Testing GR with GWs: challenges and perspectives

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erc

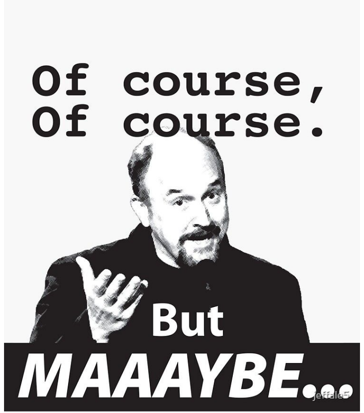
1. Testing GR

Confrontation of GR with experiments

C. Will, *Living Review* 1403.7377

Chandrasekhar to C. Will: "Why do you want to test GR? We know that GR is right"

E. Berti



"Why test GR with LVK? Let's wait for XG detectors. Let's do populations"

"Testing GR with LVK creates the expertise for XG"

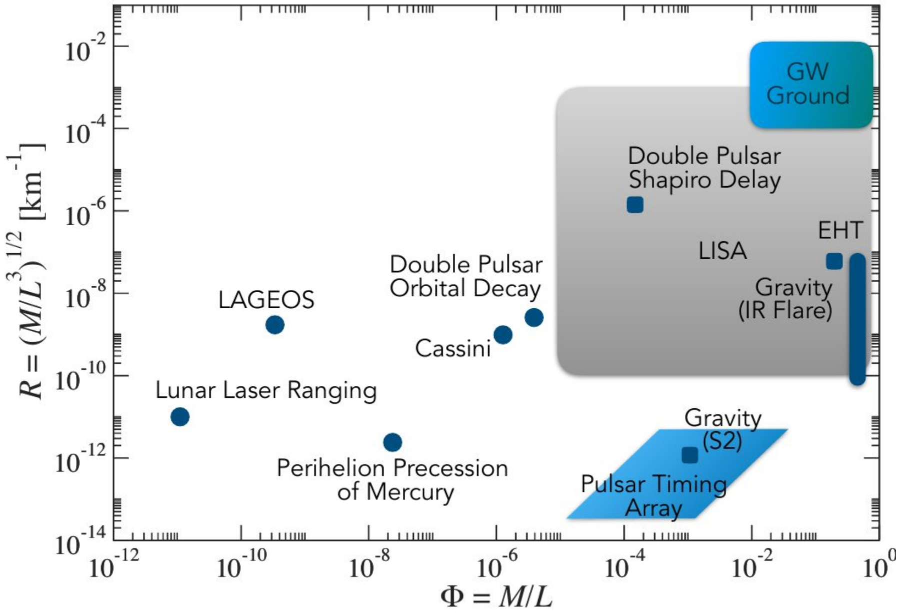
"I don't believe in combining events. We must wait for smoking guns"

"Only comparing GR with alternative theories is meaningful. Work out b-GR waveforms!"

"Beyond-GR theories are too many. Only null tests with my money!"

"GR is only wrong at Planck scales: unprobed by event-horizon physics"

Probing strong gravity with gravitational waves



GW detectors probe gravity in systems with:

Strong **gravitational potential**

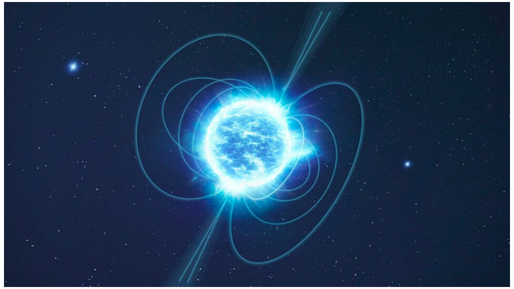
Strong **curvature**

The Riemann curvature at the event horizon of a Schwarzschild metric scales as:

$$\sqrt{R^{abcd}R_{abcd}} \sim M^{-2}$$

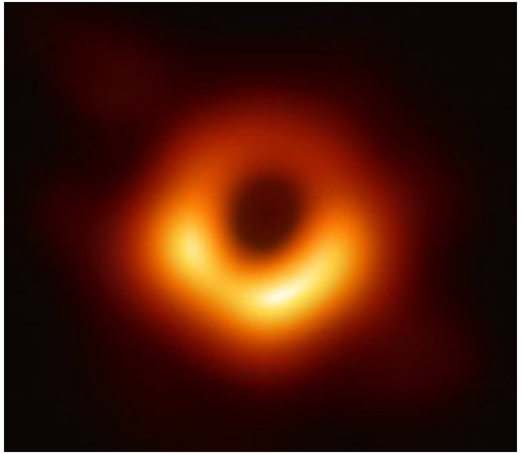
Baker+ (2014), [1412.3455](1412.3455)
EXTREME GRAVITY AND FUNDAMENTAL PHYSICS, [1903.09221](1903.09221)

Fundamental predictions of GR (+SM)



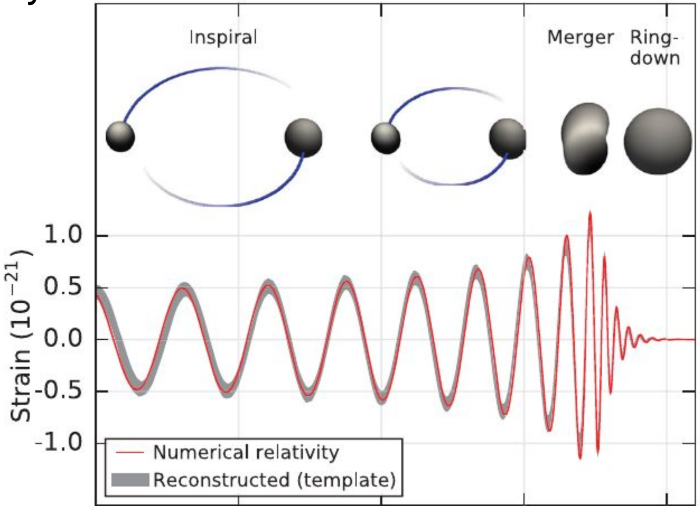
Uniqueness Theorems:

BHs are **Kerr BHs**, uniquely described by their mass and spin



Compact objects:

BH's, NS's and collapsing stars are the standard compact objects observable by ET



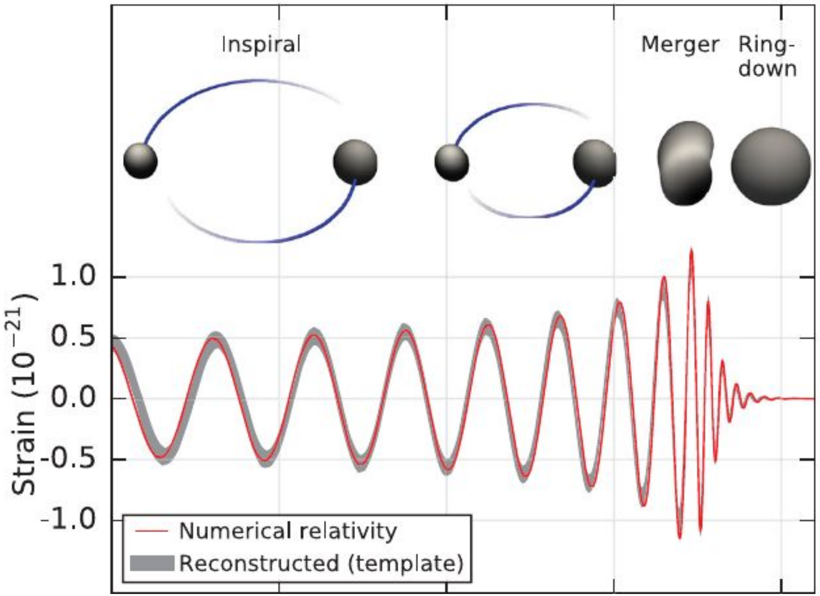
Gravitational waves:

