Axion inflation in the regime of strong backreaction

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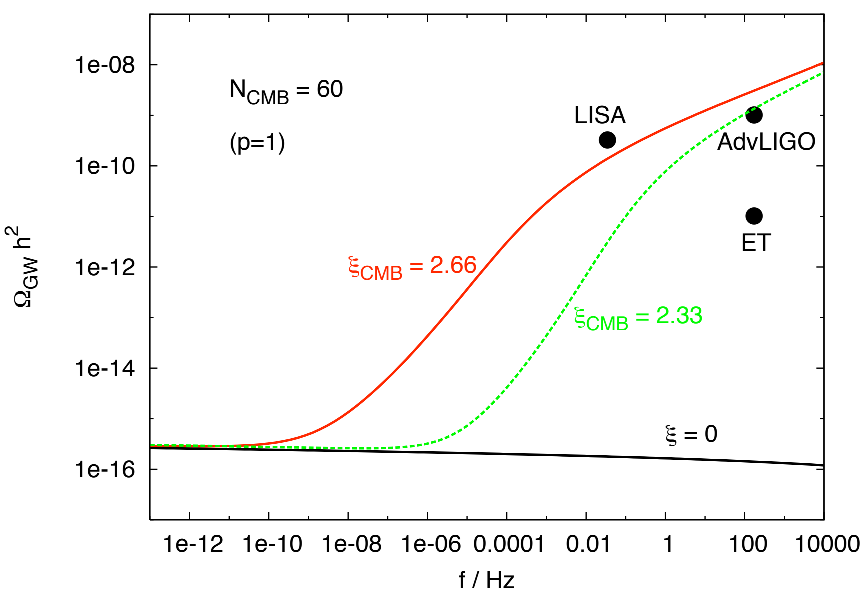
- Vanilla slow-roll inflation produce a GW signal too low to be observed in the foreseeable future (at least, at sub-CMB scales)

- Larger signal in various inflationary models / mechanisms
  Braglia et al '24, LISA Cosmology Working Group.
  Talk by Ricciardone

- Highly studied (and natural) mechanism is from an axion inflaton of strong backreaction, where our knowledge is still incomplete.



Barnaby, Pajer, MP '11

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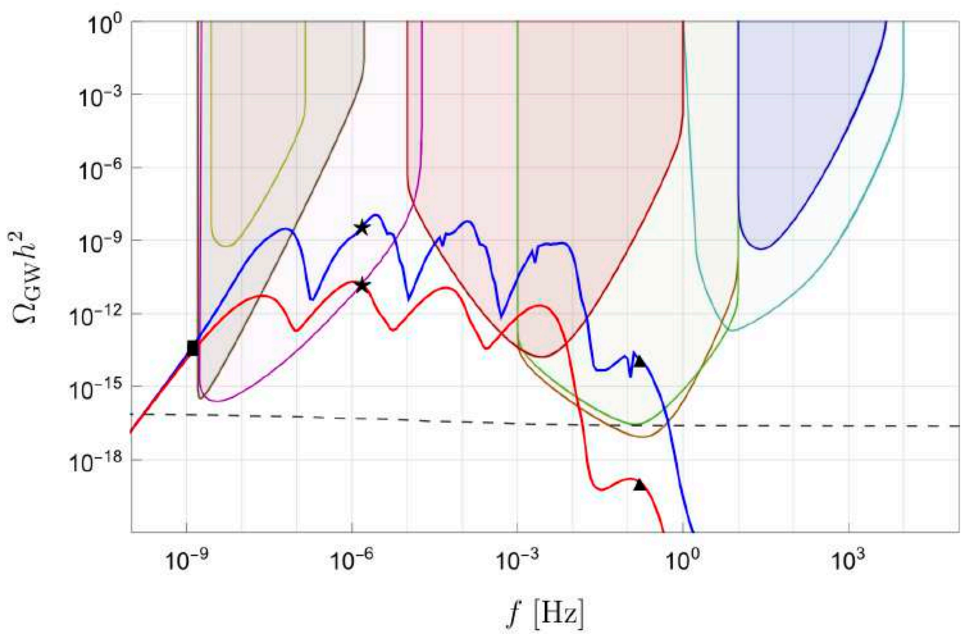


?

García-Bellido, Papageorgiou
MP, Sorbo '23

Shift symmetry $\phi \to \phi + C$. E.g. axion (natural) inflation

Freese, Frieman, Olinto '90

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + V_{\text{shift}}(\phi) + \frac{c_A}{f}\partial_\mu \phi\, \bar{\psi}\gamma^\mu \gamma_5 \psi + \frac{c_\psi}{f}\phi\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

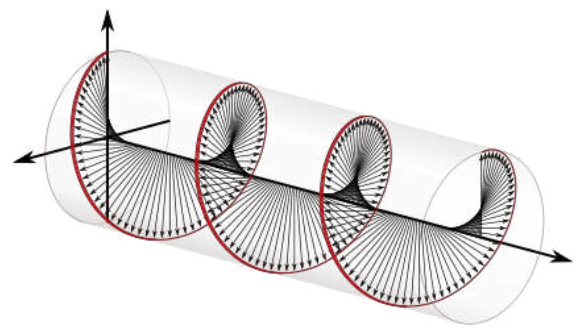
- Couplings shift symmetric, no large radiative corrections to V

- $\phi F \tilde{F}$ breaks partity, $\neq$ results for two polarization

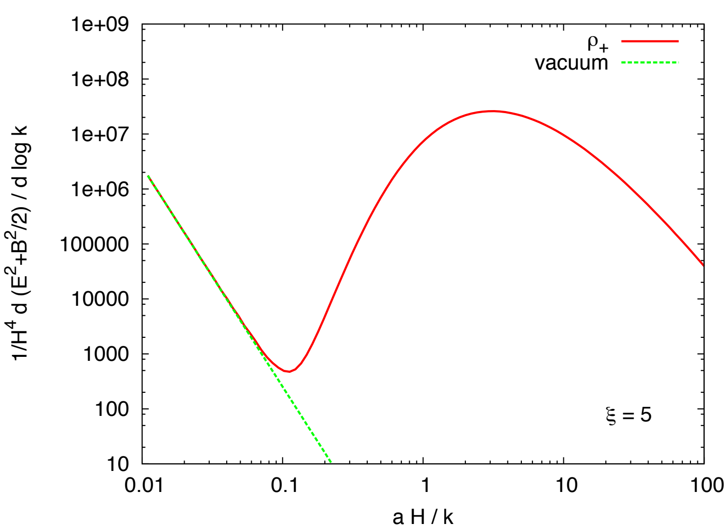
$$\left(\frac{\partial^2}{\partial \tau^2} + k^2 \mp \frac{a\, k\, \dot{\phi}}{f/c_A}\right) A_\pm(\tau, k) = 0$$

$+$ left handed

$-$ right handed



Physical ρ in one mode



- Renormalize away vacuum energy in UV

Ballardini, Braglia, Finelli, Marozzi, Starobinsky, '19

- One tachyonic helicity at horizon crossing

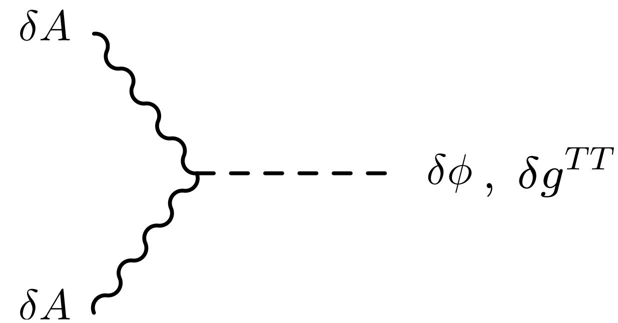
- Then diluted by expansion

- Max amplitude $A_+ \propto e^{\pi \xi}$, $\xi \equiv \frac{c_A \dot{\phi}}{2 f H}$

Continuous supply! At any moment during inflation, gauge modes with $\lambda \simeq H$ are produced $\to$ possible signals at several scales

Amplified gauge fields source scalar and tensor perturbations

Barnaby, MP '10



- GWs are chiral, $\delta g_L^{TT} \gg \delta g_R^{TT}$ Sorbo '11
- Sourced signals for $5 \sim \xi \equiv \frac{C_A\,\dot\phi}{2 f H} \simeq \sqrt{\frac{\epsilon}{2}}\,\frac{C_A\,M_p}{f} \quad \Rightarrow \quad \frac{f}{c_A} \simeq 10^{-2}\,M_p$

Problems with natural inflation

Common to identify axion inflation $\equiv$ "natural inflation"

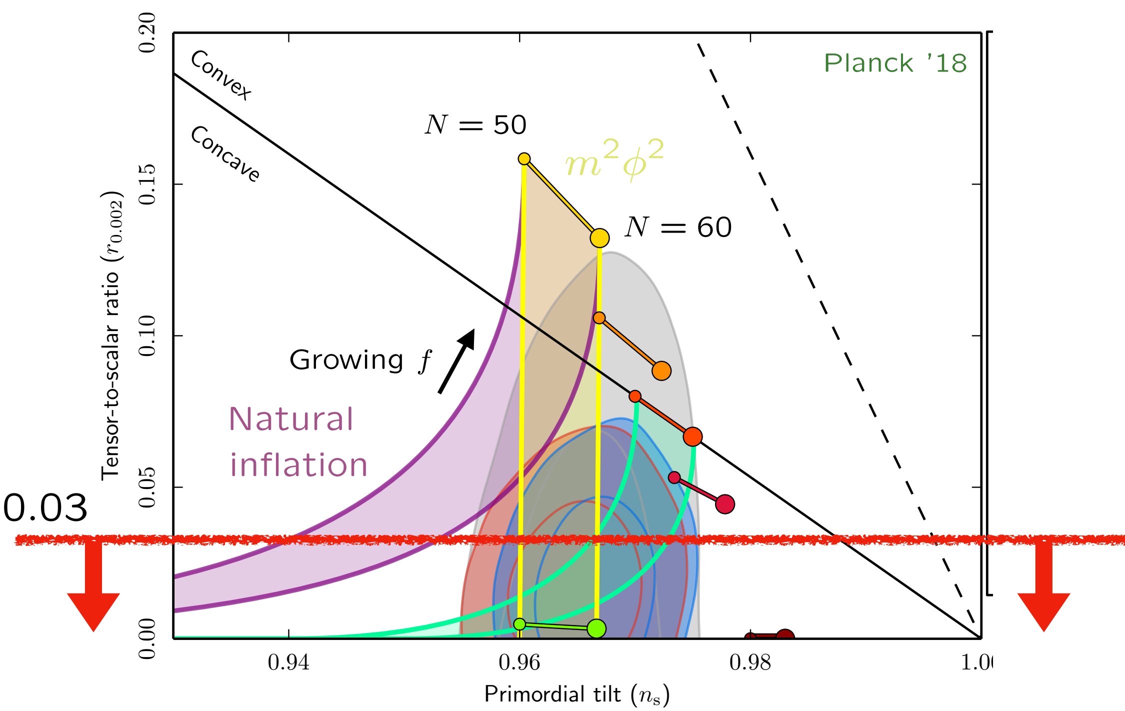
$$V = \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

