Exploring the Gravitational Wave Universe with PTAs

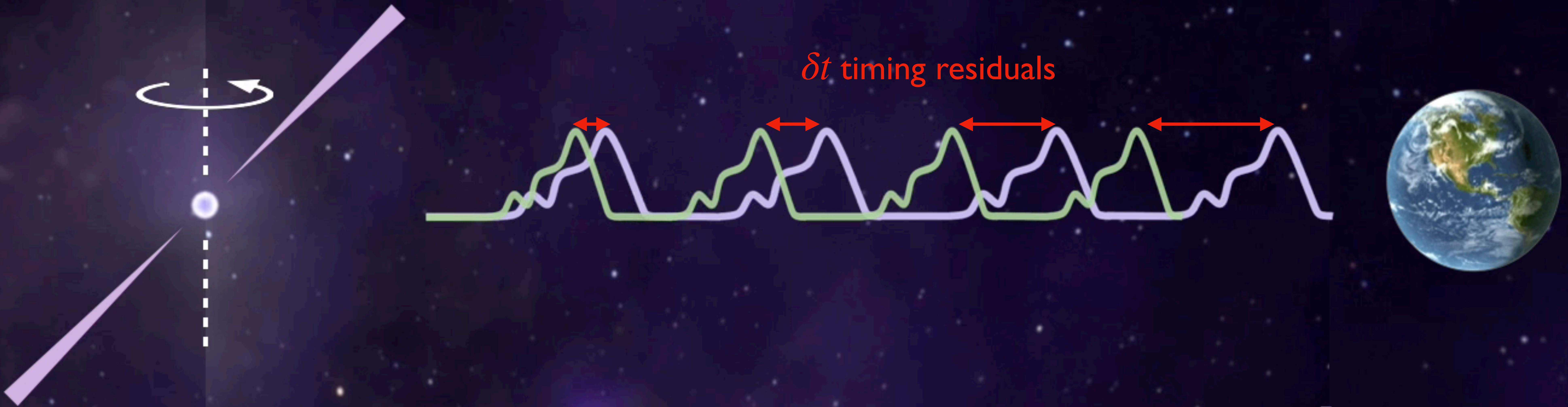
Andrea Mitridate

Fundamental physics and gravitational wave detectors, Pollica | Sep 19, 2024

DESY.

NANOGrav
Physics Frontiers Center

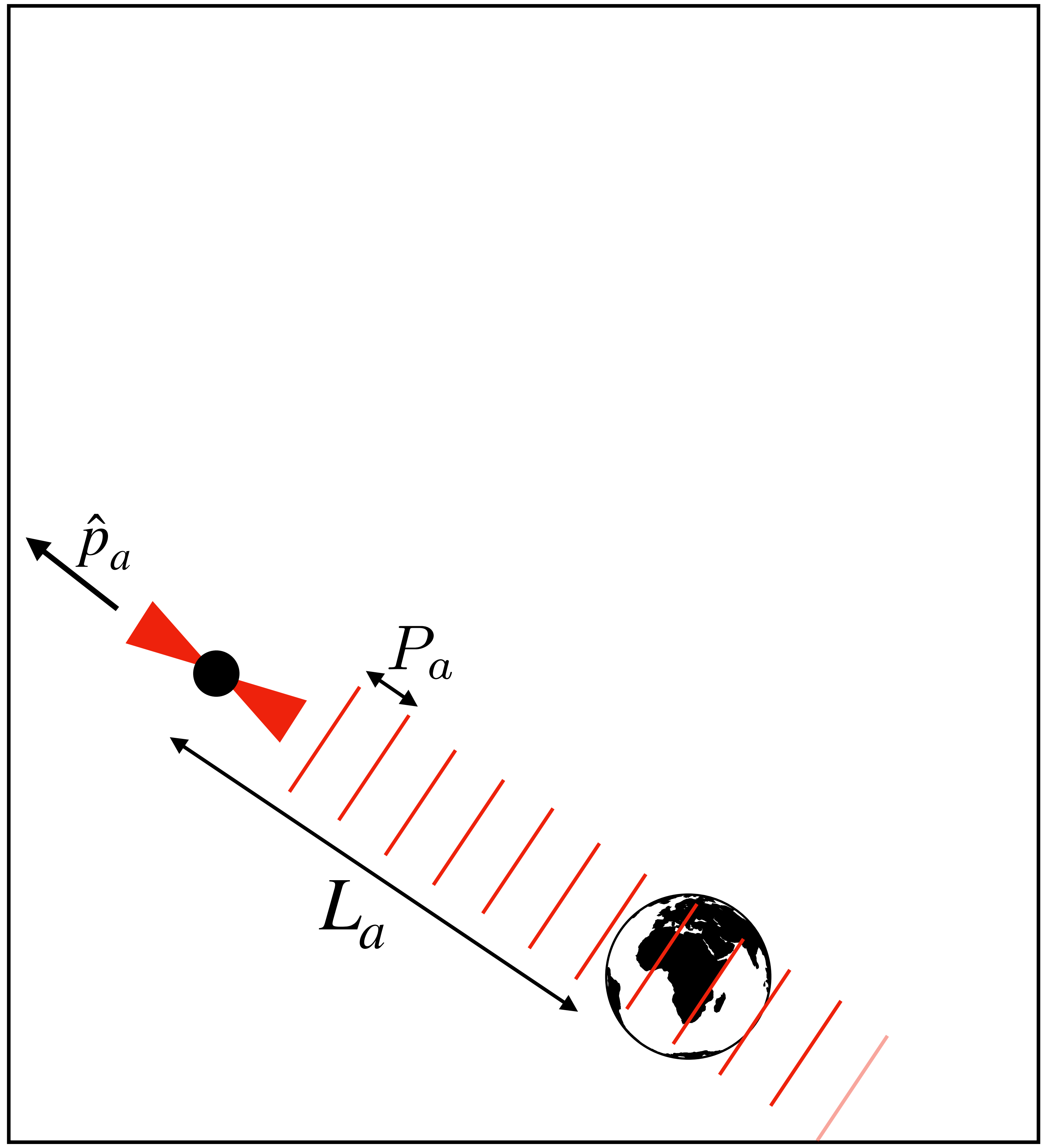
TIMING RESIDUALS



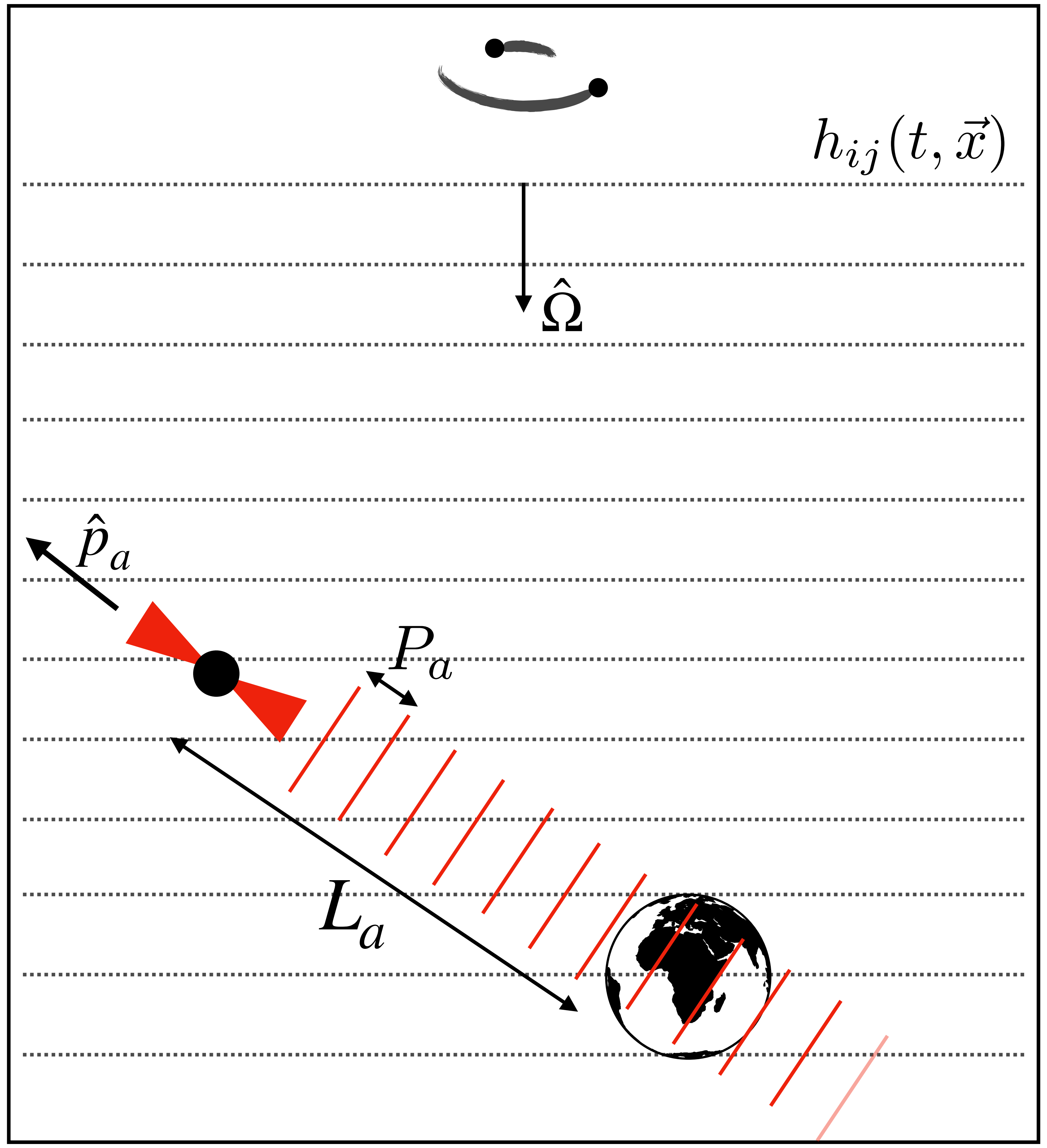
δt timing residuals

Pulses Recorded
by Radio Telescope

GW SIGNALS

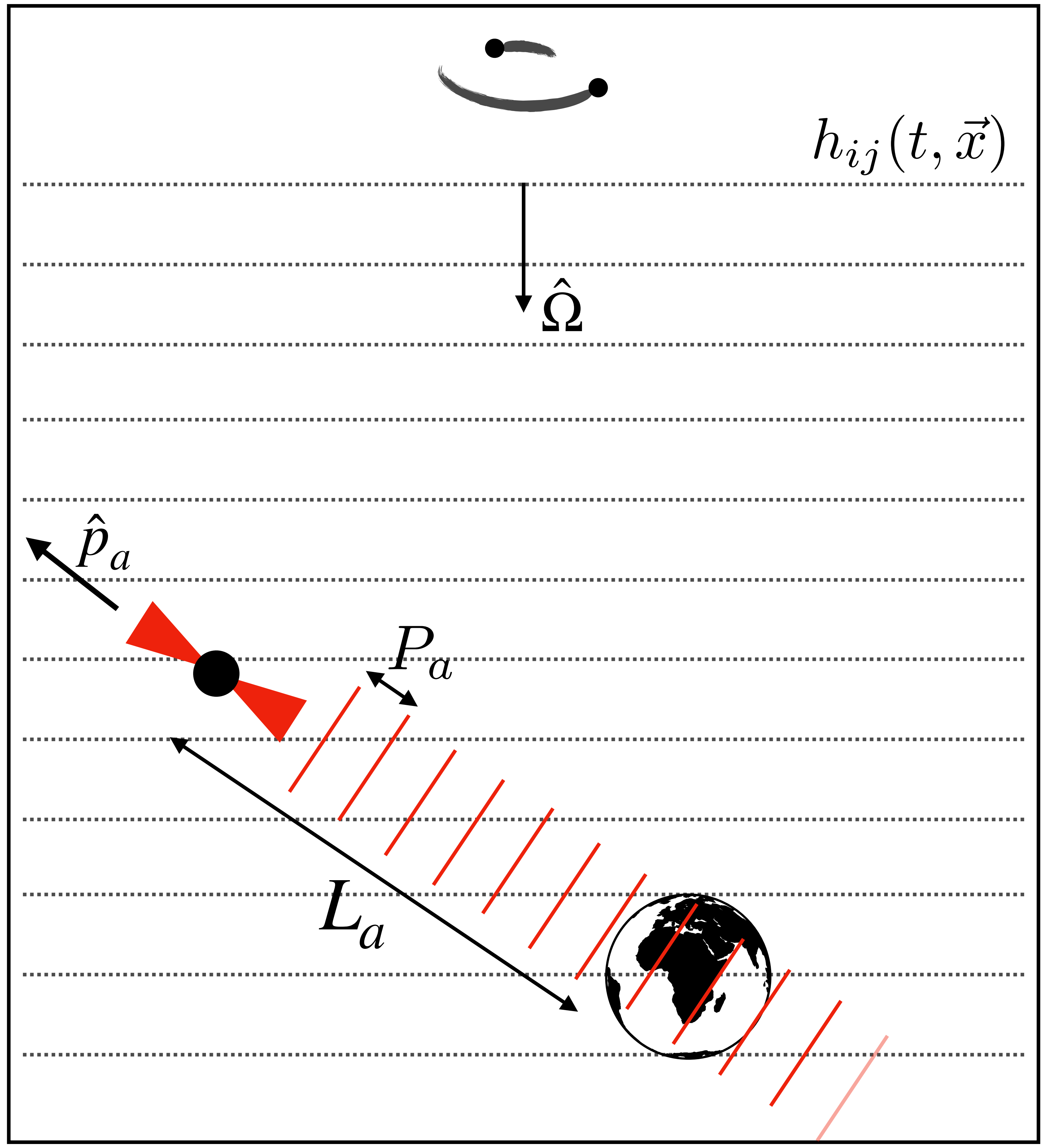


GW SIGNALS



GW SIGNALS

$$\frac{\delta P_a(t)}{P_a} = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)}\left[h_{ij}(t,\vec{0}) - h_{ij}(t-L_a, L_a\hat{p}_a)\right]$$



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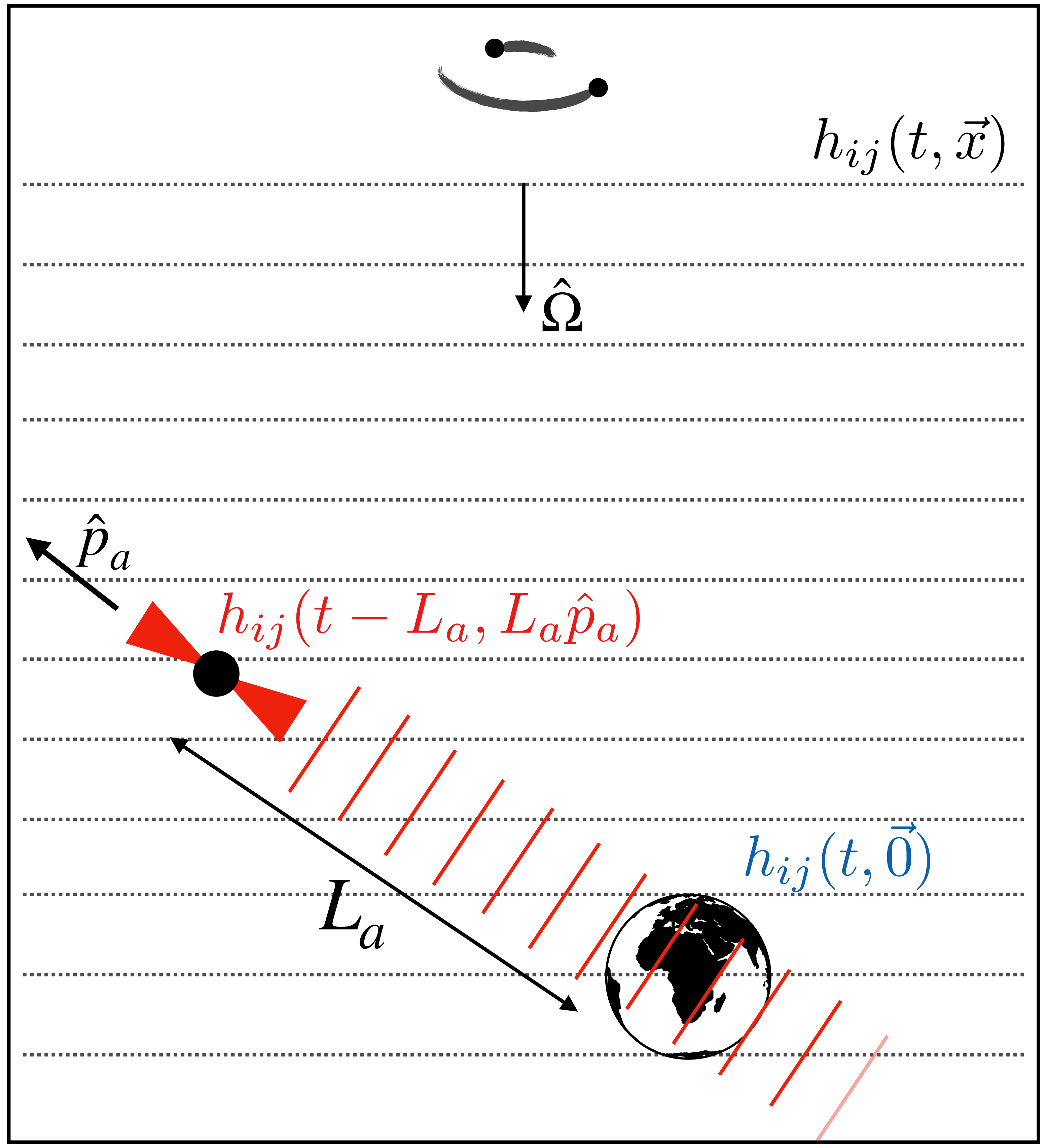
Earth term pulsar term



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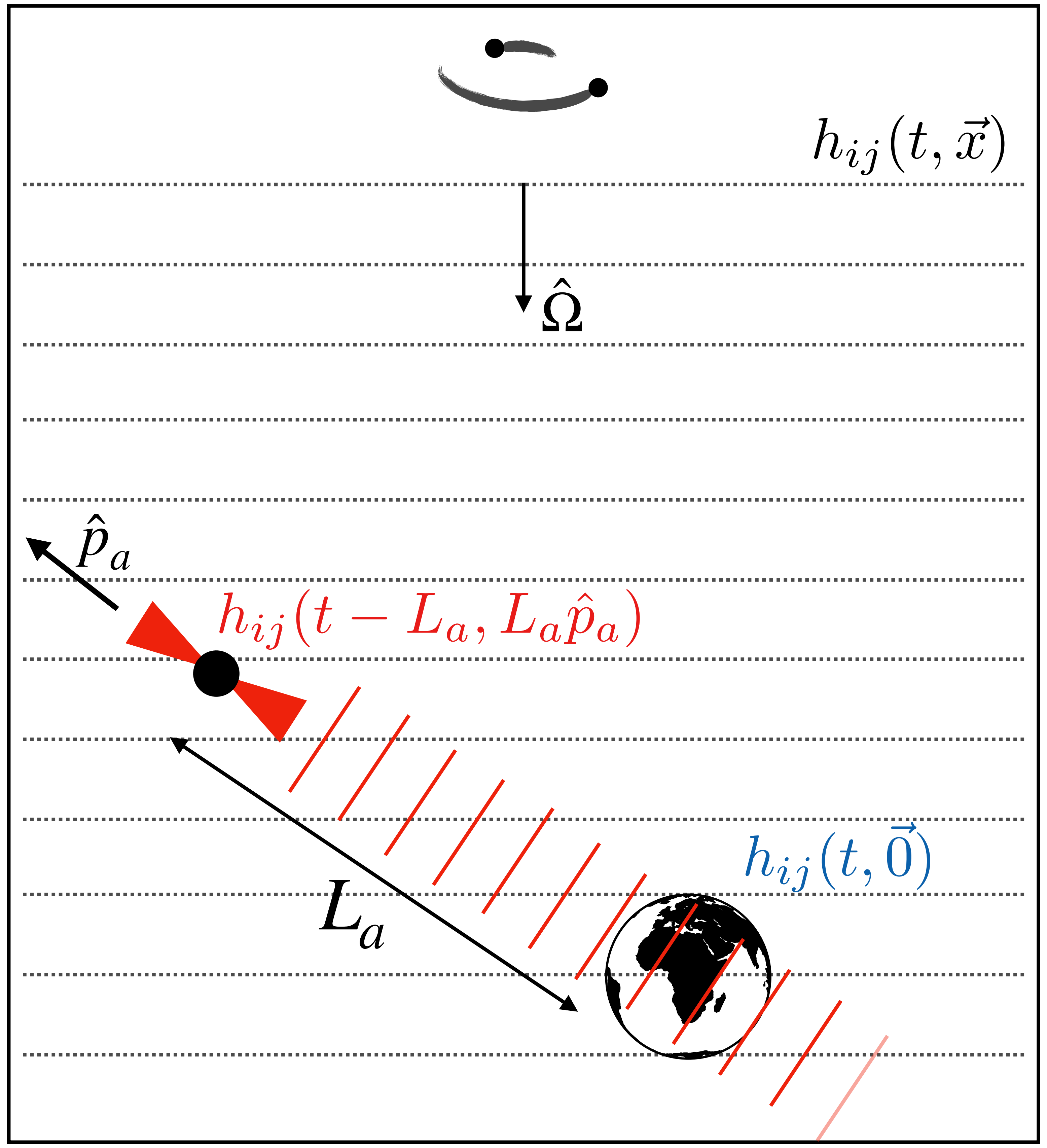
Earth term

pulsar term

geometric response



GW SIGNALS

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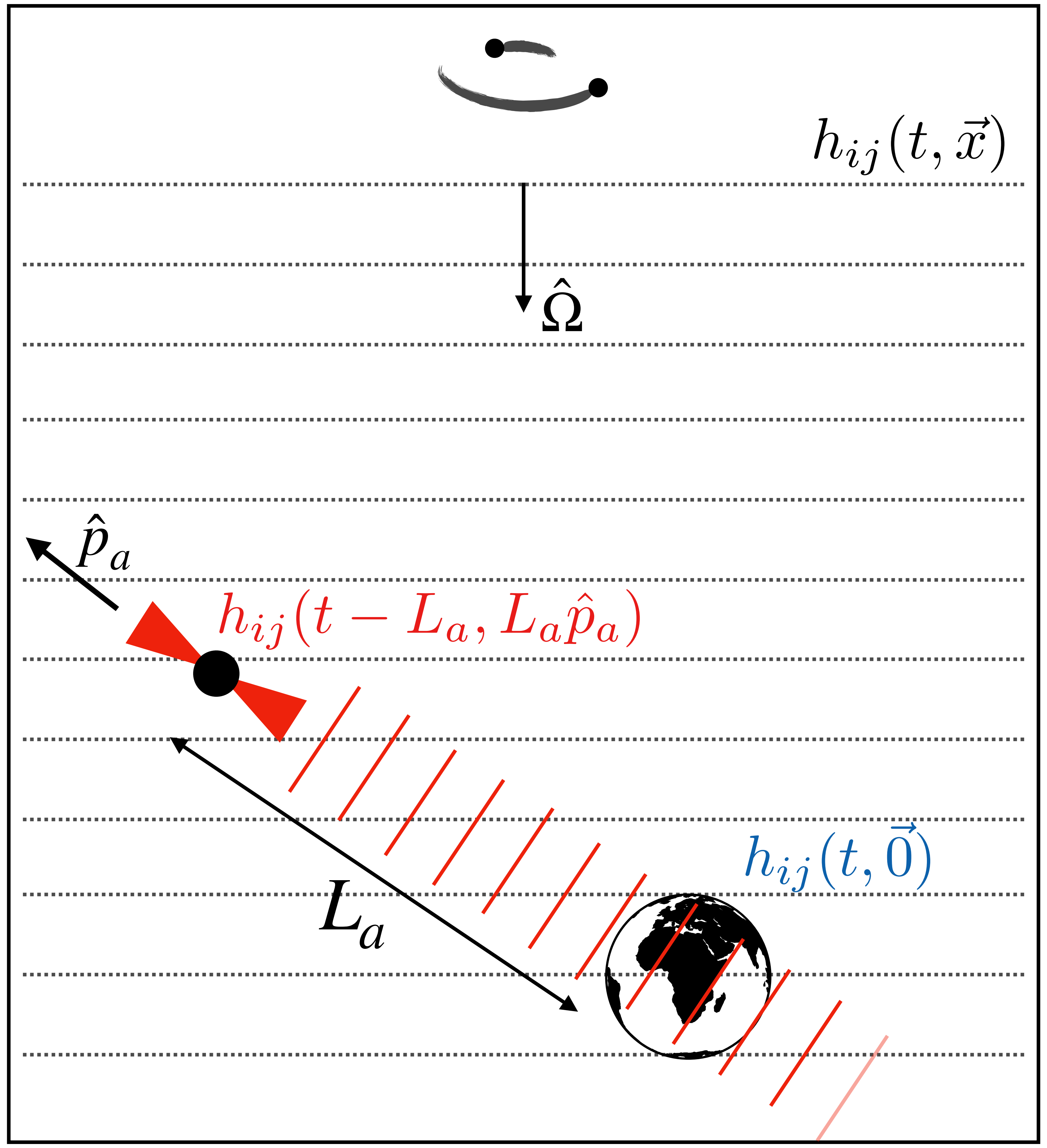
Earth term

pulsar term

geometric response

$$\delta t_a(t) = \int^t dt' \, \frac{\delta P_a(t')}{P_a}$$



CONTINUOUS WAVE

$$h_{ij}(t,\vec{x}) = \sum_A h^A(t - \hat{\Omega}\cdot\vec{x})\, e^A_{ij}(\hat{\Omega})$$

assuming:

- constant orbital frequency, ω
- plane of the orbit is ⊥ to the line of sight

$$h^+(x) = h_0 \cos(\omega x + \phi) \qquad h^\times(x) = h_0 \sin(\omega x + \phi)$$

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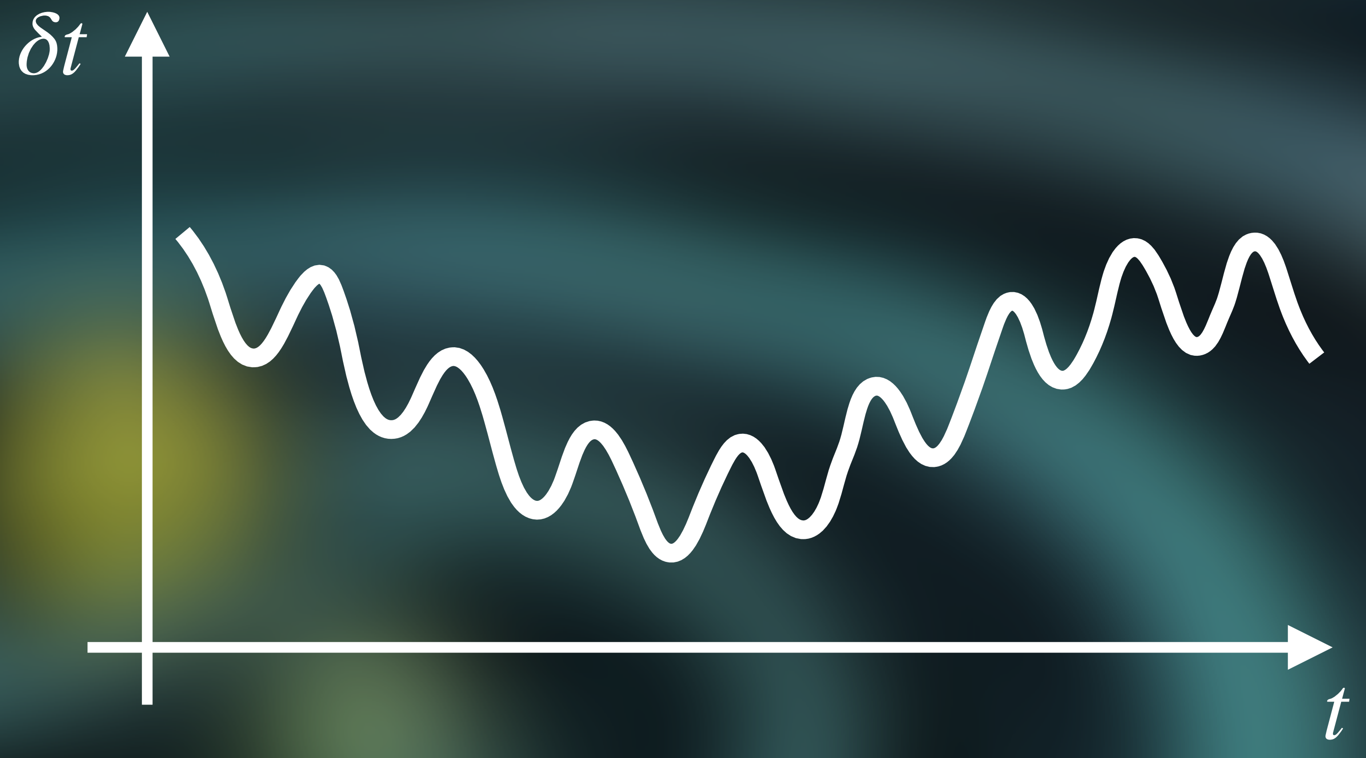
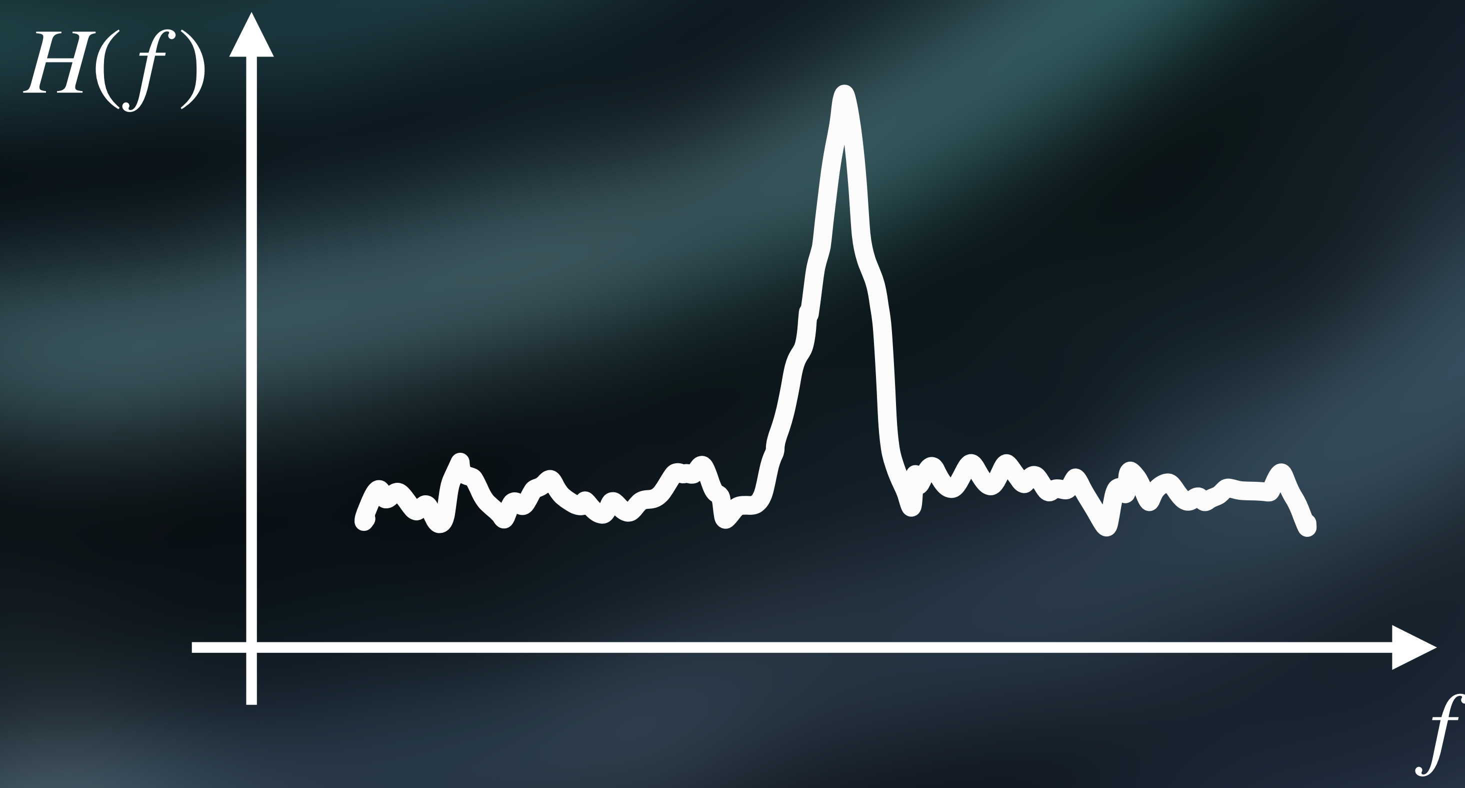
$$\delta t \propto F_A(h_A^e - h_A^p)$$





GW BACKGROUND

$$h_{ij}(t,\vec{x}) = \sum_{A} \int df \int d\hat{\Omega}\; \tilde{h}_A(f,\hat{\Omega})\; e^{A}_{ij}(\hat{\Omega})\; e^{-2\pi i f (t-\hat{\Omega}\cdot\vec{x})}$$

GW BACKGROUND

$$h_{ij}(t,\vec{x}) = \sum_A \int\! df \int\! d\hat{\Omega}\; \underbrace{\tilde{h}_A(f,\hat{\Omega})}_{\text{“Fourier” components}}\; \overbrace{e^A_{ij}(\hat{\Omega})}^{\text{polarization tensors}}\; \underbrace{e^{-2\pi i f(t-\hat{\Omega}\cdot\vec{x})}}_{\text{plane waves}}$$

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the complex functions $\tilde{h}^A(f,\hat{\Omega})$ are treated as random variables

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$$\langle \tilde{h}_A(f,\hat{\Omega})^* \tilde{h}_A(f',\hat{\Omega}') \rangle = \delta_{AA'}\delta(\hat{\Omega},\hat{\Omega}')\delta(f-f')\underbrace{H(f)}_{\text{GWB power spectrum}}$$

GW BACKGROUND

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the complex functions $\tilde{h}^A(f,\hat{\Omega})$ are treated as random variables

$$\langle \tilde{h}_A(f,\hat{\Omega})\rangle = 0 \qquad\qquad \langle \delta t_a(t)\rangle = 0$$

→

$$\langle \tilde{h}_A(f,\hat{\Omega})^* \tilde{h}_A(f',\hat{\Omega}')\rangle = \delta_{AA'}\delta(\hat{\Omega},\hat{\Omega}')\delta(f-f')\underbrace{H(f)}_{\text{GWB power spectrum}}$$

$$\langle \delta t_a(t_i)\delta t_b(t_j)\rangle \propto \Gamma_{ab} \int\! df\; H(f)\; e^{2\pi i f(t_i-t_j)}$$

GW BACKGROUND

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timing residual in the a-th pulsar at time t_i

GW BACKGROUND

$$\langle \underset{\big\uparrow}{\delta t_a(t_i)}\delta t_b(t_j)\rangle \propto \Gamma_{ab} \underbrace{\int df \; H(f) \; e^{2\pi i f(t_i - t_j)}}_{\text{the signal is time-correlated}}$$

timing residual in the a-th pulsar at time t_i

GW BACKGROUND

$$\underset{\substack{\uparrow\\ \text{timing residual in the } a\text{-th pulsar at time } t_i}}{\langle \delta t_a(t_i)\delta t_b(t_j)\rangle} \propto \Gamma_{ab} \underbrace{\int df\; H(f)\; e^{2\pi i f(t_i-t_j)}}_{\text{the signal is time-correlated}}$$

the **power spectrum** of the GWB-induced signal is **common across pulsars**

GW BACKGROUND

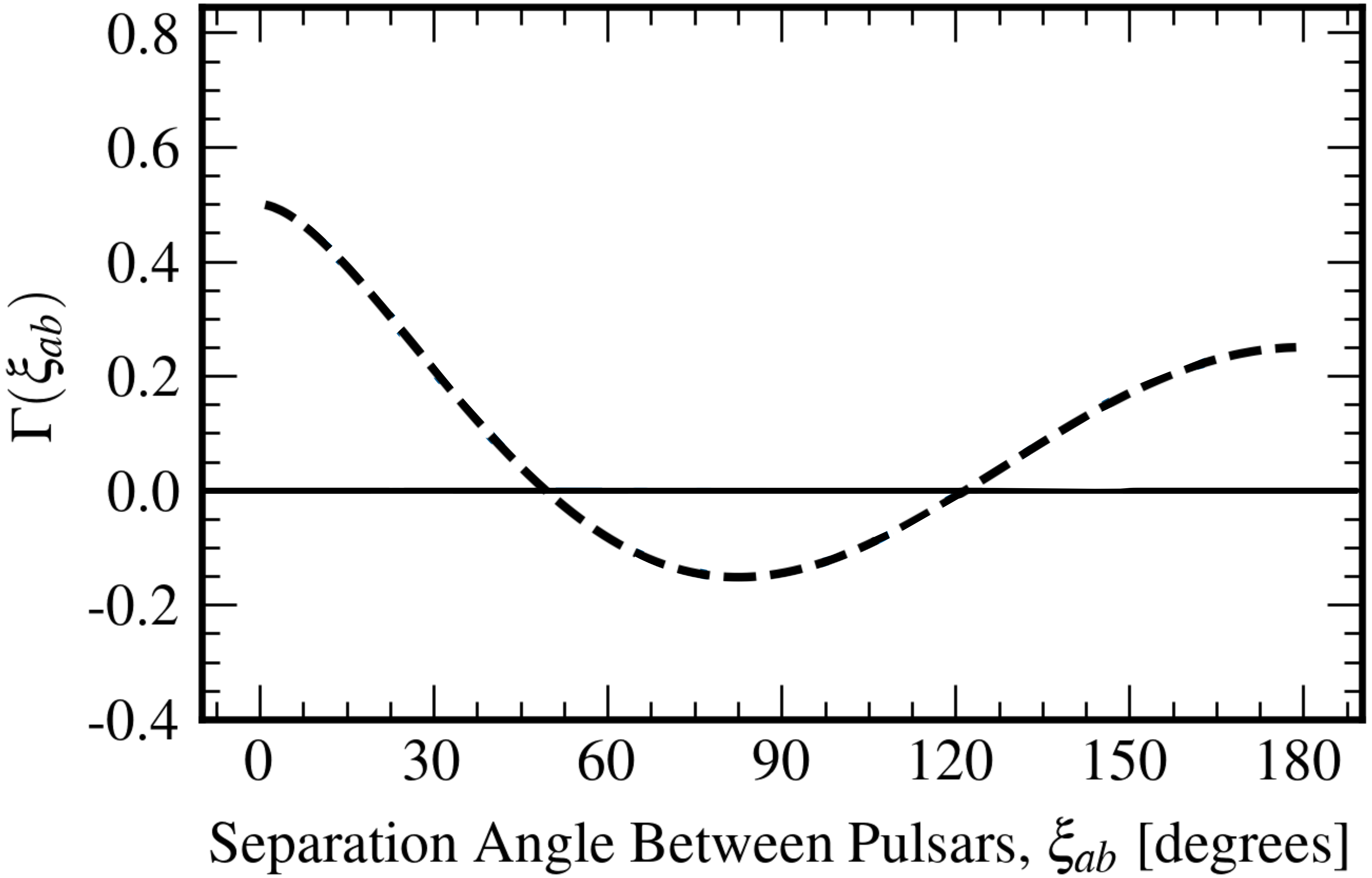
$$\langle \delta t_a(t_i)\delta t_b(t_j)\rangle \propto \boxed{\Gamma_{ab}} \int df\ H(f)\ e^{2\pi i f(t_i - t_j)}$$

the signal is **correlated among pulsars**

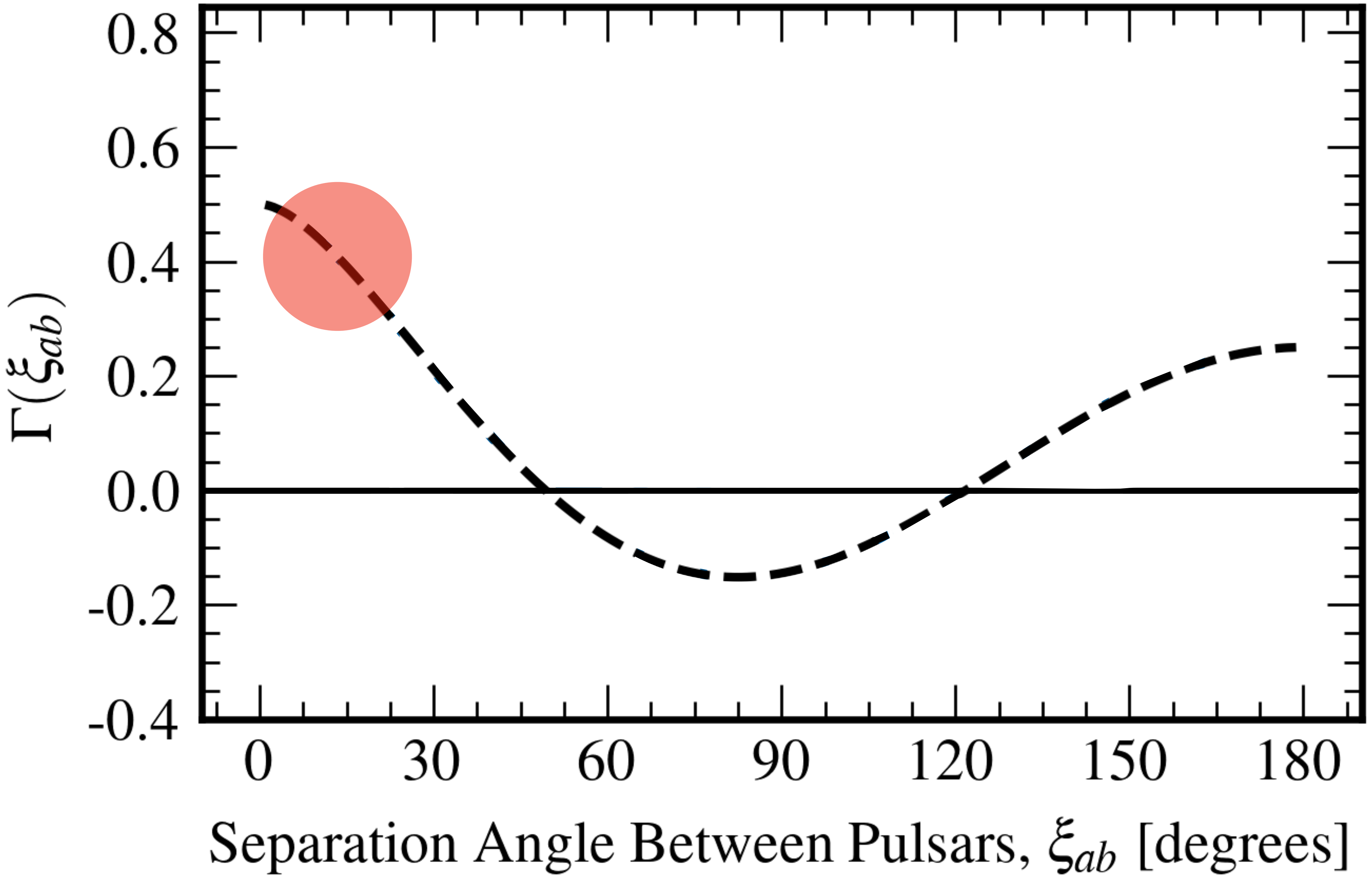
$$\Gamma_{ab} = \int d\hat{\Omega} \sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) [1 + \delta_{ab}]$$

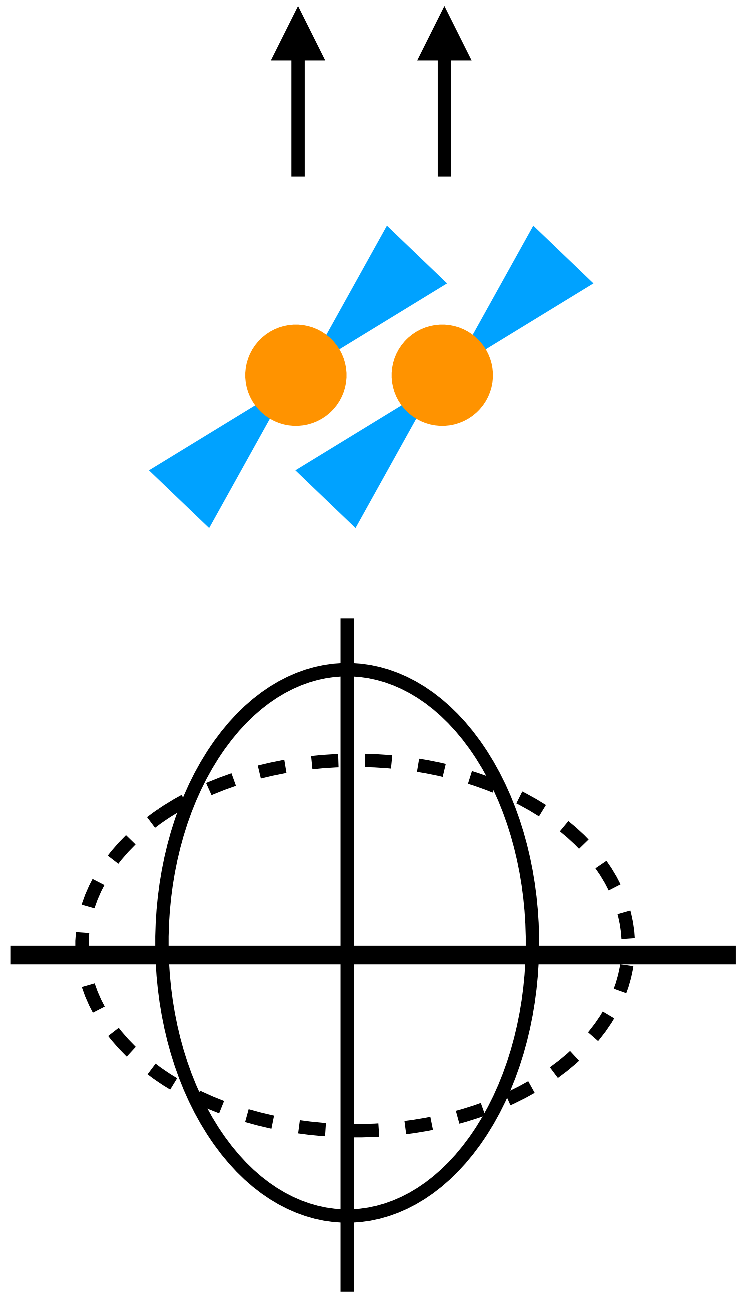
$$F_a^A(\hat{\Omega}) = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)} e_{ij}^A(\hat{\Omega})$$

