

# Probing Cosmology with Gravitational Waves

## - Current Status and Future Prospects with **LISA & ET**

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# Latest Updates

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**In June 2021, the European Strategy Forum on Research Infrastructures (ESFRI) decided to include the Einstein Telescope (ET) in the update of its roadmap for 2021.**

**In June 2022 formal establishment of ET collaboration (today 1400 members)**

**In June 2023 Italian government present the Italian candidacy to host the Einstein Telescope,**

**On 25 January 2024 LISA has been adopted by ESA and construction will start in January 2025**

**On 25 March 2024 ASI presented the LISA mission to the scientific community**

[Link to article](#)



ASTROPHYSICS AND COSMOLOGY | FEATURE

## Gravitational waves: a golden era

23 August 2023

A Maleknejad, F Rompineve

### Outlook

Precision detection of the gravitational-wave spectrum is essential to explore particle physics beyond the reach of particle colliders, as well as for understanding astrophysical phenomena in extreme regimes. Several projects are planned and proposed to detect GWs across more than 20 decades of frequency. Such a wealth of data will provide a great opportunity to explore the universe in new ways during the next decades and open a wide window on possible physics beyond the SM.

# Where we are - LVK

The third observing run (B) from April 2019 to March 2020

Total number of gravitational waves observed to date (with probability of astrophysical origin > 0.5): ~ 90 (mostly BBHs, 2 BNS and 2 NS-BH)

GWTC-3 catalogue: arXiv:2111.03606

The fourth run O4b has started with Virgo online

↓  
**10 April**

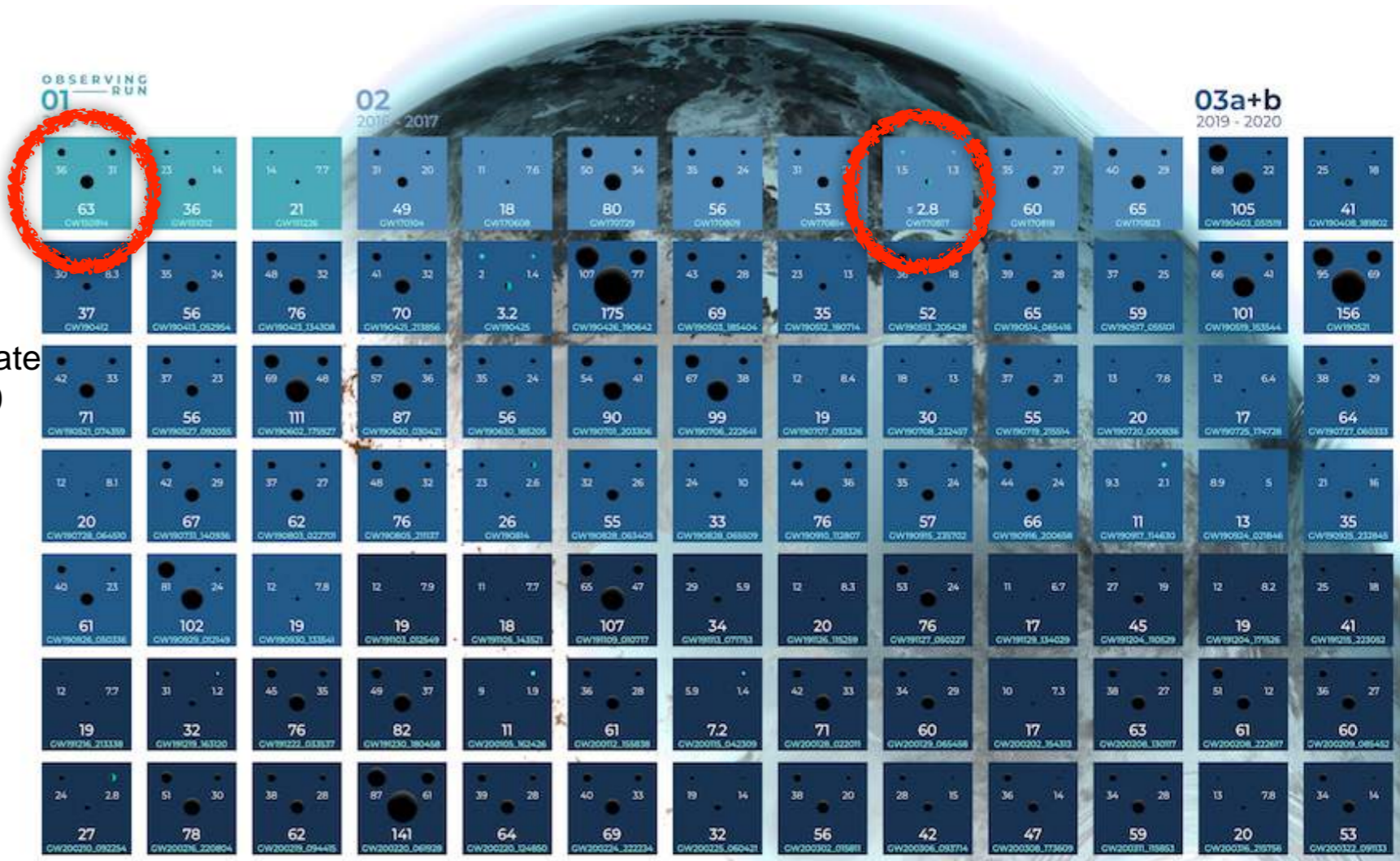
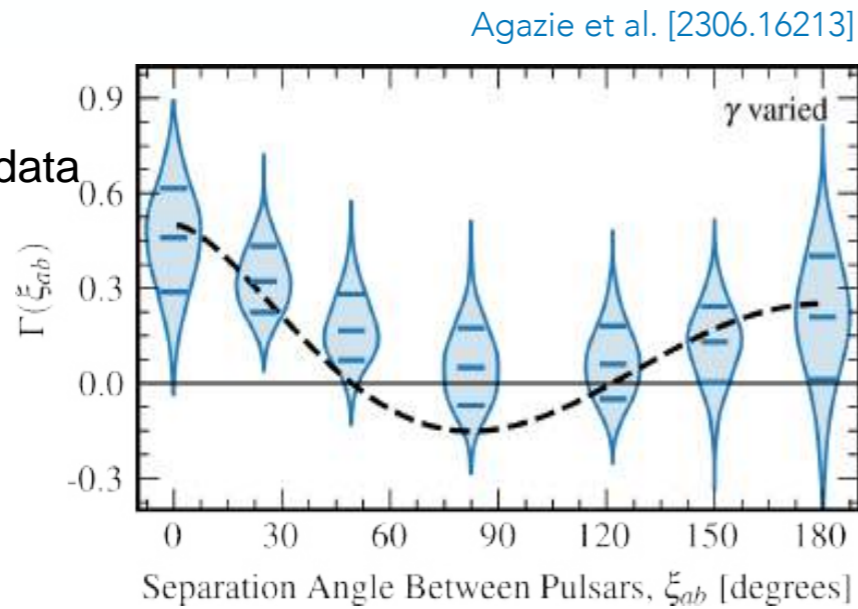


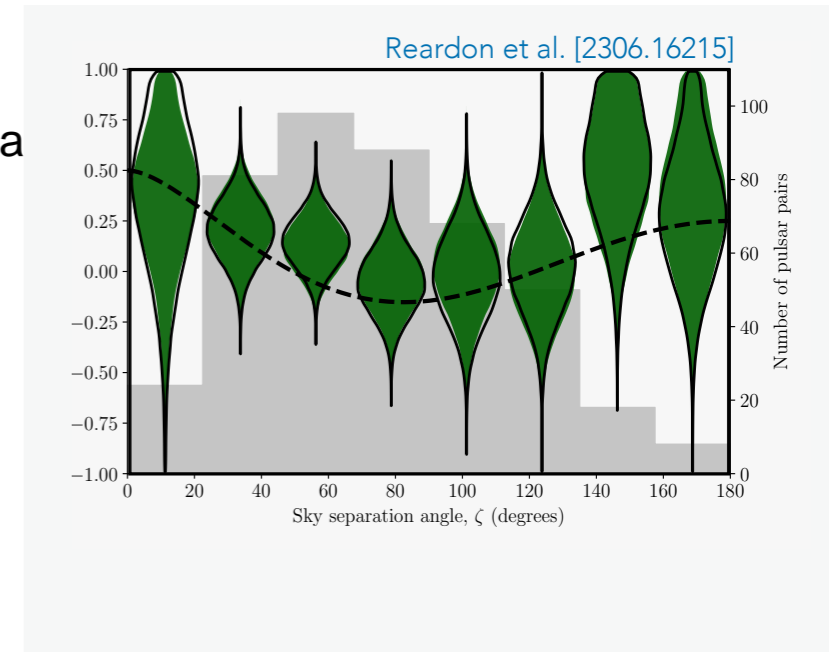
Image credit: LIGO / Virgo / KAGRA / C. Knox / H. Middleton

# Where we are - PTA

**NANOGRav:**  
68 pulsars, 16 yrs of data  
 $\sim 3 - 4\sigma$  significance

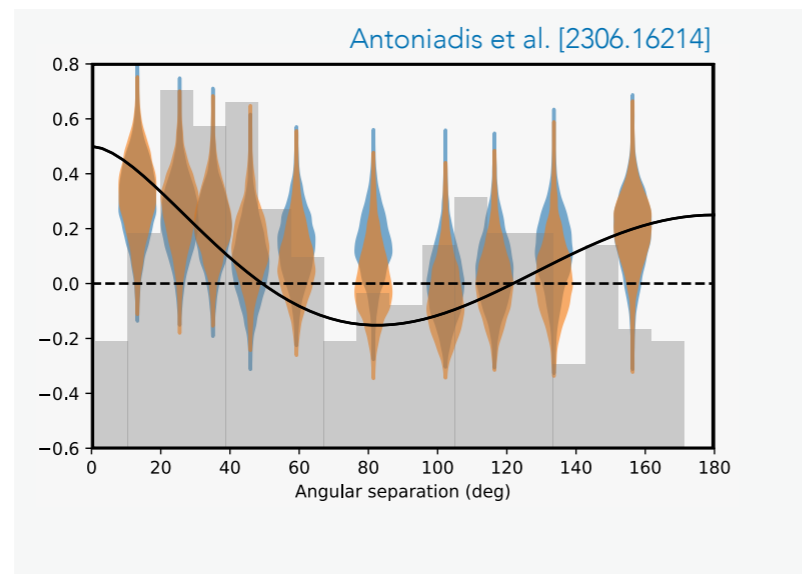


**PPTA:**  
32 pulsars, 18 yrs of data  
 $\sim 2\sigma$  significance

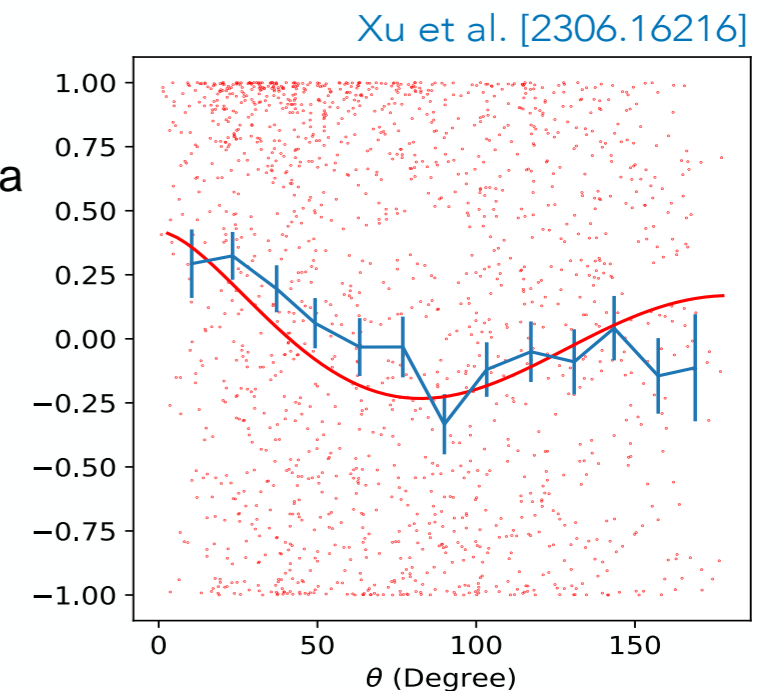


## First Detection of a SGWB

**EPTA+InPTA:**  
25 pulsars, 24 yrs of data  
 $\sim 3\sigma$  significance



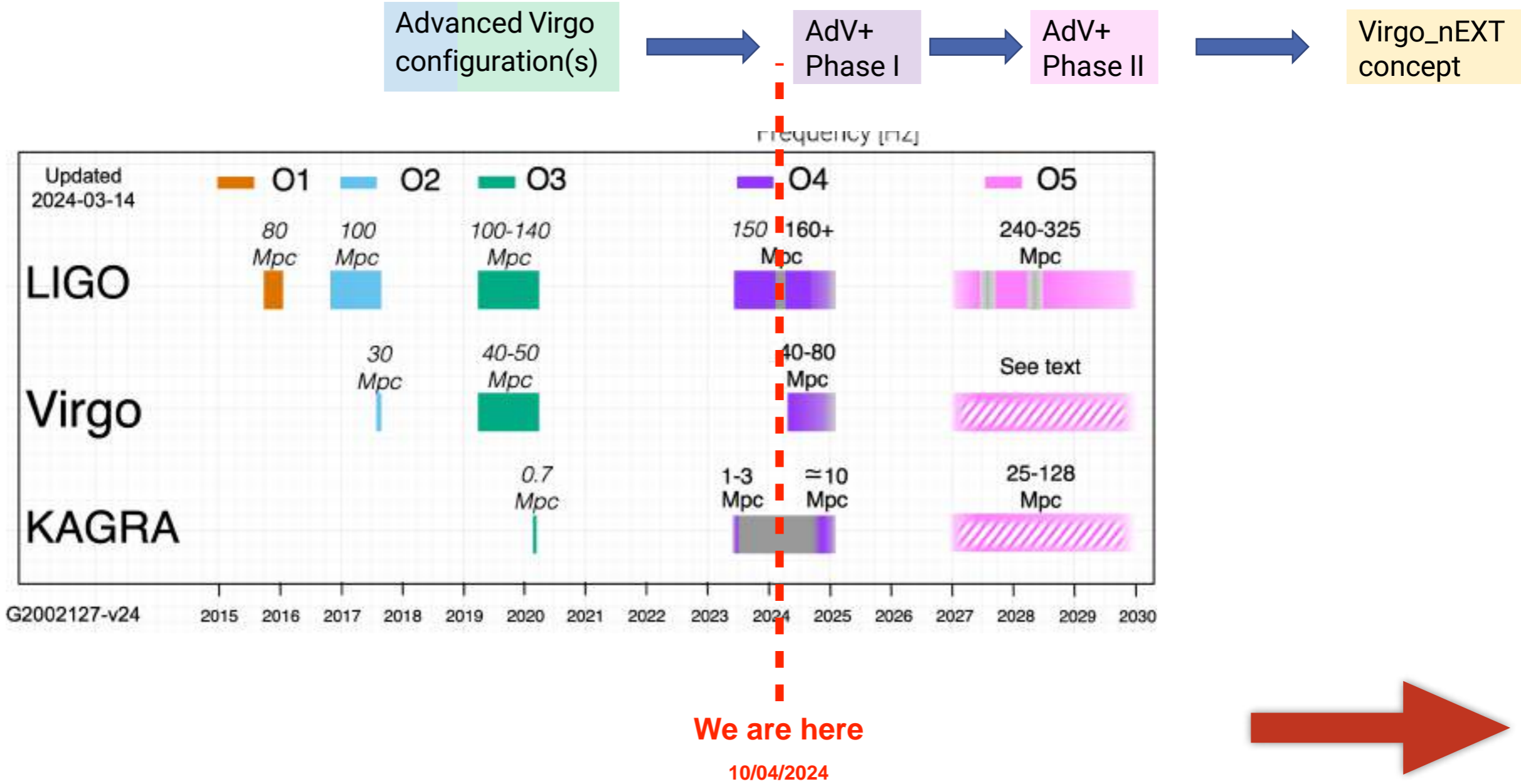
**CPTA:**  
57 pulsars, 3 yrs of data  
 $\sim 4.6\sigma$  significance



Bayesian reconstruction of normalized inter-pulsar correlations  
Violins plot = marginal posterior densities (plus median and 68% credible values)

See K. Schmitz, Stas and Matias talks

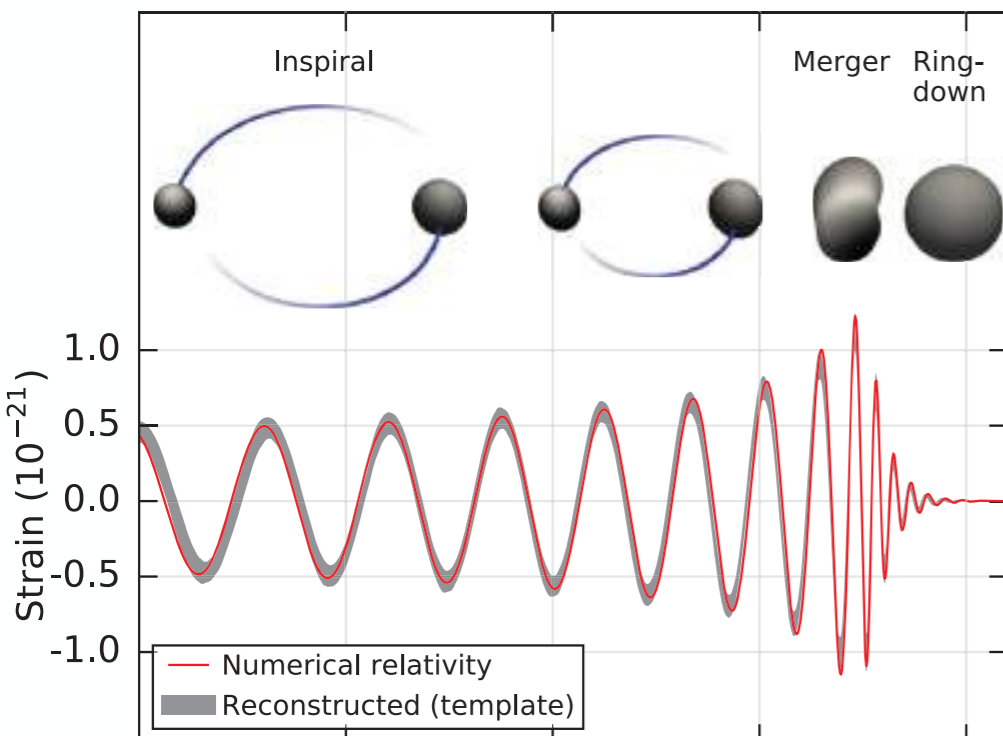
# What are the plans for the LVK runs?



# Sources of Gravitational Waves

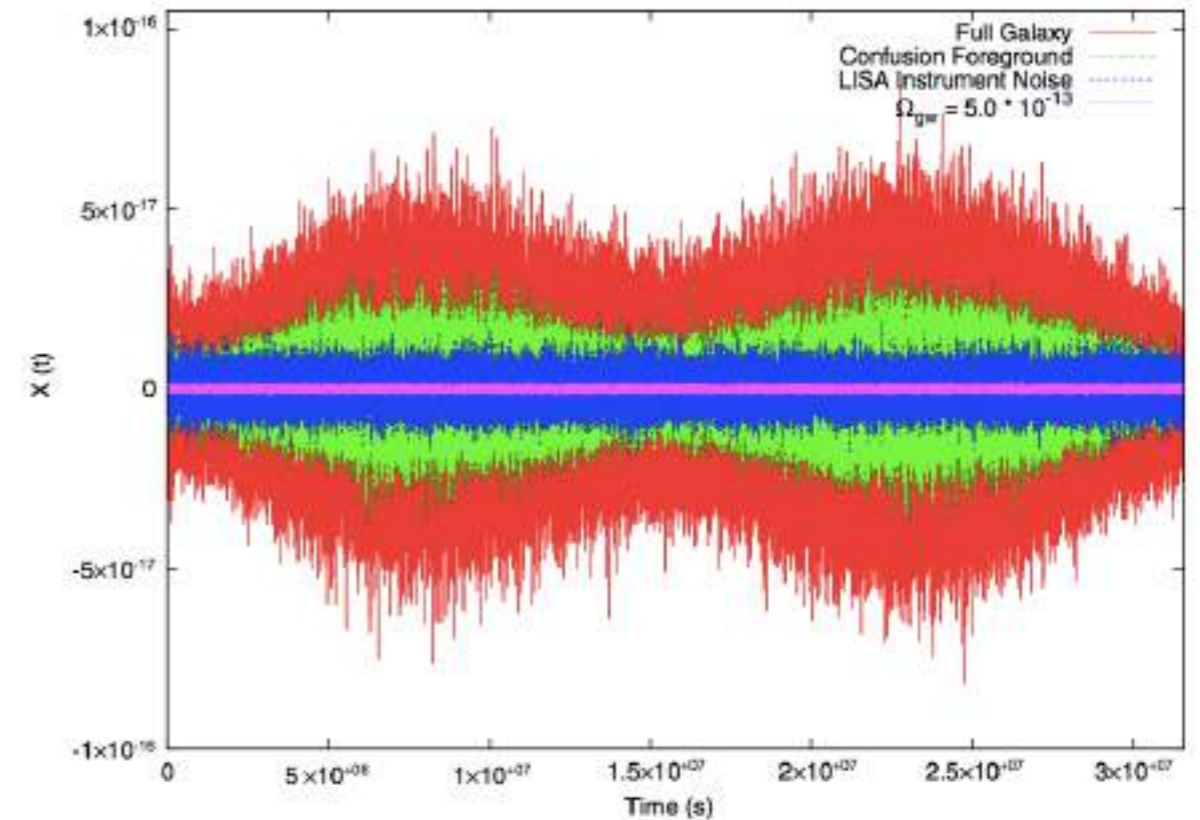
## Resolved Sources:

- Black Holes
- Neutron Stars
- White Dwarfs
- Supernovae
- ...