Freese, Frieman, Olinto '90

★ Ruled out

★ $f > M_p$ in range shown

$r \lesssim 0.03$

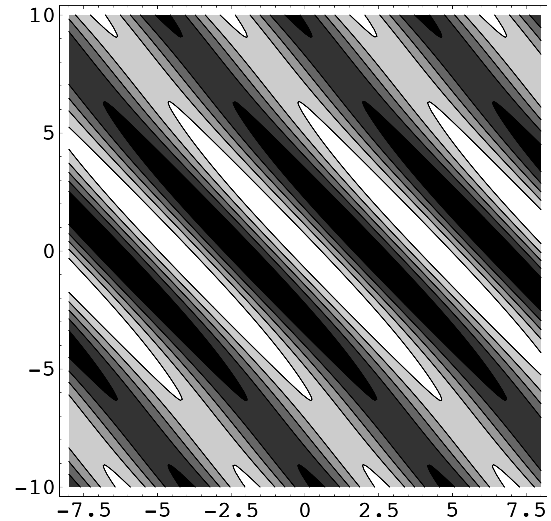


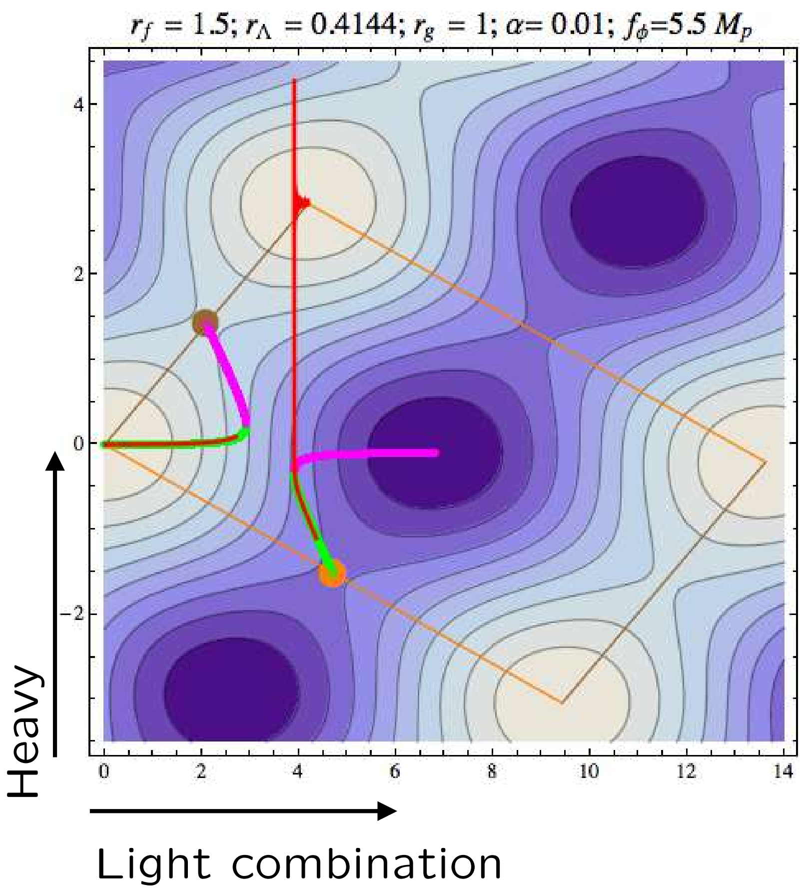
- Aligned natural inflation in agreement with data & testable at CMB-S4

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

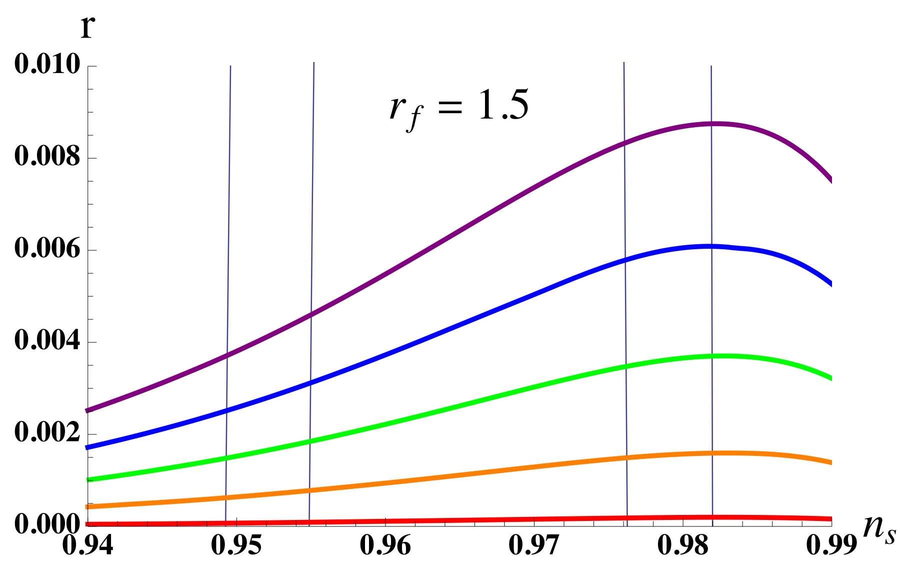
$f_{\rm eff} \gg f_i,\, g_i$ if $\dfrac{f_1}{g_1} \simeq \dfrac{f_2}{g_2}$

Kim, Nilles, MP '05





New class of solutions where inflation ends as a waterfall MP, Unal '15



Analytic trajectory and CMB phenomenology worked out in Greco, MP '24

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

Model invariant under simultaneous rotation of $\vec{\phi} \equiv \{\theta,\, \rho\}$, $\vec{v}_i \equiv \left\{f_i^{-1},\, g_i^{-1}\right\}$

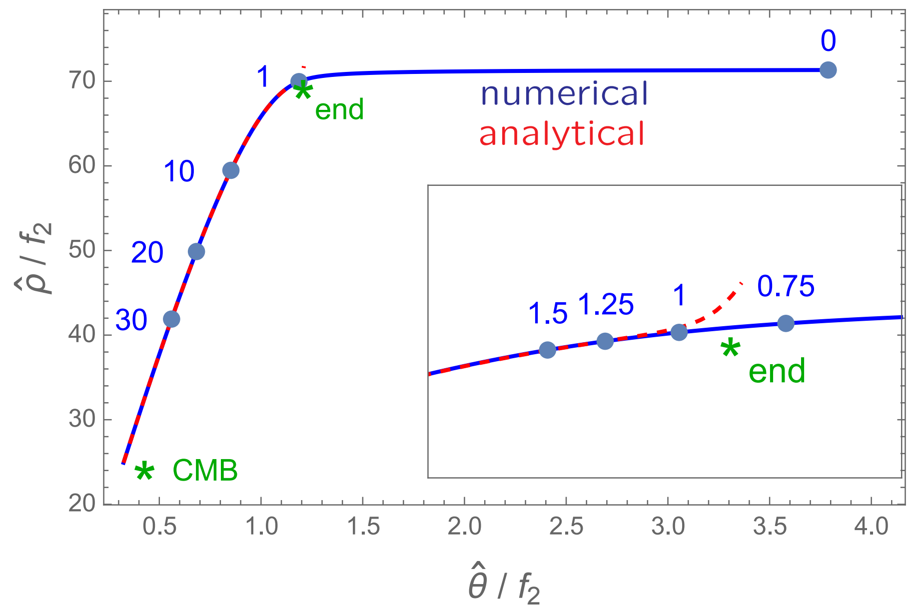
Described in terms of invariant $n_1 \equiv \vec{v}_1 \cdot \vec{v}_1$, $n_2 \equiv \vec{v}_2 \cdot \vec{v}_2$, $\mathcal{C} \equiv -\vec{v}_1 \times \vec{v}_2$

as well as $\Lambda^4 \equiv \Lambda_1^4 + \Lambda_2^4$, $r_\Lambda \equiv \dfrac{\Lambda_2^4}{\Lambda_1^4}$

Conditions for trajectory, $\vec{\nabla} V \cdot \vec{\phi}_{\text{heavy}} = 0$

and its stability $m^2_{\text{heavy}} > 0$ greatly simplify

for strong alignment, $|\mathcal{C}| \ll n_1,\, n_2$



$$n_s - 1 \simeq -\frac{M_p^2\, r_\Lambda\, \mathcal{C}^2}{r_\Lambda\, n_2 - n_1}$$

$$r \simeq \frac{2\,|n_s - 1|^2}{\mathcal{C}^2\, M_p^2} \left[\sqrt{n_2}\, \arcsin\left(\sqrt{\frac{r_\Lambda^2 n_2^2 - n_1^2}{n_1\,(n_2 - n_1)}}\right) - \sqrt{n_1}\, \arccos\left(\frac{1}{r_\Lambda}\sqrt{\frac{n_1\left(n_1 - r_\Lambda^2\, n_2\right)}{n_2\,(n_2 - n_1)}}\right)\right]^2 \mathrm{e}^{-|n_s - 1| N}$$

