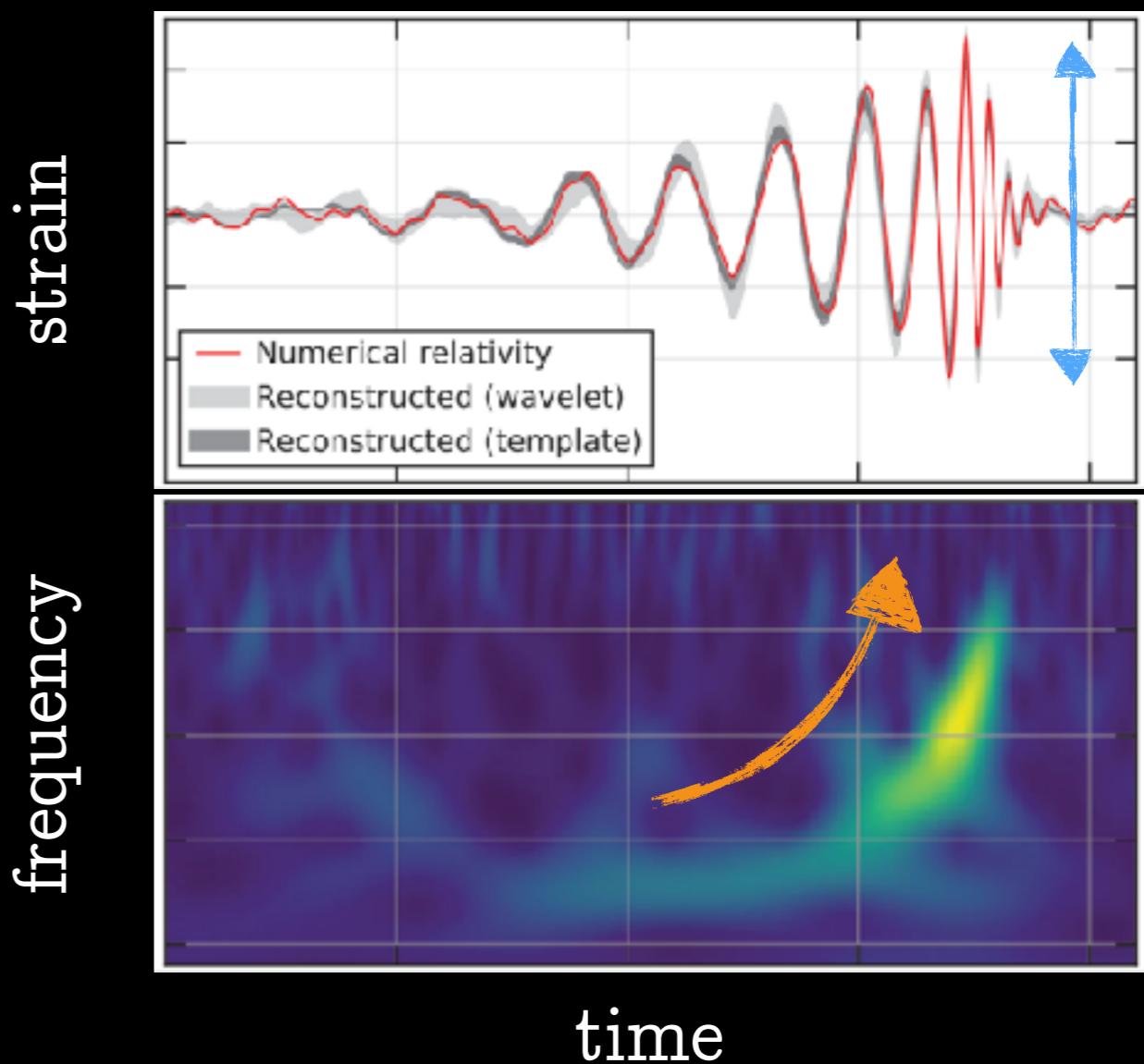


$$d_L(z)$$

[GW Hubble diagram]

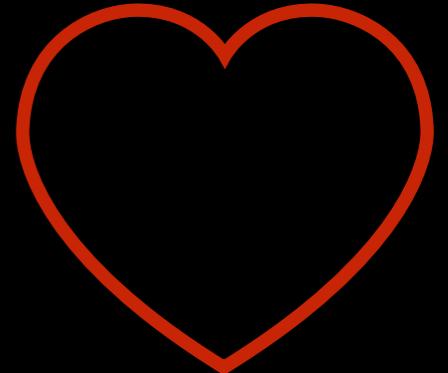
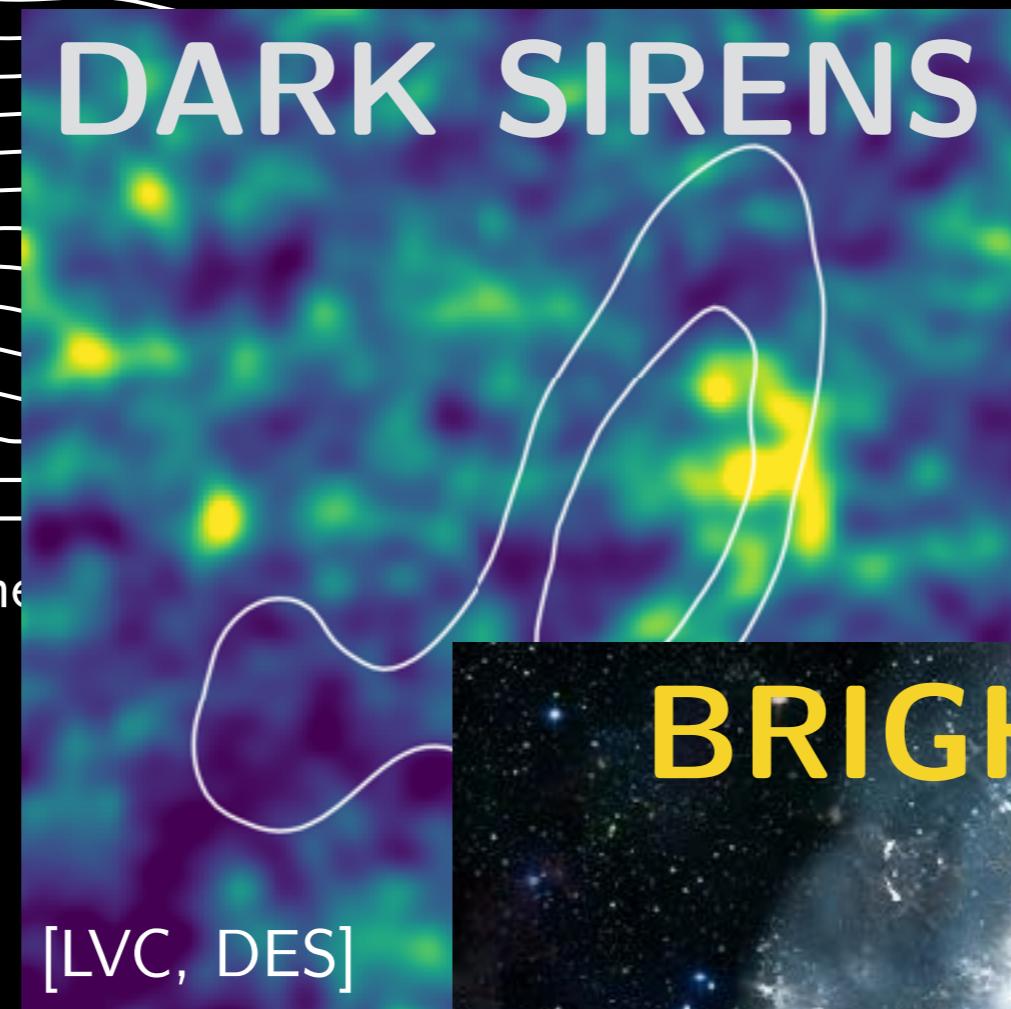
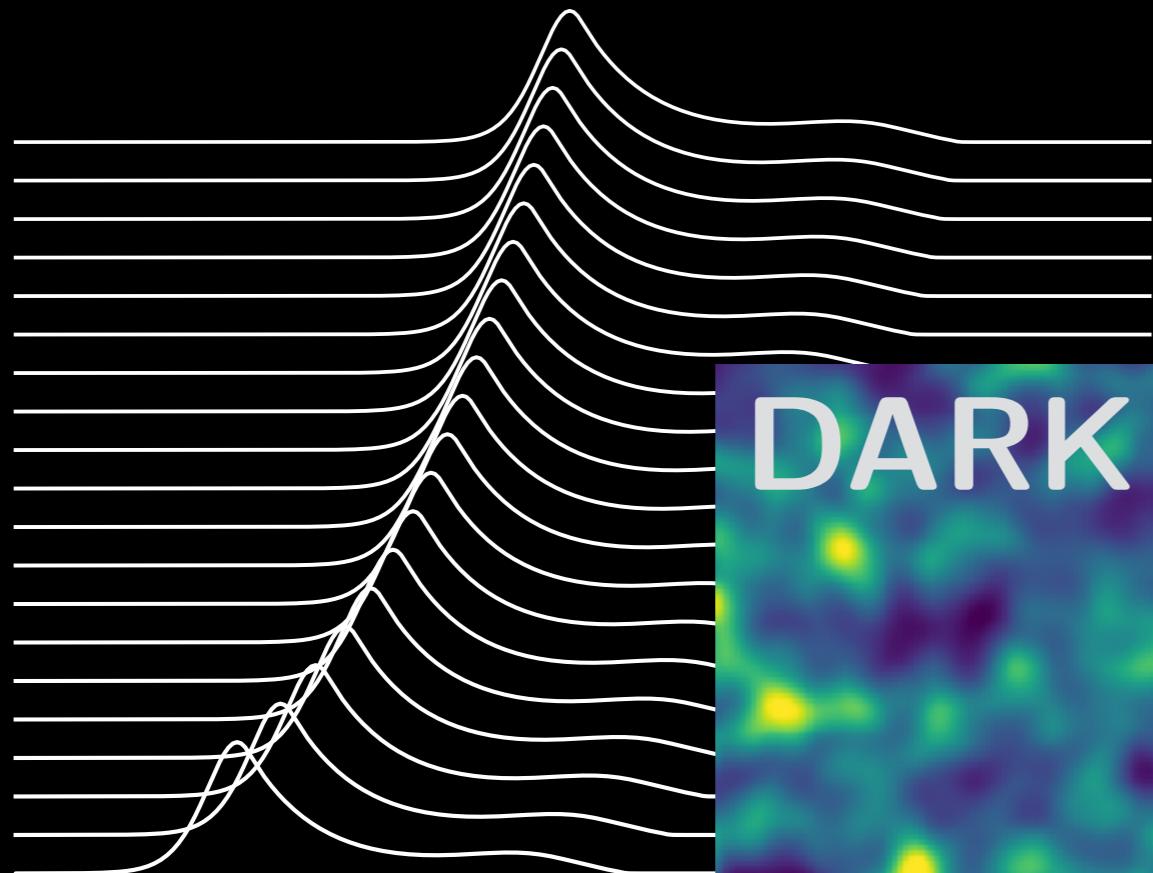
$$m_{\text{det}} = (1 + z)m$$

[Interplay with astrophysics]



# SPECTRAL SIRENS

Luminosity distance



Love sirens

# Cosmography with next-generation gravitational wave detectors

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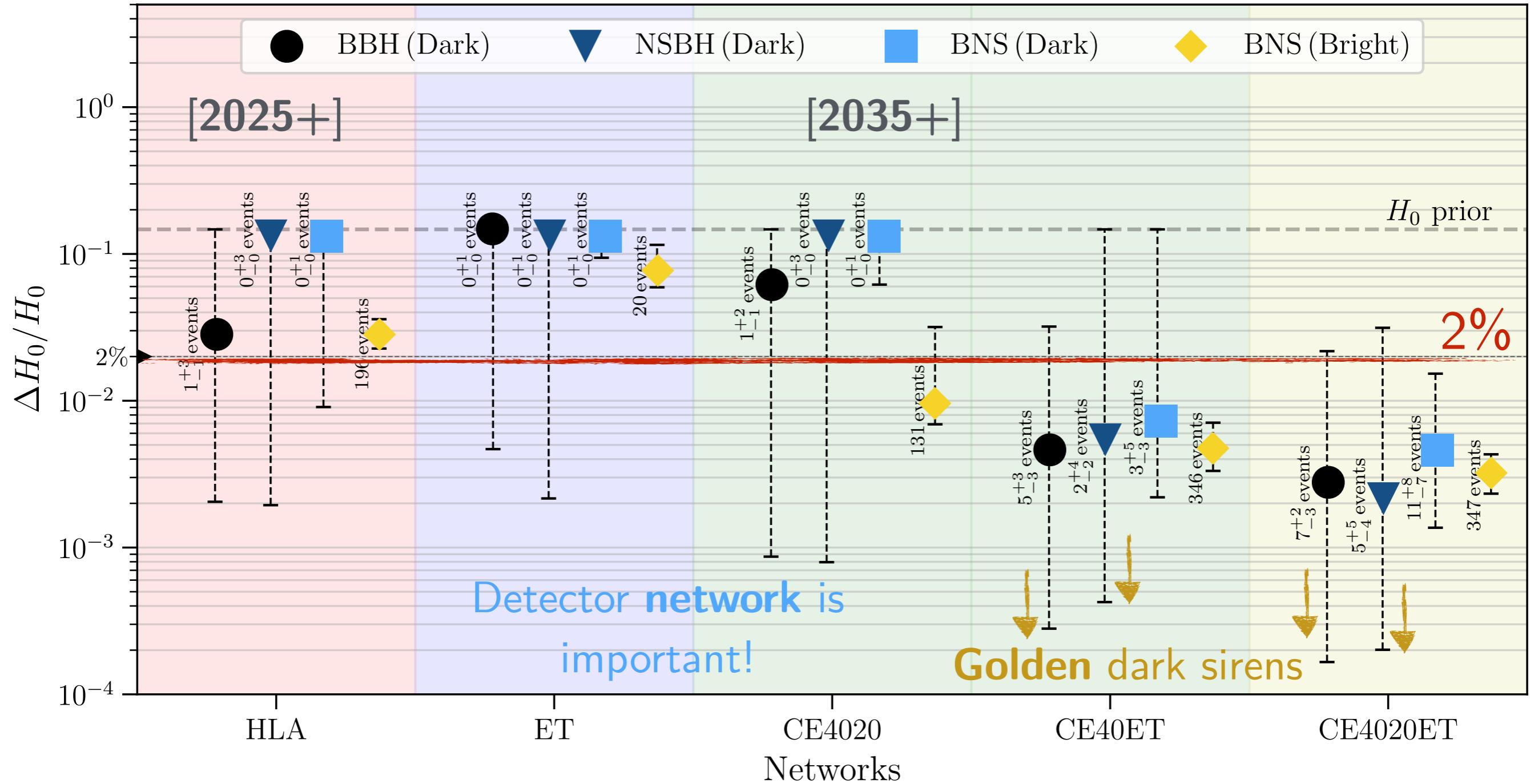
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## Abstract