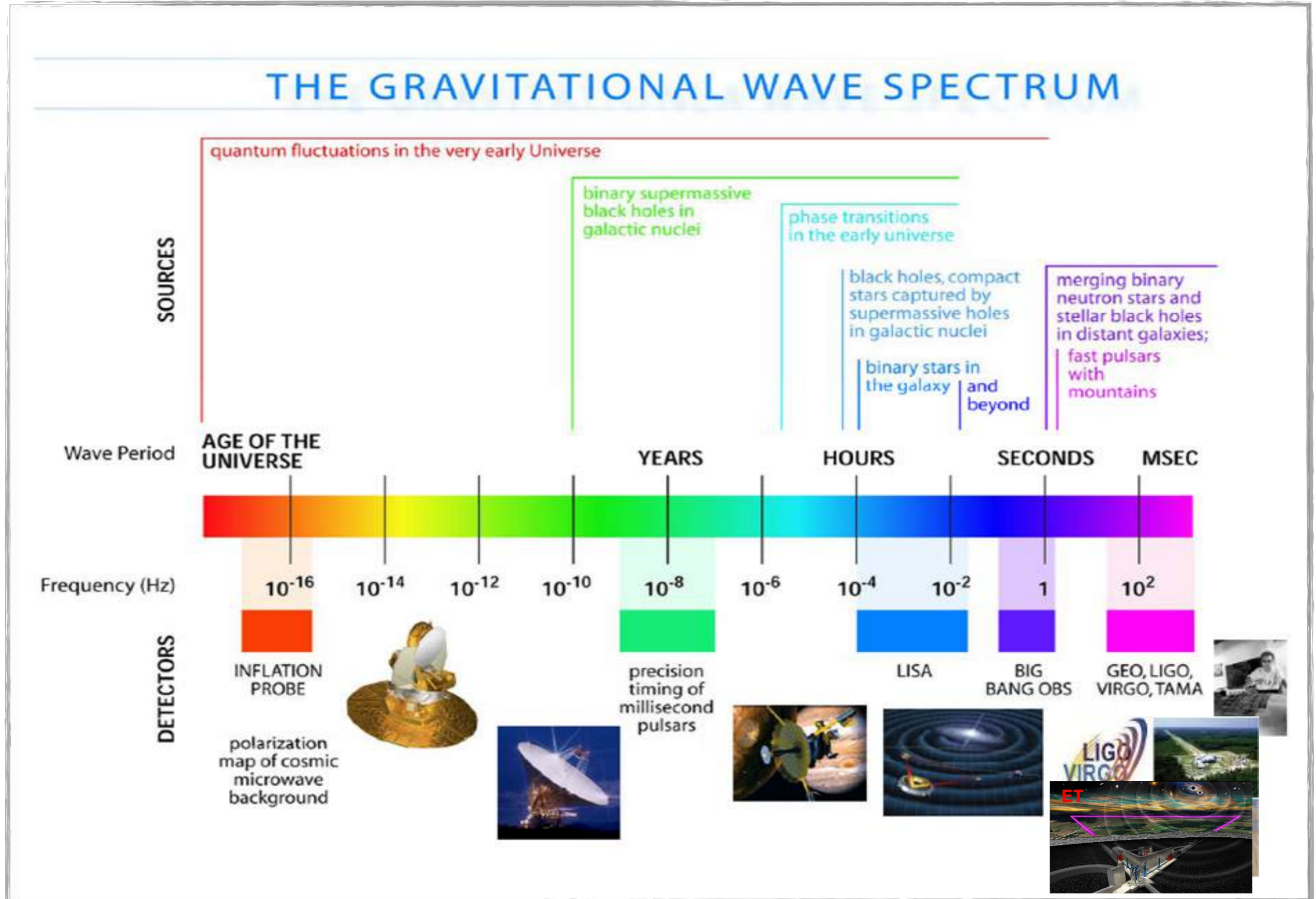


## Unresolved Sources:

- Stochastic Backgrounds
  - Astrophysical
  - Cosmological

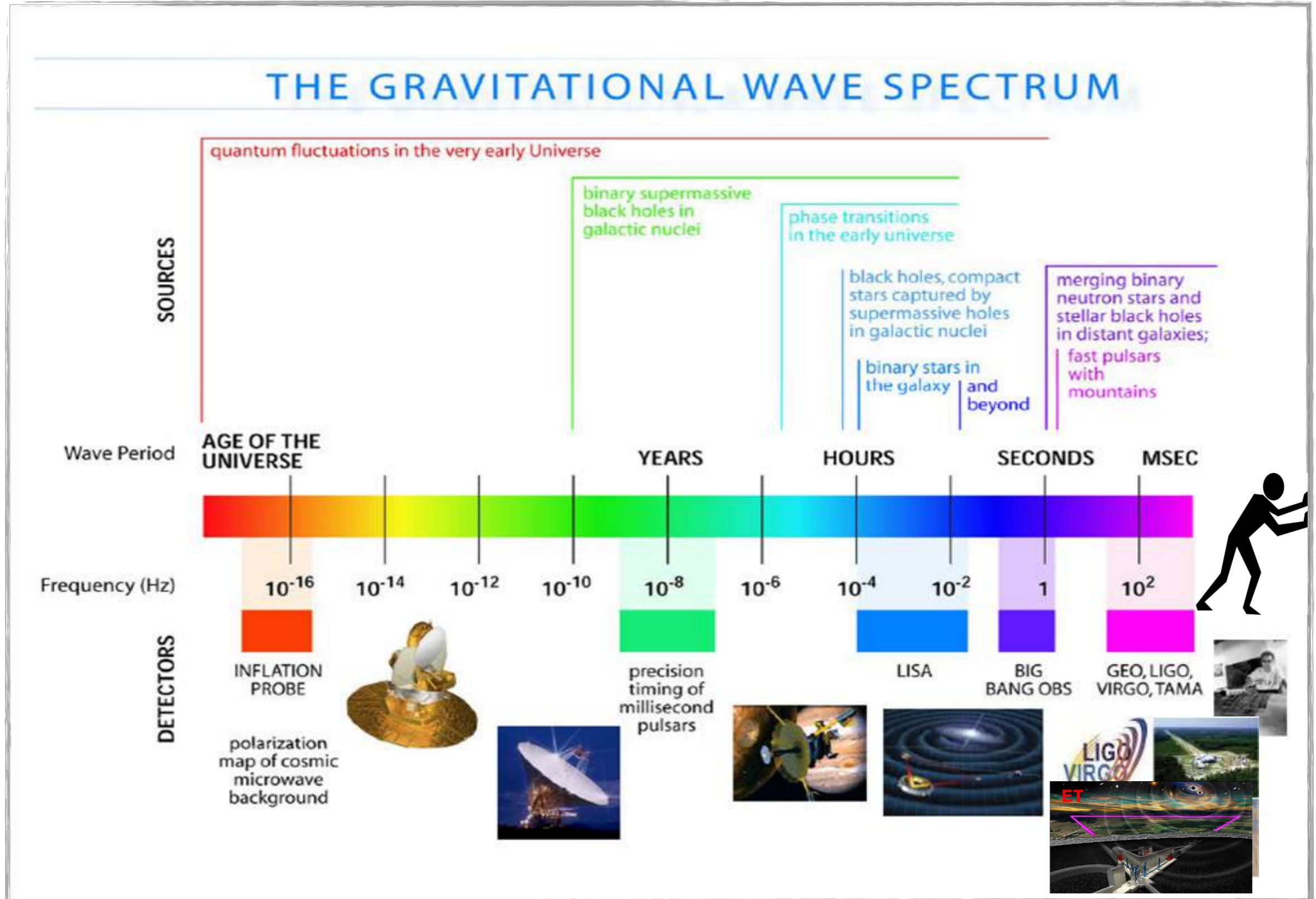


# The Gravitational Wave Spectrum

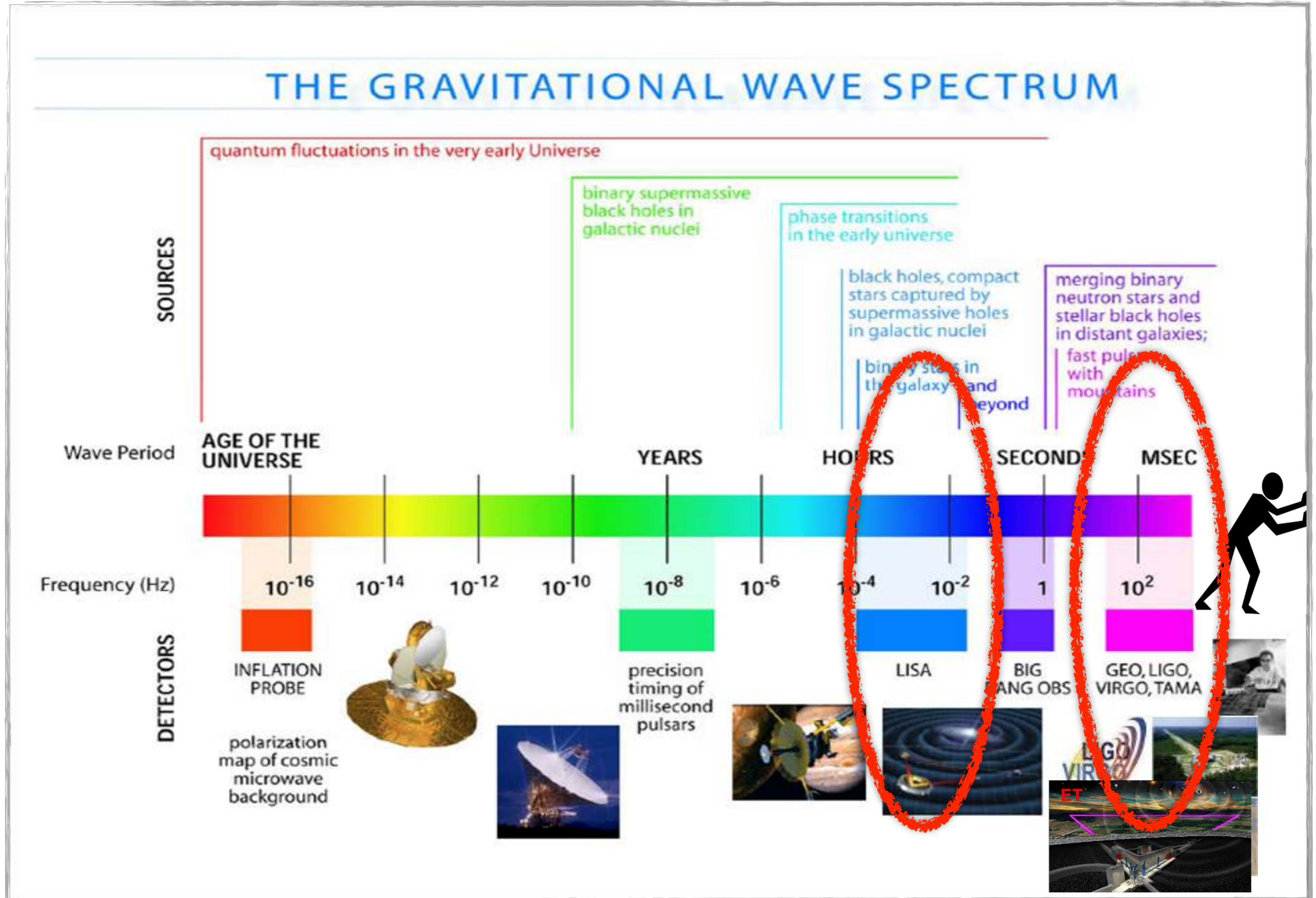




# The Gravitational Wave Spectrum

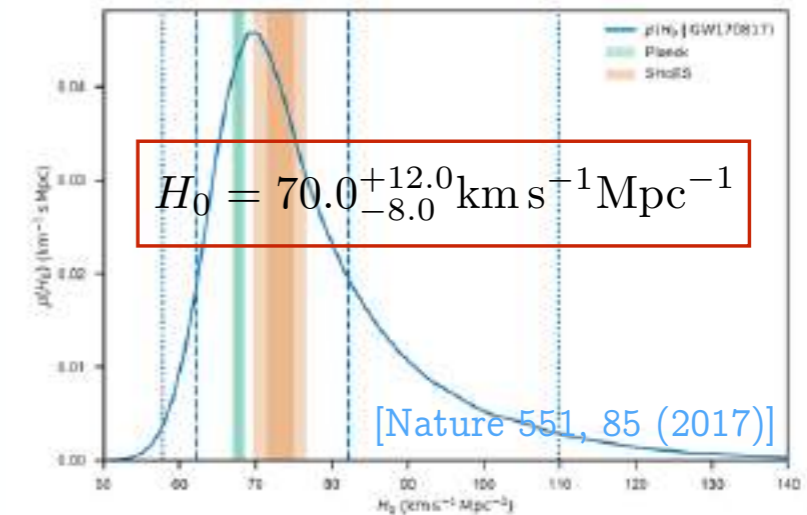


# The Gravitational Wave Spectrum



# What we have learned about Cosmology so far?

- We obtained the first measurement of the Hubble constant using GWs



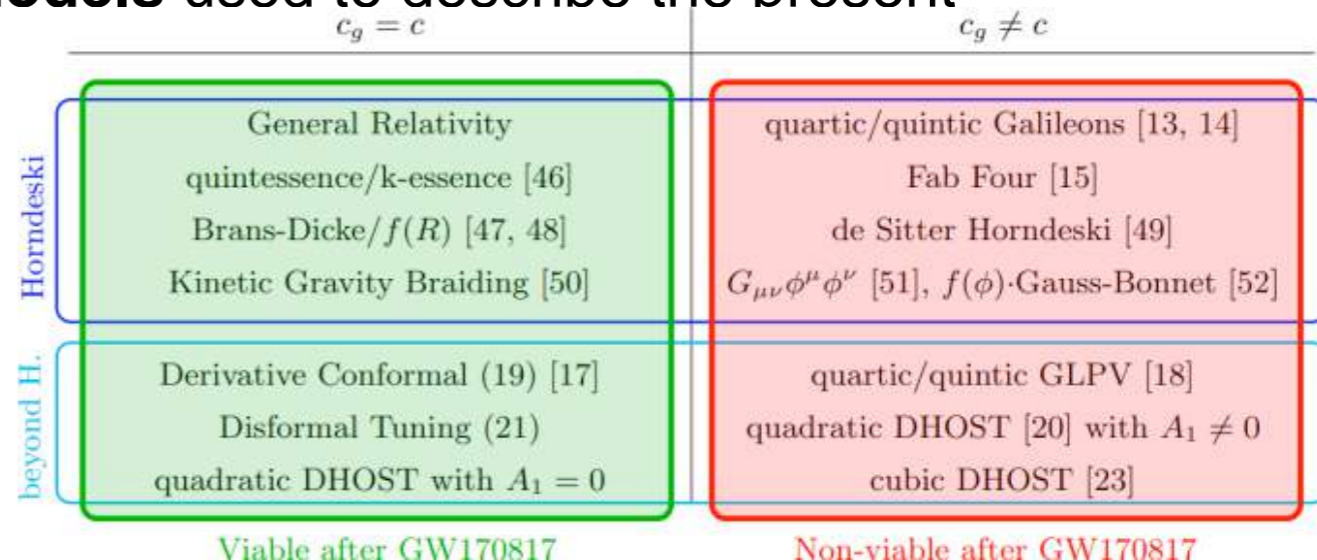
- We have tested and bounded deviations from GR (e.g., graviton mass, post-Newtonian coefficients, modified dispersion relations, etc.)

$$m_g \leq 1.27 \times 10^{-23} \text{ eV}/c^2$$

- The speed of GWs is the same as the speed of light

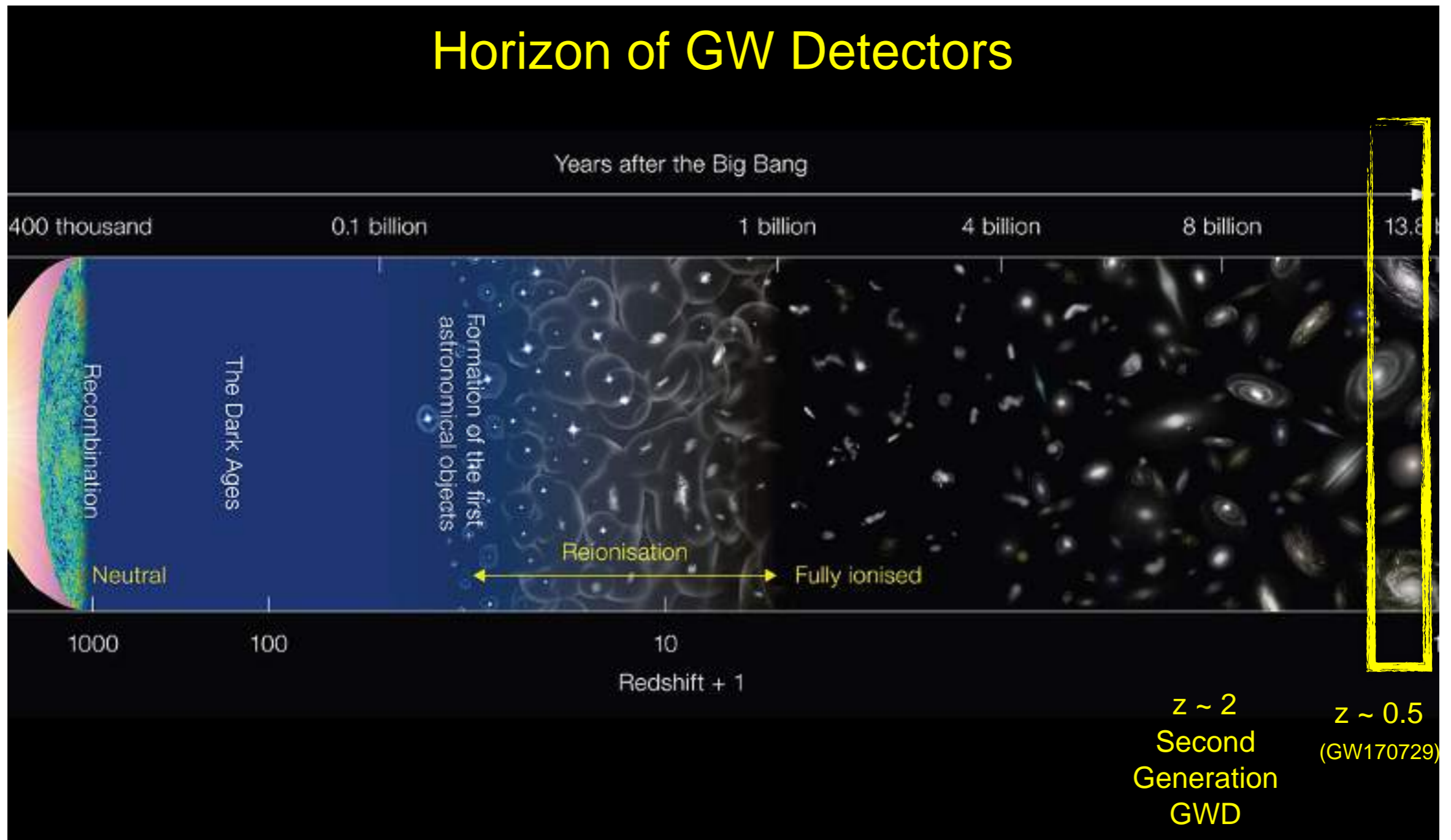
$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 6 \cdot 10^{-16}$$

- We have ruled out many Modified Gravity models used to describe the present acceleration of our Universe



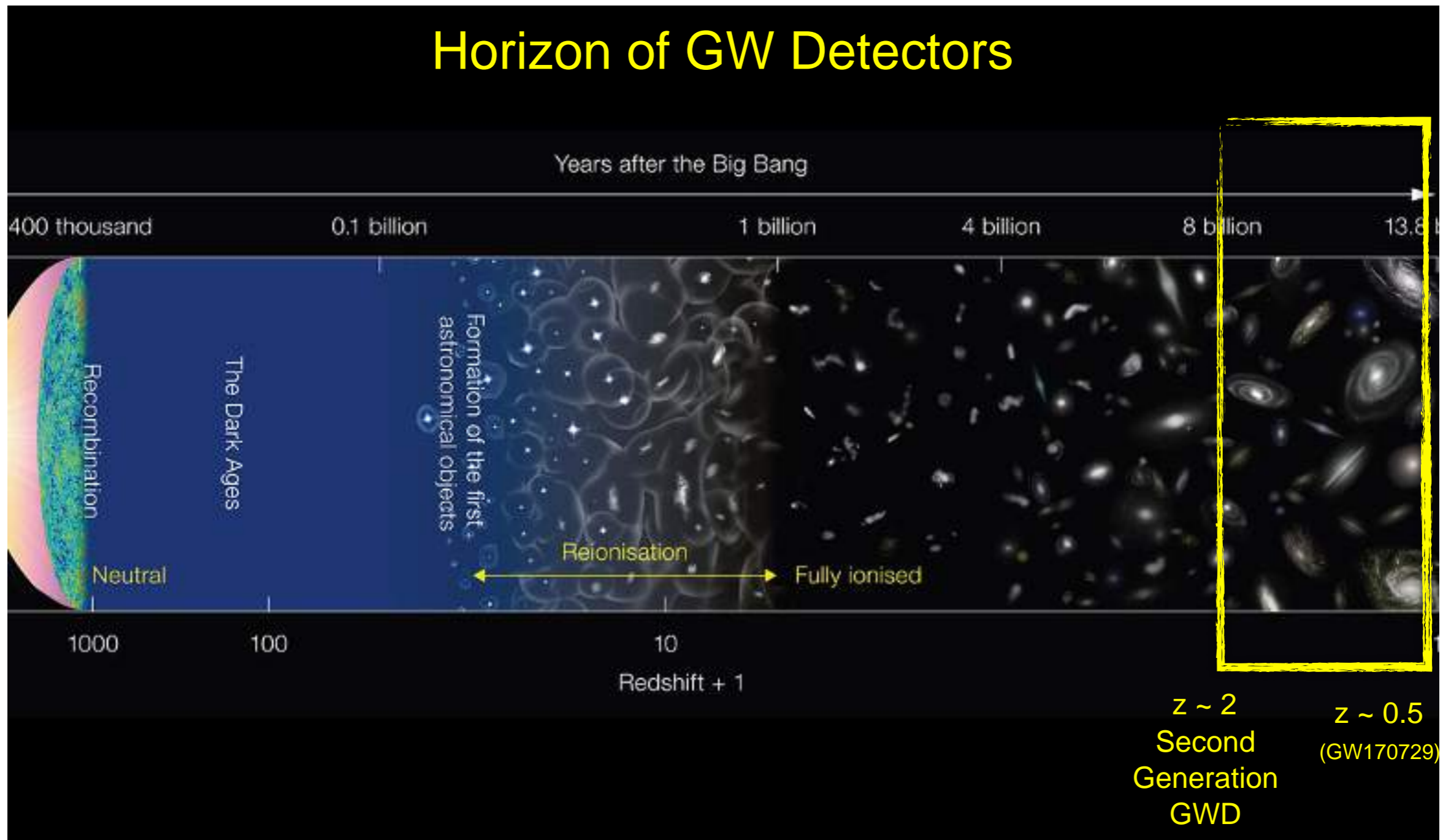
[LVK arxiv:2112.06861](https://arxiv.org/abs/2112.06861)

# Where is the horizon?



Adv LIGO - Virgo - KAGRA:  
BBHs only up to  $z \sim 2$   
BNSs in the very local Universe

# Where is the horizon?



Adv LIGO - Virgo - KAGRA:  
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BNSs in the very local Universe

# Open questions in Cosmology

Early Universe

Primordial GWs

Cosmic Structure

Dark Matter

**Primordial BHs?**

Axion particles

Late Universe

Dark Energy

Modified Gravity?