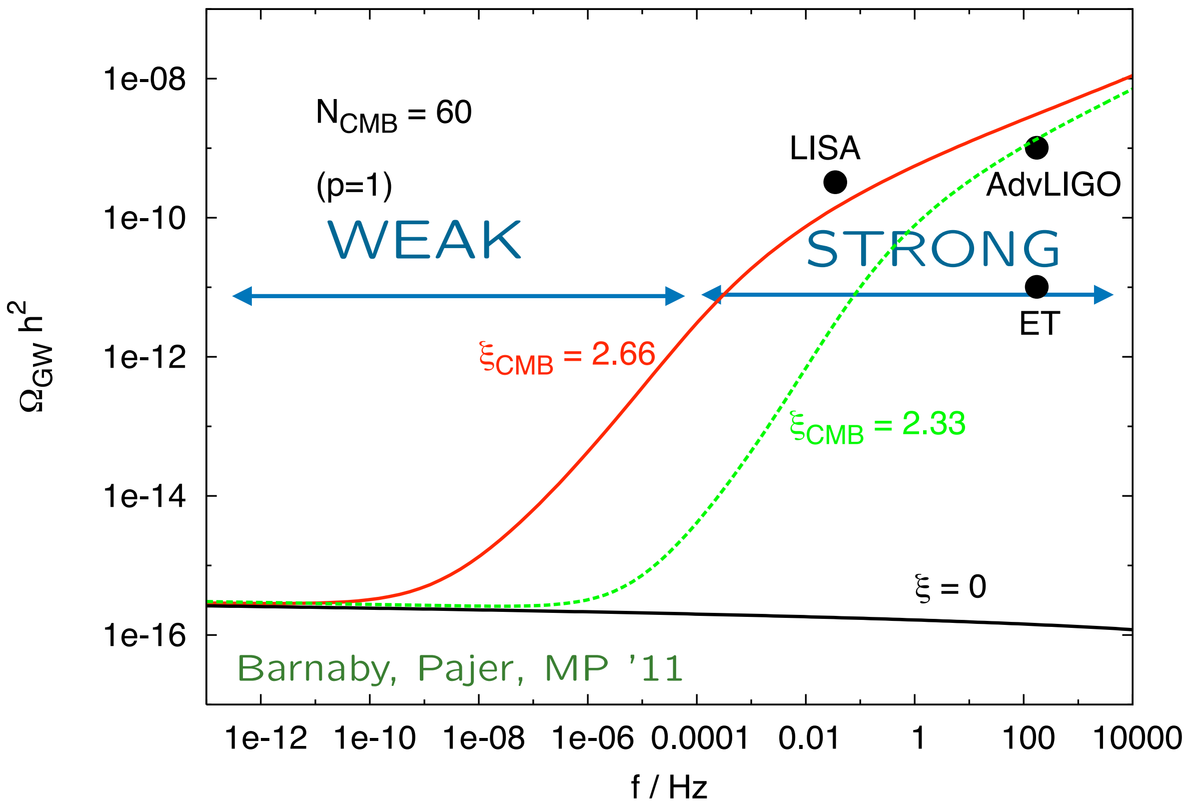
- $\frac{c_A}{f}\phi F\tilde{F}$ constrained at CMB scales by scalar NG

$$\xi \equiv \frac{c_A\,\dot{\phi}}{2\,f\,H} \lesssim 2.5$$

Barnaby, MP '10
Planck '15

- ξ grows during inflation, and $A_+ \propto e^{\pi\xi}$, can produce observable GW at PTA / interferometers scales (and PBH)

Cook, Sorbo '11; Barnaby, Pajer, MP'11; Domcke, Pieroni, Binétruy '16; . . .



Results in the weak backreaction regime

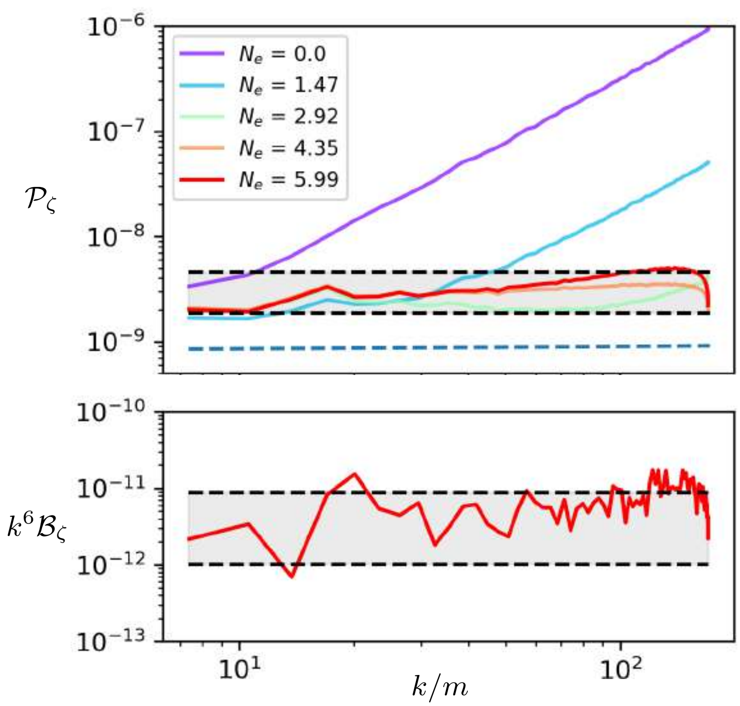
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{c_A}{f}\vec{E}\cdot\vec{B}$$

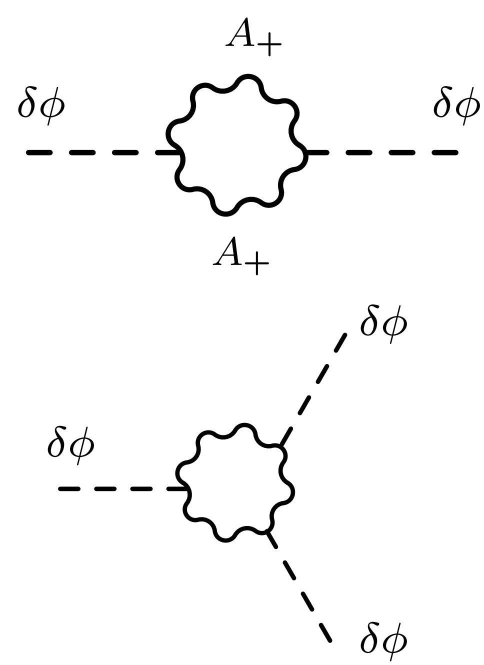
$$H^2 = \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\phi} + V + \frac{\vec{E}^2 + \vec{B}^2}{2}\right]$$

$\equiv$ negligible backreaction terms on background dynamics (electromgnetic notation)

- Analytic results for power spectrum $P \propto \langle \delta\phi^2 \rangle$ and bispectrum $B \propto \langle \delta\phi^3 \rangle$ in this regime based on several approximations: constant ξ and H, specific UV regularization Barnaby, MP '10

- Excellent agreement with full lattice simulations Caravano, Komatsu, Lozanov, Weller '22





Strong backreaction regime: the Anber and Sorbo solution

- Strong backreaction regime studied by Anber, Sorbo '09
- Old idea of warm inflation $\ddot{\phi} + 3H\dot{\phi} + \boxed{V' = -\Gamma\,\dot{\phi}}$ Berera '95

  Dissipation reduces the inflaton motion. Can allow inflation in

  steep potentials (large V') and for reduced field excursion (small $\Delta\phi$)

  Both aspects might be beneficial in the recent swampland program

- Anber-Sorbo mechanism simple & well defined QFT realization

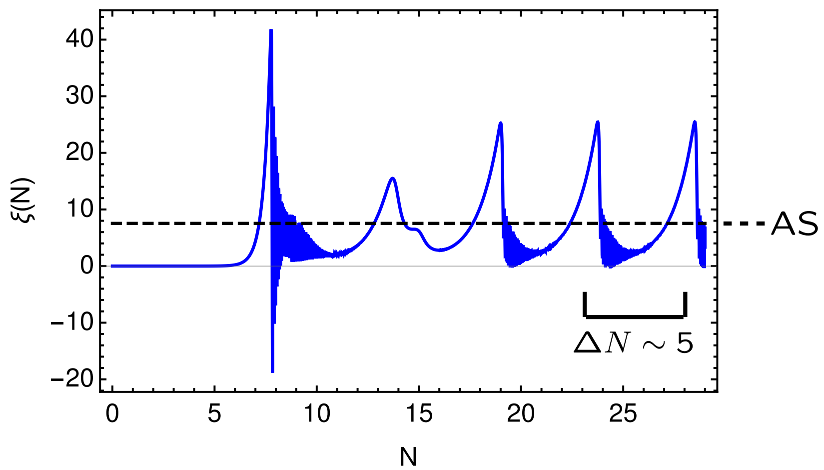
$$\ddot{\phi} + 3H\dot{\phi} + V' = \frac{c_A}{f} F\,\tilde{F}\left[\dot{\phi}\right]$$

Perfect balance assumed at all times $\rightarrow$ Steady state evolution

$$\Rightarrow \quad V' \simeq -2.4 \cdot 10^{-4}\,\frac{c_A\,H^4}{f}\,\frac{e^{2\pi\xi}}{\xi^4} \quad , \quad \xi \equiv \frac{c_A\,\dot{\phi}}{2fH}$$

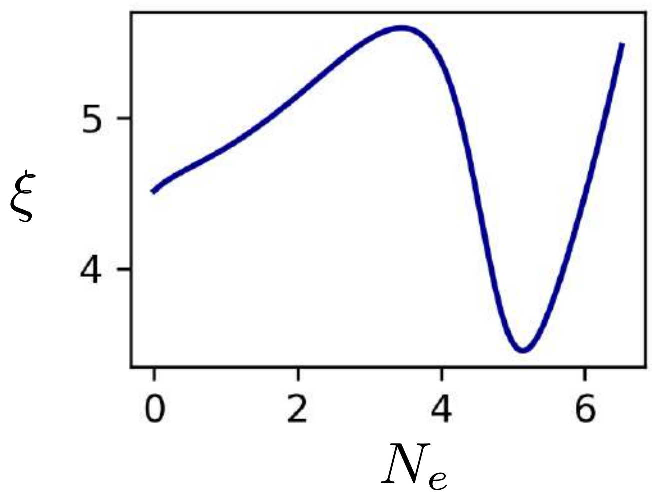
- Oscillatory behaviour from simplified numerical solutions of the system (set of gauge modes + homogeneous inflaton)

