## **FUN WITH PTA INTERPRETATIONS**

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**Fundamental physics and GW detectors**

**Pollica September 13, 2024**

## **New era in fundamental (particle) physics**

Standard model is valid to high energies

No clear hint where new physics is





## **New era in fundamental (particle) physics**

At the same time:

SM is incomplete, does not explain dark matter, baryon asymmetry, inflation, …

All linked to very early Universe dynamics





## **New era in fundamental (particle) physics**

At the same time:

SM is incomplete, does not explain dark matter, baryon asymmetry, inflation, …

All linked to very early Universe dynamics

Gravitational waves are messengers from this era

▶ great opportunity





#### **PTA: First observation of stochastic GW background**

Could be of primordial origin, though a large astrophysical contribution (SMBHB) is likely

In any case:

Testing ground for model building, parameter reconstruction, …

- ▶ what can we learn from GW data
- ▶ how else can we probe and distinguish models







Audible axions: GWs from rolling axions & PTA

#### GWs from domain walls

▶ spectral distortions as complementary probe of GW sources

#### PTA GWs from supermassive pBH

Not today:

- ▶ Ultra-high frequency GWs
- ▶ Supercooled PTs
- ▶ Strongly coupled PTs from holography





# Audible Axions

#### **Axion/ALP with dark photon** photon *X<sup>µ</sup>* of an unbroken *U*(1)*<sup>X</sup>* gauge symmetry under which the Standard Model fields are under which the Standard Model fields are uncharged 12

Take an axion *ϕ*

Dark photon *X*

$$
\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\alpha}{4f} \phi X_{\mu\nu} \widetilde{X}^{\mu\nu} \right]
$$

tially homogeneous, the equation for the equation for  $\mathcal{L}_\mathbf{t}$ 

In radiation domination, i.e. after inflation! ing that gives rise to the Nambu-Goldstone field . The We assume a cosine-like potential for the axiom with the axiom with the axion with the



#### **Axion/ALP with dark photon** photon *X<sup>µ</sup>* of an unbroken *U*(1)*<sup>X</sup>* gauge symmetry under which the Standard Model fields are under which the Standard Model fields are uncharged 12 nhat*i*

- Take an axion *ϕ* Dark photon *X* Vacuum Misalignment  $\overline{\phantom{a}}$  since  $\overline{\phantom{a}}$  since  $\overline{\phantom{a}}$  since  $\overline{\phantom{a}}$  $S =$ z<br>Z  $d^4x\sqrt{-g}$  1 2  $\overline{-g}\left[\frac{1}{2}\partial_{\mu}\phi\,\partial^{\mu}\phi-V(\phi)\right]$  $-\frac{1}{4}$ 4  $X_{\mu\nu}X^{\mu\nu} - \frac{\alpha}{4}$  $\frac{\alpha}{4f}\phi X_{\mu\nu}\widetilde{X}^{\mu\nu}$  $1_{\mathbf{v}} \mathbf{v} \mathbf{u}$   $\alpha$   $\mathbf{v} \mathbf{v}$   $\tilde{\mathbf{v}} \mathbf{u}$  $\overline{\phantom{a}}$  $\bar{X}^{\prime}\left[\frac{1}{2}\partial_{\mu}\phi\,\partial^{\mu}\phi\right]$  $\mu^{\mu}\phi-V(\phi)$  $\overline{1}$  $i = \frac{1}{4} \Lambda_{\mu\nu} \Lambda^{\dagger} = \frac{1}{4} \phi \Lambda_{\mu\nu} \Lambda^{\dagger}$  $\mathbf{L}$  and  $\mathbf{L}$  for the axion with  $\mathbf{L}$
- In radiation domination, i.e. after inflation! poten3al when it 3lts, we generically expect ini3al condi3ons of  $\phi_i = \theta f, \quad \phi'_i \approx 0, \quad \theta \sim \mathcal{O}(1)$ where *f* is the scale of the global symmetry break**includes**  $V(\phi) = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{a}\right)\right]$  $\phi_i = \theta f$ ,  $\phi'_i \approx 0$ ,  $\theta \sim \mathcal{O}(1)$  $V(\phi) = m^2 f^2$  $\left[1-\cos\left(\frac{\phi}{f}\right)\right]$ *f* ◆ *,* (2) however our results do not depend crucially on the





tize the dark gauge field as

tially homogeneous, the equation for the equation for  $\mathcal{L}_\mathbf{t}$ 

#### **ALP dynamics - with dark photon** Equation of motion poten  $\bullet$  , and all we call the set of  $\bullet$  and  $\bullet$ the **w**  $\phi_i = \theta f, \quad \phi'_i \approx 0, \quad \theta \sim \mathcal{O}(1)$  $V(\phi)$ *ϕ*′′+ 2*aHϕ*′+ *a*<sup>2</sup> *V*′(*ϕ*)