Cosmic Tensions

$H_0$  measurements?

Isotropy

Testing  
Cosmological  
Principle

**RESOLVED SOURCES**

**SGWB**

# Open questions in Cosmology

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Early Universe

Primordial GWs

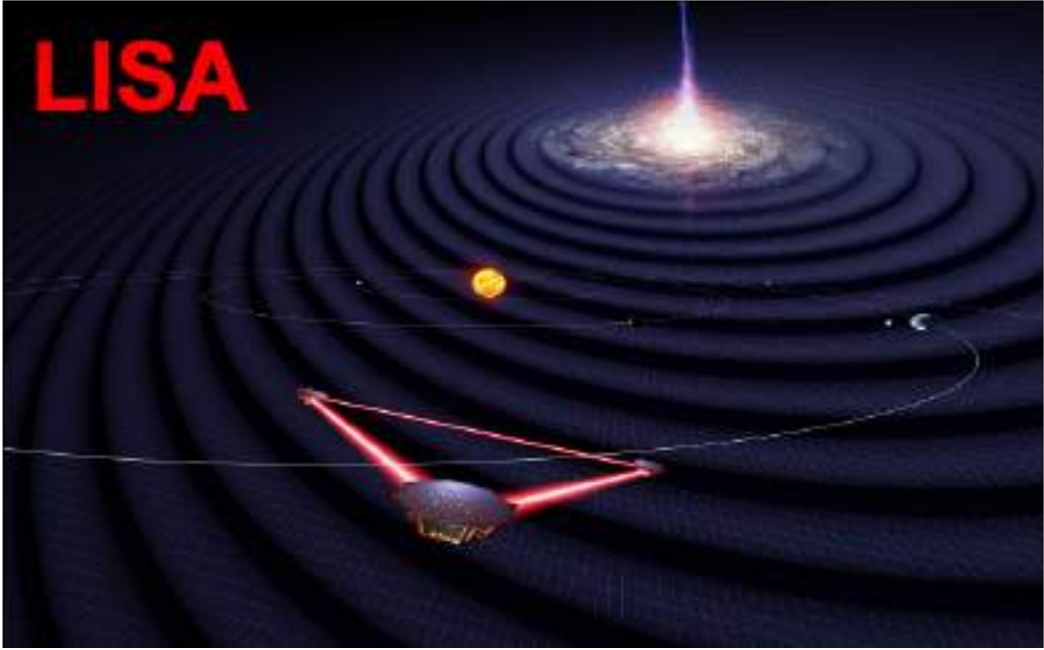
Cosmic Tensions

$H_0$  measurements?

**RESOLVED SOURCES**

**SGWB**

# Prospects for next Generation GW Interferometers

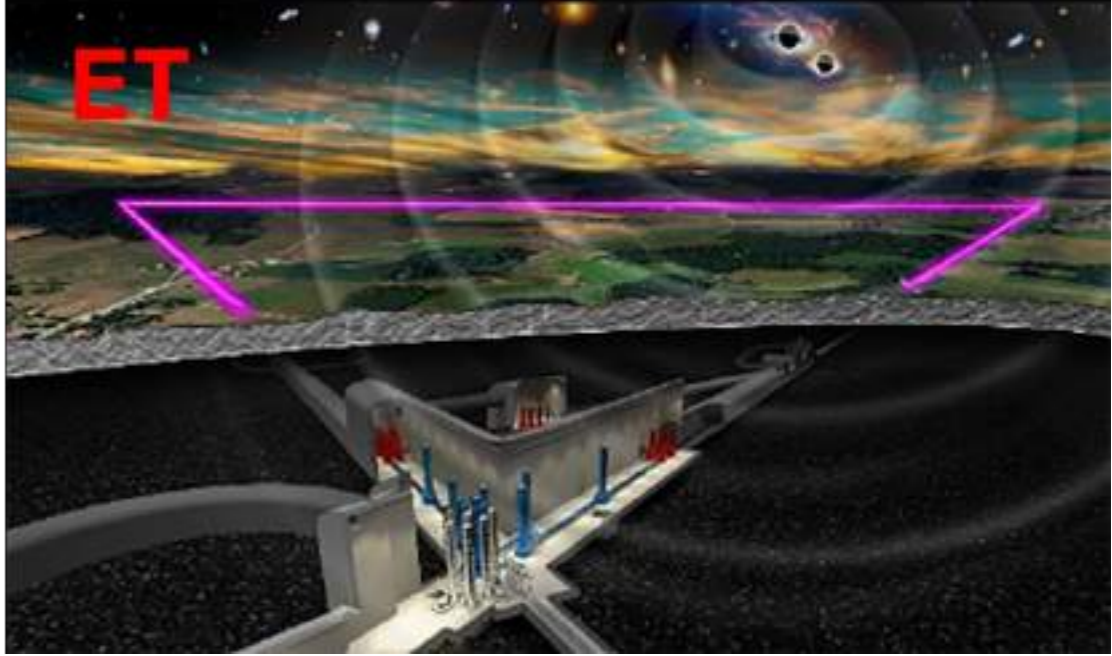


**Geometry: Constellation of 3 spacecraft in an equilateral configuration (a giant interferometer)**

Mission duration: **4 y science mission**  
**10 y nominal mission**

Arm Length: **2.5 million km**

**Expected Launch: 2034**



**Geometry: Ground-based Triangular detector (HF+LF)**

Arm Length: **10 km**

**Expected to be operative in: 2034**

**ET collaboration officially launched**

+ CE, DECIGO, BBO, Taiji, TianQin, etc

Without forgetting AdV -> Virgo-nEXT

New PTA data

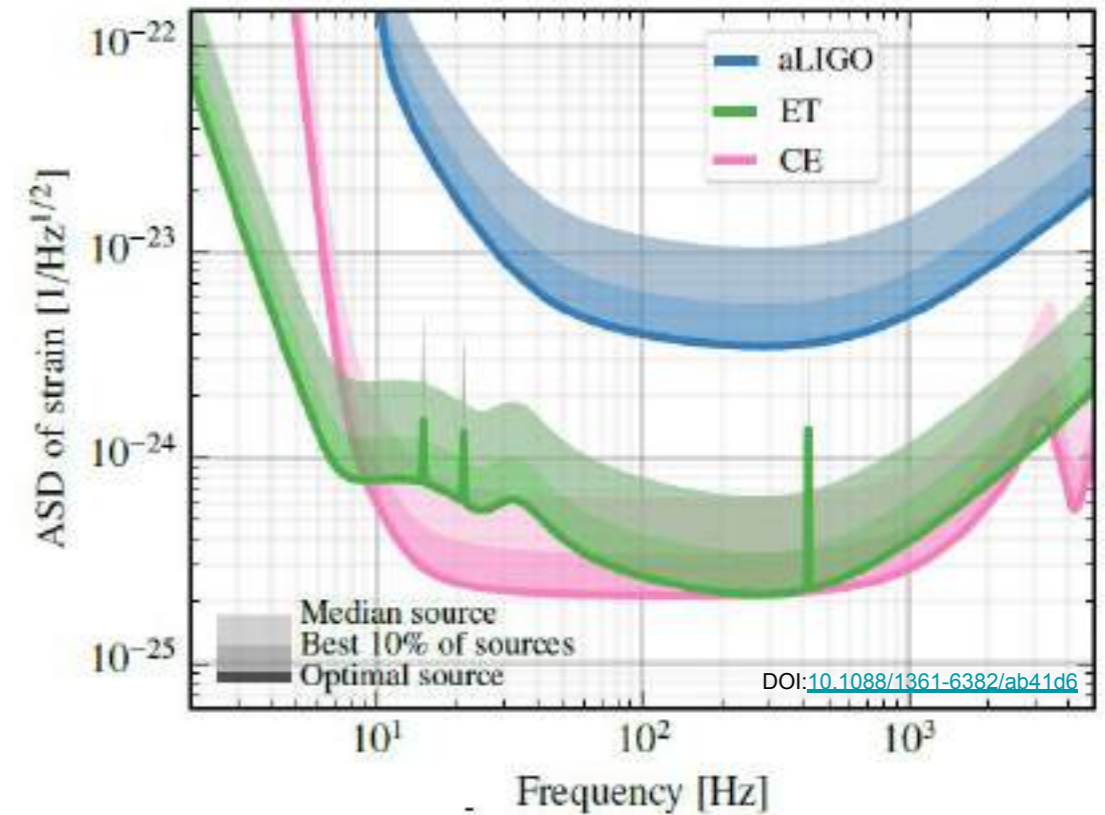


# Einstein Telescope

But current LVK detectors have limitations: need to jump to next generation detectors:

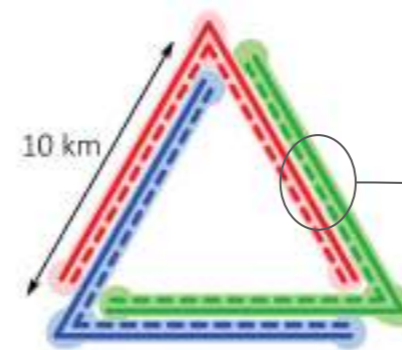
- **Einstein Telescope (ET):**
  - EU proposal for 3G observatory
  - Design study (baseline) triangle with arms of 10km
- **Cosmic Explorer (CE):**
  - US proposal for 3G observatory
  - L-shaped 40km interferometer

ET and CE will provide an improvement in sensitivity by one order of magnitude and a significant enlargement of the bandwidth



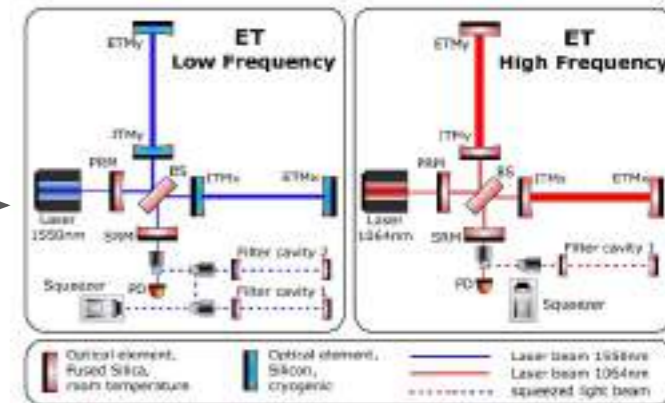
## The current design of ET:

- **single site** located **200-300 meters underground** in order to significantly reduce seismic noise;
- **triangular shape**, consisting of three nested detectors
  - providing redundancy
  - resolving the GW polarizations and a null stream
- **'xylophone'** configuration: each detector consists of two interferometers
  - one tuned toward **high frequencies (HF)**, and using high laser power
  - one tuned toward **low-frequency (LF)**, working at cryogenic temperatures and low laser power



ET Design Report Update 2020

ET Design Report Update 2020



# Einstein Telescope - possible designs

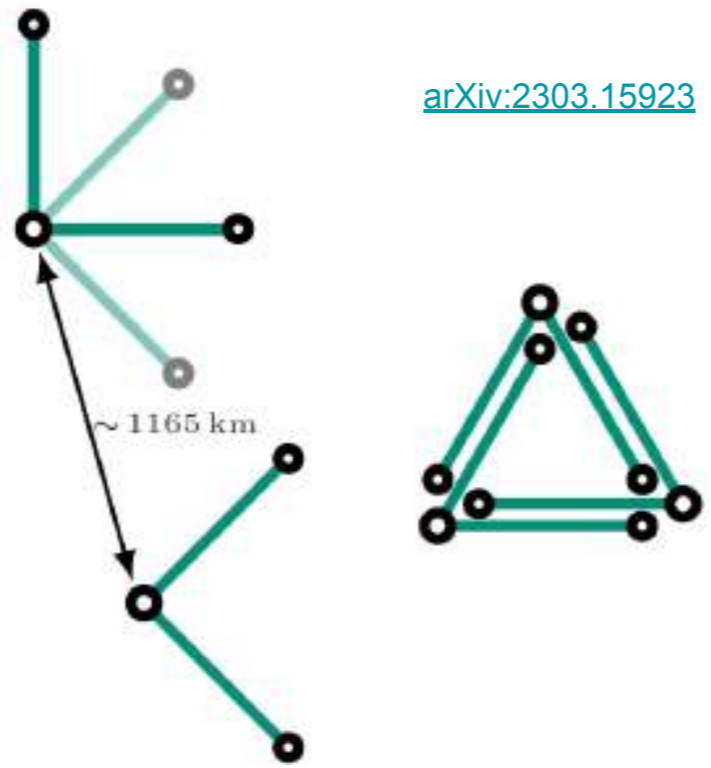
In the last years, proposals for **different designs** were made as they may bring **scientific advantages** with respect to the baseline design.

Different geometries are studied:

- 1 Triangular detector vs 2 L-shaped detector
- different arm length (10, 15, 20 km)
- Cryo LF may be challenging to achieve: HF+LF vs HF only

For **2L-shape** two different orientations are proposed:

- parallel
- 45° angle

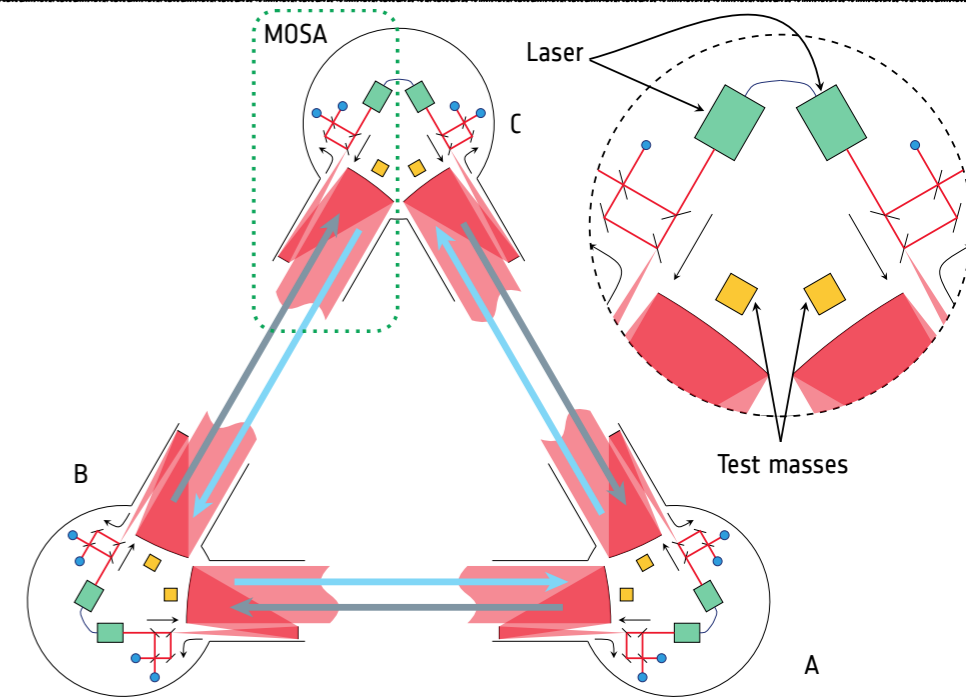
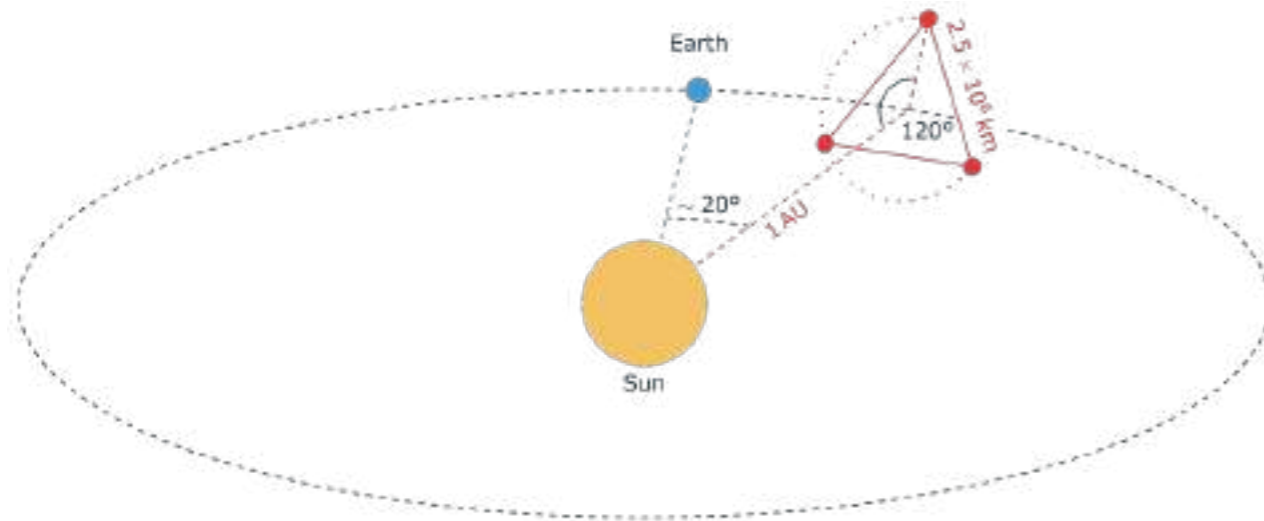


PAPER • OPEN ACCESS  
**Science with the Einstein Telescope: a comparison of different designs**  
 Marica Branchesi<sup>1,2</sup>, Michele Maggiore<sup>3,4</sup>, David Alonso<sup>5</sup>, Charles Badger<sup>6</sup>, Biswajit Banerjee<sup>1,2</sup>, Freija Beirnaert<sup>7</sup>, Enis Belgacem<sup>3,4</sup>, Swetha Bhagwat<sup>6,9</sup>, Guillaume Boileau<sup>10,11</sup>, Ssohrab Borhanian<sup>1,2</sup>  
[+ Show full author list](#)  
 Published 28 July 2023 · © 2023 The Author(s)  
[Journal of Cosmology and Astroparticle Physics, Volume 2023, July 2023](#)  
 Citation Marica Branchesi et al JCAP07(2023)068  
 DOI 10.1088/1475-7516/2023/07/068

• 197 pages  
 • 75 authors



# LISA



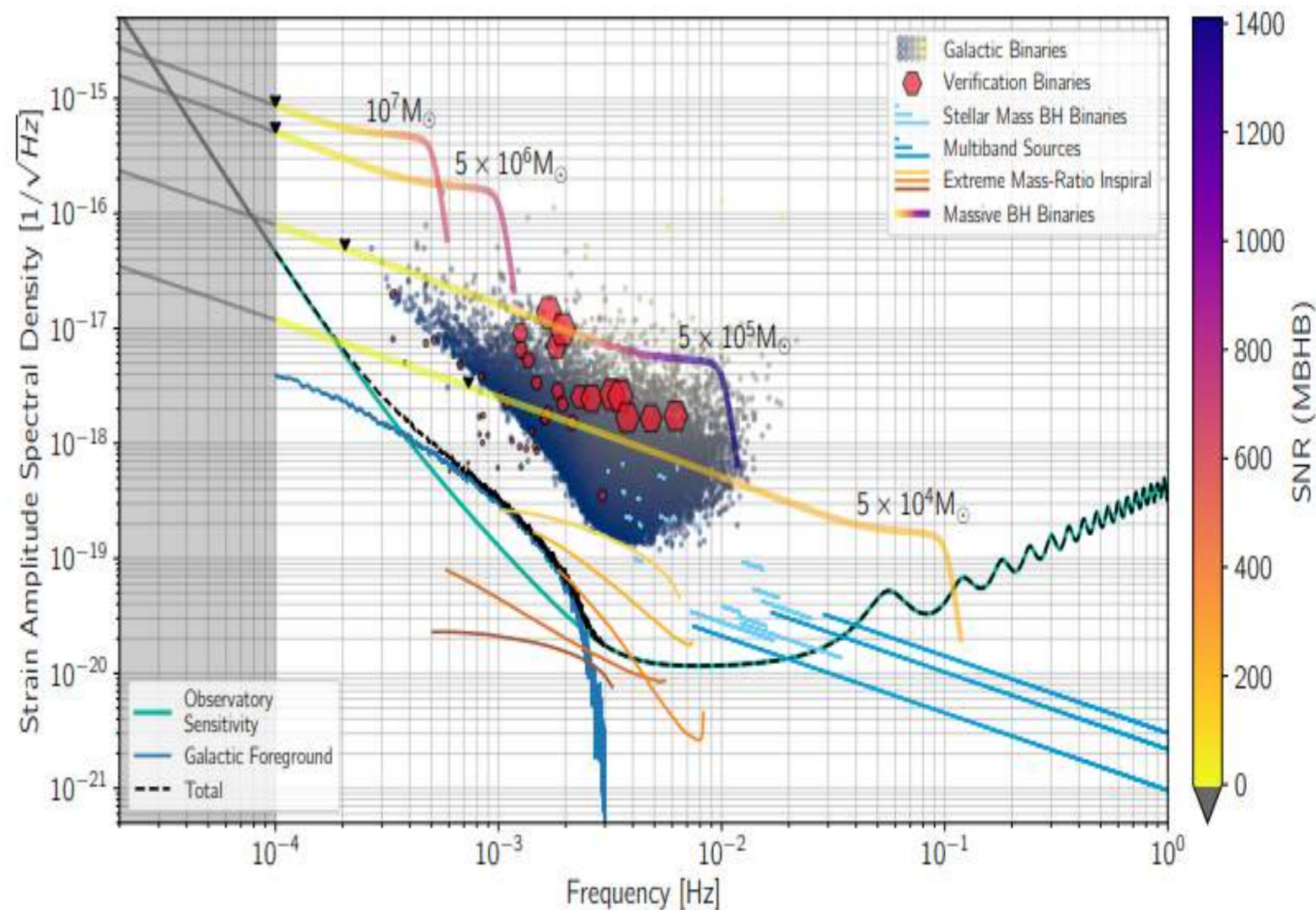
Mission	
Duration	4.5 years science orbit • >82% duty cycle • ~6.25 years including transfer and commissioning
Constellation	Three drag-free satellites forming an equilateral triangle • $2.5 \times 10^6$ km separation • trailing/leading Earth by $\sim 20^\circ$ • inclined by $60^\circ$ with respect to the ecliptic
Orbits	Heliocentric orbits • semimajor axis $\sim 1$ AU • eccentricity $e \approx 0.0096$ • inclination $i \approx 0.96^\circ$