Cheng, Lee, Ng '15; Notari, Tywoniuk '16; Dall'Agata, González-Martín, Papageorgiou, MP '19 Gorbar, Schmitz, Sobol, Vilchinskii '21



- Confirmed by full lattice simulation $\phi(t, \vec{x})$, $A^\mu(t, \vec{x})$

Caravano, Komatsu, Lozanov, Weller '22



- Interpreted as delayed effect between the moment the gauge quanta are produced and the moment they backreact on $\phi(t)$

Domcke, Guidetti, Welling, Westphal '20

Analytical study: $\phi(t) = \bar{\phi}(t) + \delta\phi(t)$, $A^\mu\left(t, \vec{k}\right) = \bar{A}^\mu\left(t, \vec{k}\right) + \delta A^\mu\left(t, \vec{k}\right)$

of the homogeneous inflaton & gauge modes around the AS solution

MP, Sorbo '22

$$\delta\phi'' + 2aH\delta\phi' + a^2V''\delta\phi = -\frac{\alpha}{fa^2}\int\frac{d^3k}{(2\pi)^3}\frac{k}{2}\frac{\partial}{\partial\tau}\left[\bar{A}\,\delta A^* + \bar{A}^*\,\delta A\right]$$

$$\delta A'' + \left(k^2 - \frac{k\,\bar{\phi}'}{f}\right)\delta A = \frac{\alpha\,\bar{A}}{f}\,\delta\phi'$$

- Formally solve 2nd eq for δA as a function of $\delta\phi'$

$$\delta A(\tau, k) = \frac{\alpha\,k}{f}\int^\tau d\tau' G_k\left(\tau, \tau'\right)\,\bar{A}\left(\tau', k\right)\delta\phi'\left(\tau'\right)$$

- Insert solution in 1st eq $\;\rightarrow\;$ integro-differential eq for $\delta\phi$

$\xi_\gamma \equiv \xi\,\gamma$

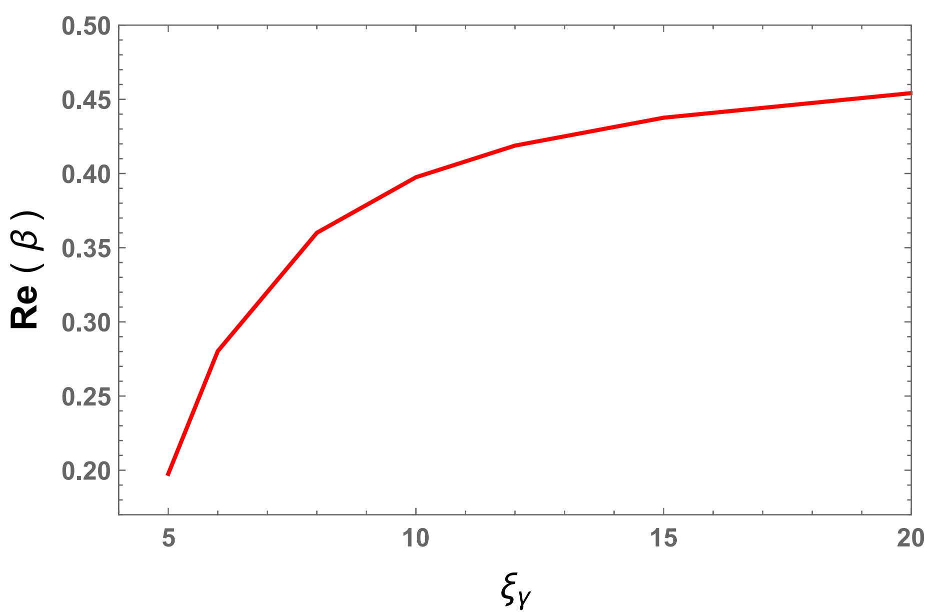
$$\delta\phi''(\tau) + 2aH\delta\phi'(\tau) + a^2 V''\delta\phi(\tau) \simeq$$

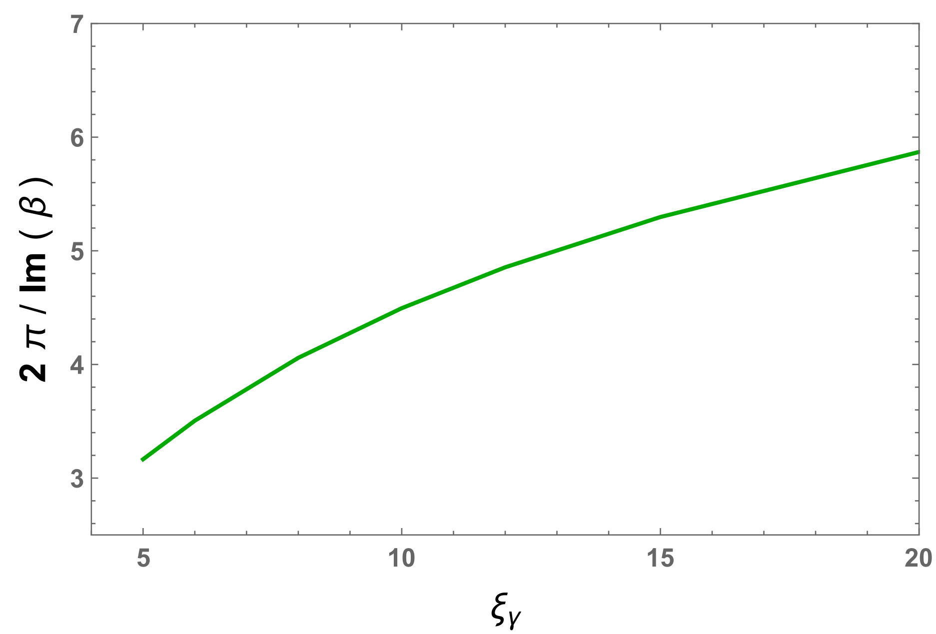
$$\frac{\alpha^2}{f^2 a^2}\frac{e^{2\pi\xi}}{2^8\pi^2\xi^5}\int^{\tau}\frac{d\tau'}{(-\tau')^4}\delta\phi'(\tau')\,\frac{\partial}{\partial\tau}\int_0^{4\xi_\gamma^2} dy\, y^3\sqrt{\tau\tau'}\left[e^{-4\sqrt{y}} - e^{-4\sqrt{y}\sqrt{\frac{-\tau}{-\tau'}}}\right]$$

- Look for $\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\mathrm{Re}\,\beta}\cos(\mathrm{Im}\,\beta \times N + \mathrm{phase})$

Inserting this and doing the integrals $\rightarrow$ homogeneous eq in time (all terms scale as $\tau^{-\beta-2}$). Therefore left wih an algebraic equation for complex β.

Several roots; behavior most unstable mode:





Gradient Expansion Formalism

Sobol, Gorbar,Vilchinskii '19
Gorbar, Schmitz, Sobol, Vilchinskii '21
Durrer, Sobol, Vilchinskii '23

- Rater than $\vec{A}\left(\vec{k}\right)$ e.o.m., tower of eqs. for

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n\, \boldsymbol{E}\rangle \qquad \mathcal{B}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{B}\cdot \mathrm{rot}^n\, \boldsymbol{B}\rangle \qquad \mathcal{G}^{(n)} = -\frac{1}{2a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n\, \boldsymbol{B} + \mathrm{rot}^n\, \boldsymbol{B}\cdot \boldsymbol{E}\rangle$$

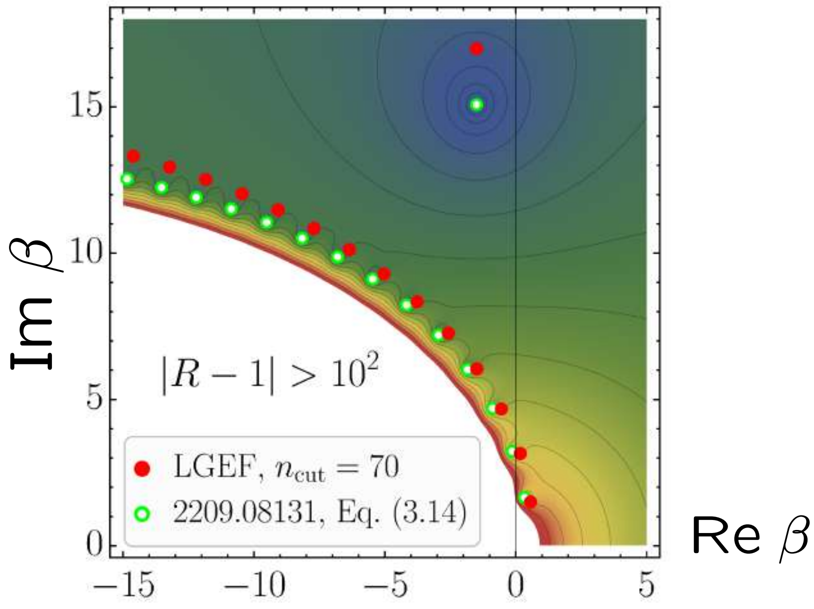
conveniently integrated numerically

- correlators initialized according to AS solution

MP, Schmitz, Sobol, Sorbo, von Eckardstein,'23

1) Analytic linear system

for $\delta\dot{\phi}$, $\delta\mathcal{E}^{(m)}$, $\delta\mathcal{B}^{(m)}$, $\delta\mathcal{G}^{(m)}$