Advancements in cosmology through next-generation (XG) ground-based gravitational wave (GW) observatories will bring in a paradigm shift. We explore the pivotal role that GW standard sirens will play in inferring cosmological parameters with XG observatories, not only achieving exquisite precision but also opening up unprecedented redshifts. We examine the merits and the systematic biases involved in GW standard sirens utilizing binary black holes, binary neutron stars, and neutron star-black hole mergers. Further, we estimate the precision of bright sirens, golden dark sirens, and spectral sirens for these binary coalescences and compare the abilities of various XG observatories (A<sup>3</sup>, cosmic explorer, Einstein telescope, and their possible networks). When combining different sirens, we find sub-percent precision over more than 10 billion years of cosmic evolution for the Hubble expansion rate  $H(z)$ . This work presents a broad view of opportunities to precisely measure the cosmic expansion rate, decipher the elusive dark energy and dark matter, and potentially discover new physics in the uncharted Universe with XG GW detectors.

# $H_0$ (also) with dark sirens

H: Hanford (US)  
 L: Livingston (US)  
 A: Aundha (India)  
 ET: Einstein Telescope (EU)  
 CE: Cosmic Explorer (US)

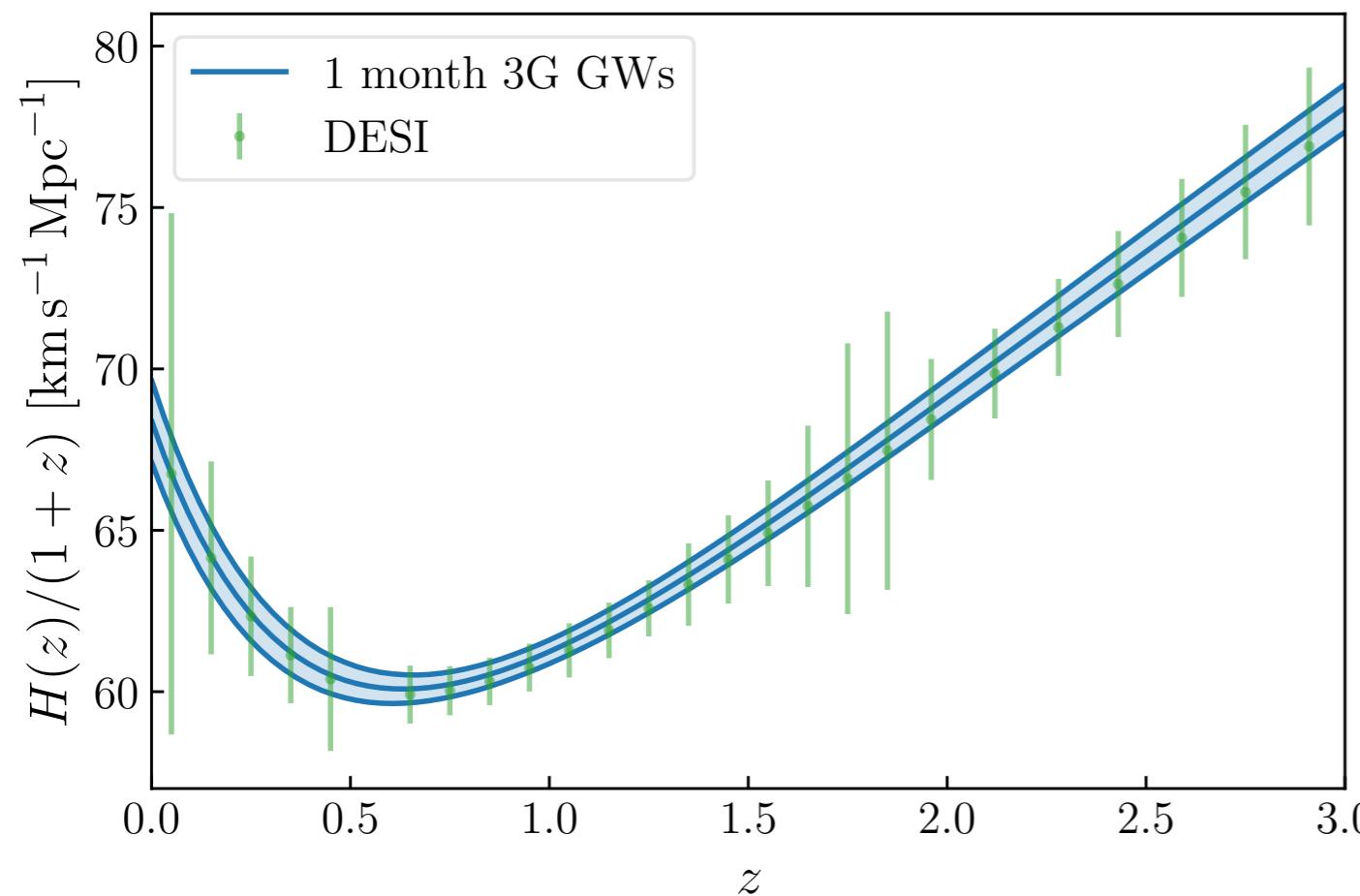


[Chen, Ezquiaga & Gupta (CQG'24)]

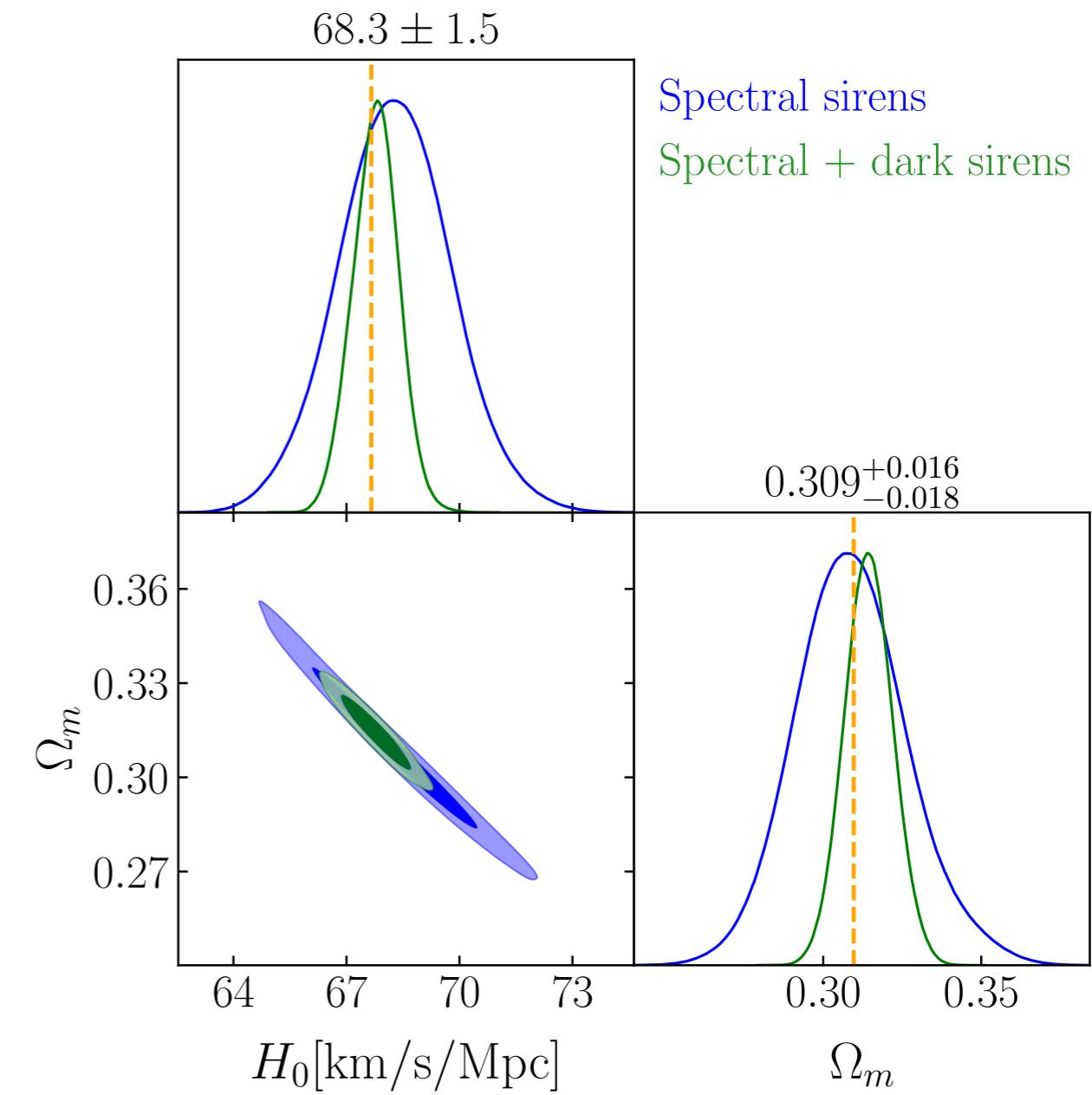
# Expansion rate at high redshift $H(z)$

Combining sirens **sub-percent** precision across cosmic history!

Spectral sirens are competitive  
with cosmic surveys



[Ezquiaga & Holz (PRL'22)]



[Chen, Ezquiaga & Gupta (CQG'24)]

