Axion inflation in the regime of strong backreaction

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In collaboration with: $M\no$ Person, $M\no$ th

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- R. Namba, G. Nardini, E. Pajer, A. Papageorgiou, M. Pieroni,
- S. Renaux-Petel, A. Ricciardone, K. Schmitz, M. Shiraishi, O. Sobol,
- L. Sorbo, G. Tasinato, C. Unal, V. Vaskonen, R. von Eckardstein
- *•* Vanilla slow-roll models of inflation produce a GW signal too low to be observed in the foreseeable future • Vanilla slow-roll inflation produce a GW signal too low to be observed in the foreseeable future (at least, **•** Various sidw-roll inflation produce a GVV signal too low to be Braglia et al '24, Lisa cosmology Working Group.
Braglia et al '24, Lisa cosmology Working Group. De la Gr • Vanilla slow-roll inflation produce a GW signal too low to be Talk by Ricciardone observed in the foreseeable future (at least, at sub-CMB scales) observed in the foreseeable future (at least, at sub-CMB scales) $\mathcal{L}(\mathsf{S})$
- *•* Larger signal in various inflationary models / mechanisms Braglia et al '24, LISA Cosmology Working Group. Talk by Ricciardone *•* Larger signal in various inflationary models / mechanisms *•* Larger signal in various inflationary models / mechanisms *•* Larger signal in various inflationary models / mechanisms Bragild et al '24, LISA Cosmology Working Group. Braglia et al '24, LISA Cosmology Working Group. Braglia et al '24, LISA Cosmology Working Group. Talk by Ricciardone **Figure 10** Highly studied (and natural) means of the studies of th $D \cos \theta$ is at all $D \theta$ l IC • Larger signal in various initiationary models / mechanisms
• Productional in the Conception Medical Conceptions *•* Highly studied (and natural) mechanism is from an axion inflaton Braglia et al '24, LISA Cosmology Working Group. *•* Larger signal in various inflationary models / mechanisms Braglia et al -24, LISA Cosmology Working Group. Talk by Ricciardone
	- *•* Highly studied (and natural) mechanism is from an axion inflaton n strong backreaction, where our knowledge is suit incomplet Llighty studied (and natural) mechanism is from an avion inflaten of strong backreaction, where our knowledge is still incomplete. • Highly studied (and natural) mechanism is from an axion inflaton of strong backreaction, where our knowledge is still incomplete. *•* Highly studied (and natural) mechanism is from an axion inflaton

	of strong baskreastion, where our knowledge is still incomn coupled to gauge fields. In the gauge fields \sim

Shift symmetry $\phi \to \phi + C$. E.g. axion (natural) inflation γ γ | ∪. ∟.g. α_{Al} \overline{O} *A[±]* (⌧*, k*) = 0 .g. axion (natural) Garretson, Field, Carroll '92 + left handed righthanded Anber, Sorbo '06 $T₁$ $T₂$ $T₃$ $T₄$ α avion (natural) inflation Freese, $\overline{}$

ndial inflation Freese, Frieman, Olinto '90

Olinto '90

$$
\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^{2} + V_{\text{shift}} (\phi) + \frac{c_{A}}{f} \partial_{\mu} \phi \bar{\psi} \gamma^{\mu} \gamma_{5} \psi + \frac{c_{\psi}}{f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}
$$

- **•** Couplings shift symmetric, no large radiative corrections to V ric no l mmetric, no large radiative corrections to V Garretson, Field, Carroll '92 radia
• $F_{\rm eff}$ \sim $F_{\rm eff}$ \sim $F_{\rm eff}$ \sim $F_{\rm eff}$ **•** Couplings shift symmetric, no large radiative corrections to V *•* Couplings shift symmetric, no large radiative corrections to *V*
- \mathbb{L} and the couplings with only these couplings with only these couplings \mathbb{L} *•* One tachyonic helicity at horizon crossing $\frac{1}{2}$ Then displays the displays of $\frac{1}{2}$ Its for two pola and two pola for @⌧ ² ⁺ *^k*² ⌥ blariz *A[±]* (⌧*, k*) = 0 2 *f H* ⇠ ⌘ vo polarization **2** polarization *a k ^d* \sqrt{P} • $\phi F\tilde{F}$ breaks partity, \neq results for two polarization

$$
\left(\frac{\partial^2}{\partial \tau^2} + k^2 \mp \frac{ak\,\dot{\phi}}{f/c_A}\right) A_{\pm}(\tau, k) = 0
$$
 + left handed
- right handed
Physical ρ in one mode

✓ @² ✓ @² @⌧ ² ⁺ *^k*² ⌥ ⇠ ⌘ *cA* ˙ + left handed + left handed Equation for one Fourier mode + left handed right handed

- m en ϵ ers
Ballardir ¹^{p_t} **••** *•* Renormalize away vacuum energy in UV
Rallardini, Braglia, Finelli Ballardini, Braglia, Finelli, Marozzi, Starobinsky, '19 \sim 1970 \pm 1970 \pm 1970 \pm
- *•* One tachyonic helicity at horizon crossing *•* One tachyonic helicity at horizon crossing • One tachyonic helicity at horizon crossing

¹ *•* Max amplitude *^A*⁺ / ^e˙
	- *•* Then diluted by expansion *•* Then diluted by expansion *•* Then diluted by expansion

• max amplitude *^A*⁺ / ^e⇡⇠ *•* max amplitude *^A*⁺ / ^e⇡⇠ *•* Max amplitude *^A*⁺ / ^e˙ *•* The produced *A*⁺ modes source inflaton perturbations *•* Max amplitude *^A*⁺ / ^e⇡ ⇠ *,* ⇠ ⌘ *cA* ˙ 2*fH •* The produced *A*⁺ modes source inflaton perturbations

pply! At any moment during inflation, gauge modes with through inverse decay. These modes are highly non-gaussian. **Possibic signals at several searcs** $\lambda \simeq H$ are produced \rightarrow possible signals at several scales Continuous supply! At any moment during inflation, gauge modes with Continuous supply! At any moment during inflation, gauge modes with Continuous supply! At any moment during *•* The sourced perturbations are highly non-gaussian. This imposes