ALP starts rolling when  $H \sim m_\phi$ 

*fa*<sup>2</sup>

−∇2*<sup>ϕ</sup>* <sup>−</sup> *<sup>α</sup>*

ALP is damped due to exponential production of dark photons

 $\mathbf{X}'\cdot(\nabla\times\mathbf{X})=0$ 

- ▶ Reduced relic abundance enlarge natural DM parameter space
- ▶ Or production of vector DM

Agrawal, Marques-Tavares, Xue, 2018 And others…

 $\phi_i = \theta f$ 



 $\phi$ 

#### **How does this work?**  the total energy density.

Equation of motion (in momentum space)  $a_n = \frac{1}{2}$ Equation of motion (in momentum space

$$
X_\pm''(\tau,\bm{k})+\left(k^2\pm k\frac{\alpha}{f}\phi'(\tau)\right)X_\pm(\tau,\bm{k})=0
$$

The rolling ALP induces a tachyonic instability

$$
X_{\pm}^{\prime\prime} + \omega_{\pm}(\tau)X_{\pm} = 0 \qquad \text{with} \quad \omega_{\pm} = k^2 \mp k\frac{\alpha}{f}\phi^{\prime}
$$

Exponential growth of a range of dark photon modes

$$
X(\tau) \propto e^{|\omega|\tau} \quad \text{for} \quad k \sim \frac{\alpha \phi'}{2f}
$$



#### **Dark photon spectrum**  $\phi'$

 $\Omega$ 

Initial condition violates parity (field rolls to the left or to the right) first, the system of the system of the line of the system of  $\mathcal{C}^*$  and  $\mathcal{C}^*$  and  $\mathcal{C}^*$  and  $\mathcal{C}^*$ mitial condition violates parity (field rolls to the<br>The sight)

 $\mathop{\mathsf{O}}$ re da $k$ k photøn helicity dominates A certain range of modes undergoes growth  $\alpha \phi'$  k 13 7  $\begin{bmatrix} 1 & 50 \end{bmatrix}$ 10-25<br>10-25<br>10-25 10-15  $10^{-10}$  $10^{-5}$ 1 t,  $\mathcal{L}$ /  $\overline{\mathrm{SO}}_{\mathrm{c}}$  .  $\cancel{\sim}$  $\omega$ ộ (re) đa $k$ k ph  $\overline{\alpha}$ **f** 0r  $\widetilde{k}$  $k(\tau)$  $0 < \mathcal{R}^{\omega_+}(w)$ *f , k m*  $\stackrel{\textstyle \star}{\sim} \alpha\theta$  $\kappa$  $\kappa =$  $\frac{f}{2f} \approx \frac{1}{2}$ *m*  $B(z)$  is  $L^2$  if  $\alpha$  if  $\phi'$  is a function of  $\alpha$  $\frac{1}{2}$   $\frac{1}{2}$  helicity dominates under<sup>g</sup>oes g  $\omega$  and  $\omega$ 2 5 Machado, Batzinger, Stefanek, PS, 1861.01950  $10^{-25}$  $10^{-20}$  $10^{-15}$  $10^{-10}$  $10^{-5}$ 1  $k / (m a_{\rm osc})$  $\mathcal{L}% _{G}^{(h,\sigma),(h,-h)}(\theta)=\mathcal{L}_{G}^{(h,\sigma),(h,-h)}(\mathcal{M},\mathcal{M},\mathcal{M},\mathcal{M},\mathcal{M},\mathcal{M},\mathcal{M})$  $\bf G$  $\Join$  $\diagup$  $d{\log}$  $\cancel{\sim}$  $\phi'$  $\omega + \alpha \omega = k^2 - k^2$  $\overline{\boldsymbol{\alpha}}$  $\frac{d}{dx}$   $\phi'$  $\tilde{k}(\tau)$  $0 < k < \frac{\alpha \phi'}{c}$  $\frac{f}{f}$  , *k m*  $\lesssim \alpha\theta$  $\tilde{k} =$  $\alpha\phi'$  $\frac{\alpha\phi'}{\alpha\,c}\lesssim \frac{\alpha\theta}{\alpha}$ *m*