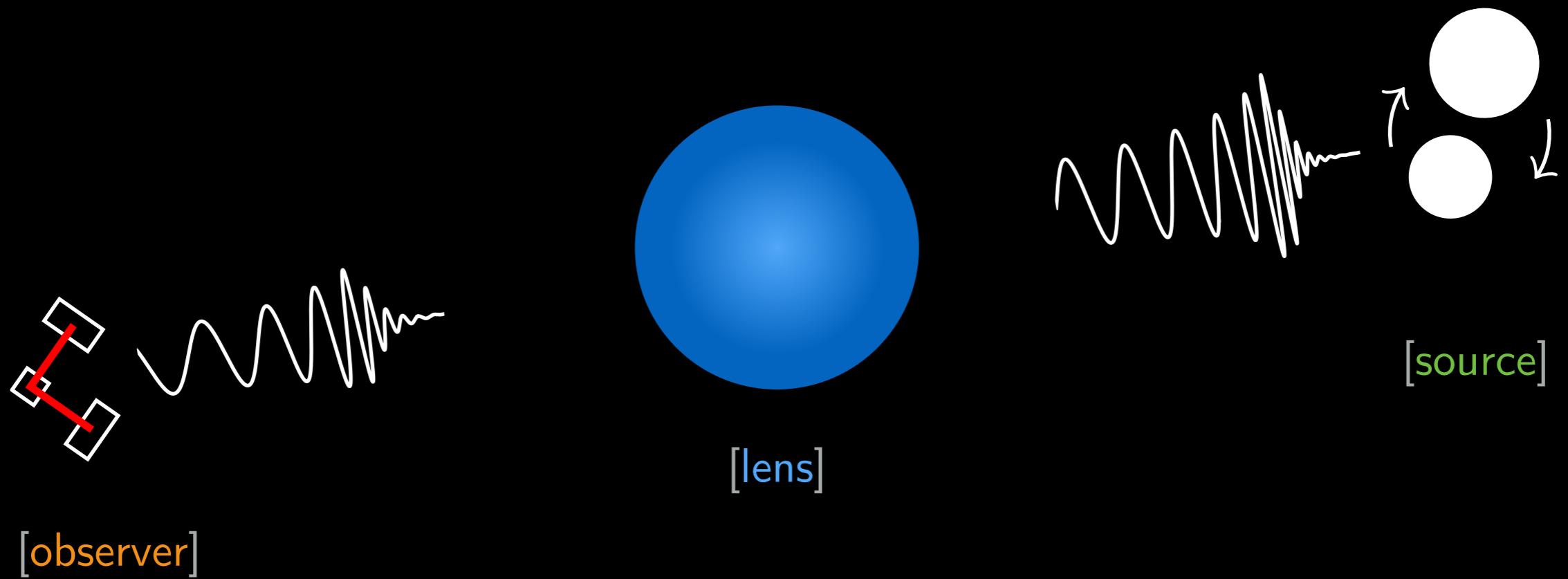


# Gravitational lensing

- Solve GW propagation on a **curved background**

$$\square \bar{h}_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0$$

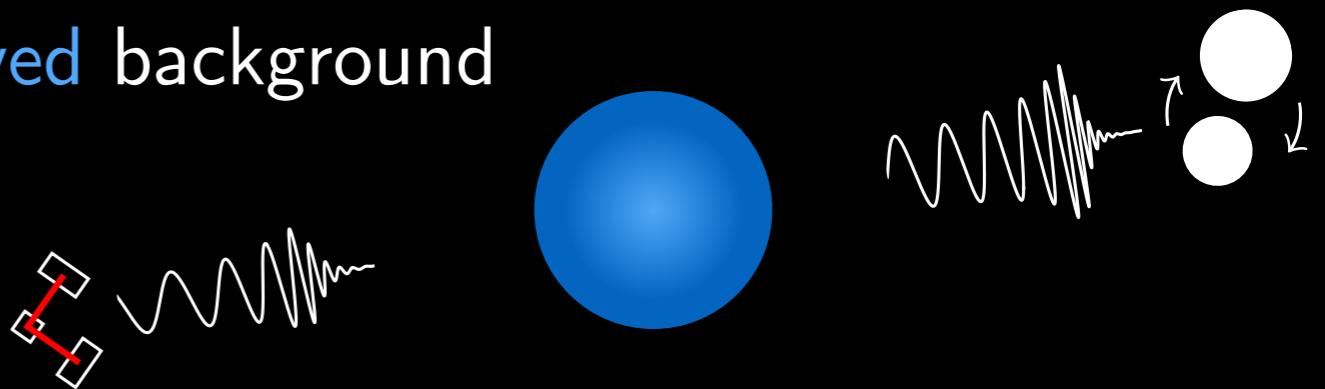
- We want to make a mapping between the **source** and the **observer** through the **lens**



# Gravitational lensing

- Solve GW propagation on a curved background

$$\square \bar{h}_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = 0$$



- Within *weak-gravity* & *thin lens* approximations, in *Fourier* space:

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F(\textcolor{blue}{w}, \vec{y}) = \frac{w}{2\pi i} \int d^2x \exp[i\textcolor{blue}{w}T_d(\vec{x}, \vec{y})]$$

[Dimensionless variables]  $\vec{x} \equiv \vec{\theta}/\theta_*$  ,  $\vec{y} \equiv \vec{\theta}_S/\theta_*$  ,  $w \equiv \tau_D \theta_*^2 \omega$

$$T_d \equiv t_d/\tau_D \theta_*^2 \quad \tau_D \equiv (1+z_L) D_L D_S / c D_{LS}$$

# Stationary Phase Approximation

- Solve integral in the limit of highly oscillatory integrand

$$F(w, \vec{y}) = \frac{w}{2\pi i} \int d^2x \exp[iwT_d(\vec{x}, \vec{y})]$$

- Stationary points define the **images**:

$$\left. \frac{\partial t_d}{\partial \theta_a} \right|_{\vec{\theta}=\vec{\theta}_j} = 0$$

$$T_d(\vec{\theta}) \approx T_d(\vec{\theta}_j) + \frac{1}{2} \sum_{(a,b)=1}^2 \delta\theta_a \delta\theta_b \frac{\partial^2 T_d(\vec{\theta}_j)}{\partial \theta_a \partial \theta_b} + \dots$$

- Hessian matrix determines magnifications

$$\mu(\theta_j) = 1/\det(T_{ab}(\theta_j))$$

$$T_{ab} \equiv \tau_D^{-1} \partial^2 t_d / \partial \theta_a \partial \theta_b$$

# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

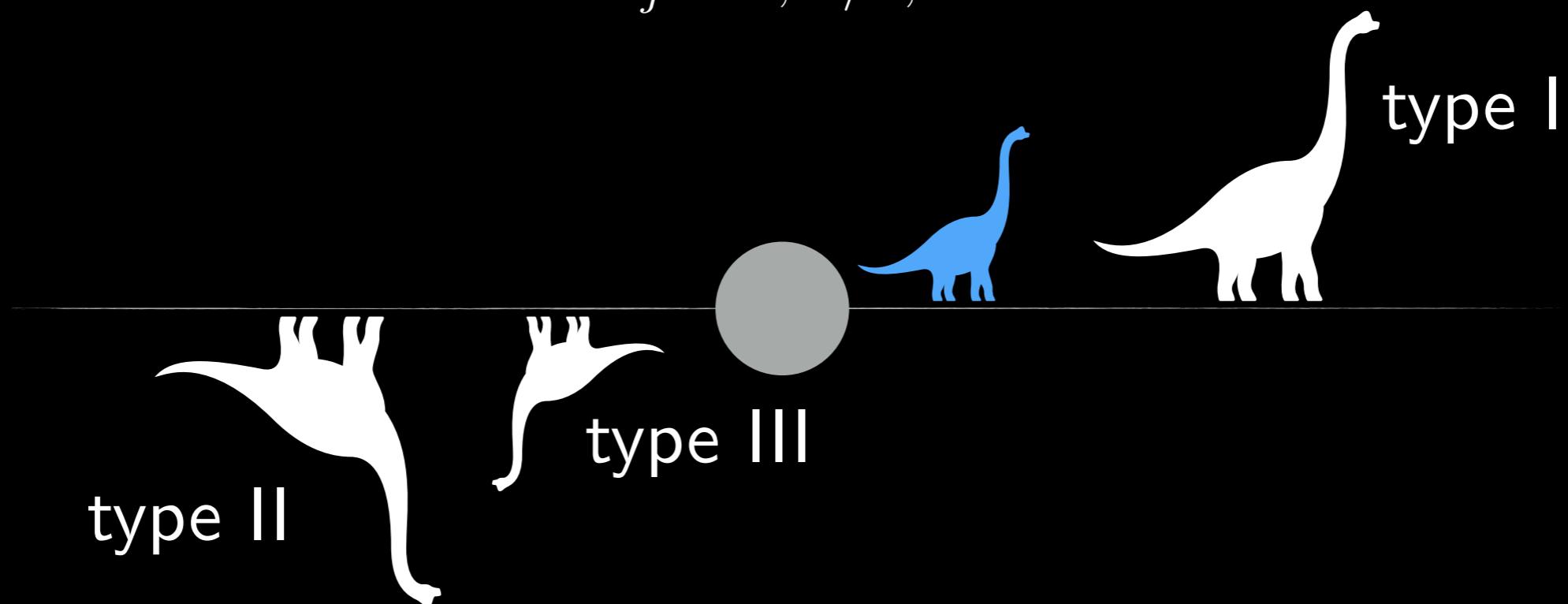
$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega \textcolor{green}{t}_j - i\pi n_j)$$

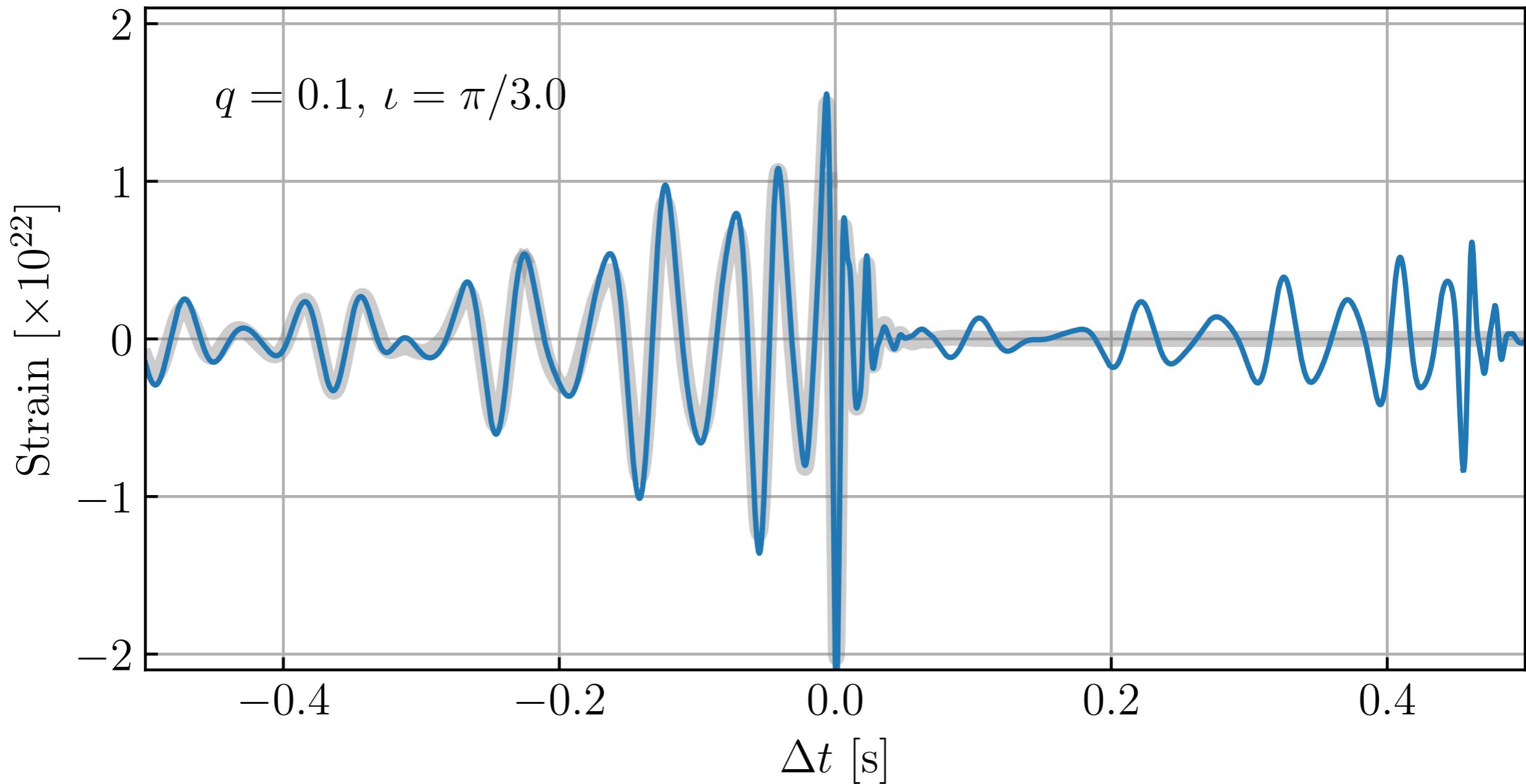
Magnification  
Time delay  
Phase shift