Amplified gauge fields source scalar and tensor perturbations and tensor perturbations and tensor perturbations ا
Lurhations δA γ and tensor perturbations $\frac{1}{2}$ mpmoa gaag and tensor perturbations *I* mplified gauge fields source scalar rriphited gauge fiew
• Amplified gauge fields source scalar and relations and relations and relations and relations and relations relat *•* GWs are chiral, *gT T ^L gT T*

• GWs are chiral, *gT T*

 $\mathsf{Barnaby}, \mathsf{MP}$ '10 δA ¹ δA δA A ----- $\delta \phi$, δg^{TT} \mathcal{S} \mathcal{A} $\frac{10}{\delta A}$ σ $\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$ $\sum_{i=1}^n$ $\frac{1}{2}$ sourced modes are highly non-Gaussian, imposes $\frac{1}{2}$ **f** $\frac{1}{2}$ $\mathcal{F}_{\mathcal{F}}$ $\boldsymbol{\mathcal{S}}$ *A , gT T ^R* Sorbo '11

 \mathcal{L}

, gT T L GWs • GWs are chiral, $\delta g_L^{TT} \gg \delta g_R^{TT}$ Sorbo '11 2*fH* $\gg \delta g^{TT}_R$ 56 **GWs are chiral,** $\delta \omega$ <u>'</u> *CA Mp* hiral, $g_L^{TT} \gg \delta g_R^{TT}$ Sorbo '11 $\gg \delta g^{TT}_{\scriptscriptstyle D}$ \mathcal{G}_I \mathcal{G}_I *•* Sourced signals for

S ourced signals for
$$
5 \sim \xi \equiv \frac{C_A \dot{\phi}}{2fH} \simeq \sqrt{\frac{\epsilon}{2}} \frac{C_A M_p}{f} \Rightarrow \frac{f}{c_A} \simeq 10^{-2} M_p
$$

5 T

Problems with natural inflation Fig. 7. Marginalized joint two-dimensional 68 % and 95 % CL regions for combinations of (✏¹ ,✏² ,✏3) (upper panels) and (✏*^V* , ⌘*V*, ⇠² Problems with natural inflation contours). $A_{\rm eff}$ about a null set α inflation still via α such as α Problems with natural inflation

• Aligned natural inflation in agreement with data & testable at CMB-S4 Kim, Nilles, MP '05 **Forman in agreement with data & testable at CMB-S4** on in agreement with data & testable at CMB-S4

$$
V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right) \right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right) \right]^{5}
$$

 $f_{\text{eff}} \gg f_i, g_i$ if $\frac{f_1}{g_1} \simeq \frac{f_2}{g_2}$

s as a wat rfall *g*2 ⌘i ends as a waterfall MP, Unal '15 ends as a waterfall MP, Unal '15

Analytic trajectory and CMB phenomenology worked out in Greco, MP '24 Analytic trajectory and CMB phenomenology worked out in Greco, MP '24 Analytic trajectory and CMB phenomenology worked out in Greco, MP '24 $\frac{1}{2}$ **roca** MD '24 Analytic trajectory and CMB phenomenology worked out in Greco, MP $^{\prime}$ 2 Analytic trajectory and CMB phenomenology wo

$$
V = \Lambda_1^4 \left[1 - \cos \left(\frac{\theta}{f_1} + \frac{\rho}{g_1} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(\frac{\theta}{f_2} + \frac{\rho}{g_2} \right) \right]
$$

Model invariant under simultaneous rotation of $\vec{\phi} \equiv \{\theta, \rho\}$, $\vec{v}_i \equiv \left\{f_i^{-1}, g_i^{-1}\right\}$ Model invariant under simultaneous rotation of $\vec{\phi} \equiv {\theta, \rho}$, $\vec{v_i} \equiv {\{f_i^{-1}, g_i^{-1}\}}$ \mathcal{L}_i Model invariant under simultaneous rotation of $\vec{\phi} \equiv \{\theta, \rho\}$, $\vec{v_i} \equiv \left\{f_i^{-1}, g_i^{-1}\right\}$ as well as ⇤⁴ ⌘ ⇤⁴ ¹ + ⇤⁴ ² *, r*⇤ ⌘ านไ **Model invariant under si** as a set that $\vec{\phi} \equiv \{\theta, \rho\}$

Described in terms of invariant $n_1 \equiv \vec{v}_1 \cdot \vec{v}_1$, $n_2 \equiv \vec{v}_2 \cdot \vec{v}_2$, $C \equiv -\vec{v}_1 \times \vec{v}_2$ as well as $\Lambda^4 \equiv \Lambda_1^4 + \Lambda_2^4$, $r_\Lambda \equiv \frac{r_2}{44}$ *•* Can have *f*e↵ *> Mp* even if *fi, gi* sub-Planckian (issue 1) Conditions for trajectory, $\vec{\nabla}V \cdot \vec{\phi}_{heavy} = 0$ $\begin{bmatrix} 10 \\ 20 \end{bmatrix}$ (issue 2) for strong alignment, $|C| \ll n_1$, n_2 do well do $N = N_1 + N_2$, $N = \frac{1}{N_1^4}$ $N_2 = \frac{1}{N_1^4}$ $N_3 = \frac{1}{N_1^4}$ $N_4 = \frac{1}{N_1^4}$ $m_{\text{heavy}}^2 > 0$ greatly simplify $_{\text{40}}$ $40\frac{3}{4}$ $\left\{\n\begin{array}{c}\n30 \\
7\n\end{array}\n\right\}$ $\begin{bmatrix} 30 \\ 40 \end{bmatrix}$ 2 $\sqrt{2}$ 1)
 $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $10¹$ 2 $\sqrt{2}$ as well as $\Lambda^4 \equiv \Lambda_1^4 + \Lambda_2^4$, $r_\Lambda \equiv$ Λ^4_2 Λ_1^4 * end 20 1 0 30 60 $\begin{matrix} \frac{1}{2} \\ \frac{1}{2} \end{matrix}$ - ρ / f2 * 1.5 1.25 end greatly simplify in the limit of strong alignment, *|C|* ⌧ *n*1*, n*² numerical analytical property Described in terms of invariant $n_1 \equiv \vec{v}_1 \cdot \vec{v}_1$, $n_2 \equiv \vec{v}_2 \cdot \vec{v}_2$, $C \equiv -\vec{v}_1 \times \vec{v}_2$ greatly simplify in the limit of strong alignment, *|C|* ⌧ *n*1*, n*² numerical control analytical as well as $\Lambda^4 \equiv \Lambda_1^4 + \Lambda_2^4$, $r_\Lambda \equiv \frac{\Lambda_2}{\Lambda_1^4}$ $\frac{1}{2}$ 2 and its stability $m_{\mathsf{heavy}}^2 > 0$ greatly simplify for strong alignment, *|C|* ⌧ *n*1*, n*² for strong alignment, *|C|* ⌧ *n*1*, n*² A ⁴ for strong alignment, *|C|* ⌧ *n*1*, n*² \cdots *M*² *^p ^r*⇤ *^C*² *<i>r* \mathcal{L} *x* CMB