HELLINGS & DOWNS CURVE



HELLINGS & DOWNS CURVE





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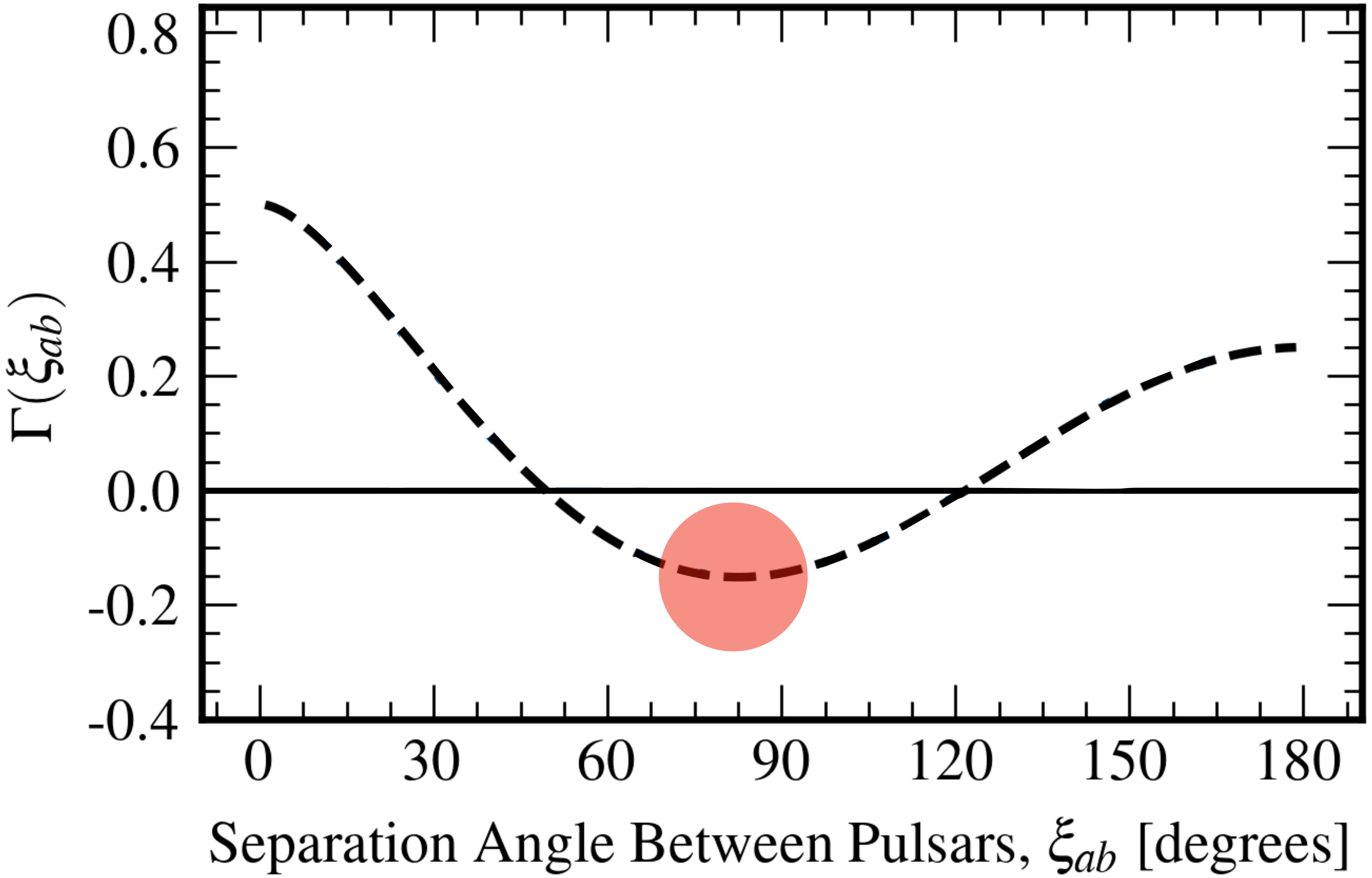

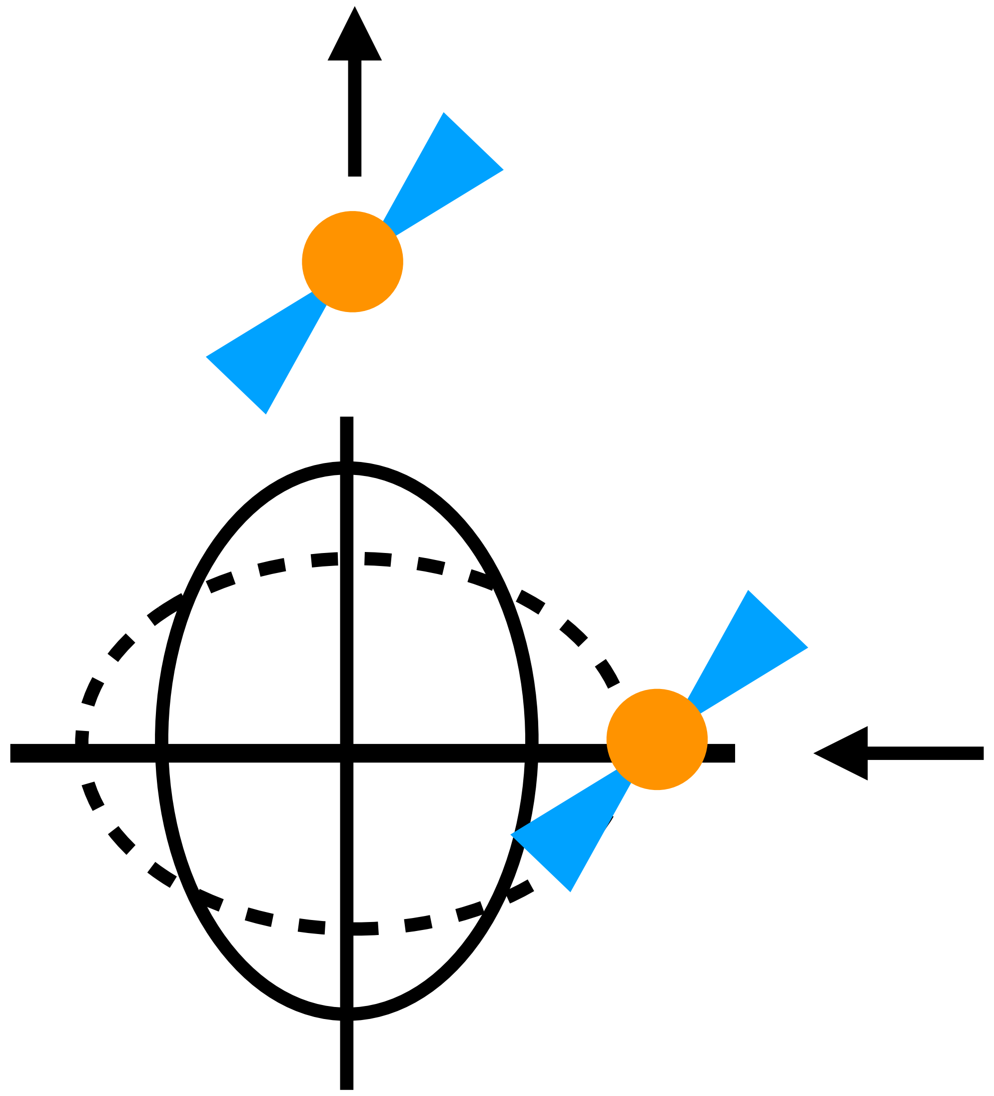

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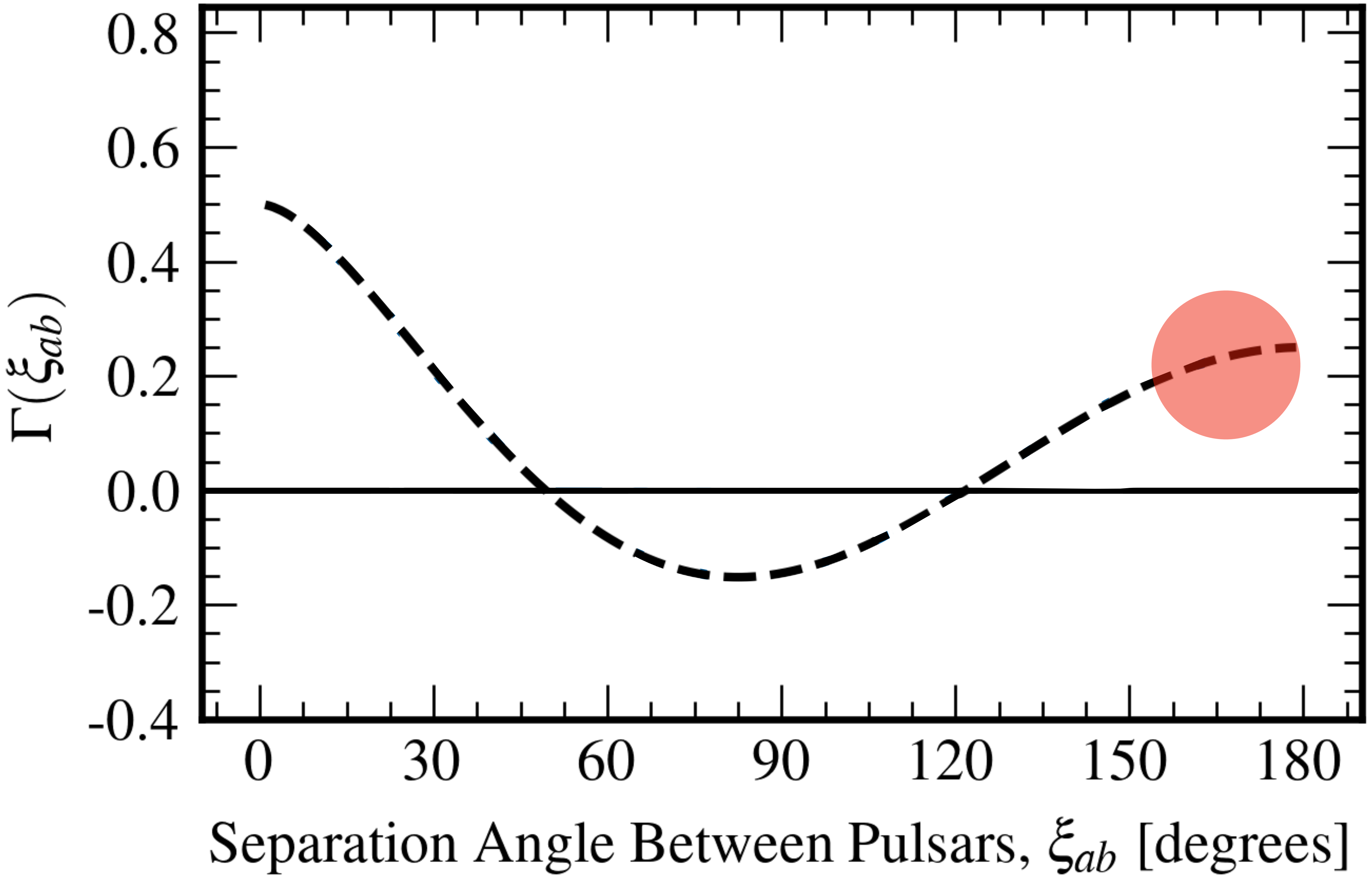
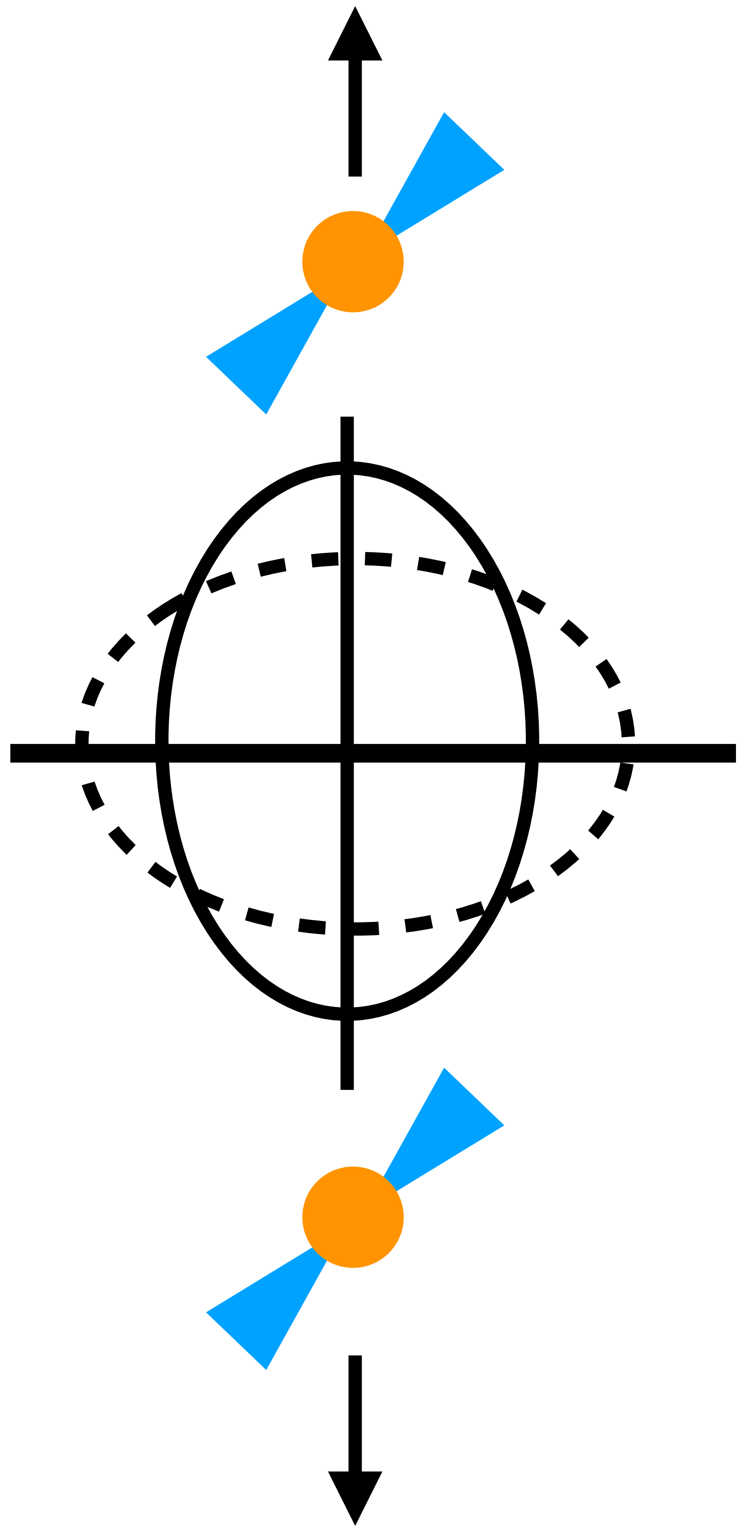
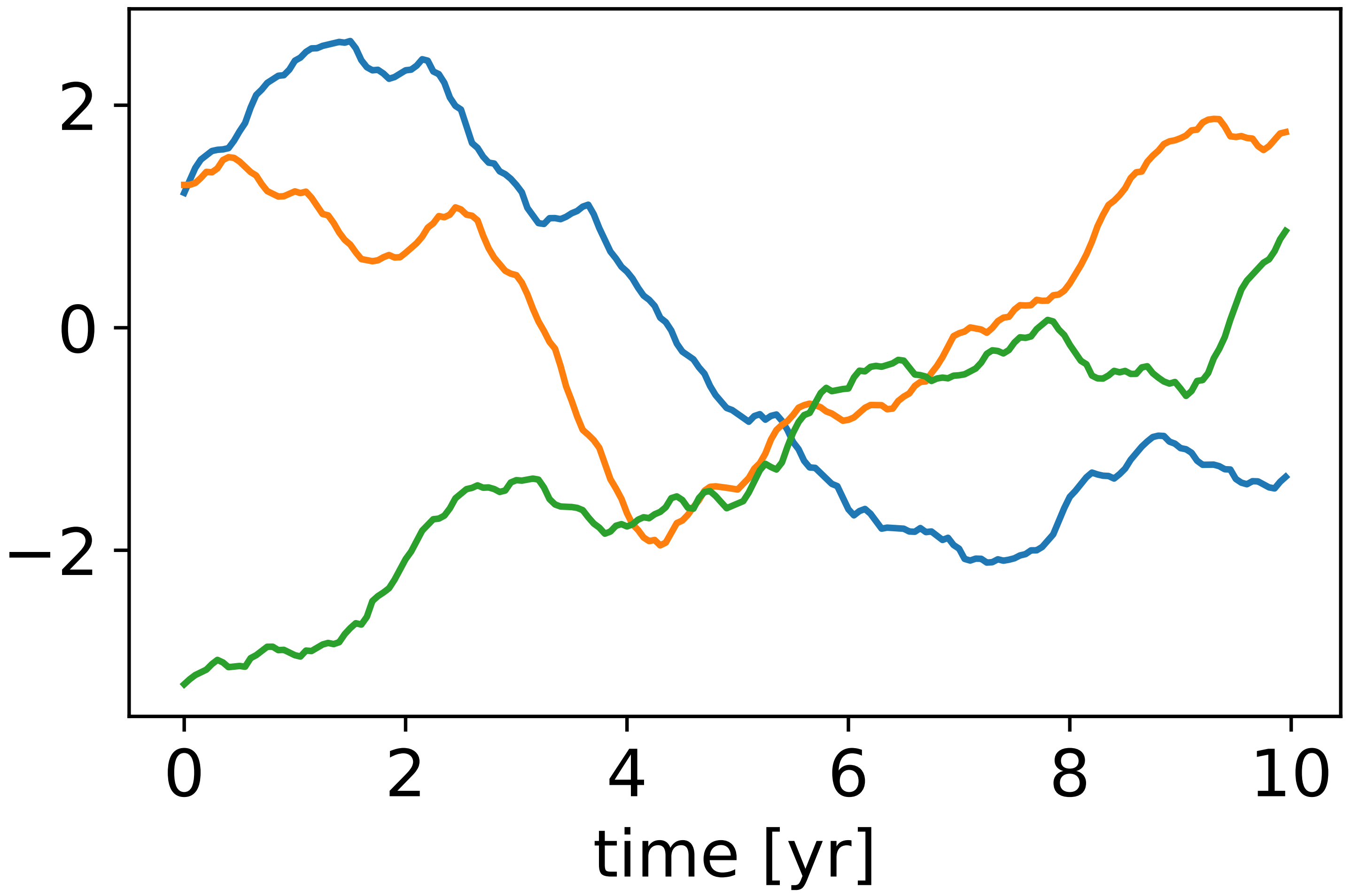




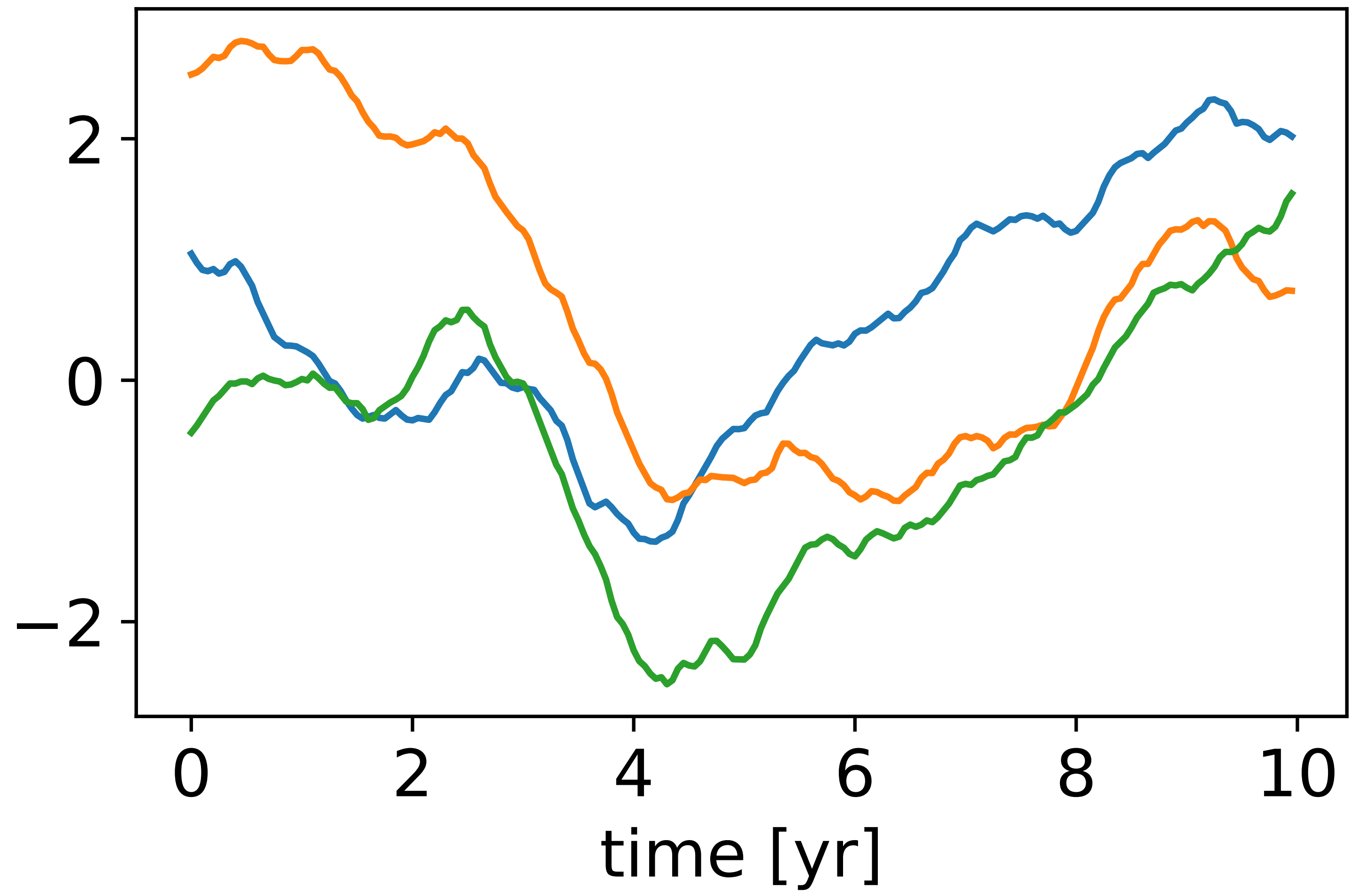
CORRELATIONS EXAMPLE

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



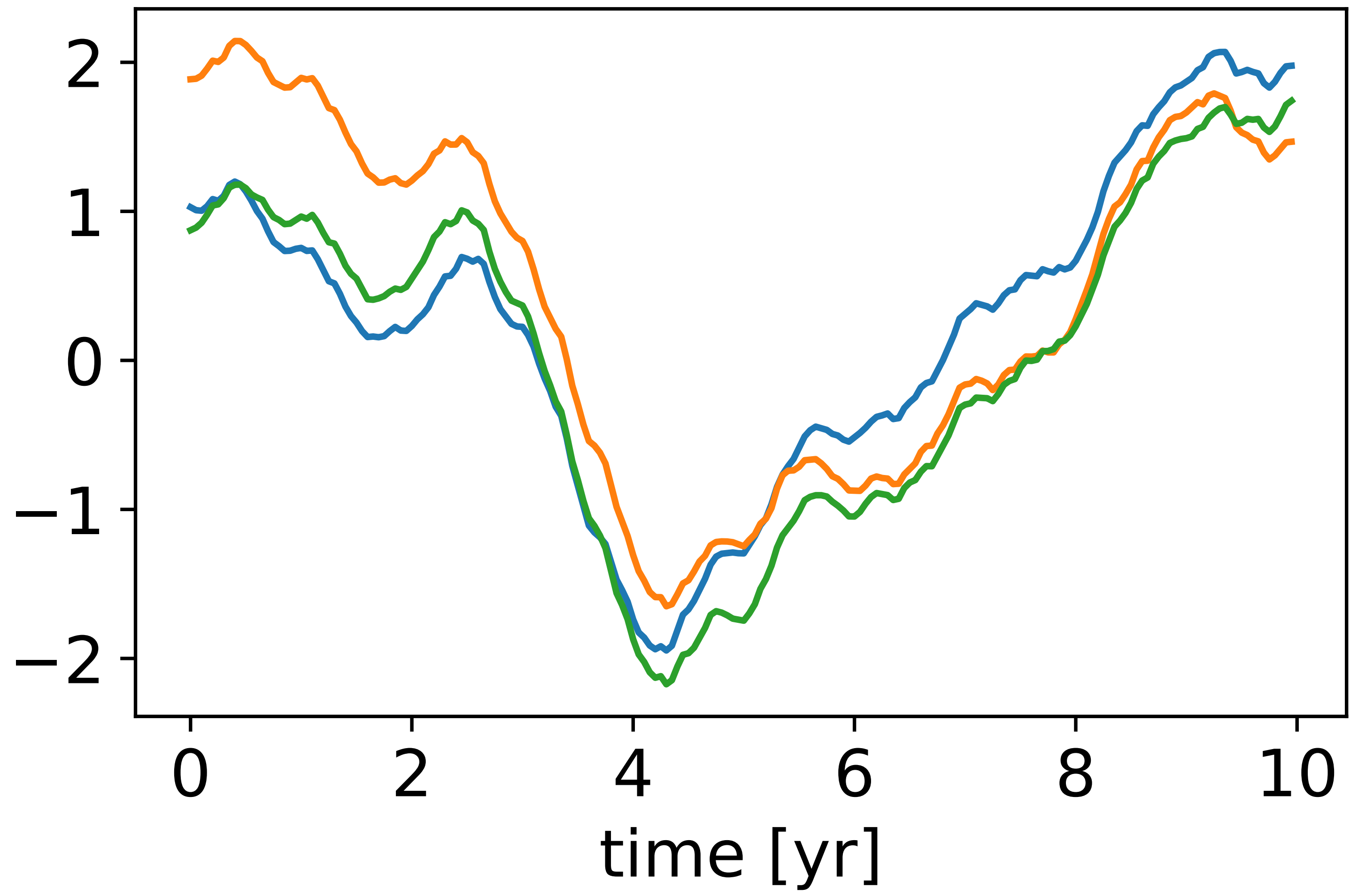
uncorrelated

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix}$$



moderately correlated

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0.95 & 0.95 \\ 0.95 & 1 & 0.95 \\ 0.95 & 0.95 & 1 \end{pmatrix}$$



strongly correlated

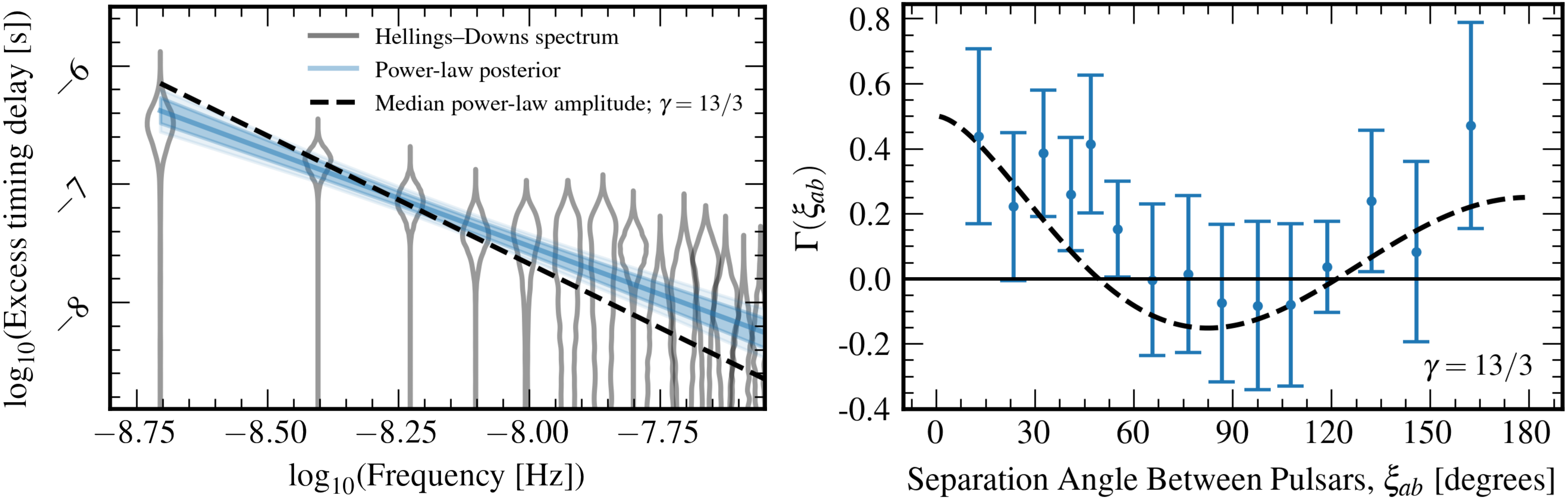
slide from Joe Romano

searching for a GWB means searching for **stochastic delays** in the TOAs that are **common across pulsars** and **spatially correlated**

EVIDENCE FOR GWB

NANOGrav 15-year

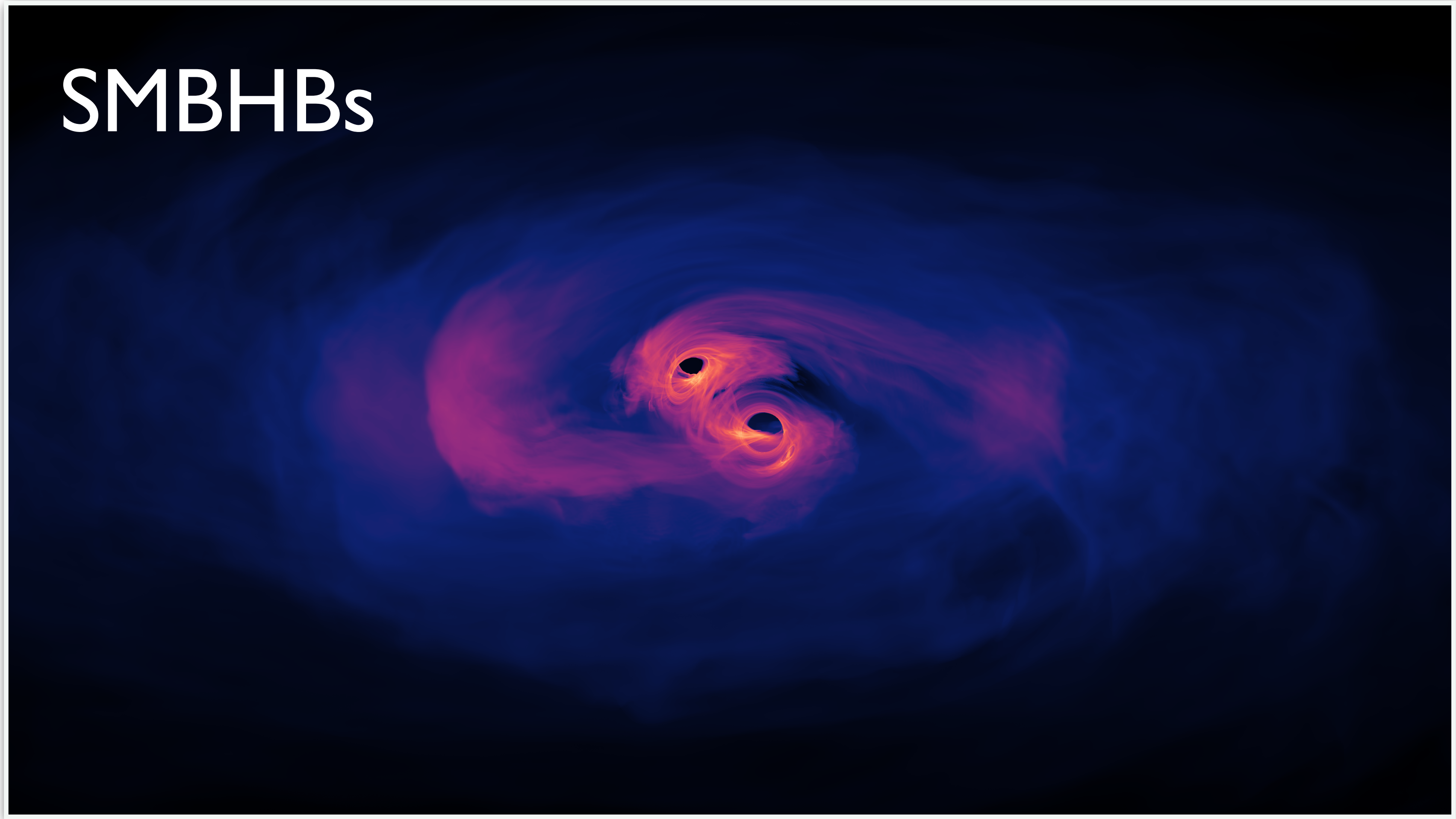
Agazie et al. [2306.16213]



we find evidence for Hellings & Downs correlation with a p-value of $5 \times 10^{-5} - 1.9 \times 10^{-4}$ (approx. $3.5 - 4\sigma$)

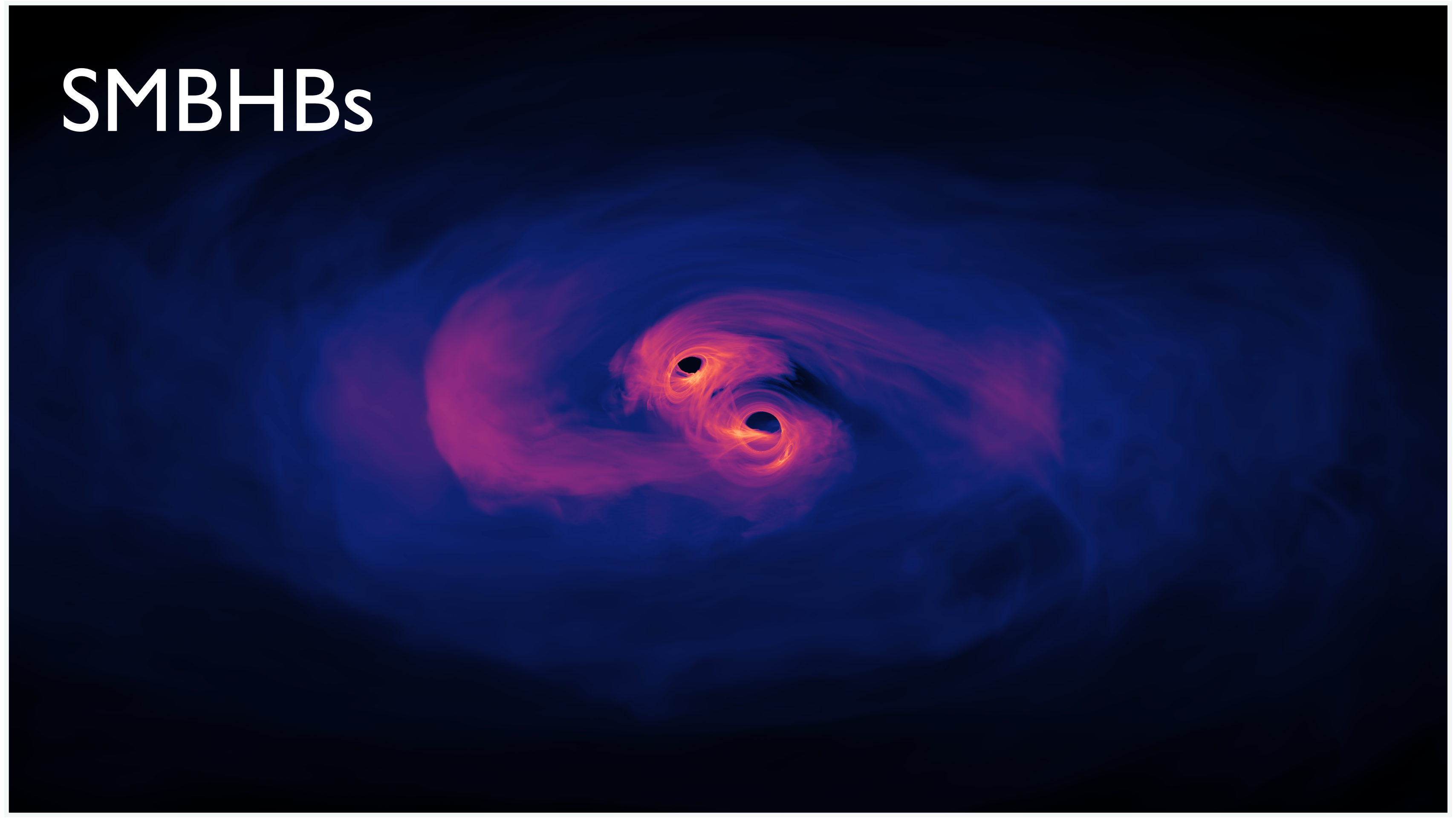
what is the source?

ASTRO OR COSMO?

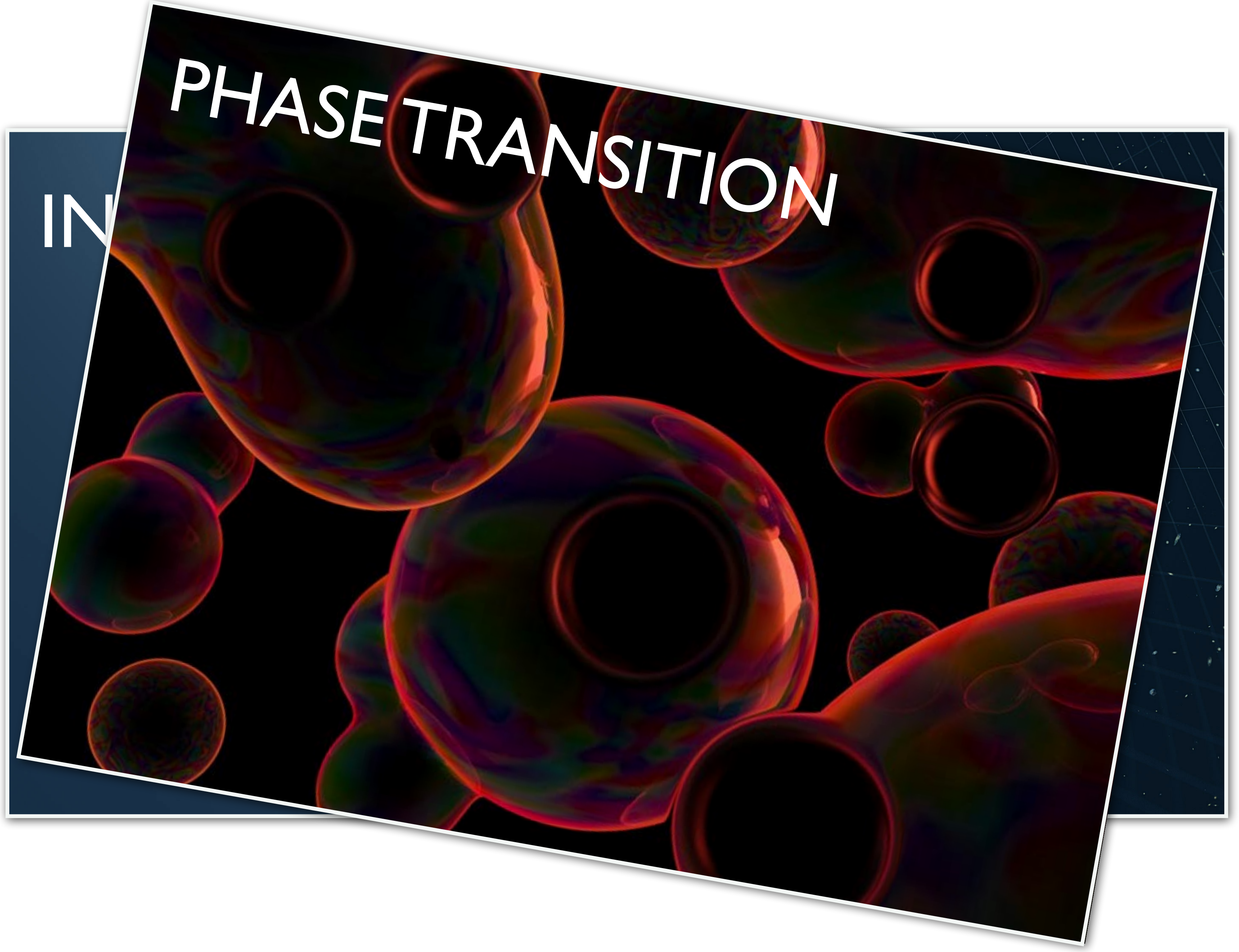


VS

ASTRO OR COSMO?



vs



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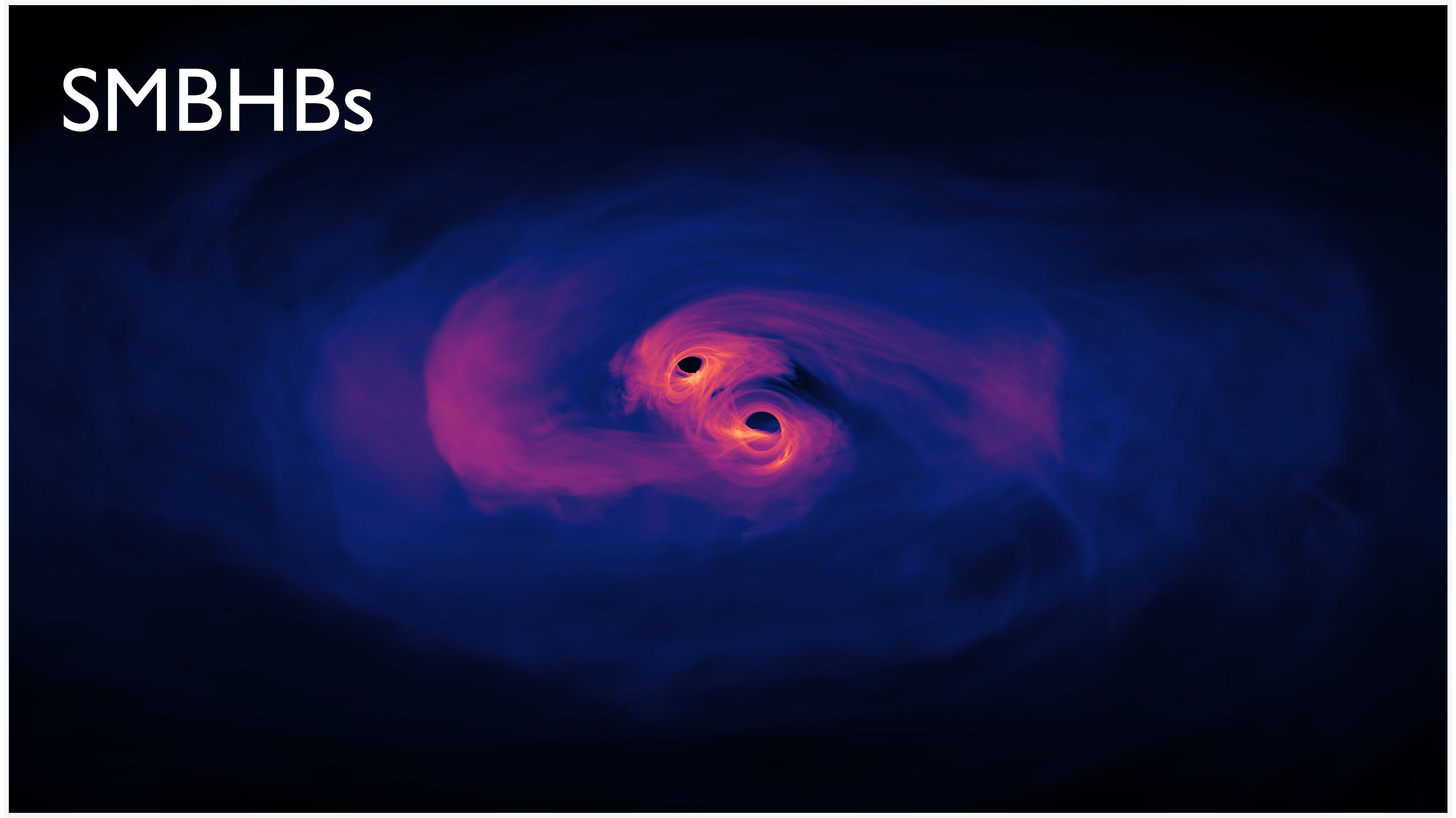

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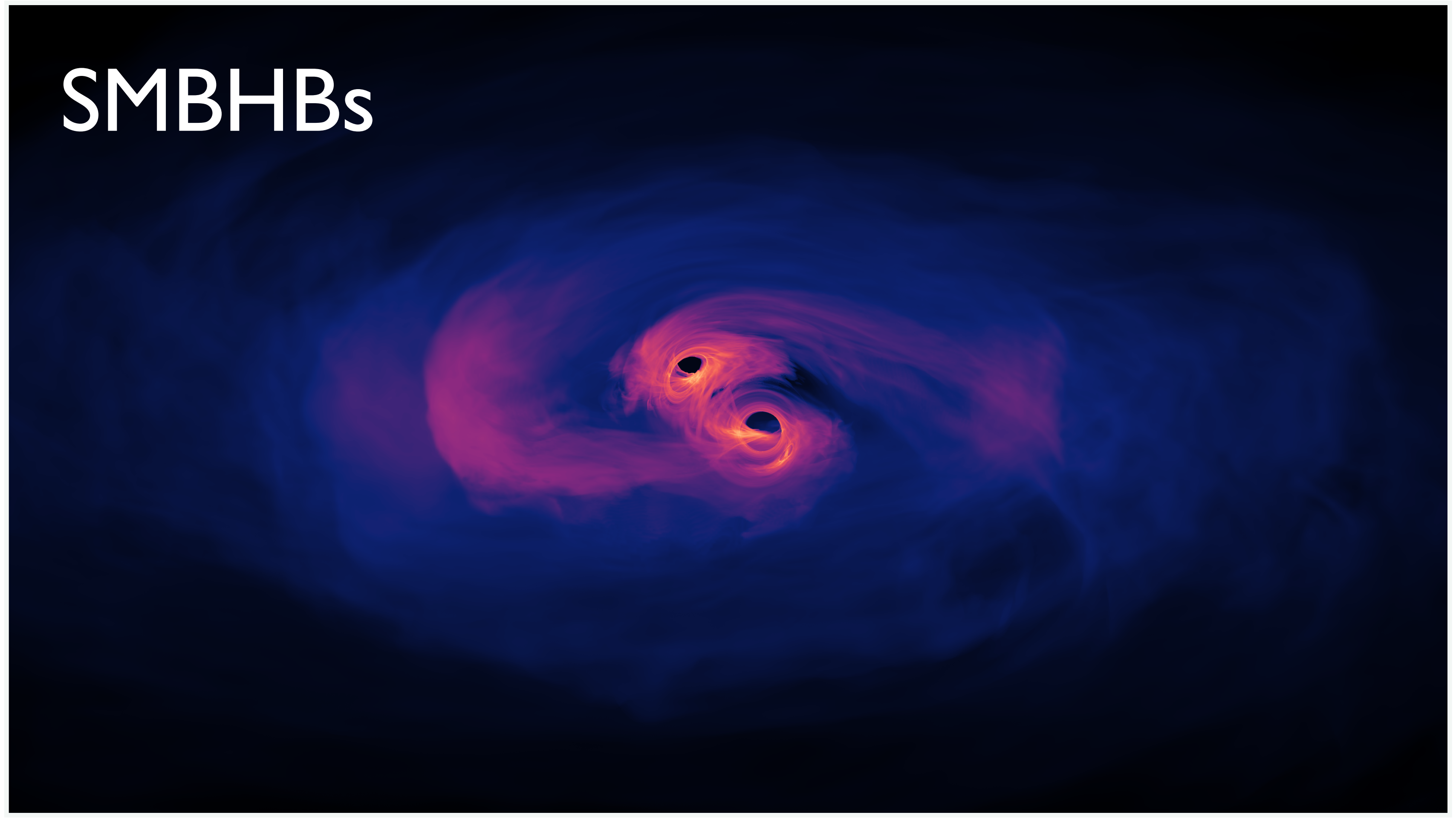