Payload	
Lasers	2 per spacecraft • 2 W output power • wavelength 1064 nm • frequency stability $300 \text{ Hz}/\sqrt{\text{Hz}}$
Optical Bench	2 per spacecraft • double-sided use • high thermal stability (Zerodur)
Interferometry	heterodyne interferometry • $15 \text{ pm}/\sqrt{\text{Hz}}$ precision • Inter-spacecraft ranging to $\sim 1$ m
Telescope	2 per spacecraft • 30 cm off-axis telescope • high thermal stability
Gravitational Reference System	2 per spacecraft • acceleration noise $< 3 \text{ fm}/(\text{s}^2 \sqrt{\text{Hz}})$ • 46 mm cubic AuPt test mass • Faraday cage housing • electrostatic actuation in 5 degree of freedom

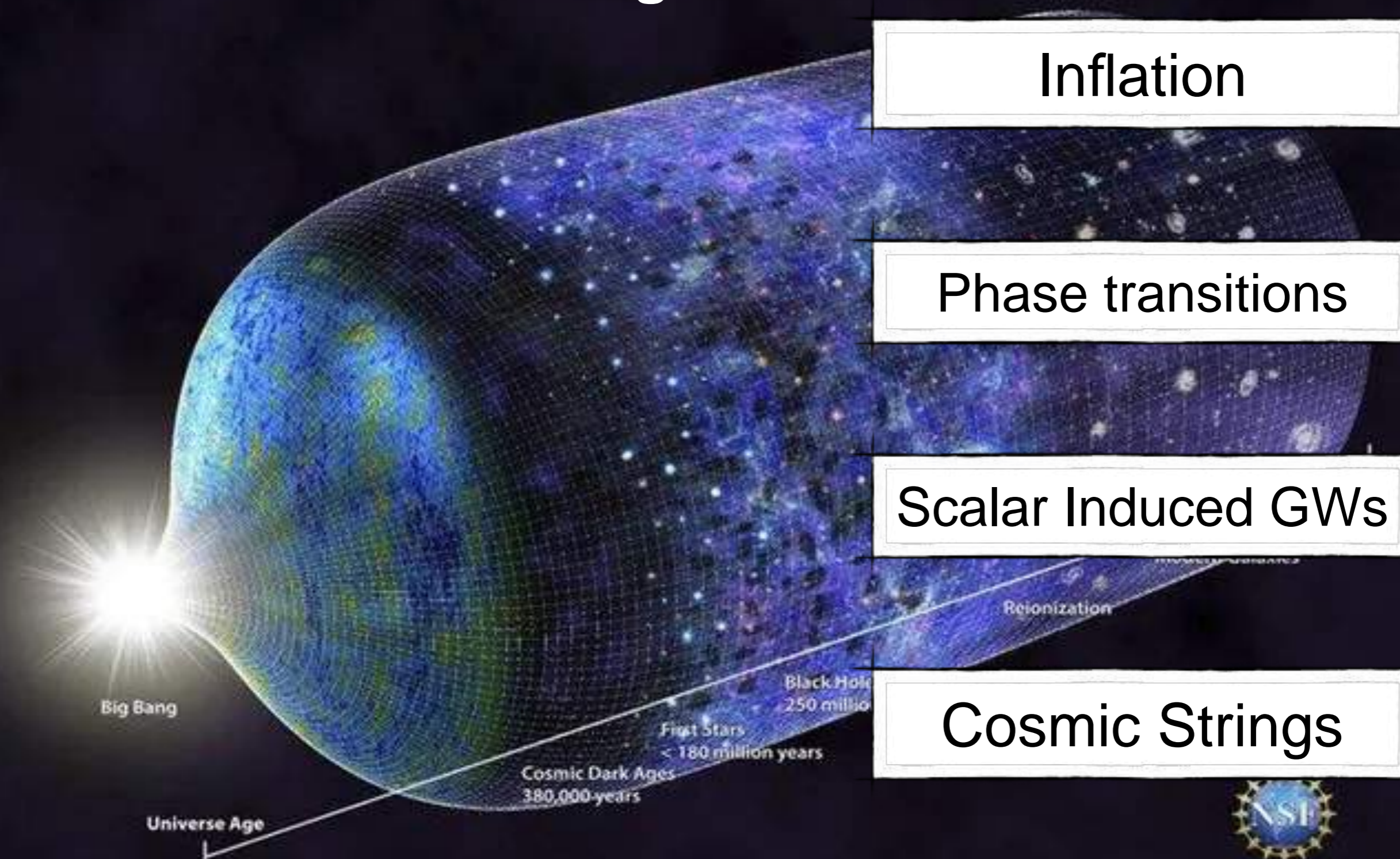
# LISA - Science Objectives

## Science Objectives

- Study the formation and evolution of **compact binary stars** and the structure of the Milky Way Galaxy
- Trace the origins, growth and merger histories of **massive Black Holes** across cosmic epochs
- Probe the properties and immediate environments of Black Holes in the local Universe using **extreme mass-ratio inspirals** and **intermediate mass-ratio inspirals**
- Understand the astrophysics of **stellar-mass Black Holes**
- Explore the **fundamental nature of gravity** and Black Holes
- Probe the rate of **expansion of the Universe** with standard sirens
- Understand **stochastic gravitational wave backgrounds** and their implications for the early Universe and TeV-scale particle physics
- Search for gravitational wave bursts and **unforeseen sources**

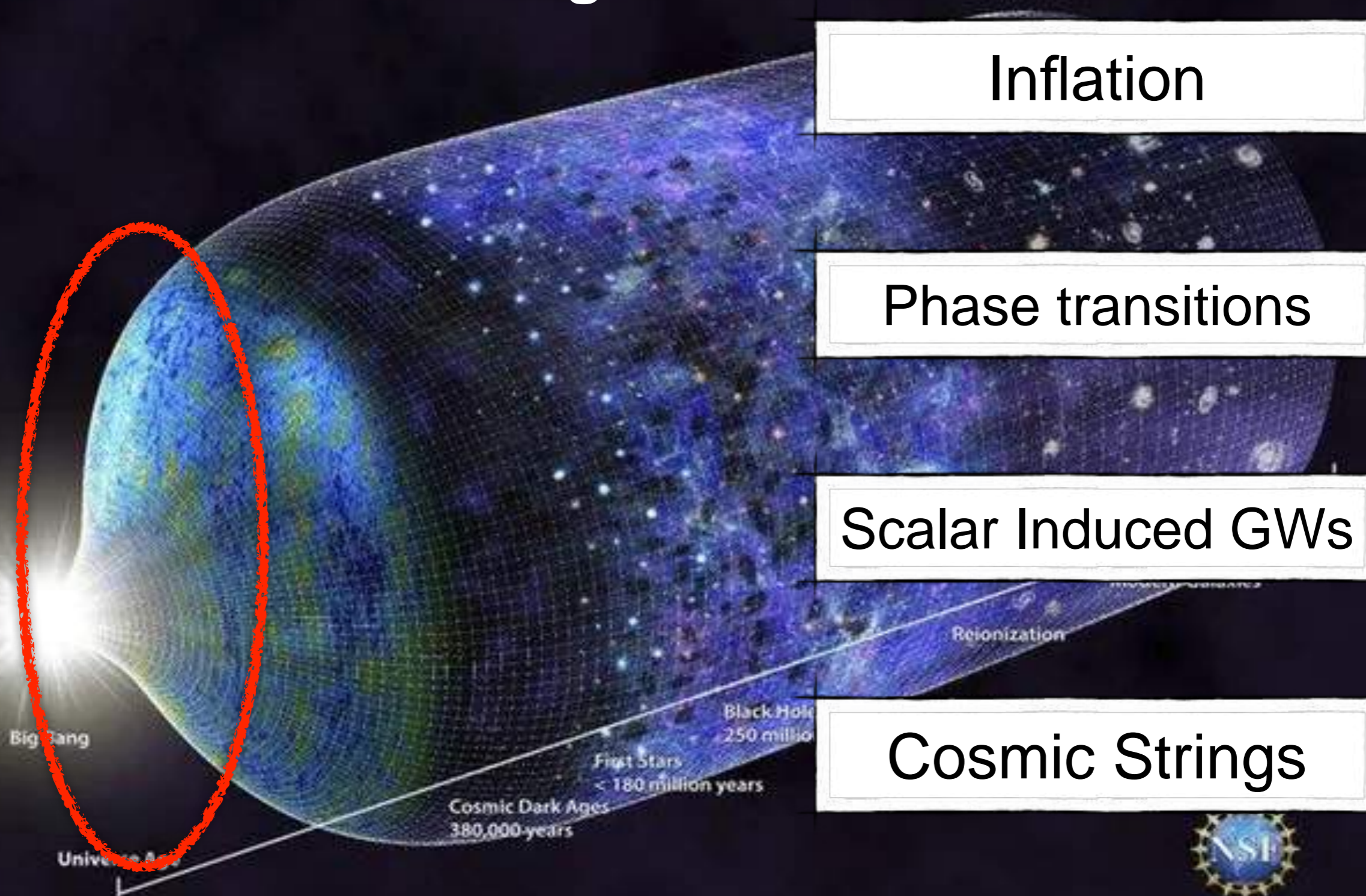


## Cosmological Sources



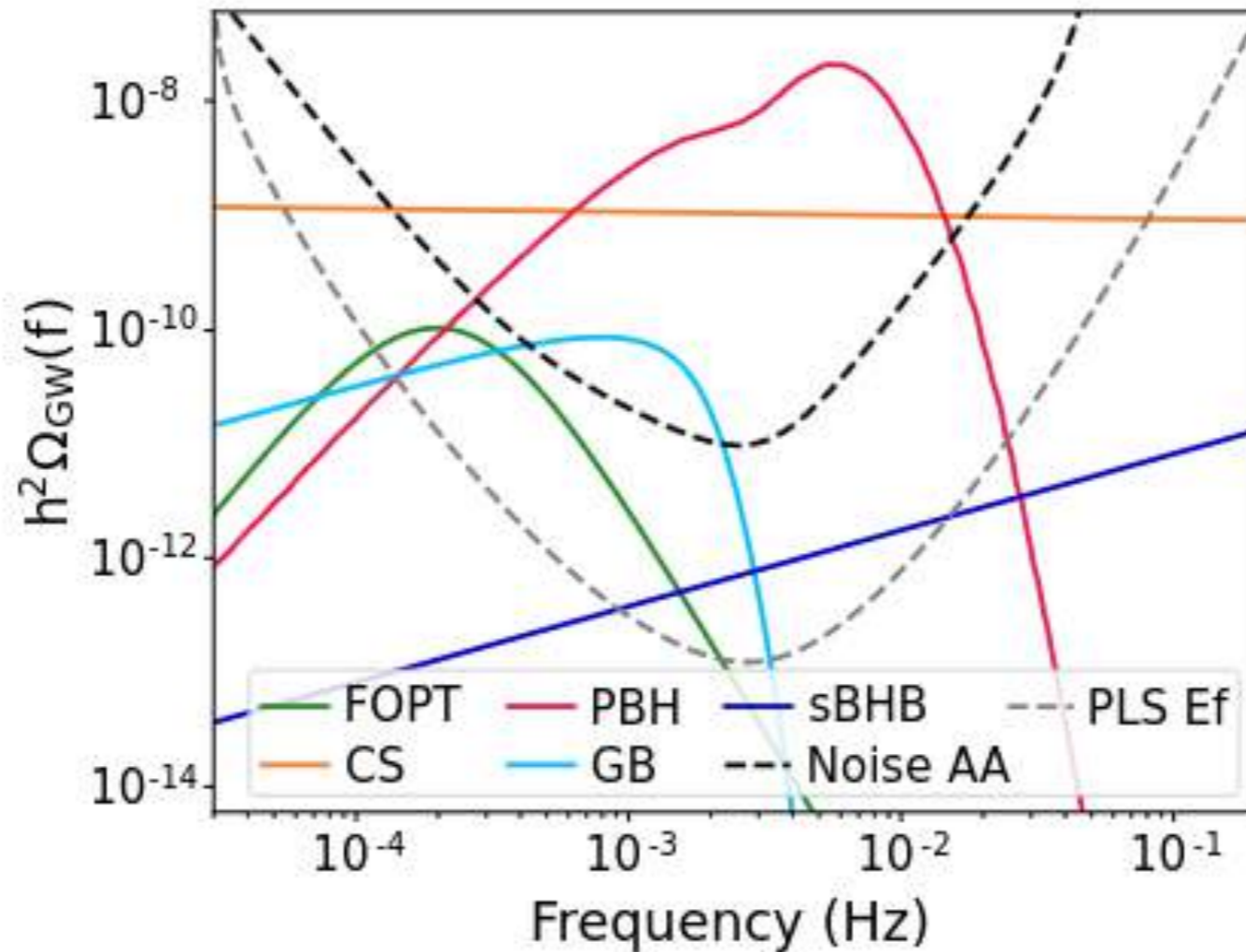
Stochastic (i.e., *persistent, incoherent*) GWB of cosmological origin: **probe of the early Universe at energy scales above the ones achievable at current particle colliders**

## Cosmological Sources



Stochastic (i.e., *persistent, incoherent*) GWB of cosmological origin: **probe of the early Universe at energy scales above the ones achievable at current particle colliders**

# GWB sources in the LISA band

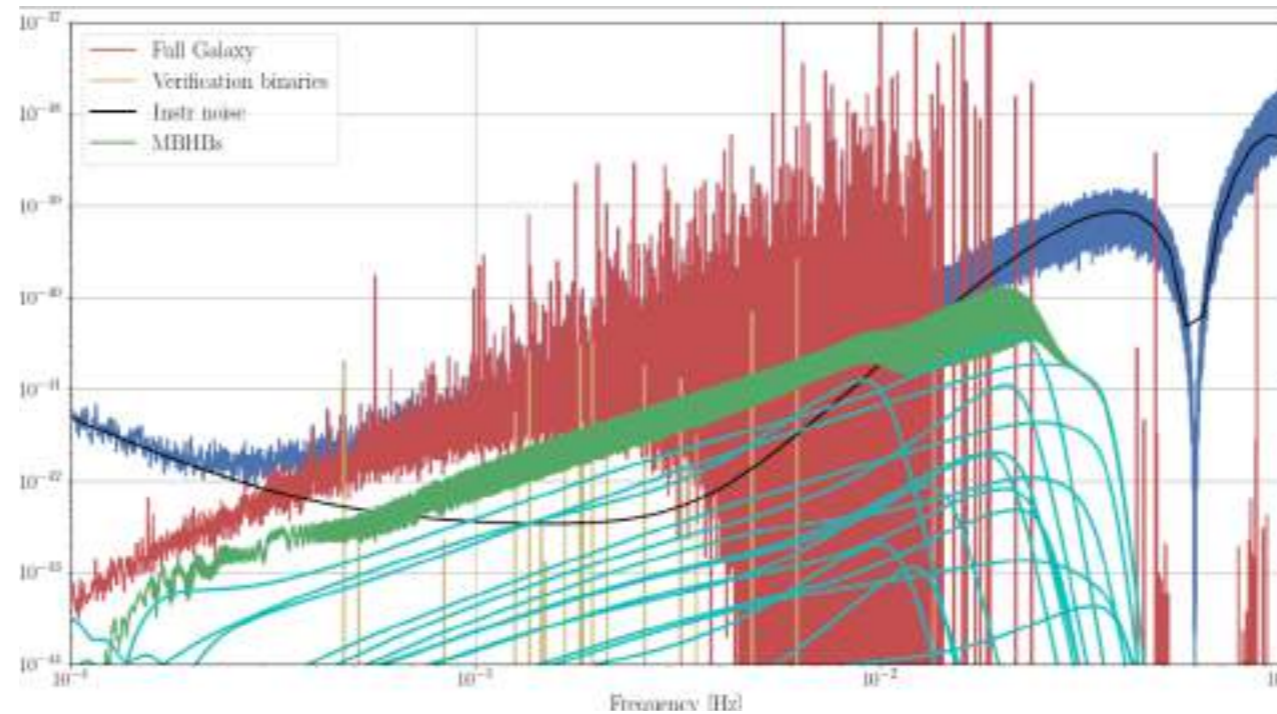
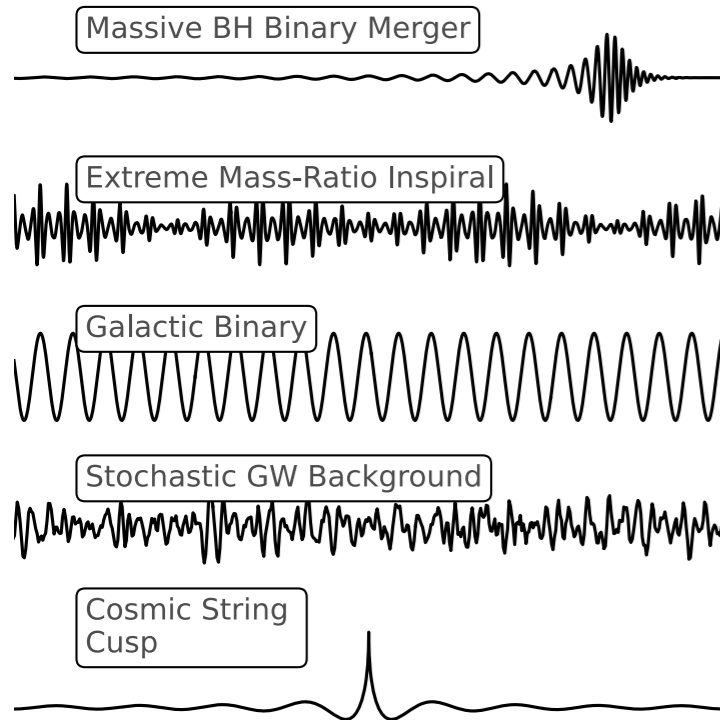


\* Figure from M. Colpi et al., ArXiv:2402.07571

At least two AGWB components (sBHBs and CGBs) are guaranteed signals for LISA!

The Cosmological GWB would be an invaluable source of information for **HEP**

# How to approach the data analysis



- Large number of overlapping sources
- Residuals from sources subtraction
- Confusion from unresolved sources
- One doesn't cross-correlate like LVK
- Prediction of the instrumental noise?
- Artefacts: gaps in the data, glitches...

LISA Ground Segment - Italia

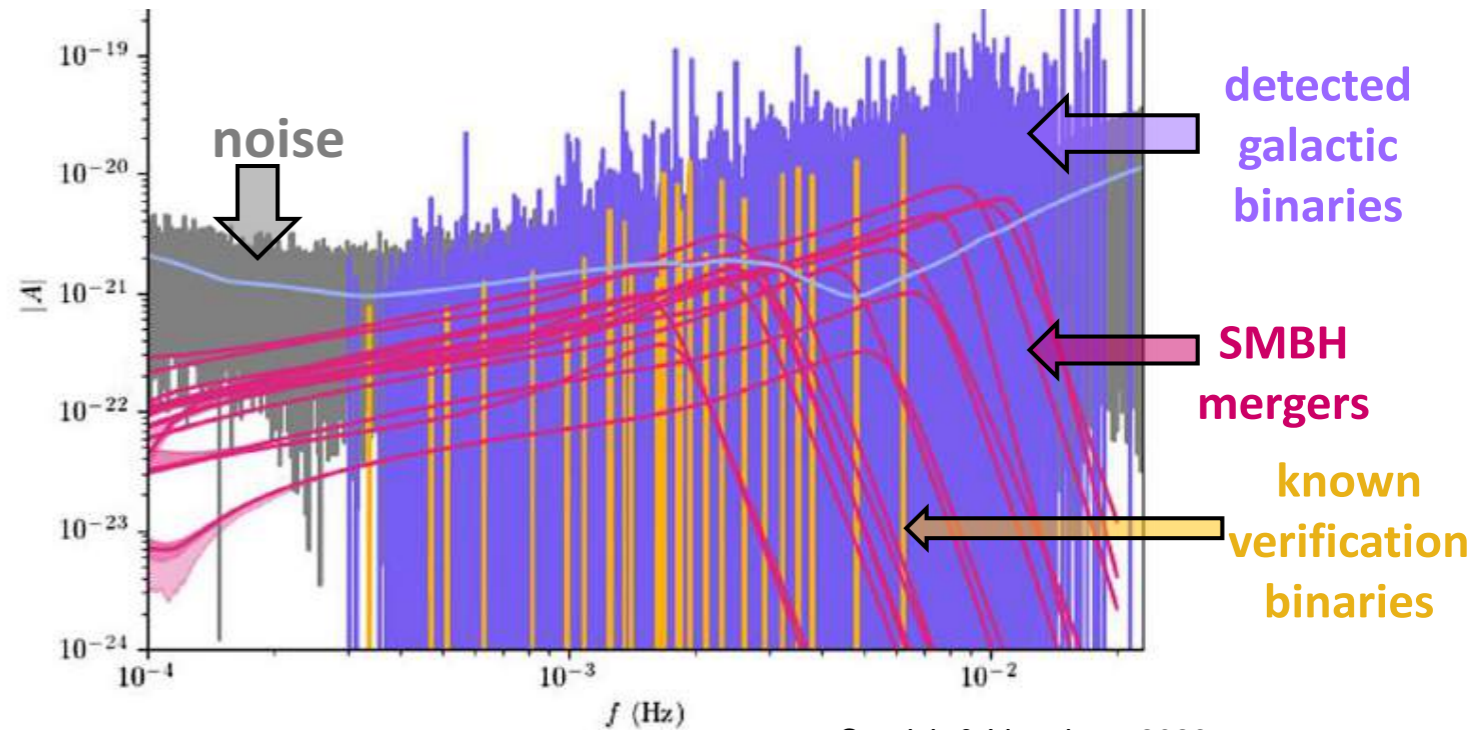
**Develop a GLOBAL FIT pipeline for LISA**

(Similar approach for ET)

Cornish, Littenberg, 23

Marsat+,24

Katz, Karnesis et al, '23



Cornish & Littenberg 2023



# Template Search Algorithm - SGWBinner

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SURVEY (INFLATIONARY) MODELS



GROUP THEM ACCORDING TO THE SPECTRAL  
SHAPE OF  $\Omega_{\text{GW}}$



BUILD A TEMPLATE BANK



FORECAST CONSTRAINTS ON TEMPLATES  **SGWBinner  
Code**



DRAW CONCLUSIONS ABOUT EARLY UNIVERSE  
PHYSICS [LISA CosGW project Arxiv: 2407.04356 \(inflation\)](https://arxiv.org/abs/2407.04356)

# Template-based reconstruction with SGWBinner

Based on two LISA COSWG projects:

1906.09244: C. Caprini, D. Figueroa, R. Flauger, M. Pieroni, G. Nardini, M. Peloso, A. R., G. Tasinato

2009.11845: R. Flauger, N. Karnesis, G. Nardini, M. Pieroni, A. Ricciardone, J. Torrado

We look for **best approximation** of the signal **with a multi-PL**

$$h^2 \Omega_{\text{GW}}(f, \vec{\theta}) = \sum_i 10^{\alpha_i} \left( \frac{f}{\sqrt{f_{\min,i} f_{\max,i}}} \right)^{p_i} \Theta(f - f_{\min,i}) \Theta(f_{\max,i} - f)$$

$N$  bins  $\rightarrow 4N (f_{\min,i}, f_{\max,i}, \alpha_i, p_i) + 2N$  (noise) parameters.