> $n_s - 1 \simeq M_p^2 r_\Lambda C^2$ $r_\mathsf{A}\,n_2 - n_1$ n_s-1 : $M_p^2 r$ $\frac{2}{r_1}\frac{1}{n_2}$ –

$$
r \simeq \frac{2 |n_s - 1|^2}{\mathcal{C}^2 M_p^2} \left[\sqrt{n_2} \arcsin \left(\sqrt{\frac{r_\Lambda^2 n_2^2 - n_1^2}{n_1 (n_2 - n_1)}} \right) - \sqrt{n_1} \arccos \left(\frac{1}{r_\Lambda} \sqrt{\frac{n_1 (n_1 - r_\Lambda^2 n_2)}{n_2 (n_2 - n_1)}} \right) \right]^2 e^{-|n_s - 1|N|}
$$

*

20

CMB

0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0

 $\hat{\theta}$ / f_2

 \sqrt{AB} scales by scalar NG *^f FF*˜ constrained at CMB scales by scalar NG, ⇠ ⌘ *cA* ˙ \overline{a} at \overline{c} *•* PBH • $\frac{c_A}{f} \phi F \tilde{F}$ constrained at CMB scales by scalar NG constrained at CMB scales by scalar NG *E* constrained
. $\frac{c_A}{f} \phi FF$ const c_A \angle *FF*^{\tilde{F}} constrained at CMB scales by s

Rinde, MP '10 $\zeta = \frac{4H}{2fH} \gtrsim 2.5$ Planck '15 Garcia-Bellido, Marchia-Bellido, Marchia-Bellido, Marchia-Bellido, Indiana, In Planck '15 $\xi \equiv \frac{c_A \, \dot{\phi}}{2\, f \, H}$ $\xi \equiv \frac{c_A \phi}{2 f H} \lesssim 2.5$ Barnaby, MF Planck '15 *•* The amplified gauge fields also produce GW, though *A*+*A*⁺ ! *gL •* The amplified gauge fields also produce GW, though *A*+*A*⁺ ! *gL* ζ \mathcal{L} \mathcal{J} \mathcal{L}

⇠ ⌘ *cA* ˙

uring inflation, and $A_+\propto e^{n\varsigma}$, can produce $\left| \right|$ $\left| \right|$ $\mathcal{G} = \mathcal{G}$ T_A / interferomaters scales $\bullet\;$ ξ grows during inflation, a **•• UNSURVANIC GWV ACT TAY, ANCHOLOGIST SUATS**
••• Cook, Sorbo '11: Barnaby, Pajer, MP'11 during inflation and $A_{\perp} \propto e^{\pi \xi}$ can produce observable GW at PTA / interferometers scales (and PBH) • ξ grows during inflation, and $A_+ \propto e^{\pi \xi}$, can produce Cook, Sorbo '11; Barnaby, Pajer, MP'11; Cook, Sorbo '11; Barnaby, Pajer, MP'11; observable GW at PTA / interferometers scales $($ and PBH $)$

at PTA / interferometers scales (and PBH) \sim PTA \sim PTA \sim PTA \sim PTA \sim Domcke, Pieroni, Binétruy '16; ...

Results in the weak backreaction regime $\overline{ }$ 3*M*² $\overline{1}$ 1 red 3*M*² *p H*² = 3*M*² *p* 2 *Pesults in the weak backreact* ts in t ˙ + *V* + $\mathcal{C}^{\mathcal{C}}$ (electromgnetic notation) *cA* ¨+ 3*H*˙ ⁺ *dV* Results in the weak backreaction \mathbf{E} (e.g. \mathbf{E} notation)

$$
\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{c_A}{f}\vec{E} \cdot \vec{B}
$$
\n= negligible backreaction
\non background dynam
\n
$$
H^2 = \frac{1}{3M_p^2} \left[\frac{1}{2} \dot{\phi} + V + \frac{\vec{E}^2 + \vec{B}^2}{2} \right]
$$
\n(electromometric notation)

 \equiv negligible backreaction terms on background dynamics ⌘ negligible backreaction terms ⌘ negligible backreaction terms on background dynamics on background dynamics (electromgnetic notation) (electromgnetic notation) $-\frac{dV}{d\phi} = \frac{c_A}{f} \vec{E} \cdot \vec{B}$ \equiv negligible backreaction terms $\left[\frac{1}{\phi} + V + \frac{\vec{E}^2 + \vec{B}^2}{\phi} \right]$ (electromanetic notation) *d* ل بال
موجد عبد on background dynamics 3*M*² *p* 2 *in secrigionia agricunos*
(electromanetic notation) ¨+ 3*H*˙ ⁺ *dV* in this regime based on several approximations: constant ⇠ and *H*, specific