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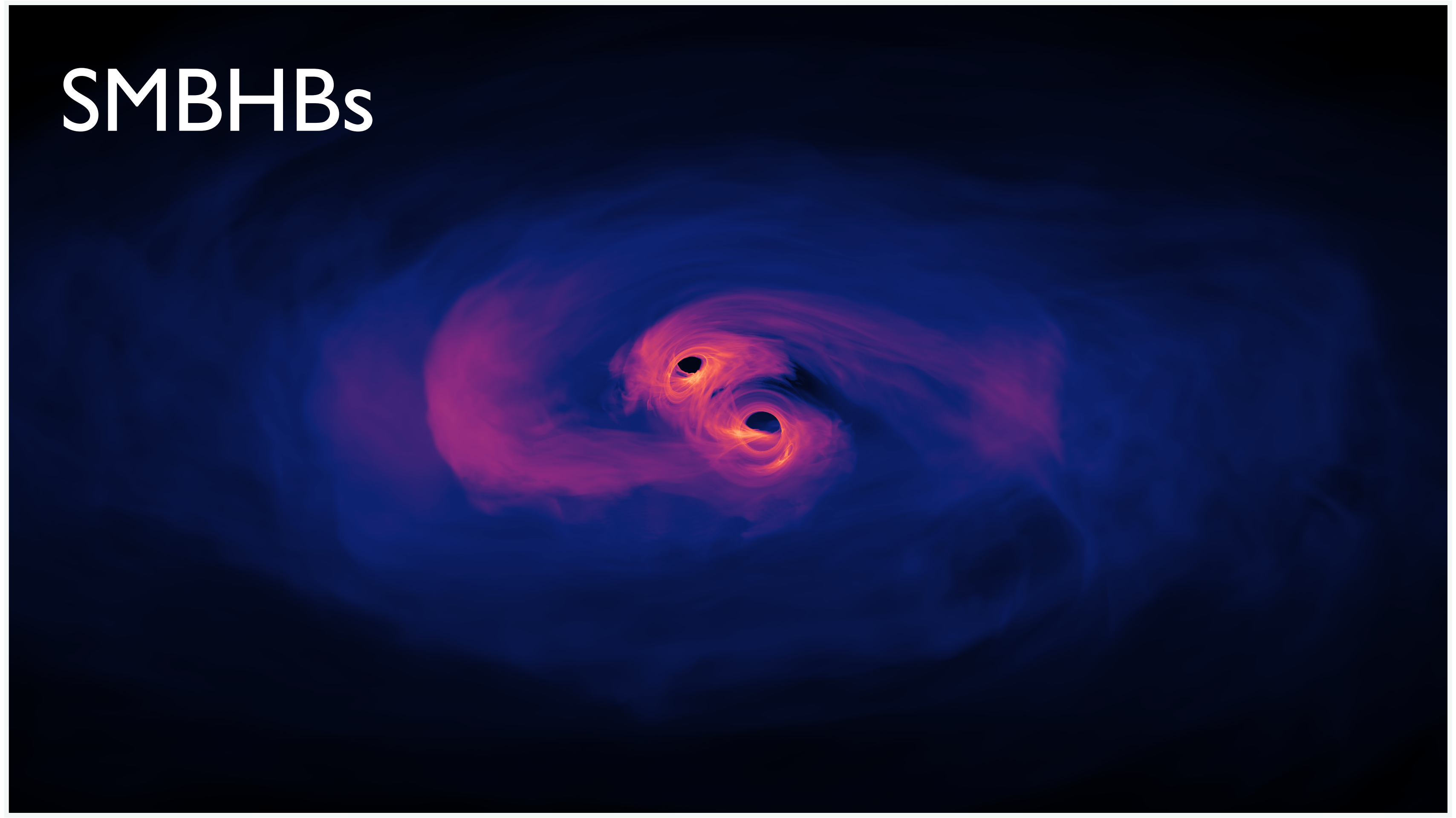
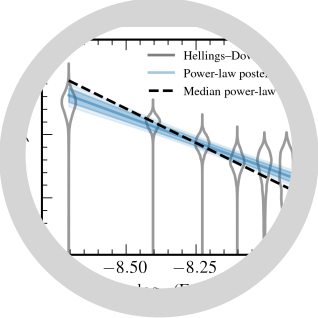
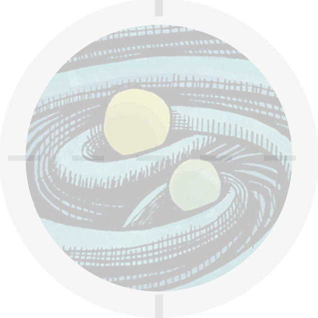
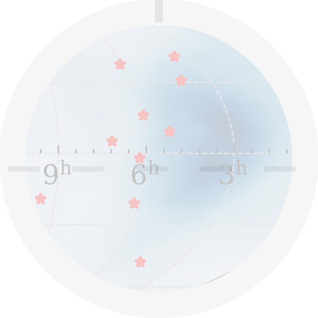


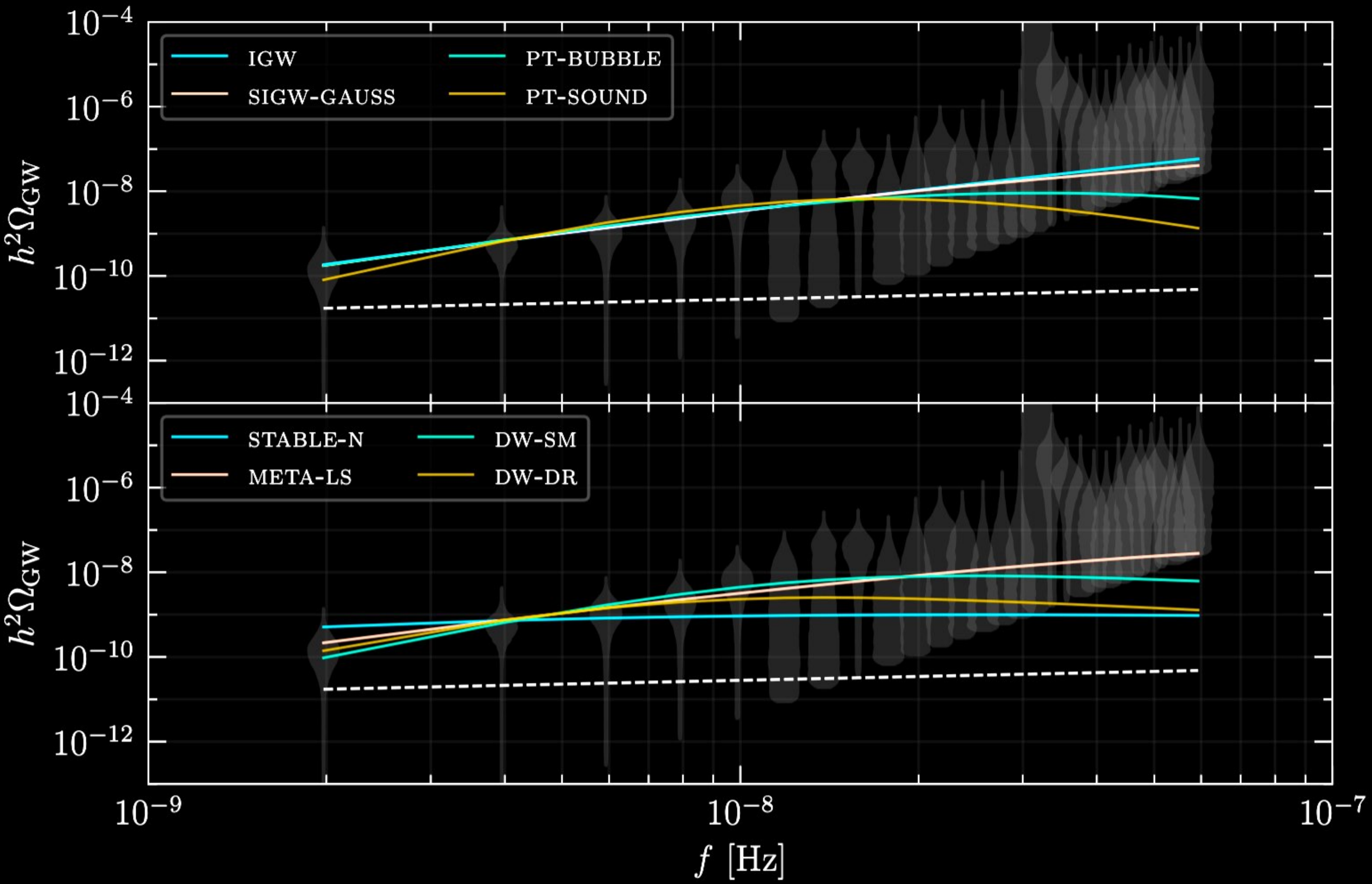
vs

HOW TO TELL THE DIFFERENCE

what are the distinguishing features of astrophysical and cosmological signals?

astrophysical	VS	cosmological
$\Omega_{GW} \propto f^{2/3}$ at intermediate frequencies turn over at low and high frequencies		$\Omega_{GW} \propto f^{3}$ at low frequencies (?) ???
GW signal from individual sources can be resolved from the background		no resolvable sources
sizable anisotropies in the GWB sky		~ isotropic

Afzal et al. (**NANOGrav**) [2306.16219]



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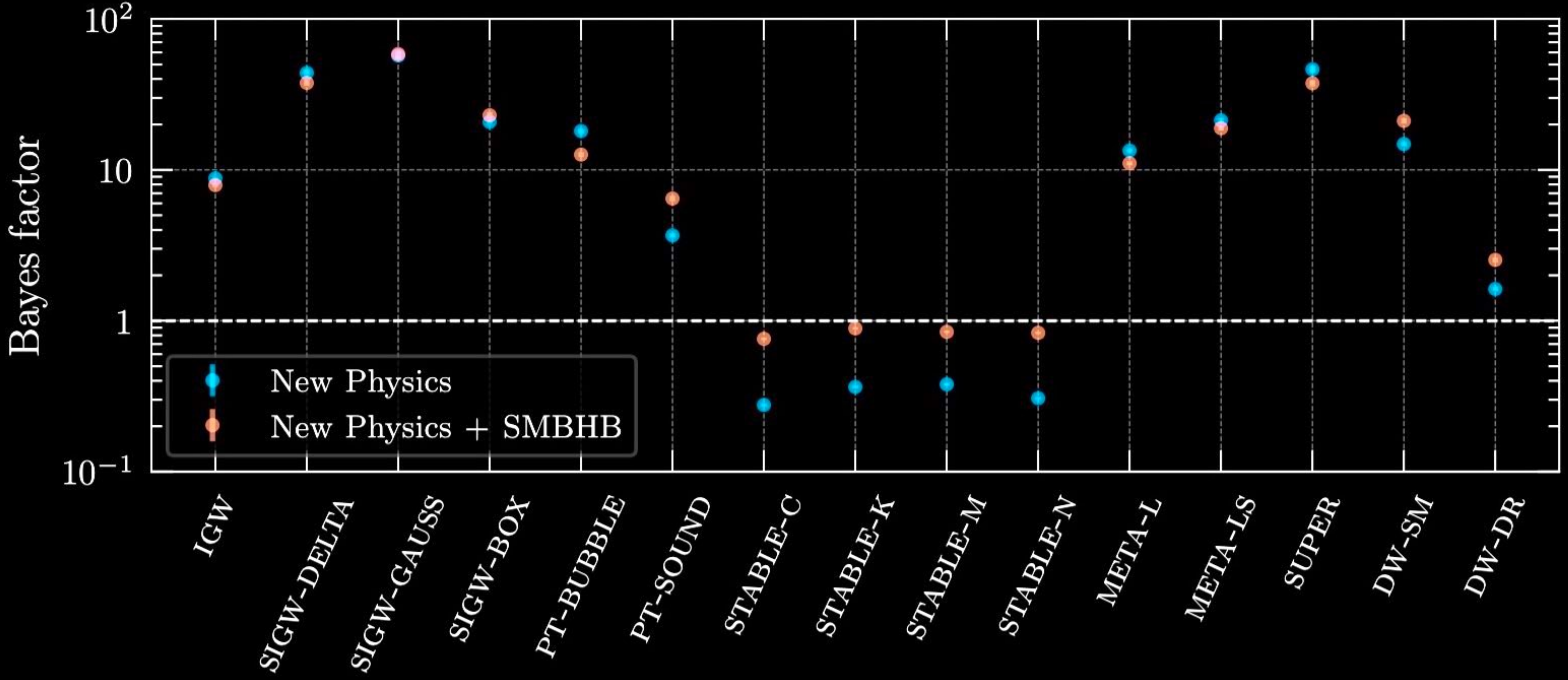

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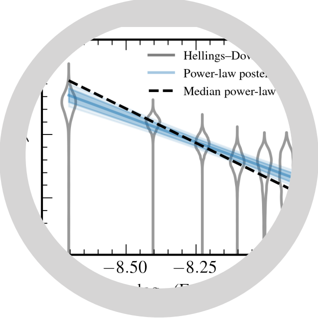
astrophysical ← VS → **cosmological**

$\Omega_{\rm GW} \propto f^{2/3}$ at intermediate frequencies
turn over at low and high frequencies



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???

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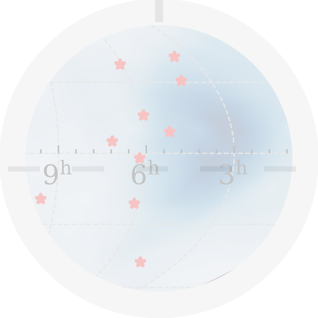
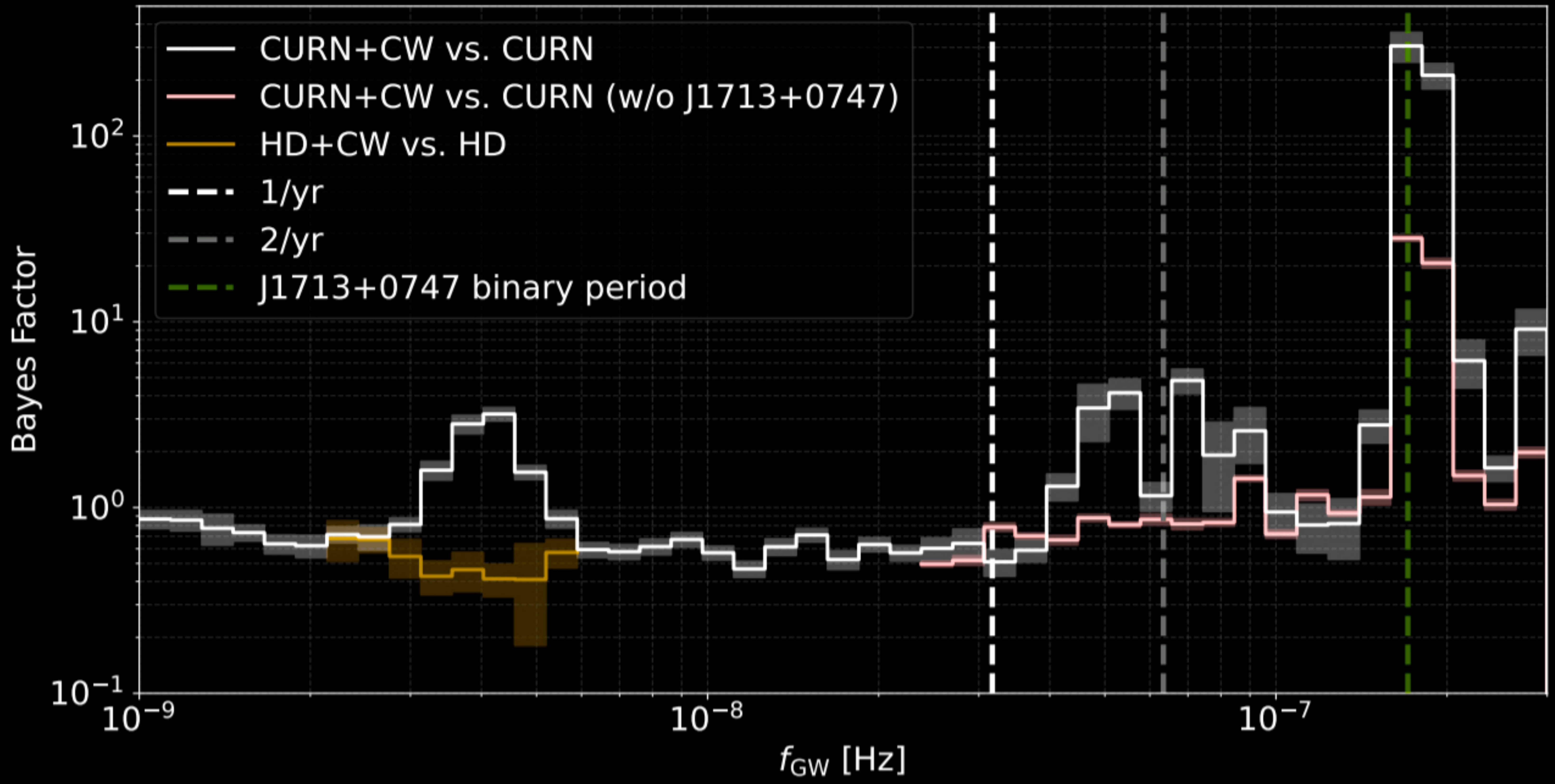
no resolvable sources

sizable anisotropies in the GWB sky



~ isotropic

Agazie et al. (**NANOGrav**) [2306.16222]



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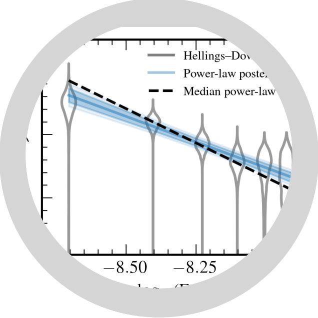
$\Omega_{\mathrm{GW}} \propto f^{2/3}$ at intermediate frequencies turn over at low and high frequencies		$\Omega_{\mathrm{GW}} \propto f^{3}$ at low frequencies (?) ???
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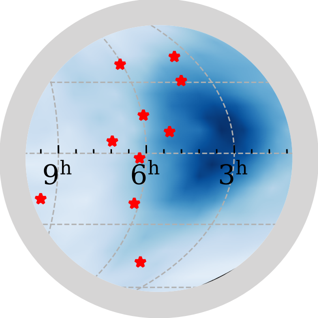
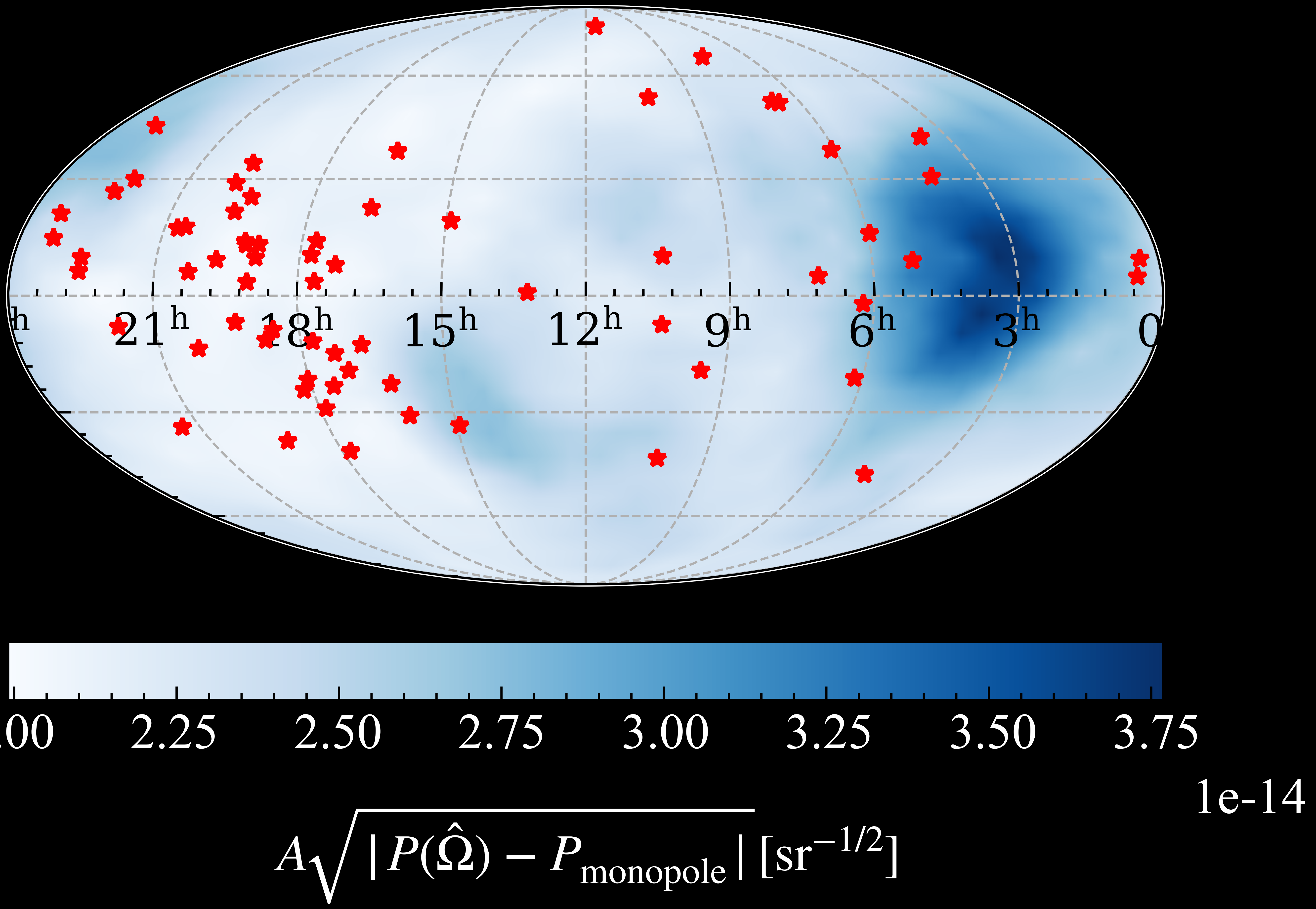
sizable anisotropies in the GWB sky		~ isotropic


21^{h}
18^{h}
15^{h}
12^{h}
9^{h}
6^{h}
3^{h}
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
1e-14
$A\sqrt{|P(\hat{\Omega}) - P_{\mathrm{monopole}}|}\,[\mathrm{sr}^{-1/2}]$

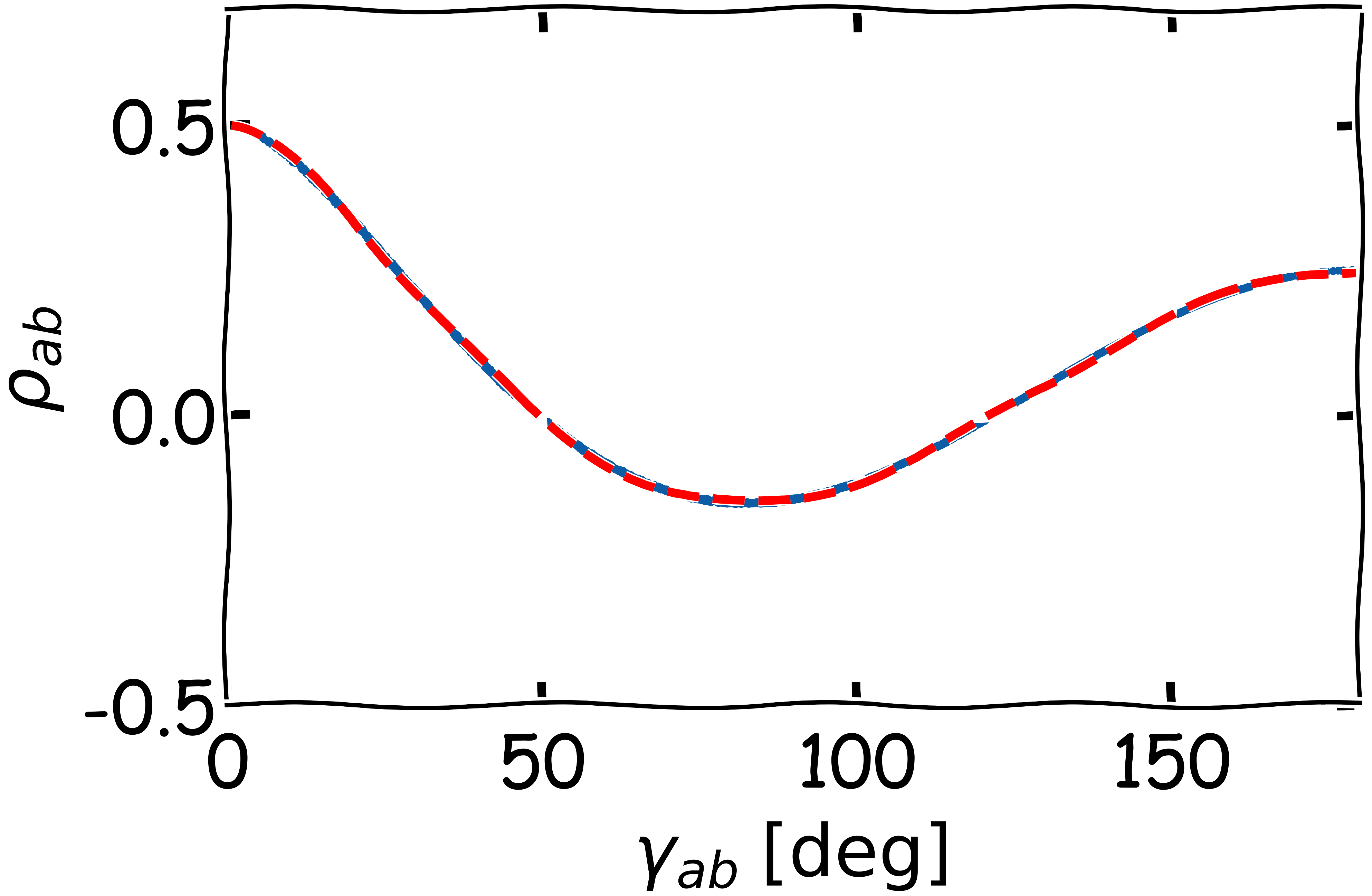
a deep dive on anisotropies

THE BASIC IDEA

isotropic GWB

$$\langle \tilde{h}_A^*(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle \propto \delta(\hat{\Omega},\hat{\Omega}')$$

↓



THE BASIC IDEA

anisotropic GWB

$$\langle \tilde{h}^*_A(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle \propto \delta(\hat{\Omega},\hat{\Omega}')P(\hat{\Omega})$$

$$\downarrow$$

$$\langle \rho_{ab} \rangle \propto \sum_A \sum_k F^A_{a,k} F^A_{b,k} \, P_k \longleftarrow$$

GWB power

pulsars
correlations

geometric
response

$$F^A_{a,k} = \frac{\hat{p}^i_a \hat{p}^j_a}{2(1+\hat{\Omega}_k \cdot \hat{p}_a)} e^A_{ij}(\hat{\Omega}_k)$$

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THE BASIC IDEA

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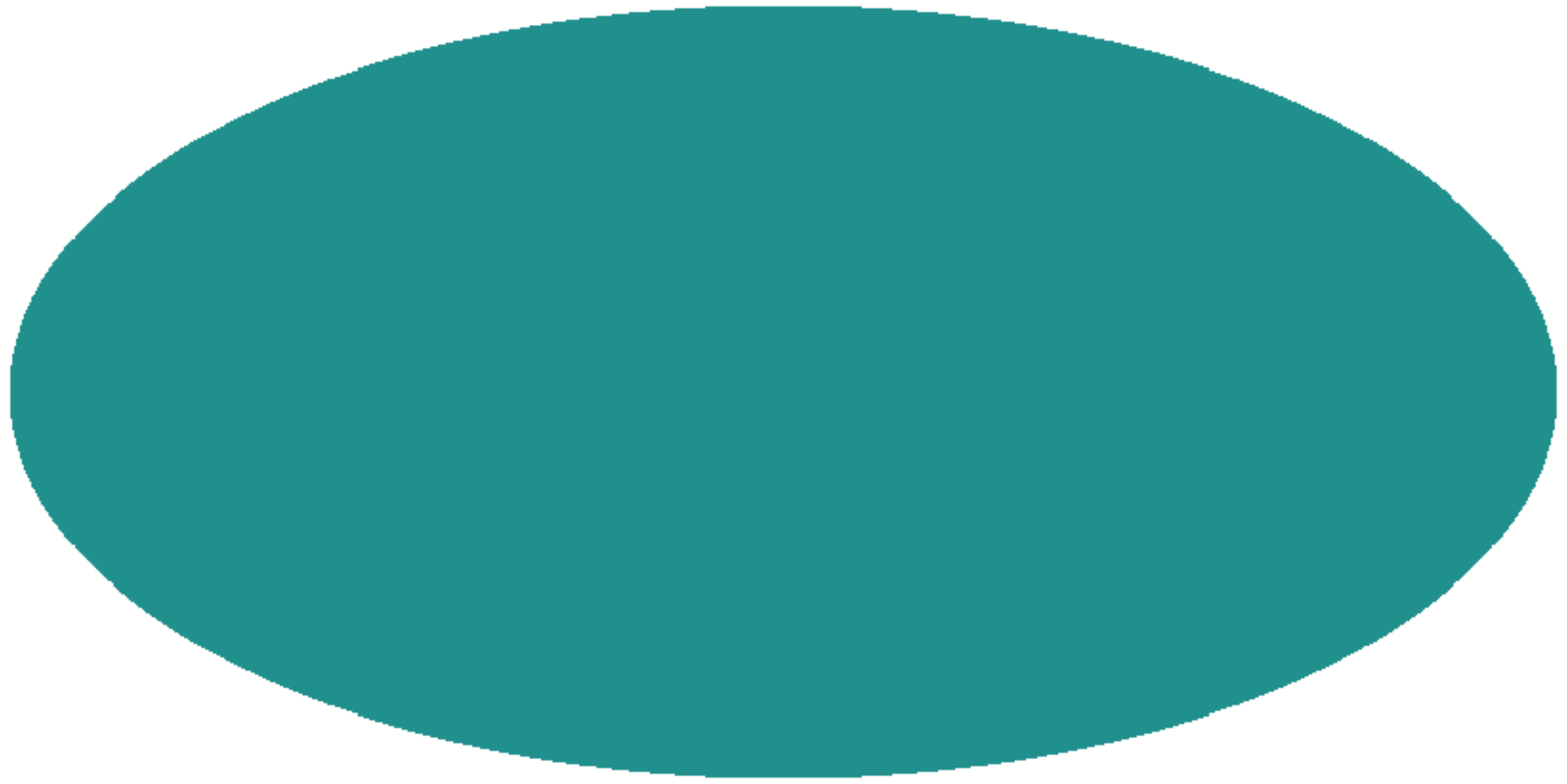
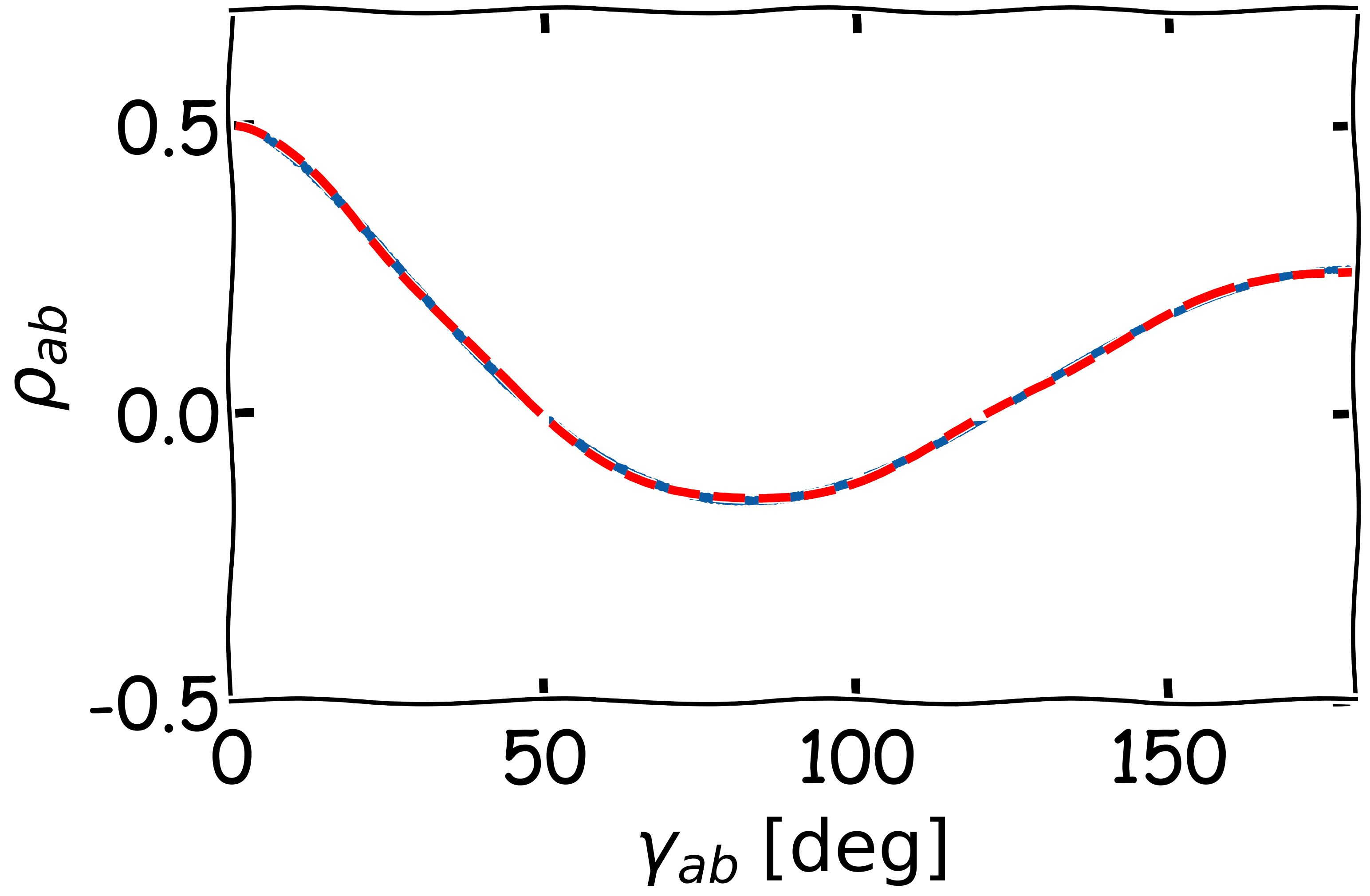
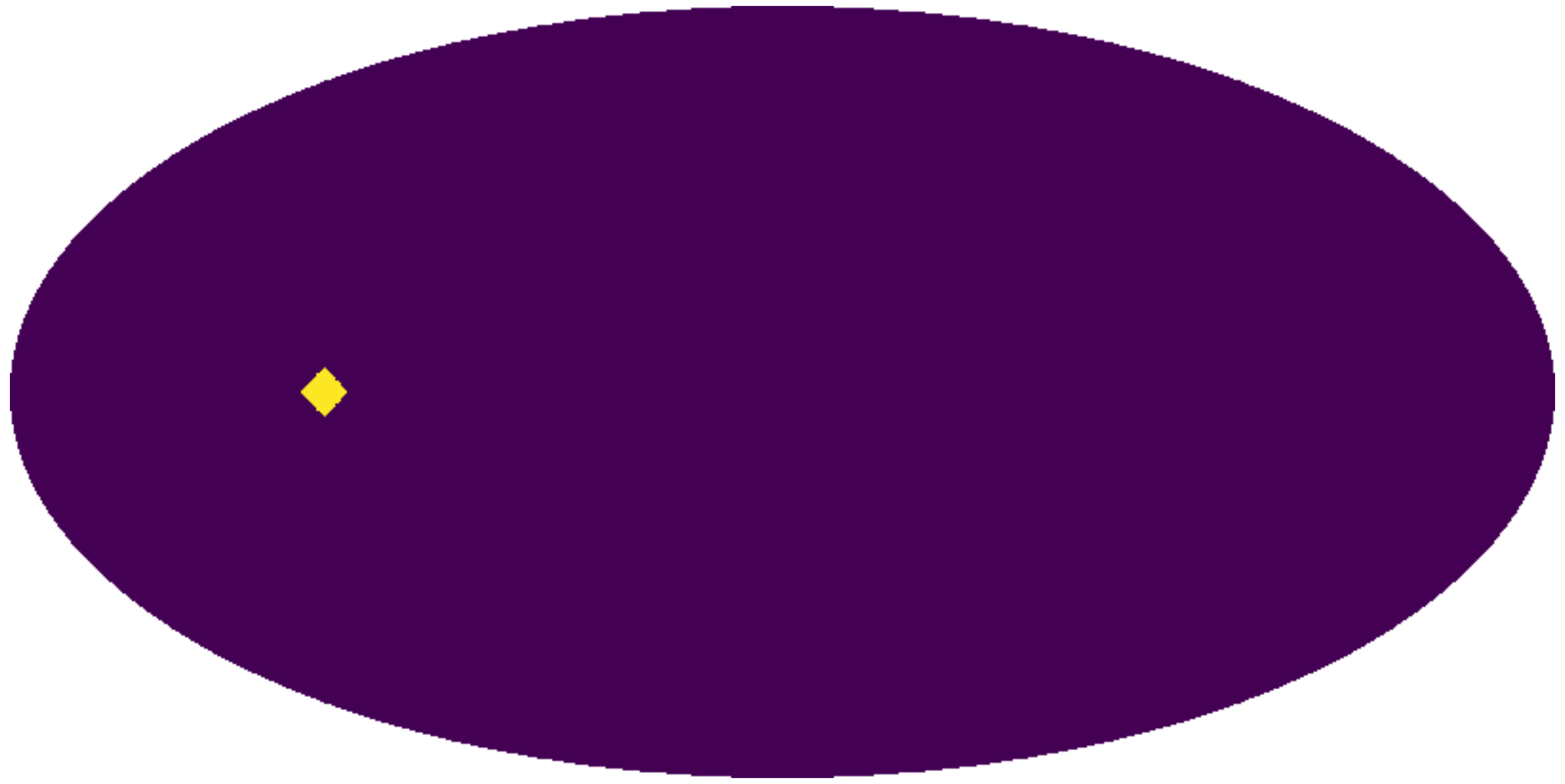


P_k





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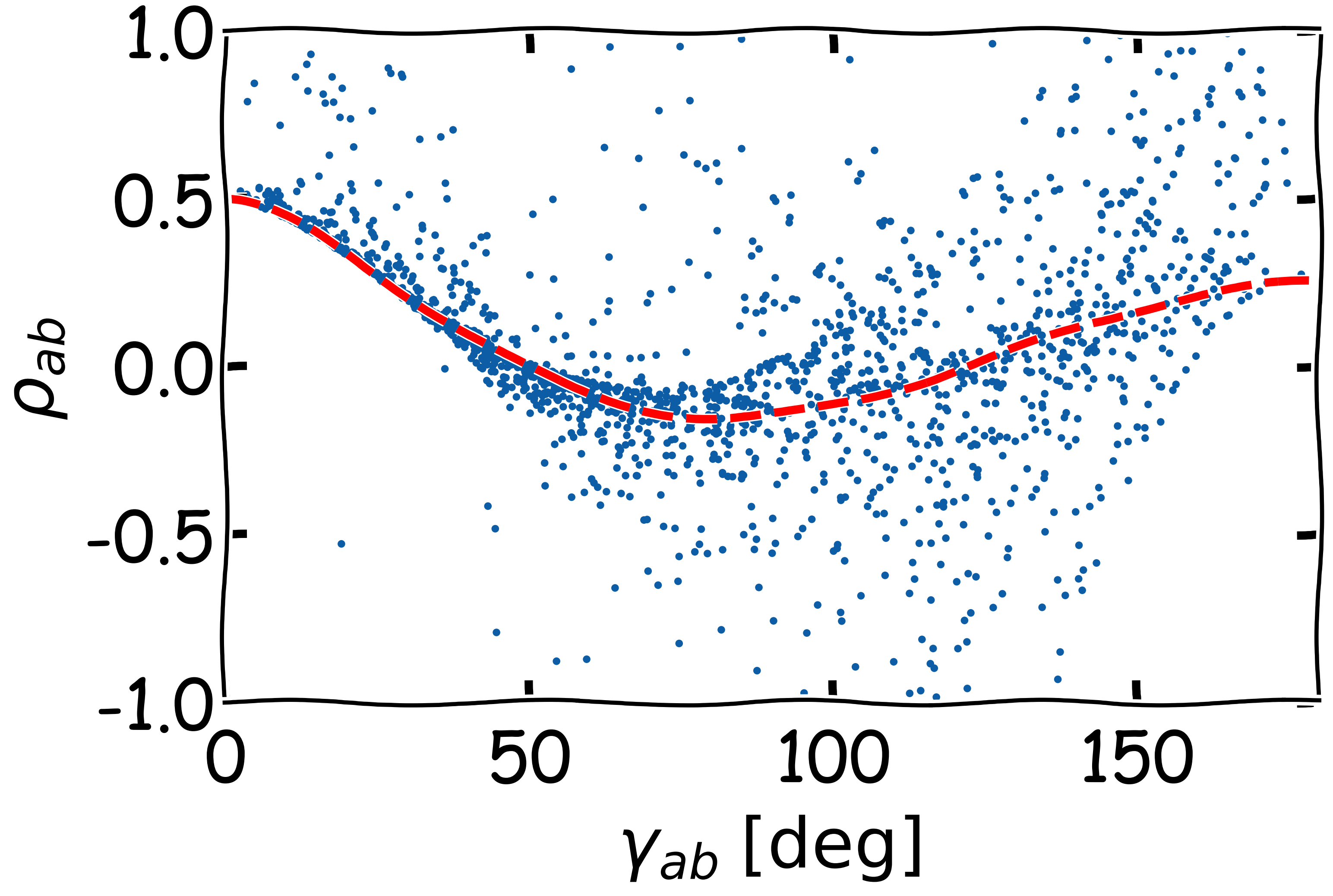

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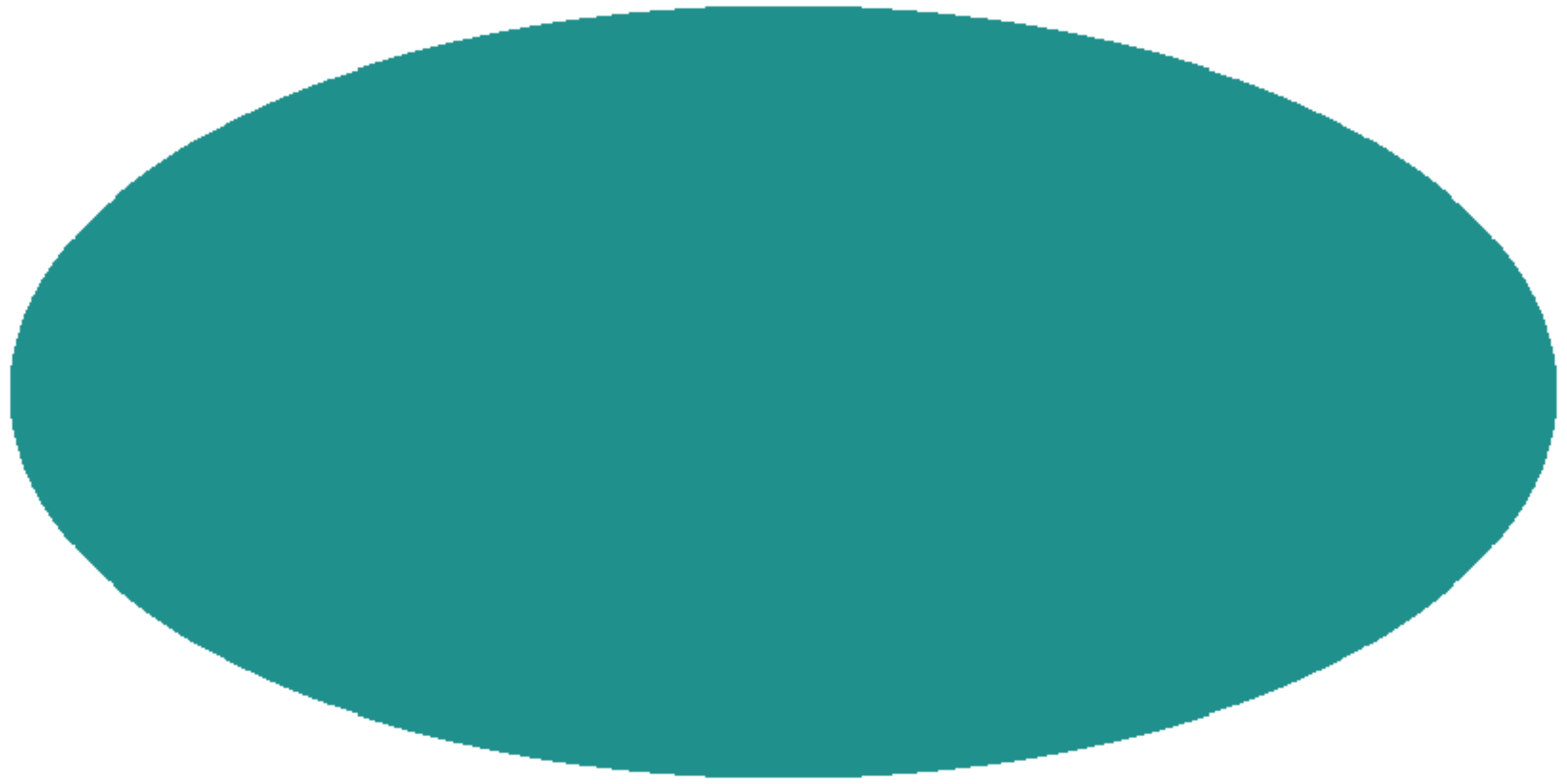