Propagation of **nonlinear** gravity waves

2 polarizations

Binary black holes and tests of GR

GW150914 discovery paper



Tests of GR with LVK event catalogs:

- LKV GWTC-2 (2020), 2010.14529
- LVK GWTC-3 (2021), 2112.06861

Ghosh (2022), 2204.00662

Test	Section	Quantity
RT	4.1	p-value
IMR	4.1	Fractional deviation in remnant mass and spin
PAR	4.2	PN deformation parameter
SIM	4.2	Deformation in spin-induced multipole parameter
MDR	4.3	Magnitude of dispersion, $\|A_\alpha\|$
POL	4.4	Bayes Factors between different polarization hypotheses
RD	4.5	Fractional deviations in frequency (PYRING)
		Fractional deviations in frequency and damping time (PSEOB)
ECH	4.5	Signal-to-noise Bayes Factor

Null tests

Limited by challenges in modelling strong-field dynamics in beyond GR theories

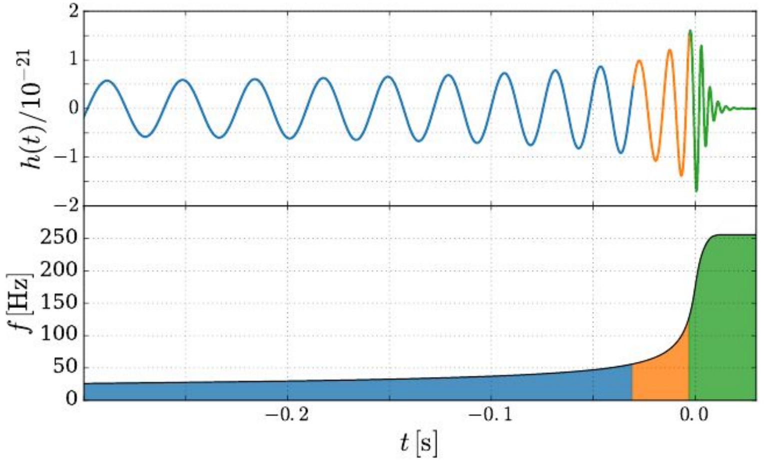
Introduce ad-hoc deviation parameters within waveform models such that

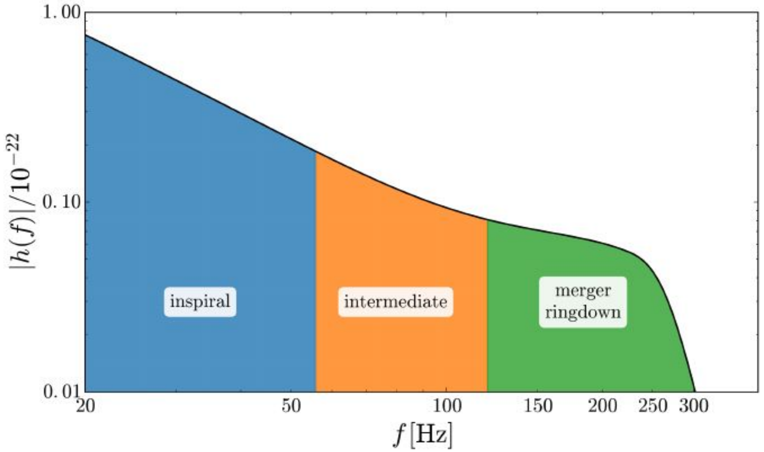
$$\text{GR} : x = 0$$

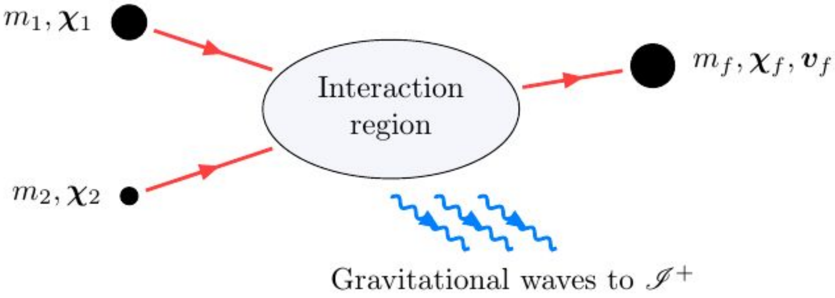
Yunes++ (2016) 1603.08955

Theoretical Effect	Theoretical Mechanism	Theories	ppE b	Order	Mapping
Scalar Dipolar Radiation	Scalar Monopole Activation	EdGB [143, 145, 152, 153]	-7	-1PN	β_{EdGB} [143]
	BH Hair Growth	Scalar-Tensor Theories [60, 154]	-7	-1PN	β_{ST} [60, 154]
Anomalous Acceleration	Extra Dim. Mass Leakage	RS-II Braneworld [155, 156]	-13	-4PN	β_{ED} [144]
	Time-Variation of G	Phenomenological [140, 157]	-13	-4PN	$\beta_{\dot{G}}$ [140]
Scalar Quadrupolar Radiation Scalar Dipole Force Quadrupole Moment Deformation	Scalar Dipole Activation due to Grav. Parity Violation	dCS [143, 158]	-1	$+2$PN	β_{dCS} [149]
Scalar/Vector Dipolar Radiation Modified Quadrupolar Radiation	Vector Field Activation due to Lorentz Violation	EA [111, 112], Khronometric [113, 114]	-7 -5	-1PN 0PN	$\beta_{Æ}^{(-1)}$, $\beta_{\text{KG}}^{(-1)}$ [115] $\beta_{Æ}^{(0)}$, $\beta_{\text{KG}}^{(0)}$ [115]
Modified Dispersion Relation	GW Propagation	Massive Gravity [159–162]	-3	$+1$PN	β_{MDR} [148, 159]
		Double Special Relativity [163–166]	$+6$	$+5.5$PN	
		Extra Dim. [167], Horava-Lifshitz [168–170]	$+9$	$+7$PN	
		gravitational SME ($d = 4$) [82]	$+3$	$+4$PN	
		gravitational SME ($d = 5$) [82]	$+6$	$+5.5$PN	
		gravitational SME ($d = 6$) [82]	$+9$	$+7$PN	
		Multifractional Spacetime [171–173]	3–6	4–5.5PN	

IMR consistency test



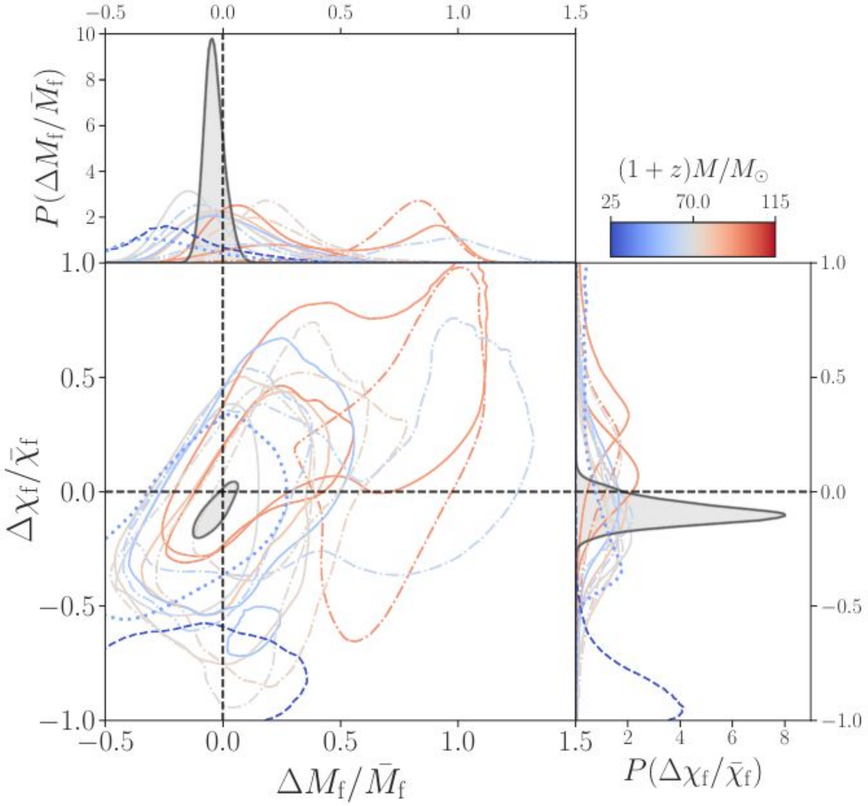




GR is a deterministic theory: consistency of final mass and spin are determined by the progenitors