- ² $\propto \left\langle \delta \phi^2 \right\rangle$ and $\langle \delta \phi^2 \rangle$ and bi $\sqrt{g(2)}$ and $\sqrt{g(2)}$ • Analytic results for power spectrum $P \propto \left\langle \delta \phi^2 \right\rangle$ and bispectrum $B \propto \left\langle \delta \phi^3 \right\rangle$ in this regime based on several approximations: constant ζ and *H*, specific UV regularization Barnab in this regime based on several approximations: constant ξ and H , specific rization Rarnahy • Analytic results for power spectrum $P \propto \left\langle \delta \phi^2 \right\rangle$ and bispectrum in this regnite based on several approximations. constant ζ and \therefore constant ξ and H spec Barnaby, MP '10 *H* bispectrum $B \propto$ $\langle \delta \phi^3 \rangle$ $B = \sqrt{3/2}$ and bionest on background dynamics (16) and bionootrups $D_{\text{sc}}/s/3$ on background dynamics $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ $\propto \langle \rho \phi^2 \rangle$ and bispec R mations: constant ξ and H , specifier ¨+ 3*H*˙ ⁺ *dV d* $B \propto \langle \delta$ *H*² = $3 \propto \big\langle \alpha$ veral approximations: constant ξ and H, sp ˙ + *V* + $2\sqrt{3}$ and bionactrum $P_{\alpha\beta}/(843)$ ¨+ 3*H*˙ ⁺ *dV* \mathbf{u} _{\mathbf{u}} \mathbf{v} proximations. Barnaby, MP '10
	- ce sin • Excellent agreement with full lattice simulations **Excellent agreement with full lattice simulations**

• Analytic results in this regime based on several approximations: Caravano, Komatsu, rttice Simulations

Lozanov, Weller '22 constant Caravano, Komatsu,
Figures 1,07319V, Weller, 132 ons background, Kome *H*² = **• Excellent agreement with full lattice simulations** Caravano, Komatsu,

 \sim Sorborg were. *•* Dissipation reduces the inflaton motion. Can allow inflation in **b** a calcon containecture in the solid and a backreaction regime: the Strong backreaction regime: the Anber a *•* Old idea of warm inflation Strong backreaction regime: the Anber and Sorbo solution

- Strong backreaction regime studied by Anber, Sorbo '09 can inflate also if *V* is steep MP, Sorbo '22 *•* With dissipation MP, Sorbo '22
- Old idea of warr $\overline{ }$ $\overline{$ can inflate also if *V* is steep ea of warm initiation $\varphi + 3H\varphi + V = -\Gamma \varphi$ Berera '95 Dissipation reduces the inflaton motion. Can allow inflation in steep potentials (large V') and for reduced field excursion (small $\Delta \phi$) Both aspects might be beneficial in the recent swampland program Both aspects might be beneficial in the recent swampland program Both aspects might be beneficial in the recent swampland program • Old idea of warm inflation $\ddot{\phi} + 3H\dot{\phi} + V' = -\Gamma \dot{\phi}$ Berera '95 Both aspects might be beneficial in the recent swampland program *•* With dissipation ld idea of warm inflat $-3H\dot{\phi} + V' = -\Gamma \dot{\phi}$ F can inflate also if *V* is steep Berera '95 steep potentials (large *V* ⁰) and for reduced field excursion (small). $\frac{1}{2}$ warm inflation $\phi + 3$ Berera '95 Both aspects might be beneficial in the recent swam \overline{O} ld idea of warm inflation $\ddot{a} + 3H\dot{a} + V' = \overline{C}\dot{a}$ Berera '05 *•* Dissipation reduces the inflaton motion. Can allow inflation in Both aspects might be beneficial in the recent swampland program
- **Sorbo mechanism simple & we** can hand the *nechanism* sin *•* Anber-Sorbo mechanism simple & well defined QFT realization *•* Anber-Sorbo mechanism simple & well defined QFT realization *•* Anber-Sorbo mechanism simple & well defined QFT realization **bo mechanism simple & well defined QFT realization** • Anber-Sorbo mechanism simple & well defined QF

Could help with string th. arguments for *<...* and *V* ⁰ *>...*

$$
\ddot{\phi} + 3H\dot{\phi} + V' = \frac{c_A}{f} F \tilde{F} \left[\dot{\phi} \right]
$$

Perfect balance assumed at all times \rightarrow Steady state evolution

$$
\Rightarrow V' \simeq -2.4 \cdot 10^{-4} \frac{c_A H^4}{f} \frac{e^{2\pi\xi}}{\xi^4} , \qquad \xi \equiv \frac{c_A \dot{\phi}}{2fH}
$$

• Oscillatory behaviour from simplified numerical solutions of the system Jiul simplified numerical solutions of the system

(set of gauge modes + homogeneous inflaton) Cheng, Lee, Ng['] $\overline{}$

 N_{otheri} , $T_{\text{systemi}}/16$; Dall'Agata, González-Martín, *•* Oscillatory Cheng, Lee, Ng '15; Notari, Type in the top of Cheng, Lee, Ng '15; L Dall'Agata, González-Martín, Dan Agata, Gonzalez-Martin, Papageorgiou, MP '19; Notari, Typywoniuk in simplified (homogeneous) numerical solu- $\sum_{i=1}^{n}$ Cheng, Lee, Ng '15; Notari, Tywoniuk '16; *•* Oscillatory behaviour from simplified (homogeneous) numerical solu-⌦ ↵ *,* ⌦ D an M gata, Gonzalez-Martin, P **Gorbar, Schmitz,** u gioc /ilchinskii '21 *Ea*n, rywoniak 10,
Il'Aaata González-Martín Gorbar, Schmitz, Sobol, Vilchinskii '21

Sobol, Vilchinskii '21

• Confirmed by full lattice simulation $\phi(t, \vec{x})$, $A^{\mu}(t, \vec{x})$

guanta are produced and the moment they backread number of e-folds *Ne*, in the case of negligible backreaction number of negligible *N*

20

Nomcke, Guidetti, Welling, Westphal '20 • Interpreted as delayed effect between the moment the gauge quanta are produced and the moment they backreact on (*t*). Interpreted as delayed effect between the moment the gauge quanta are produced and the moment they backreact on $\phi(t)$.