- Each image type (I, II and III) acquire a different phase shift

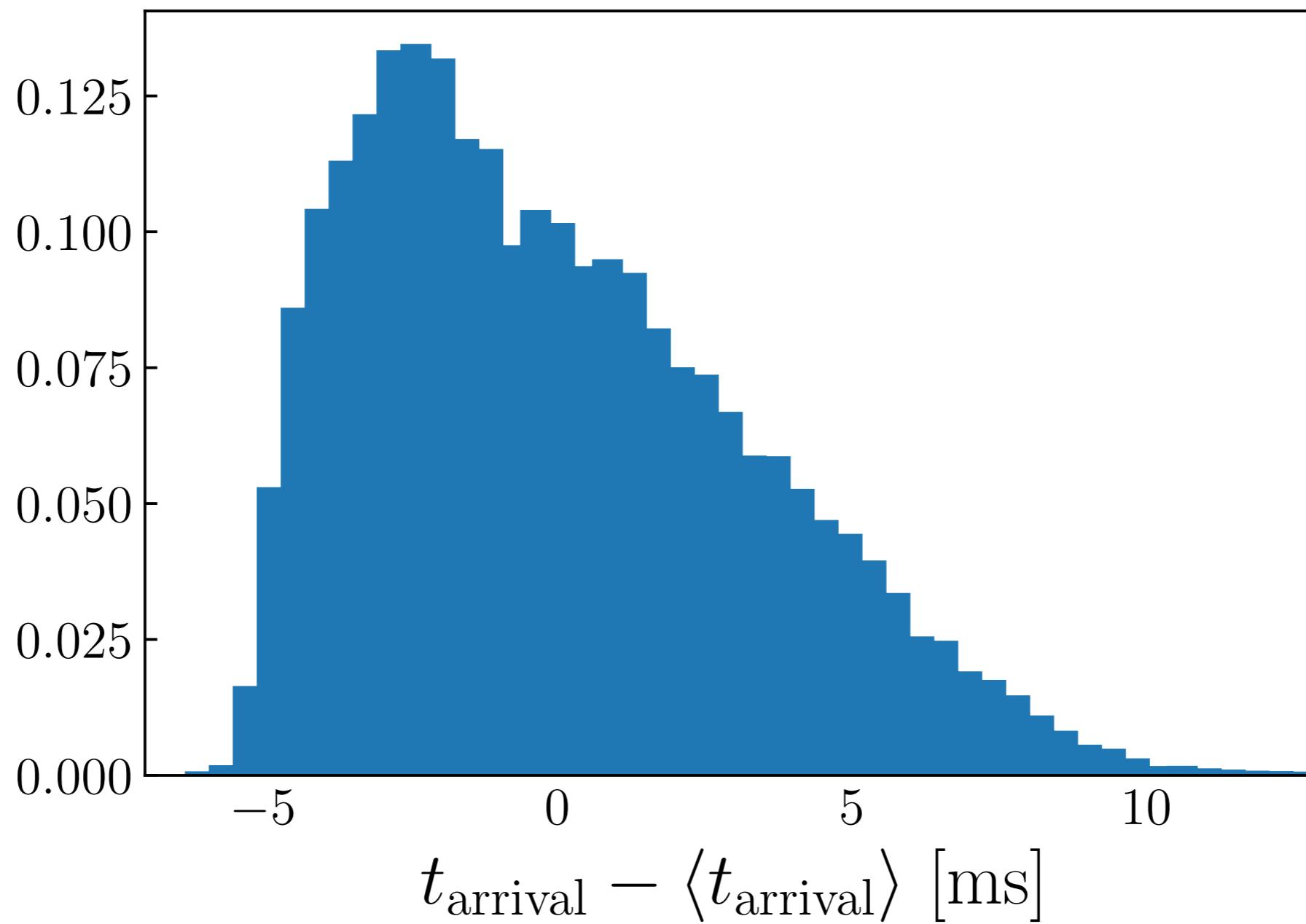
$$n_j = 0, 1/2, 1$$



# Repeated, coherent signals

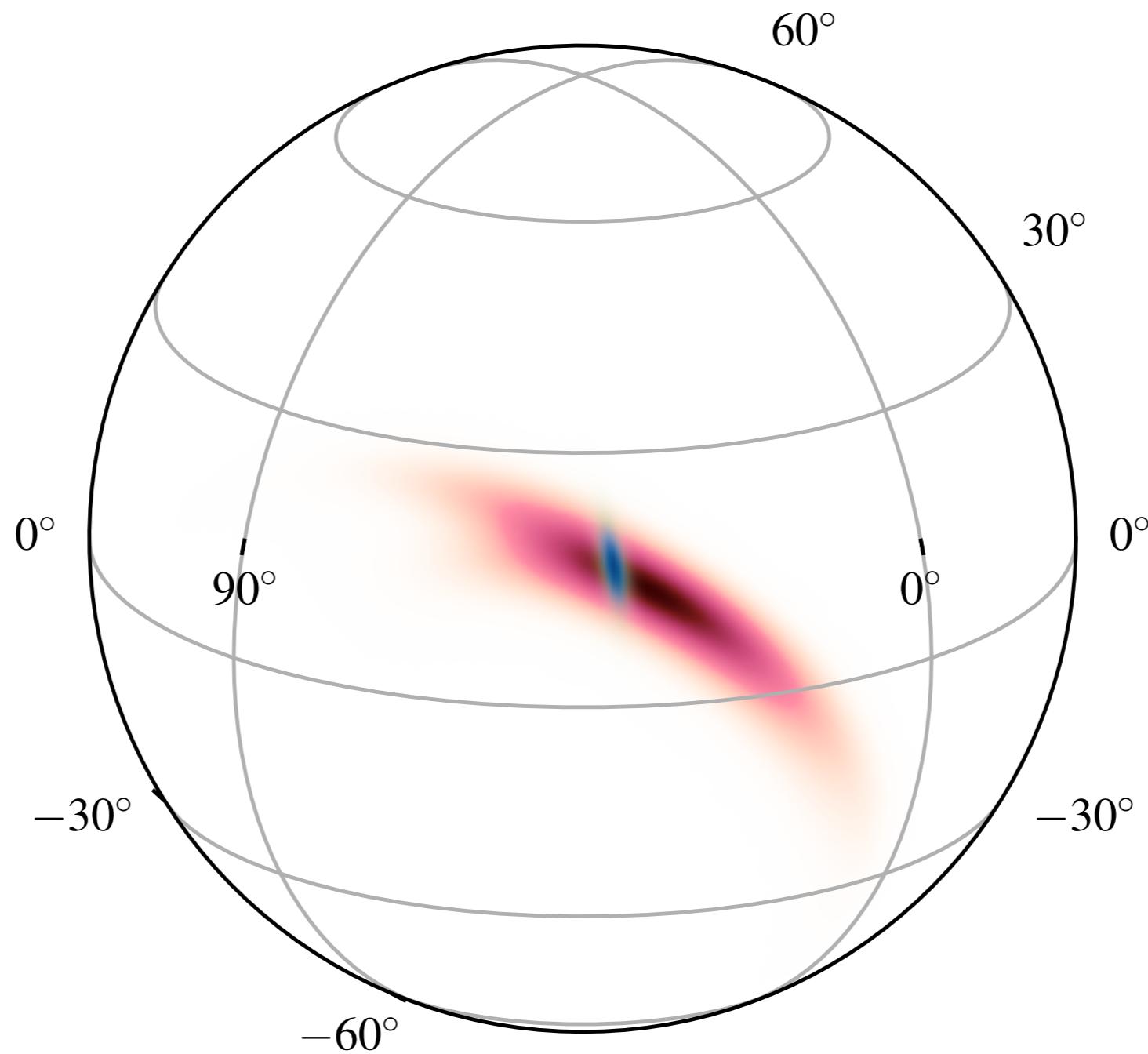


# Precise timing



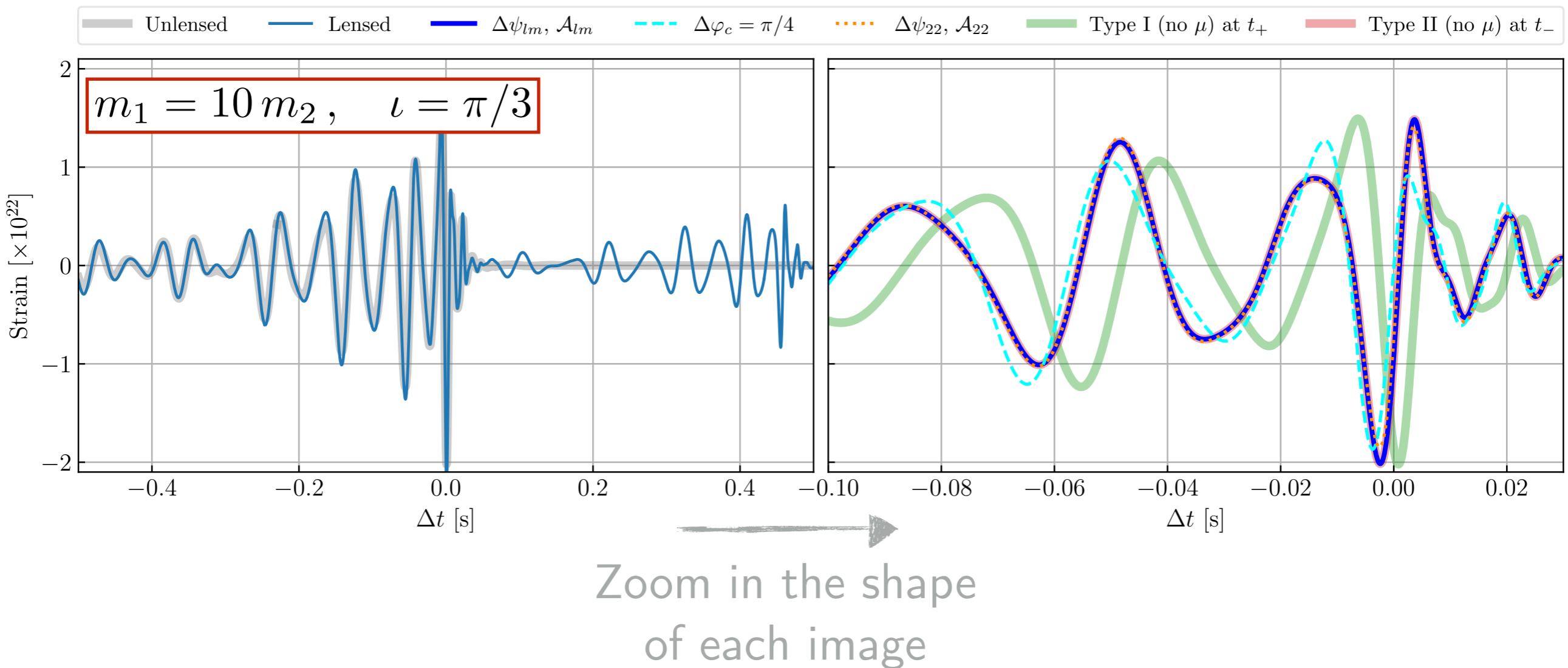
# Poor sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1\text{Gpc}}{D}}$$



# Waveform distortions in type II images

- Lensing imprints *small* but *characteristic* modifications in the signals that cannot be mapped to other astrophysical parameters



# Wave optics

$$\Delta t_d \cdot \omega$$

- Time delay scales with the lens mass

$$\Delta t_d(y=1) \simeq 4 \left( \frac{(1+z_L)M_L}{100M_\odot} \right) \text{ ms}$$

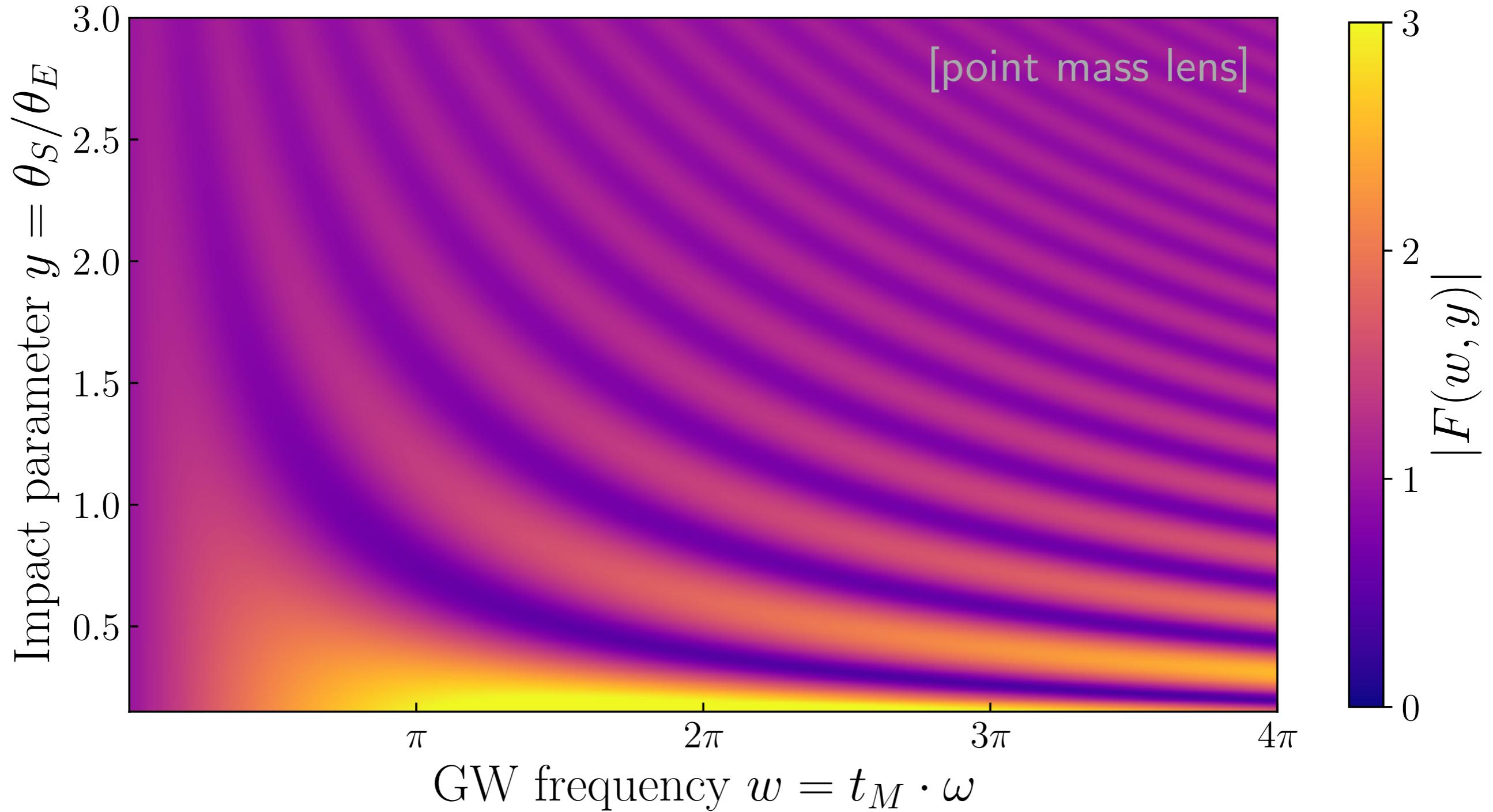
[point mass lens]

- GW frequency scales with binary mass (*has astrophysical size!*)

$$f \sim \frac{1}{2\pi} \frac{1}{2t_{\text{Sch}}} \sim 800 \text{Hz} \left( \frac{10M_\odot}{M} \right)$$

- Wave optics regime:  $\Delta t_d \cdot \omega \sim 1$
- Low-frequency limit has small lensing  $\quad \omega \rightarrow 0 \quad \Rightarrow \quad F \rightarrow 1$