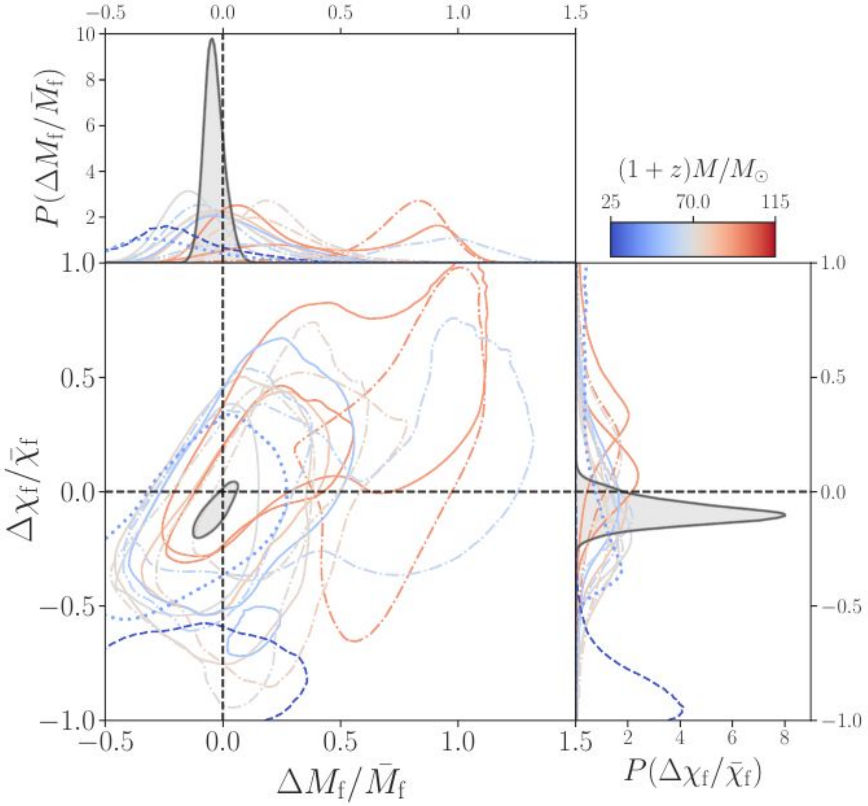
Ghosh++ (2017), 1704.06784

LVK results for IMR



Event	f_c^{IMR} [Hz]	ρ_{IMR}	ρ_{insp}	$\rho_{postinsp}$	Q_{GR} [%]
GW150914	132	25.3	19.4	16.1	55.7
GW170104	143	13.7	10.9	8.5	29.0
GW170809	136	12.7	10.6	7.1	26.6
GW170814	161	16.8	15.3	7.2	22.9
GW170818	128	12.0	9.3	7.2	26.8
GW170823	102	11.9	7.9	8.5	93.3
GW190408_181802	164	15.0	13.6	6.4	11.4
GW190412	213	19.1	18.2	5.9	69.0†
GW190421_213856	82	10.4	8.1	6.6	78.7*
GW190503_185404	99	13.7	11.5	7.5	53.2
GW190513_205428	125	13.3	11.2	7.2	35.0
GW190519_153544	78	15.0	10.0	11.2	85.6*
GW190521_074359	105	25.4	23.4	9.9	0.0
GW190630_185205	135	16.3	14.0	8.2	58.8
GW190706_222641	67	12.7	7.8	10.1	96.5*
GW190727_060333	96	12.3	10.0	7.2	98.7*
GW190814	207	24.8	23.9	6.9	99.9
GW190828_063405	132	16.2	13.8	8.5	21.5
GW190910_112807	92	14.4	9.6	10.7	29.3*

LVK results for IMR



expect biases if
SNR is below threshold
in insp. and/or postinsp.

Event	[illegible]	[illegible]	[illegible]	[illegible]	Q_{GR} [%]
GW1509[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]7
GW170104	[illegible]	[illegible]	[illegible]	[illegible]	[illegible].0
GW170809	136	12.[illegible]	[illegible]	[illegible]	[illegible].6
GW170814	161	16.8	15.3	[illegible]	[illegible]2.9
GW170818	128	12.0	9.3	7.2	26.8
GW170823	102	11.9	7.9	8.5	93.3
GW190408_181802	164	15.0	13.6	6.4	11.4
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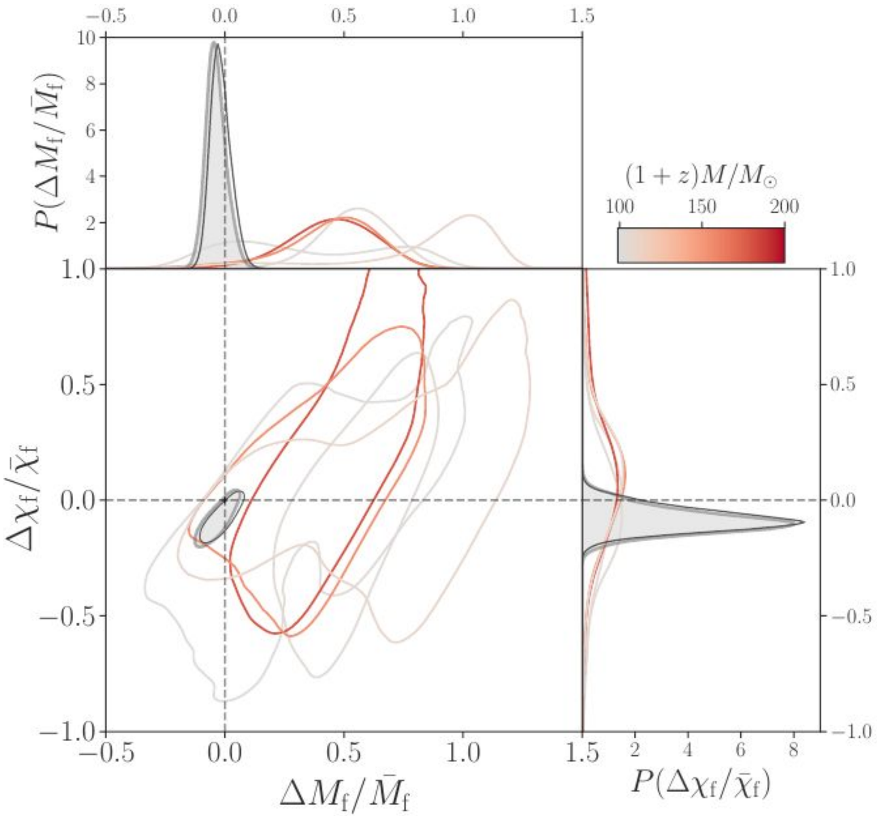
LVK results for IMR