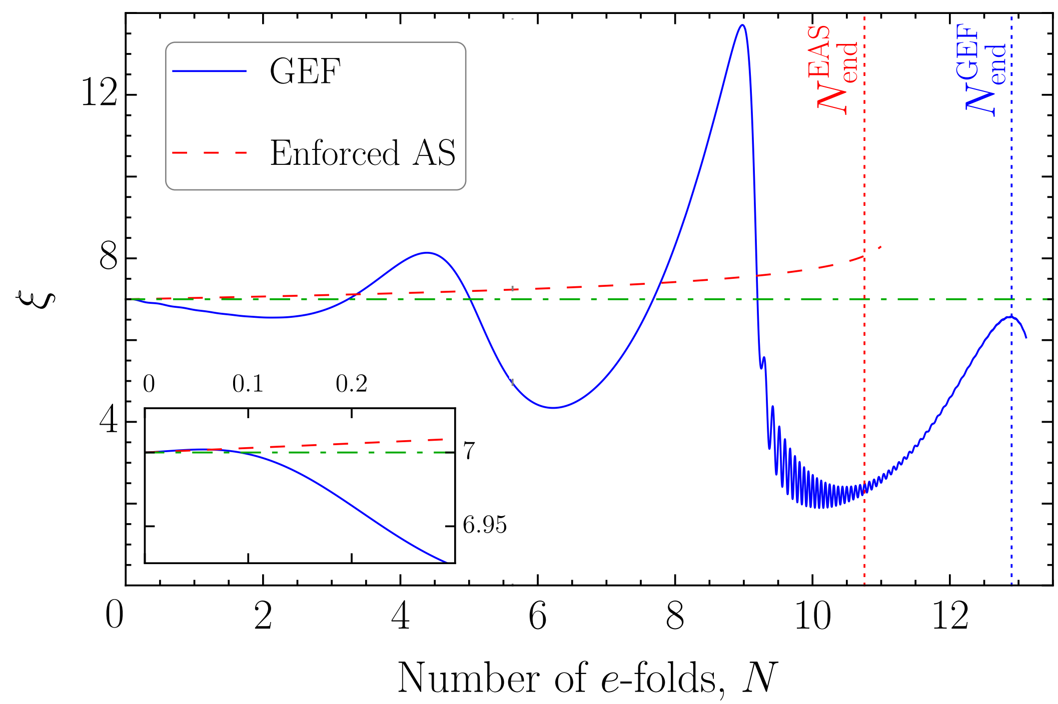


obtain eigenvalues numerically

vs. analytic solution (MP, Sorbo '22)

2) Fully numerical solution starting from AS

(existing ones from weak backreaction)



Burts of GW production

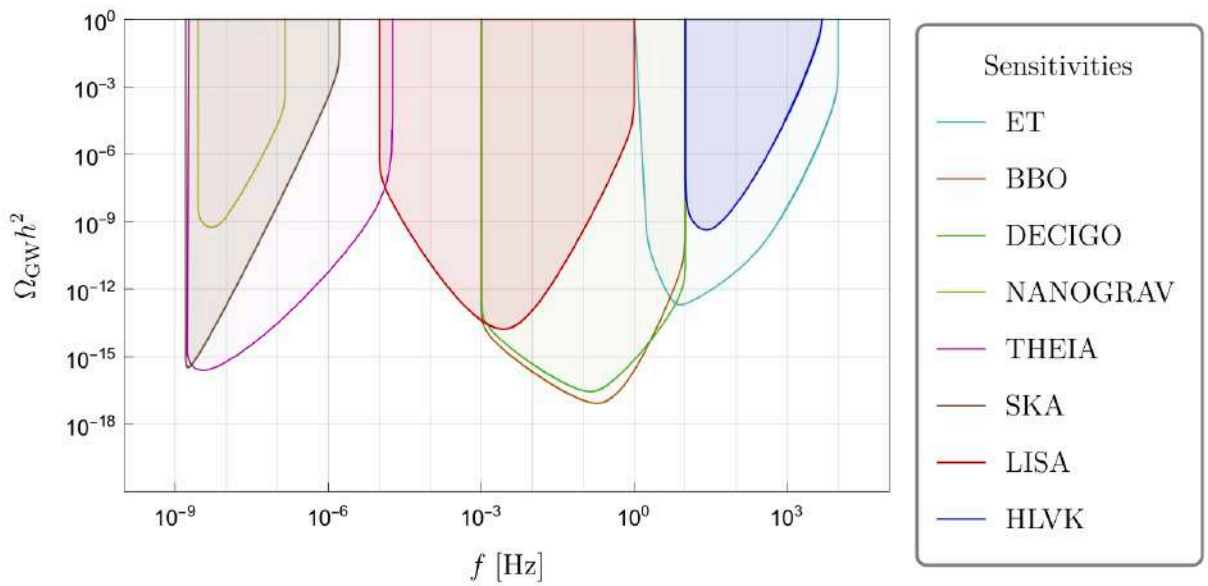
- Expect gauge field amplification and related phenomenology enhanced at scales O (H) when $\dot{\phi}$ is maximum $\rightarrow$ Recurrent peaks in power spectra

- GW easier than $\delta\rho$. Might have correlated peaks in same or across different GW experiments:

  Pulsar Timing Arrays

  Astrometry

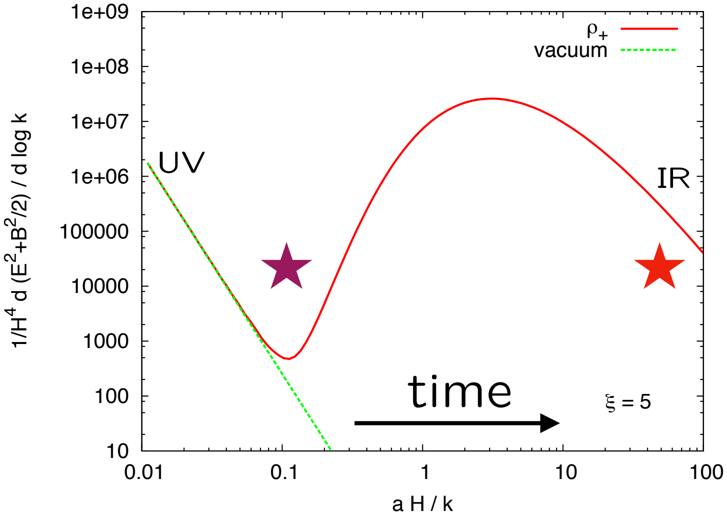
  Interfrometers



- Usual analytic approximations invalid as ξ varies too quickly as individual modes are being produced. Numerical code, with $\phi(t)$ and 400 gauge modes $A(t, k_i)$, covering the dynamical range of 60 e-folds of inflation

Garcia-Bellido, Papageorgiou, MP, Sorbo '23

Physical ρ in one mode



$$A'' + \left(k^2 - 2\xi\, aHk\right) A = 0$$

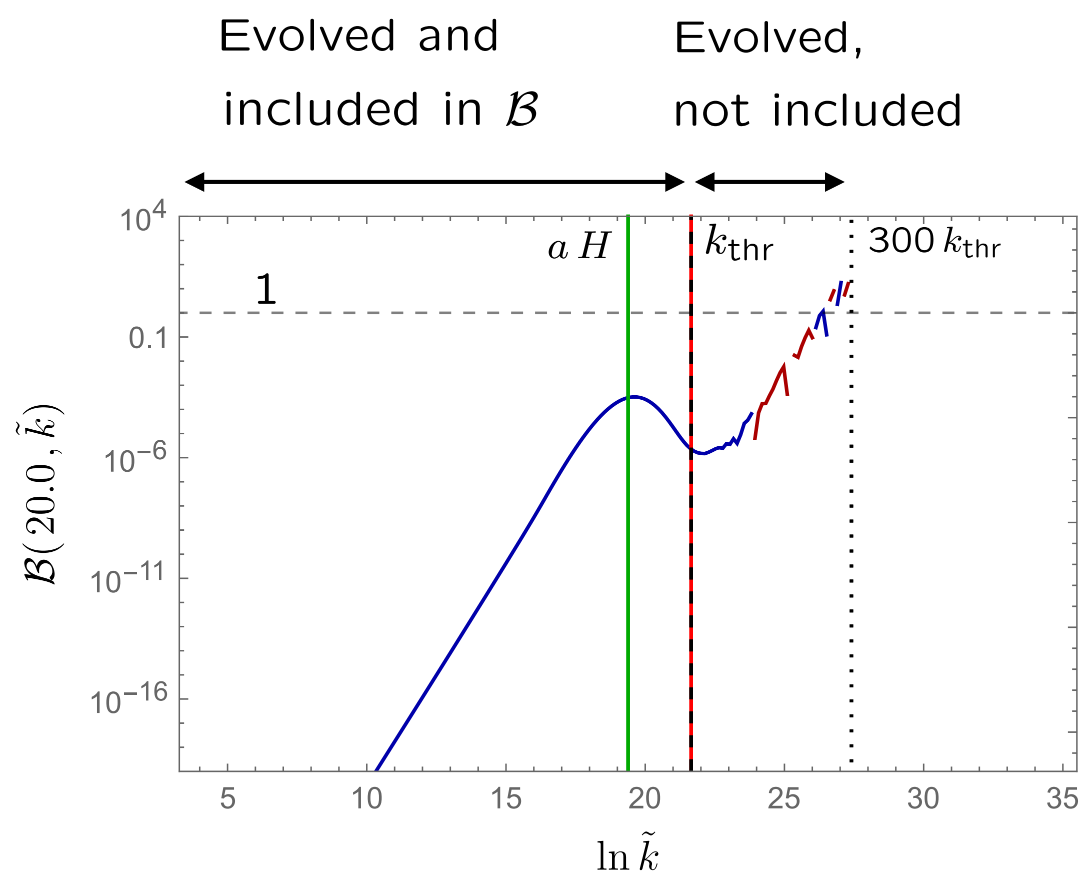
★ Unstable growth for $k < k_{\rm thr} \equiv 2\xi\, a(t)\, H(t)$