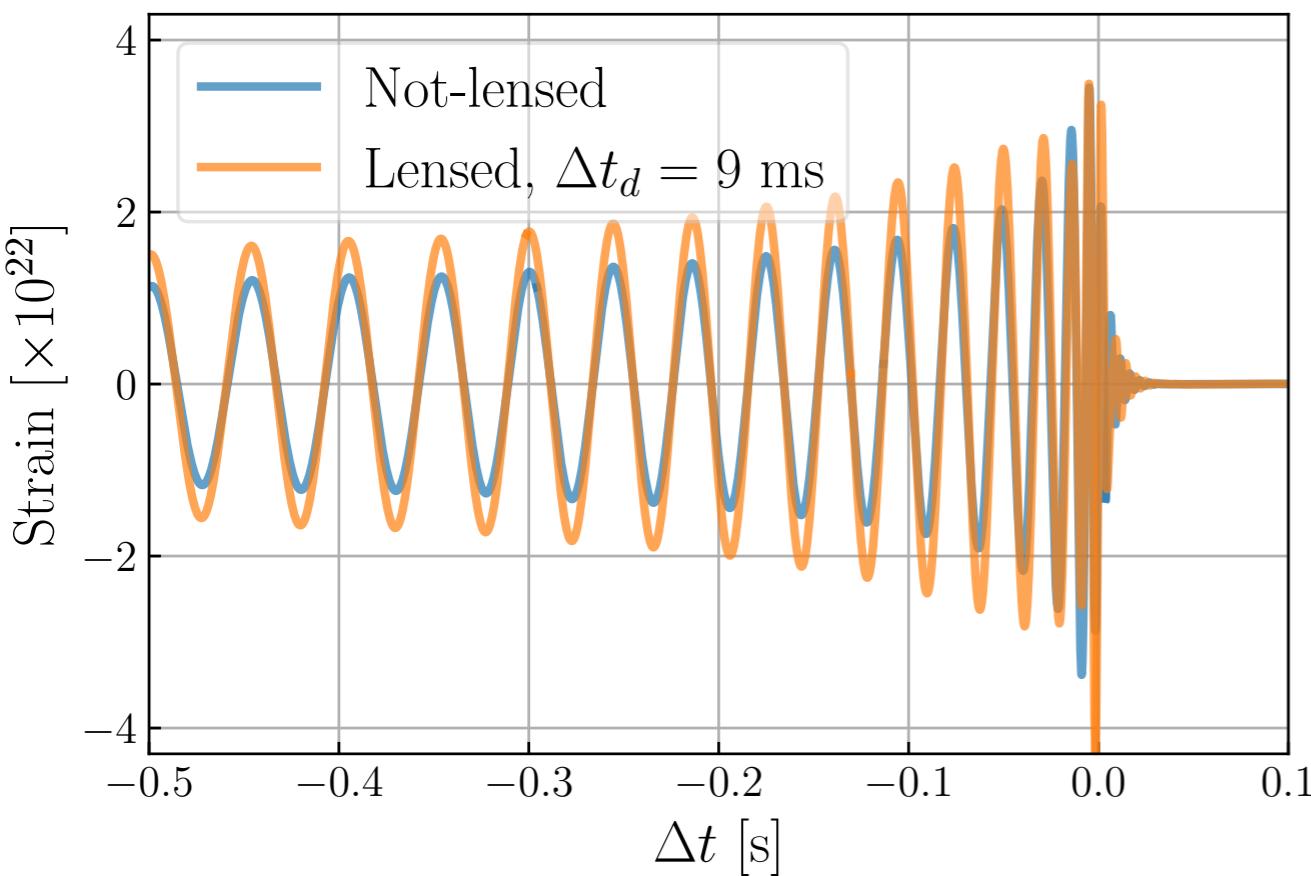
# Wave optics: point lens



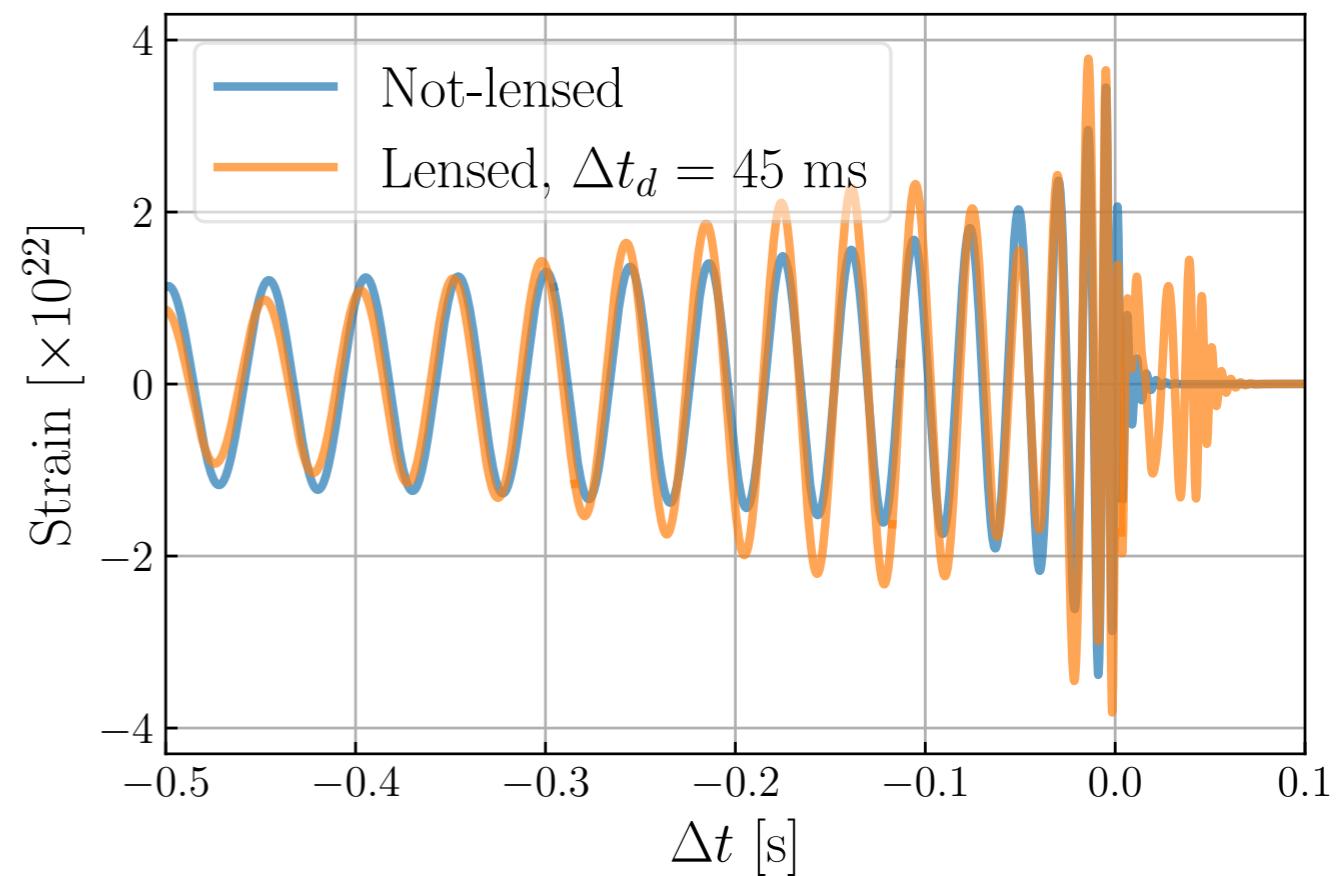
# Wave optics: point lens

$$\Delta t_d(y=1) \simeq 4 \left( \frac{(1+z_L)M_L}{100M_\odot} \right) \text{ ms}$$

Diffraction



Interference



Many recent works: probing compact objects ([Dai et al.'18](#), [Diego'19](#), [Tambalo et al.'22](#), ...), strong lensing + microlensing ([Seo et al.'21](#), [Mena et al.'22](#), ...), breaking mass-sheet degeneracy ([Cremonese, Ezquiaga, Salzano'21](#)), solving diffraction integral ([Feldbrugge&Turok'20](#), [Tambalo et al.'22](#)) ... +++

# Caustics

- For point sources, there are **singular** points in the lens mapping

$$\det \left( \frac{\partial^2 T_d(\theta_j)}{\partial \theta_a \partial \theta_b} \right) \rightarrow 0 \quad \Rightarrow \mu(\theta_j) \rightarrow \infty$$

- Caustics exhibit ***universal*** behaviors (described by catastrophe theory)

$$\mu_{\pm} \sim 1/\sqrt{\Delta\theta_S} \sim \Delta t^{-1/3}$$

- SPA is **broken** when approaching to a caustic
- **Maximum magnification** set by diffraction



# Observational Signatures of Highly Magnified Gravitational Waves from Compact Binary Coalescence

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(Dated: July 26, 2024)

Gravitational lensing has empowered telescopes to discover astronomical objects that are otherwise out of reach without being highly magnified by foreground structures. While we expect gravitational waves (GWs) from compact binary coalescences to also experience lensing, the phenomenology of highly magnified GWs has not been fully exploited. In this letter, we fill this gap and explore the observational signatures of these highly magnified GWs. We find that these signatures are robust against modeling details and can be used as smoking-gun evidence to confirm the detection of lensing of GWs without any electromagnetic observation. Additionally, diffraction becomes important in some cases, which limits the maximum possible magnification and gives waveform signatures of lensing that can only be observed by GW detectors. Even with current-generation observatories, we are already sensitive to these highly magnified GWs and can use them to probe the high-redshift Universe beyond the usual horizon.