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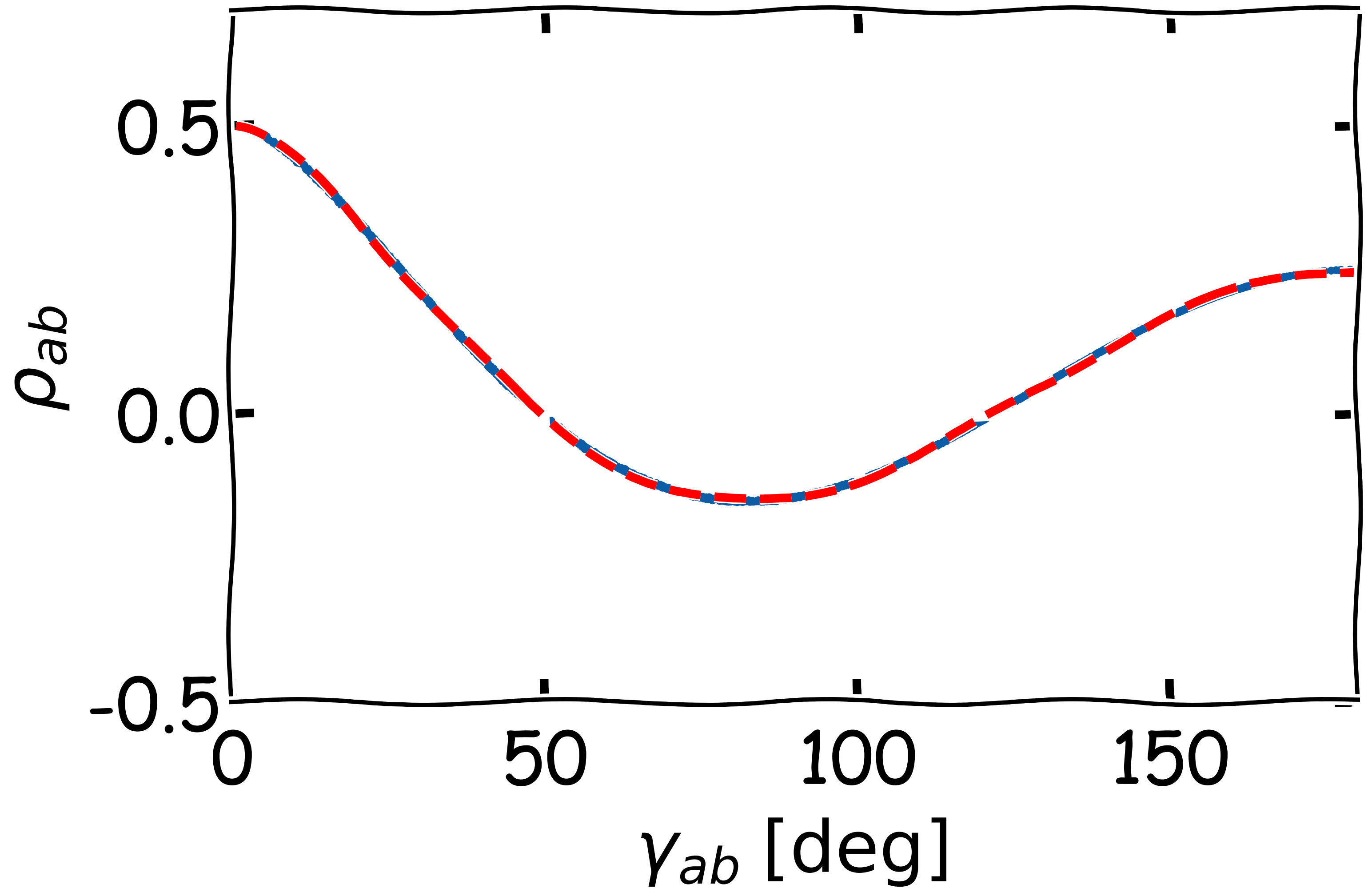

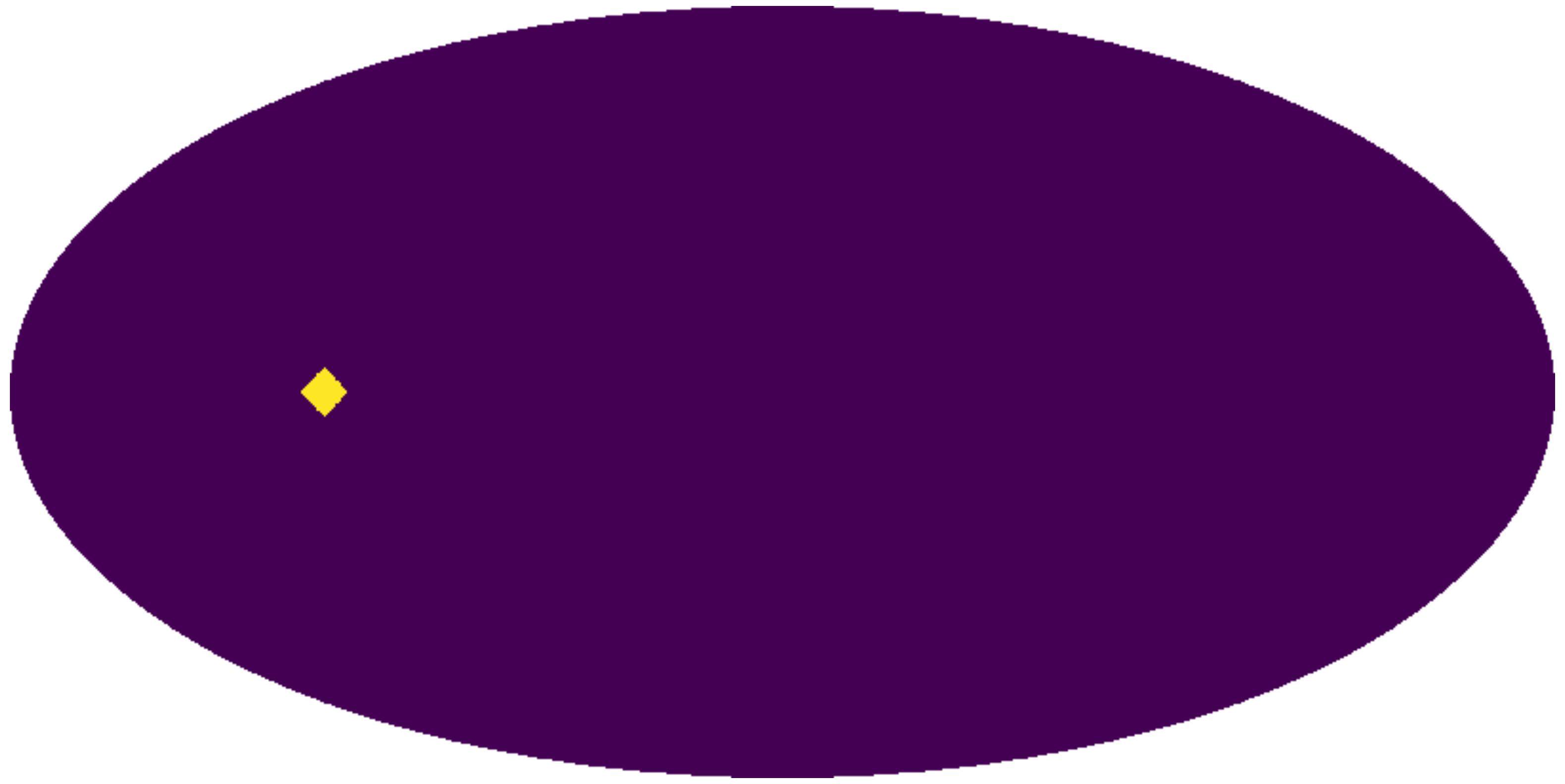

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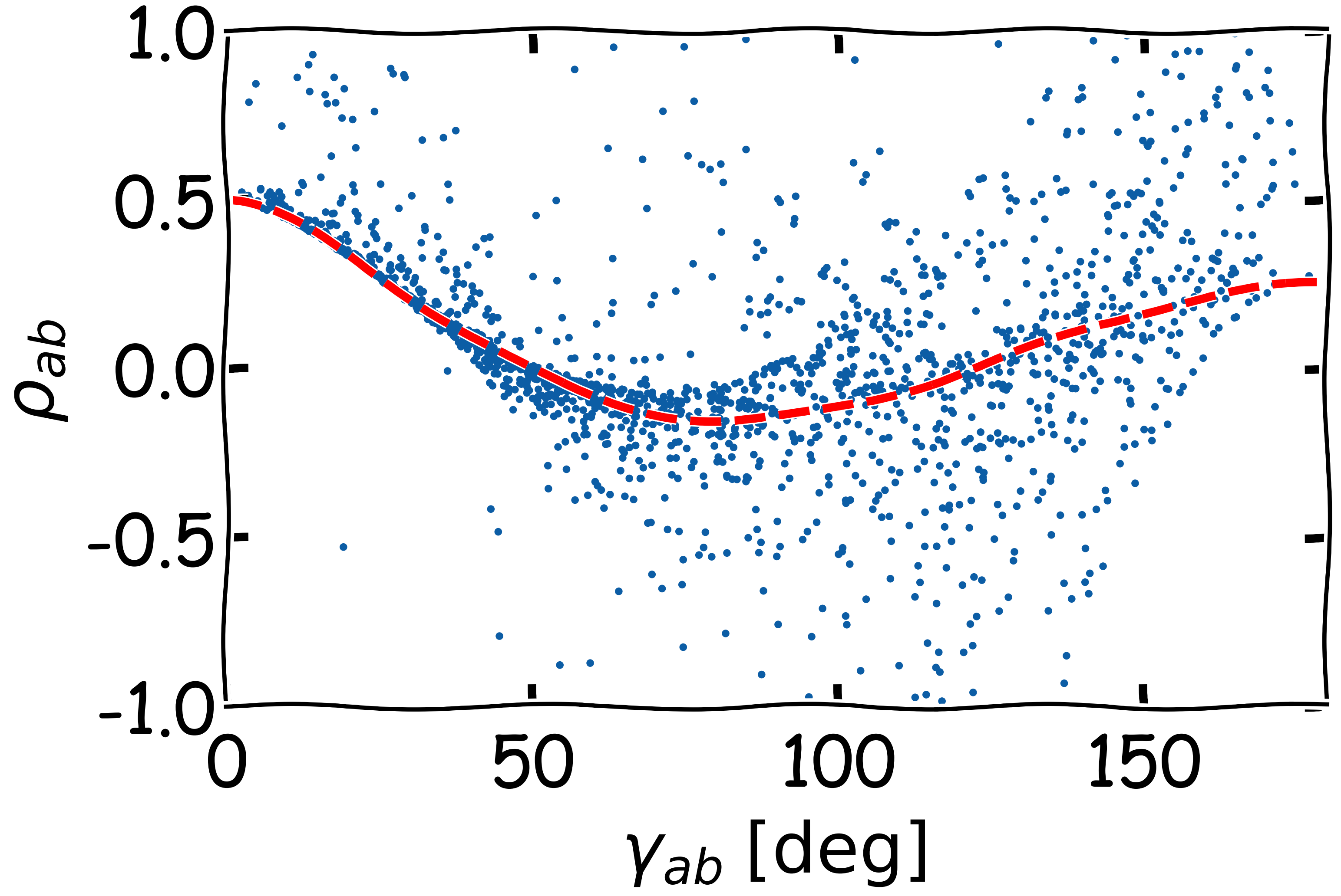

FREQUENTIST SEARCHES FOR ANISOTROPIES

three ingredients

the data

measured cross correlations

ρ_{ab}

the noise

cross correlations uncertainties

Σ_{ab}

the sky map

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likelihood to observe a set of correlations, ρ, given a sky map $\mathbf{P}$

$$p(\rho|\mathbf{P}) = \frac{\exp[-\frac{1}{2}(\rho - \mathbf{RP})^T \mathbf{\Sigma}^{-1}(\rho - \mathbf{RP})]}{\sqrt{\det(2\pi\mathbf{\Sigma})}}$$

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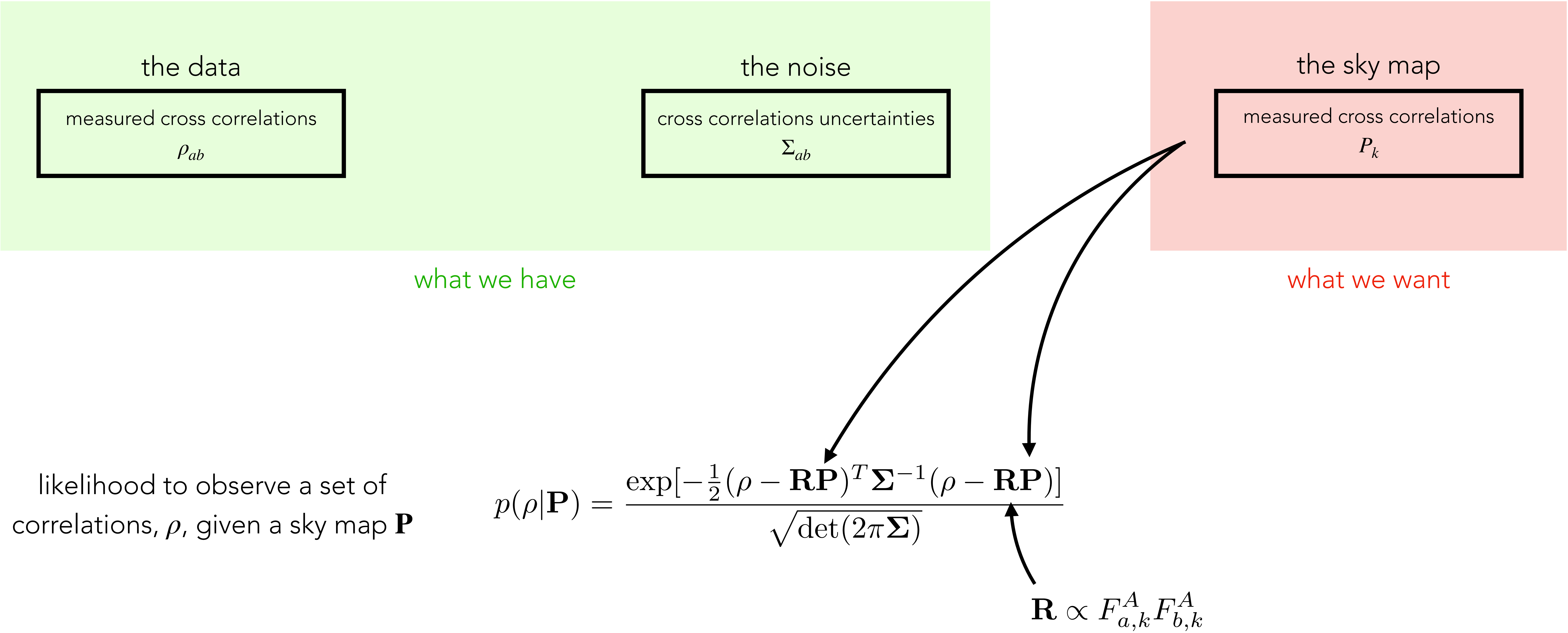

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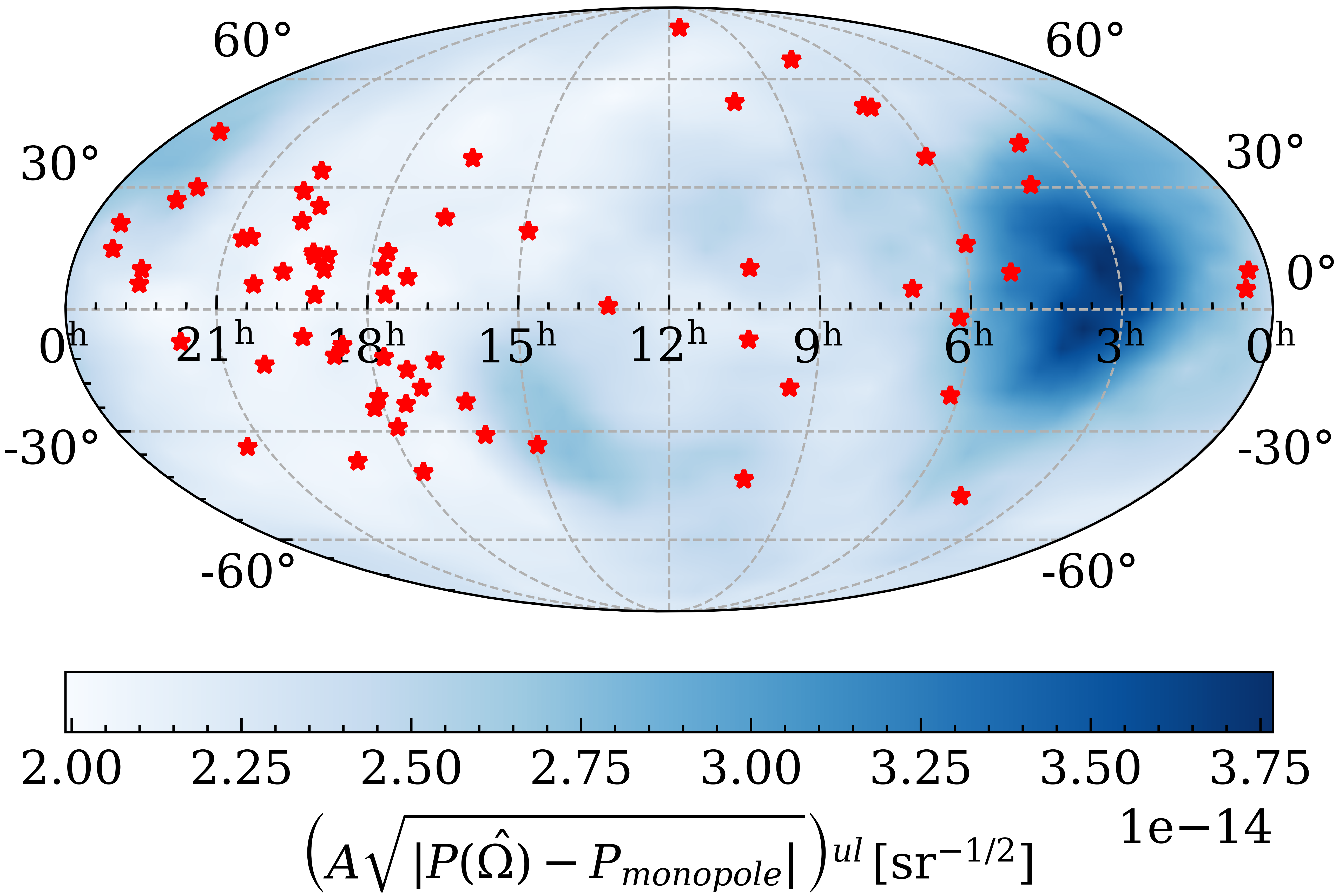
→

given $\{\rho, \mathbf{\Sigma}\}$ maximize p with respect to $\mathbf{P}$

FREQUENTIST SEARCHES FOR ANISOTROPIES



DETECTION STATISTIC

given a reconstructed sky map, $\hat{\mathbf{P}}$, how do we quantify the evidence for anisotropies? we need a detection statistic!

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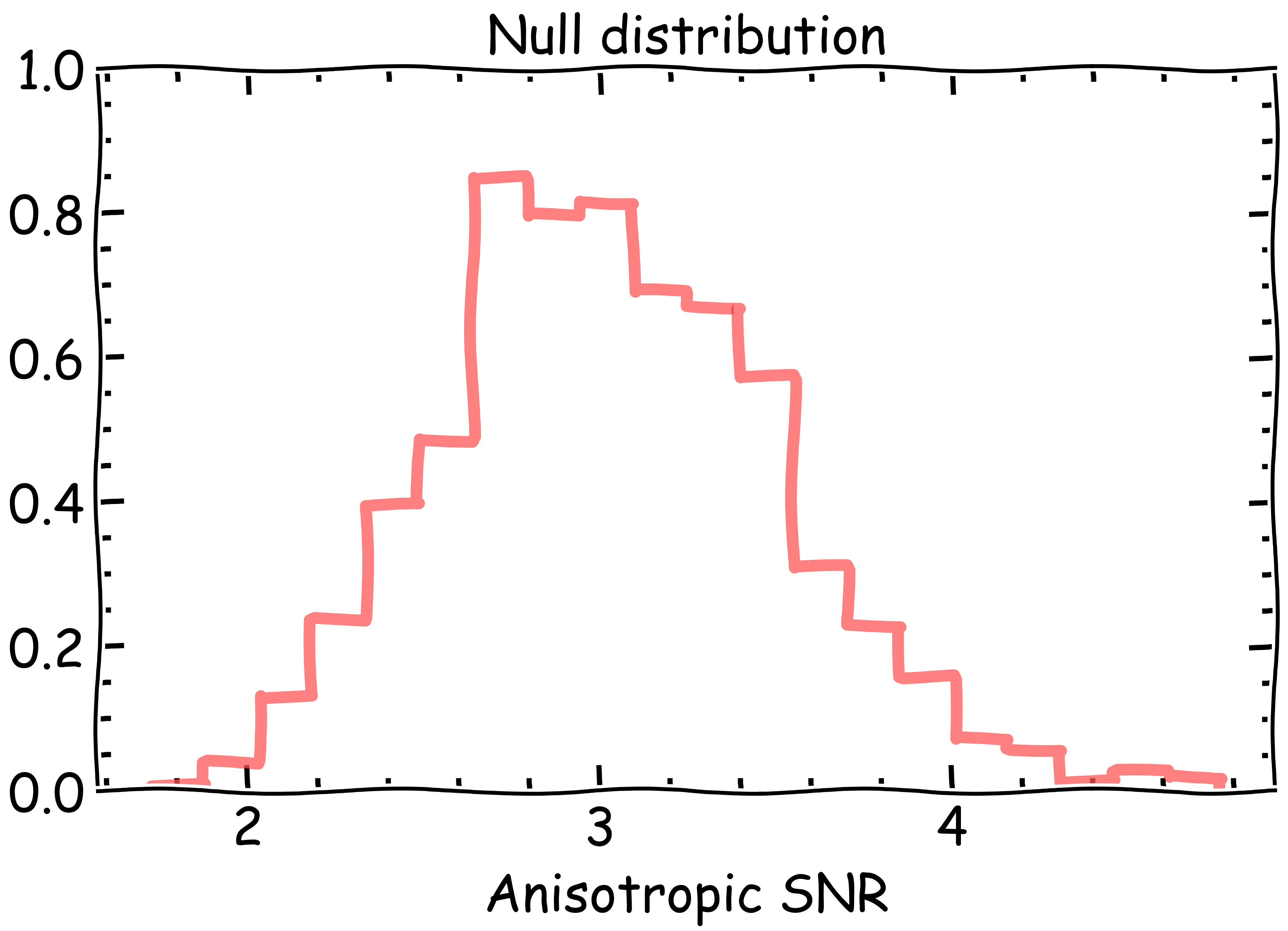

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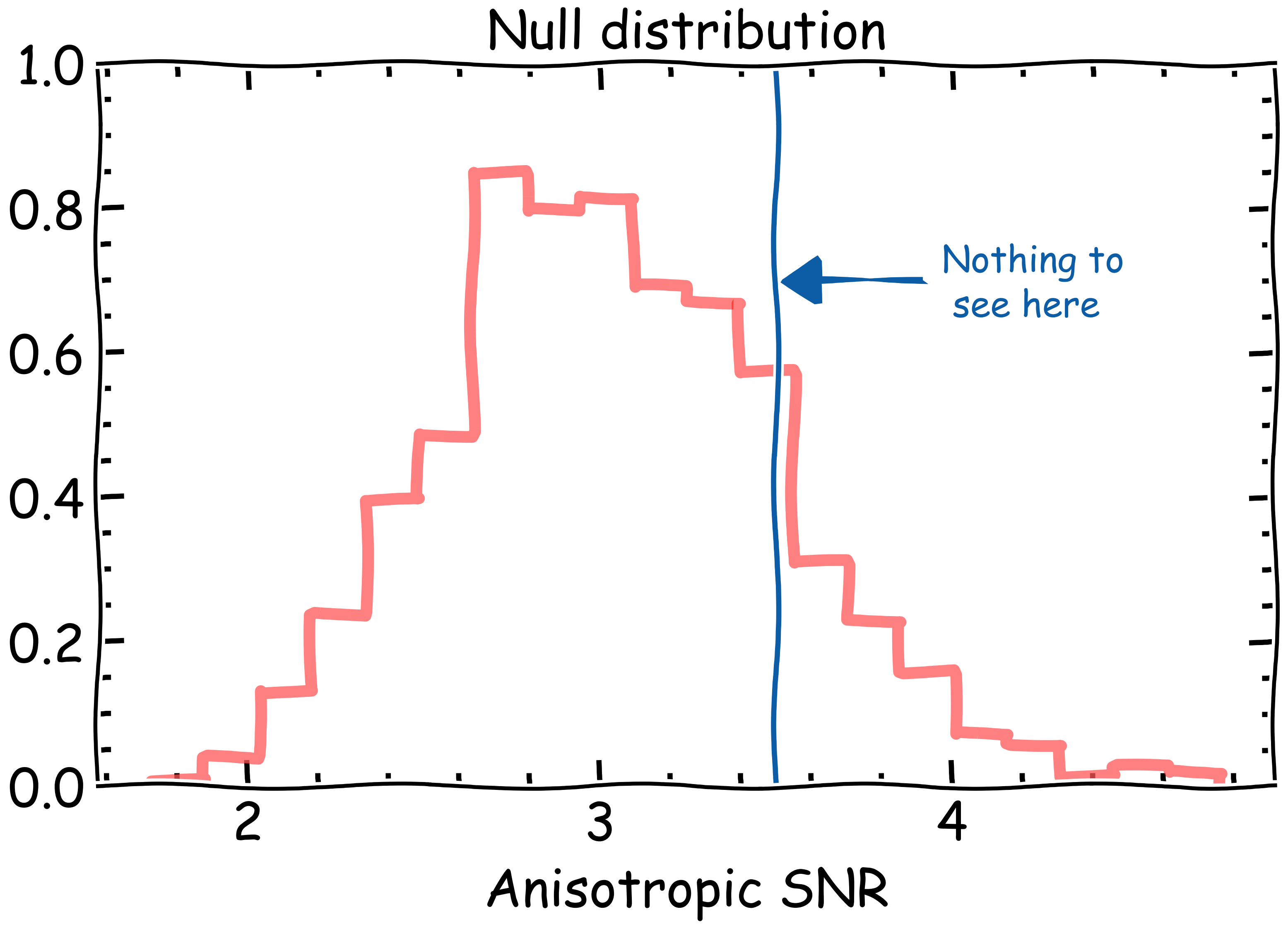

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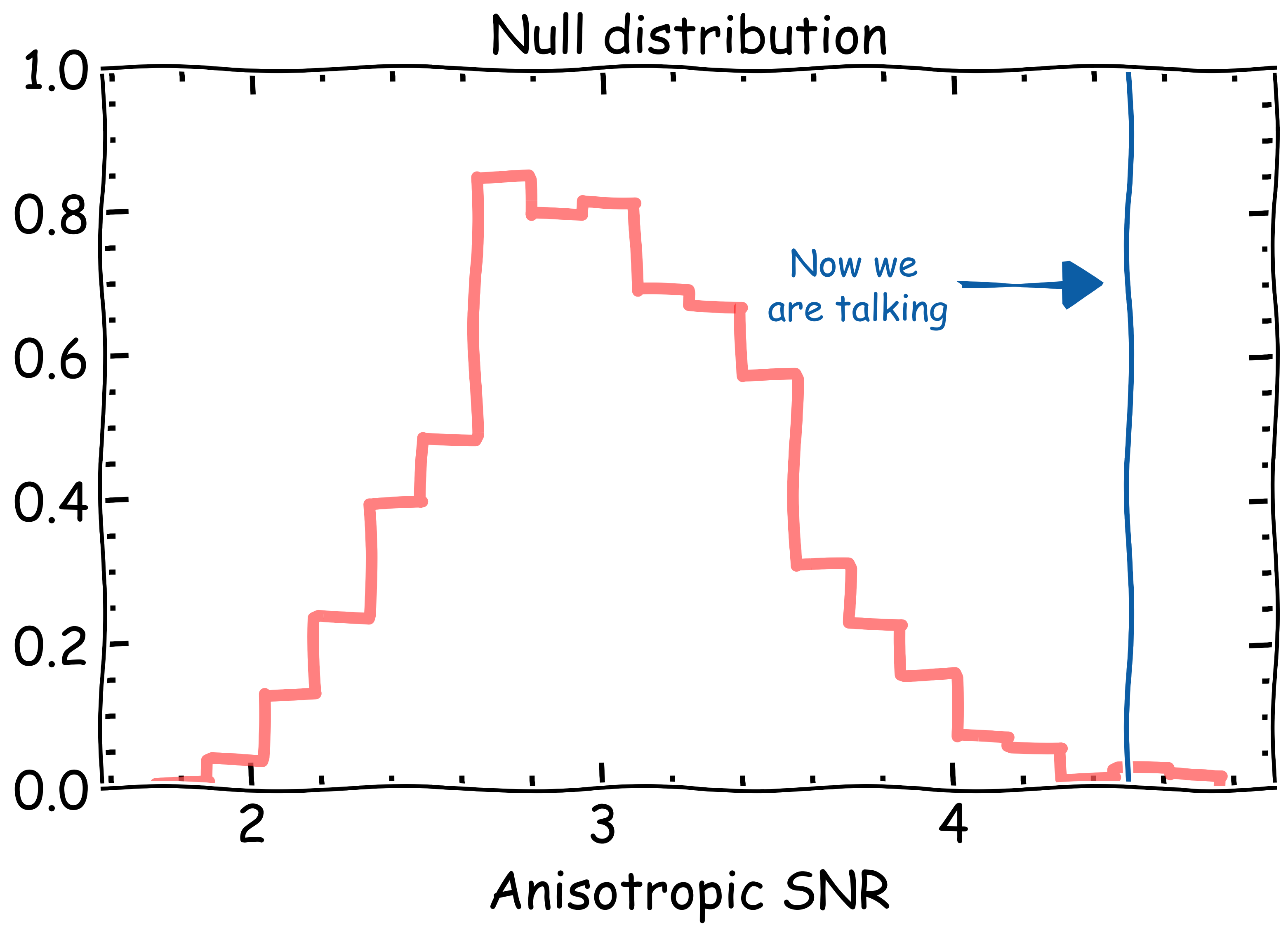
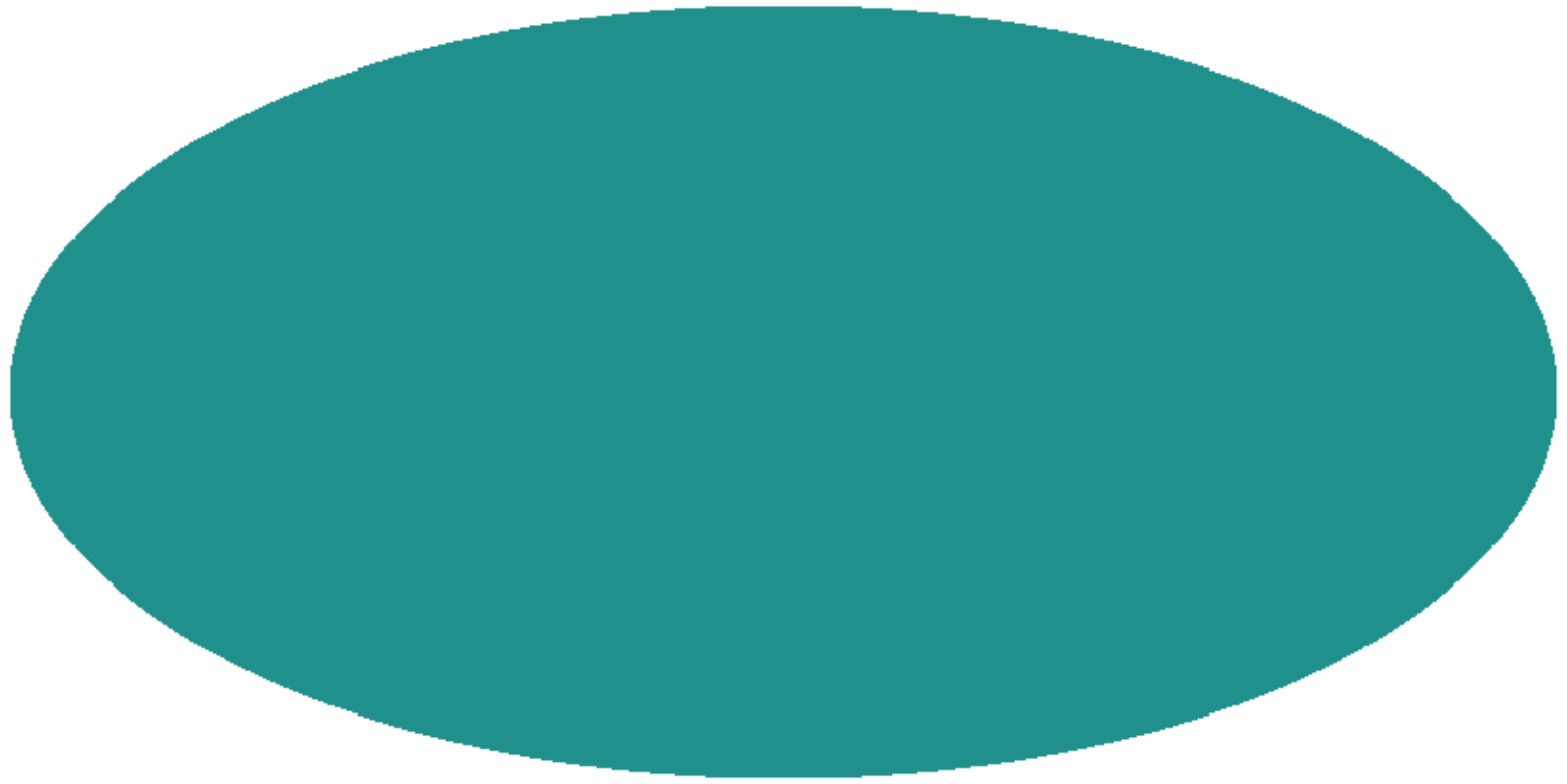


GENERATING NULL DISTRIBUTIONS



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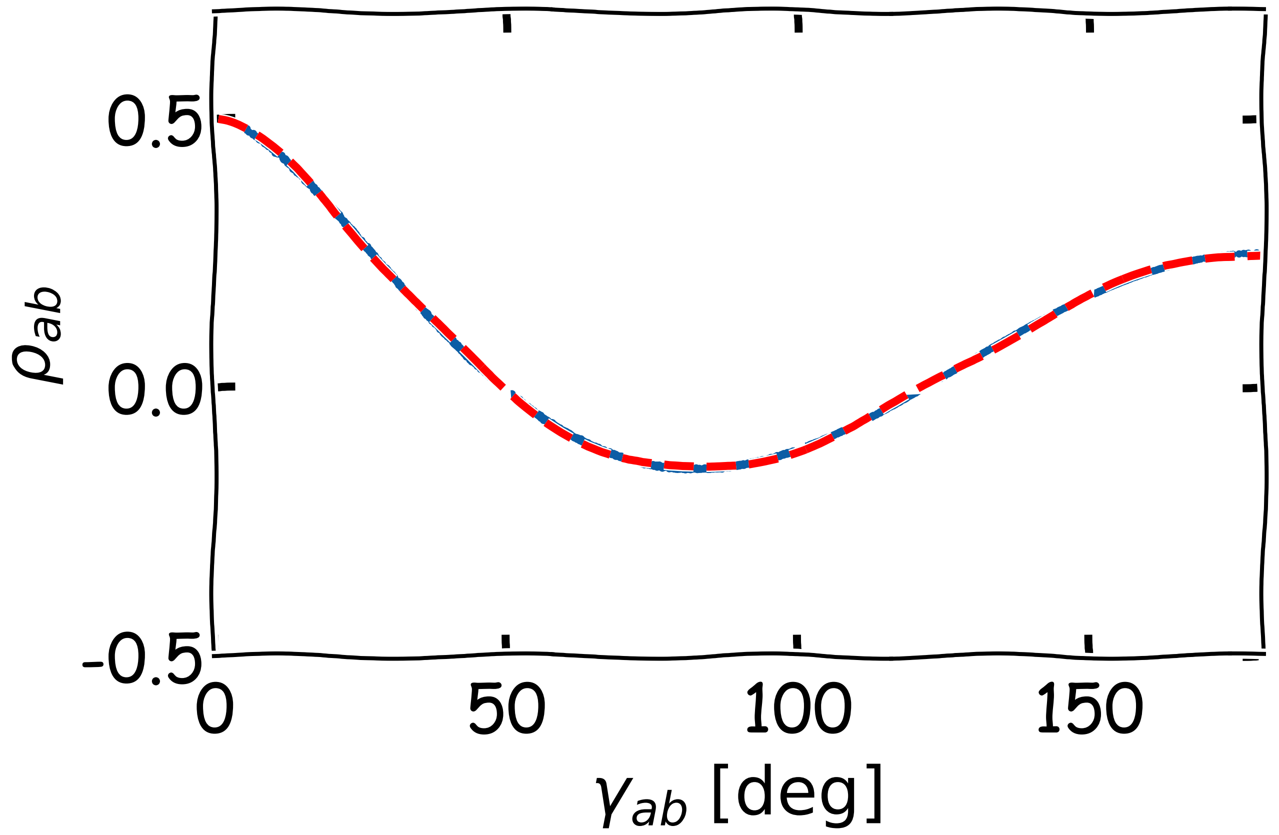

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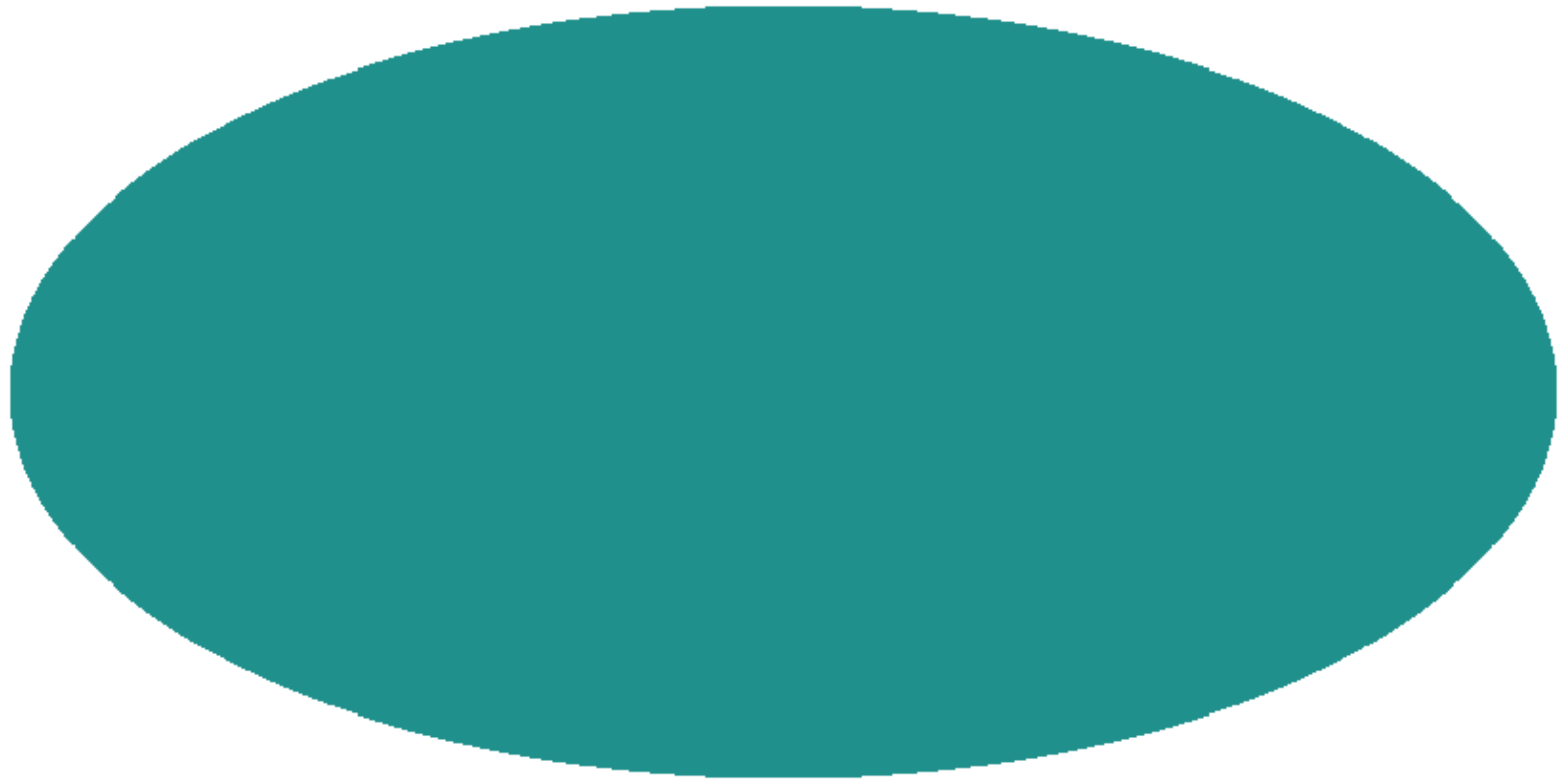

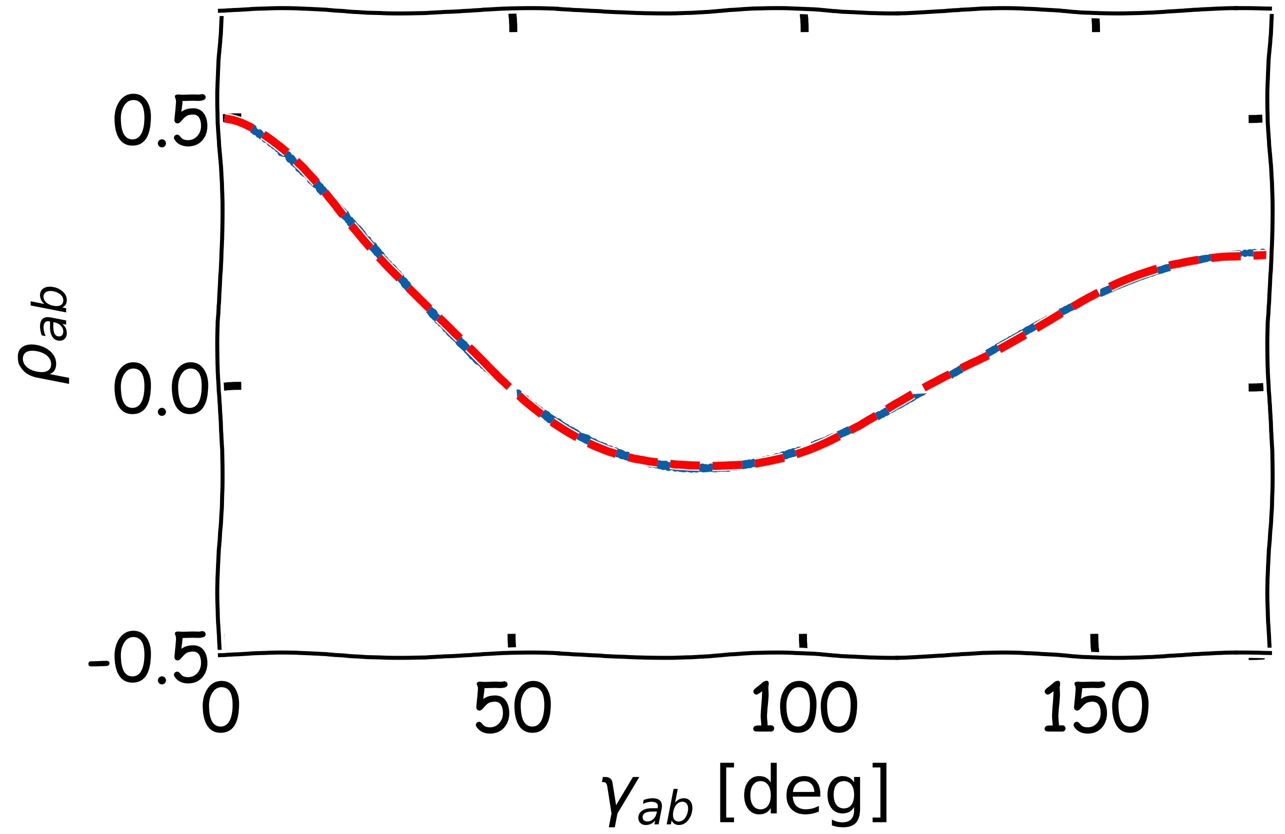

one realization of the null hypothesis

$$\rho_{ab} = \underset{\substack{\big\uparrow \\ \text{“true value”}}}{\mathrm{HD}_{ab}} + \underset{\substack{\big\uparrow \\ \text{noise}}}{\mathcal{N}(0, \Sigma_{ab})}$$