★ Decreased by resdshift after ~ 7 e-folds

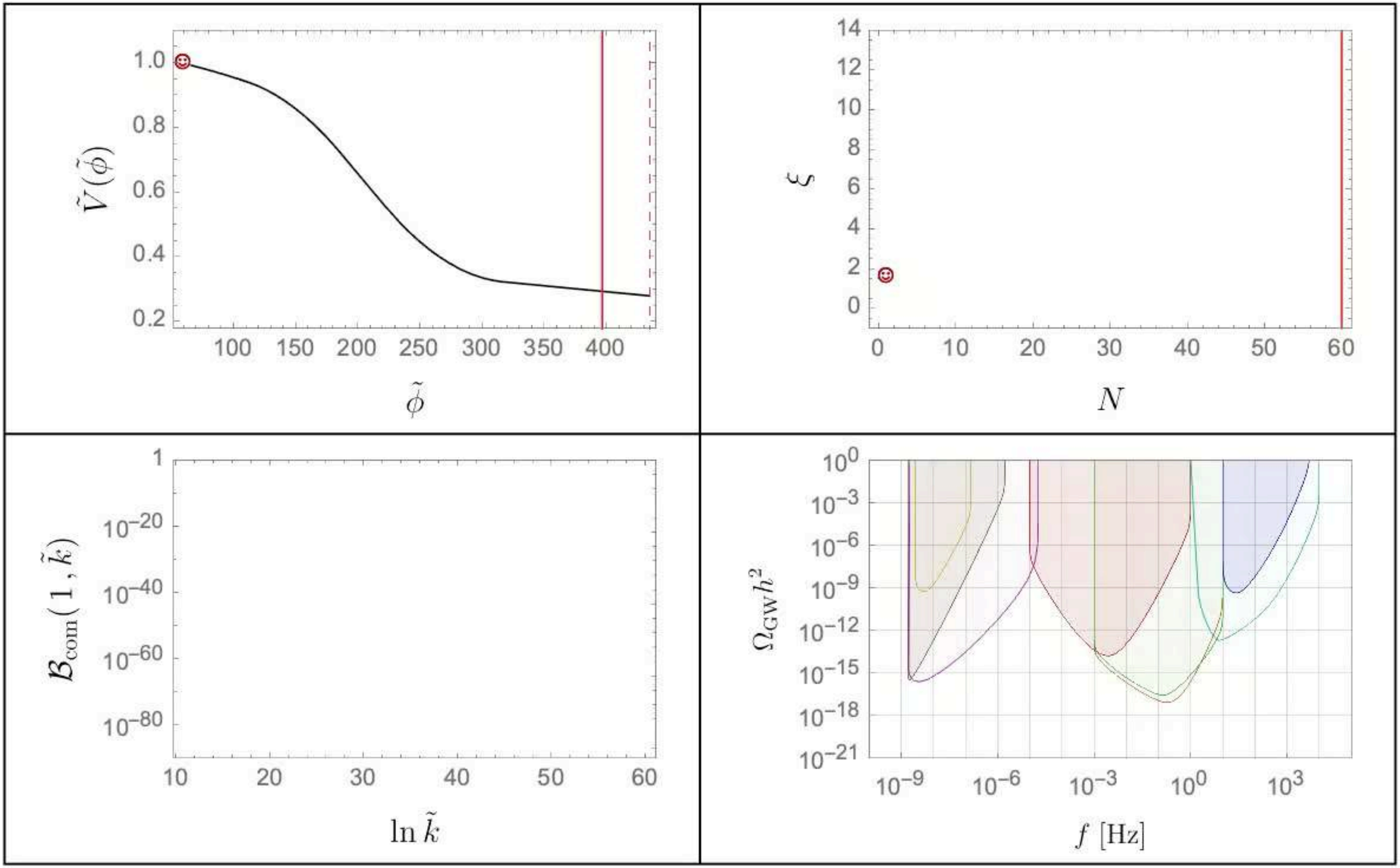
Code integrates modes leaving the horizon all throughout inflation, $k_{\rm max}/k_{\rm min} \simeq e^{60} \simeq 10^{26}$. Each mode followed while dynamically relevant

$$\ddot{\phi} + 3H\dot{\phi} + V' = \frac{c_A}{f}\vec{E}\cdot\vec{B}$$

$$\longrightarrow \quad \frac{c_A\,\vec{E}\cdot\vec{B}}{f\,V'} \equiv \int d\ln k\, \mathcal{B}(N,\, k)$$

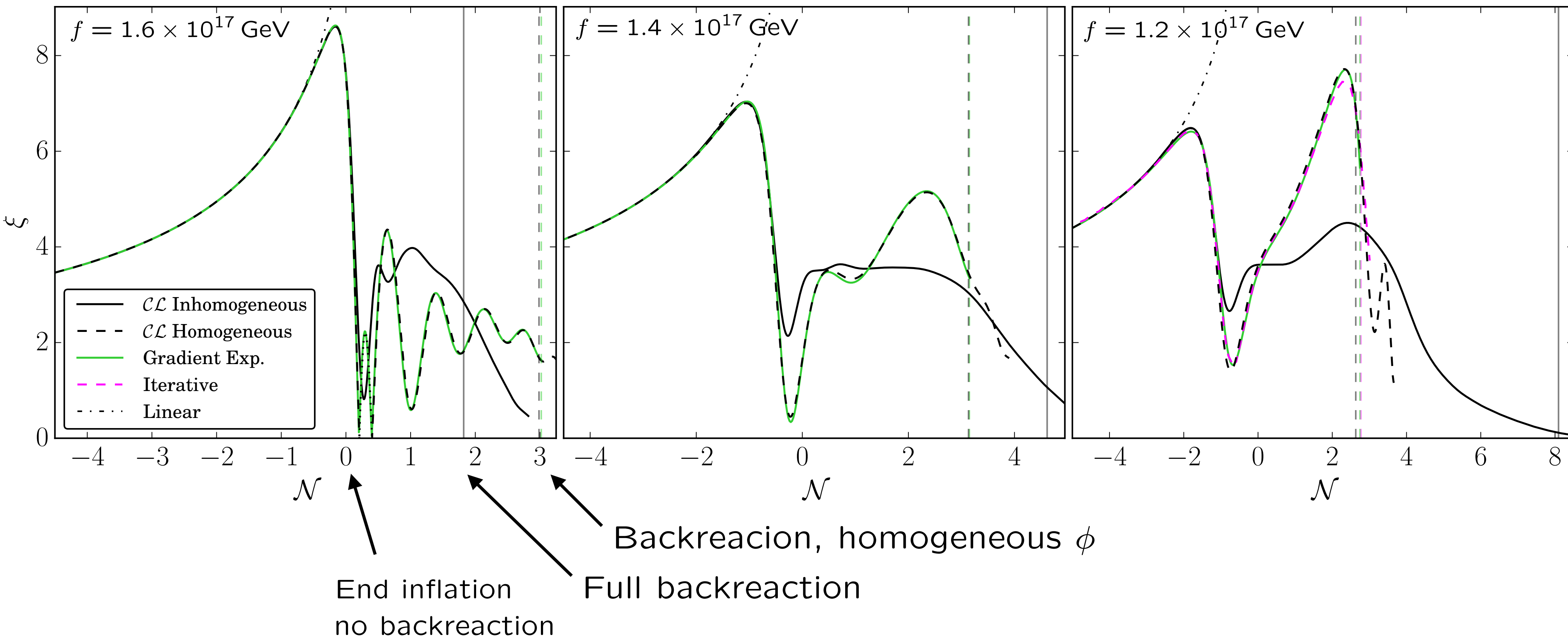


Garcia-Bellido, Papageorgiou, MP, Sorbo '23



Inhomogeneous backreaction

- Very recent lattice simulation Figueroa, Lizarraga, Urio Urrestilla '23 with greater dynamical range than Caravano et al '22



- Oscillations of ξ reduced after the first one
- End of inflation simulated, oscillations reduced also in homogeneous case

- Scalar non-G @ CMB scales $\rightarrow$ $f/c_A \gtrsim 10^{16}$GeV
- GW less produced ($1/M_p$ vs c_A/f)

Barnaby, MP '10

In this regime, negligible backreaction on background dynamics

Under perturbative control; confirmed by lattice

MP, Sorbo, Unal '16

Caravano, Komatsu, Lozanov, Weller '22

General lesson: Several mechanisms for additional GW, result in a decrease of r once also extra density perturbations are accounted for

Observation of GW through the CMB ⟷	$V^{1/4} \simeq 10^{16}\,\text{GeV} \left(\frac{r}{0.01}\right)^{1/4}$ $\Delta\phi \gtrsim M_p \left(\frac{r}{0.01}\right)^{1/2}$

Robust!

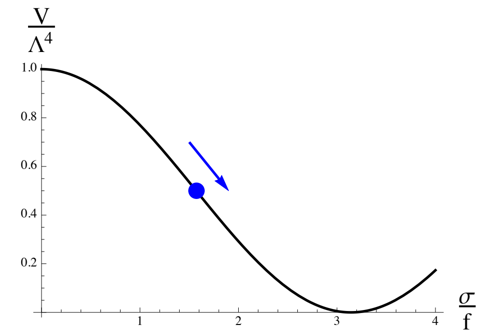
How robust ? Cost for evading it ?