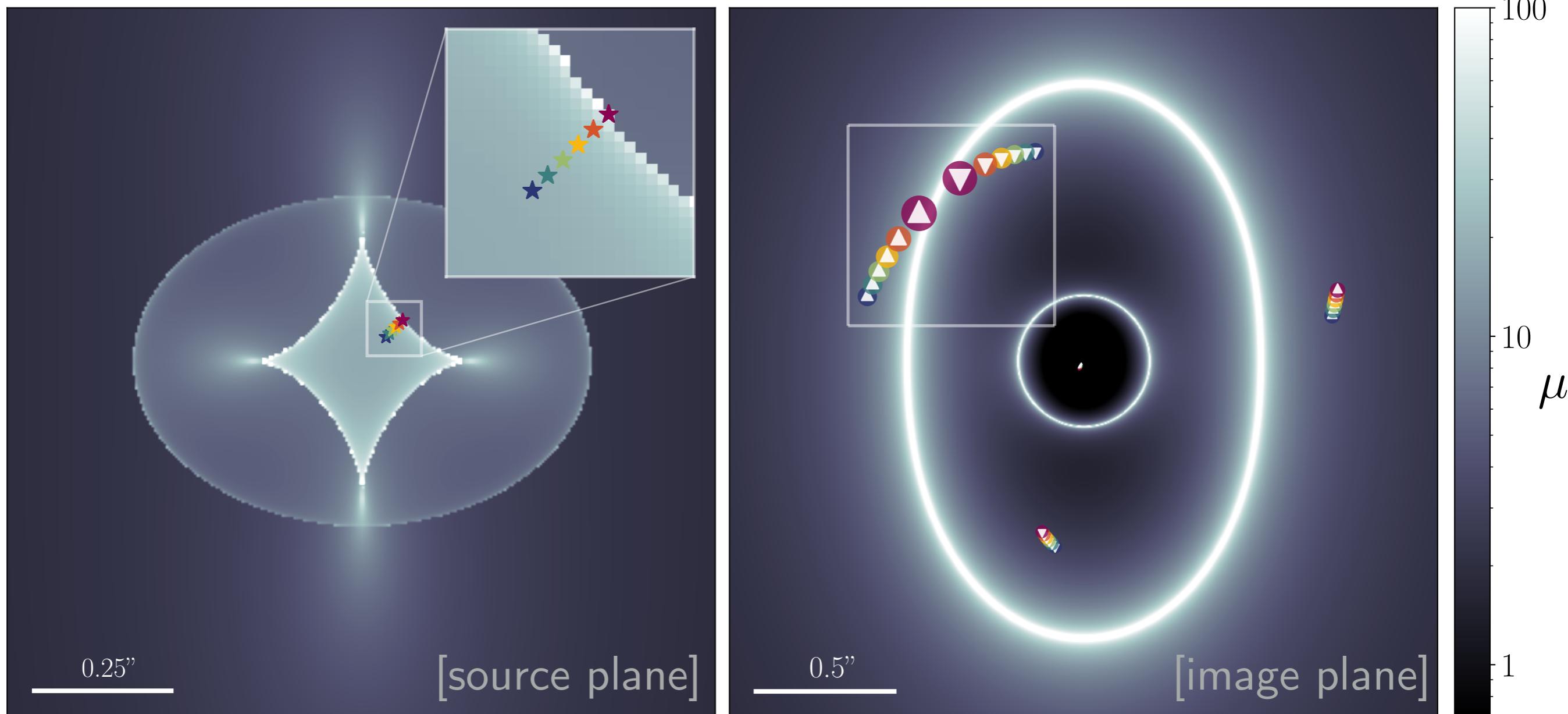


Rico Lo



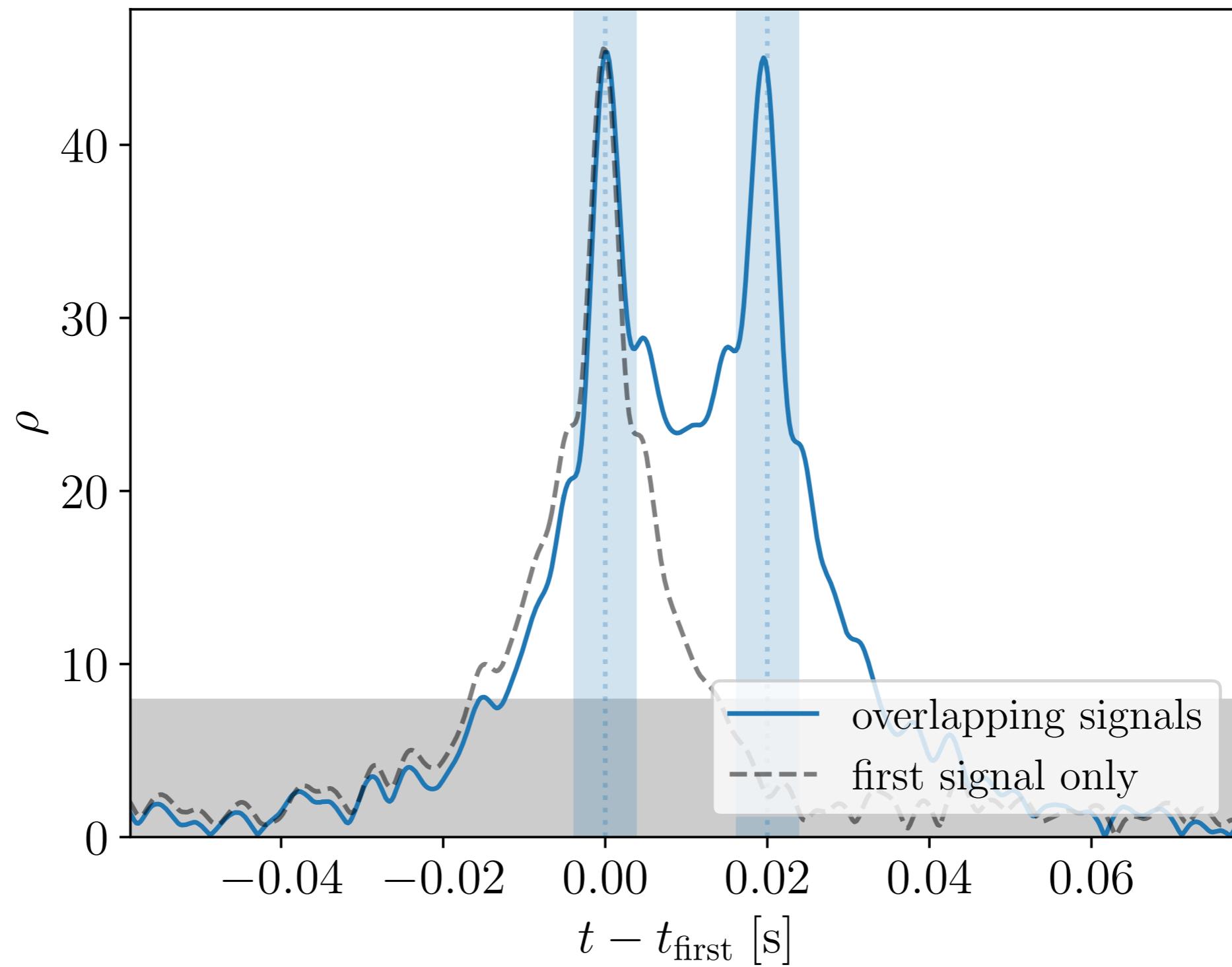
Luka Vujeva

# Fold caustics around galaxies

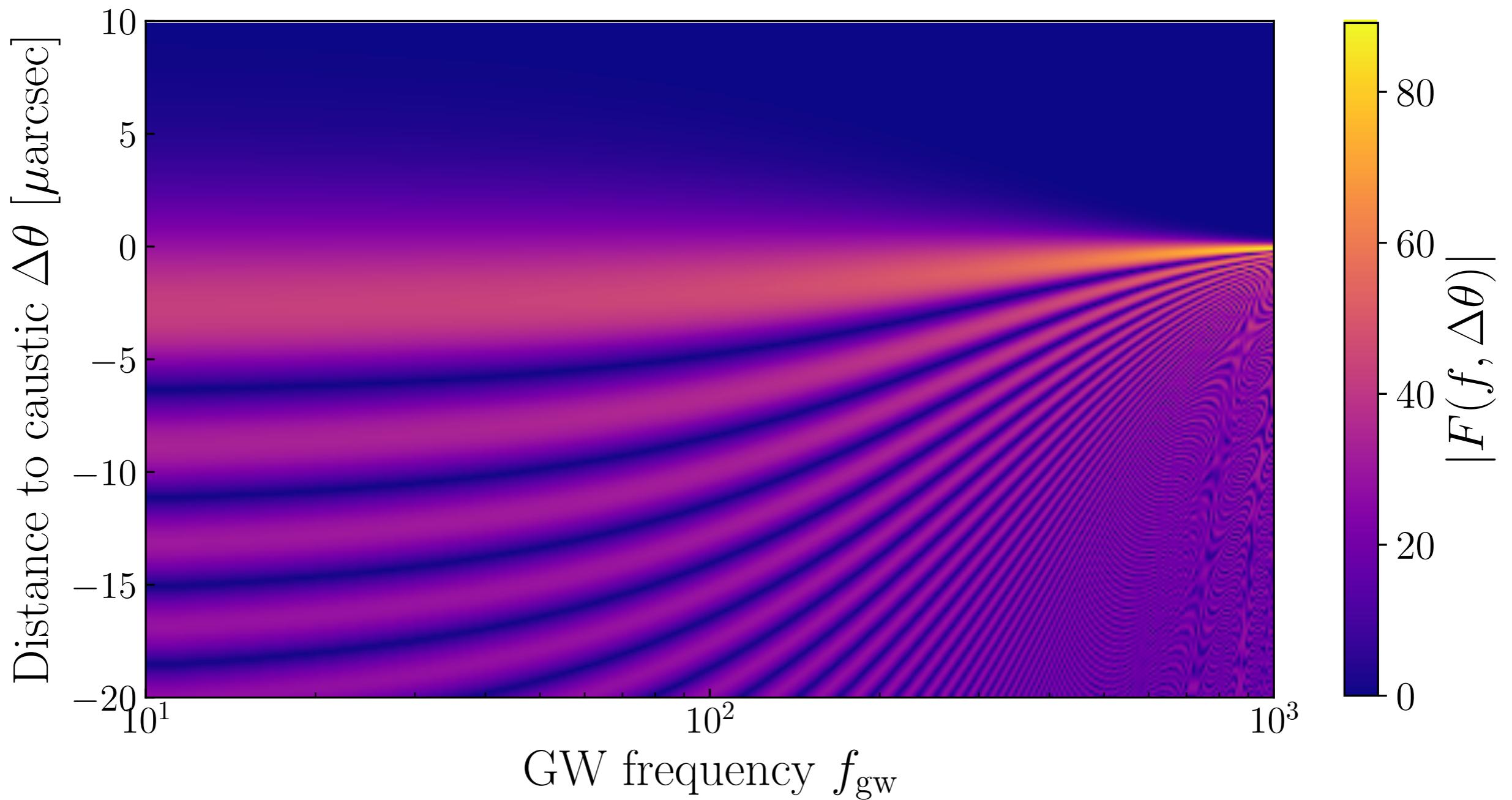


[galaxy lens with a cored singular  
isothermal ellipsoid density profile]