expect systematics if the detector frame mass exceeds 100 solar masses

Event					Q_{GR} [%]
GW1509					7
GW170104					.0
GW170809	136	12.			5.6
GW170814	161	16.8	15.3		2.9
GW170818	128	12.0	9.3	7.2	26.8
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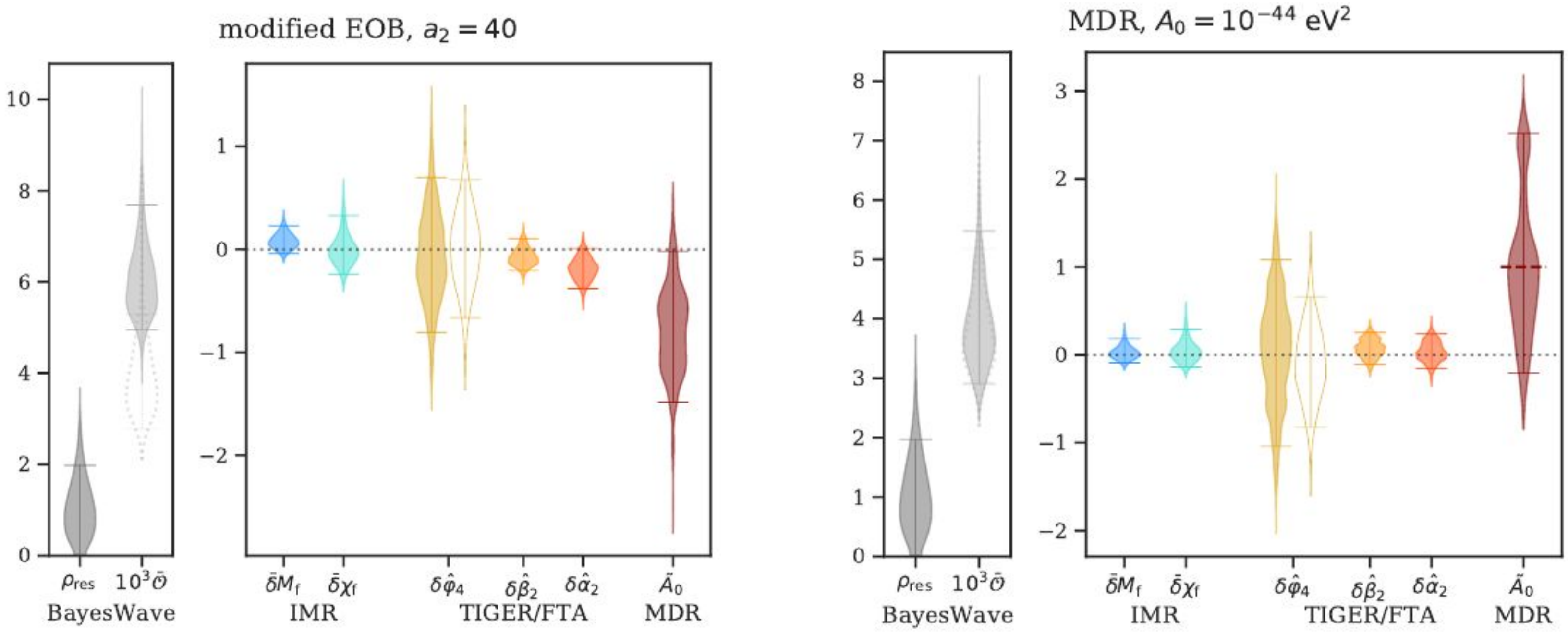
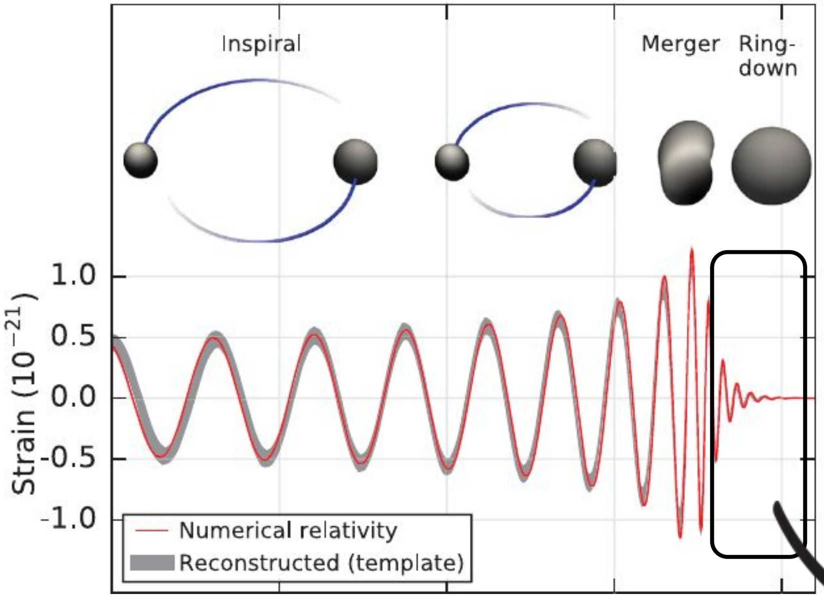
What are you testing?

Johnson-McDaniel++ (2021) [2109.06988](https://arxiv.org/abs/2109.06988)



Quasi normal modes and the no-hair theorem

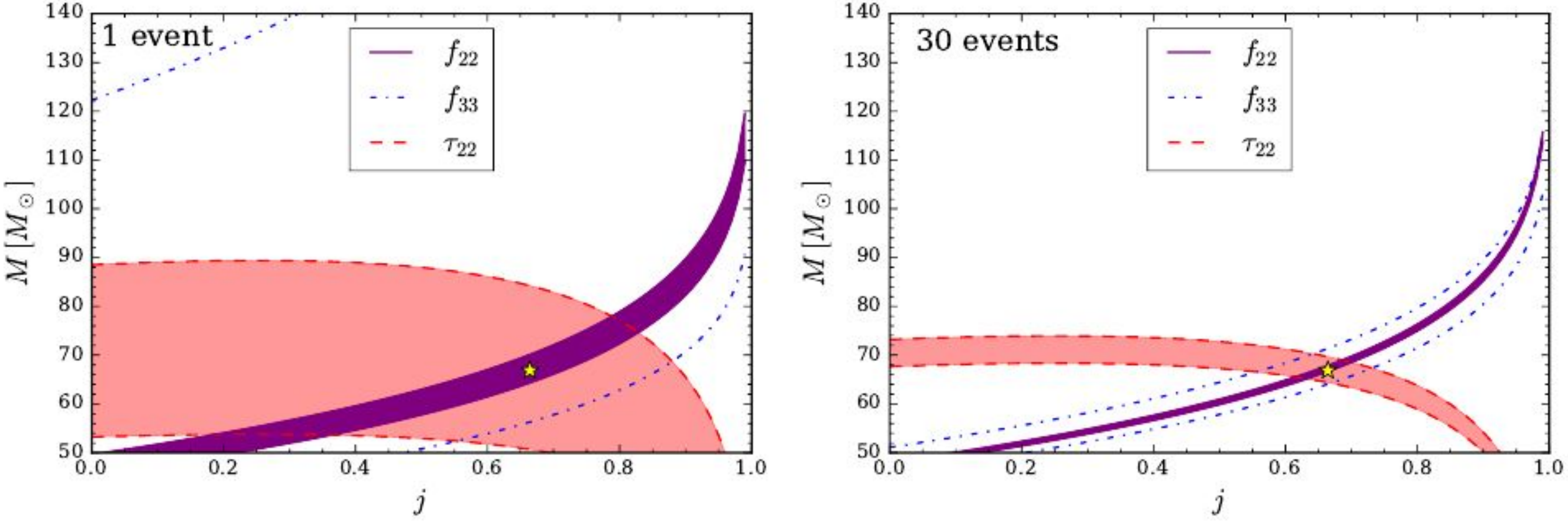


As a consequence of the no-hair theorem, **the quasi normal modes depend only on the mass and spin** of the BH remnant