Analytical study:
$$
\phi(t) = \overline{\phi}(t) + \delta\phi(t)
$$
, $A^{\mu}(t, \vec{k}) = \overline{A}^{\mu}(t, \vec{k}) + \delta A^{\mu}(t, \vec{k})$
of the homogeneous inflaton & gauge modes around the AS solution
MP, Sorbo '22

$$
\delta\phi'' + 2aH\delta\phi' + a^2V''\delta\phi = -\frac{\alpha}{fa^2} \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \frac{\partial}{\partial \tau} \left[\bar{A}\delta A^* + \bar{A}^*\delta A \right]
$$

$$
\delta A'' + \left(k^2 - \frac{k\bar{\phi}'}{f} \right) \delta A = \frac{\alpha \bar{A}}{f} \delta\phi'
$$

• Formally solve 2nd eq for δA as a function of $\delta A'$ • Formally solve 2nd eq for δA as a function of $\delta \phi'$

$$
\delta A(\tau, k) = \frac{\alpha k}{f} \int^{\tau} d\tau' G_k(\tau, \tau') \, \bar{A}(\tau', k) \, \delta \phi'(\tau')
$$

• Insert solution in 1st eq $\;\rightarrow$ integro-differential eq for $\delta\phi$ *•* Insert solution in 1st eq ! integro-di↵erential eq for

 $\delta \phi''\left(\tau\right)+2aH\delta \phi'\left(\tau\right)+a^2V''\delta \phi\left(\tau\right)\simeq$ *•* Look for / (⌧)1+⇣

$$
\frac{\alpha^2}{f^2 a^2} \frac{e^{2\pi\xi}}{2^8 \pi^2 \xi^5} \int^{\tau} \frac{d\tau'}{(-\tau')^4} \delta \phi'(\tau') \frac{\partial}{\partial \tau} \int_0^{4\xi_{\gamma}^2} dy \, y^3 \sqrt{\tau \tau'} \left[e^{-4\sqrt{y}} - e^{-4\sqrt{y}} \sqrt{\frac{-\tau}{-\tau'}} \right]
$$

(⌧) + *a*2*V* ⁰⁰ (⌧) '

 $\overline{1}$

 $\xi_{\gamma} \equiv \xi \gamma$

 \mathcal{L} and the first scale as \mathcal{L}

² ln (*H*⌧) +

• Look for $\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\text{Re}\beta} \cos(\text{Im}\,\beta \times N + \text{phase})$ $\delta \phi \propto \left(-\tau \right)^{-\beta} \equiv a^{\textsf{Re}\,\beta} \, \cos \left(\textsf{Im}\, \beta \times N + \textsf{phase} \right) \, ,$

Inserting this and doing the integrals \rightarrow homogeneous eq in time (all terms $\overline{1}$ scale as $\tau^{-\beta-2}$). Therefore left wih an algebraic equation for complex $\beta.$

Gradient Expansion Formalism 55.5 Gradient expansion for \sim are and the way to the gradient-expansion in position and *Expansion Formalism Sobol, Gorbar, Vilchinskii '19* Gradient Expansion F **Sandient Expansion Formalism**

Rat

• ∧ $\vec{A}(\vec{k})$ e.o.m., tower of eqs. for $\frac{1}{2}$ • Rater than $\vec{A}(\vec{k})$ e.o.m., tower of eqs. for Durrer, Sobol, Vilchinskii '23 $\vec{A}(\vec{k})$ e a m \vec{A} rates for \vec{k} tower of eqs. for **Example 1** • Rater than $A(k)$ e.o.m., tower of eqs. for

Durrer, Sobol, Vilchinskii '23

² $\frac{23}{2}$ Gorbar, Schmitz, Sobol, Vilchinskii '21 Gorbar, Schmitz, Sobol, Vilchinskii '21 Formalism Sobol, Gorbar, Vilchinskii '19 Durrer, Sobol, Vilchinskii '23 malism sopor, Gorbar, Vilchinskii 19
Gorbar, Schmitz, Sobol, Vilchinskii '21 *a***l**, Vilchinskii '23 *, B*(*m*) *, G*(*m*) $\frac{1}{2}$ ~

$$
\mathcal{E}^{(n)} = \frac{1}{a^n} \left\langle \mathbf{E} \cdot \text{rot}^n \mathbf{E} \right\rangle \qquad \mathcal{B}^{(n)} = \frac{1}{a^n} \left\langle \mathbf{B} \cdot \text{rot}^n \mathbf{B} \right\rangle \qquad \mathcal{G}^{(n)} = -\frac{1}{2a^n} \left\langle \mathbf{E} \cdot \text{rot}^n \mathbf{B} + \text{rot}^n \mathbf{B} \cdot \mathbf{E} \right\rangle
$$

conveniently integrated numerically

 $\overline{\text{convenient}}$ integra ~ ~ *k* **CONVeniently integrated numerically** MP Schmitz S t the linear perturbation theory breaks down (relation of \mathbb{R}^n from its initial value becomes down its initial value becomes down (relation of \mathbb{R}^n from its initial value becomes down its initial value become vs. analytic solution (MP, Sorbo '22) and the solution (MP, Sorbo '22) and (MP, Sorbo '22) and (MP, Sorbo '22)

- correlators initialized according to AS solution von Eckardstein *a*¹ *a s rotomatics*** ***a c s n s n s n s n s n s n s n s n s n s n s n n <i>n n n <i>n n* • correlators initialized according to AS solution
- 1) Analytic linear system 2) Fu
 $f_{\alpha r}$ sinsc(m) sp(m) sp(m) (e) $\delta \phi$, $\delta \mathcal{E}^{(m)}$, $\delta \mathcal{B}^{(m)}$, $\delta \mathcal{G}^{(m)}$ ∂ \mathcal{L}^{\times} , ∂ \mathcal{D}^{\times} , ∂ \mathcal{G}^{\times} *• •* correlators initial cyclotic according to $\mathcal{P}(m)$ solution correlation of $\mathcal{P}(m)$ (oxisting ones from $f(x)$) for $\delta\dot{\phi}, \delta\mathcal{E}^{(m)}, \delta\mathcal{B}^{(m)}, \delta\mathcal{G}^{(m)}$ (existing one)

z) Funy numental solution start
existing ones from weak back 2) Fully numerical solution starting from AS
(existing ones from weak backreaction) [~] *·* ^r MP, Schmitz, Sobol, Sorbo, MP, Schmitz, Sobol, Sorbo, –5–