Q PRISMA<sup>+</sup>



The exponential growth amplifies quantum fluctuations in the dark photon fields which source a chiral gravitational wave background and allowing the control wave background first and which priotes in the race with FILE EXPONENTIATION GIVWEN ANIIPMIES GRANITANII NACERALIONS IN the dork photon fields which seures a shiral growitational are uaik prioton neius winch source a chilai gravitational wave detectors wave bac





#### **GW probes of audible ALPs** Listening for Invisible Axions

Machado, Ratzinger, Stefanek, PS, 1912.01107



#### **GW probes of audible ALPs**

Machado, Ratzinger, Stefanek, PS, 1912.01107



PTA region



#### **Did PTAs hear the audible axion?**



#### **Fits including SMBHB (from 2308.08546)**





cluded by super-radiance constraints.

the fact that the latter undergoes rapid thermalization that would destroy the conditions required for exponential particle production.10 The GW formation ends when the GW formation ends when

including also SMBH binaries with environmental e↵ects.

trum at frequencies *f<fH*(*T*⇤) and is given by Eq. (20)

#### **ALP dynamics - once more** - ONG

 $\phi_i = \theta f, \quad \phi'_i \approx 0, \quad \theta \sim \mathcal{O}(1)$ 

Equation of motion





Once a significant population of dark photons is produced, the back-scattering into ALP fluctuations becomes nonnegligible

Requires fully numerical treatment on the lattice



#### **Important to get correct relic abundance prediction**



result shows a suppression of the lattice result shows a suppression of the final axion abundance by ʿ 102 comp<br>The final axion abundance by ʿ 102 compared by ʿ 102 compared by ʿ 102 compared by ʿ 102 compared by ʿ 102 com

From 2012.11584 with W. Ratzinger, B. Stefanek

For also Kitajima, Sekiguchi, Takahashi, 2018 Agrawal, Kitajima, Reece et al, 2020





#### **Corrections to GW signal**



Qualitative features unchanged, but polarisation is washed out at large couplings

> From 2012.11584 with W. Ratzinger, B. Stefanek see also 2010.10990 by (Kitajima, Soda, Urakawa)

> > 22

 $JG$ U



#### **Notes**

Model variations: Audible Relaxion, Axion kinetic misalignment (see extra slides)

Also works for:

▶ Scalar dark sectors, e.g.

$$
V(\phi, \psi) = \frac{1}{4}\lambda \phi^4 + \frac{1}{2}g^2 \phi^2 \psi^2
$$

see also Cui et al, 2310.13060

 $(e.g. axiom fragmentation)$ 

see e.g. Chatrchyan, Jaeckel, 2004.07844, Fonseca, Morgante, Sato, Servant, 1911.08472, ... see e.g. Chatrchyan, Jaeckel, 2004.07844, Fonseca, Morgante, Sato, Servant, 1911.08472, …



### **Spectral distortions?**

Around  $10^4 \lesssim z \lesssim 10^6$ , photon number is frozen

Any energy added to the photons leads to a so called *μ* distortion

Energy source we consider here: Gravitational damping of dark sector fluctuations



Ramberg, Ratzinger & PS, 2209.14313

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#### **Spectral distortions from dark sector anisotropies**

Assume decoupled dark sector,  $\Omega_d \ll 1$ 

Large fluctuations  $\delta_d = \delta \rho_d / \rho_d \sim 1$ 

> ▶ Gravitationally induced sound waves in photons *ϵ*ac

Resulting *μ* distortions

$$
\mu = \int d\log k \ \epsilon_{ac}^{\rm lim}(k) \mathcal{W}(k),
$$





#### **Example source: Annihilating domain walls**



Already probes allowed parameter space

Complementary to GW probes, can break degeneracy

▶ For all low scale sources (PTs, strings, AA,...)