$$f_{lmn} \equiv f_{lmn}(M, \chi)$$
$$\tau_{lmn} \equiv \tau_{lmn}(M, \chi)$$

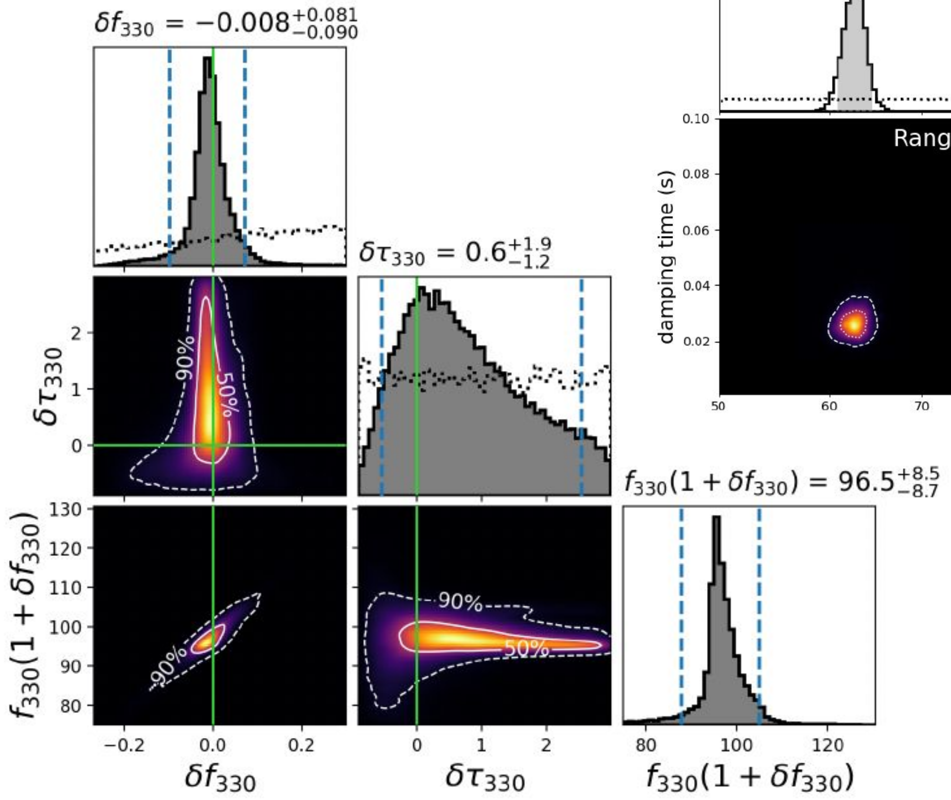
Black hole spectroscopy

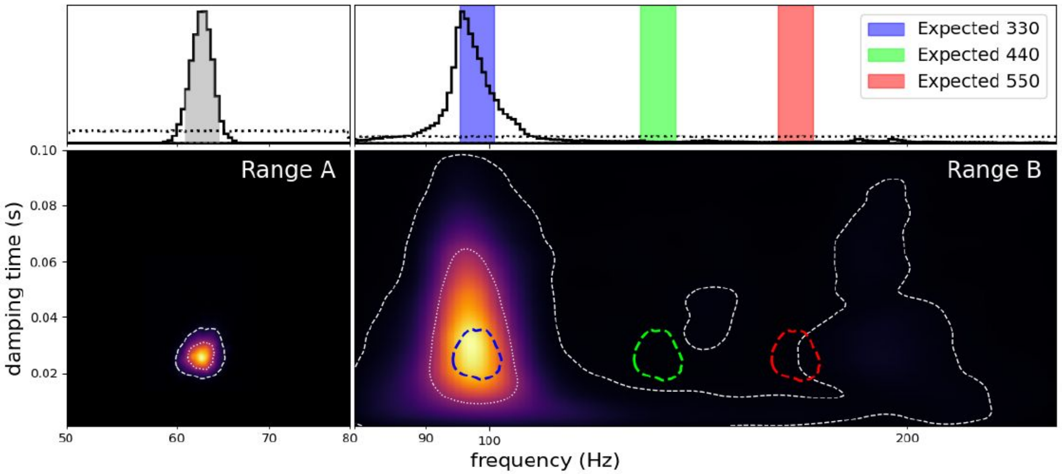
Measure **at least two** quasi-normal modes
Check that they are consistently inverted into mass and spin



Brito++ (2018) 1805.00293

GW190521: a case study for LVK



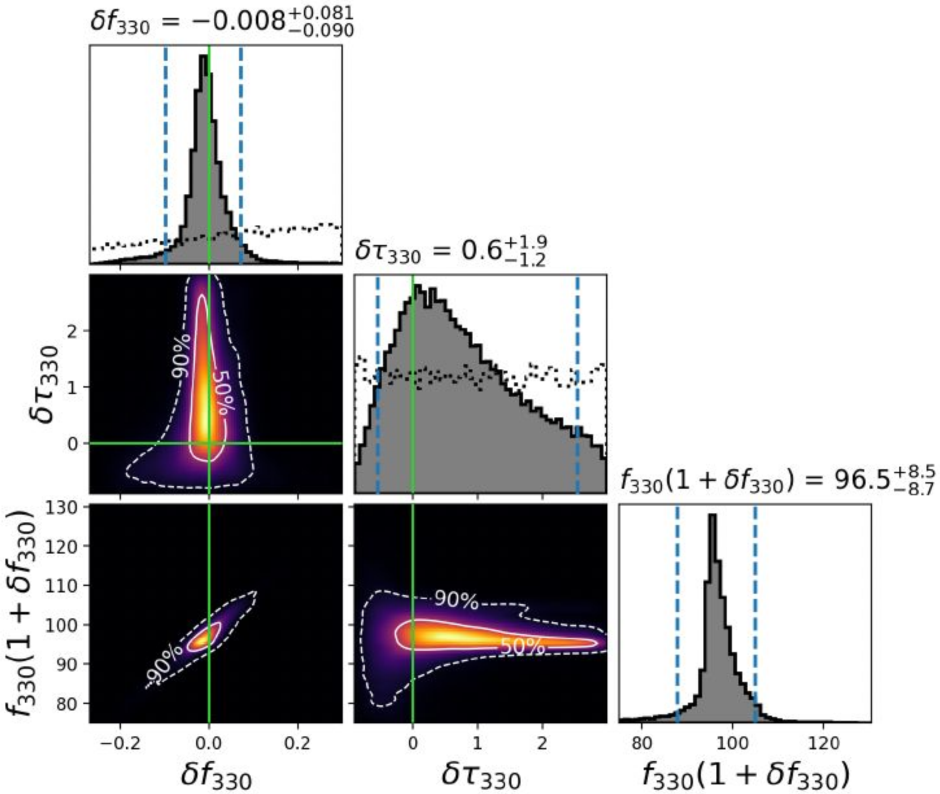


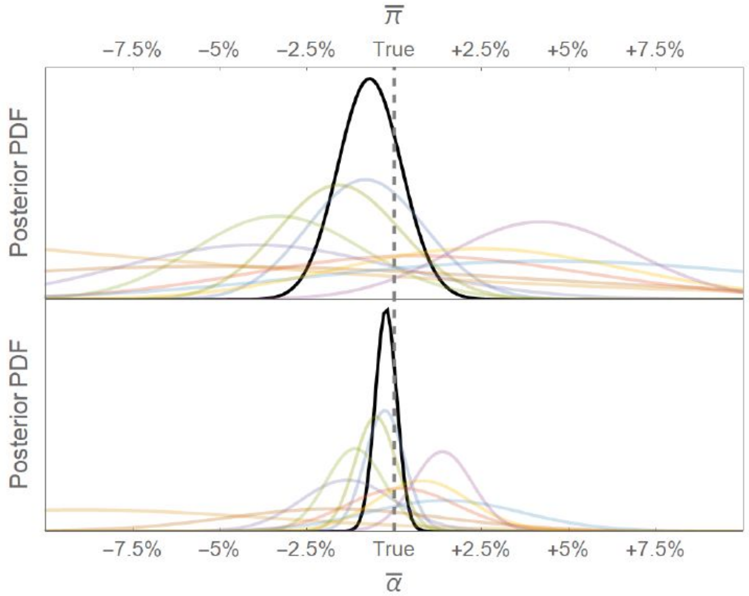
Deviation in the subdominant frequency (3,3,0) within 10%