[~] *·* ^r (existing ones from weak backreaction)

Burts of GW production Burts of GW production Burts of GW production Burts of GW production Burts of GVV production Garcia-Bellido, Papageorgiou, MP, Sorbo '23

 $\frac{1}{2}$

• Expect gauge field amplification and related phenomenology enhanced at scales O (*H*) when ˙ is maximum ! Recurrent peaks in power spectra *•* Expect gauge field amplification and related *•* Expect gauge field amplification and related phenomenology enhanced at scales O (*H*) when phenomenology enhanced at scales O (*H*) when ϕ is maximum \rightarrow Recurrent peaks in power sp $\dot{\phi}$ is maximum \rightarrow Recurrent peaks in power spectra Burts of GW production iomenology enhanced at scales $\mathsf{O}\left(H\right)$ when Garcia-Bellido, Papageorgiou, MP, Sorbo '23 • Expect gauge field amplification and related φ is maximum \rightarrow Recurrent peaks in power spectra • Expect gauge field amplification and re phenomenology enhanced at scales O (*H*) when **•** Experience gauge field and related and priorioriono,

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- GW easier than $\delta \rho$. Might have correlated peaks in same or across GW easier than $\delta \rho$. Might have correlat • GW easier than $\delta \rho$. Might have correlated peaks in same or acr easier than *op*. Might have correlated per different GW experiments: $\frac{10^{-3}}{\sqrt{2}}$ Expect gauge field and related and relativities amerent G anierent GW experiments.

Pulsar Timing Arrays, Pulsar Timing Arrays, Association of the Sensitivities $\frac{1}{2}$ GW experiments. amerent GW experim phenomenology enhanced at $\frac{1}{2}$ $\frac{1}{2}$ when $\frac{1$ phenomenology enhanced at $\frac{1}{2}$ when $-ET$ $\begin{array}{ccc} \textbf{p} & \textbf{p} & \textbf{p} & \textbf{p} & \textbf{p} \\ \hline \textbf{p} & \textbf{p} & \textbf{p} & \textbf{p} & \textbf{p} & \textbf{p} \\ \end{array}$ $\left| \begin{array}{c|c} \hline \end{array} \right|$ when $\left| \begin{array}{c|c} \hline \end{array}$ Pulsar Timing Arrays $\frac{10^{10}}{\frac{2}{5}}$ $\frac{1}{10^{19}}$ Pulsar Timing Arrays, $\sum_{\substack{a=10^{-9} \text{ g} \text{ is a}}}$
Astrometry $\sum_{a=10^{-12} \text{ g} \text{ is a}}^{\infty}$ $\begin{array}{c|c|c|c} \hline \text{10} & \text{10} & \text{10} \\ \hline \text{10} & \text{10} & \text{10} \\ \hline \text{NANOCBAV} & \text{10} & \text{NANOCBAV} \end{array}$ *•* GW easier than ⇢. Might have correlated peaks in same or across \mathcal{L} $\mathcal{$ *•* GW easier than ⇢. Might have correlated peaks in same or across *•* GW easier than ⇢. Might have correlated peaks in same or across Interfrometers $\frac{1}{10^{-10}}$ di↵erent GW experiments: $\overline{}$ LISA $P_{\text{p}} = \frac{10^{-9}}{10^{-6}}$ and $P_{\text{p}} = \frac{10^{-3}}{10^{-3}}$ and $P_{\text{p}} = \frac{10^{3}}{10^{-3}}$ and $P_{\text{p}} = \frac{10^{3}}{10^{-3}}$ $-$ HLVK \mathcal{F} [Hz] f [Hz] f [Hz]
	- Usual analytic approximations invalid as ξ varies too quickly as individual modes are being produced. Numerical code, with $\phi \left(t \right)$ and 400 gauge modes *A* (*t, ki*), covering the dynamical range of 60 e-folds of inflation Garcia-Bellido, Papageorgiou, MP, Sorbo '23 modes *A* (*t, ki*), covering the dynamical range of 60 e-folds of inflation *•* Usual analytic approximations invalid as ⇠ varies too quickly as individual *•* Usual analytic approximations invalid as ⇠ varies too quickly as individual modes are being produced. Numerical code, with (*t*) and 400 gauge modes are being produced. Numerical code, with (*t*) and 400 gauge modes *A* (*t, ki*), covering the dynamical range of 60 e-folds of inflation modes *A* (*t, ki*), covering the dynamical range of 60 e-folds of inflation

Physical ρ in one mode

$$
A'' + \left(k^2 - 2\xi aHk\right)A = 0
$$

Unstable growth for $k < k_{\text{thr}} \equiv 2\xi a(t) H(t)$ *^k*² ²⇠ *aHk* Code integrates modes leaving the horizon all throughout inflation, $\begin{bmatrix} \mathbb{R}^d & \star \end{bmatrix}$ Unstable growth for $k < k_{\text{thr}} \equiv 2 \varepsilon a(t) H(t)$ \bigstar integrates modes leaving the horizon all throughout inflation, \mathcal{L} *aH k*thr 300 *k*thr 1

 $\overrightarrow{ }$ Decreased by resdshift after \sim 7 e-folds **★** Decreased by resdshift after ~7 e-fol \sim 7 e-folds *k* 7 e-folds