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## **Axion/ALP domain walls**

Madge et al, [2306.14856](https://arxiv.org/abs/2306.14856)



Visible, no spectral distortions Figure 6. Fit results of the aligned QCD axion DW model from section 2.2.2 to NANOGrav (blue), IPTA (orange) and their combination (green). *Left:* Best-fit GW spectrum alongside the free-spectrum fit (violins). *Right:* 68 % and



## **Invisibly decaying DWs**

Madge et al, [2306.14856](https://arxiv.org/abs/2306.14856)



fit region in terms of the axion mass *m<sup>a</sup>* and annihilation temperature *T*ann. In the region between the dashed

lines, our description of the GW spectrum in terms of the scaling regime is valid. The region below the solid lines

is excluded by *N*e↵ for *f<sup>a</sup>* = 10<sup>5</sup> GeV (black) and for *f<sup>a</sup>* = 10<sup>7</sup> GeV (grey). The dotted line shows the projected

Also: PBH formation (Y. Gouttenoire 2023) Figure 5. Fit results of the ALP DW model from section 2.2.2 to NANOGrav (blue), IPTA (orange) and their **Combination (green). Also: PBH TOMMATION (Y. Gouttenoire 2023). Also: Also: 68 % And 96 % CLC & Free-spectrum fit (** 

sensitivity of PIXIE. The full triangle plot including 1D posteriors including 1D posteriors is shown in Fig. 1



#### **One more: Primordial black holes**





Depta et al,

2306.17836

### **PBH: No clustering**

#### Viable region at very large masses

already pointed out by Atal, Sanglas, Triantafyllou, 2012.14721

However: Fewer than one pBH contributes to signal on average there

- ▶ Not a stochastic BG
- ▶ Not even a signal most of the time





Depta et al, 2306.17836

## **PBH: With clustering**

Now an actually viable region emerges

Assuming a suitable production mechanism

- ▶ Needs to evade mu distortion bounds
- ▶ Non-gaussian!





Depta et al, 2306.17836

## **Superlarge PBH**

Less crazy: Astroindependent bounds on clustered pBH from PTAs

Expect anisotropies in GW background :)







GWs are new window to early, dark Universe

Today:

- ▶ Audible axions are cool
- ▶ Spectral distortions are cool
- ▶ Supermassive pBH are also cool, but maybe a bit crazy ;)

Many things to be done (simulations!), much data will come in the future -> Exciting times!





Extra slides :)

#### **Spectral distortions as probes of low scale GWs**



Tensor fluctuations (GWs) also source  $\mu$  distortions

▶ But difficult to test. Better to directly go for the scalar fluctuations (that also source the GWs)





## **High frequency GW searches**

Higher Frequency  $\rightarrow$  shorter wavelength

▶ Experiment may fit in your laboratory

Gravity couples to everything

▶ Any very sensitive device could potentially be a detector

Current interest:

▶ Cavities for axion searches





Berlin et al, 2112.11465

▶ Gertsenshtein effect: GWs convert to photons in strong magnetic field

Sources? Primordial BH, superradiance, or…?





#### **E&M on curved backgrounds is confusing however**

E and B fields not uniquely defined everywhere in detector, depend on chosen coordinate frame  $E_a = F_{a0}$ ?

Observables should be independent!

Proposed coordinate independent perturbation scheme

Applied to:

- ▶ Thin rod
- ▶ Sphere

Including mechanical deformations

Compared with commonly used

approximations  $\rightarrow$  can identify range of validity and provide error estimate

the spherical cavity of radius *R*, as a function of !*R*. Here, the sound velocity is chosen to be *Wolfram Ratzinger, Sebastian Schenk, PS, 2404.08572* 



 $E_{\underline{a}} = \hat{e}^{\mu}_{\underline{a}} F_{\mu\nu} u^{\nu}$  ! ̂

only a plus polarisation of the gravitational wave, *h*<sup>+</sup> = 1 and *h* = 0. As an example, the





#### **Audible relaxion** culation of the dark photon and GW spectra are provided in Audible P culation of the dark photon and Gw spectra are provided and Gw spectra are provided and Gw spectra are provided in Augustia Reiz

Audible relaxion Audible relaxion "<sup>\*</sup> consider the relaxion " dark photon field *Xµ*,

> $-\mathcal{L} \supset V(H, \phi) + \frac{r_X}{4}$  $\phi$  $f_{\phi}$  $X_{\mu\nu}\widetilde{X}^{\mu\nu}$  $r_X \phi_{X} \tilde{\chi}_{UV}$ where  $4J_{\phi}$  <sup>4</sup>

with the potential of the relaxion field *"* and Higgs dou- $V(H, \phi) = V_{\text{roll}}(\phi) + \mu_H^2(\phi)|H|^2 + \lambda |H|^4 + V_{\text{br}}(H, \phi)$ 