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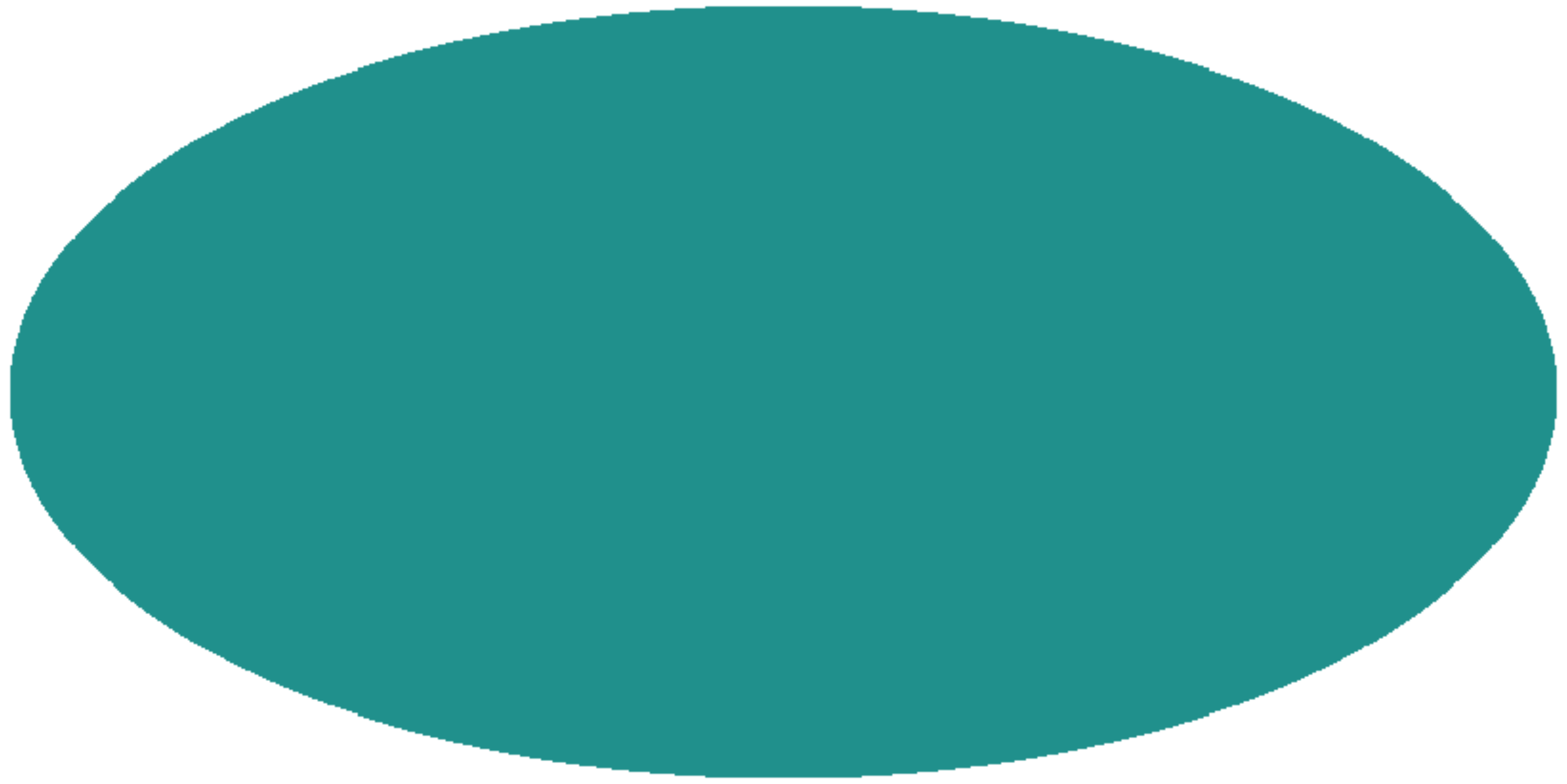

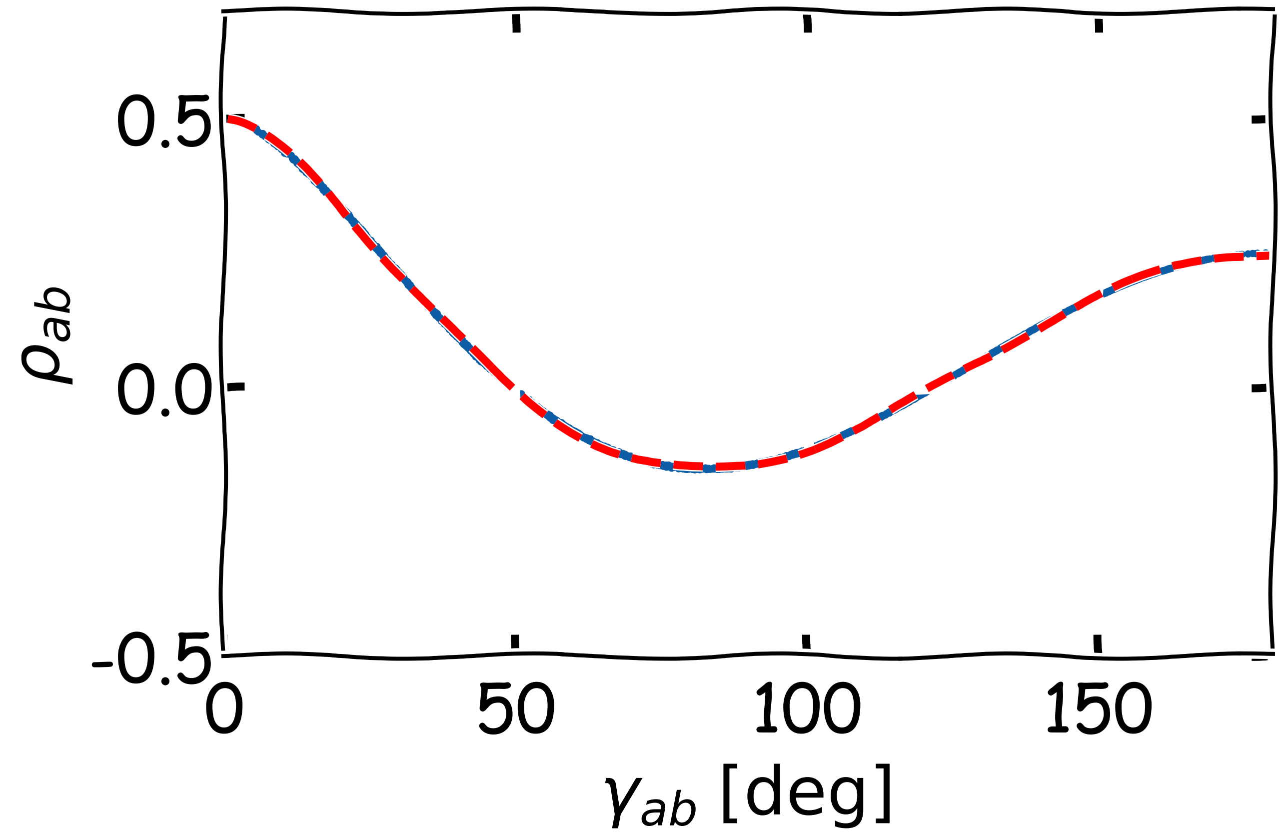

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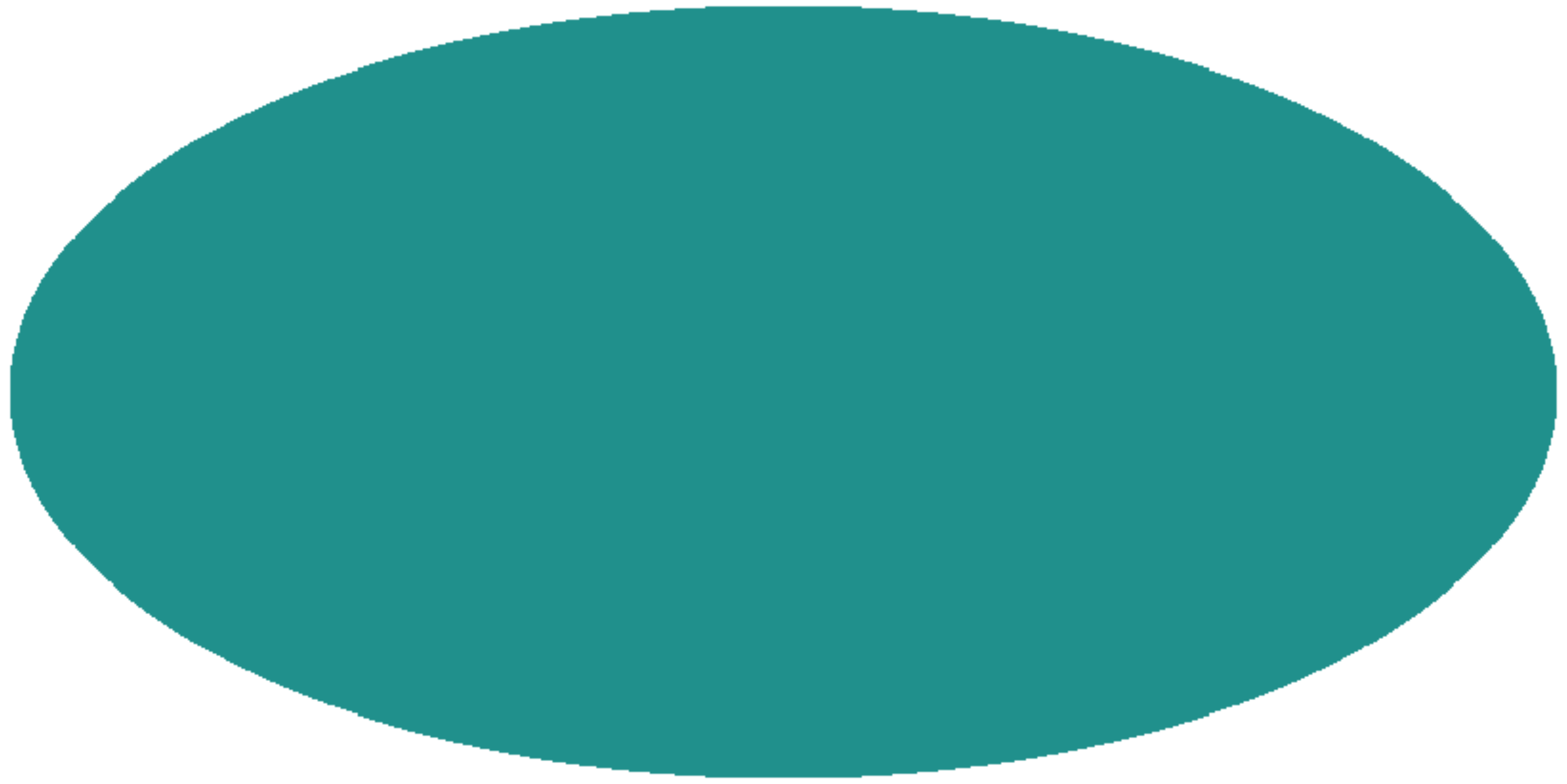
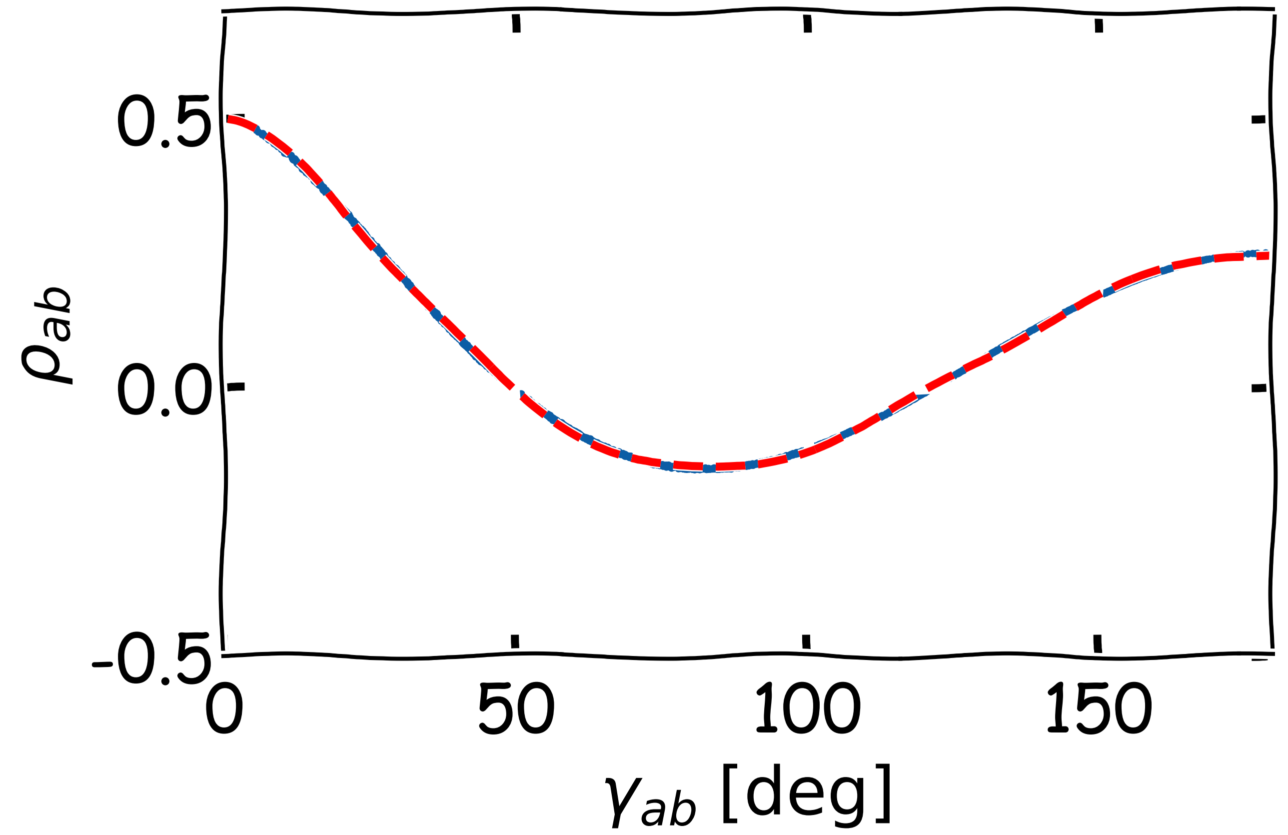
"true value"

noise

reconstruct sky map
and get SNR

GENERATING NULL DISTRIBUTIONS





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repeat
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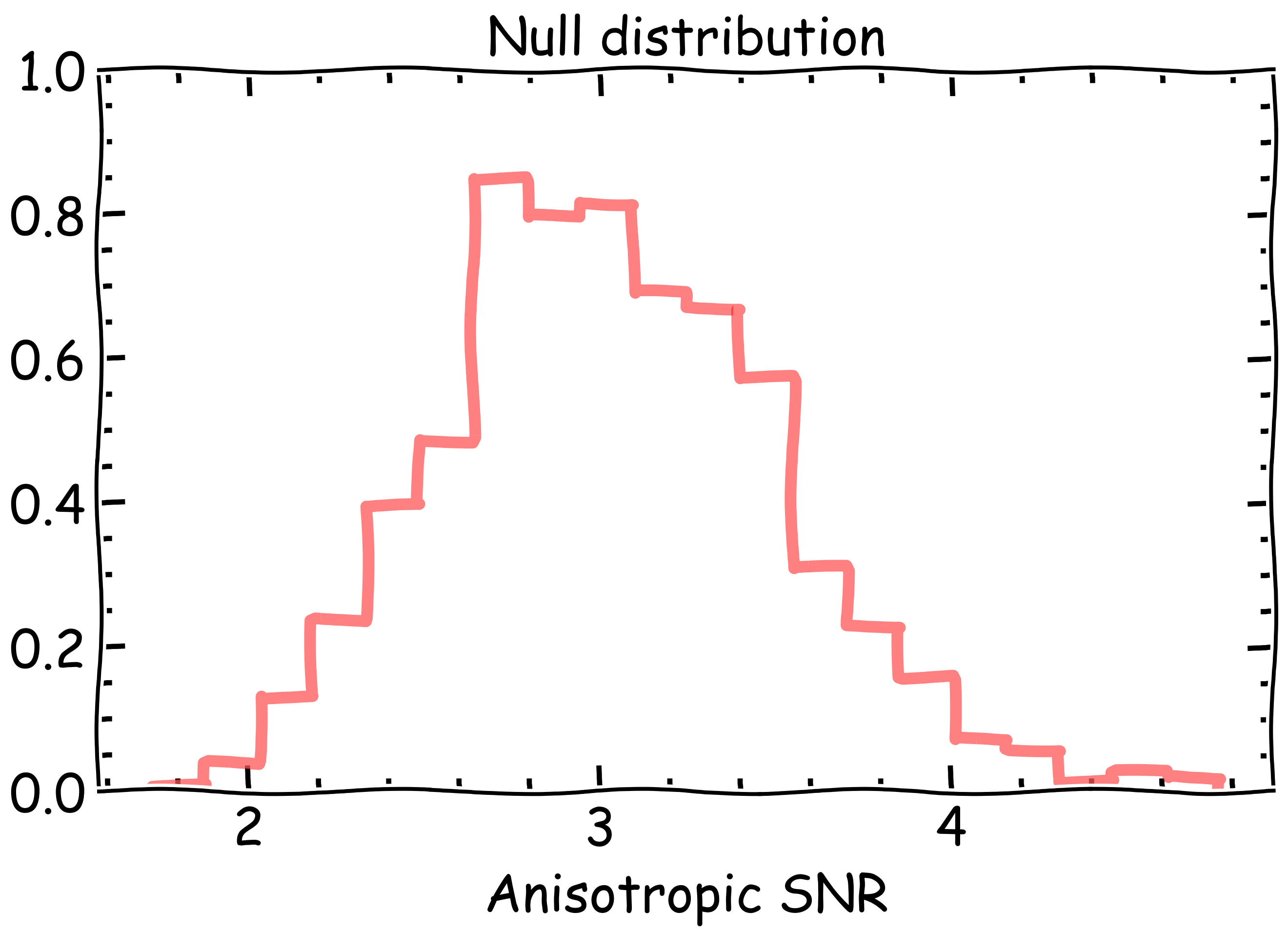
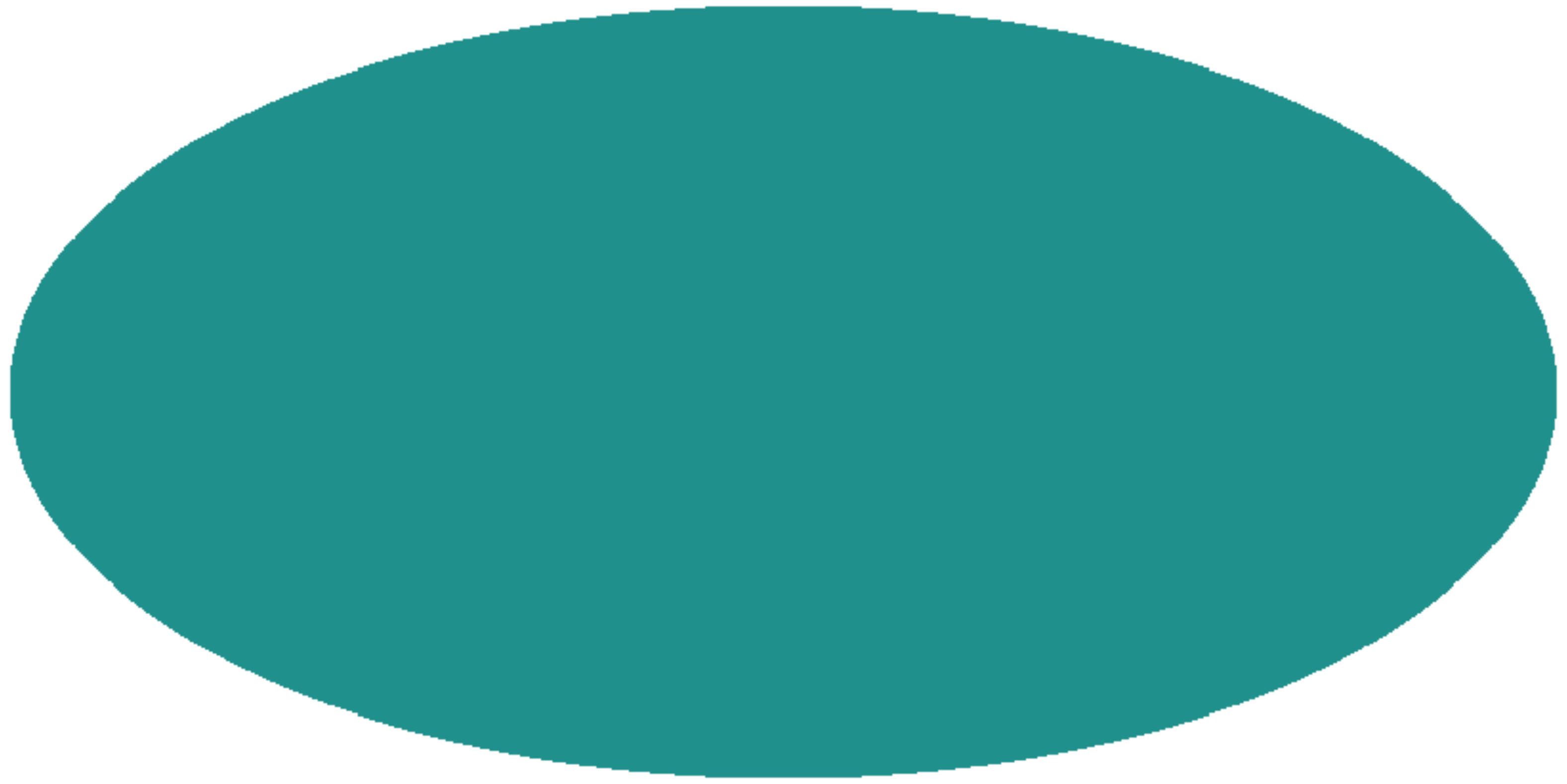


WHY IS LIFE SO HARD?!



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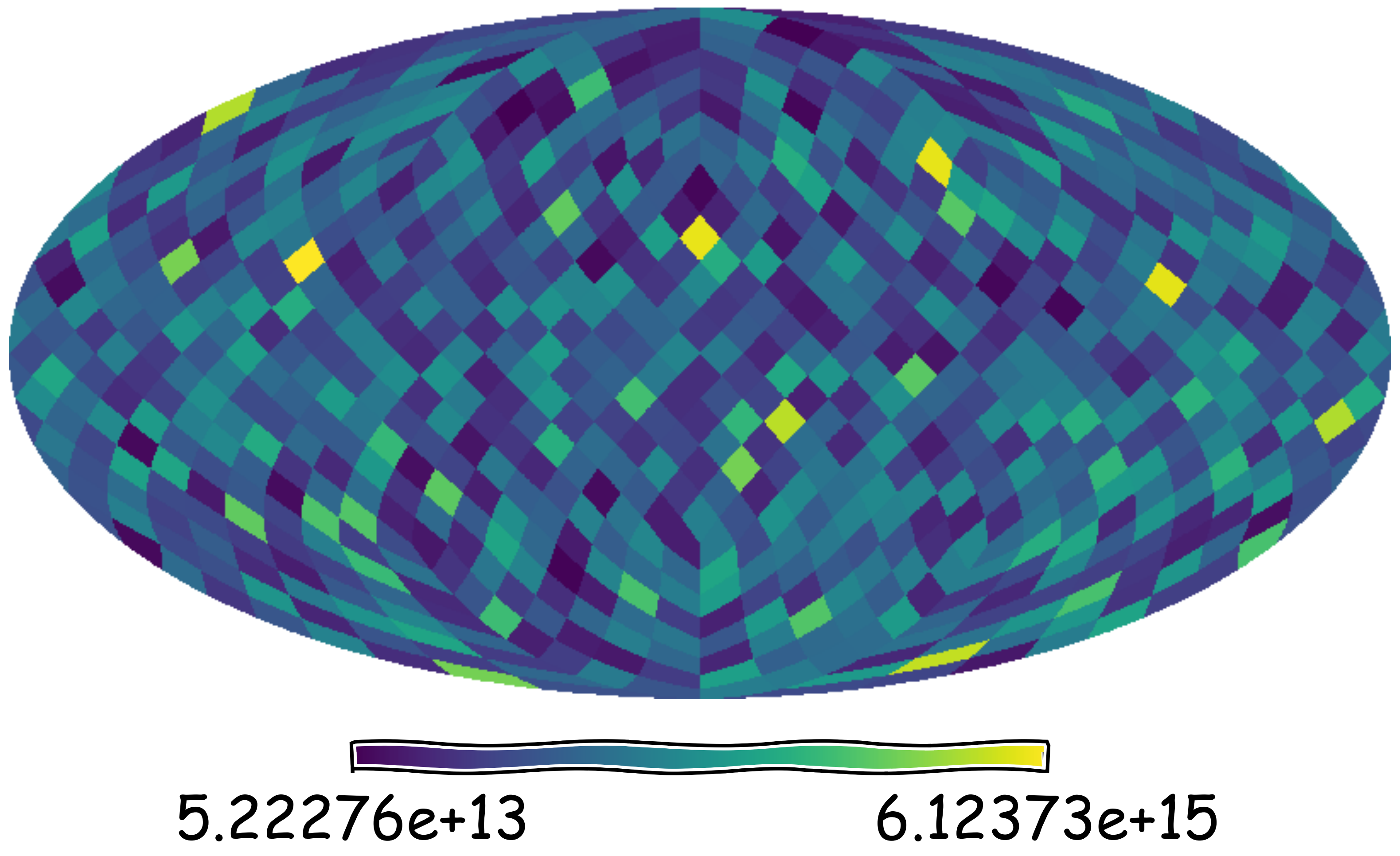

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each of this pixel can be thought as
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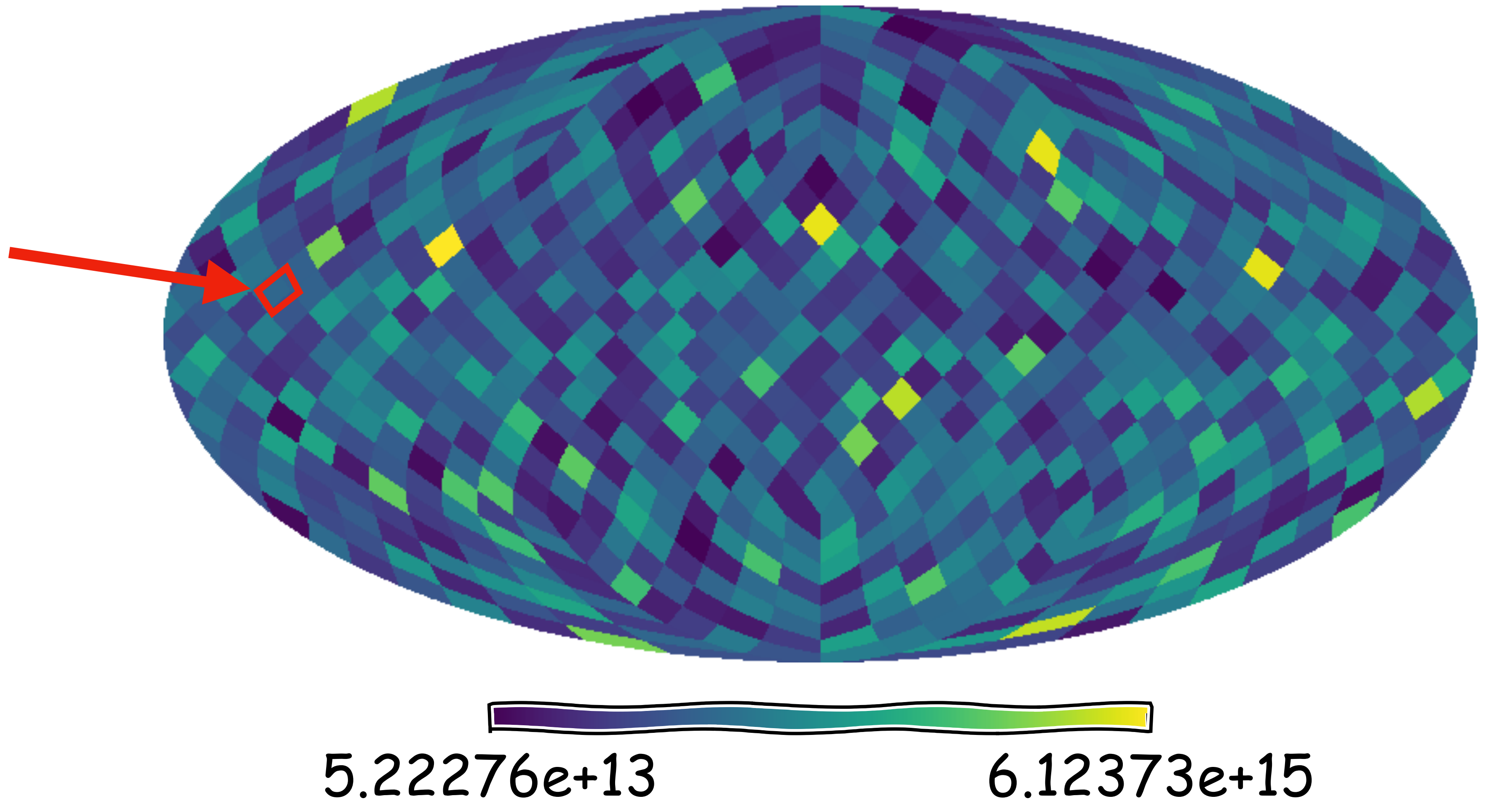
$$h_{ij}(t,\vec{x}) = \sum_A h_A\, e^A_{ij}(\mathbf{k})\, e^{i2\pi f(t-\mathbf{k}\cdot\mathbf{x}+\phi)}$$



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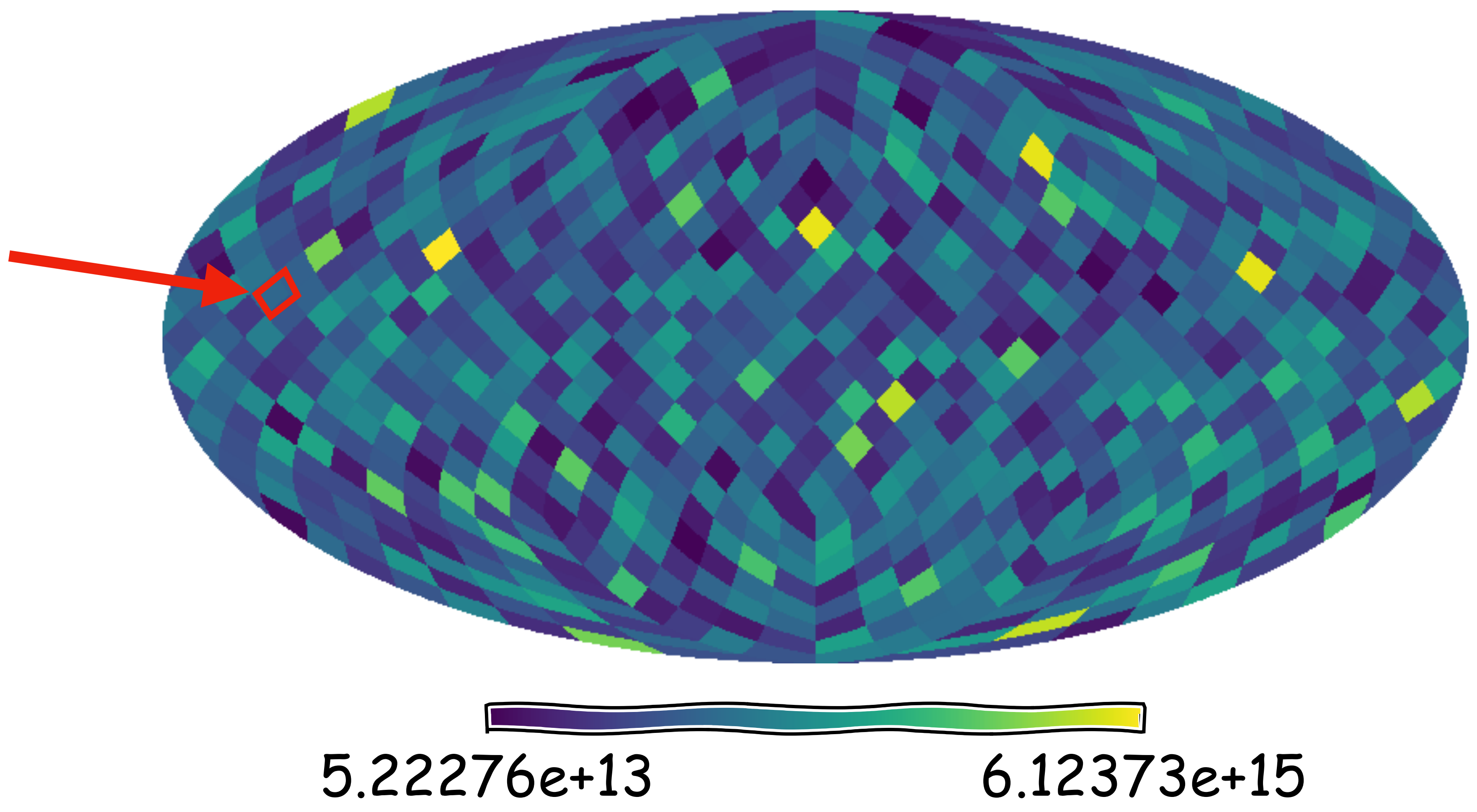

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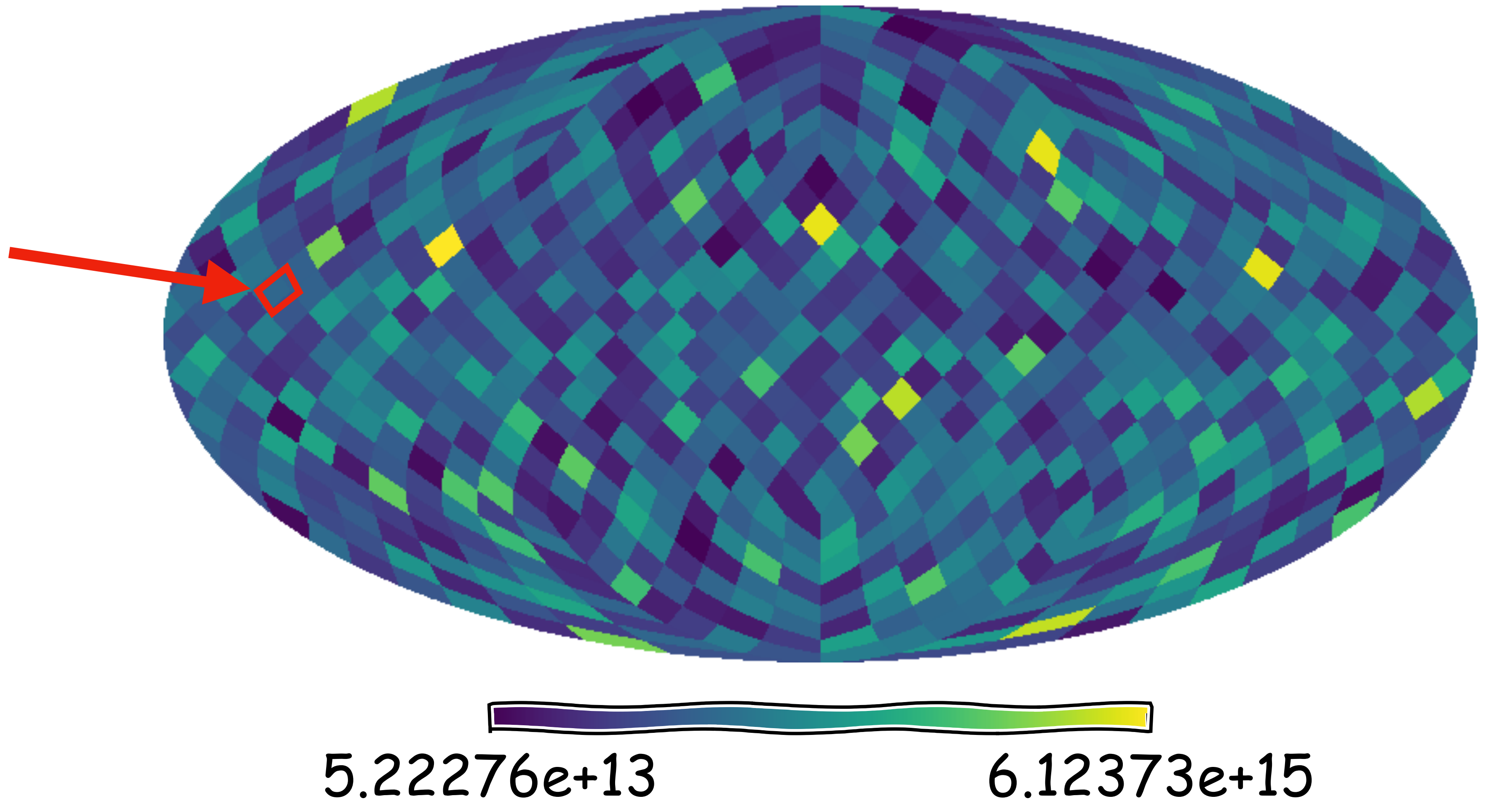

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$h^A_{k,i}$ $\phi^A_{k,i}$

one **amplitude** and **phase** for each, pixel, frequency, and polarization

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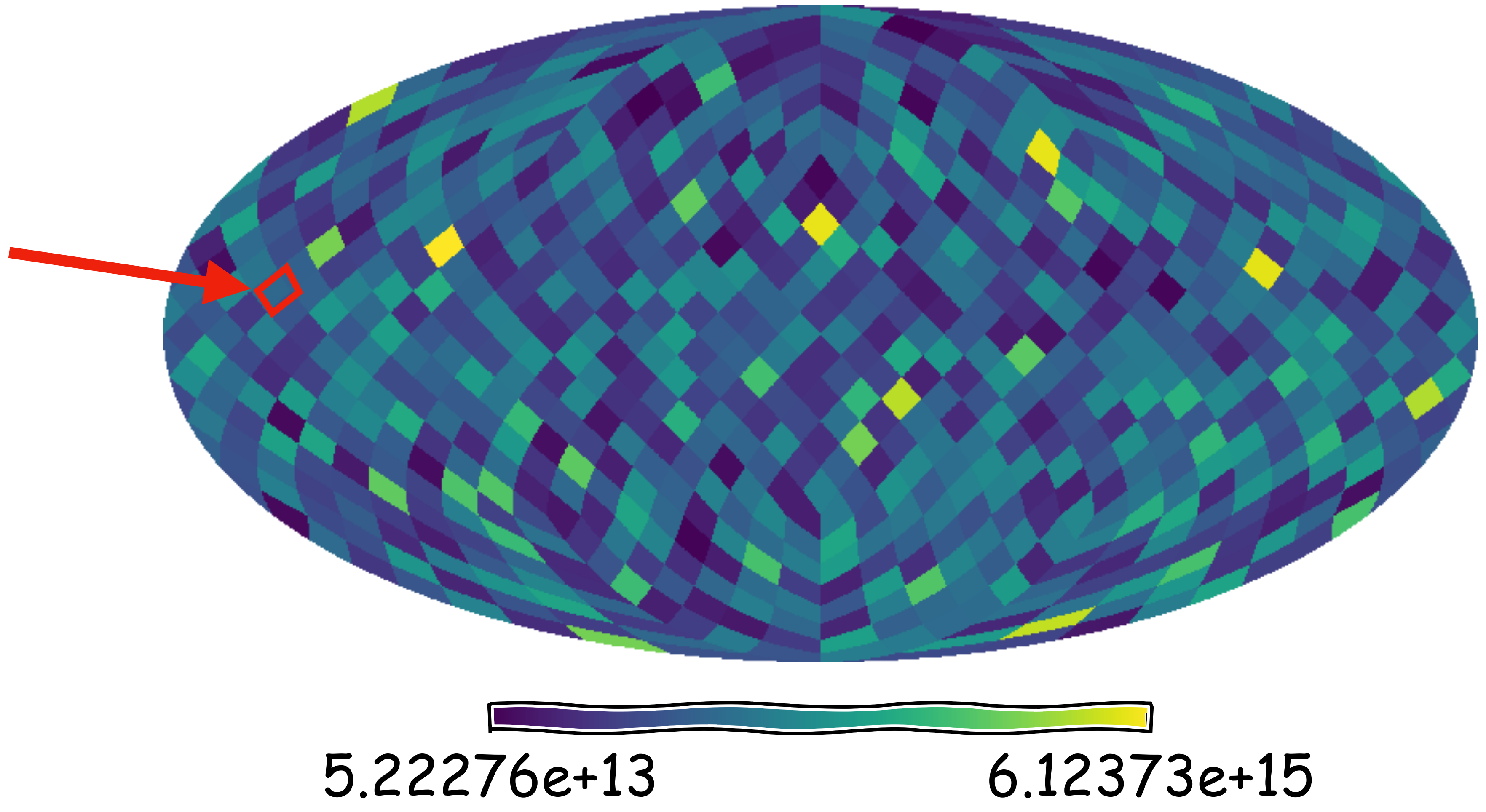

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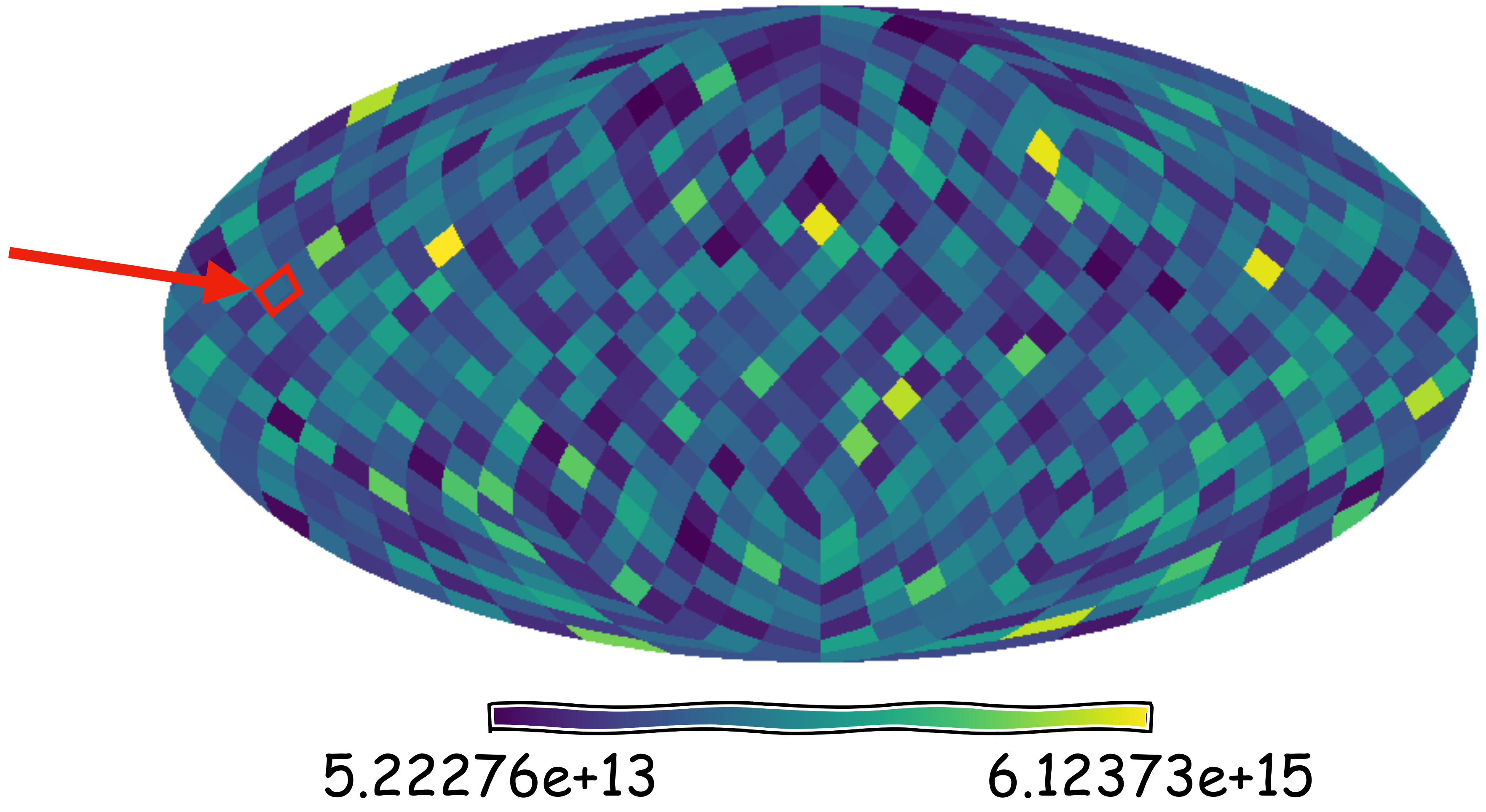

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ugly equation incoming

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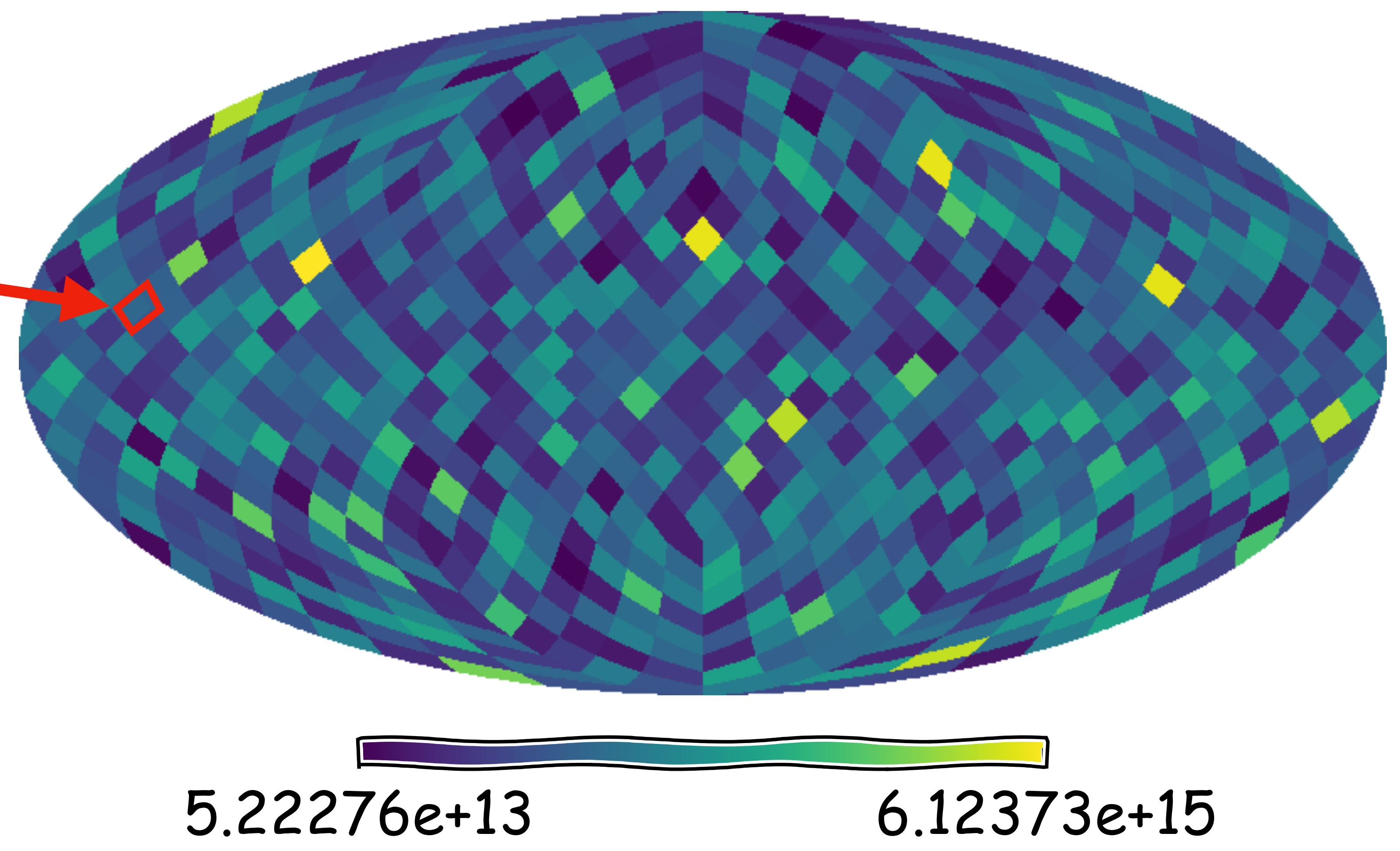