- No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton)
- Relativistic source (GW are produced by quadrupole moment; ζ by energy density)
- Source active only for limited time (GW observed only on a small window; ζ provides constrains on many more scales)

These 3 ingredients present in Namba, MP, Shiraishi, Sorbo, Unal '15

$$\mathcal{L} = \underbrace{-\frac{1}{2}(\partial\varphi)^2 - U(\varphi)}_{\text{inflaton sector}} \underbrace{- \frac{1}{2}(\partial\sigma)^2 - V(\sigma) - \frac{1}{4}F^2 - \frac{c_A\,\sigma}{4f}F\tilde{F}}_{\text{extra sector}}$$

$$V(\sigma) = \frac{\Lambda^4}{2}\left[\cos\left(\frac{\sigma}{f}\right) + 1\right]$$

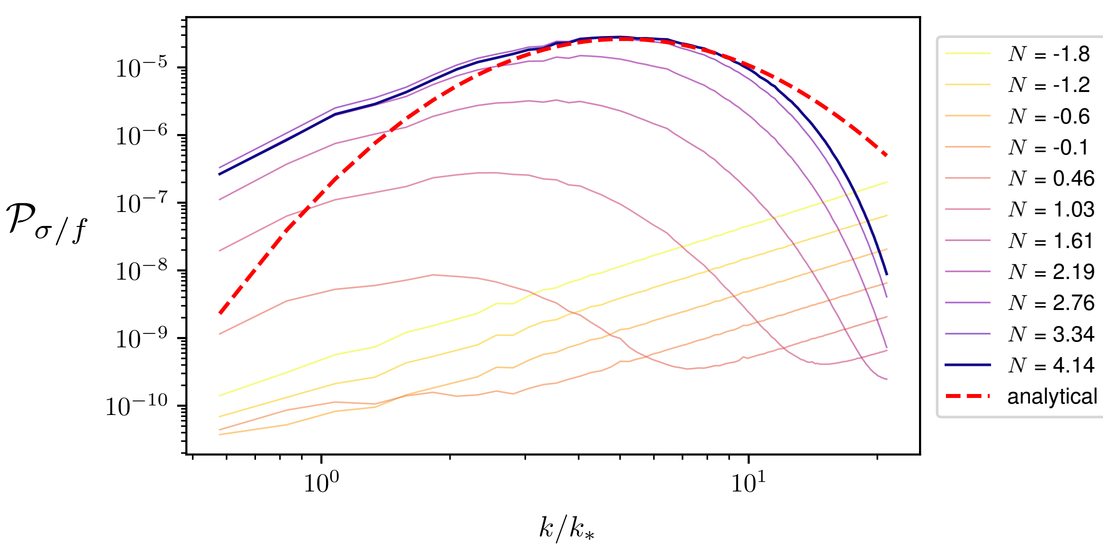


Axion rolls for $\Delta N = \frac{3H^2}{m^2} = O(1)$ e – folds of inflation

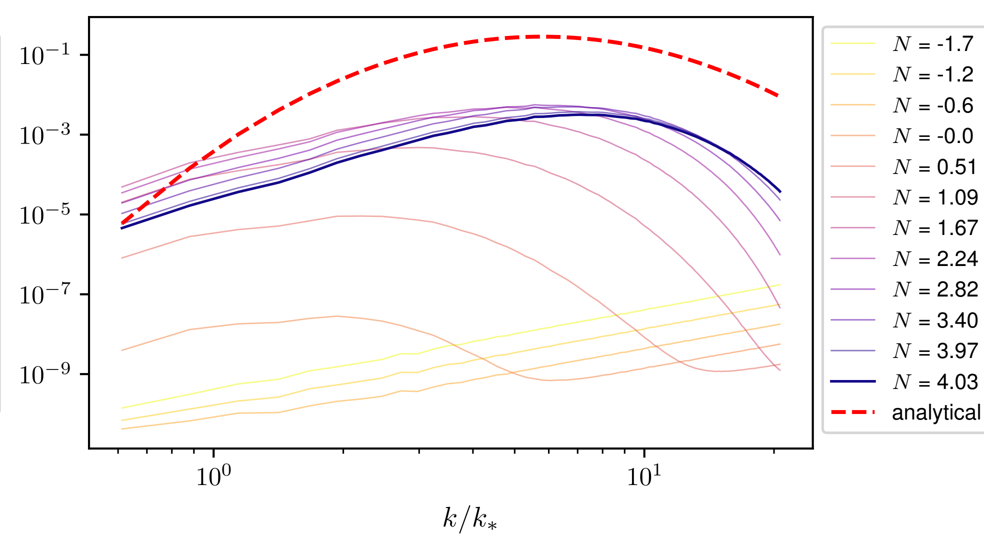
- Gives visible r at small r_{vacuum} / scale of inflation
- Relevant dynamics covered by the lattice

Caravano, MP '24

Weak backreaction ($\xi_{\text{max}} = 5$)

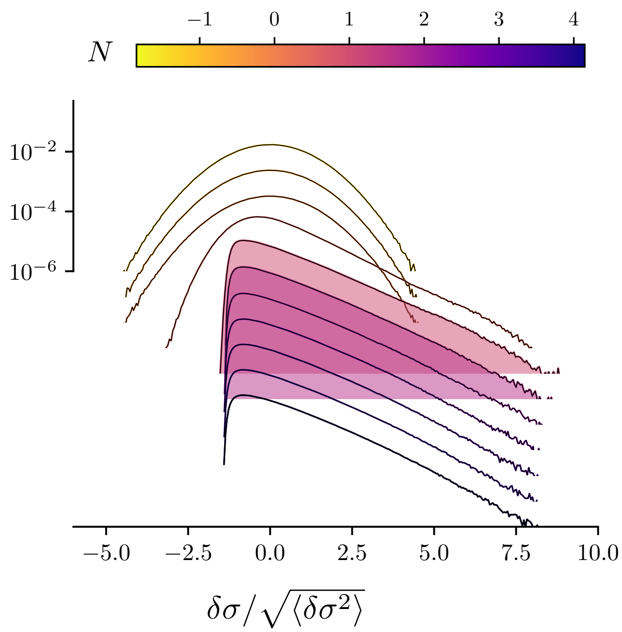


Strong backreaction ($\xi_{\text{max}} = 6$)

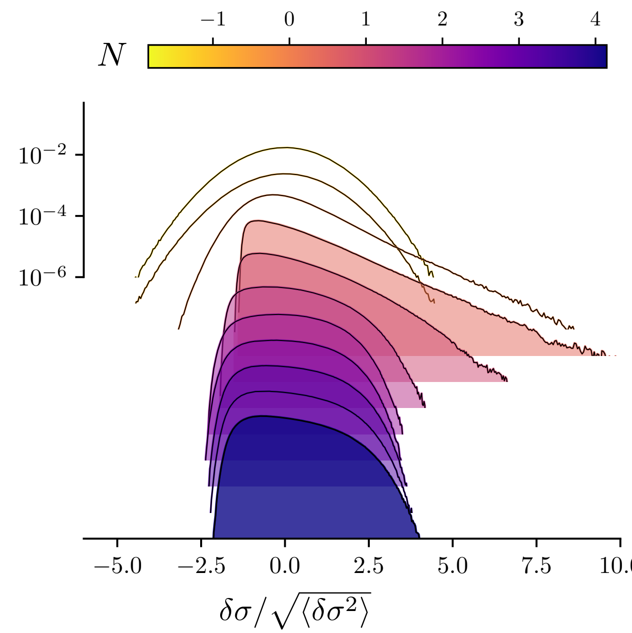


Statistics (1-point probability density function)

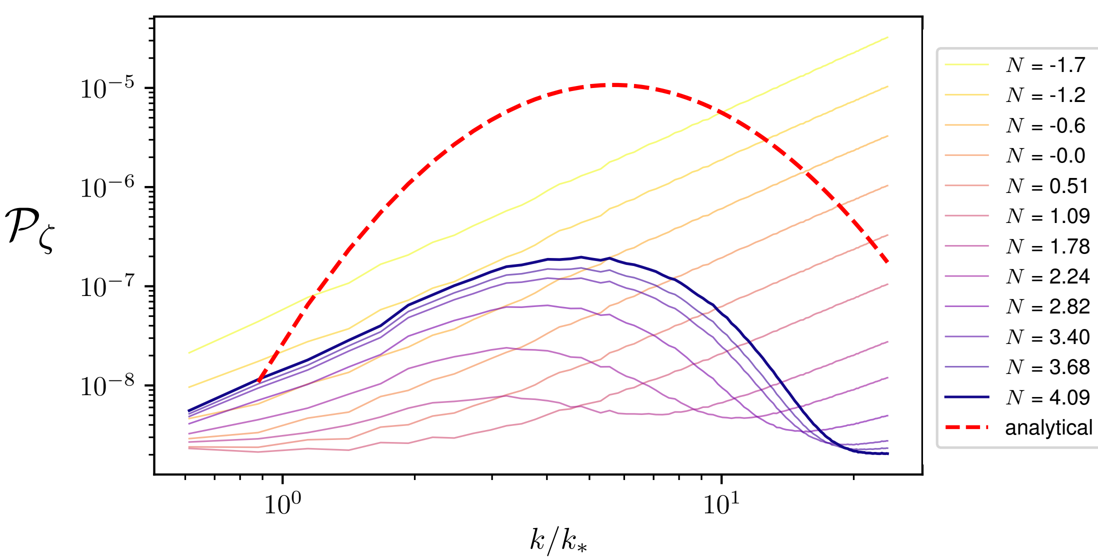
Weak backreaction ($\xi_{\rm max} = 5$)

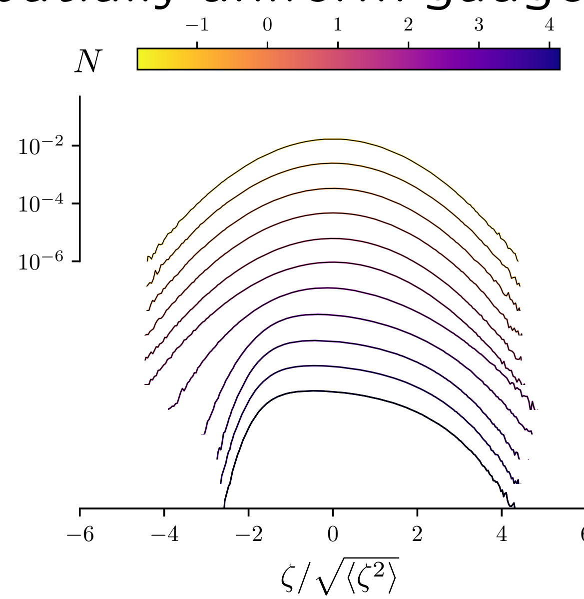


Strong backreaction ($\xi_{\rm max} = 6$)



Scalar petrurbations (added linearly, in the spatially uniform gauge)