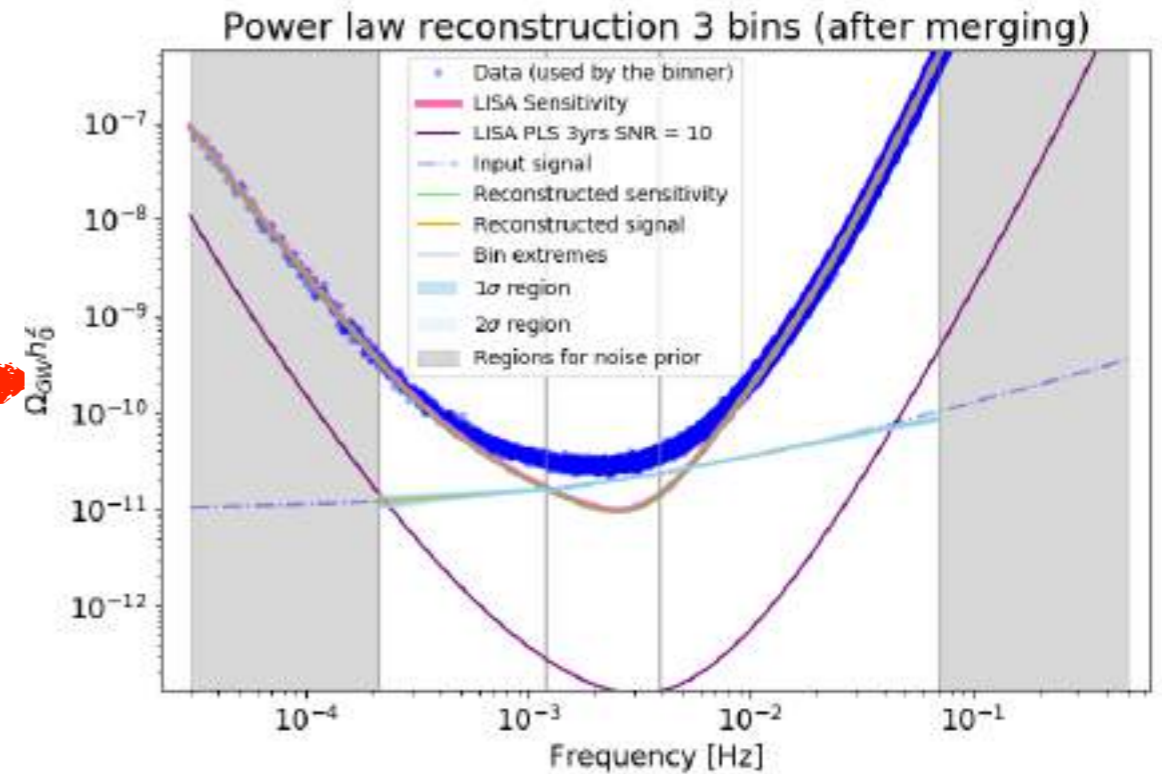
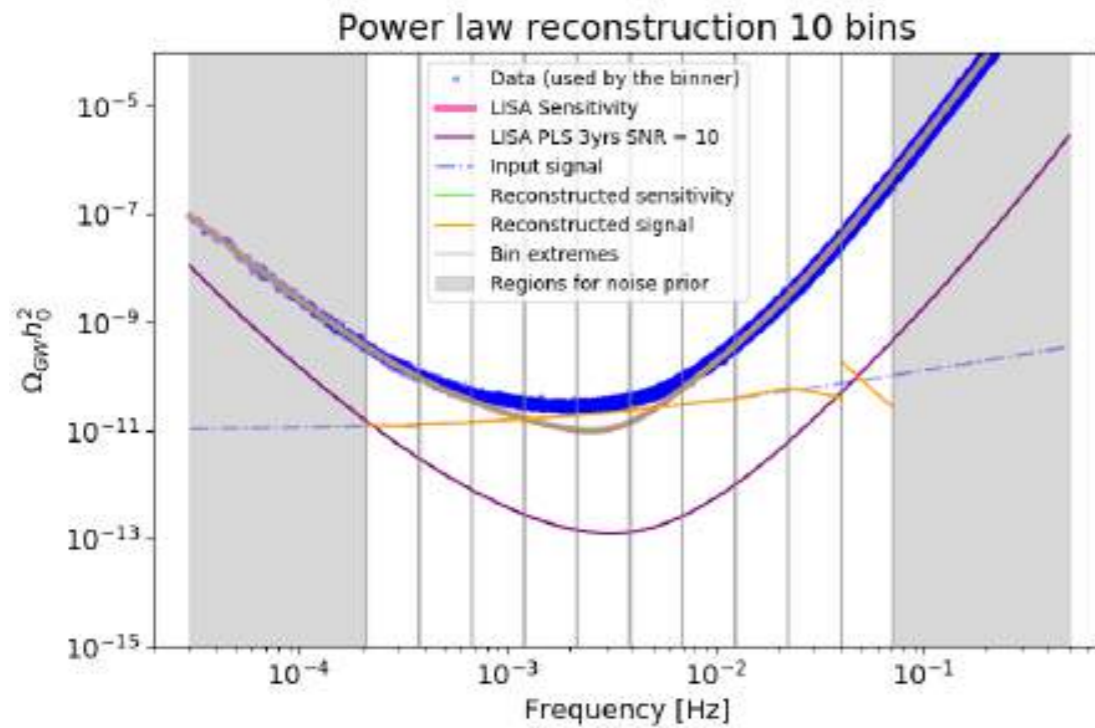
The basic procedure is composed of four steps

- Build a robust prior for the noise model (to force bin-by-bin measurements)
- Split the frequency range in a set of bins and reconstruct the signal
- Merge as many bins as possible (to avoid overfitting)
- Define a procedure to compute the error on the reconstruction
- Final MCMC run with common noise parameters

# SGWBinner code algorithm

Simple application

$$h^2 \Omega_{\text{GW}}(f) = h^2 \Omega_{\text{GW, const}}(f) + h^2 \Omega_{\text{GW, BHB+NSB}}(f) = 10^{-11} + 5.4 \times 10^{-12} \left( \frac{f}{0.001} \right)^{2/3}$$



After the merging procedure only 3 bins with small error bands are left.

# Template-based reconstruction with SGWBinner

LISA will provide three time-domain data streams  $d_i(t)$  that we divide in segment of duration  $\tau$  and then we Fourier transform

$$\tilde{d}_i(f) = \int_{-\tau/2}^{\tau/2} d_i(t) e^{-2\pi i f t} dt \quad \mathbf{l = LISA channel (X, Y, Z or A, E, T)}$$

$$\tilde{d}_i(f) = \sum_{\nu} \tilde{n}_i^{\nu}(f) + \sum_{\sigma} \tilde{s}_i^{\sigma}(f)$$

Resolved sources  $\langle \tilde{d}_i(f) \rangle \neq 0$

**SGWB**

$$\langle \tilde{d}_i(f) \rangle = 0, \quad \langle \tilde{d}_i(f) \tilde{d}_j^*(f') \rangle = \frac{\delta(f - f')}{2} \left[ \sum_{\nu} P_{N,ij}^{\nu}(f) + \sum_{\sigma} P_{S,ij}^{\sigma}(f) \right]$$

noise and signal power spectra

# Time Delay Interferometry

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**Time-Delay Interferometry (TDI) is employed to suppress laser frequency noise**

TDI is a post-processing technique that, combining measurements performed at different times, produces synthesized data streams representing laser-noise-free virtual interferometers.

$\eta_{ij}(t)$  phase measurement performed in spacecraft  $i$  at time  $t$  of a signal emitted from spacecraft  $j$  at time  $t - L_{ij}$

$L_{ij}$  Distance between spacecraft

$D_{ij}$  delay operator defined as  $D_{ij}x(t) \equiv x(t - L_{ij})$

**In practice, TDI consists in defining variables as a linear combination of single-link measurements and delay operators.**

The most common choice for LISA data analysis is to use three Michelson-like variables  $X, Y, Z$

$$X = (1 - D_{13}D_{31})(\eta_{12} + D_{12}\eta_{21}) + (D_{12}D_{21} - 1)(\eta_{13} + D_{13}\eta_{31})$$

# Time Delay Interferometry

Then you can build (quasi-)orthogonal channels

$$A = \frac{Z - X}{\sqrt{2}}, \quad E = \frac{X - 2Y + Z}{\sqrt{6}}, \quad T = \frac{X + Y + Z}{\sqrt{3}}$$

the T channel strongly suppresses GW signals compared to instrumental noise,

Assuming TDI suppresses laser noise, the main residual noise sources for LISA (also known as secondary noises) are:

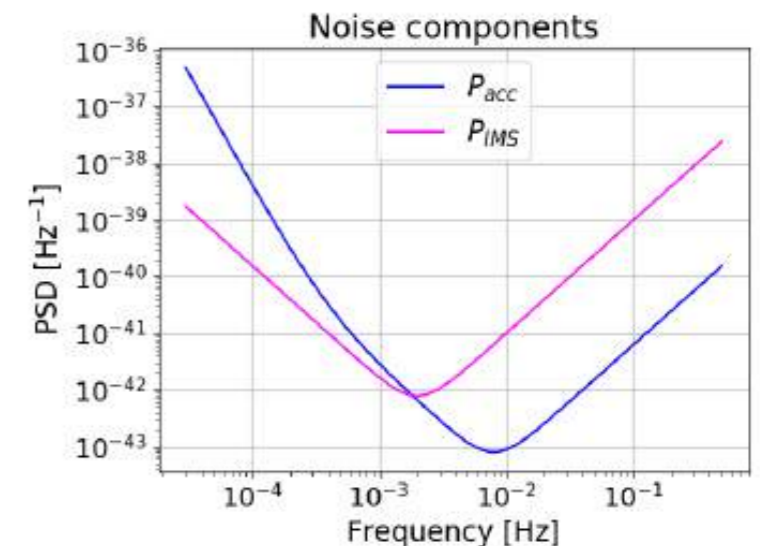
- **Test Mass (TM) noise**, representing deviations from free-fall in the TM trajectories,
- **Optical Metrology System (OMS) noise**, representing uncertainties in the determination of the TM positions.

$$P_{N,ij}(f) \equiv \sum_{\nu} P_{N,ij}^{\nu}(f) = [T_{ij,lk}^{\text{TM}}(f)S_{lk}^{\text{TM}}(f) + T_{ij,lk}^{\text{OMS}}(f)S_{lk}^{\text{OMS}}(f)]$$

With

$$S_{lk}^{\text{TM}}(f) = A_{lk}^2 \left(1 + \left(\frac{0.4\text{mHz}}{f}\right)^2\right) \left(1 + \left(\frac{f}{8\text{mHz}}\right)^4\right) \left(\frac{1}{2\pi fc}\right)^2 \left(\frac{\text{fm}^2}{\text{s}^3}\right)$$

$$S_{lk}^{\text{OMS}}(f) = P_{lk}^2 \left(1 + \left(\frac{2 \times 10^{-3}\text{Hz}}{f}\right)^4\right) \times \left(\frac{2\pi f}{c}\right)^2 \times \left(\frac{\text{pm}^2}{\text{Hz}}\right)$$



# Signal Power Spectral Density

$$P_{S,ij}(f) \equiv \sum_{\sigma} P_{S,ij}^{\sigma}(f) = \mathcal{R}_{ij}(f) [S_{\text{Gal}}(f) + S_{\text{Ext}}(f) + S_{\text{Cosmo}}(f)]$$

**Spectral density for galactic binaries**

$$h^2\Omega_{\text{GW}}(f) \equiv \frac{4\pi^2 f^3}{3H_0^2} S_h(f)$$

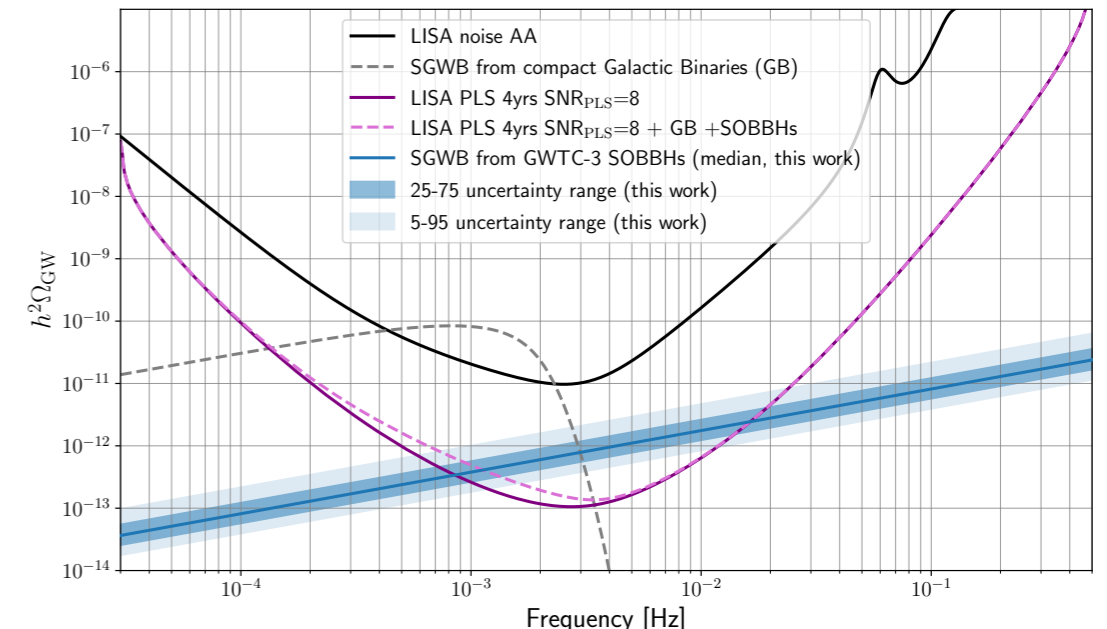
$$S_{\text{Gal}}(f) = A_{\text{Gal}} \left( \frac{f}{1 \text{ Hz}} \right)^{-\frac{7}{3}} \times e^{-(f/f_1)^\alpha} \times \frac{1}{2} \left[ 1 + \tanh \frac{f_{\text{knee}} - f}{f_2} \right]$$

$$\log_{10}(f_1) = a_1 \log_{10}(T_{\text{obs}}) + b_1$$

$$\log_{10}(f_{\text{knee}}) = a_k \log_{10}(T_{\text{obs}}) + b_k$$

**Spectral density for Stellar Origin Black Holes**

$$h^2\Omega_{\text{Ext}} = 10^{\log_{10}(h^2\Omega_{\text{Ext}})} \left( \frac{f}{0.001 \text{ Hz}} \right)^{2/3}$$



S. Babak et al., '23

# Data Generation and Likelihood

We generate  $N_d$  Gaussian realisations for the signal and all noise components, with zero mean and variances defined by their respective power spectral densities.

We perform a Fourier transform in each of the segments to get the frequency domain data

$$\bar{D}_{ij}^k \equiv \sum_{s=1}^{N_d} \tilde{d}_i^s(f_k) \tilde{d}_j^s(f_k) / N_d$$

We coarse grain and average over segments

## Full Likelihood

$$\ln \mathcal{L}(\vec{\theta}) = \frac{1}{3} \ln \mathcal{L}_G(\vec{\theta} | D_{ij}^k) + \frac{2}{3} \ln \mathcal{L}_{LN}(\vec{\theta} | D_{ij}^k)$$

$$\ln \mathcal{L}_G(\vec{\theta} | D_{ij}^k) = -\frac{N_d}{2} \sum_k \sum_{i,j} w_{ij}^k \left[ 1 - D_{ij}^k / D_{ij}^{Th}(f_{ij}^k, \vec{\theta}) \right]^2$$

$$\ln \mathcal{L}_{LN}(\vec{\theta} | D_{ij}^k) = -\frac{N_d}{2} \sum_k \sum_{i,j} w_{ij}^k \ln^2 \left[ D_{ij}^{Th}(f_{ij}^k, \vec{\theta}) / D_{ij}^k \right]$$

to take into account the mild non-Gaussianity introduced by the data generation, and to avoid biased results

[S. Hamimeche and A. Lewis 0801.0554]

$D_{ij}^{Th}(f_k, \vec{\theta})$  theoretical model for the data (containing both signal and noise)



# Forecast parameters

Parameters  $\vec{\theta} = \{\vec{\theta}_s, \vec{\theta}_n\}$

$$\vec{\theta}_n = \{A, P\}, \quad \vec{\theta}_s = \{\vec{\theta}_{\text{fg}}, \vec{\theta}_{\text{cosmo}}\}$$

Astrophysical parameters

$$\vec{\theta}_{\text{fg}} = \{h^2 \Omega_{\text{Gal}}, h^2 \Omega_{\text{Ext}}\}$$

Cosmological parameters

$$\vec{\theta}_{\text{cosmo}} \quad (\text{template dependent})$$

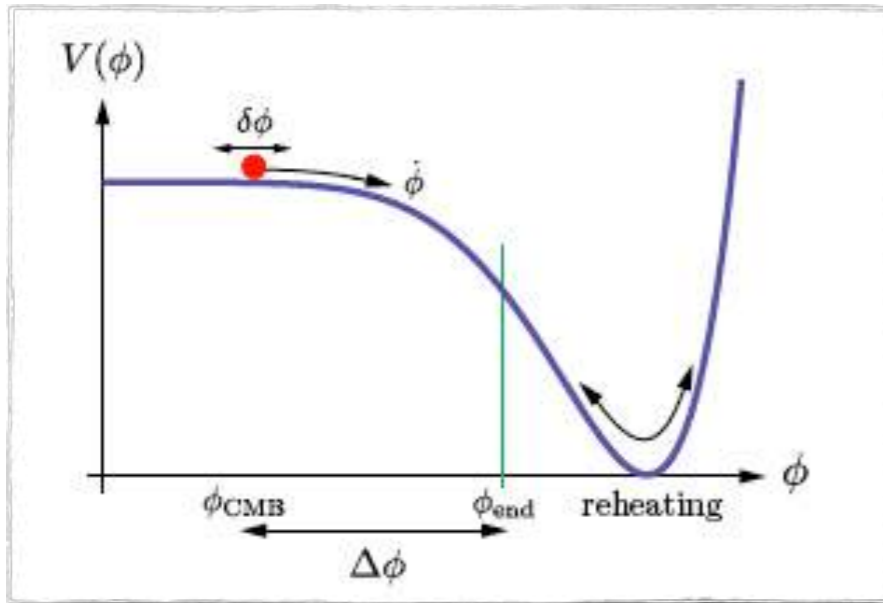
## Fisher Matrix

$$F_{\alpha\beta} \equiv T_{\text{obs}} \sum_{i \in \{\text{AET}\}} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\partial \ln C_{ii}}{\partial \theta^\alpha} \frac{\partial \ln C_{ii}}{\partial \theta^\beta} df$$

$$C_{ij}(f_k, \theta_0) = \tilde{d}_i^k \tilde{d}_j^{k*}$$

$$\text{SNR} \equiv \sqrt{T_{\text{obs}} \sum_{i \in \{\text{AET}\}} \int_{f_{\text{min}}}^{f_{\text{max}}} \left( \frac{S_{i,\text{GW}}}{S_{i,\text{N}}} \right)^2 df}$$

# Inflation and Primordial GWs



- Period of accelerated (exponential) expansion driven by a scalar field (inflaton) that rolls down on its flat potential

Solve Standard Big-Bang shortcomings

Generation of TENSOR and scalar perturbations

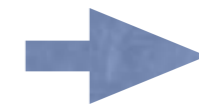
Stretches the microphysics scales to super-horizon sizes