Dark photon  $f$ riction essential for trapping relaxion after reheating *<sup>H</sup>*(*"*)*|H|* <sup>2</sup>+*⁄|H<sup>|</sup> ⁄ Dain prioton*  $\mathbf{r}$  +  $\mathbf{r}$   $\math$ *<sup>V</sup>*br(*H, "*) = <sup>≠</sup><sup>4</sup> *|H|* <sup>2</sup> cos *f" .* (5c) Here, *c* is an order one number, *g* is a dimensionless pa-*<u>bhoton</u> µ*2 *<sup>H</sup>*(*"*) = <sup>2</sup> <sup>≠</sup> *<sup>g</sup>" ,* (5b) br *|H|* <sup>2</sup> cos *v*2 Here, *c* is an order one number, *g* is a dimensionless parameter, is the Higgs mass cut-o scale, br is the relaxion after reheating vacuum expectation value, and *f"* is the decay constant



→ Potentially observable GW signal  $\mathbf{C}$  of  $\mathbf{C}$  years (blue) is indicated by the respective coloured regions. In the purple coloured regions. In the pur coloured region, a sub-range of the viable real be excluded using can be excluded using current NANOGRAV data from the viable real be excluded using current NANOGRAV data from the viable real be excluded using current NANO rameter, is the Higgs mass cut-o scale, br is the  $\rightarrow$  Potentially observab  $\alpha$  is the decay constant  $\beta$  is the decay constant  $\rightarrow$  Potentially observable mass parameter *µ*<sup>2</sup>(*"*). Once *µ*<sup>2</sup> crosses zero, the Higgs

br, we end

B<sup>1</sup>  $p_{\text{other}}$  Detringer DC 040E 1010E Banerjee, Madge, Perez, Ratzinger, PS, 2105.12135

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## **GWs from kinetic misalignment**

Consider the case of large initial .<br>h *ϕ*

Detectable signal also for smaller decay constants

Fix ALP mass to fit DM relic abundance

Also consistent with Axiogenesis!



#### From Madge, Ratzinger, Schmitt, PS, 2111.12730

See also Co, Harigaya, Pierce, 2104.02077



#### **Detectable region - update**



 $\mathcal{S} = \mathcal{S} \cup \mathcal{S}$  also variable space in the mass variable mass variable  $\mathcal{S} = \mathcal{S} \cup \mathcal{S} \cup \mathcal{S}$ From 2012.11584 with W. Ratzinger, B. Stefanek



#### **Example source I: Dark sector phase transition**



Note:  $\Omega_d$  fixed to satisfy  $N_{\text{eff}}$  constraints *Ramberg, Ratzinger & PS, 2209.14313* 

 $\mathbf{Q}$  PRISMA<sup>+</sup>

## **Source III: (global) cosmic strings**



Note: Local strings mainly radiate from small loops and are thus NOT an efficient source of spectral distortions





## **Example source IV: Audible axions…**

temperature at back-reaction  $T_*$  [eV] Not yet…  $10<sup>0</sup>$  $10^{2}$  $10^{3}$  $10^6$  $10<sup>1</sup>$  $10<sup>4</sup>$  $10<sup>5</sup>$  $10<sup>7</sup>$  $\Delta N_{eff}$ Results for scalar CMB scalar toy model  $10^{18}$ **PIXIE**  $\phi_i$ [GeV] CMB GW Voyage2050 Constraints not as strong since fluctuations are **SKA** not horizon size  $10^{17}$  $10^{-22}$  $10^{-25}$  $10^{-19}$  $10^{-13}$  $10^{-10}$  $10^{-16}$  $\omega$  {  $\text{eV}$  ]

Expect better sensitivity for axion fragmentation



#### **Fit with broken power law signals**



Wolfram Ratzinger & PS, 2009.11875

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#### **Fit with Phase Transition**

![](_page_44_Figure_1.jpeg)

Generic PT parameterisation, best fit with PT at temperatures in few MeV range

Challenge for model building  $\rightarrow$  Hint for dark sector

Wolfram Ratzinger & PS, 2009.11875

![](_page_44_Picture_5.jpeg)

### **Fit with Phase Transition**

![](_page_45_Figure_1.jpeg)

Generic PT parameterisation, best fit with PT at temperatures in few MeV range

Some model parameters excluded by PTA data now!