 ϵ α de intervented α *x* α declines the α $\frac{1}{2}$ $\mathcal{L} = \mathcal{L}$ integrates modes in the horizon all throughout inflation, $\mathcal{L} = \mathcal{L}$ $\left[0.1\right]$ followed while dynamically relevant *cA* Code integrates modes leaving the horizon all throughout inflation, $k_{\sf max}/k_{\sf min} \simeq e^{60} \simeq 10^{26}$. Each mode $\begin{bmatrix} 1 & 1 \end{bmatrix}$ *k*modes reaving the second incomes in the second model in α **followed which which develops a** function, tes *r f V* ⁰ ⌘ $\overline{\mathbf{C}}$ *e*s leaving th $\sqrt{1-\frac{1}{2}}$ $\sqrt{1-\frac{1}{2$ included in *B f V* ⁰ ⌘

$$
\ddot{\phi} + 3H\dot{\phi} + V' = \frac{c_A}{f}\vec{E} \cdot \vec{B}
$$

$$
\longrightarrow \frac{c_A \vec{E} \cdot \vec{B}}{f V'} \equiv \int d\ln k \mathcal{B}(N, k)
$$

Garcia-Bellido, Papageorgiou, MP, Sorbo '23

<u>Inhomog</u> *•* Very recent lattice simulation Figueroa, Lizarraga, Urio Urrestilla '23 Inhomogeneous Dackreaction ennomogeneo Inhomogeneous backreaction

 \mathbf{r} T. \mathbf{v} with greater dynamical range than Caravano et al '22 Inhomogeneous inflaton • Very recent lattice simulation Figueroa, Lizarraga, Urio Urrest • Very recent lattice simulation Figueroa, Lizarraga, Urio Urrestilla '23 with greater dynamical range than Caravano et al '22 with greater dynamical range than Carravano et al '22

- ϵ Oscillations of ϵ roduced after the first one • Oscillations of ξ reduced after the first one Backreacion, homogeneous
- End of inflation simulated, oscillations reduced also in homogeneous case
- MB scales $\rightarrow f/c_A > 10^{16}$ GeV $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ • Scalar non-G once also extra density perturbations are accounted for $>10^{16}$ 0*.*01⌘¹*/*⁴ 0*.*01⌘¹*/*⁴ • Scalar non-G @ CMB scales $\rightarrow f/c_A \gtrsim$ \sim • Scalar non-G @ CMB scales $\rightarrow f/c_A \gtrsim 10^{16}$ GeV \gtrsim 10^{16} GeV
	- $(1/M_p \text{ VS } c_A/f)$ **Barnaby, M** $\left(\frac{-1}{p} - p \right)$ is $\left(\frac{p}{p}\right)$ *^f ^F ^F*˜ \c{ced} $(1/M_p$ vs $c_A/f)$ h *•* Scalar non-G @ CMB scales ! *f /*↵ *>* ⇣ *r •* GW less produced (1*/Mp* vs *cA/f*)

 \downarrow

 $\frac{1}{\sqrt{2}}$ Barnaby, MP '10 0*.*01⌘¹*/*²

 S everal mechanisms for a distributional S

 E eg \vert me, negligible backreaction on background dynamics MP, Sorbo, Unal '16 $\ddot{}$ a a *F*
erturbative control; confirmed by lattice *L* = **ground dynamics** MP, Sor
Caravan amics MP, Sorbo, Unal '16 **h this regime, negligible backreaction on background dynamics** MP, Sorbo, Unal '16 vancy
Such the source of the source nder perturbative control; confirmed by lattice **results** and *Lozar* **Caravano, Komat** Lozanov, Weller In this reg me, negligible backreaction on background dynamics Once density perturbation accounted for, *r* typically decreases Under perturbative control; confirmed by lattice \downarrow *•* Scalar non-G @ CMB scales ! *f /*↵ *>* es beeldes In this regime, negligible backreaction on background dynamic $\frac{1}{2}$ Caravano, Komatsu, Lozanov, Weller '22 und dynamics MP, Sorbo, Unal '16 *•* Lozanov, Weller '22
• decrease of <i>r ackreaction on backgro 0*.*01⌘¹*/*² Are we sure the surface of the surface of

ral lesson: Several mechanisms for additional GW, result in a decrease of r once also extra density perturbations are accounted for **onal GW, result** tra donsity porturbations are accounted General lesson: Several mechanisms for additional GW, result in a decrease of r accounted for General lesson: Several mechanisms for additional GVV, result in a decreanulary
once also extra density perturbations are accounted for once also extra density perturbations are accounted for smaller *^r*1*/*² ⇠ *^h*vacuum ⁺ *^h*sourced smaller *^r*1*/*² ⇠ *^h*vacuum ⁺ *^h*sourced Several mechanisms for additional GW, result in a decrease of *r* General lesson: Several mechanisms for additional GW, result in a decrease of r once also extra density perturbations are accounted for

General issue: how to increase *h*sourced more than sourced ? \mathbf{B} How robust? Cost for evading it? by energy density) **•** No direct coupling with inflaton (Sourced gravitation) *•* Relativistic source (GW are produced by quadrupole moment; ⇣ by energy density) How robust ? Cost for evading it ? hoping for *r*tot *> r*vacuum hoping for *r*tot *> r*vacuum

- No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton) *•* No direct coupling with inflaton $\frac{1}{2}$ is the coupling $\frac{1}{2}$ in the coupling $\frac{1}{2}$ • No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton) Once density perturbation accounted for, *r* typically decreases aton (Sour ce gravitatic both GW and inflaton) *P* No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton) Once density perturbation accounted for, *r* typically decreases on to an occlocaping with imaton (Soarce coupled to both GW and inflaton)
- Relativistic source (GW are produced by quadrupole moment; ζ by energy density) suc source (GW are produced by quadrupole mom ivistic source (GW are pro lativistic source (GW are produced by quadrupole moment; ζ by energy density) $\overline{}$ *v r c* (GW a isity*)*
Energy density **E** (GW are produced by quadrupole moment; ζ by energy density)
- e active only for limited time (GW observed only on a small window: · Source active only for limited time (GW observed only on a small window; • Source active only for limited time (GW observed only on a small window;
 C provides constrains on many more scales) $VIIIUOW,$ nly on a small window;