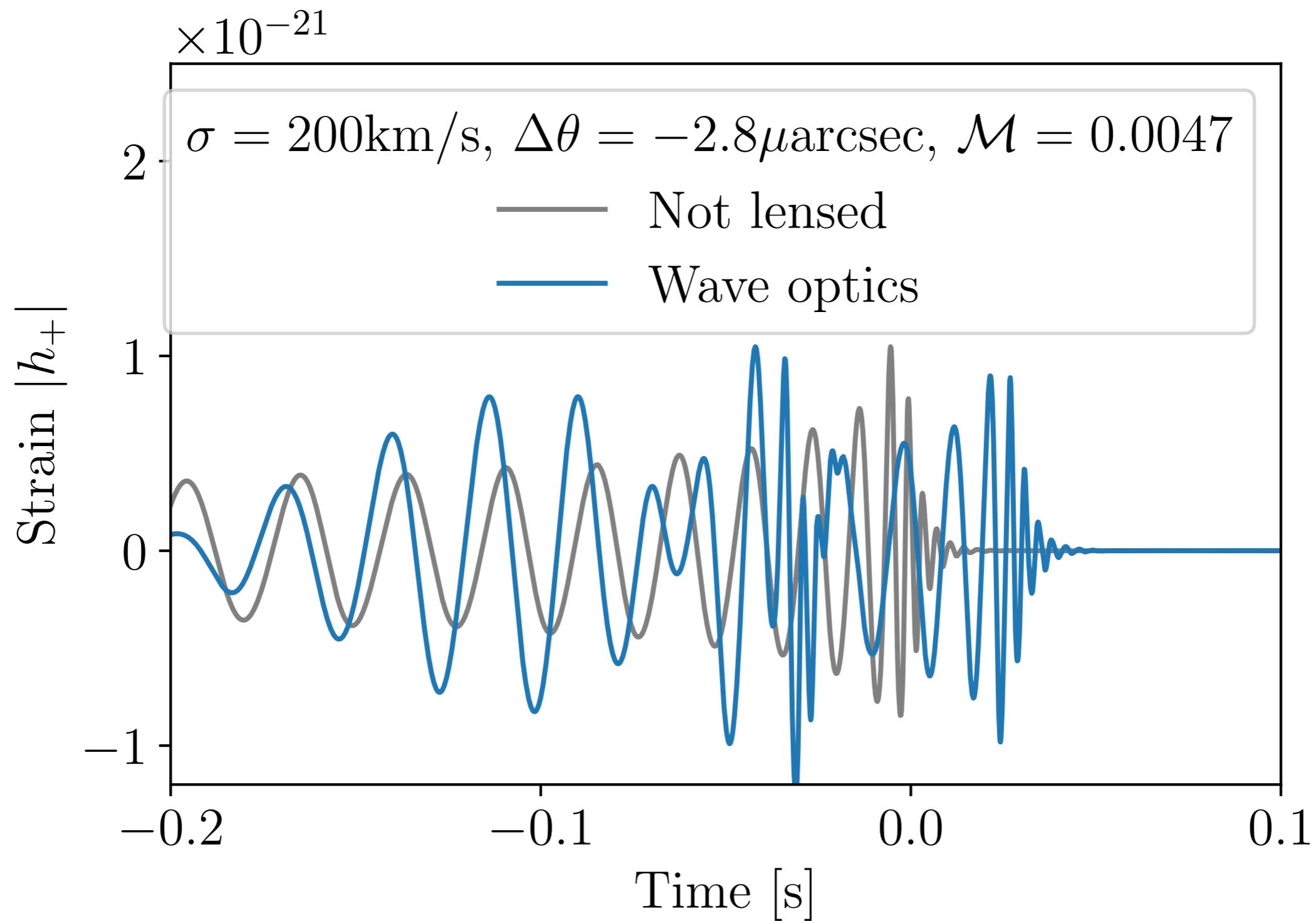
# Overlapping signals



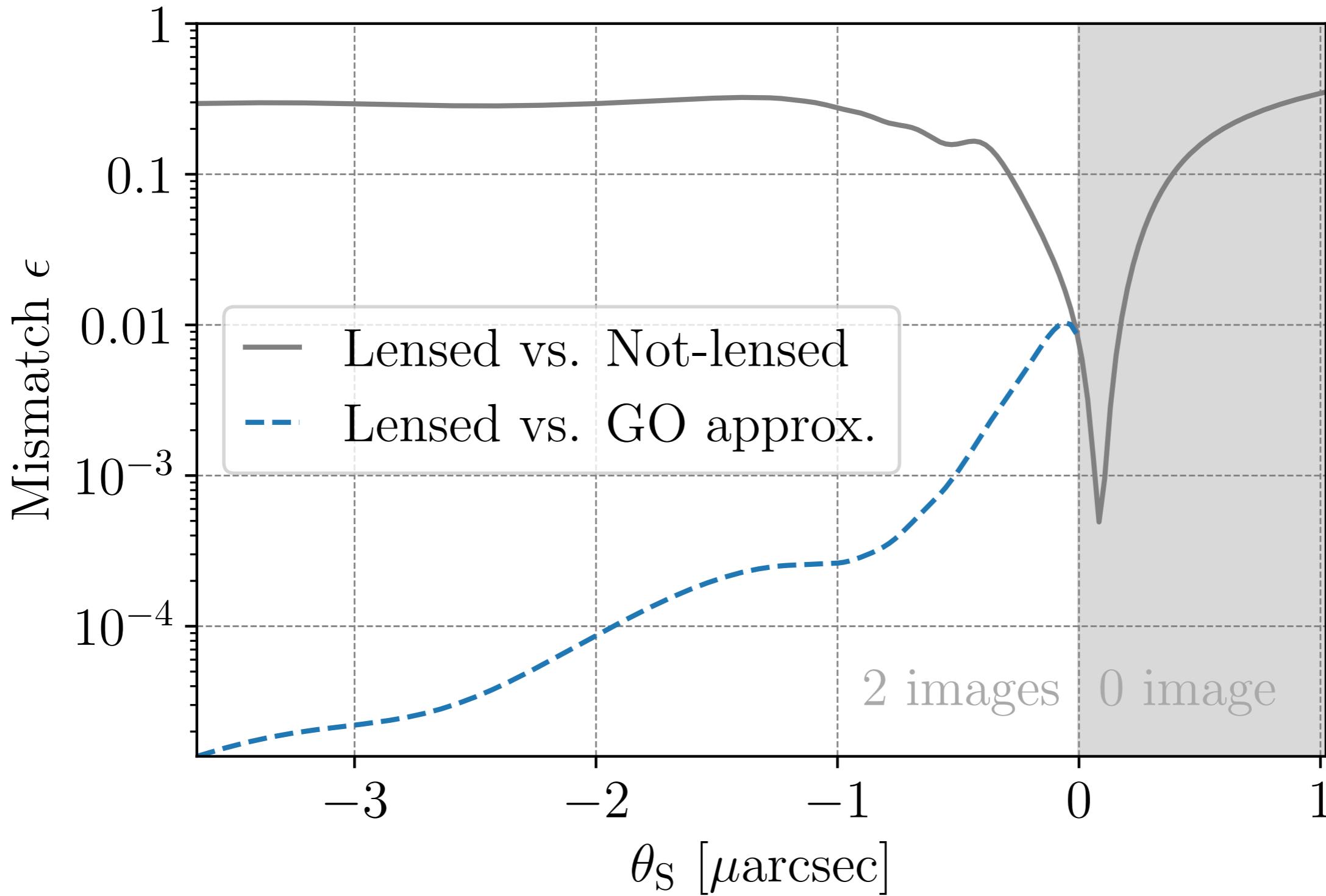
# Diffraction around fold caustics



# Waveform distortions

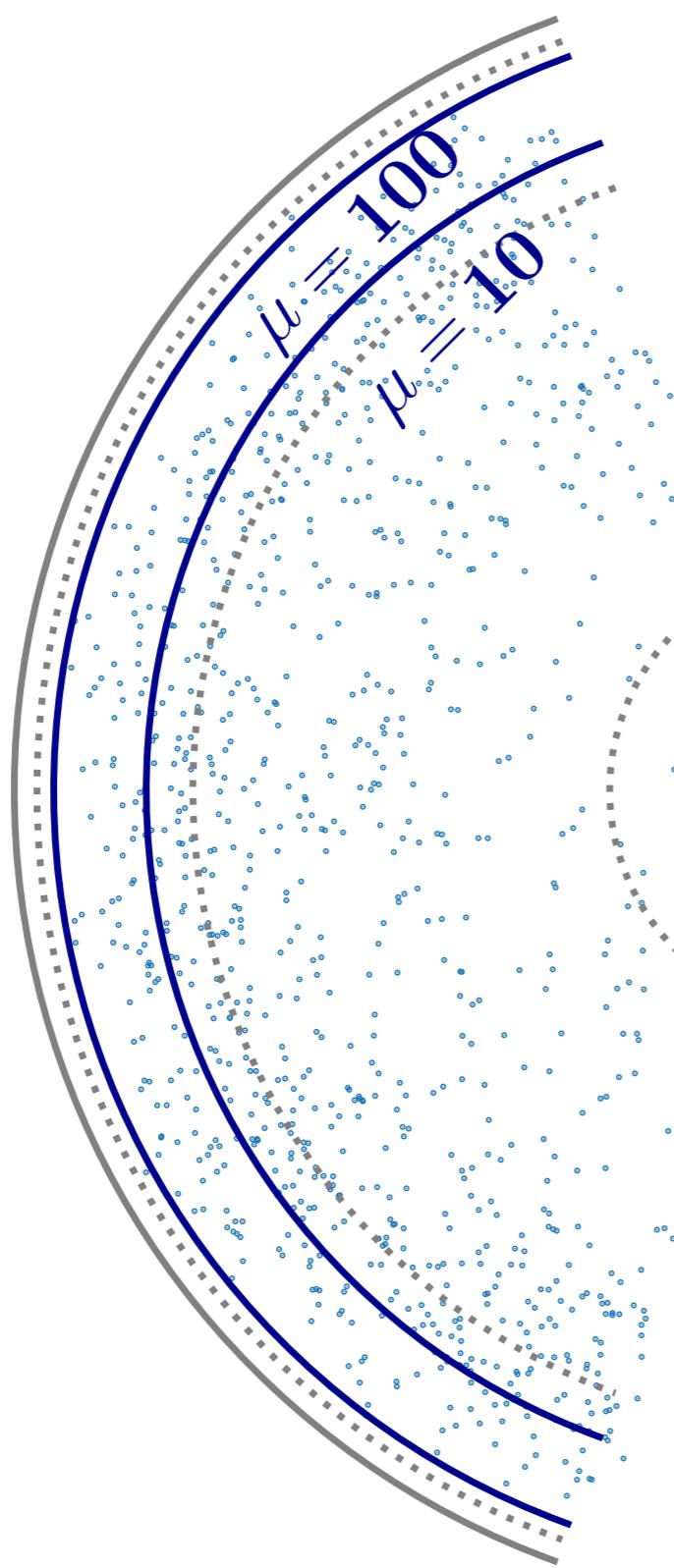


# Observing GW diffraction



[a shortcut to  
next-generation  
detectors]

## Gravitational Wave horizons



2035+

2025+

Now

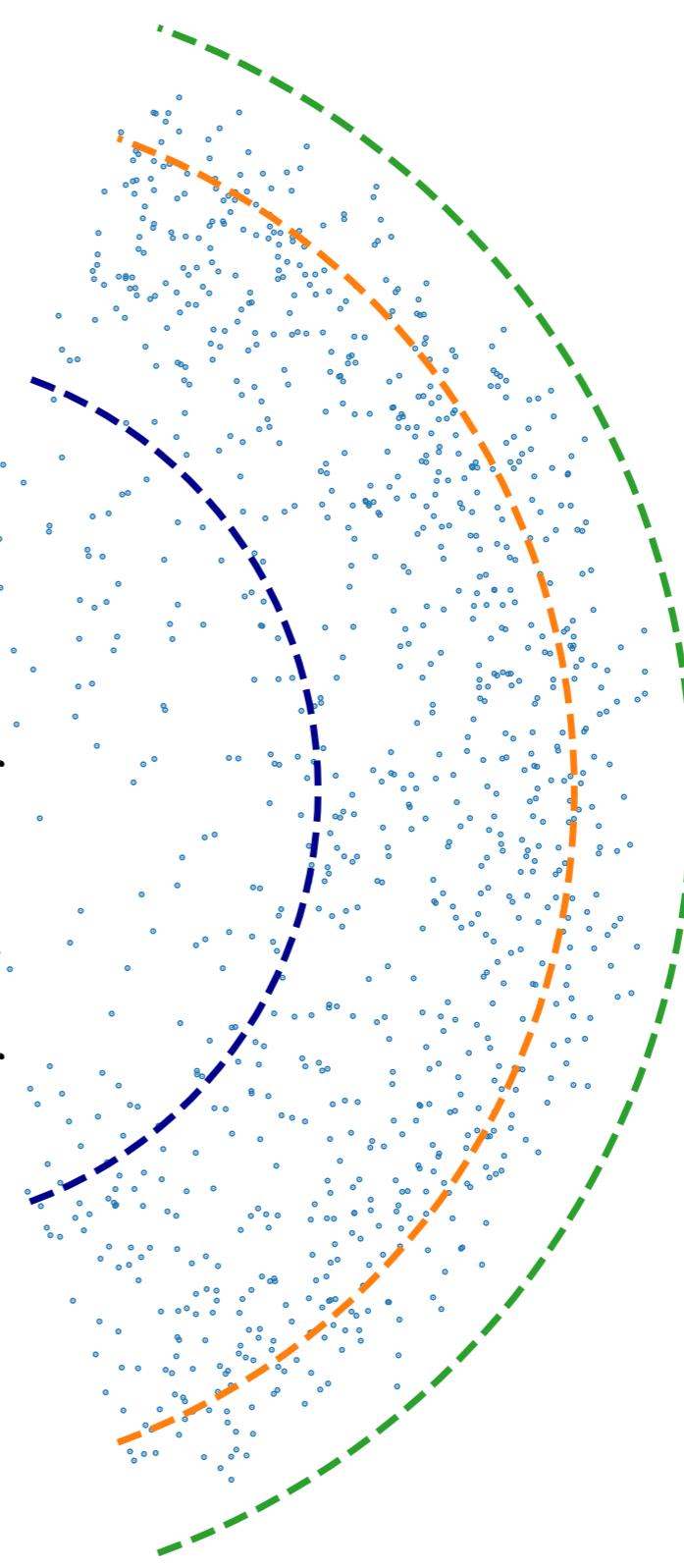
13.8 Gyr

10.0 Gyr

3.0 Gyr

3.8 Myr

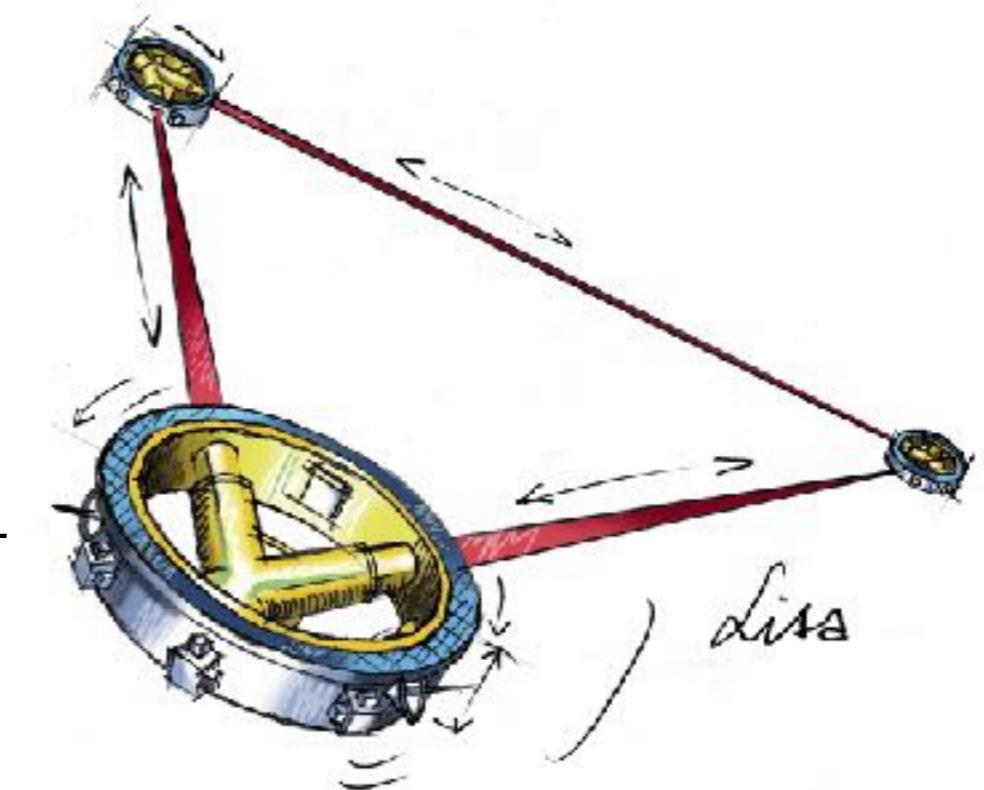
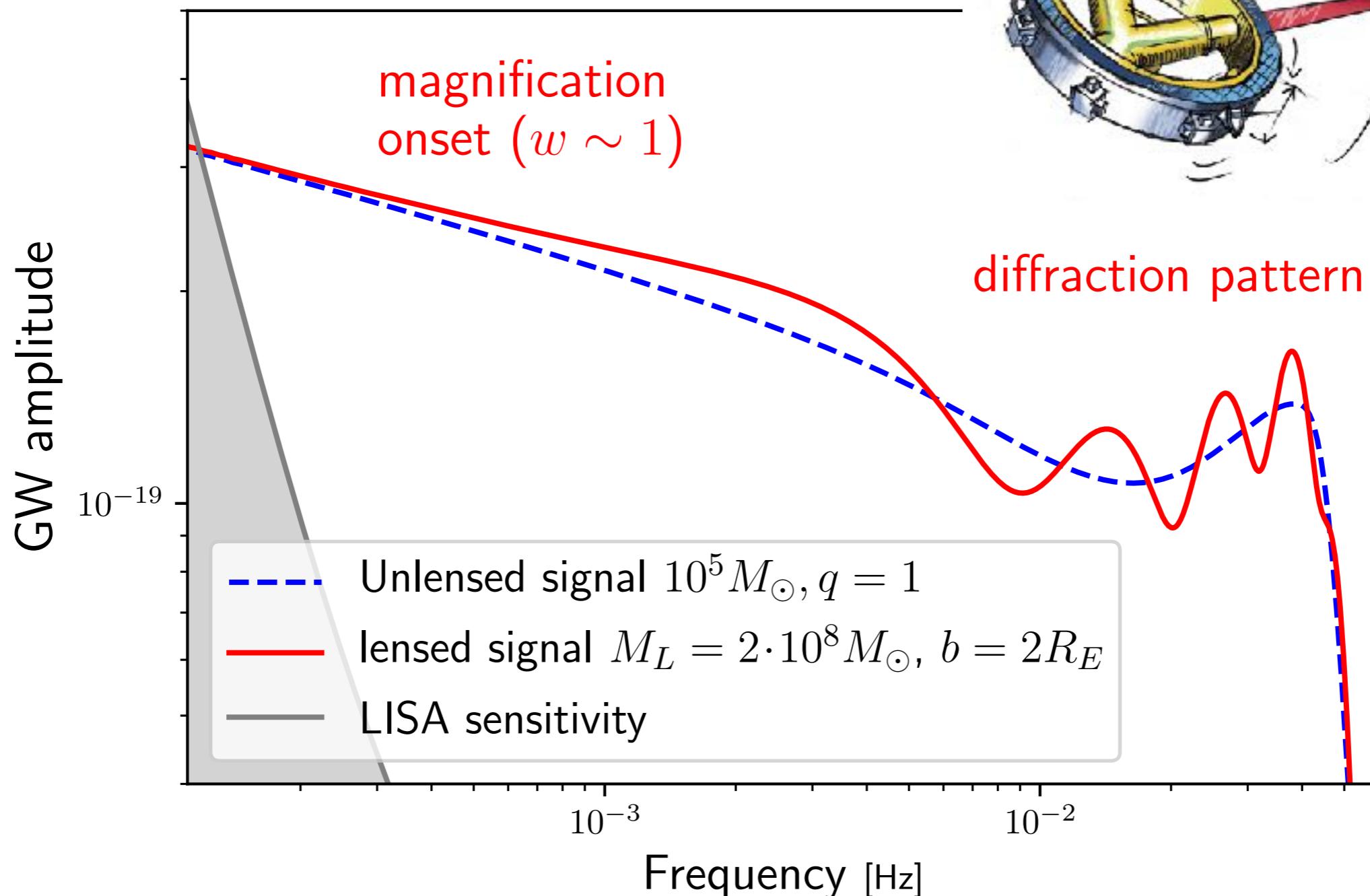
Age of the Universe