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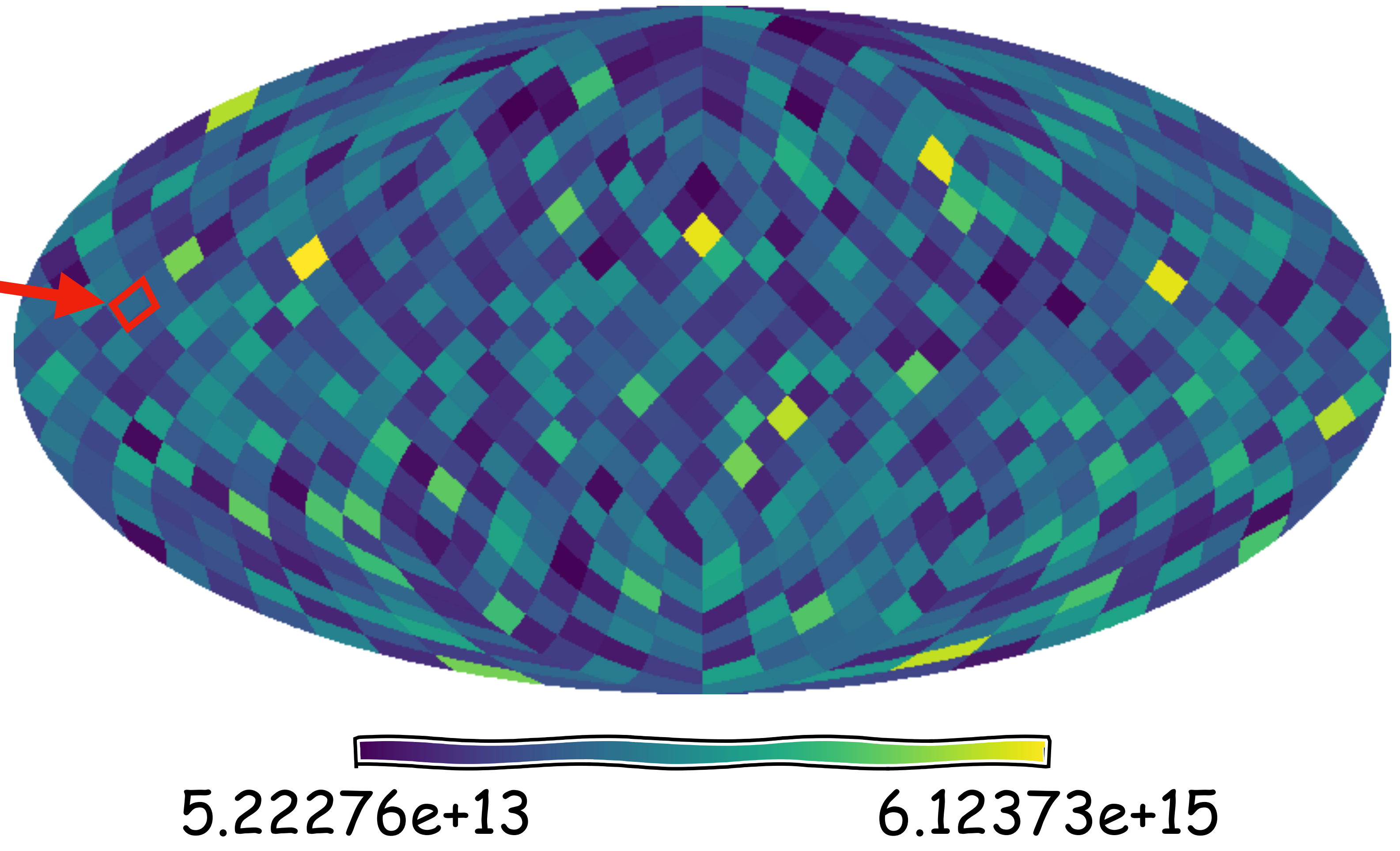

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sum over frequencies, pixels, and polarizations

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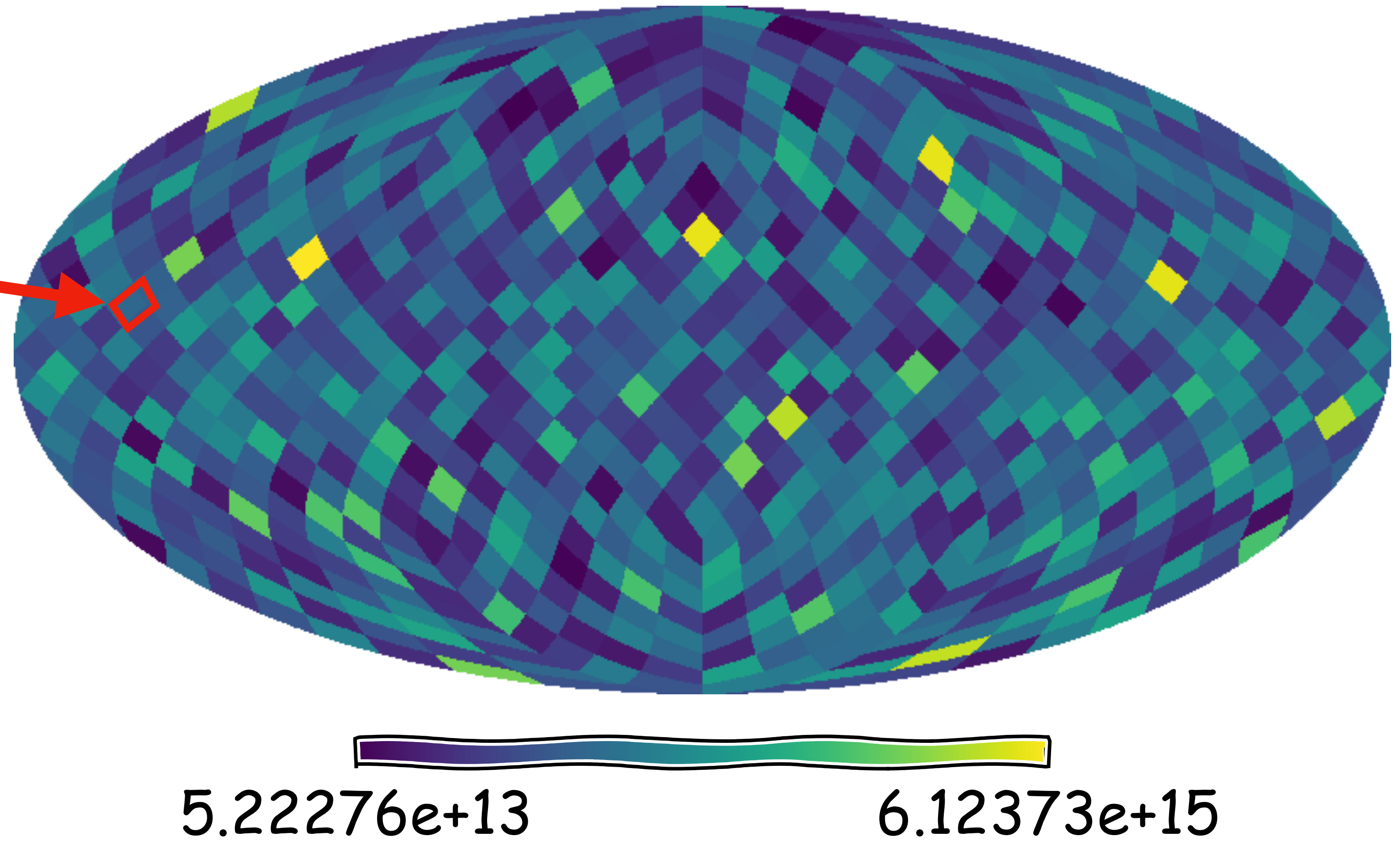

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response functions for pulsars a and b

$$R^A_a(f,\hat{\Omega}) \equiv F^A_a(\hat{\Omega}) \left[1 - e^{2\pi i f L_a(1+\hat{p}_a\cdot\hat{\Omega})}\right]$$

$$F^A_a(\hat{\Omega}) \equiv \frac{p^i_a p^j_a}{2(1+\hat{\Omega}\cdot\hat{p}_a)} e^A_{ij}(\hat{\Omega})$$

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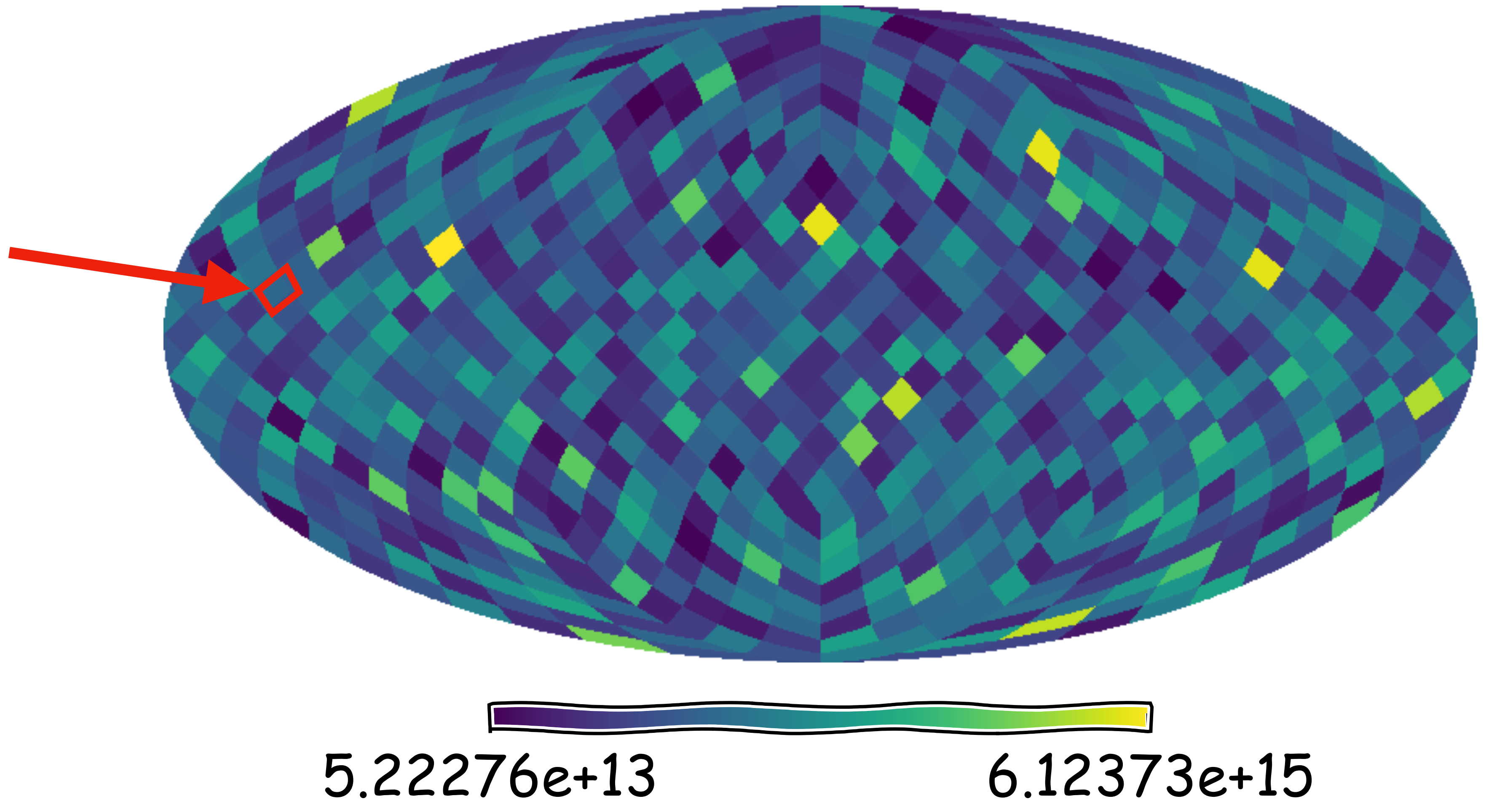

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pixel amplitudes

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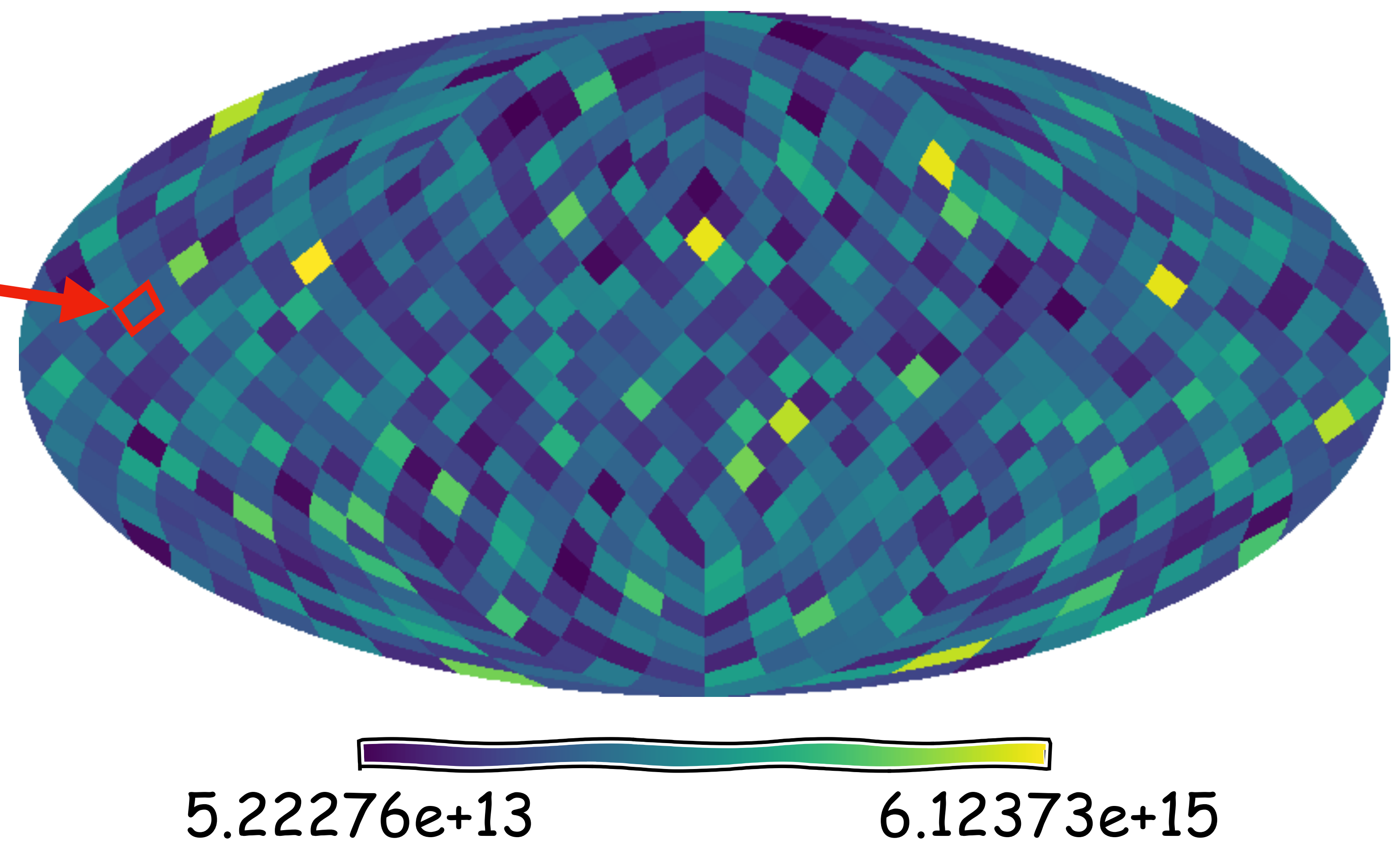

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interference between different pixels

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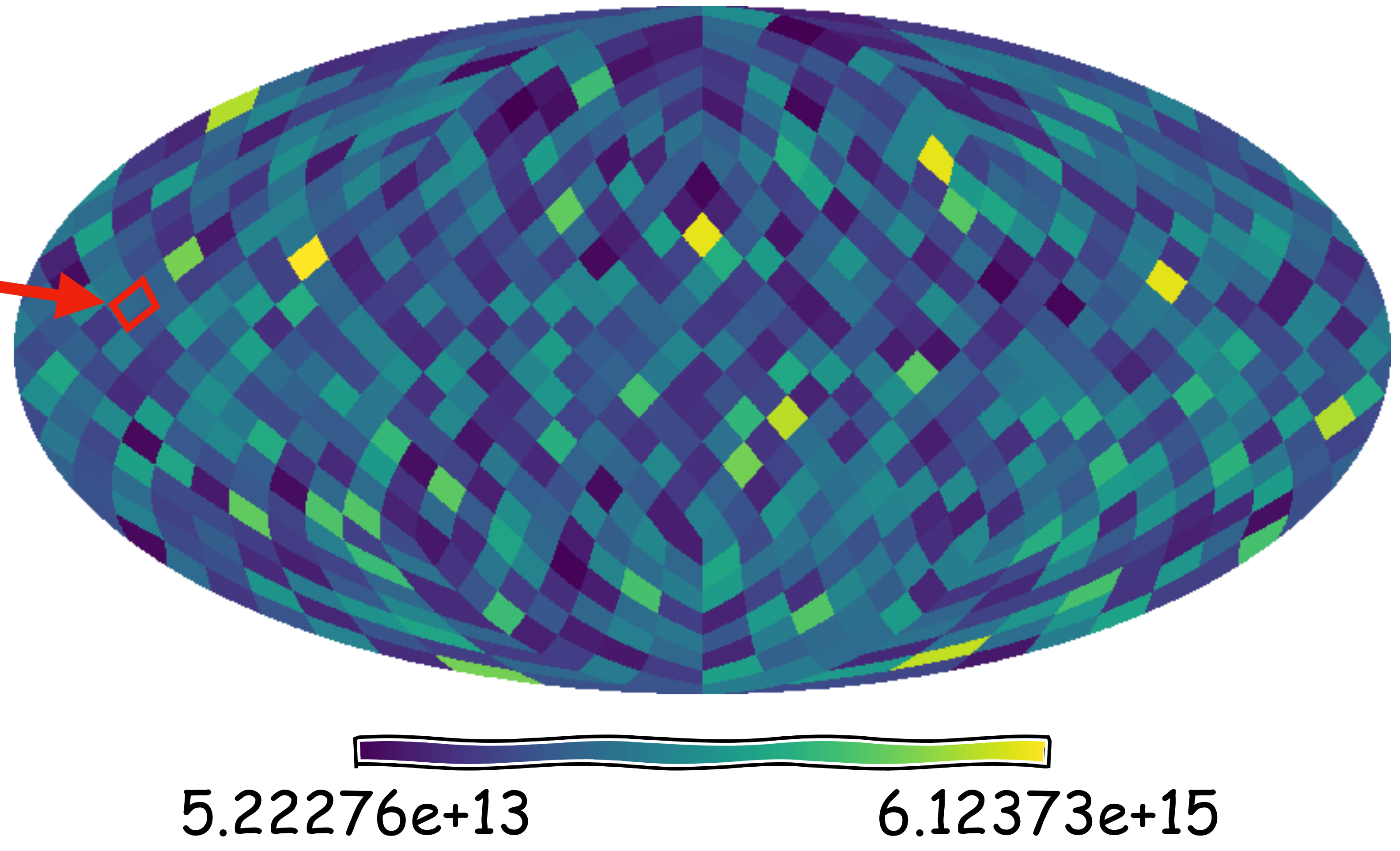

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one **amplitude** and **phase** for each, pixel, frequency, and polarization

given a set of $\{h^{A}_{k,i}, \phi^{A}_{k,i}\}$, the cross correlation coefficients can be uniquely derived

$$\rho_{ab} = \Delta f^2 \Delta\hat{\Omega}^2 \sum_{jj'}\sum_{kk'}\sum_{AA'} R^{A*}_{akj} R^{A'}_{bk'j'}\, h^{A}_{kj} h^{A'}_{k'j'}\, \exp(i\Delta\phi^{AA'}_{kk',jj'}) \boxed{\mathrm{sinc}(\pi(j-j'))} + \mathrm{c.c.}$$

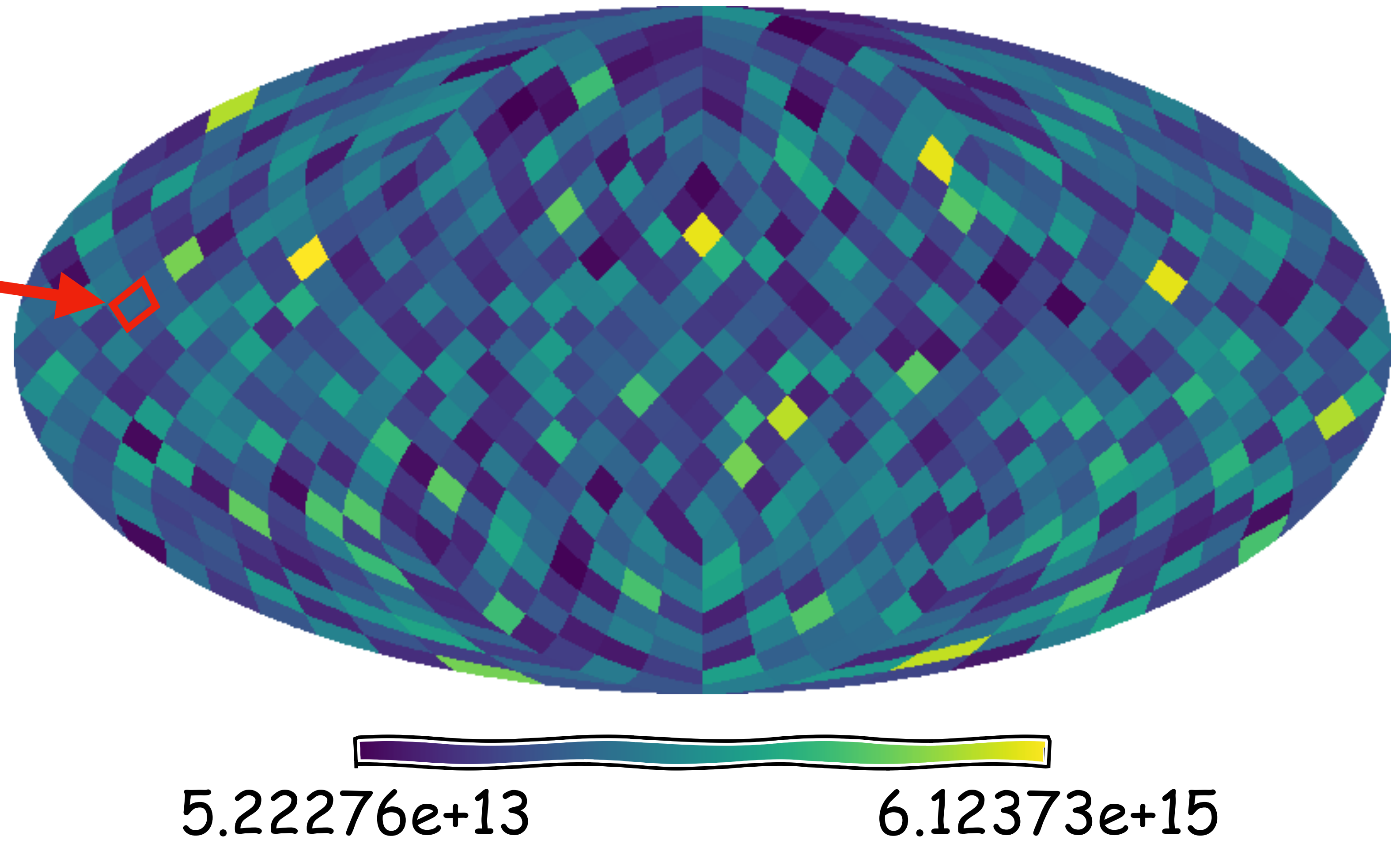
interference between different frequencies

WHY IS LIFE SO HARD?!

each of this pixel can be thought as emitting a GW plane wave

$$h_{ij}(t, \vec{x}) = \sum_A h_A \, e^A_{ij}(\mathbf{k}) \, e^{i2\pi f(t - \mathbf{k}\cdot\mathbf{x} + \phi)}$$

random variables



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$$\langle \rho_{ab} \rangle \propto \sum_A \sum_k F^A_{a,k} F^A_{b,k} \; P_k$$

if we average over many realizations of the GWB this reduces to the usual expression

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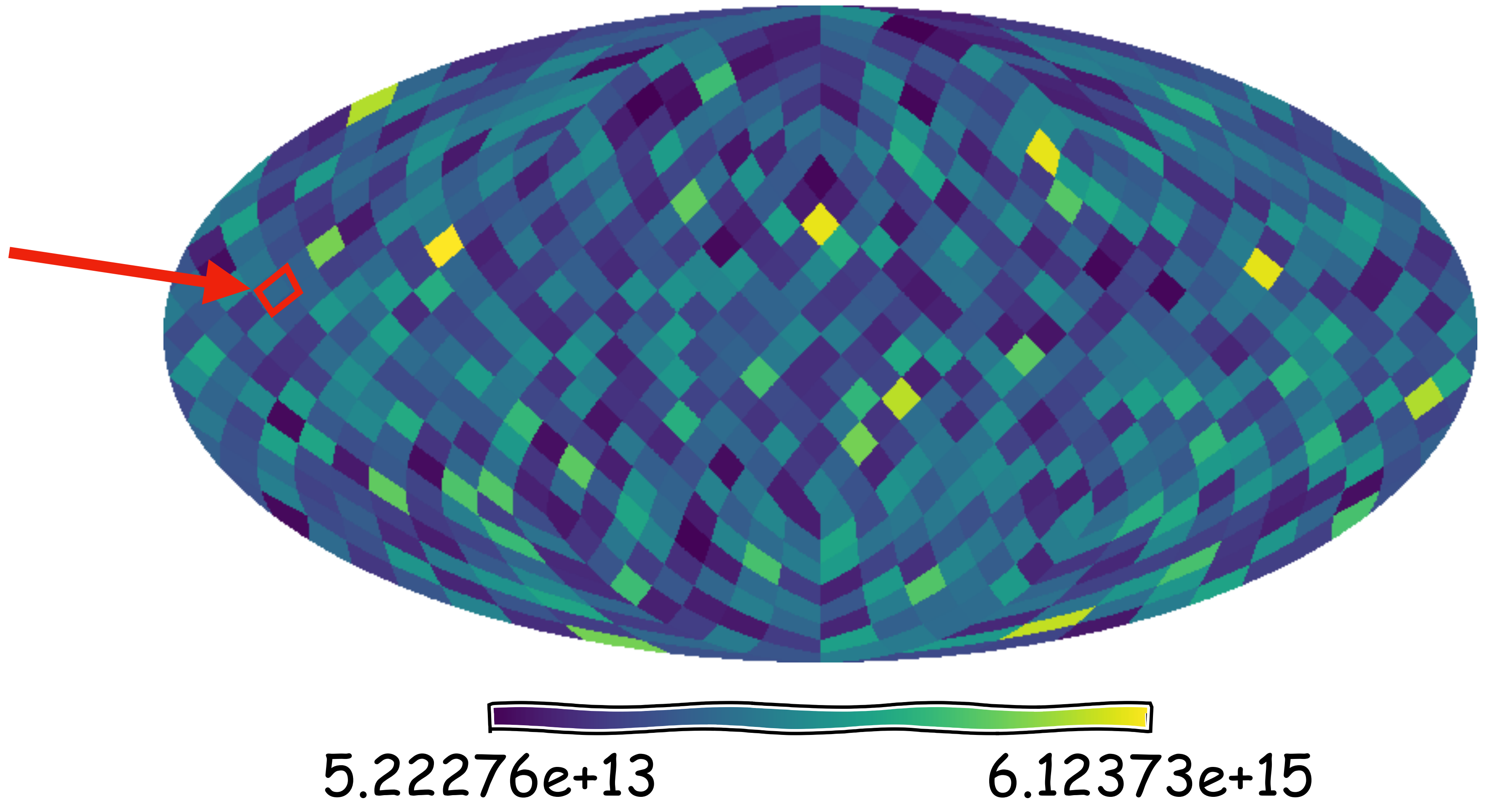

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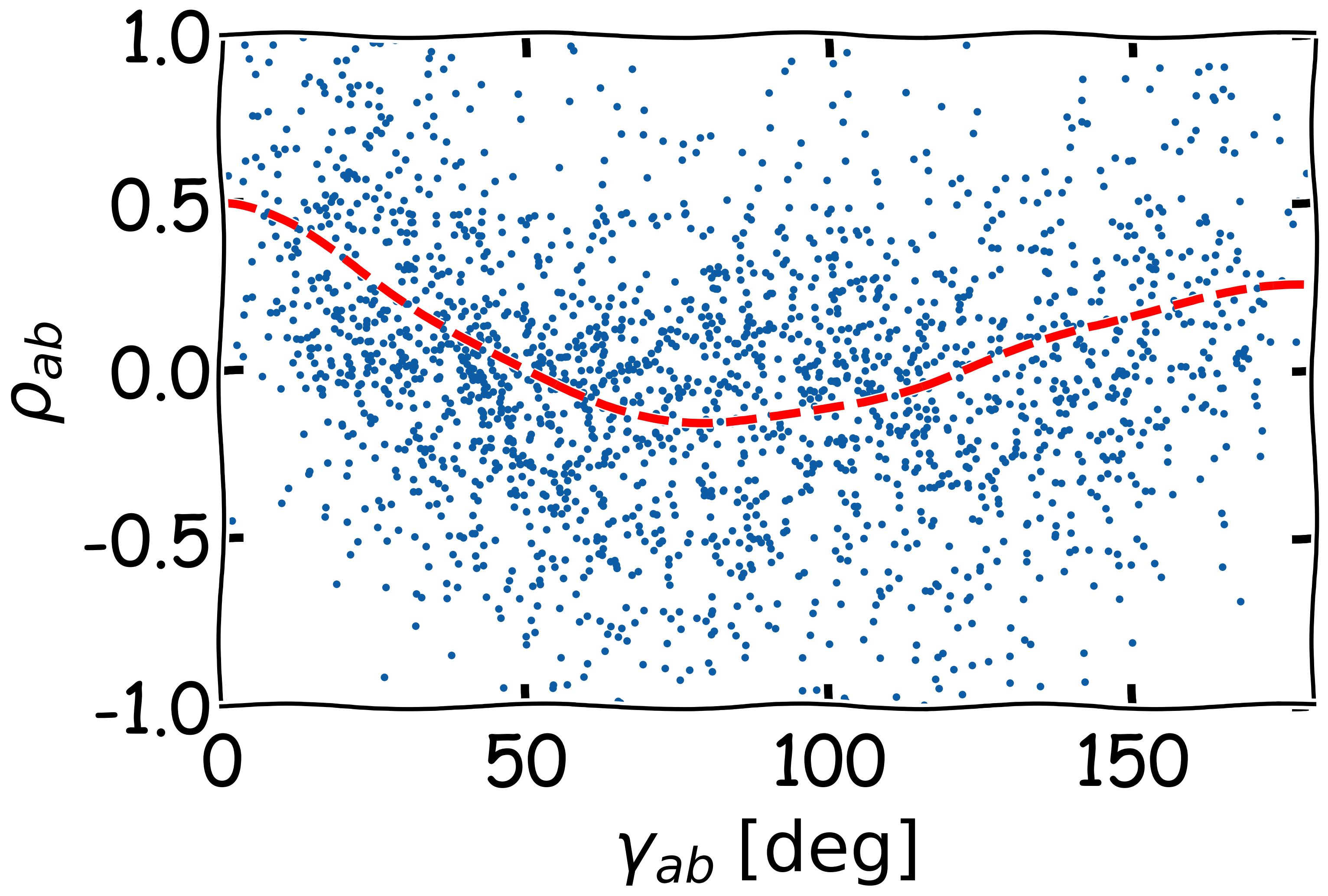

each realization of an isotropic GWB will produce cross correlation that fluctuate around the HD value

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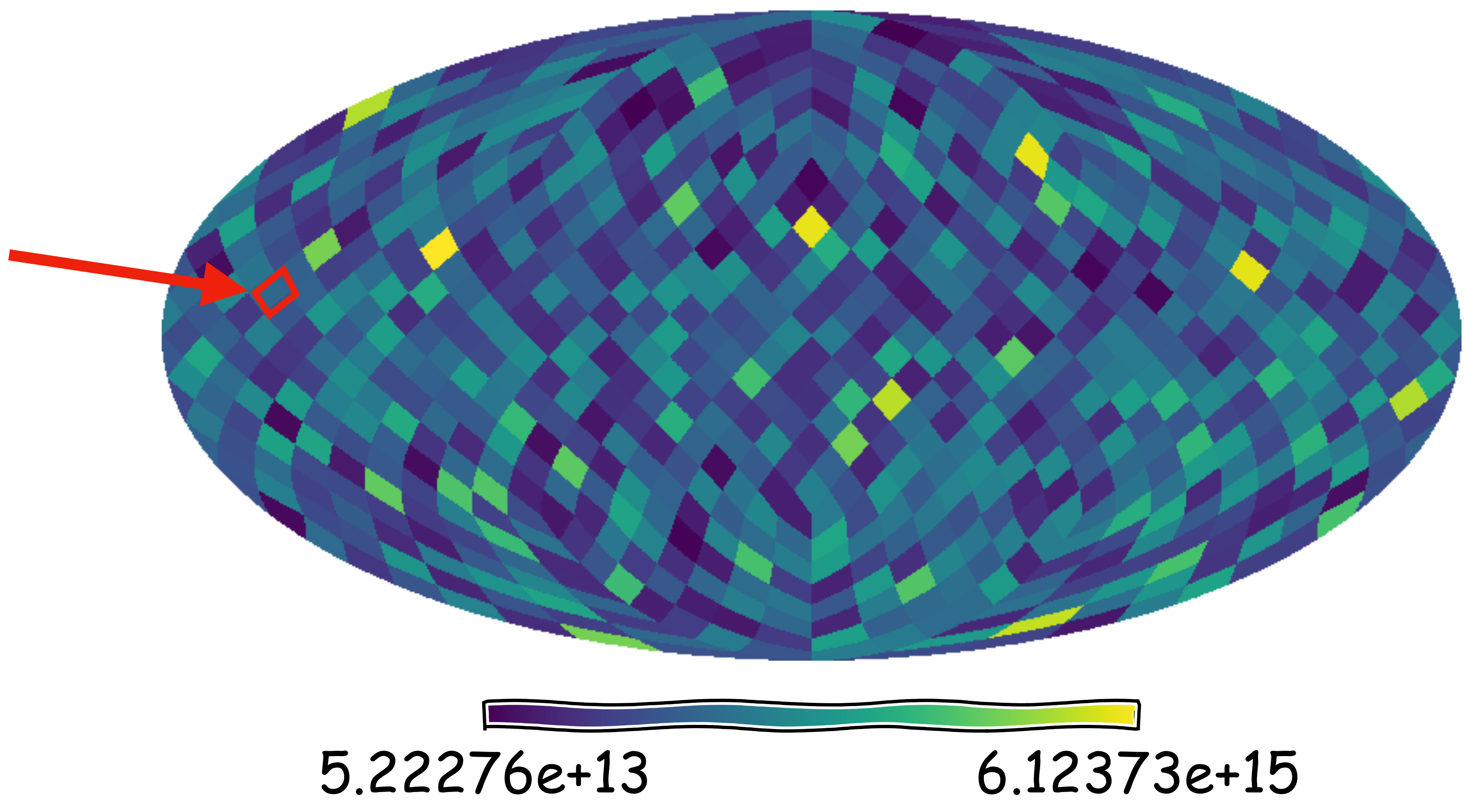

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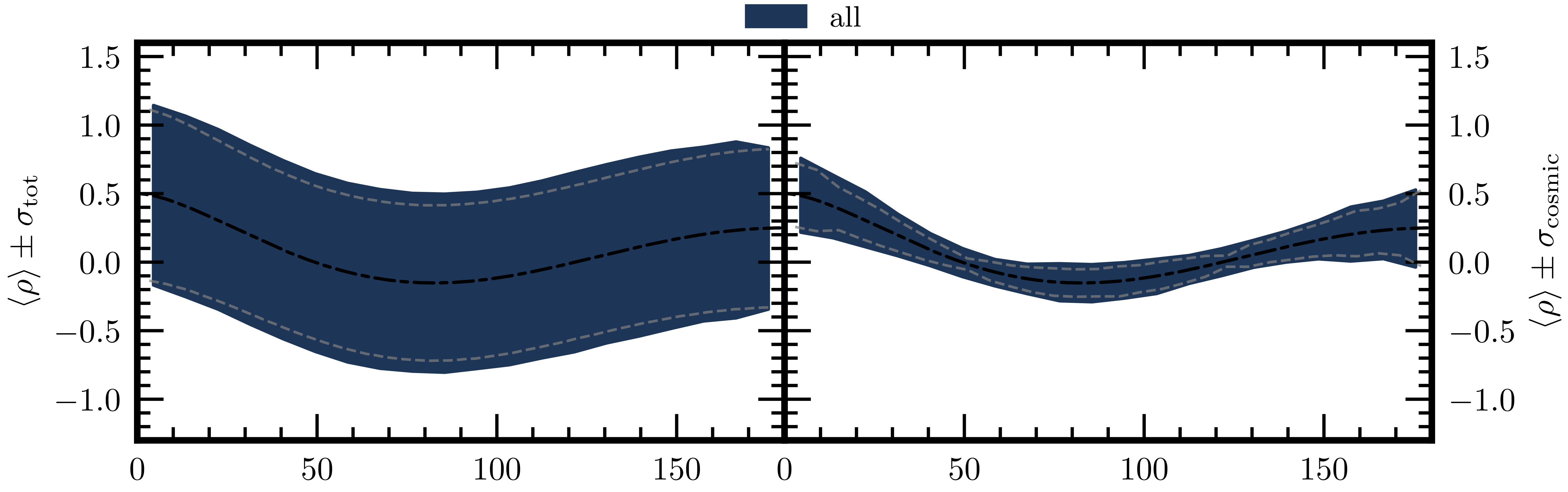

Konstandin, Lemke, AM, Perboni *"The impact of cosmic variance on PTAs anisotropy searches"*, [2498.07741]

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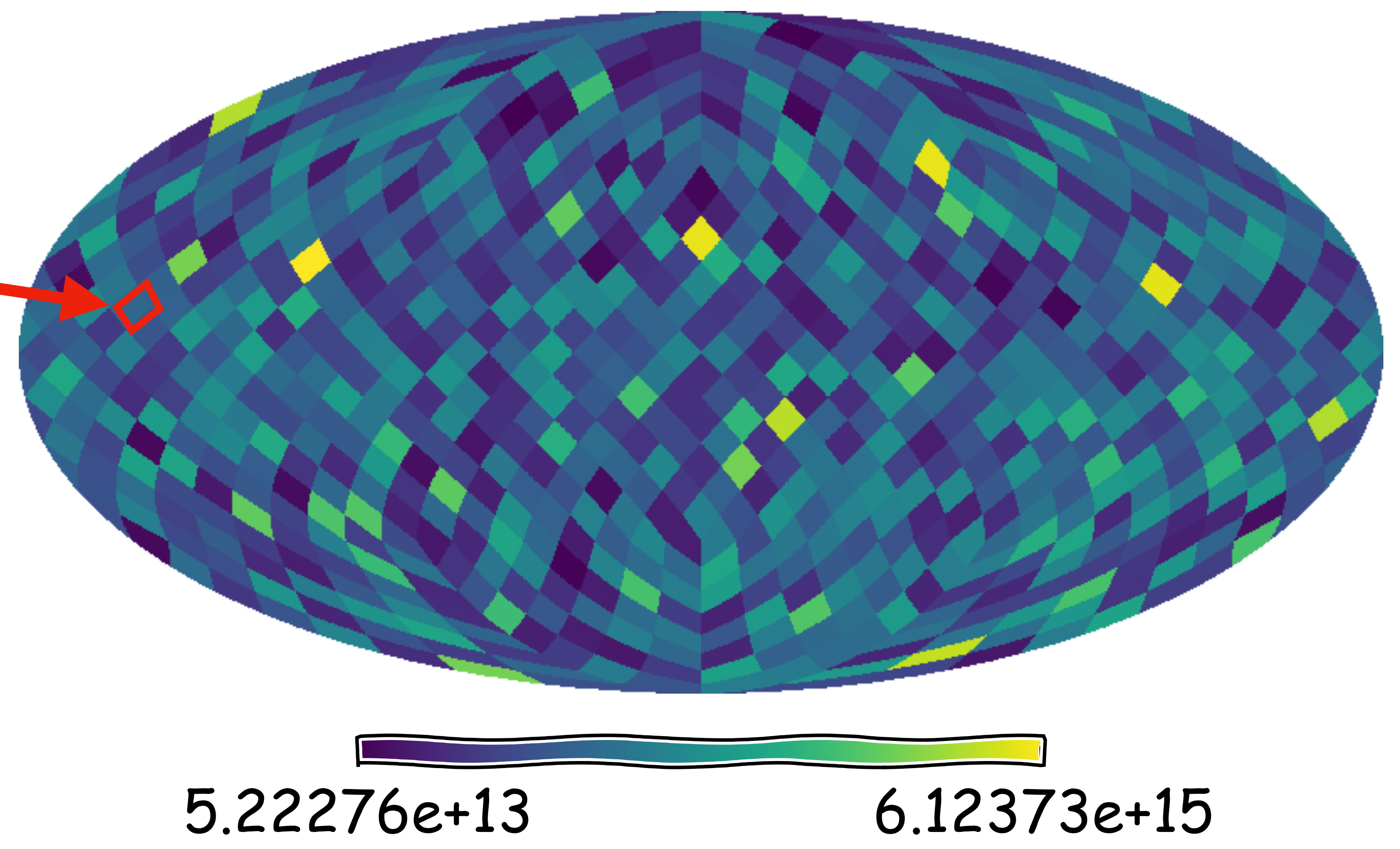

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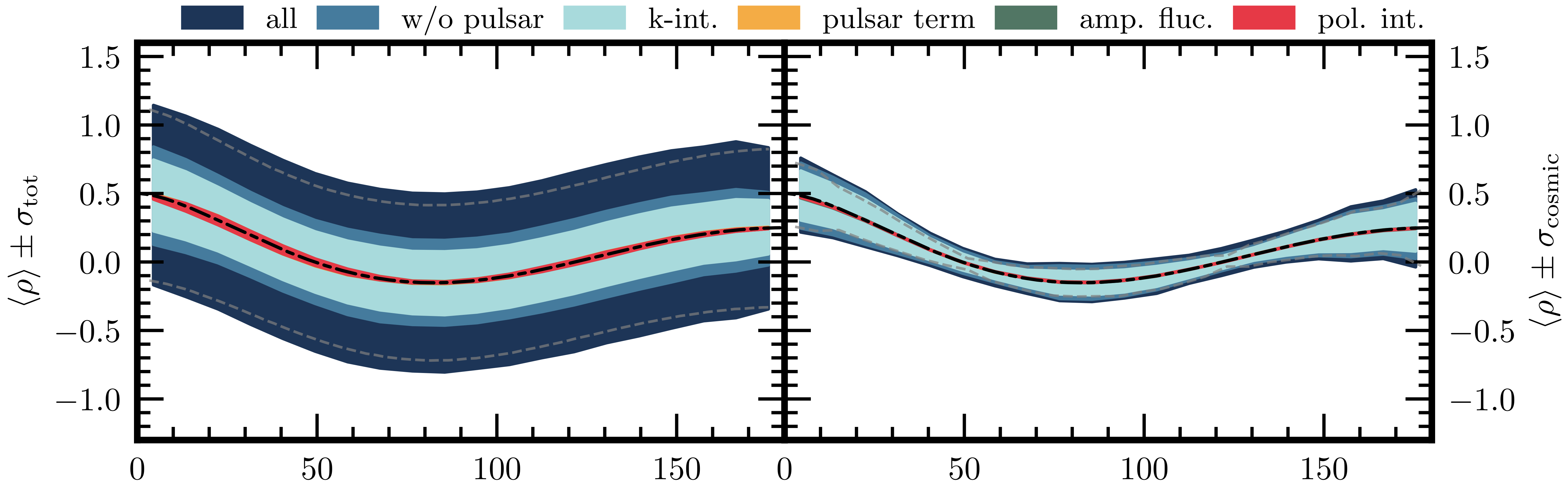

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NULL DISTRIBUTIONS v2.0

given an isotropic GWB as null hypothesis

$$\langle \tilde{h}^*_A(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle = \delta_{AA'}\delta(f-f')\delta(\hat{\Omega},\hat{\Omega}')H(f)$$

take one realization of the null hypothesis

$$\{h^A_{k,i},\phi^A_{k,i}\}$$

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➜

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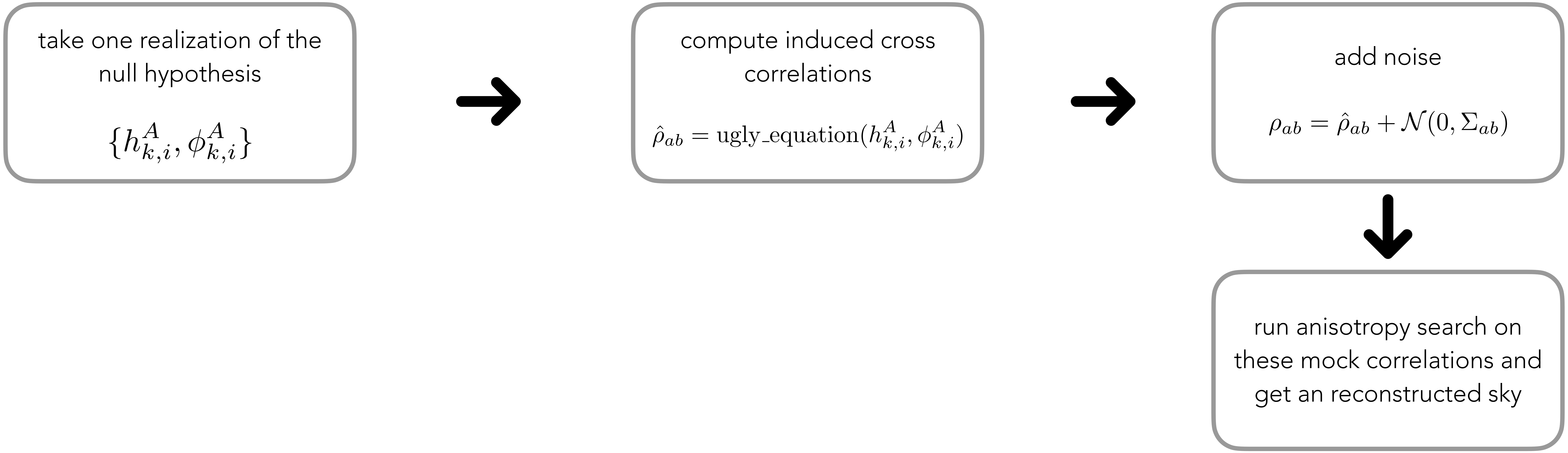