 ζ provides constrains on many more scales) provides constrains on many more scales) ⇣ provides constrains on many more scales) ⇣ provides constrains on many more scales) *•* No direct coupling with inflaton (Source gravitationally coupled $\mathcal{P}(\mathcal{P})$ is the coupling with inflaton (Source gravitationally coupled $\mathcal{P}(\mathcal{P})$ ζ provided density) many more scales) *•* \blacksquare \mathbf{B} \mathcal{F} source \mathcal{F} *Pressures* o many more scale

(@')² *^V* (') ¹ These 3 ingredients present in (@)² *^U*() ¹ *^F*² Vamba, MP, Shiraishi, Sorbo, Unal '15 *FF*˜ ront in \sim MD *f* .
iroichi Namba, MP, Shiraishi, Sorbo, Unal '15 These 3 ingredients present in Simplest *V* for a pseudoscalar: Simplest *V* for a pseudoscalar:

Axion rolls for $\Delta N=\frac{3H^2}{\epsilon}=$ 0 (1) e – observable inflation. As it rolls to the minimum of *^U*, ! *^A*[~] ! GW *,* ⇣ observable inflation. As it rolls to the minimum of *^U*, ! *^A*[~] ! GW *,* ⇣ *V* () = $\frac{\epsilon_{11}}{m^2} = O(1) e -$ 2 $3H^2$ – 0.1 e *–* fol folds of inflation Axion rolls for $\Delta N = \frac{3H}{m^2} = O(1) e$ – folds of inflation 3*H*² on rolls for $\Delta N = \frac{3H}{m^2} = O(1) e$ – folds of inflation \overline{a} ¹ $=$ O (1) e $-$ folds of inflation

- \bullet Gives visible r at small r_{vacuum} / • Gives visible r at small r_{vacuum} / scale of inflation 6*H*2*f*² **•** Gives visible *r* at small r_{vacuum} / scale of inflation
- **•** Relevant dynamic \overline{a} covered by *f* **Relevant dynamics covered by t** \mathbf{I} s
Santa Contara *f* **Example 10 Relevant dynamics covered by the lattice** ⇤⁴ *m*² *•* Relevant dynamics covered by the lattice *•* Relevant dynamics covered by the lattice

 $\frac{1}{\sqrt{2}}$ backreached, $\frac{1}{\sqrt{2}}$ is $\frac{1}{\sqrt{2}}$ Caravano, Caravano, MP '24

Testing a chiral SGWB temperature/polarization pixel-noise variances, σT /P , as Testing a chiral SGWB

• © CMB scales (TB & EB correlations)

 \int Gluscevic and Kamionkowski '10

Measurement of GW polarization at LISA / ET $M_{\rm E}$ cosmological polarization and the cosmological dipole in ϵ

 $r_{\rm e}$ response in a planar detector. Cannot detect net circular. polarization of an isotropic SGWB Two GWs related by a mirror symmetry produce the same response in a planar detector. Cannot detect net circular Circular Polarization and the Cosmic Dipole Polarization and the Cosmic Dipole Cosmic Dipole Cosmic Dipole Cos Measurement of GW polarization at LISA Two Gws relationship and the state of the state the sympath produce the sympath sympath sympath sympath sympath response in a planar detector. Can planar detector. Can in a planar detector. Can plan a plan detector. Can pl
Planar detector. Cannot detector de transponse in a planar de transponse in a planar de transponse in a planar

the succession, $\frac{1}{2}$ potential increased on $\frac{1}{2}$ **b** $\frac{1}{2}$ \vec{v}_d Isotropy in any case broken by peculiar motion of Isotropy in any case broken by peculiar motion of the solar system. Assumption, $v_d \simeq 10^{-3}$ as CMB ISOtropy in any case broken by peculiar m the solar system. Assumption, $v_d \simeq 10^{-3}$ Isotropy in any case broken by peculiar motion of Isotropy in any case broken by peculiar motion of the solar system. Assumption, $v \sim 10^{-3}$ as CMB

$$
\text{SNR}_{\text{LISA}} \simeq \frac{v_d}{10^{-3}} \frac{\Omega_{\text{GW,R}} - \Omega_{\text{GW,L}}}{1.2 \cdot 10^{-11}} \sqrt{\frac{T}{3 \text{ years}}}
$$

⌦GW*,*^R ⌦GW*,*^L ·ke, García-Bellido, MP,
Irdone, Sorbo, Tasinato Domcke, Garcia-Bellido, MP, Pieroni
Ricciardone, Sorbo, Tasinato '19 Domcke, García-Bellido, MP, Pieroni Demoke Carcía Bellide MD ¹*.*² *·* ¹⁰¹¹ ^r *^T*

(one order of magnitude greater than estimate in Seto '06) (one order of magnitude greater than estimate in Seto '06) (one order of magnitude greater than estimate in Seto '06) (to '06)

- σ and the Civid dipole confider ϵ $\sum_{n=1}^{\infty}$ order of $\sum_{n=1}^{\infty}$ order than $\sum_{n=1}^{\infty}$ order than estimate $\sum_{n=1}^{\infty}$ *•* Do the GW and the CMB dipole coincide ? *•* Do the GW and the CMB dipole coincide ?
- One order of magnitude improven *•* One order of magnitude improvement with LISA-Taiji *•* One order of magnitude improvement with LISA-Taiji *•* One order of magnitude improvement with LISA-Taiji

Orlando, Pieroni, Ricciardone '20

Measurement at ground-based interferometers

Conclusions

- *•* Potentially detectable GW at several scales from axion inflation \bullet Dotantially entially detect
- New results in the regime of strong backreaction *•* New results in the regime of strong backreaction *•* New results in the regime of strong backreaction

 \mathbf{F} study of associated signatures signatures of associated signatures \mathbf{F} Instability of steady state solution Instability of steady state solution

 First ctudies of associated signatures First studies of associated signatures First study of associated signatures

• Awaiting increased dynamical range from lattice