No consensus yet about the detection

Capano++ (2021), 2105.05238

GW190521: a case study for LVK





Chua & Vallisneri (2019), [2006.08918](2006.08918)

2. Combining multiple events

Stacking techniques

Combining Bayes Factor: Multiply individual Bayes Factors from each event	Assume independent deviations across all events	Isi++(2022) 2204.10742
Combining Likelihoods: Multiply the individual likelihoods from each event	Assume common deviations to all events	Del Pozzo++(2011) 1101.1391
Hierarchical stacking: Estimate population hyper-parameters	Assume deviations are sampled from a population	Isi++(2019) 2204.10742

More on hierarchical stacking

Isi++(2019) 2204.10742

$$x \sim \mathcal{N}(x|\mu, \sigma)$$

Assume **deviations** are sampled **from a population**

$$p(\mu, \sigma|\{d_i\})$$

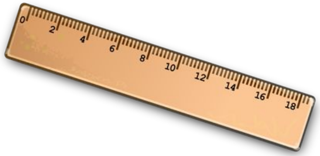
Derive a **hyper-posterior** over pop. parameters

$$\mathrm{GR}\ : \mu = \sigma = 0$$

What is the credibility of the **null hypothesis**?

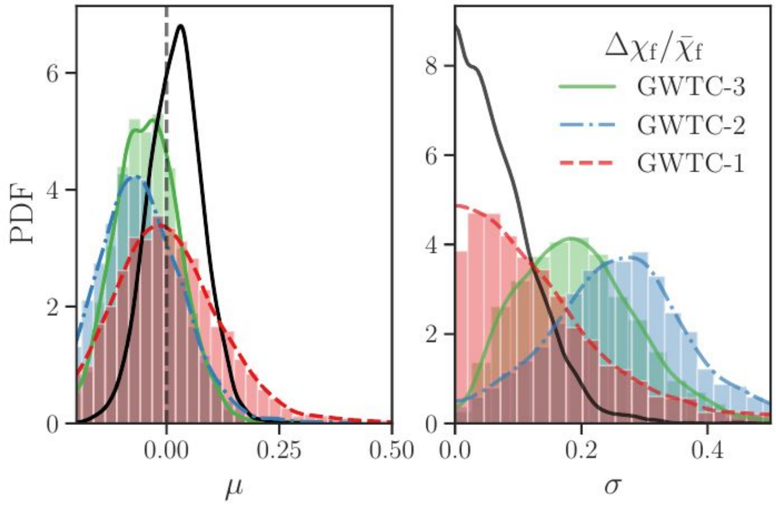
Credible quantile of the null hypothesis

$$Q_0 = \int_{p \geq p(0,0)} d\mu \, d\sigma \, p(\mu, \sigma|\{d_i\})$$

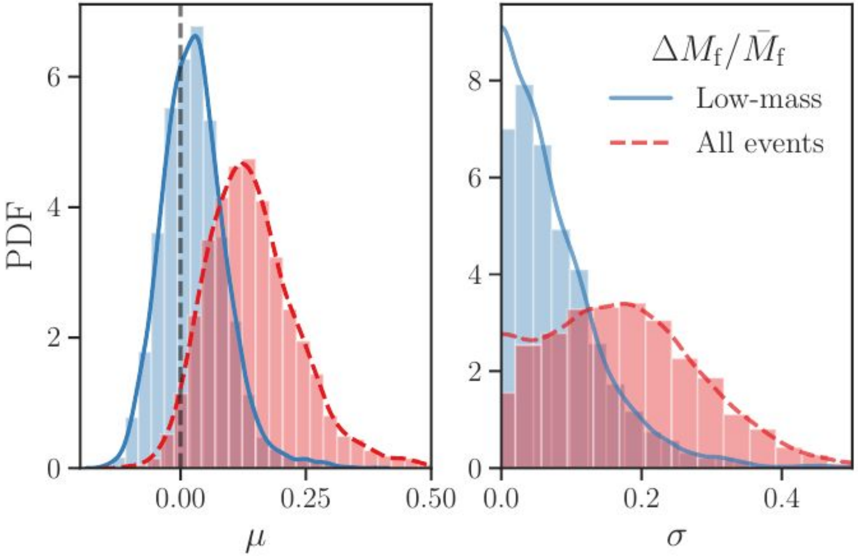


Hierarchical Stacking with GWTC-3 (1)

Final Spin

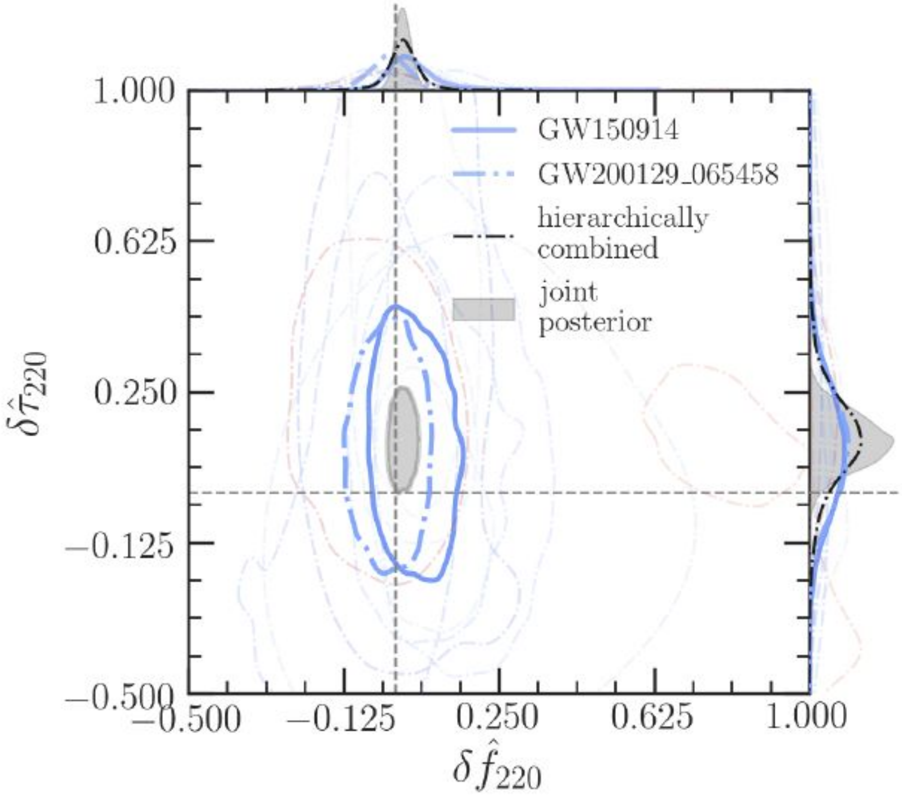


Final Mass



Note: these are 1-D marginal projection

Hierarchical Stacking with GWTC-3 (2)