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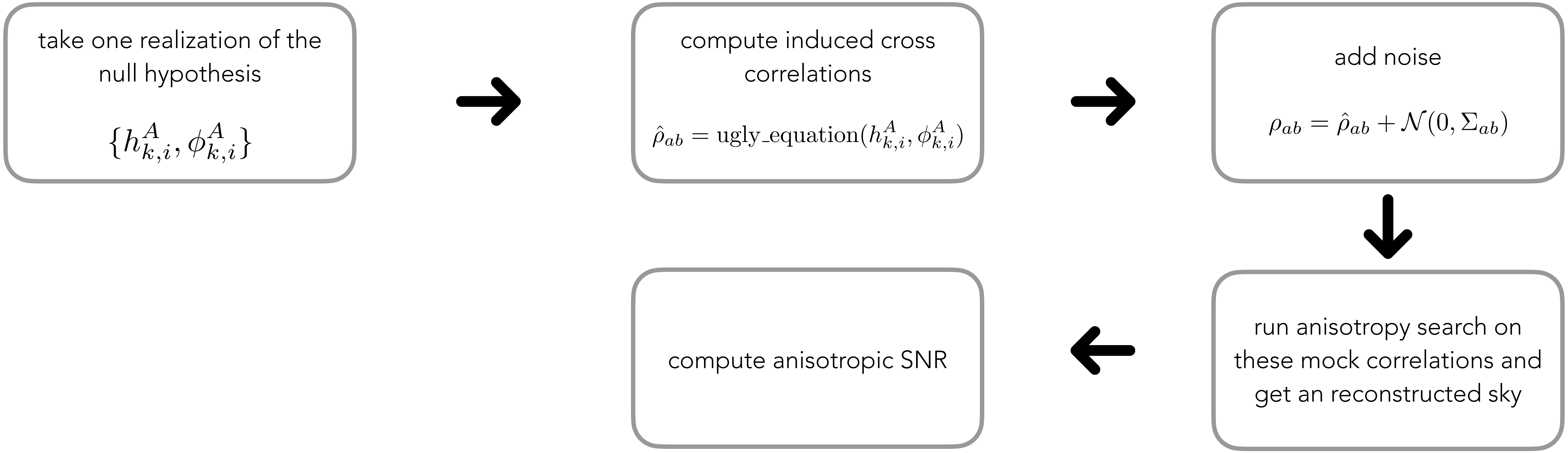

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take one realization of the null hypothesis

$$\{h_{k,i}^A, \phi_{k,i}^A\}$$

→

compute induced cross correlations

$$\hat{\rho}_{ab} = \text{ugly\_equation}(h_{k,i}^A, \phi_{k,i}^A)$$

→

add noise

$$\rho_{ab} = \hat{\rho}_{ab} + \mathcal{N}(0, \Sigma_{ab})$$

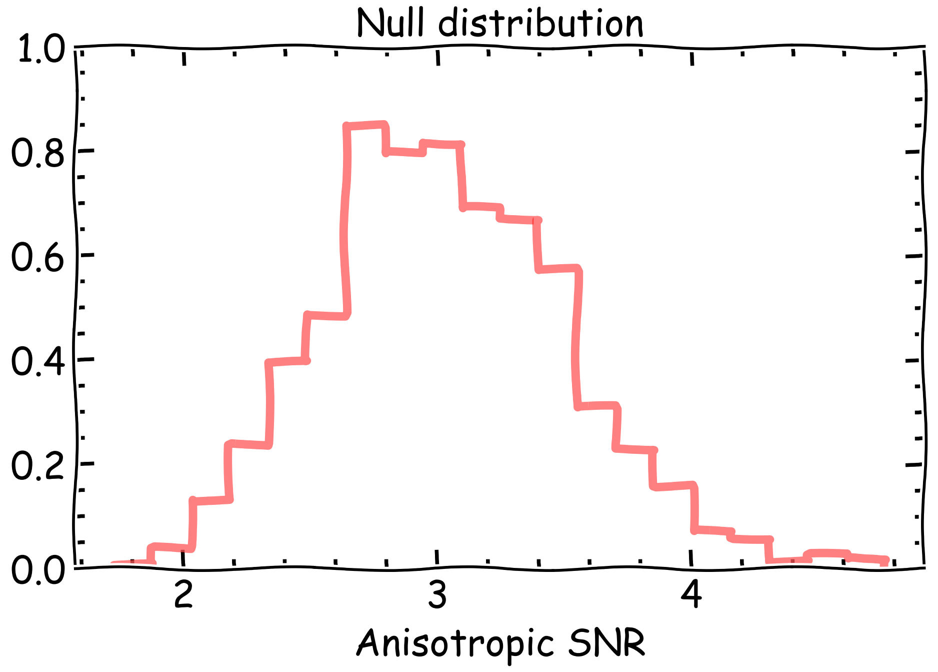
↓

run anisotropy search on these mock correlations and get an reconstructed sky

←

compute anisotropic SNR

← repeat many times



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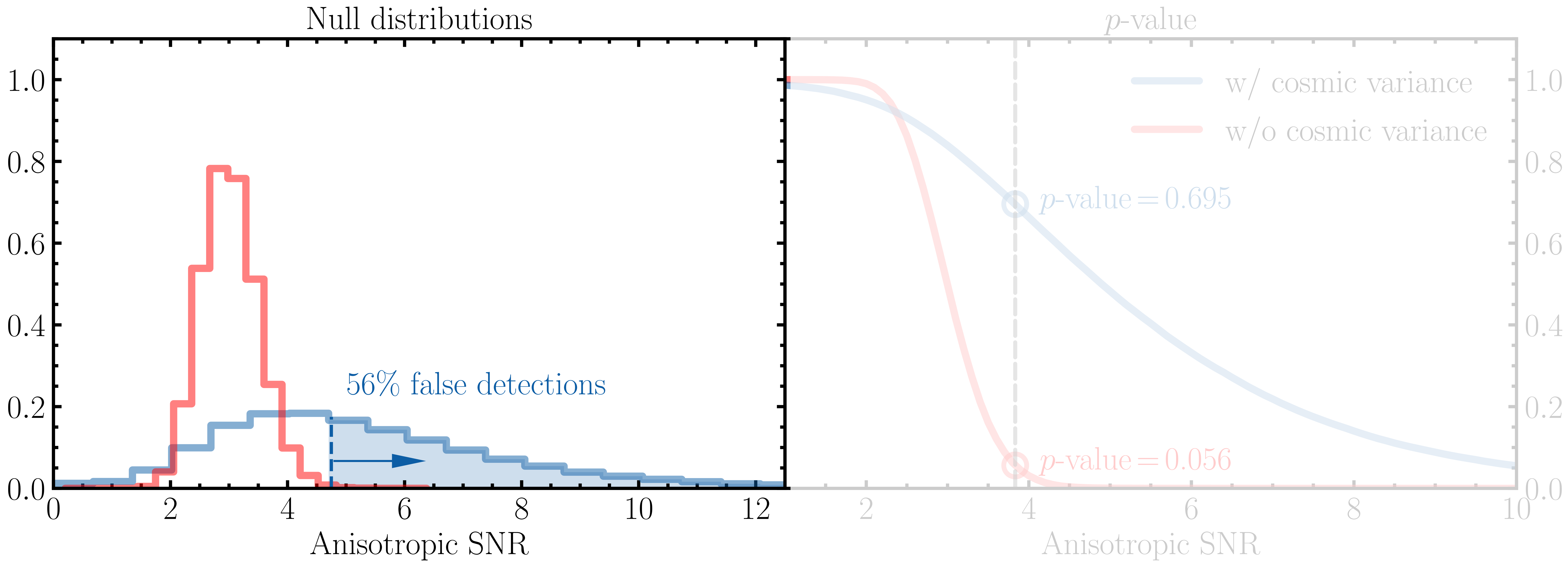

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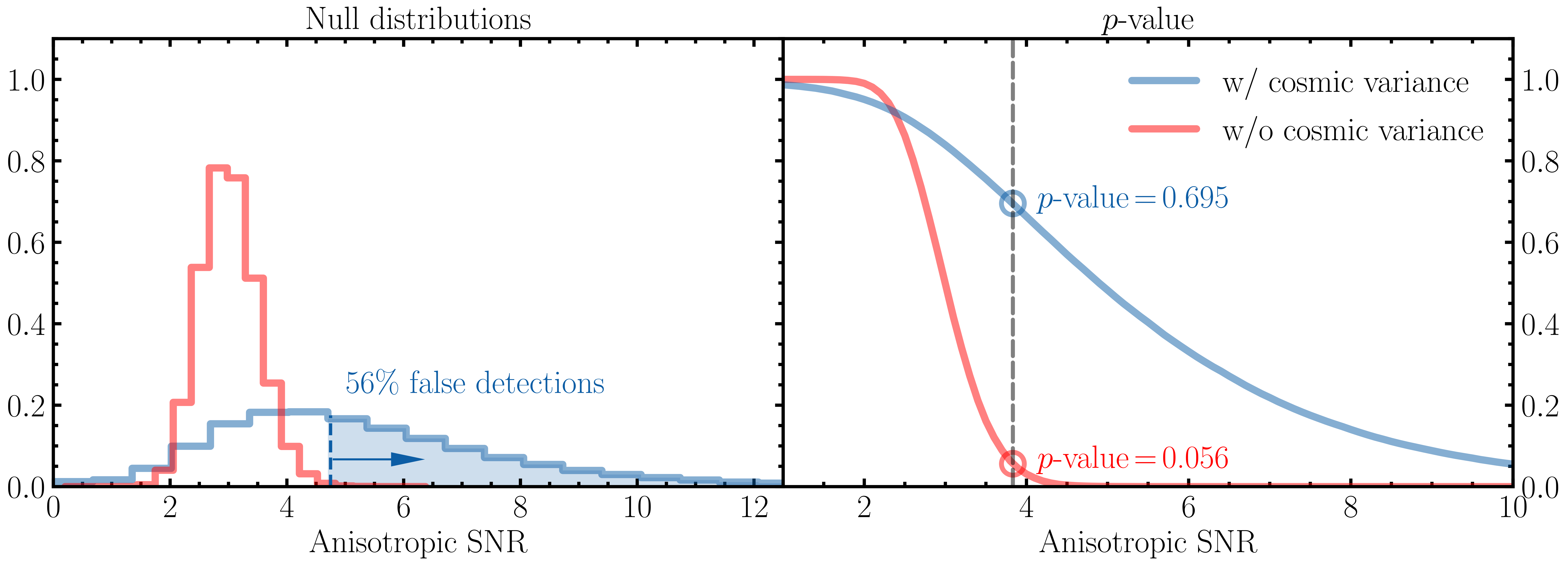

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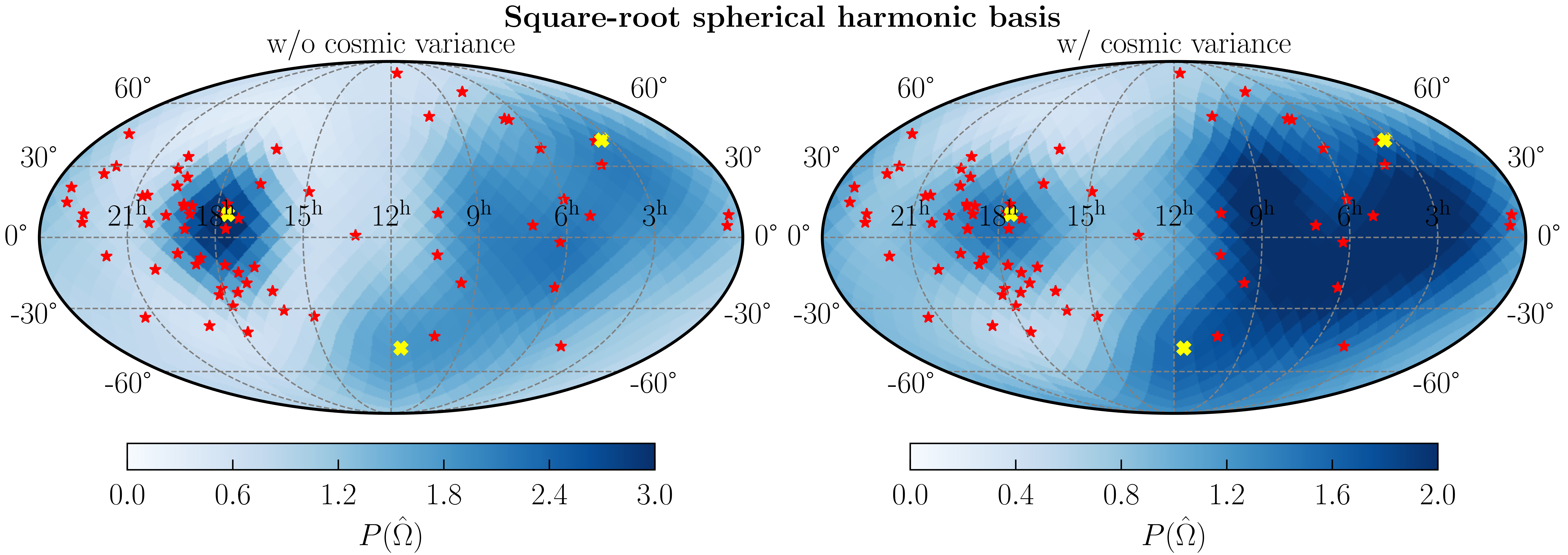
MAP RECONSTRUCTION IS ALOS NOT GREAT



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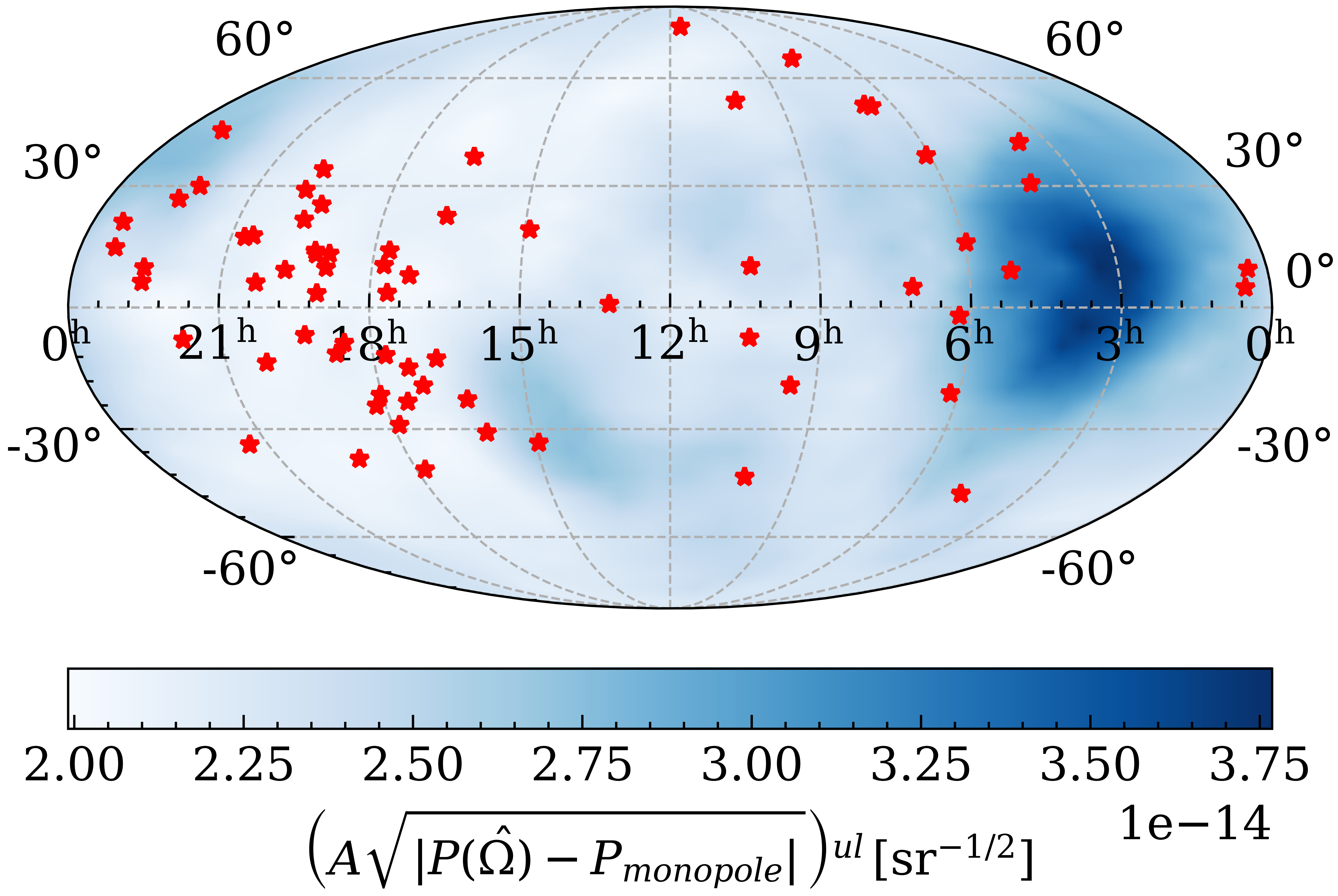
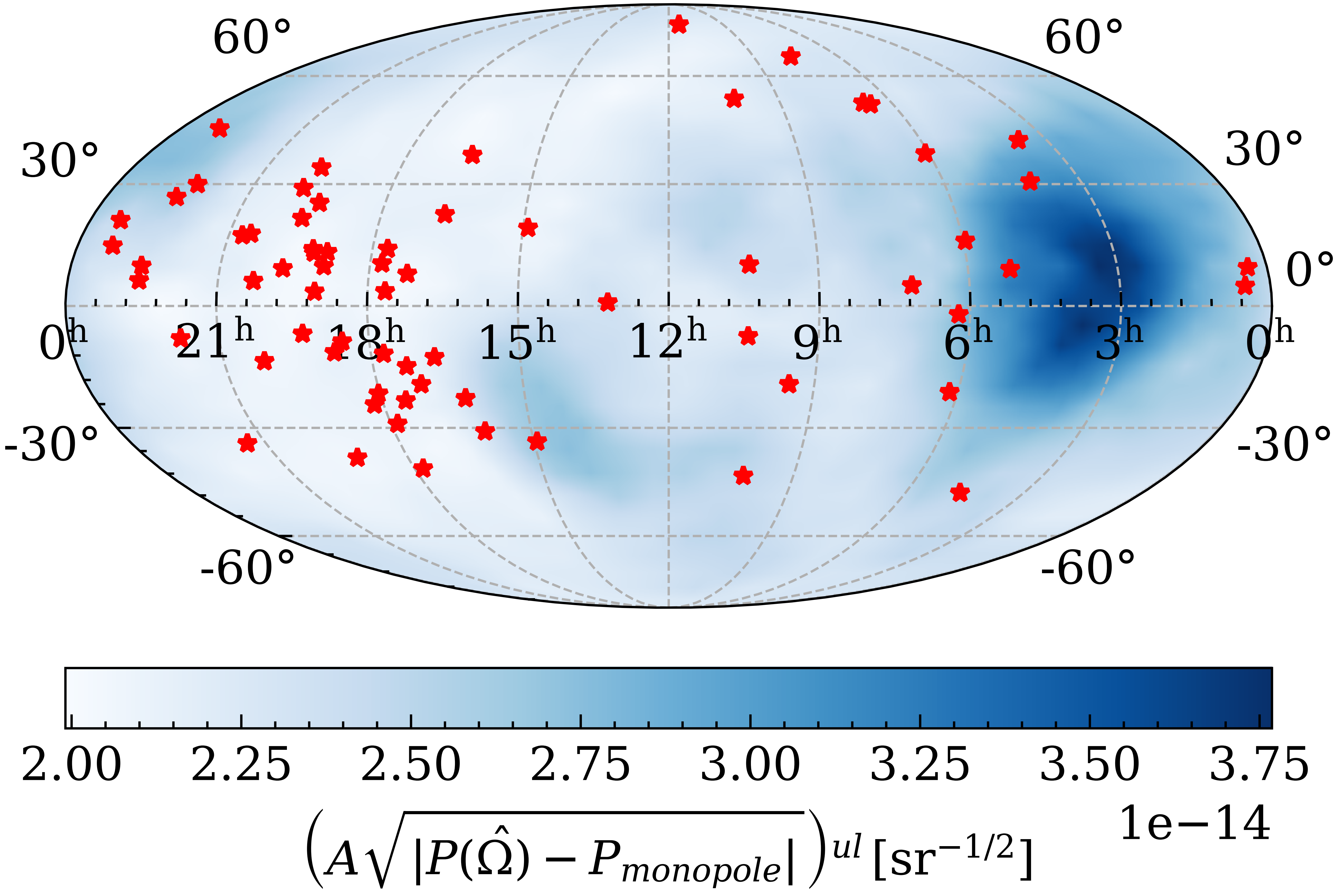
SO, HAVE WE DETECTED ANISOTROPIES?!



for the reconstructed GWB map in the NG15 data

SNR~4 $\Rightarrow p \sim 0.05$

SO, HAVE WE DETECTED ANISOTROPIES?!



for the reconstructed GWB map in the NG15 data

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THE PATH FORWARD

what does these null detections teach us?

→ is there a tension between these null detections and the SMBHB intepretation of the GWB?

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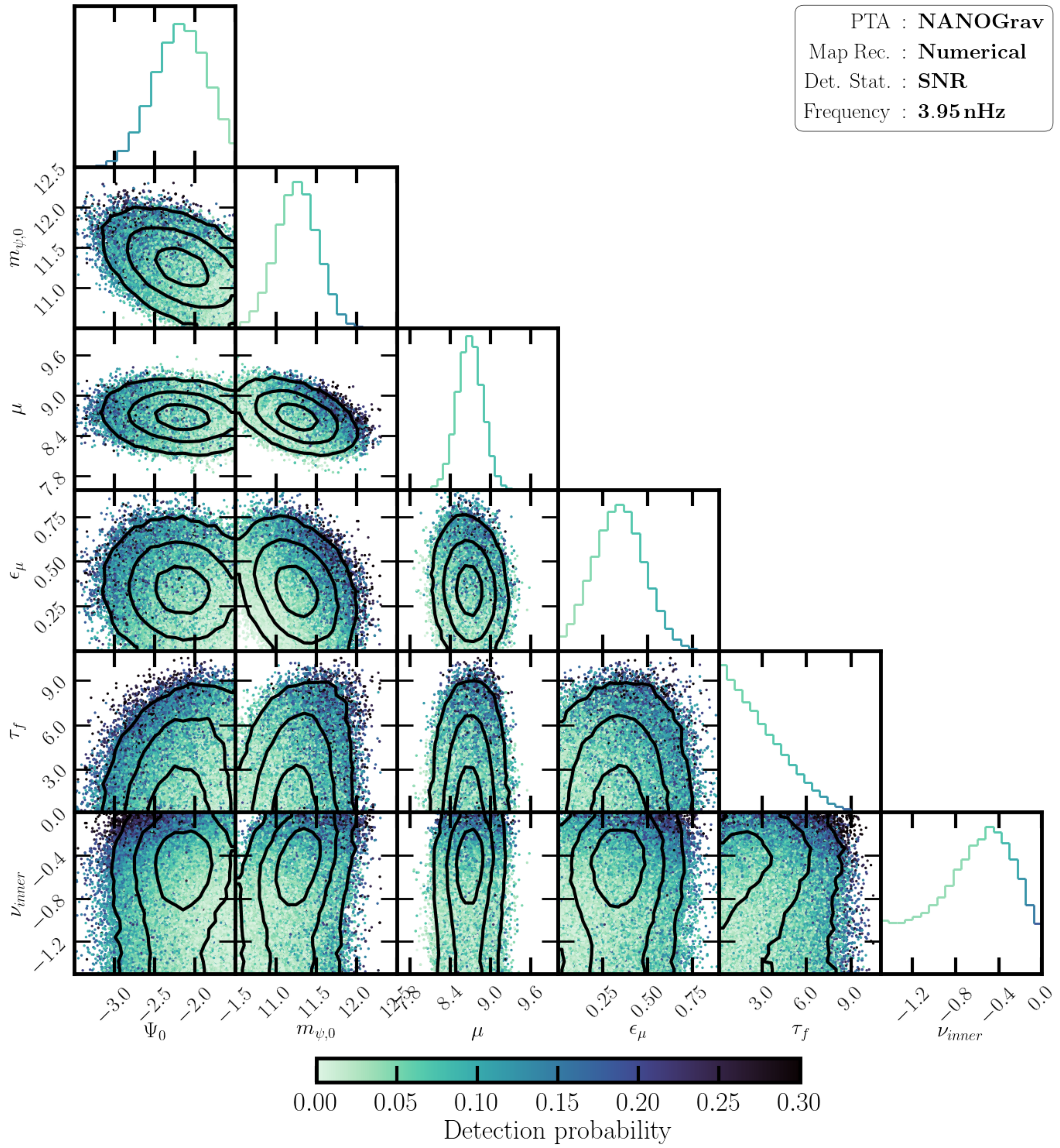
Lemke, AM, Gersbach, *"Detecting Gravitational Wave Anisotropies from Supermassive Balck Hole Binaries"*, [2407.04464]



THE PATH FORWARD

what does these null detections teach us?

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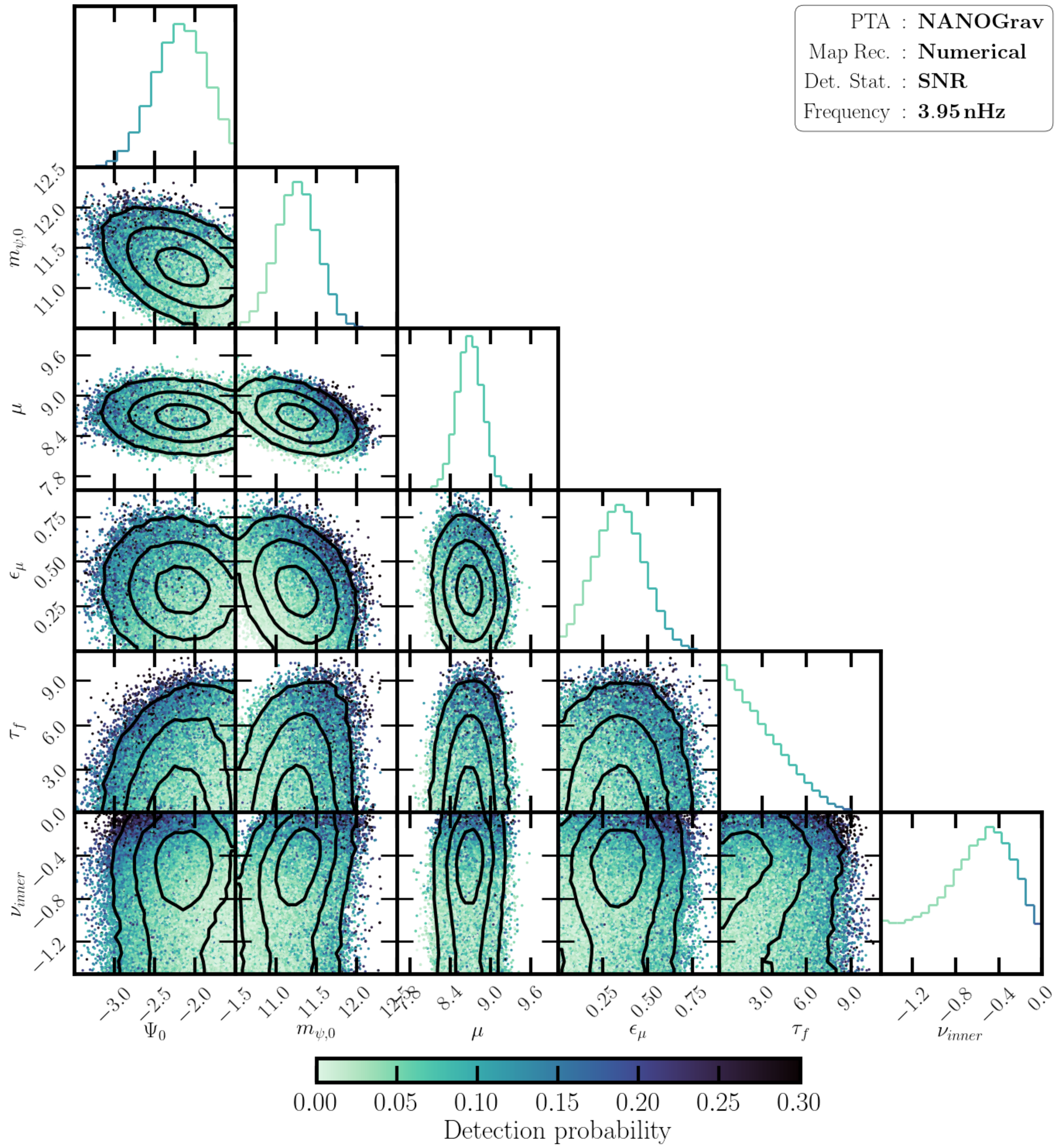

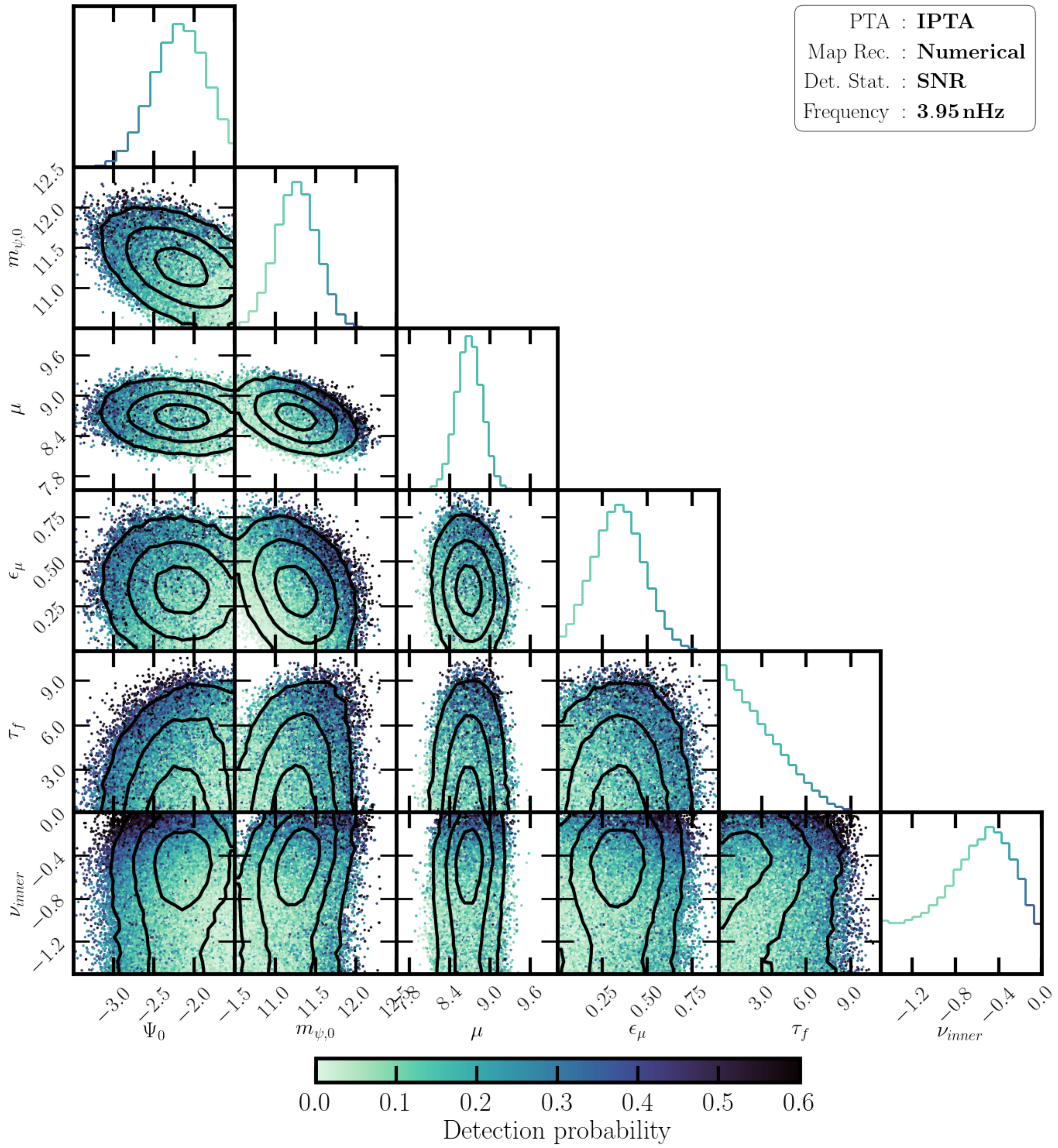

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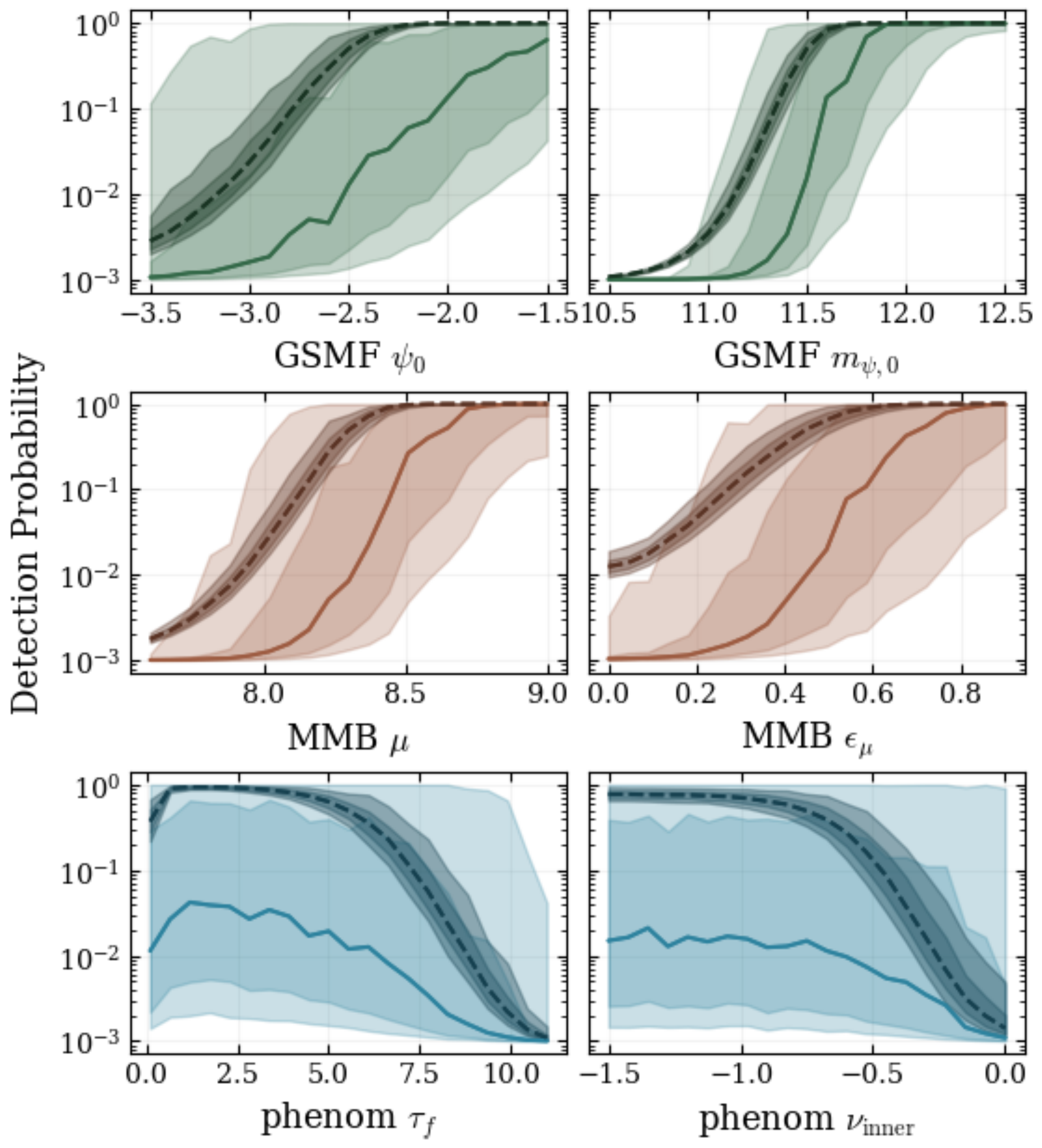
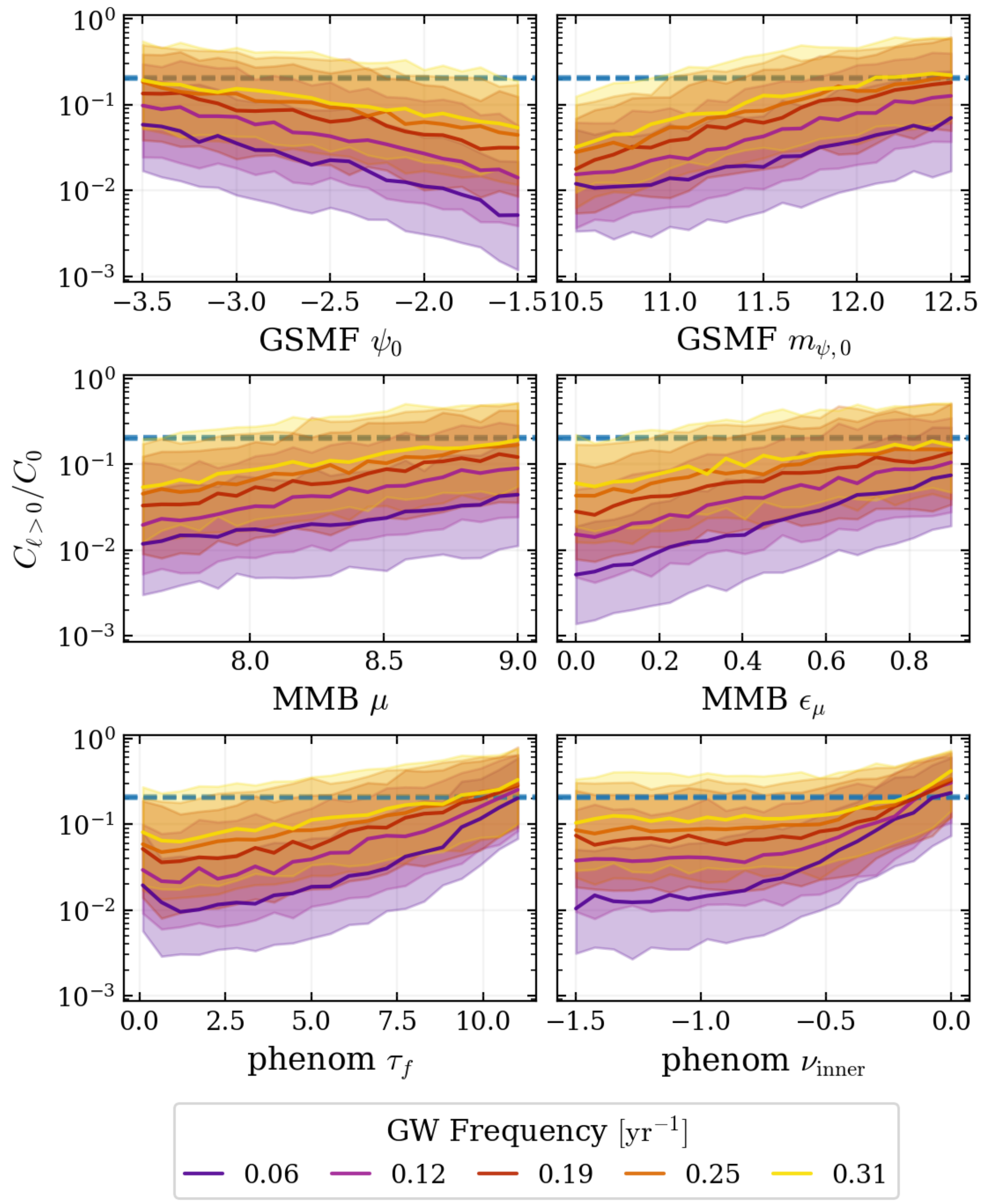
  Gardiner, Kelley, Lemke, AM, *“Beyond the Background: Gravitational-wave Anisotropy and Continuous Waves from Supermassive Black Hole Binaries”*, ApJ 965, 2, 264 (2024)





strong evidence for a GWB in the nHz band

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cosmology or astrophysics?

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CW and anisotropies will help us discriminating

strong evidence for a GWB in the nHz band

cosmology or astrophysics?