GW are represented by tensor perturbation  $h_{ij}$  of the FLRW metric

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

Transverse and Traceless

$$\partial_i h^i_j = h_{ii} = 0$$

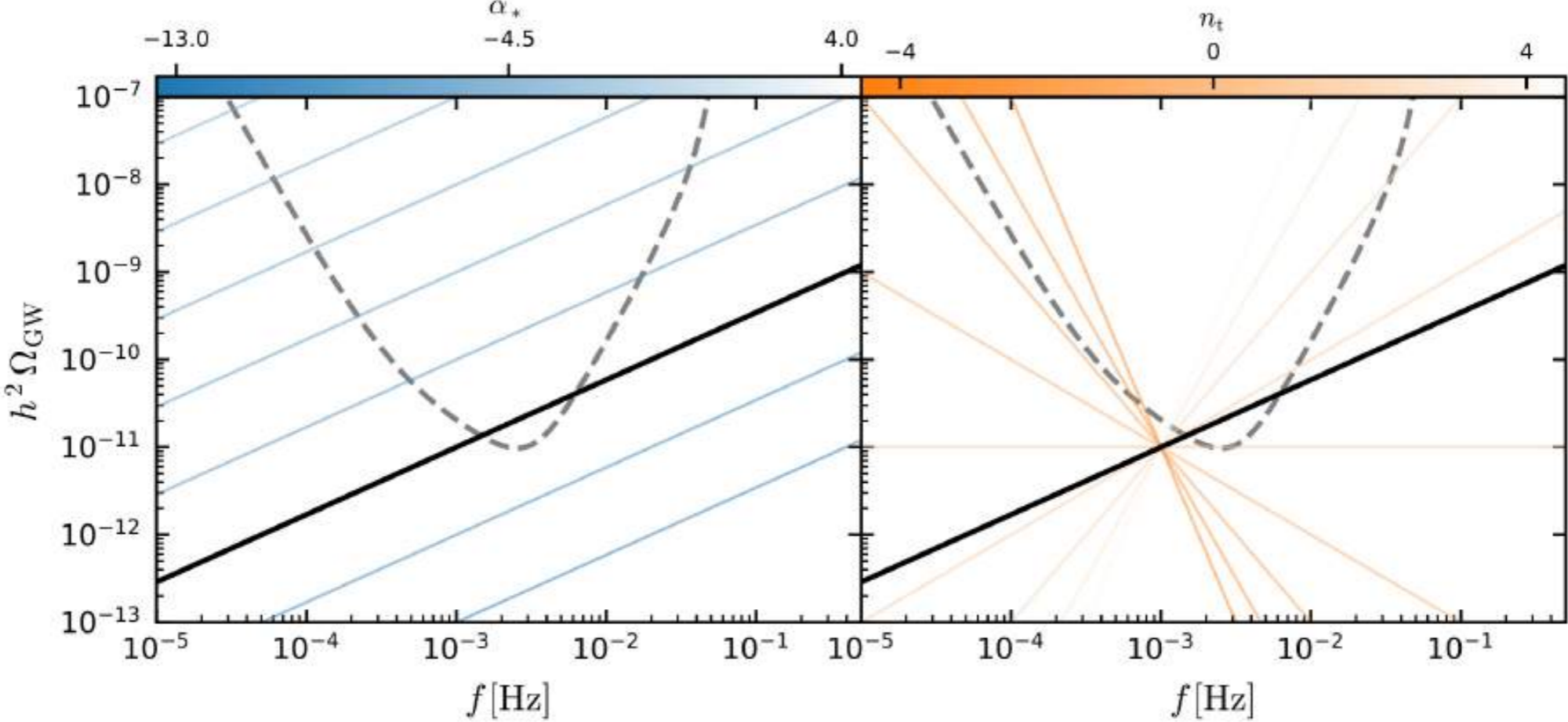


**2 D.O.F**  
(2 polarizations)

# Power Law Template

$$h^2 \Omega_{\text{GW}}^{\text{PL}}(f, \vec{p}) = 10^{\alpha_*} \left( \frac{f}{f_*} \right)^{n_t}$$

$$\vec{p} = \{\alpha_*, f_*, n_t\}$$



# Inflationary models producing such a signal

## Axion inflation:

$$\mathcal{L} \supset -\frac{\varphi}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\xi \equiv \frac{\dot{\varphi}}{2fH}$$

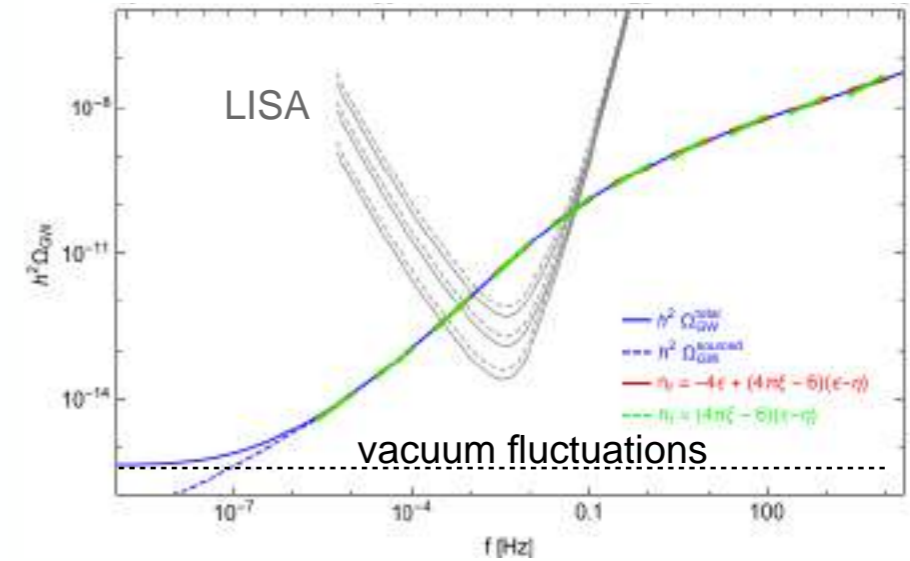
[Barnaby & Peloso 1011.1500]

[Sorbo 1101.1525]

The rolling axion strongly amplifies the gauge field, which in turn produces a strong SGWB.

See M. Peloso's talk

GW energy spectrum today



$$h^2 \Omega_* \simeq 1.5 \times 10^{-13} \frac{H_*^4}{\Lambda^4} \frac{e^{4\pi\xi_*}}{c^6}$$

$$n_t \simeq -4\epsilon_* + (4\pi\xi_* - 6)(\epsilon_* - \eta_*)$$

## Broken space diffeomorphisms:

[Ricciardone & Tasinato 1611.04516, 1711.02635]

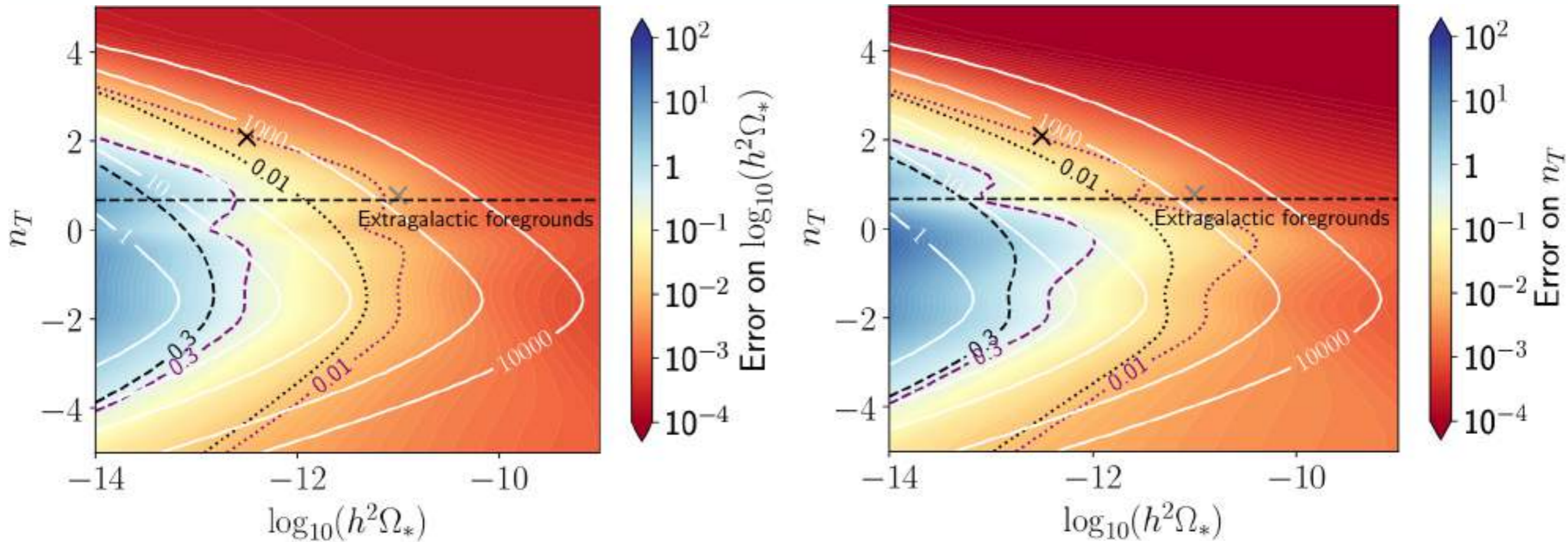
[Fujita et al 1808.02381]

The breaking of space diffeomorphisms can give rise to a **massive graviton during the inflationary epoch** which tilts the SGWB spectrum towards the **blue**

$$h^2 \Omega_* = \text{Model dependent}$$

$$n_t \simeq \frac{2}{3} \frac{m_h^2}{H_*^2} > 0$$

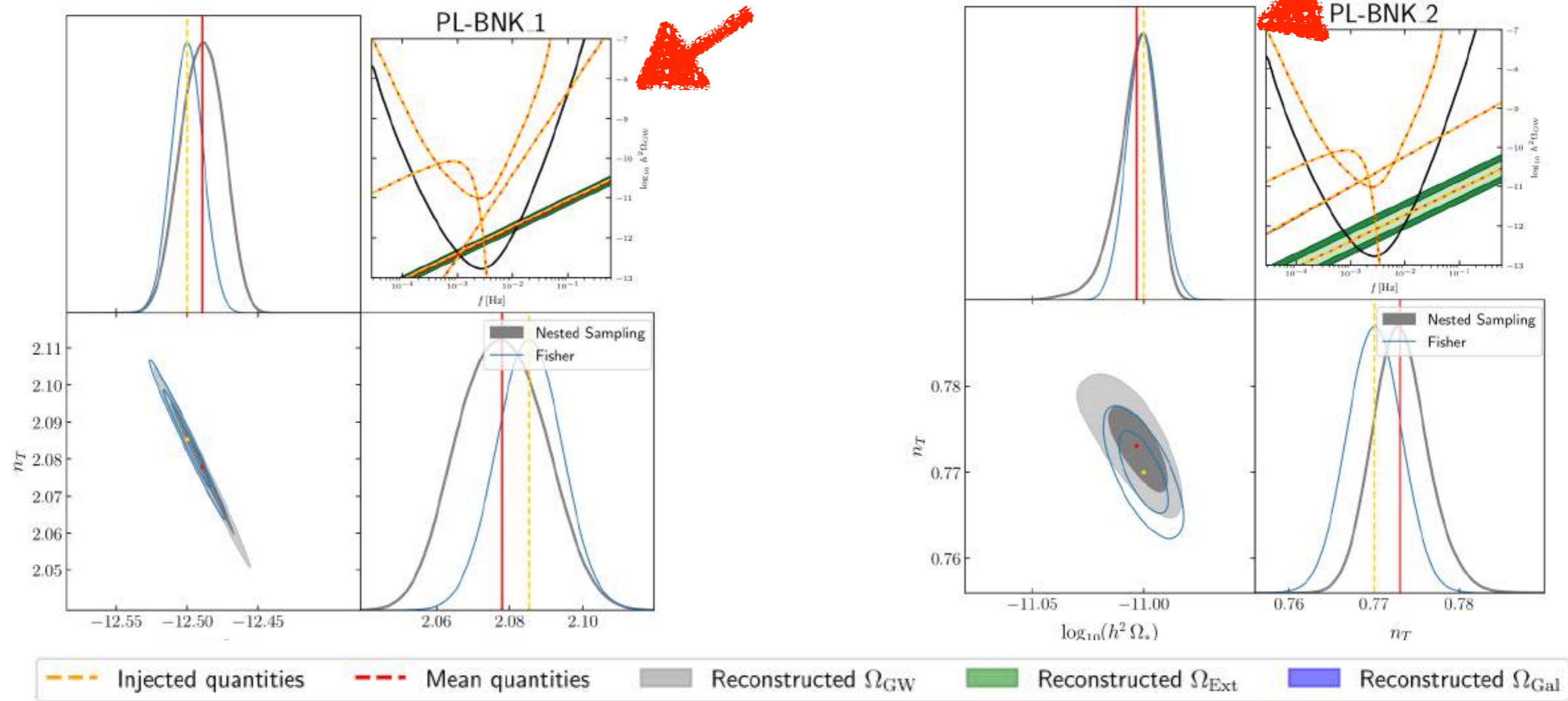
# (Fisher) Forecast for Power Law GWB



- For a flat signal (i.e.,  $n_T = 0$ ), an accuracy  $\sigma \approx O(0.01)$  on the logarithm of the amplitude requires  $h^2\tilde{\Omega}_* \gtrsim 10^{-12}$  without foregrounds
- In presence of foregrounds only slightly larger errors.
- Peculiar behavior along the line at  $n_T = 2/3$ , where the primordial SGWB is degenerate with the foreground due to the extragalactic compact binaries.

# (Nested Sampling) Forecast

Predictive posterior distribution of noise, foregrounds and primordial signal.



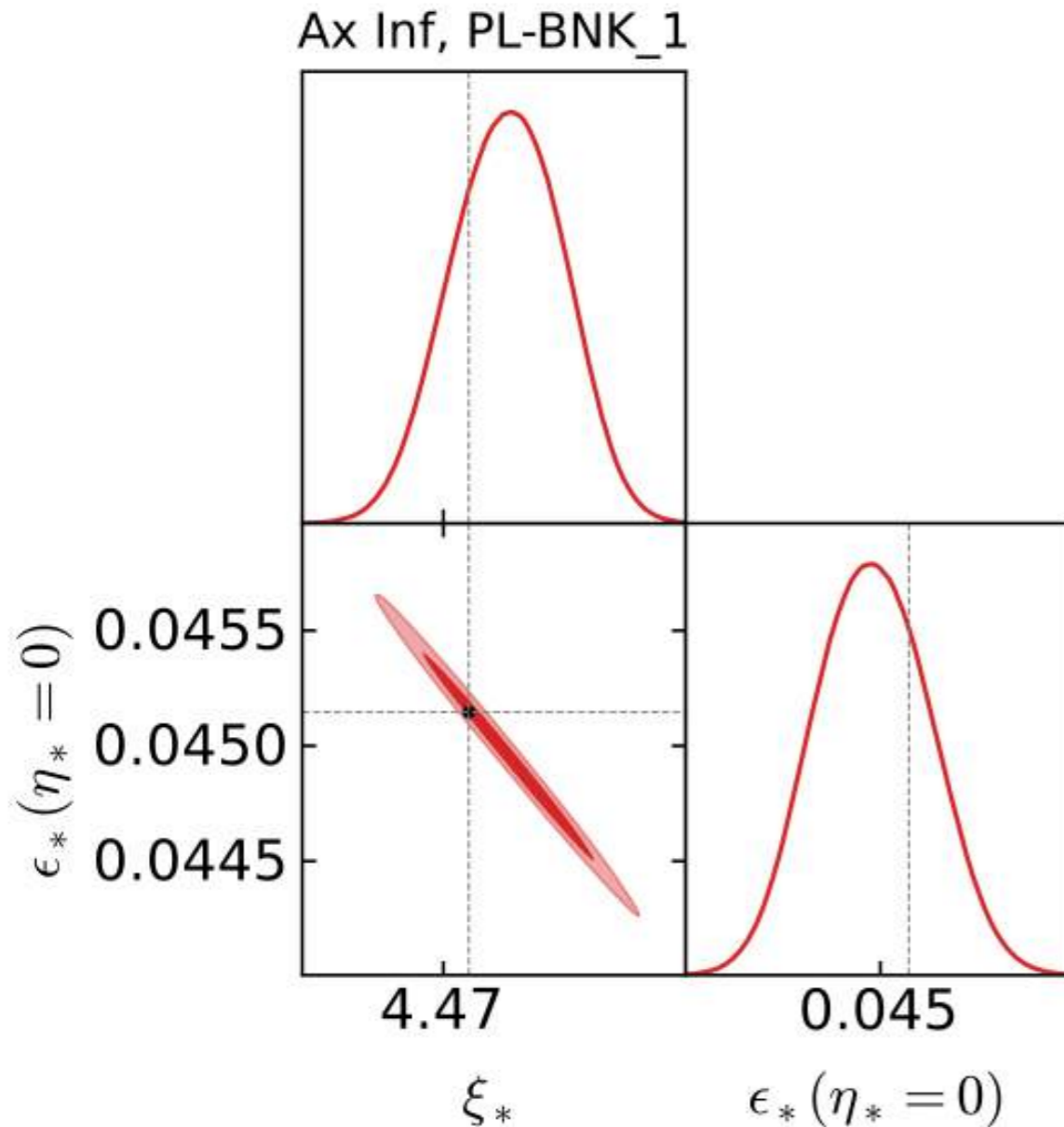
the injected values of both benchmarks are reconstructed well within the 68 % CL contours

For the galactic foreground, the reconstruction is very accurate, with the reconstructed amplitude within the 68 % CL error band (recall that we vary only the amplitude in our analysis, keeping the spectral shape of foregrounds fixed) while the error bands on the extragalactic foreground are larger, but still within the 68 % CL error band

# Benchmark 1: $\{\log_{10}(h^2 \Omega_*), n_T\} = \{-12.5, 2.085\}$

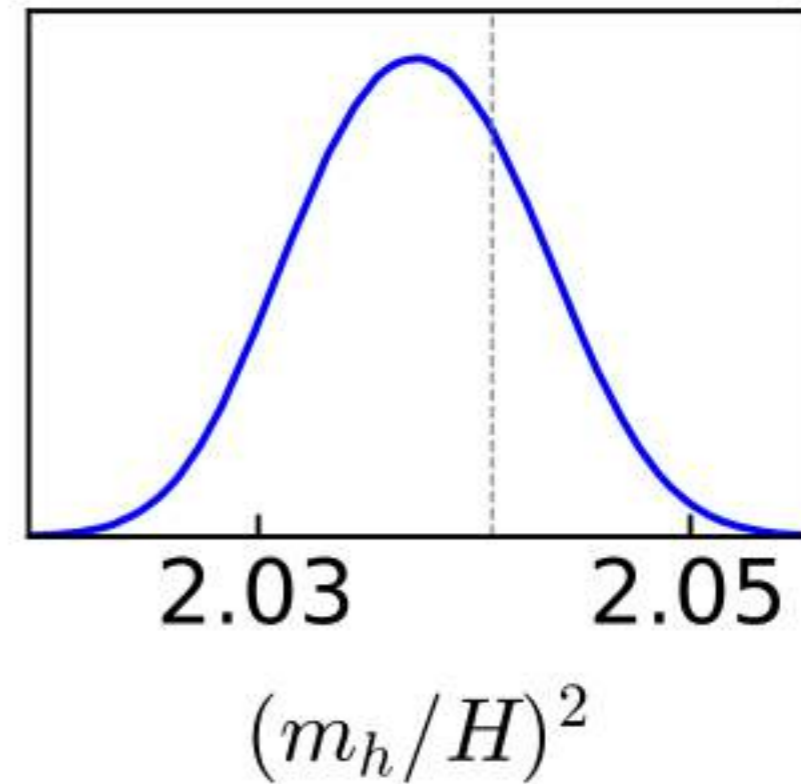
The parameter reconstruction allows to set tight constraints on axion inflation and massive-gravity inflation.

## Axion inflation:



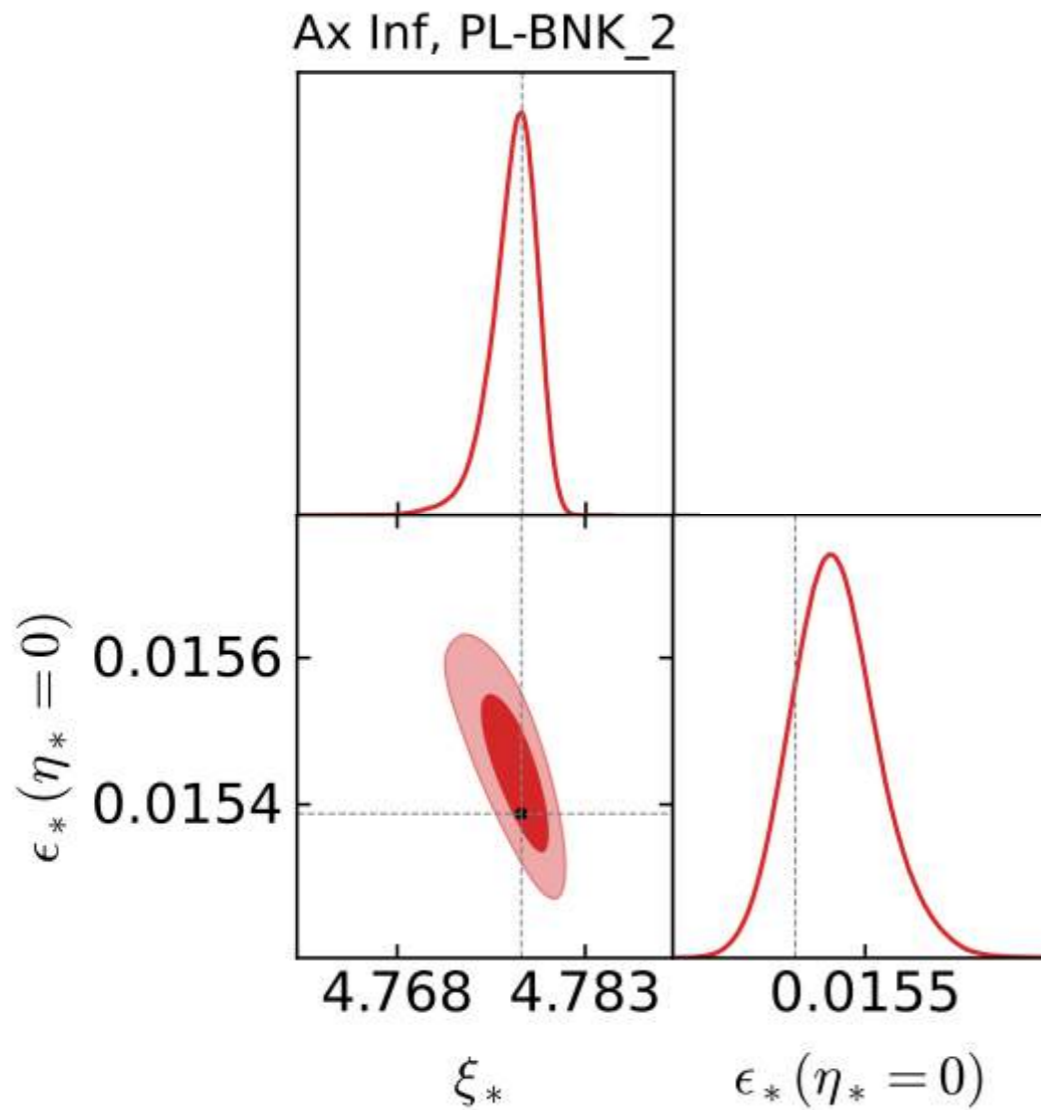
## Broken space diffeomorphisms:

Massive Grav. PL-BNK\_1



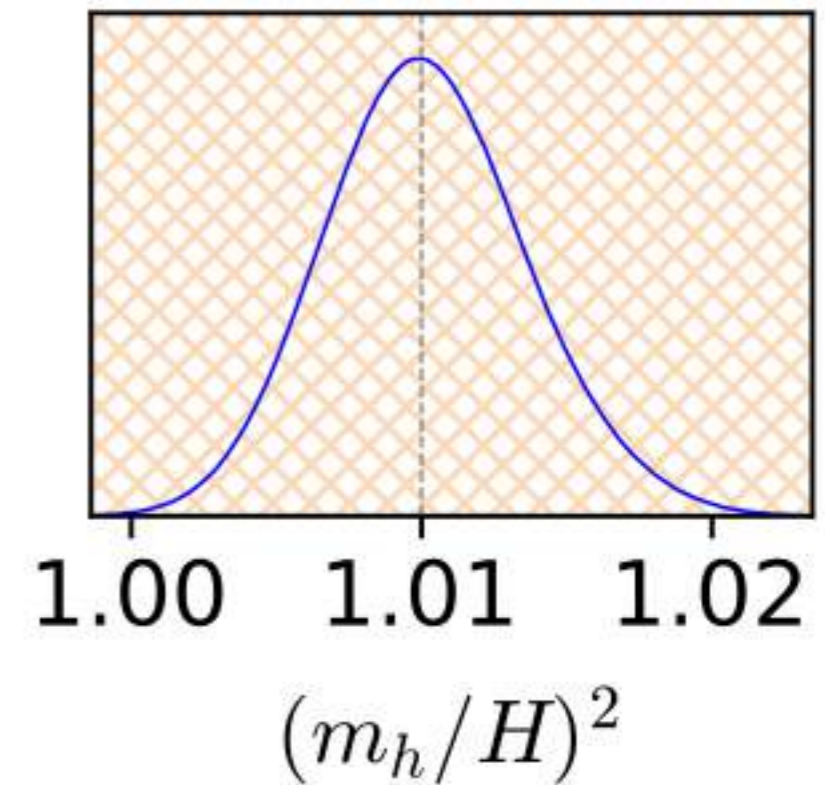
# Benchmark 2: $\{\log_{10}(h^2 \Omega_*), n_T\} = \{-11, 0.77\}$

## Axion inflation:



## Broken space diffeomorphisms:

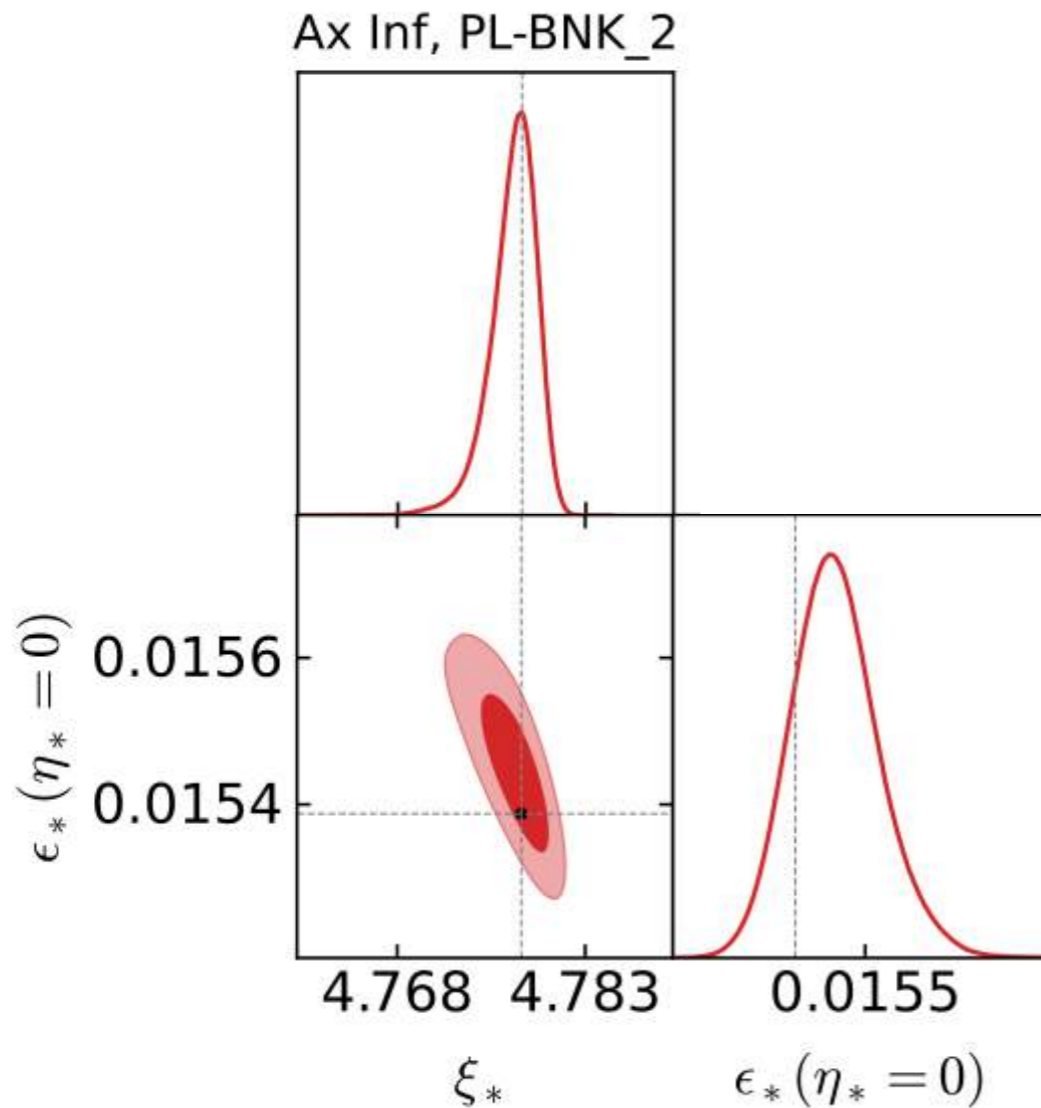
### Massive Grav. PL-BNK\_2





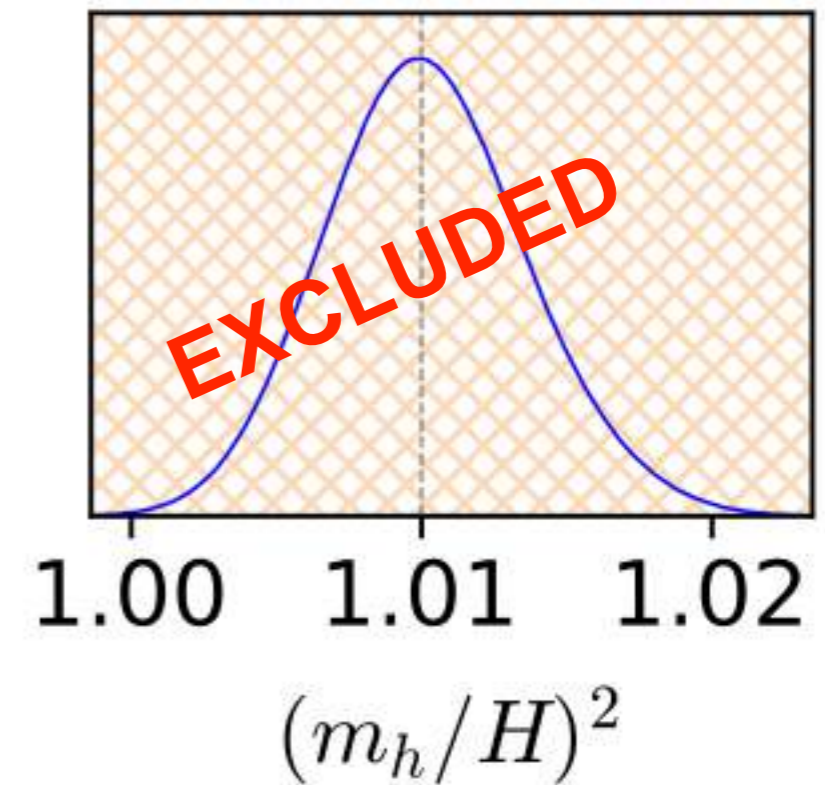
# Benchmark 2: $\{\log_{10}(h^2 \Omega_*), n_T\} = \{-11, 0.77\}$

## Axion inflation:



## Broken space diffeomorphisms:

Massive Grav. PL-BNK\_2



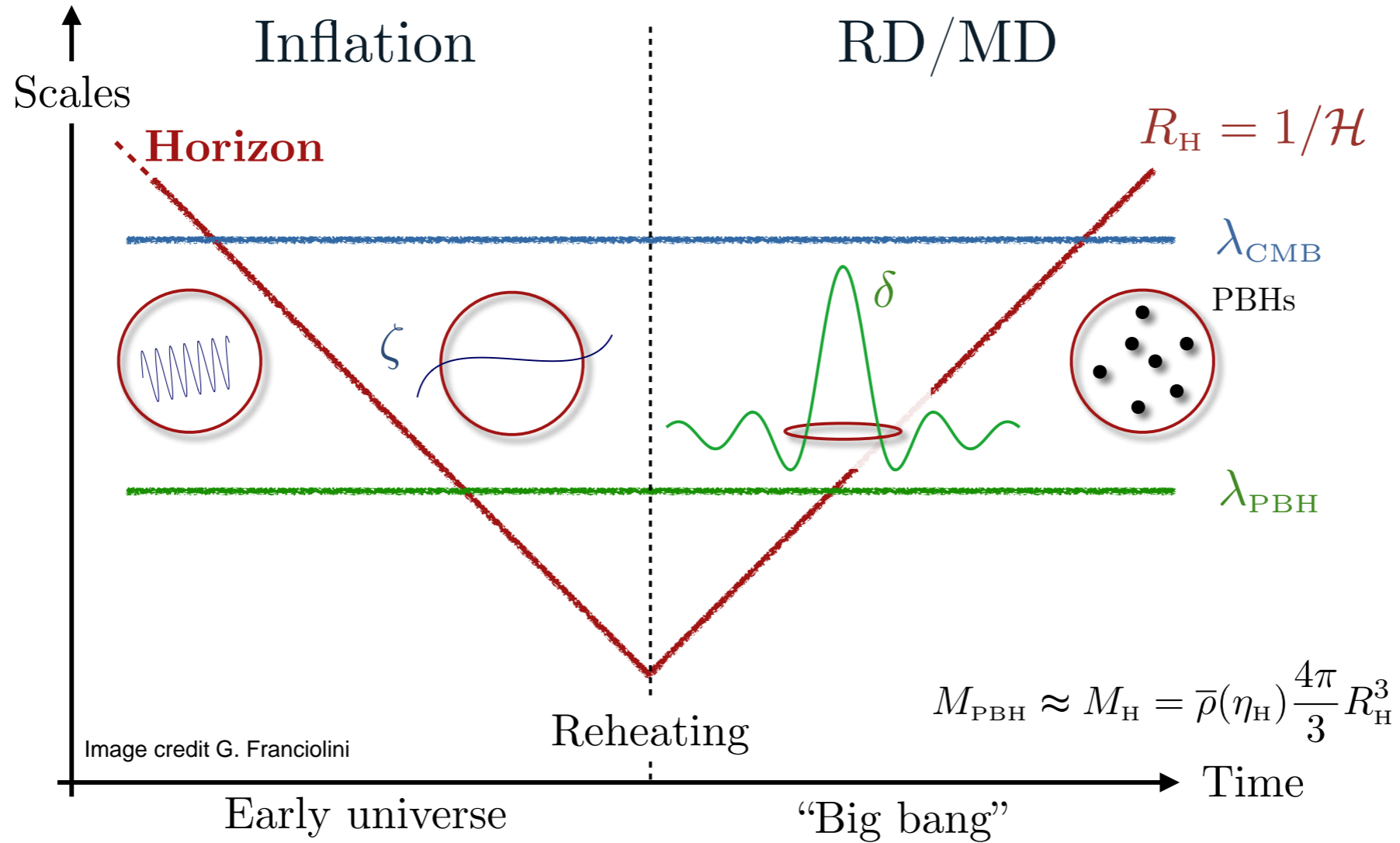
Presence of other fields (e.g. axions) in the Early Universe

Testing fundamental physics (e.g. axion decay constant) - related to high energy physics

Testing peculiar features of the SGWB - parity violation

# GWs - Primordial Black Holes and Dark Matter

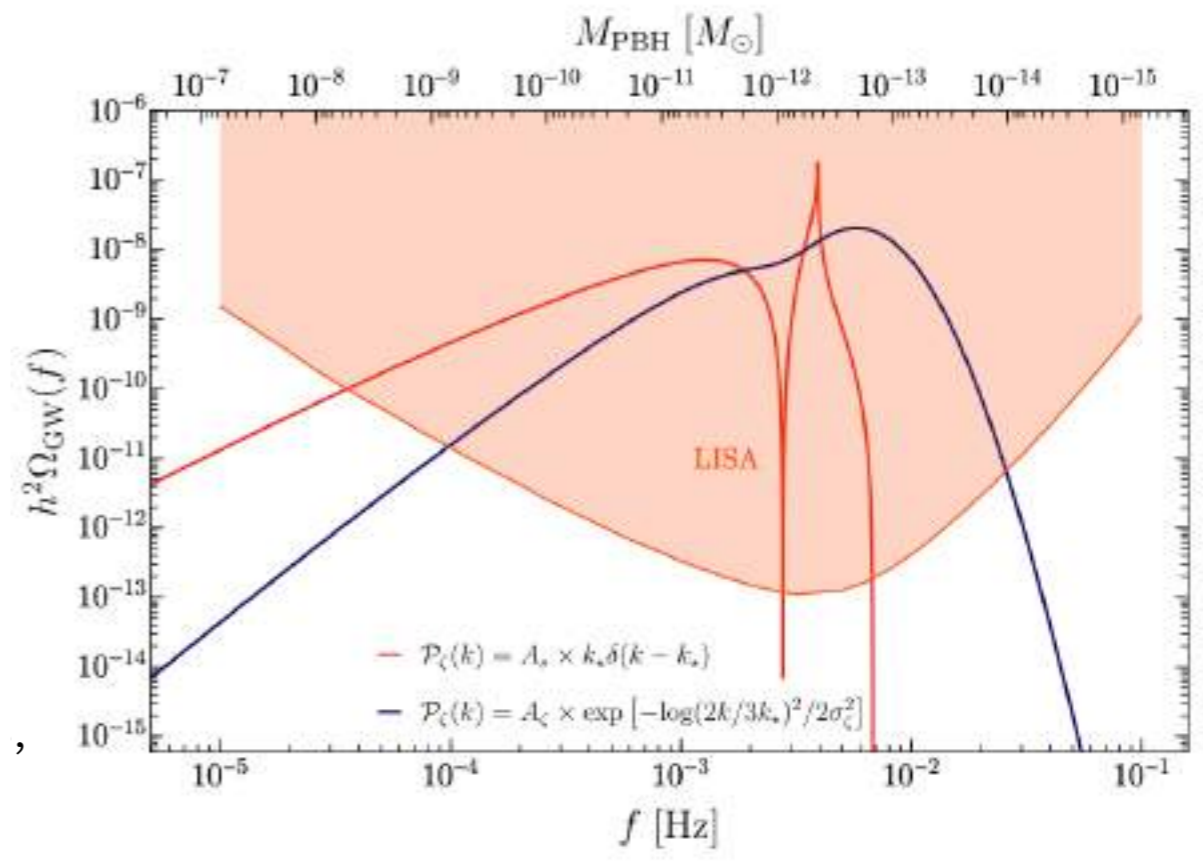
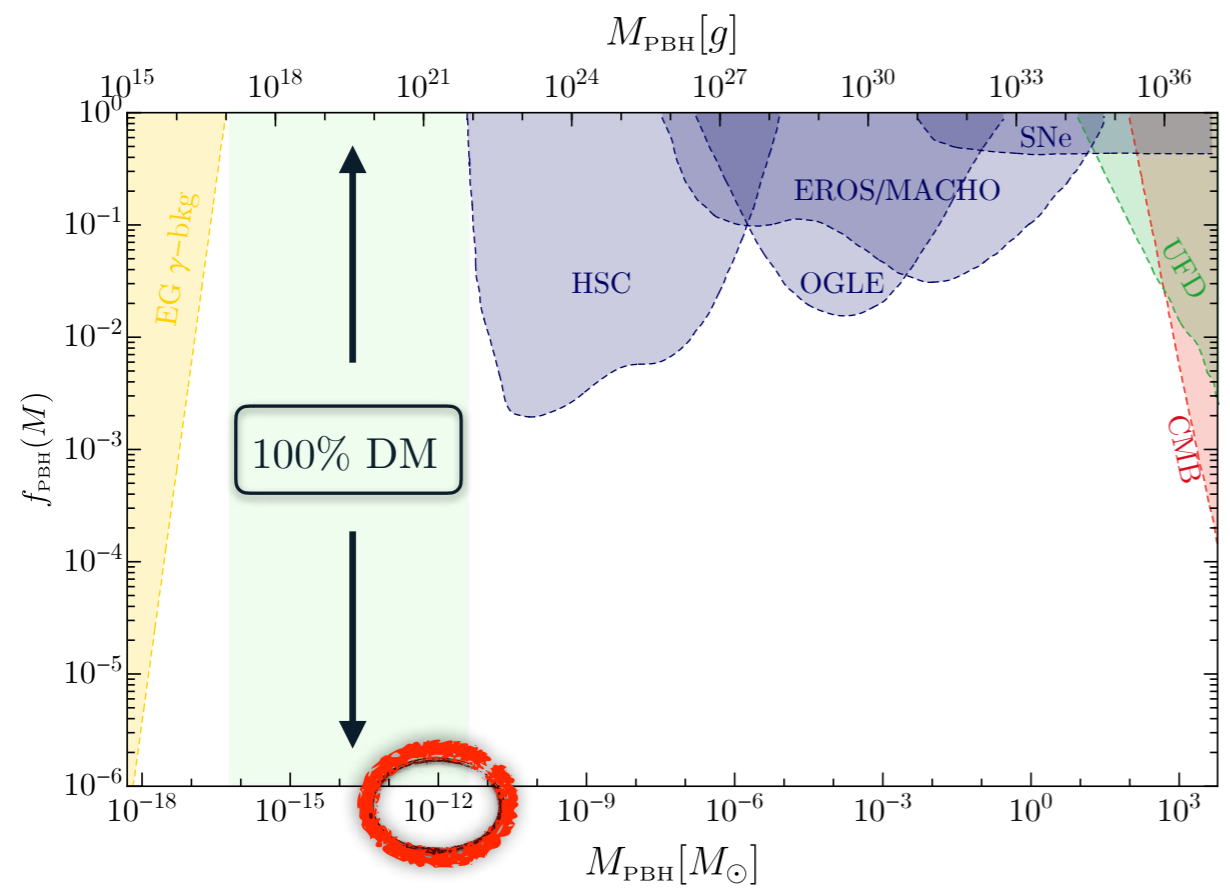
Large scalar perturbations source gravitational waves at 2nd order in perturbation theory when they re-enter the horizon during radiation era



$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i \zeta \partial_j \zeta)$$

[Tomita, K., 1967]  
 [Matarrese, S., et al., 1993]  
 [Domenech, G., review '21]

# GWs - Primordial Black Holes and Dark Matter



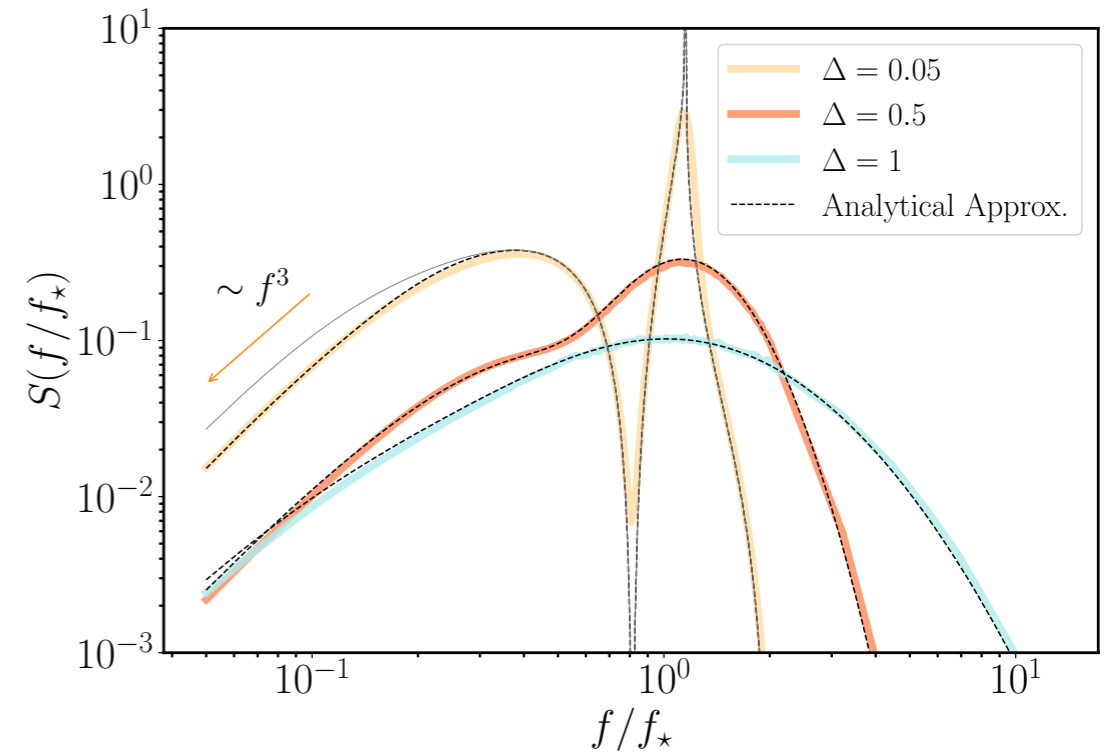
$$f \simeq 3 \cdot 10^{-9} \text{ Hz} \left( \frac{\gamma}{0.2} \right)^{1/2} \left( \frac{M}{M_{\odot}} \right)^{-1/2}$$

$$M \simeq 10^{-12} M_{\odot} \longleftrightarrow f \simeq 10^{-3} \text{ Hz}$$

[Espinosa, et al., 2018]  
 [Bartolo, N., et al., PRL 2019]  
 [De Luca, V., et al., PRL 2021]

# Scalar Power Spectrum seed

$$P_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi}\Delta} \exp\left[-\frac{\log^2(k/k_*)}{2\Delta^2}\right] \rightarrow \Omega_{\text{gw}} h^2 = 10^{-9} \left(\frac{A_\zeta}{0.01}\right)^2 \left(\frac{\Omega_r h^2}{10^{-5}}\right) S(f/f_*, \Delta)$$