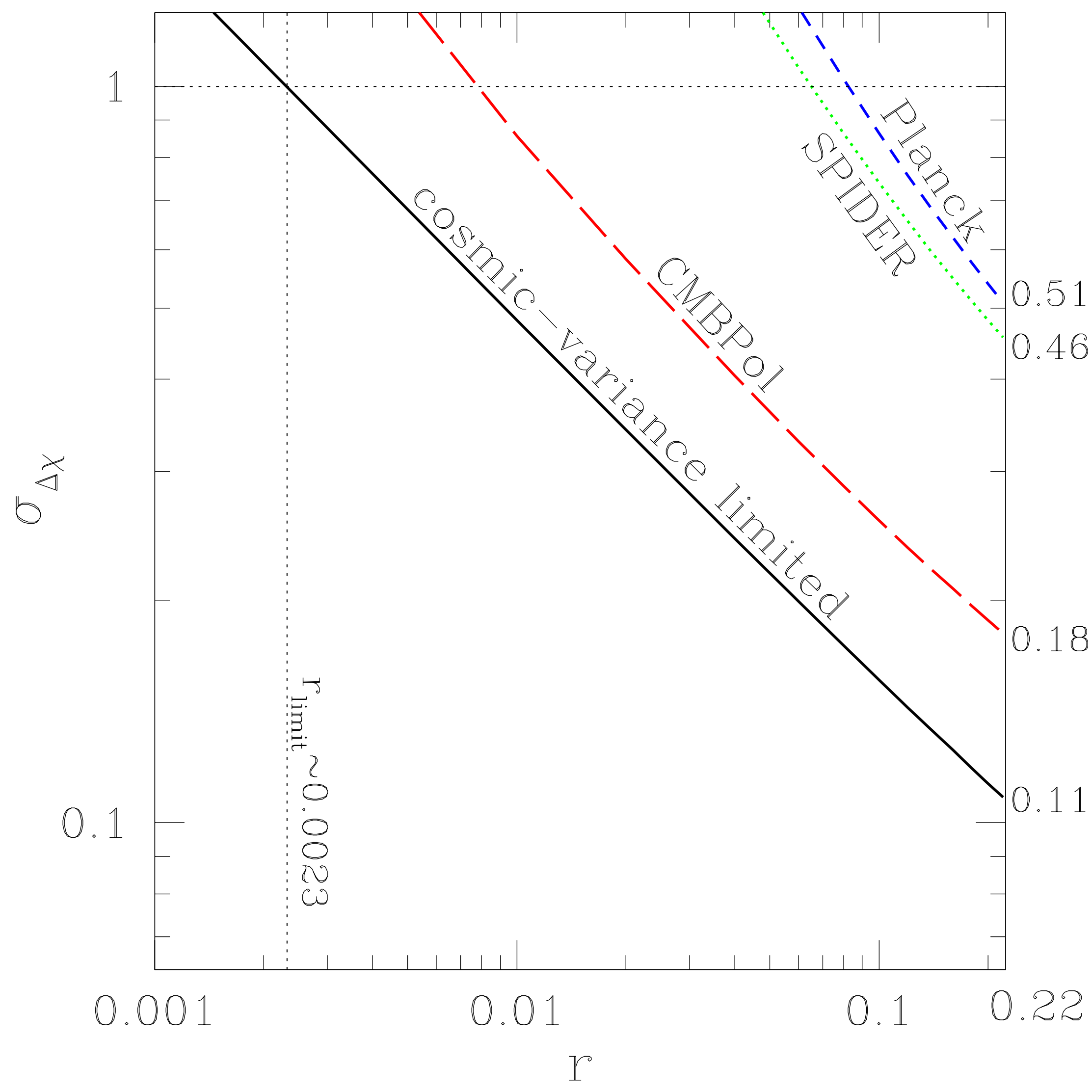


Testing a chiral SGWB

- @ CMB scales (TB & EB correlations)

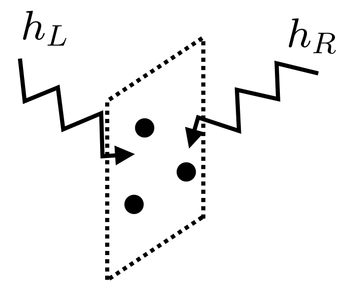
Gluscevic and Kamionkowski '10

$$\Delta\chi = \frac{P_L - P_R}{P_L + P_R}$$

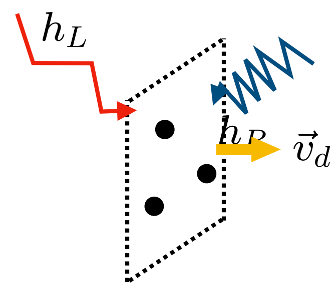


Measurement of GW polarization at LISA / ET

Two GWs related by a mirror symmetry produce the same response in a planar detector. Cannot detect net circular polarization of an isotropic SGWB



Isotropy in any case broken by peculiar motion of the solar system. Assumption, $v_d \simeq 10^{-3}$ as CMB



$$\text{SNR}_{\text{LISA}} \simeq \frac{v_d}{10^{-3}} \frac{\Omega_{\text{GW,R}} - \Omega_{\text{GW,L}}}{1.2 \cdot 10^{-11}} \sqrt{\frac{T}{3\,\text{years}}}$$

Domcke, García-Bellido, MP, Pieroni Ricciardone, Sorbo, Tasinato '19

(one order of magnitude greater than estimate in Seto '06)

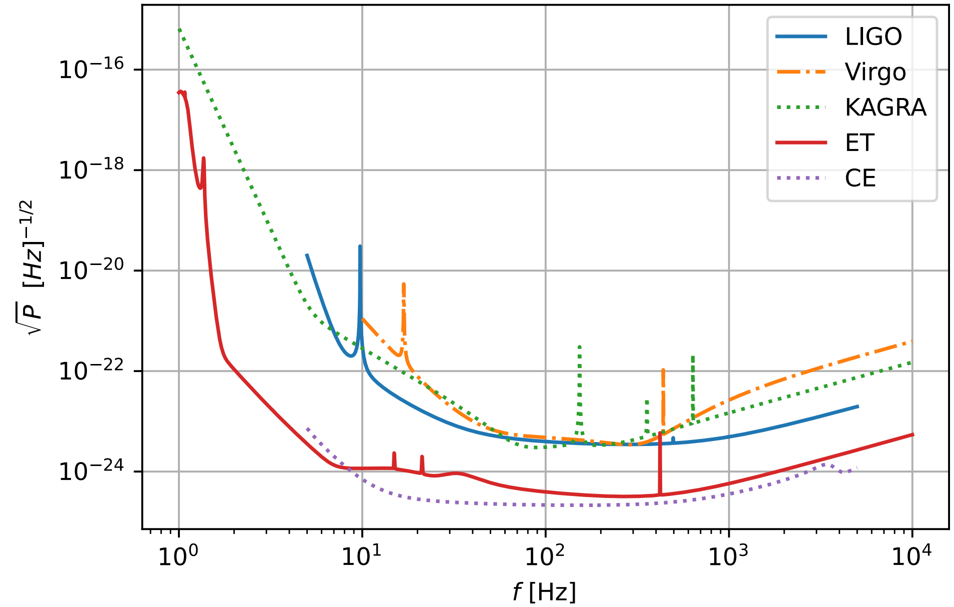
- Do the GW and the CMB dipole coincide ?
- One order of magnitude improvement with LISA-Taiji

Orlando, Pieroni, Ricciardone '20

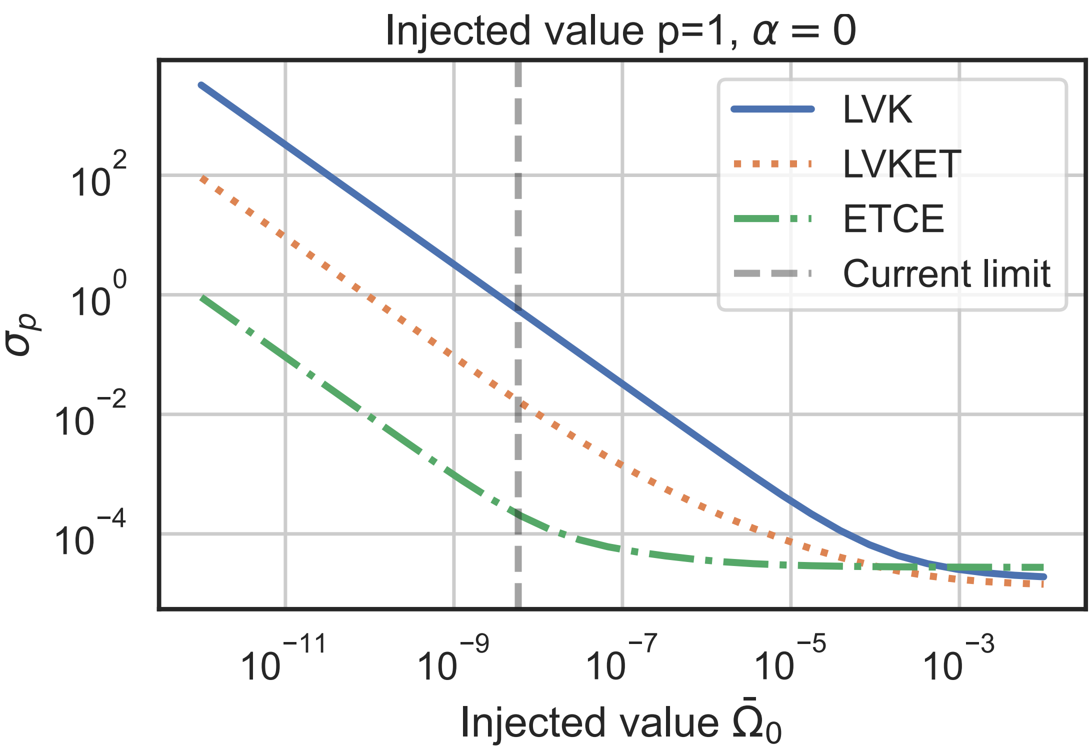
Measurement at ground-based interferometers

$$\Omega_{L/R} = \Omega_0 \left(\frac{f}{f_0}\right)^{\alpha} (1 \pm p)$$

https://dcc.ligo.org/LIGO-T1500293/public



Mentasti, Contaldi, MP '23



10 years observation

Conclusions

- Potentially detectable GW at several scales from axion inflation
- New results in the regime of strong backreaction

  Instability of steady state solution

  First studies of associated signatures

- Awaiting increased dynamical range from lattice