pSEOB ringdown test

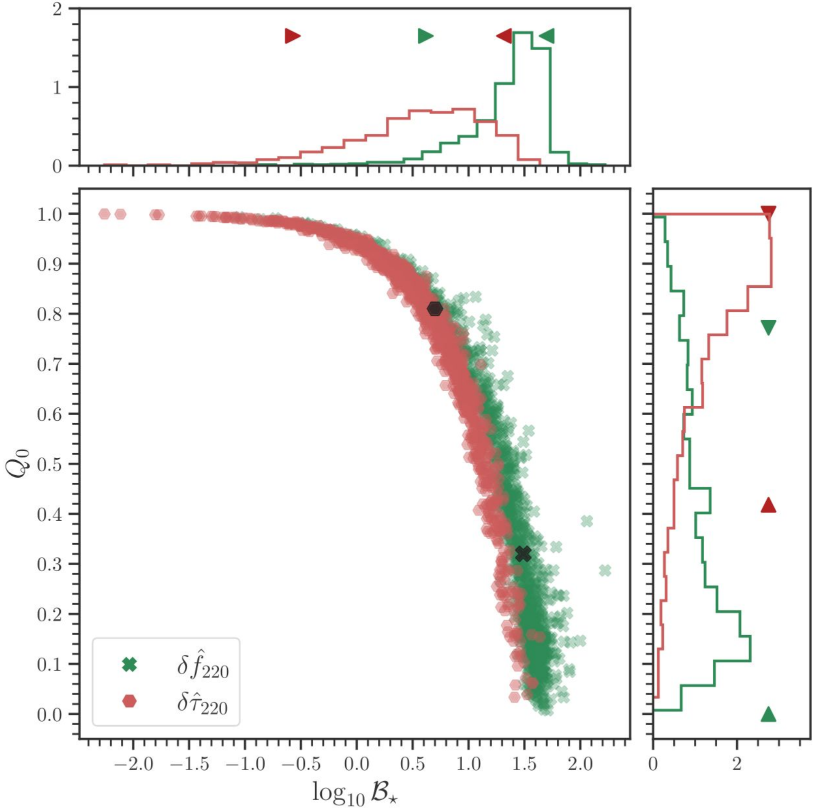
Hyperparameters 90% credible intervals

$$\mu = 0.02^{+0.04}_{-0.04} \quad \sigma < 0.06 \quad (\delta f_{220})$$
$$\mu = 0.13^{+0.13}_{-0.13} \quad \sigma < 0.19 \quad (\delta \tau_{220})$$

Credible quantile of the null hypothesis

$$Q_0 = 0.32 \quad (\delta f_{220})$$
$$Q_0 = 0.81 \quad (\delta \tau_{220})$$

Catalog variance?



From actual data:

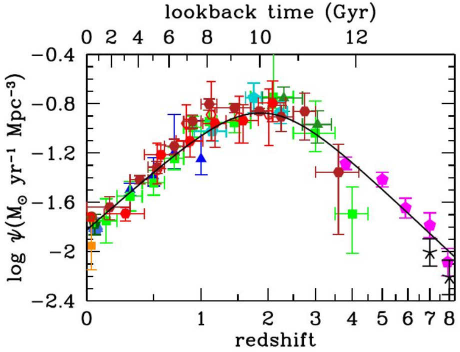
$$Q_0 = 0.32 \quad \log_{10}\mathcal{B}_\star = 1.49 \quad (\delta f_{220})$$
$$Q_0 = 0.81 \quad \log_{10}\mathcal{B}_\star = 0.70 \quad (\delta \tau_{220})$$

From 1000 bootstrapped catalogs
At 90% confidence:

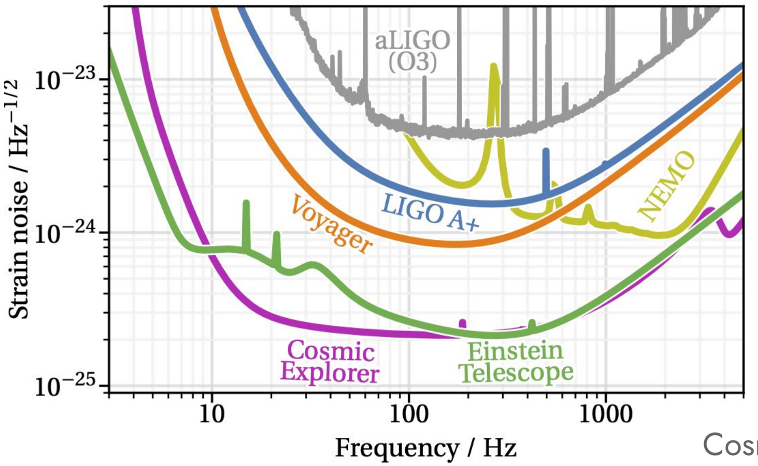
$$Q_0 < 0.77 \quad \log_{10}\mathcal{B}_\star = 1.45^{+0.25}_{-0.83} \quad (\delta f_{220})$$
$$Q_0 > 0.42 \quad \log_{10}\mathcal{B}_\star = 0.62^{+0.70}_{-1.19} \quad (\delta \tau_{220})$$

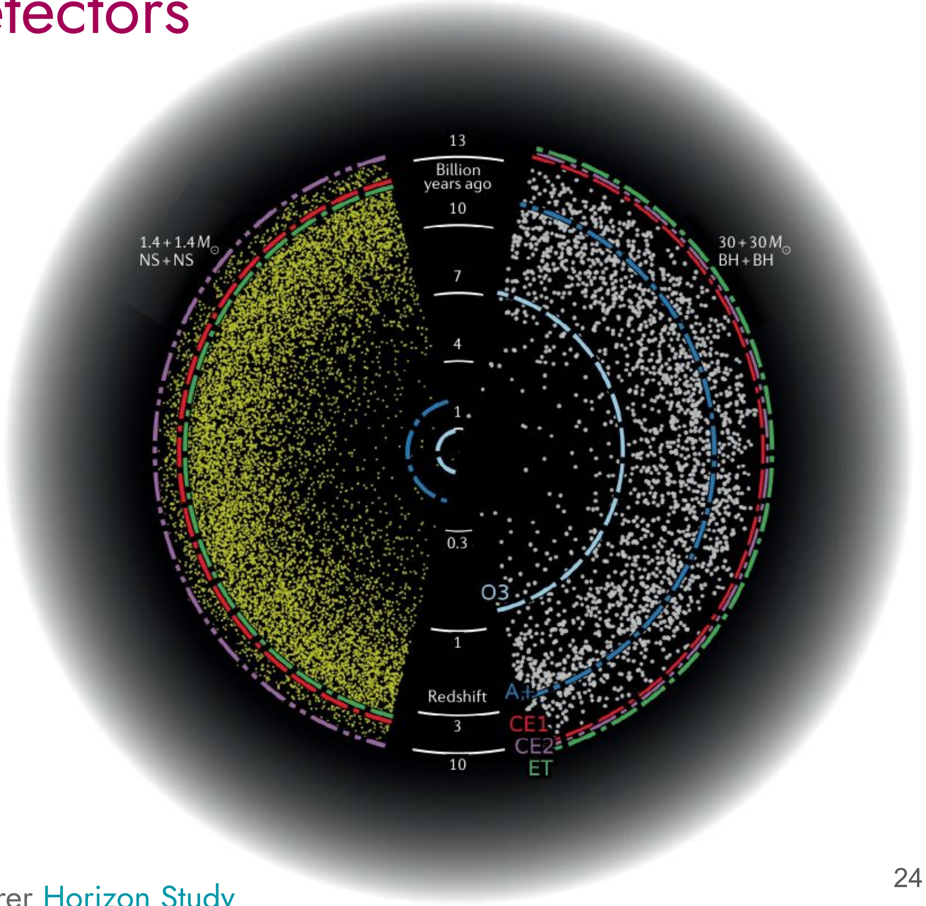
3. Next-generation detectors

3G detectors



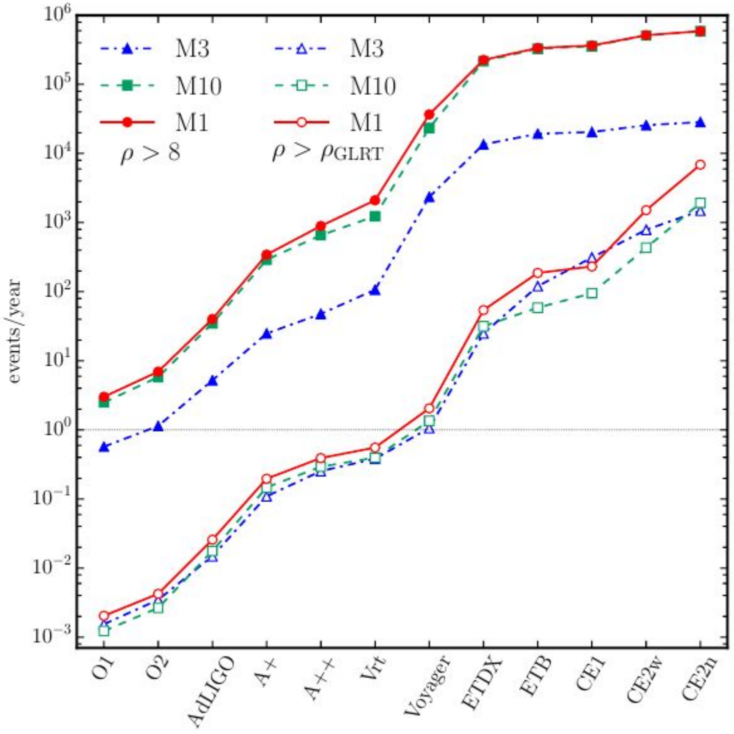
Madau & Dickinson (2014), [arXiv 1403.0007]