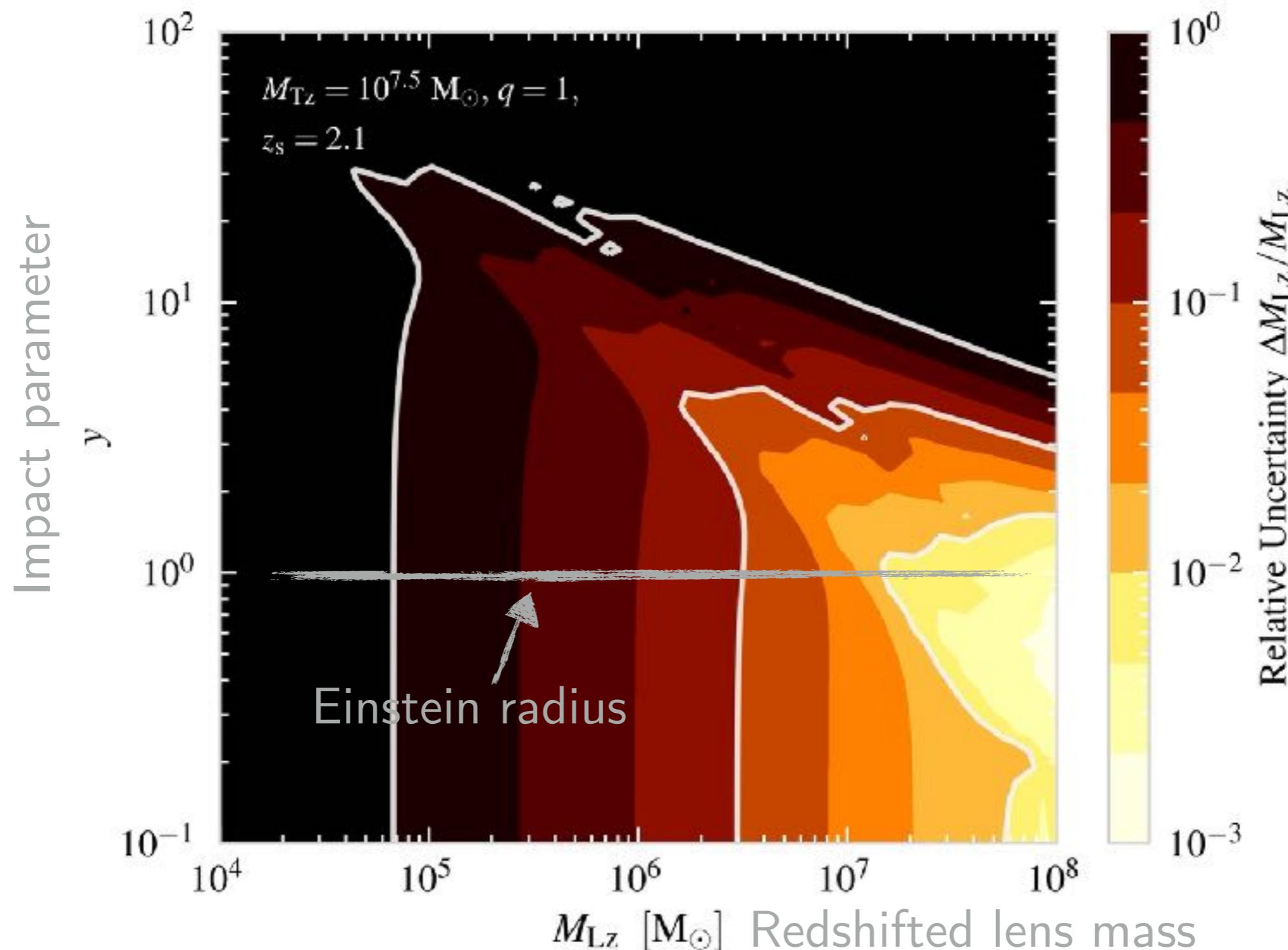
Binary black holes

\*stellar mass  
binary black holes

# Wave effects: LISA



# Increased optical depth in wave optics

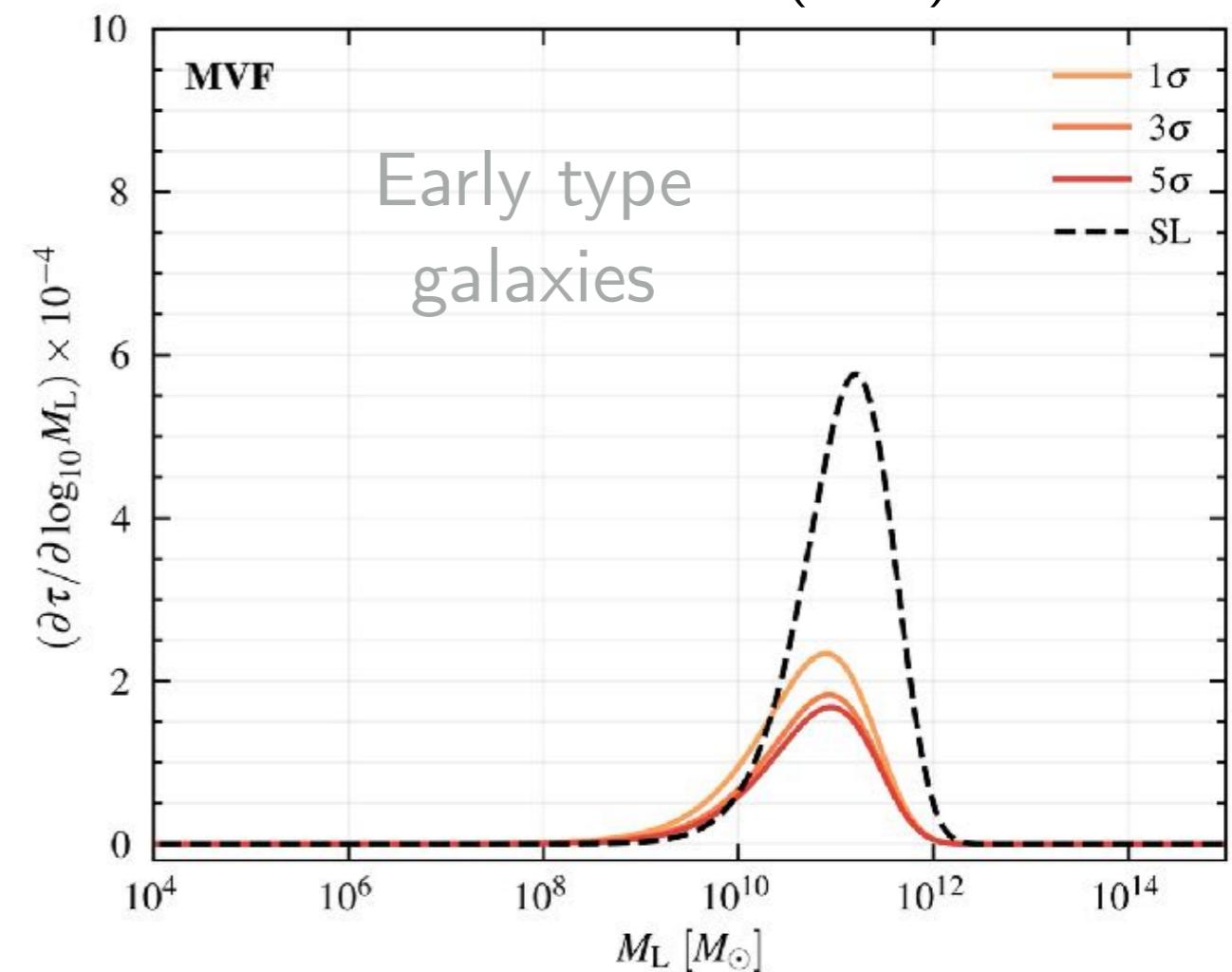
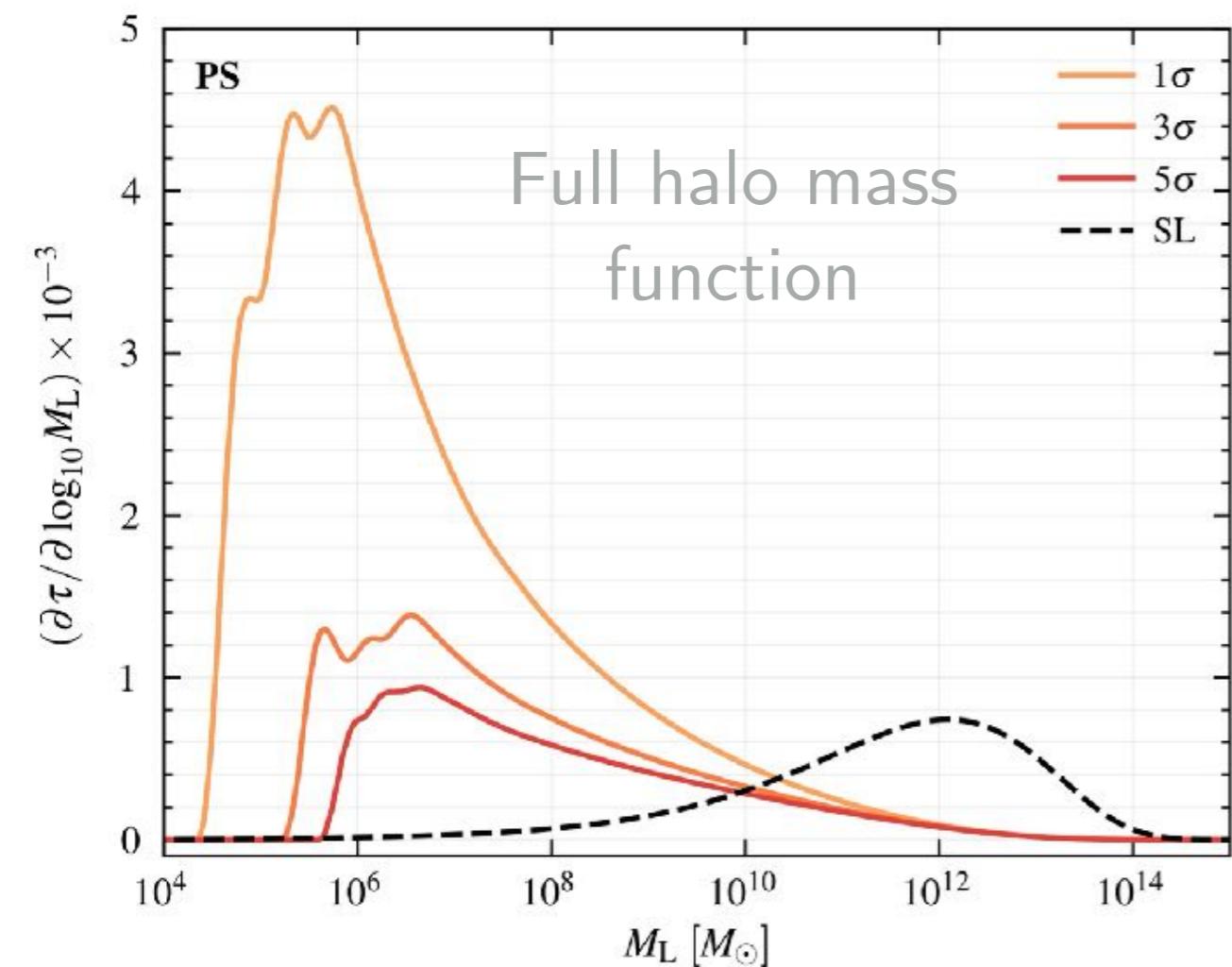


# Probing low-mass dark matter halos

- Probability of wave optics lensing:  
few percent and larger than strong lensing



Mesut Çalışkan  
(JHU)



# Conclusions

Binary black holes are precious **cosmological** probes:

1.  $H_0$  (also) with **dark sirens**
2. Expansion rate at high redshift  $H(z)$  with **spectral sirens**
3. Probing **inhomogeneous** Universe via **lensing**
4. **Highly magnified** GWs have **unique** observational signature
5. Probing **low-mass halos** with **LISA** binaries



Medfinansieret af Den Europæiske  
Unions Connecting Europe-facilitet

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# Join us!

[ezquiaga.github.io/joinus](http://ezquiaga.github.io/joinus)