Wolfram Ratzinger & PS, 2009.11875

![](_page_45_Picture_5.jpeg)

#### **At higher frequencies**

![](_page_46_Figure_2.jpeg)

LISA will probe above 10 GeV, colliders could fill gap

![](_page_46_Picture_4.jpeg)

#### **Supercooled phase transitions**

Benchmark model: Coleman-Weinberg model with vanishing tree level potential  ${\cal L} = -\frac{1}{4} F_{\mu\nu}^2 + D_\mu \Phi^\dagger D^\mu \Phi - V(\Phi,T) \, ,$ 

Two parameter model: Mass scale  $M$  and coupling  $g$ 

![](_page_47_Figure_3.jpeg)

Signal dominated by colliding bubbles and sound shells

Simulated by Lewicki and Vaskonen, 2208.11697

#### **Supercooled phase transitions** *T*rh = 0*.*55 *g* **GI LUUICU PIIASE LI AIISILIUIIS** 2306.14856

Madge et al, 2306.14856

12 year data

Large supercooling and reheating

- dark matter
- ▶ Two BBNs

Pheno: Light scalar  $m_{\phi} \approx M$ , and  $\frac{10}{M}$  [MeV] decay to electrons and photons algetrans and photons, we can consider a direction of photons, via couplings of photons, via couplings of photons, via couplings of photons, via couplings of photons, via coupling  $\alpha$ 

Higgs portal not viable, instead

FCC? Or low energy e+e- machine (e.g. MESA in Mainz)

![](_page_48_Figure_9.jpeg)

$$
\mathcal{L} \supset c_{ee} \frac{|\Phi|^2}{\Lambda^2} L H \bar{e} + c_{\gamma\gamma} \frac{|\Phi|^2}{\Lambda^2} F_{\mu\nu} F^{\mu\nu}
$$

#### Axion/ALP domain walls In addition, the global U(1) symmetry is expected to be broken  $\mathcal{L}_1$

Domain walls appear when discrete symmetries are spontaneously broken to degenerate ground states In addition to the generic ALP model, we consider  $\mathcal{L}$  models of the  $\mathcal{L}$  models that  $\mathcal{L}$  models that  $\mathcal{L}$ mechanism. Here a collection of *N* axions that individually respect a shift symmetry

Long lasting GW source, until DWs annihilate, before dominating the Universe ideally 170 is considered, where *C<sup>i</sup>* is a real-valued transformation parameter. One then assumes that *N* 1 of these linearing che annocide ideally

Review: Saikawa, 1703.02576

Axion DW:  $U(1)_{PQ} \rightarrow Z_N$  $\mathcal{P}$ **P** and  $\mathcal{P}$  and  $\mathcal{P}$  are actual symmetry breaking symmetry b e.g. around the TeV-PeV scale, the TeV-PeV scale, thus making the model testable at particle physics experiments. This also the model testable at particle physics experiments. This also the model testable at particle physi

Surface tension  $\sigma = 8m_a f_a^2$  $\frac{1}{2}$  found  $\sigma = 8m$   $\hat{f}^2$  $\text{SUE}$  is and  $\text{SUS}$  a

Annihilation triggered by QCD instantons  $\mathcal{L}$ of the QCD phase transition. This lifts the degeneracy between the di↵erent minima by *V* ' ⇤<sup>4</sup> the annihilation temperature can be predicted as  $\mathcal{I}(\mathcal{I})$ 

$$
T_{\rm ann} \sim 1 \,\mathrm{GeV} \, \left(\frac{g_*(T_{\rm ann})}{80}\right)^{-\frac{1}{4}} \left(\frac{\Lambda_{\rm QCD}}{400\,\mathrm{MeV}}\right)^2 \left(\frac{10^7\,\mathrm{GeV}}{f_a}\right) \sqrt{\frac{10\,\mathrm{GeV}}{m_a}}
$$

*<u>.</u> (2.21)* **.** (2.21) **.** (2.21) **.** (2.21) Madge et al, [2306.14856](https://arxiv.org/abs/2306.14856)

![](_page_49_Picture_9.jpeg)

#### **Pushing the limits**

![](_page_50_Figure_1.jpeg)

![](_page_50_Picture_2.jpeg)