Cosmic Explorer [Horizon Study]

BH ringdown event rate



Berti++ (2016) [1605.09286]

Number of ringdown events/year with ground based detectors

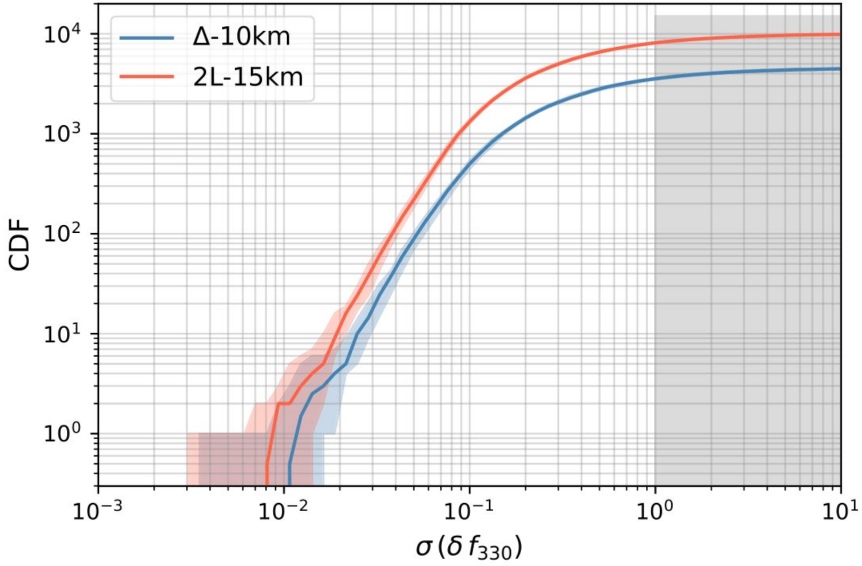
Detections: ringdown SNR > 8

Spectroscopy: ringdown SNR > GLRT (you can resolve the modes and constrain amplitudes away from 0)

3G detectors will mark a quantum leap

Prospects with ET

Configuration	$\rho_{\rm RD} \geq 12\ {\rm yr}^{-1}$	$\rho_{\rm RD} \geq 50\ {\rm yr}^{-1}$	$\rho_{\rm RD} \geq 100\ {\rm yr}^{-1}$	$\max(\rho_{\rm RD})$
Δ-10km	4594 ± 61	28 ± 7	3 ± 1	1134
2L-15km	10071 ± 88	70 ± 9	7 ± 3	1262



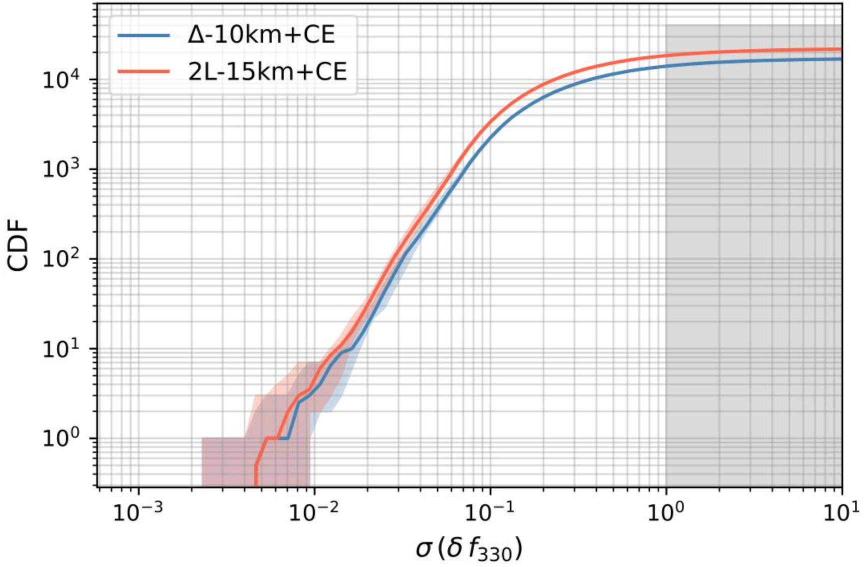
A population of stellar mass binary black holes in agreement with the LVK constraints

Generated with the pop synth code MOBSE by Mapelli et al. 2021

Bhagwat, CP++(2023) 2304.02283
"Science case with the Einstein Telescope" 2303.15923

Prospects with ET+CE

Configuration	$\rho_{\rm RD} \geq 12\ {\rm yr}^{-1}$	$\rho_{\rm RD} \geq 50\ {\rm yr}^{-1}$	$\rho_{\rm RD} \geq 100\ {\rm yr}^{-1}$	$\max(\rho_{\rm RD})$
Δ-10km+CE	17174 ± 115	161 ± 14	13 ± 5	1508
2L-15km+CE	22144 ± 122	246 ± 16	18 ± 7	1607



A population of stellar mass binary black holes in agreement with the LVK constraints

Generated with the pop synth code MOBSE by Mapelli et al. 2021

Bhagwat, CP++(2023) 2304.02283
"Science case with the Einstein Telescope" 2303.15923

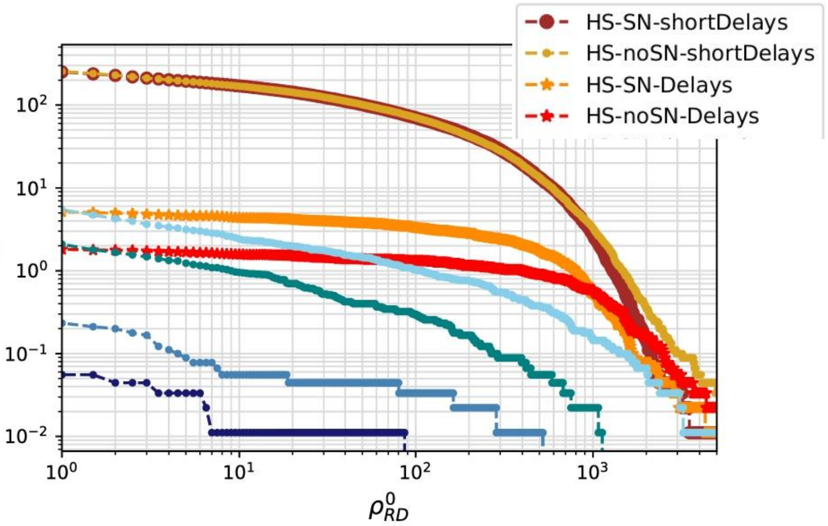
Prospects with LISA

Intrinsic rate of BBH events

POPULATION MODEL	BBH EVENTS PER YEAR
SN Delays	~ 6
no-SN Delays	~ 2.5
SN short-Delays	~ 317
no-SN short-Delays	~ 322

Barausse & Lapi (2020), [2011.01994]
Bhagwat, CP,++ (2022), [2201.00023]

Inverse-CDF of ringdown SNR per year



- *Delays*: most events have SNR O(1000)
- *short-Delays*: SNR spreads across [1,1000]

Thank you!