CW and anisotropies will help us discriminating

we need to develop tools to characterize the GWB

backup

HOW TO SAMPLE A GWB

$$\langle \tilde{h}^{A*}_{kj}\, \tilde{h}^{A'}_{k'j'} \rangle = \delta_{AA'}\delta_{kk'}\delta_{jj'}\,\frac{H_j}{\Delta\hat{\Omega}\,\Delta f}$$

$$\langle \tilde{h}^{A}_{kj}\, \tilde{h}^{A'}_{k'j'} \rangle = \langle \tilde{h}^{A*}_{kj}\, \tilde{h}^{A'*}_{k'j'} \rangle = 0$$

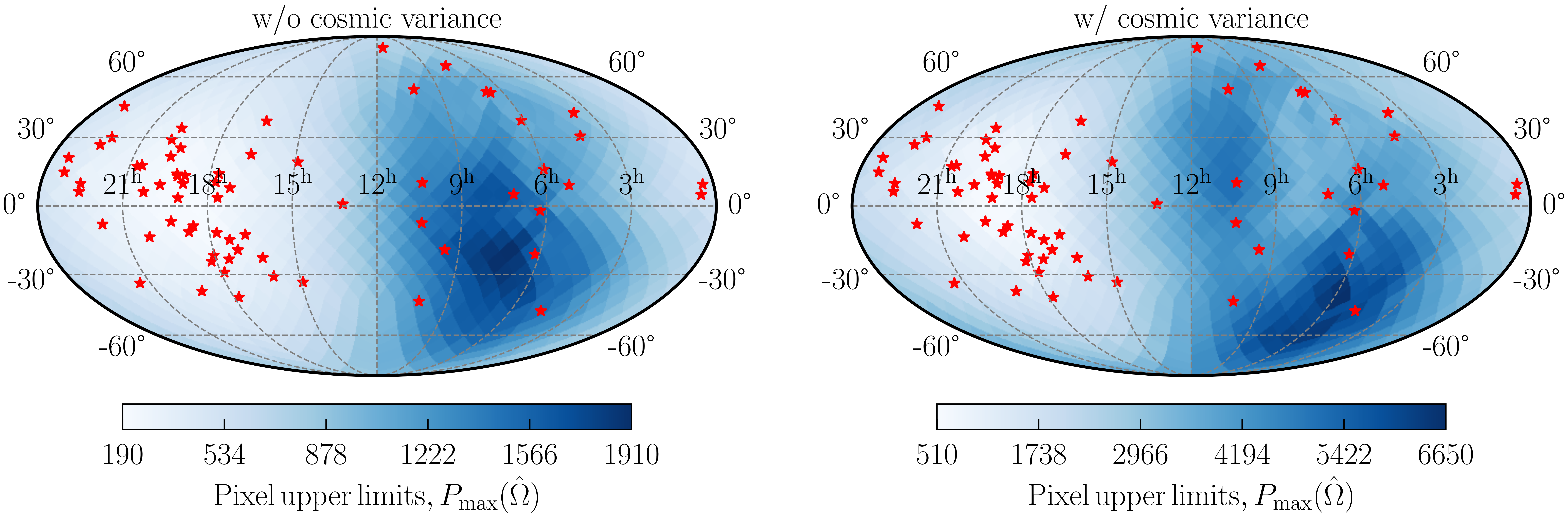
$$\Big\downarrow \qquad \tilde{h}^{A}_{kj} \equiv h^{A}_{kj}\, e^{i\phi^{A}_{kj}}$$

$$p(h) = \frac{h}{2\pi\sigma_j^2} e^{-h^2/2\sigma_j^2}$$

$$\sigma_j^2 = H_j/(2\Delta\hat{\Omega}\Delta f)$$

$$p(\phi) = \mathrm{Uniform}[0,\ 2\pi]$$

RESULTS IN THE PIXEL BASIS



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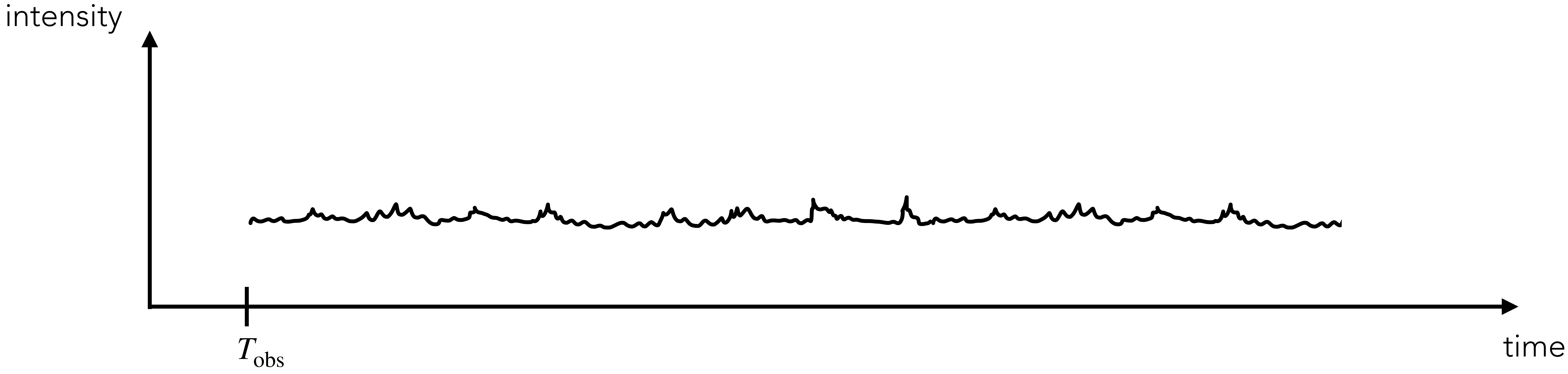
PULSAR TIMING



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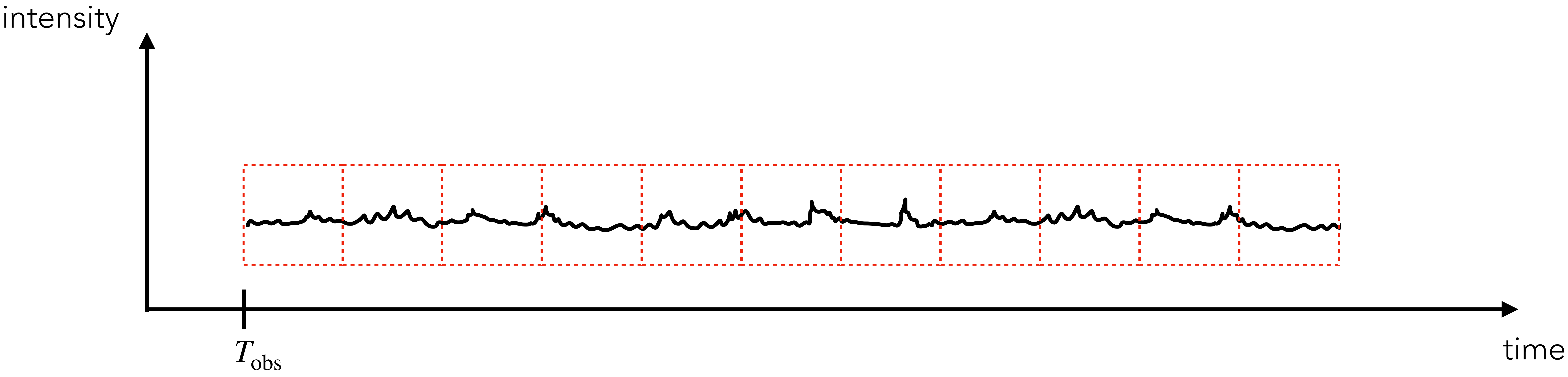

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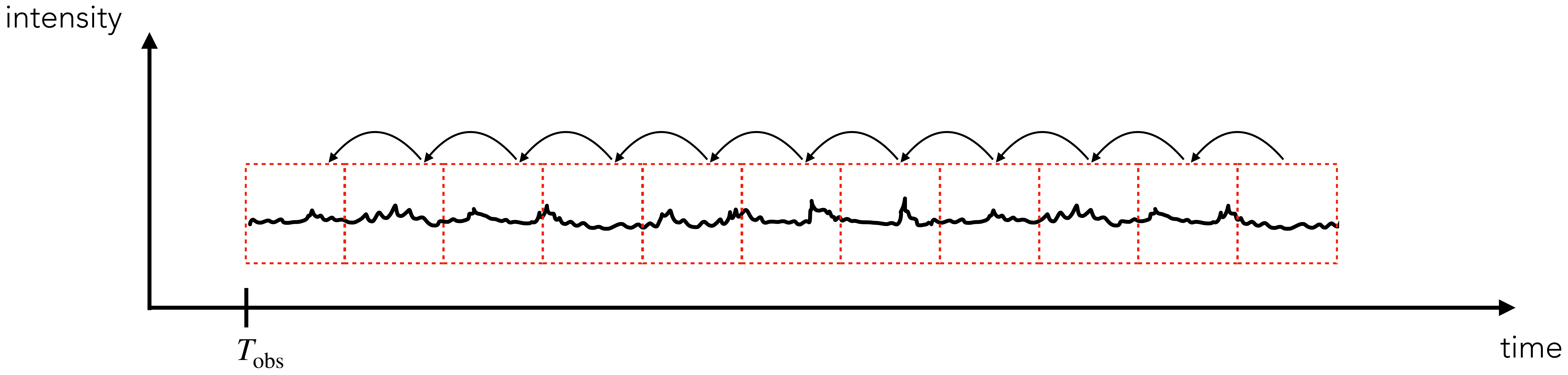

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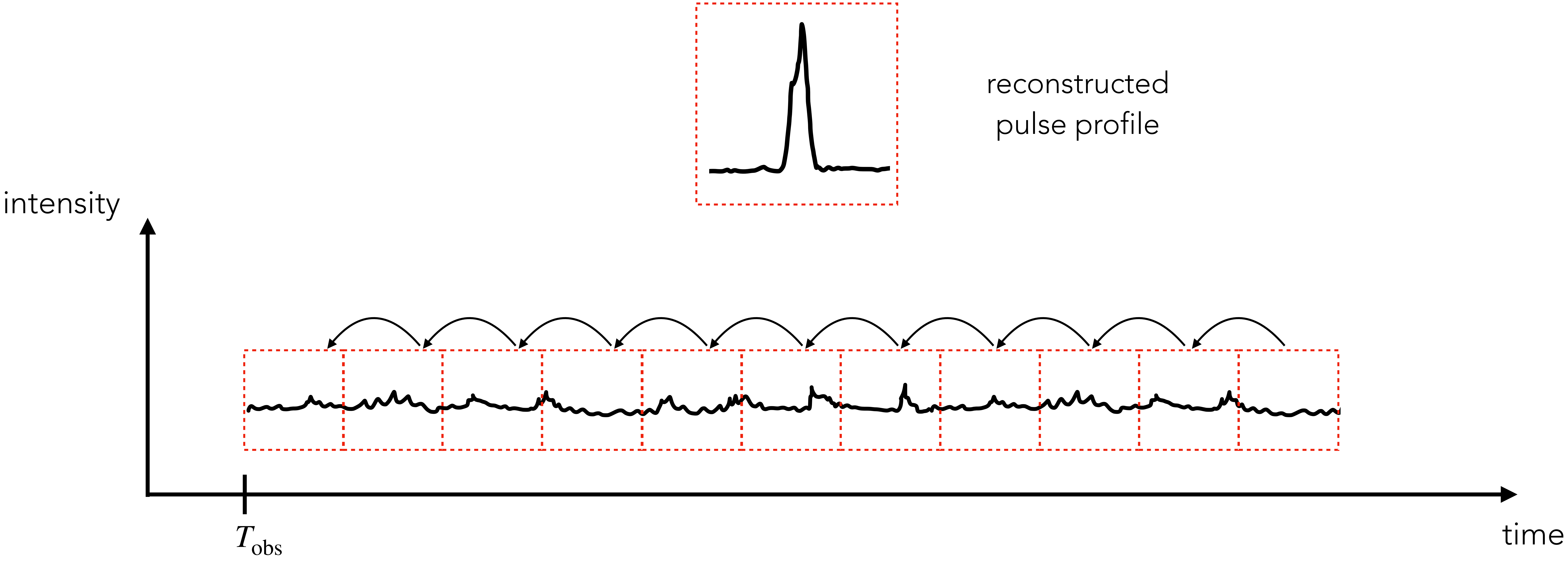
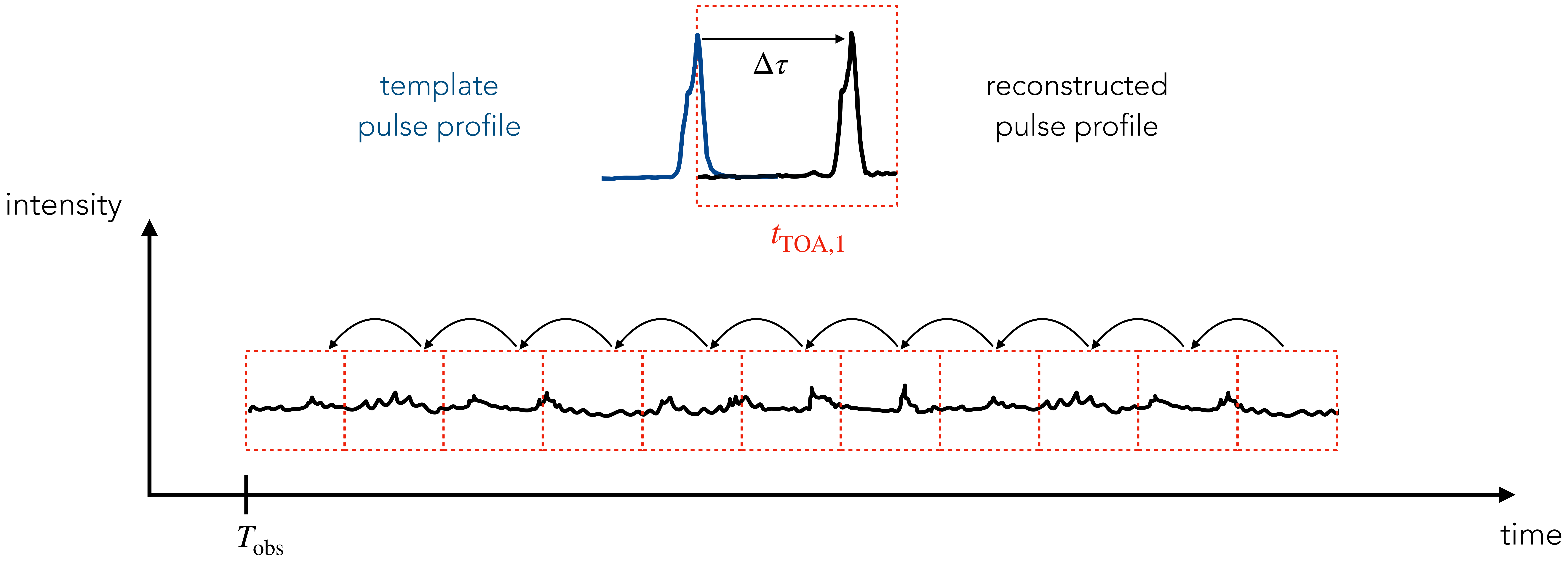


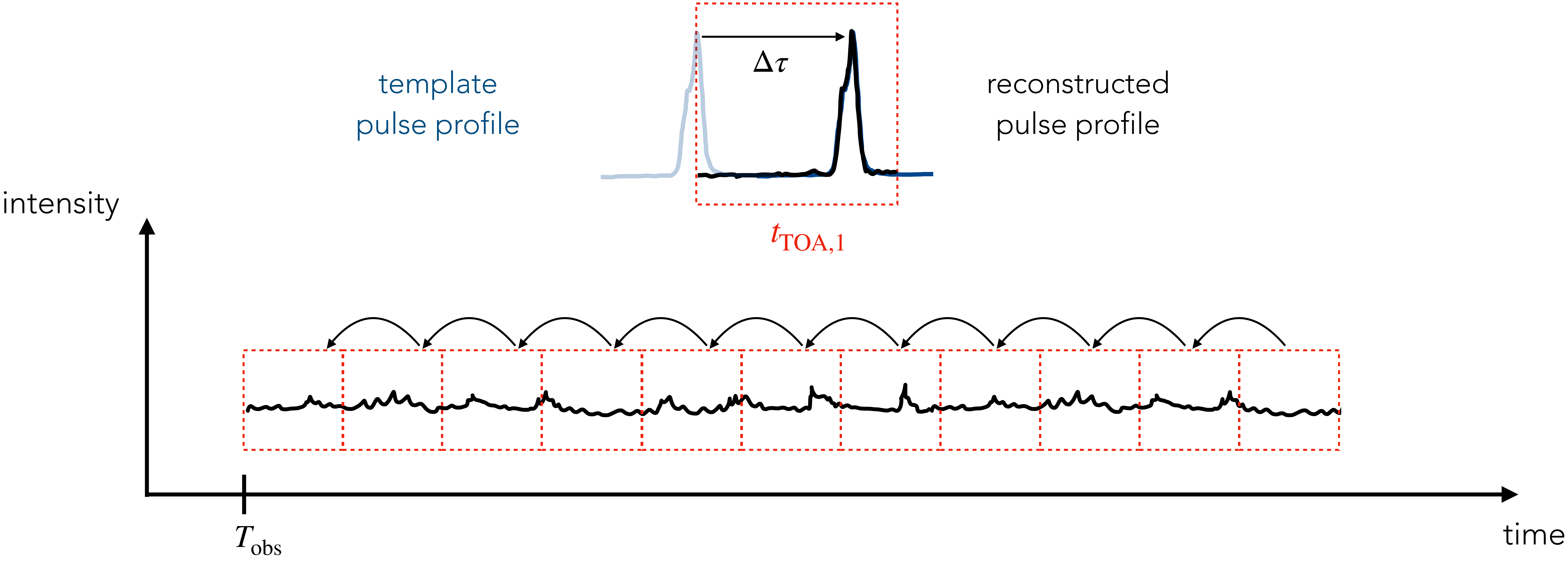
reconstructed
pulse profile

PULSAR TIMING



$$\boxed{t_{\mathrm{TOA}} = T_{\mathrm{obs}} + \Delta\tau}$$

PULSAR TIMING



$$t_{\mathrm{TOA}} = T_{\mathrm{obs}} + \Delta\tau$$

BUILDING A TIMING MODEL

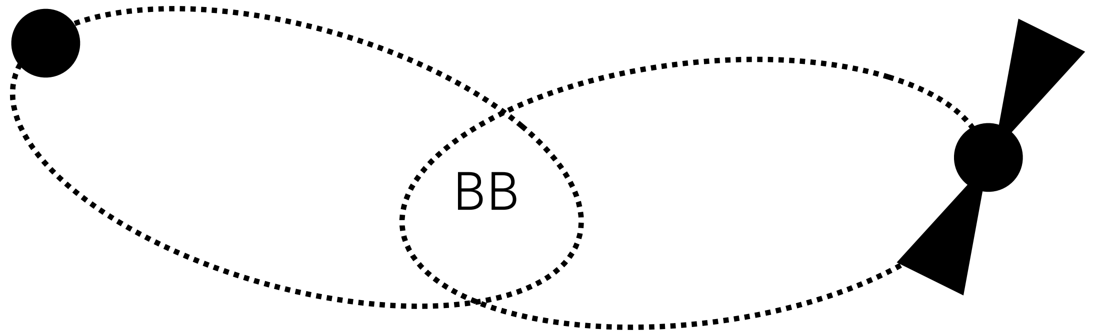
$$t_{\rm e}^{\rm psr} = t_{\rm a}^{\rm obs} - \Delta_{\odot} - \Delta_{\rm IS} - \Delta_{\rm B}$$

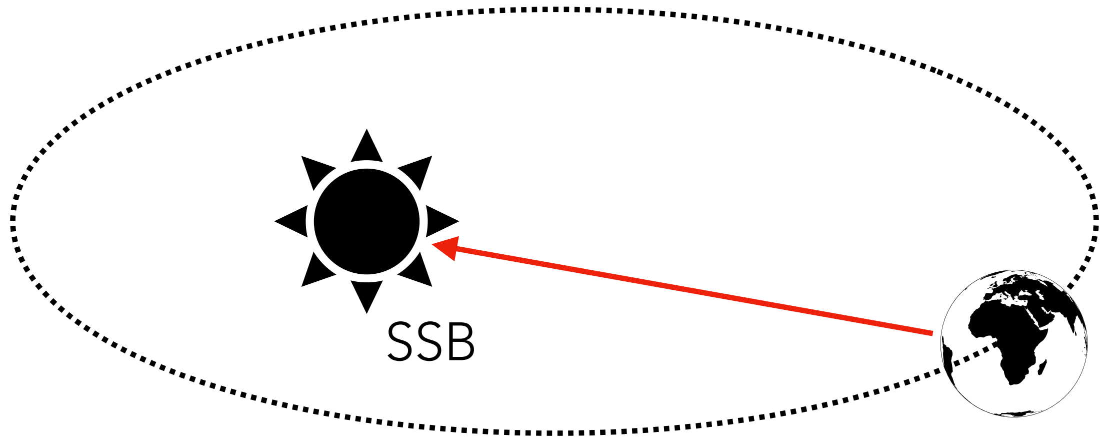
the goal is to relate the arrival time in Terrestrial Time (TT) to the emission time in the pulsar reference frame

BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \boxed{\Delta_{\odot}} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$

first step transform from the from the TT time to the SSB time





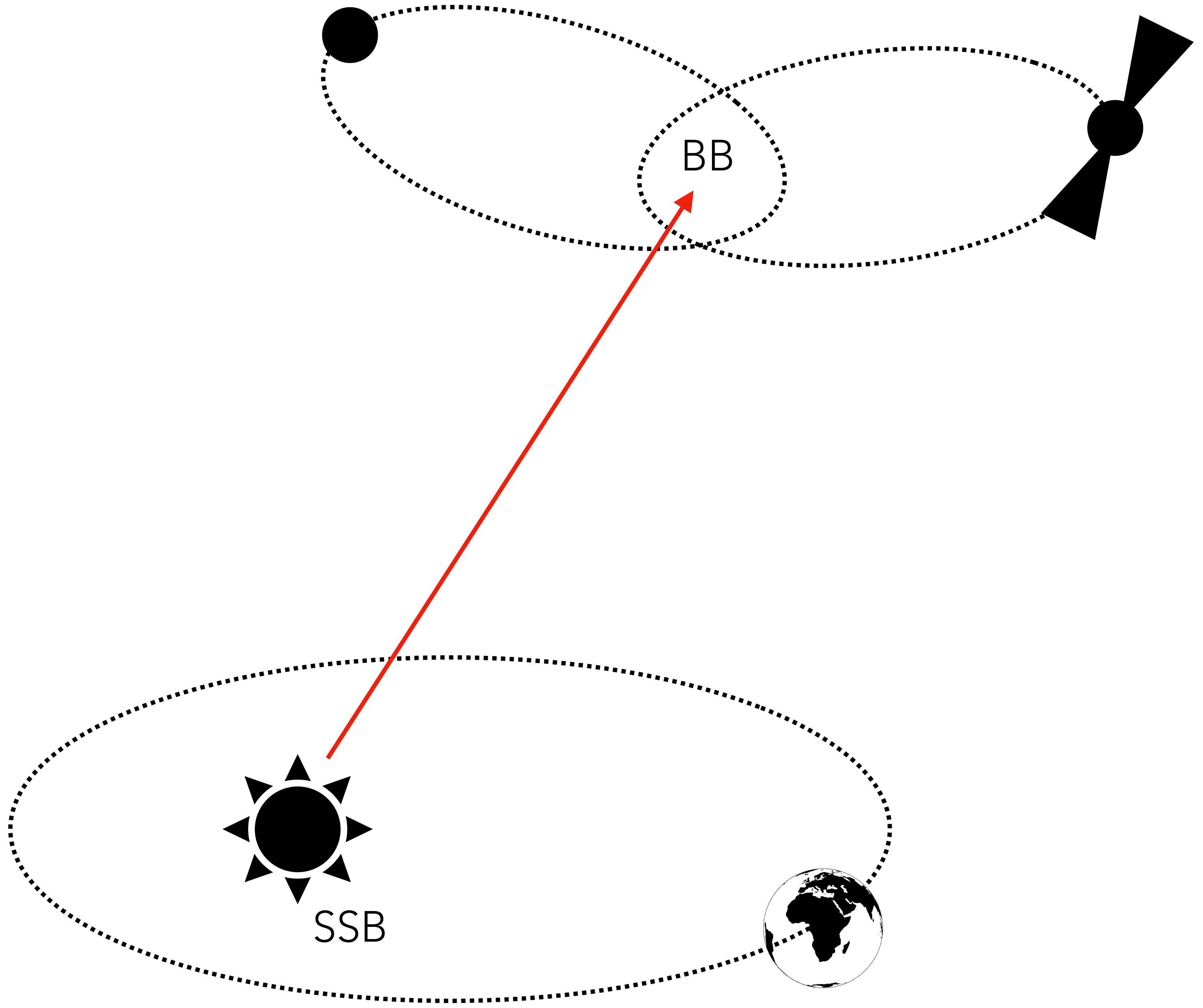
$\Delta_{\odot} = \Delta_{\mathrm{A}}$	**atmospheric delay:** group velocity of radio waves in the atmosphere differs from the vacuum speed of light
$+\ \Delta_{\mathrm{R}\odot}$	**Roemer delay:** difference in the arrival time of the pulse at the observatory and at the SSB
$+\ \Delta_{\mathrm{p}}$	**Parallax term:** delay due to the curvature of spherical wavefront
$+\ \Delta_{\mathrm{D}\odot}$	**dispersion term:** due to radio waves propagation in interplanetary medium
$+\ \Delta_{\mathrm{S}\odot}$	**Shapiro delay:** due to propagation of radio pulses through potential wells of solar systems bodies
$+\ \Delta_{\mathrm{E}\odot}$	**Einstein delay:** difference in the arrival time due to acceleration and potential of the Earth

BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \boxed{\Delta_{\mathrm{IS}}} - \Delta_{\mathrm{B}}$$

second step transform from the from SSB to the BB time

$\Delta_{\mathrm{IS}} = \Delta_{\mathrm{VP}}$	**vacuum propagation delay:** vacuum light travel time
Δ_{ISD}	**dispersion term:** due to propagation through the interstellar medium
Δ_{ES}	**Einstein delay:** due to the secular motion of the SSB

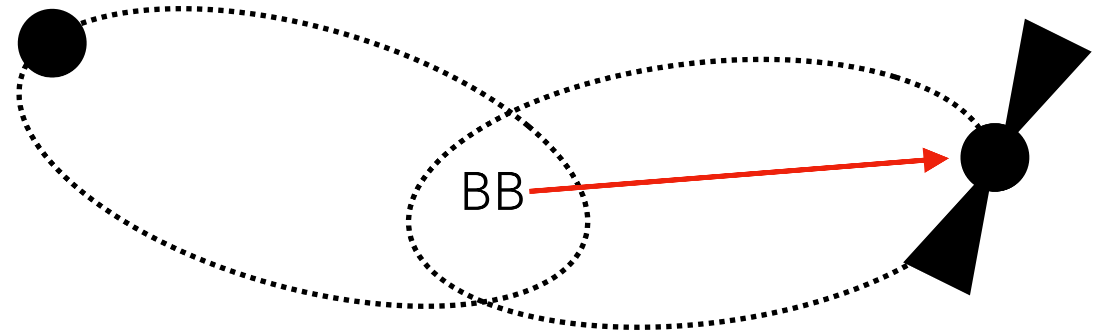


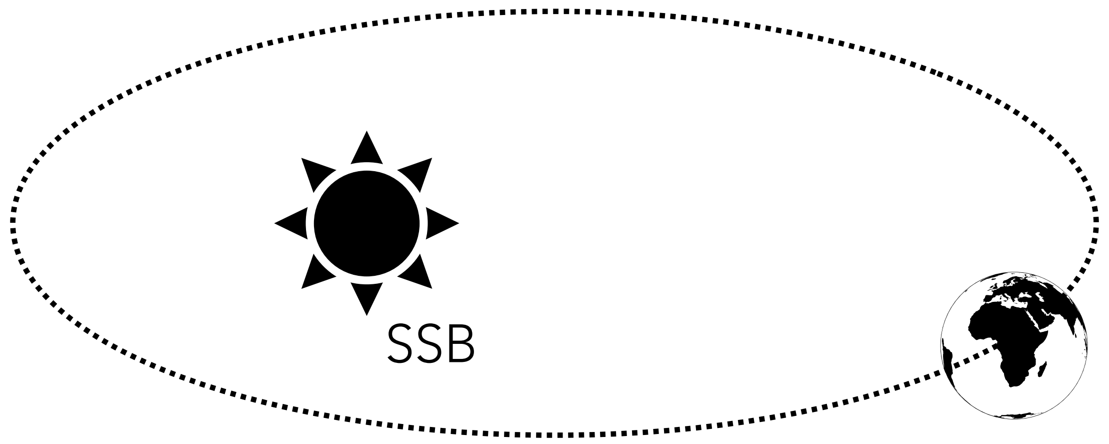
BUILDING A TIMING MODEL

$$t_{\rm e}^{\rm psr} = t_{\rm a}^{\rm obs} - \Delta_\odot - \Delta_{\rm IS} - \boxed{\Delta_{\rm B}}$$

last step transform from the from BB to the pulsar time

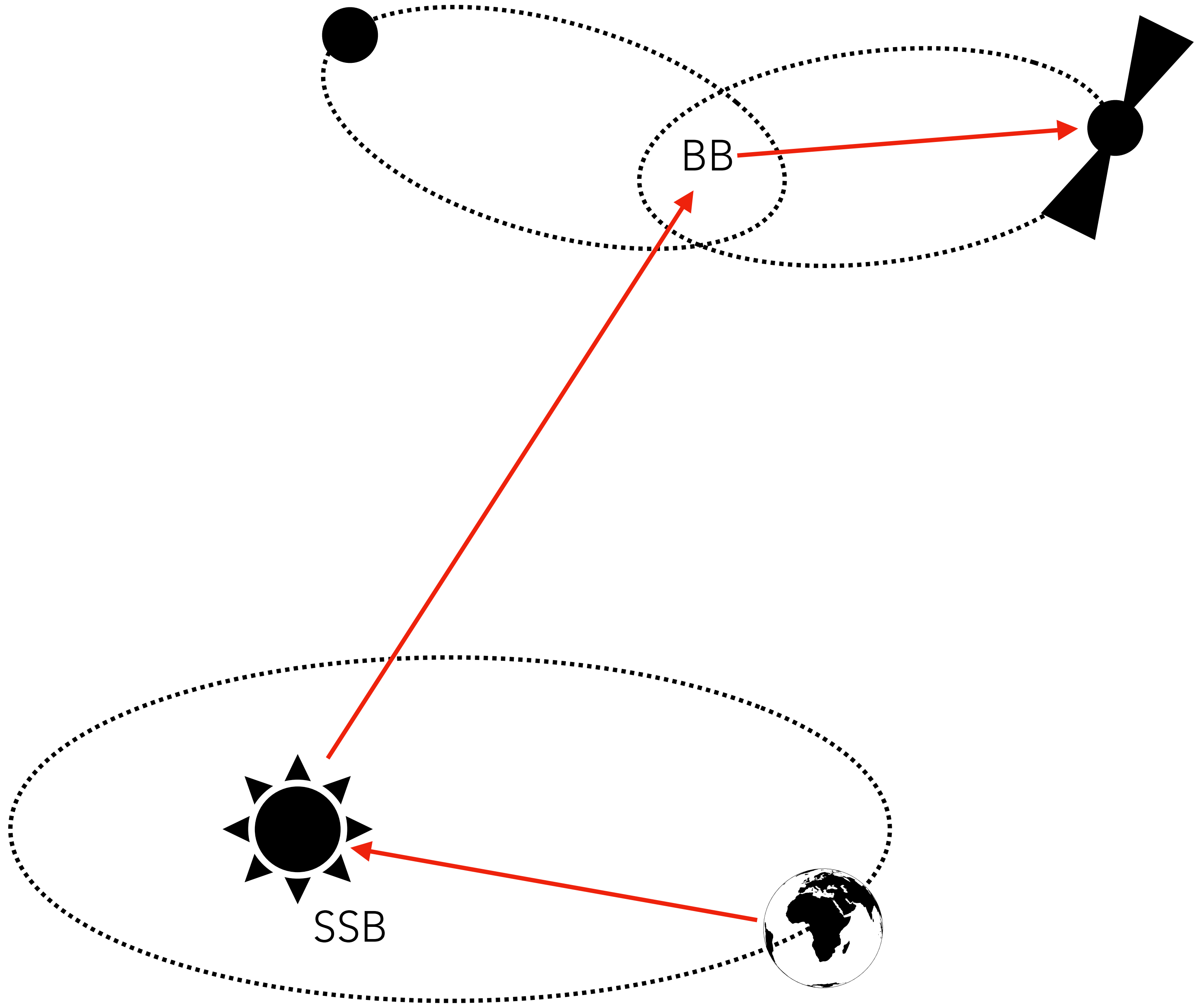
$\Delta_{\rm B} = \Delta_{\rm RB}$	**Roemer delay:** delay due to the orbital motion of the pulsar
$\Delta_{\rm EB}$	**Einstein delay:** difference in the emission time from the pulsar inertial reference frame and the binary barycenter
$\Delta_{\rm SB}$	**Shapiro delay:** effect of the gravitational potential of the binary companion



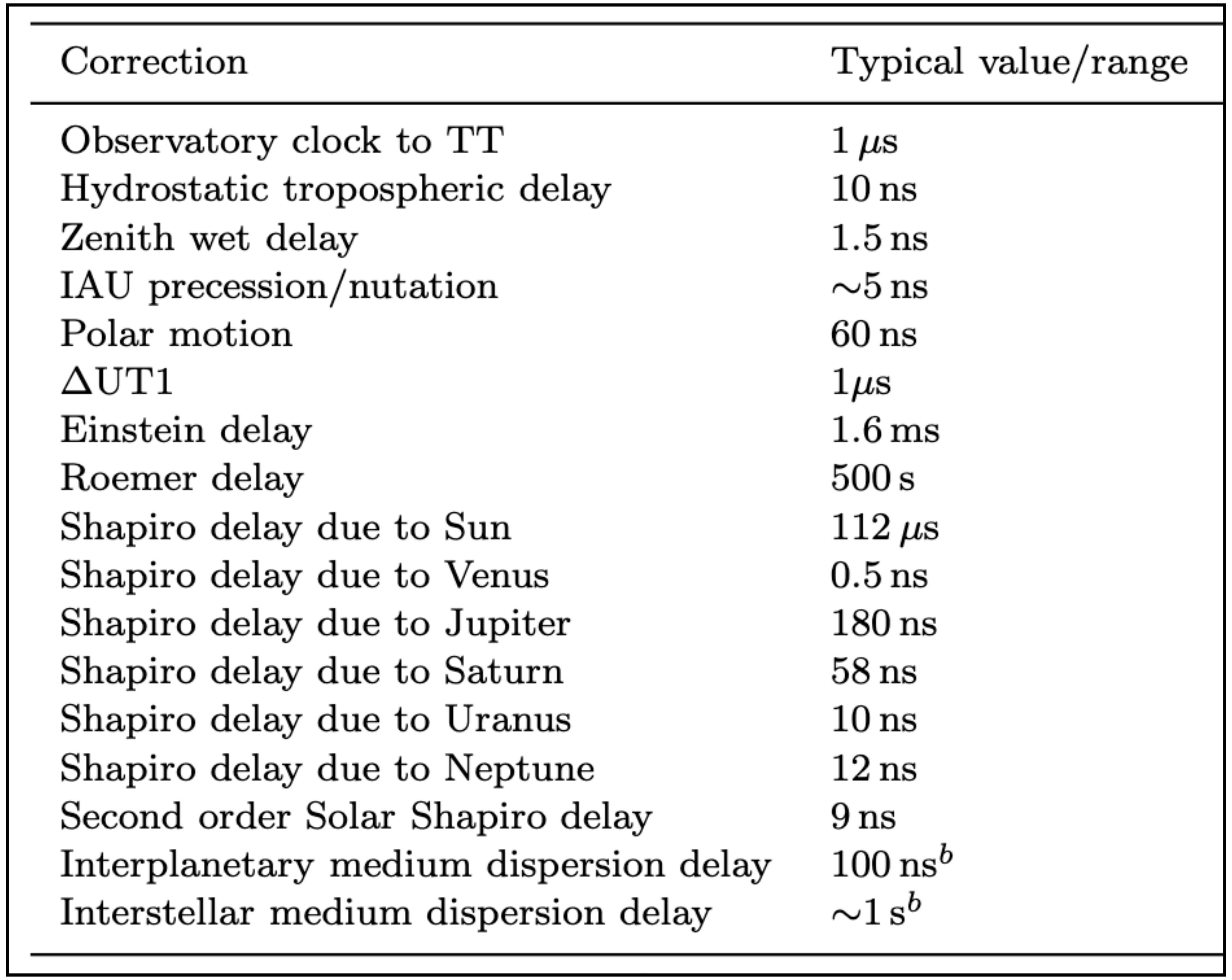


BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$



arXiv: 0603381

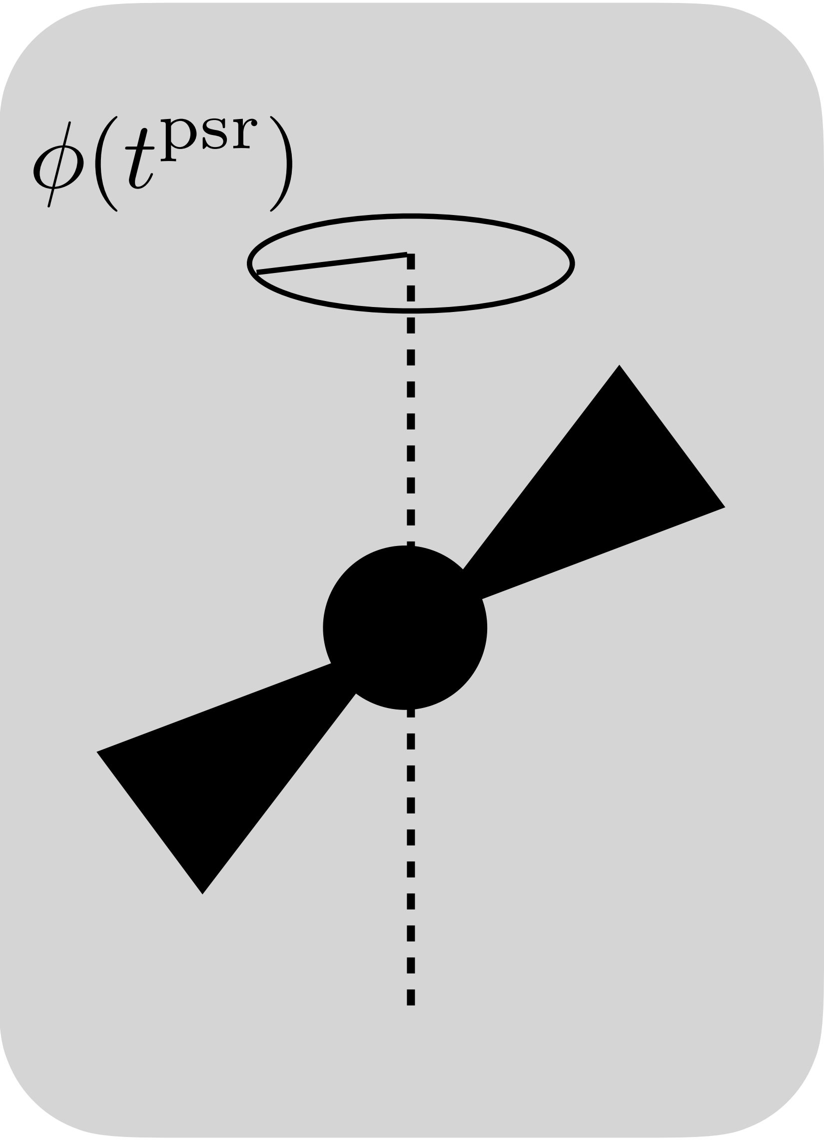


Correction	Typical value/range
Observatory clock to TT	1 μs
Hydrostatic tropospheric delay	10 ns
Zenith wet delay	1.5 ns
IAU precession/nutation	~5 ns
Polar motion	60 ns
ΔUT1	1μs
Einstein delay	1.6 ms
Roemer delay	500 s
Shapiro delay due to Sun	112 μs
Shapiro delay due to Venus	0.5 ns
Shapiro delay due to Jupiter	180 ns
Shapiro delay due to Saturn	58 ns
Shapiro delay due to Uranus	10 ns
Shapiro delay due to Neptune	12 ns
Second order Solar Shapiro delay	9 ns
Interplanetary medium dispersion delay	100 ns^b
Interstellar medium dispersion delay	~1 s^b

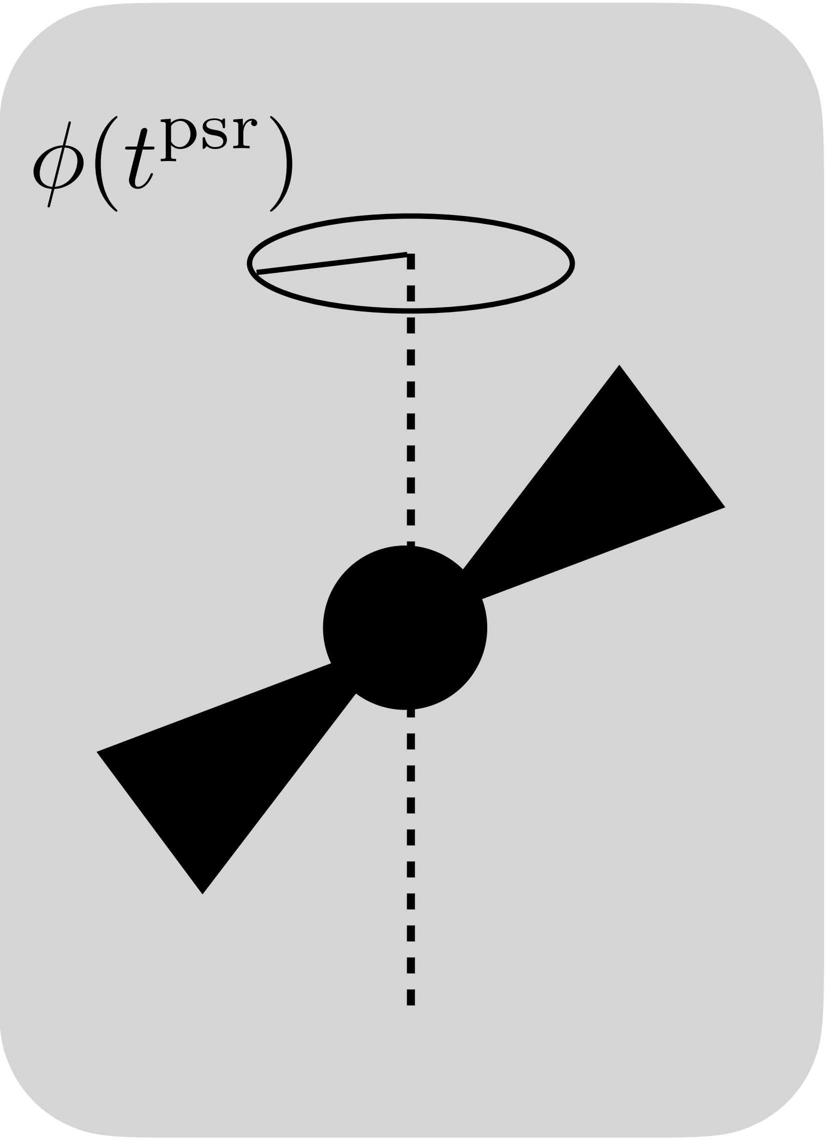
BUILDING A TIMING MODEL

$$\phi(t^{\mathrm{psr}}) = \phi_0 + \nu_0(t^{\mathrm{psr}} - t_0) + \frac{1}{2}\dot{\nu}_0(t^{\mathrm{psr}} - t_0)^2$$

unknown quantities



BUILDING A TIMING MODEL



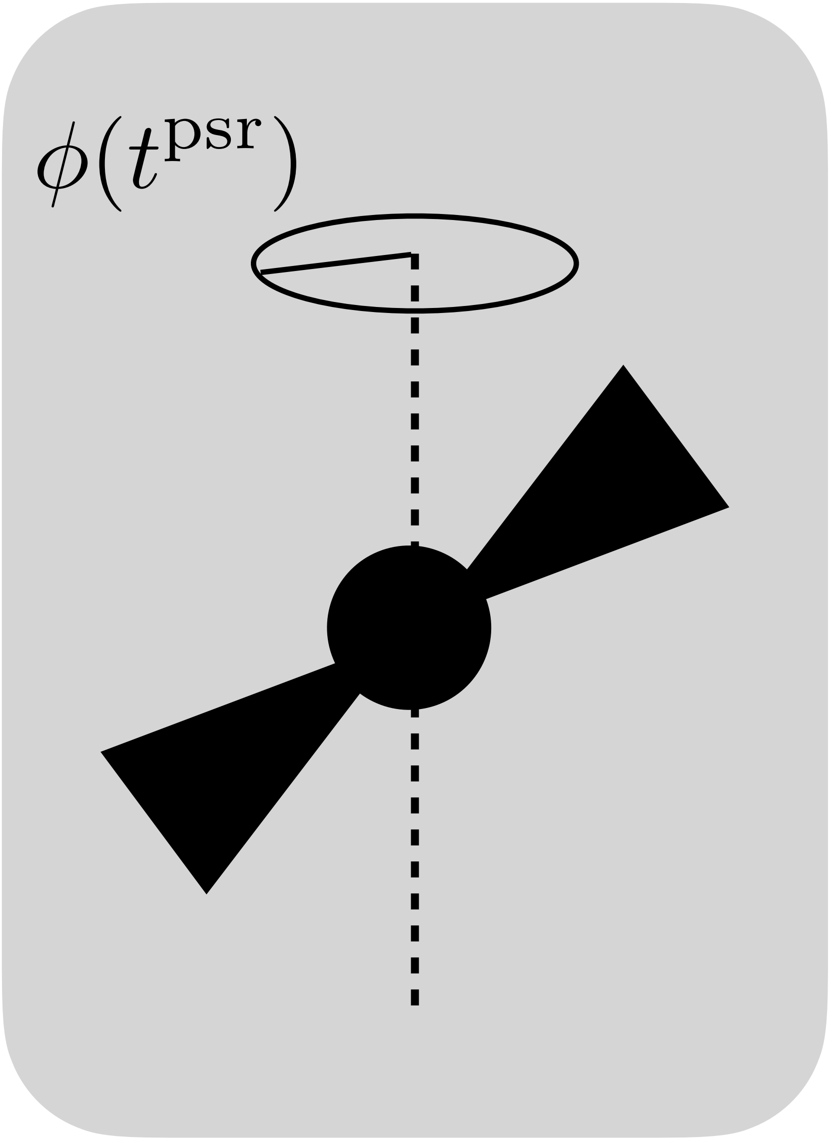
$$\phi(t^{\mathrm{psr}}) = \phi_0 + \nu_0(t^{\mathrm{psr}} - t_0) + \frac{1}{2}\dot{\nu}_0(t^{\mathrm{psr}} - t_0)^2$$

unknown quantities

unknown timing models parameters are fitted to the TOAs

$$\chi^2 = \sum_i \left(\frac{\phi(t^{\mathrm{psr}}_{\mathrm{e},i}) - N_i}{\sigma_i} \right)^2$$

BUILDING A TIMING MODEL



$$\phi(t^{\rm psr}) = \phi_0 + \nu_0(t^{\rm psr} - t_0) + \frac{1}{2}\dot{\nu}_0(t^{\rm psr} - t_0)^2$$

unknown quantities

unknown timing models parameters are fitted to the TOAs

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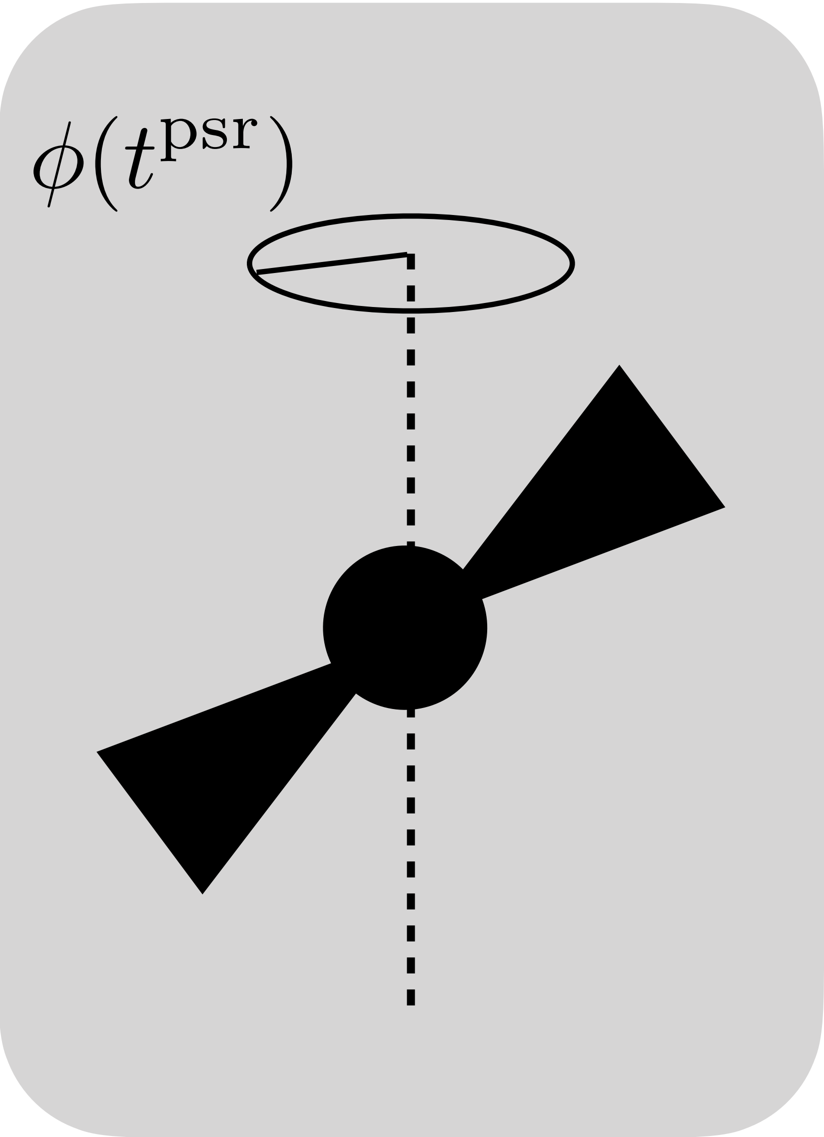
closest integer

BUILDING A TIMING MODEL



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closest integer

time of emission in the
pulsar reference frame

FACE-OFF

free parameters

$$h^2\Omega_{\mathrm{GW}} \propto \frac{A^2}{H_0^2}\left(\frac{f}{\mathrm{yr}^{-1}}\right)^{5-\gamma} \mathrm{yr}^{-2}$$

VS

free parameters

$$h^2\Omega_{\mathrm{GW}}(f;\ \Theta)$$

FACE-OFF

$$\mathcal{B} = \frac{\mathcal{Z}_{\rm NP}}{\mathcal{Z}_{\rm BHB}}$$

$$\mathcal{Z} = \int d\Theta \; P(\mathcal{D}|\Theta, \mathcal{H}) \times P(\Theta|\mathcal{H})$$

FACE-OFF

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likelihood function

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prior distributions

FACE-OFF

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prior distributions

Afzal et al. [2306.16219]