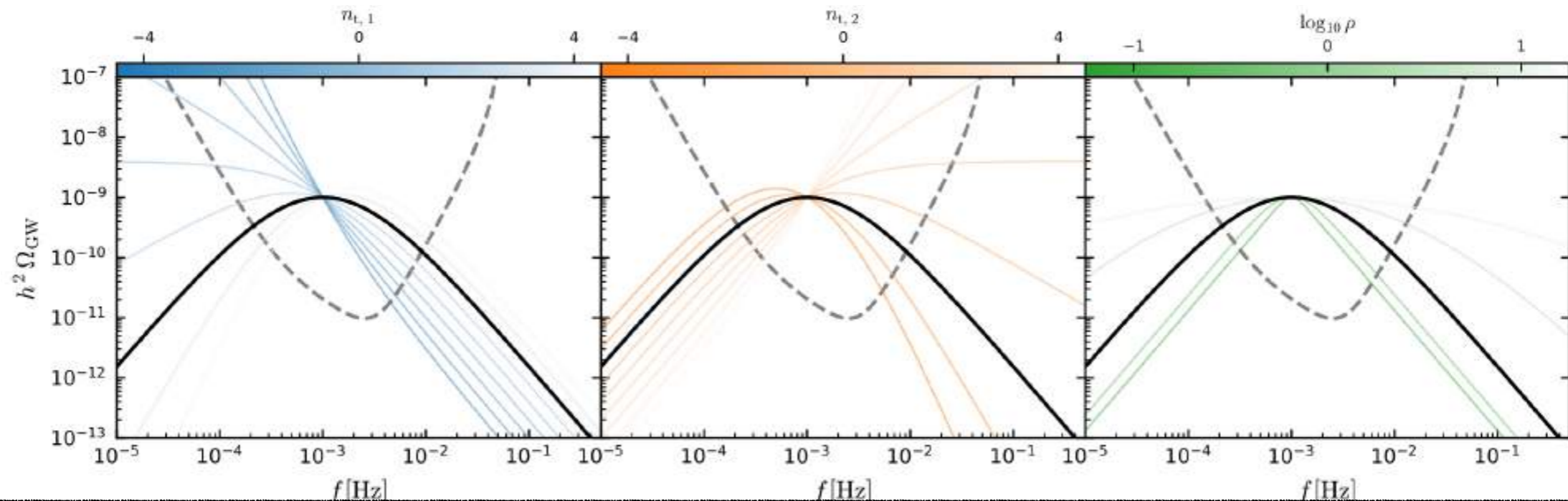


## TEMPLATE

$$h^2 \Omega_{\text{GW}}^{\text{BPL}}(f, \vec{p}) = 10^{\alpha_*} \frac{\left(\frac{f}{f_*}\right)^{n_{t,1}}}{\left\{ \frac{1}{2} \left[ 1 + \left(\frac{f}{f_*}\right)^{1/\delta} \right] \right\}^{(n_{t,1} - n_{t,2})\delta}}$$

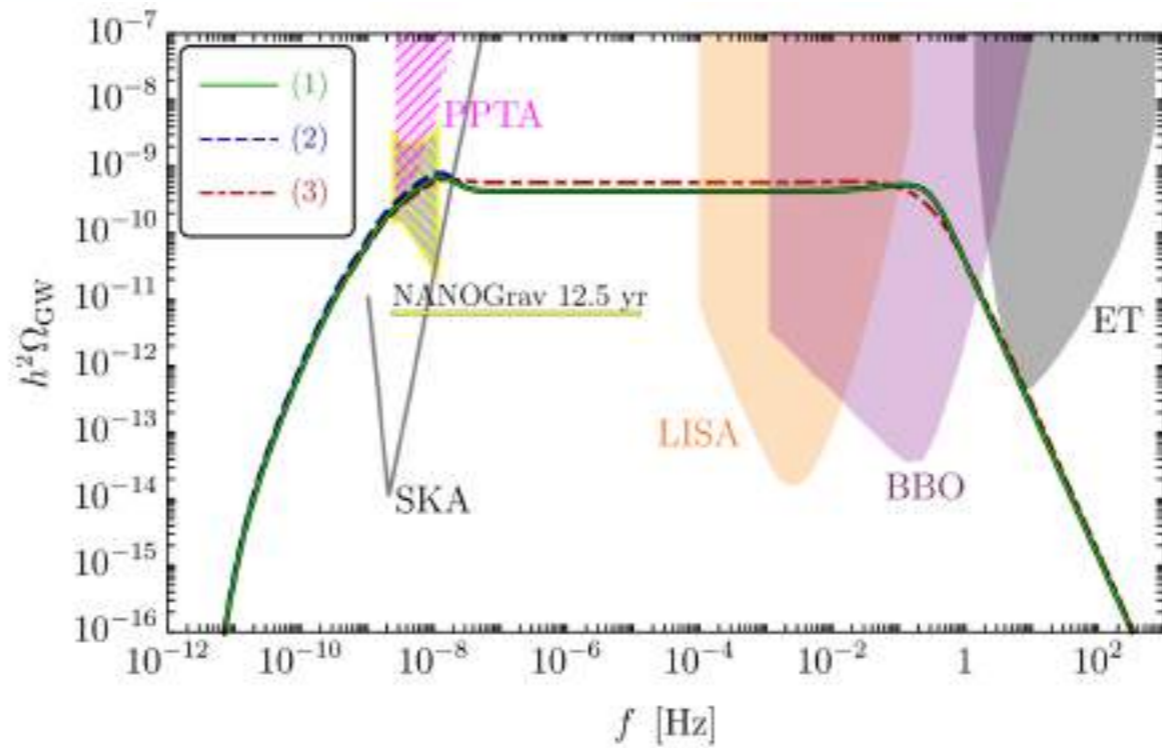
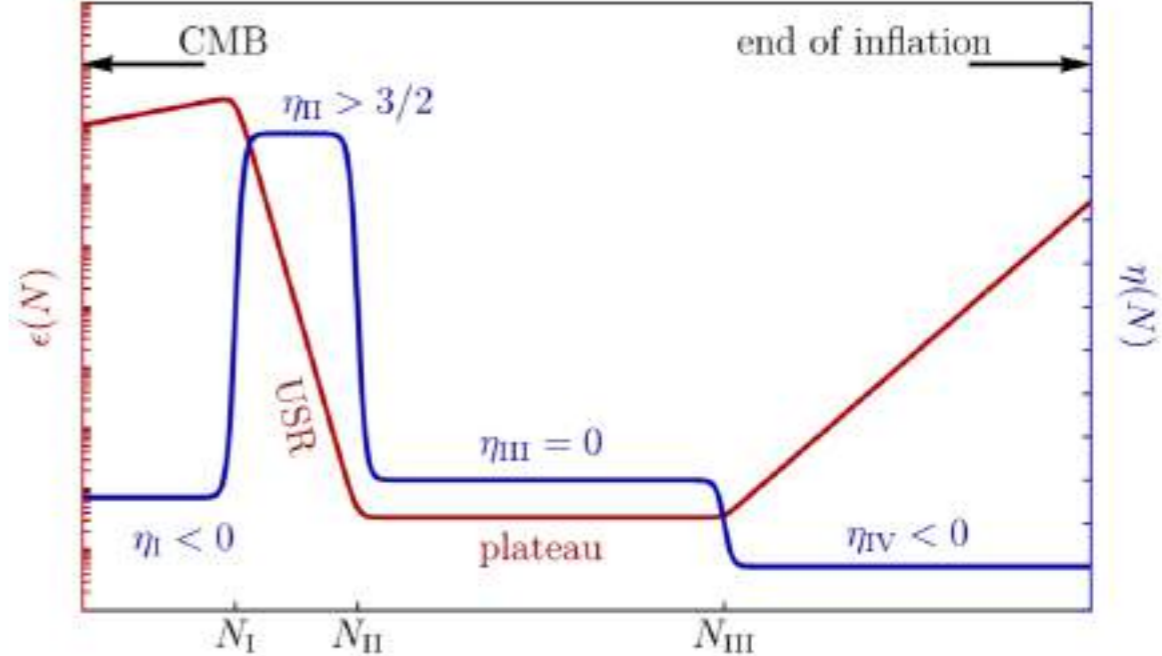
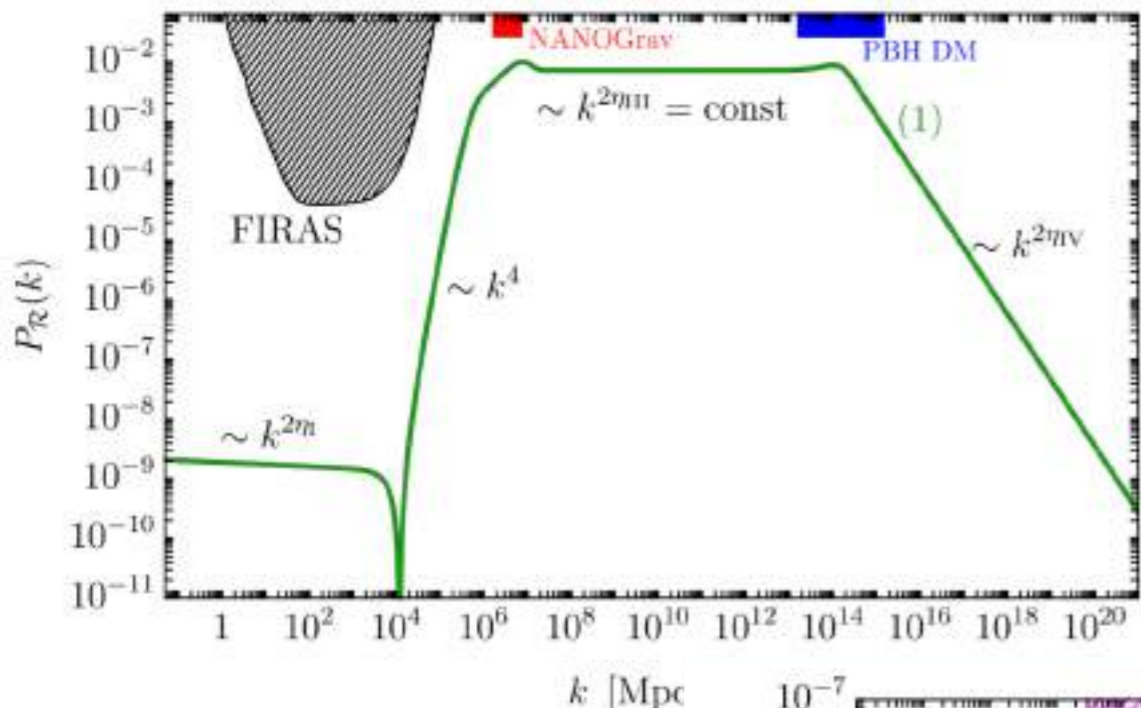
$$\vec{p} = \{\alpha_*, f_*, n_{t,1}, n_{t,2}, \delta\}$$

Pi & Sasaki 2005.12306



# Inflationary models producing such a signal

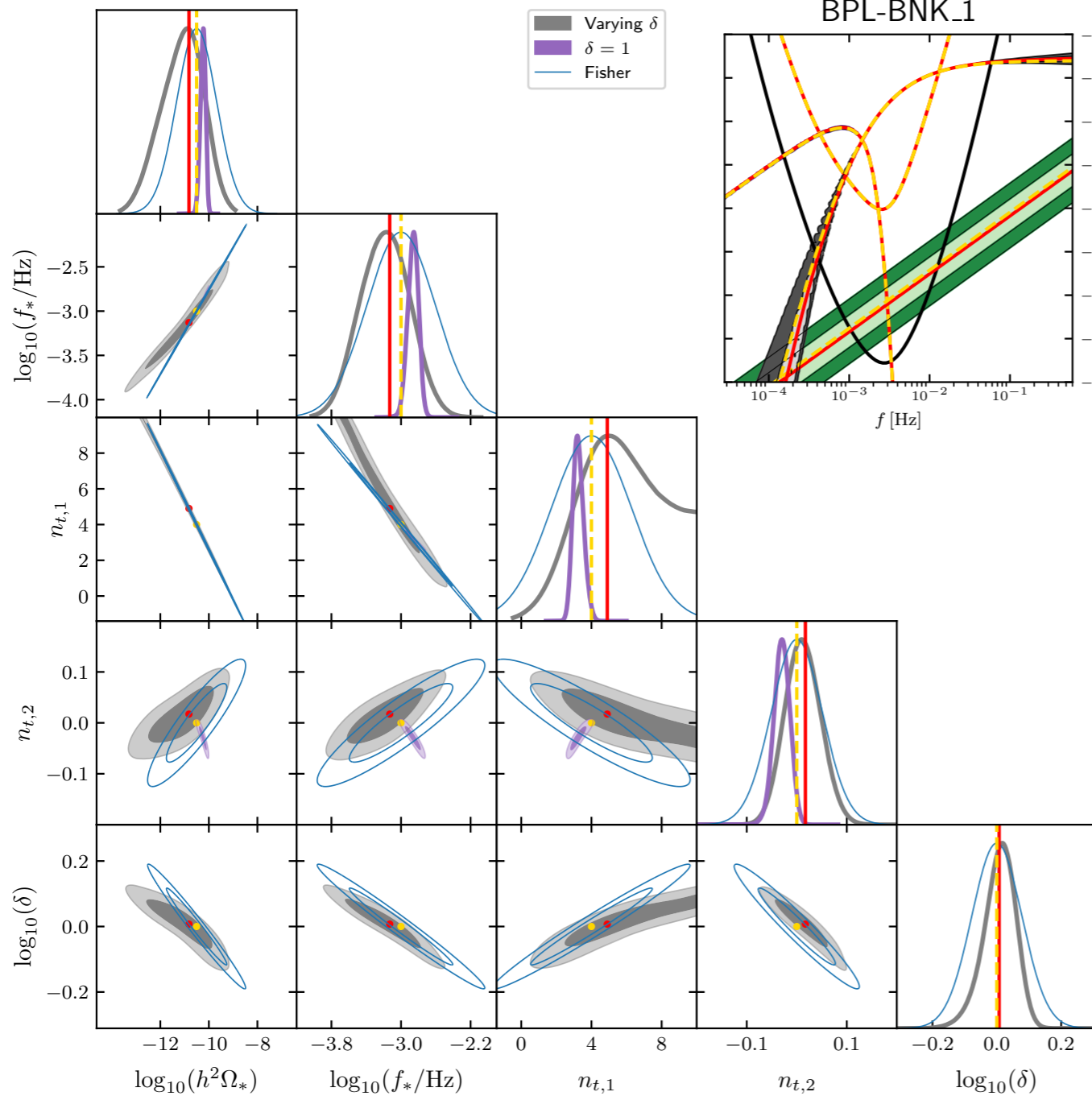
**Second slow-roll stage:** An ultra-slow-roll phase amplifies primordial scalar perturbations and is followed by a second slow-roll regime generating a plateau.



[Franciolini & Urbano 2207.10056]

See M. Redi's talk

# Forecast



**Amplitude** ratio  $H^2/\epsilon$  during the second SR stage, abundance of Primordial Black Holes  $f_{\text{PBH}}$

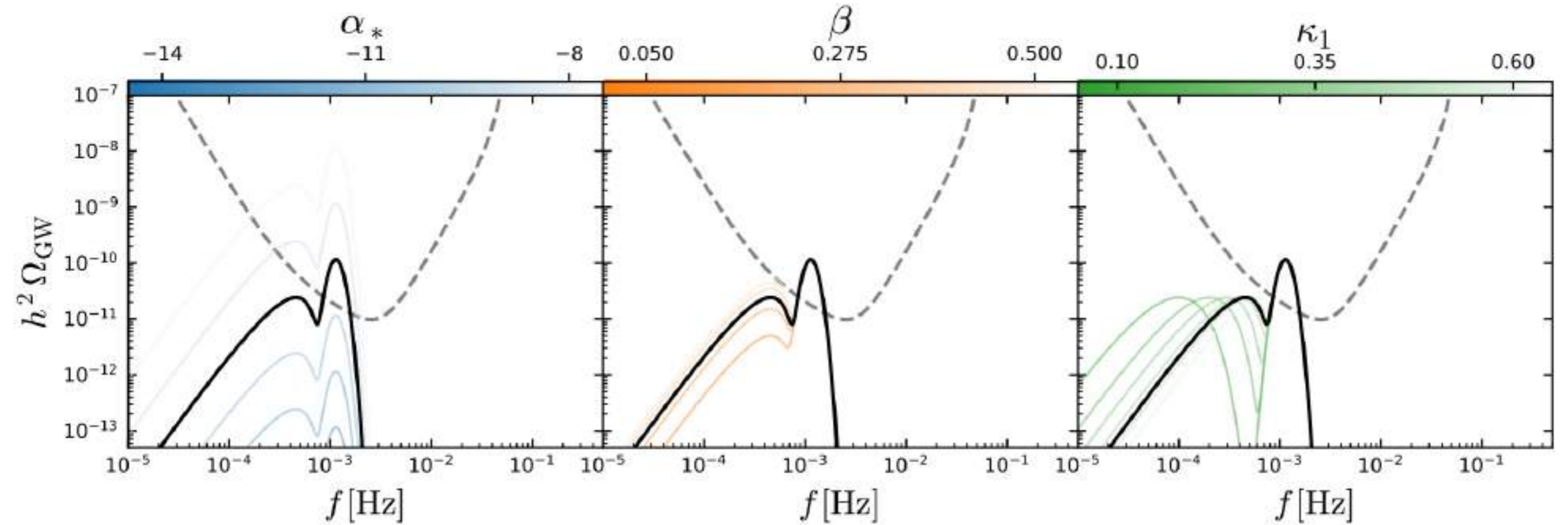
**Frequency of the turn:** time of the onset of the second SR stage, mass of Primordial Black Holes  $M/M_{\odot}$

**IR spectral index:** related to the scalar IR spectral index.

**UV spectral index.** predicted to be flat in this model

**Smoothing parameter:** related to the sharpness of the USR-SR transition.

# Double Peak signal



$$h^2 \Omega_{\text{GW}}^{\text{DP}}(f, \vec{p}) = 10^{\alpha_*} \left[ \beta \left( \frac{f}{\kappa_1 f_*} \right)^{n_p} \left[ \frac{c_1 - f/f_*}{c_1 - \kappa_1} \right]^{\frac{n_p}{\kappa_1} (c_1 - \kappa_1)} \Theta \left( c_1 - \frac{f}{f_*} \right) + \exp \left[ -\frac{1}{2\rho^2} \log_{10}^2 \left( \frac{f}{\kappa_2 f_*} \right) \right] \left\{ 1 + \text{erf} \left[ -\gamma \log_{10} \left( \frac{f}{\kappa_2 f_*} \right) \right] \right\} \right]$$

$$\vec{p} = \{ \alpha_*, f_*, \beta, \kappa_1, \kappa_2, \rho, \gamma \}$$

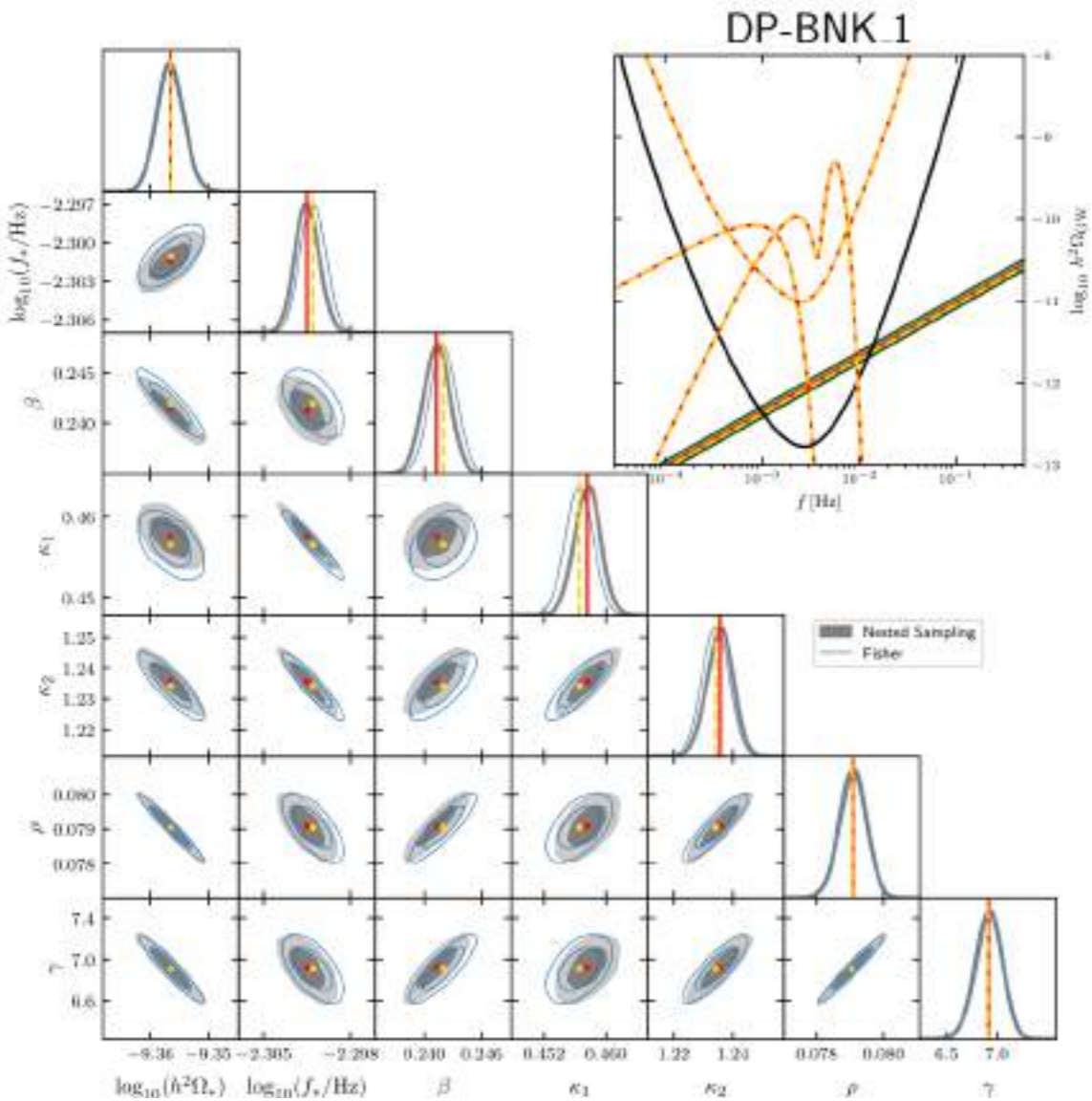
# Inflationary models producing such a signal

Broken power law  $\mathcal{P}_\zeta(k)$ :

$$\mathcal{P}_\zeta^{\text{bpl}}(k) = \frac{\mathcal{A}_s(p_1 + p_2)}{\left[ p_2 \left( \frac{k}{k_*} \right)^{-p_1} + p_1 \left( \frac{k}{k_*} \right)^{p_2} \right]}$$

Lognormal  $\mathcal{P}_\zeta(k)$ :

$$\mathcal{P}_\zeta^{\text{ln}}(k) = \mathcal{A}_s \exp \left[ -\frac{1}{2\Delta^2} \ln^2 \left( \frac{k}{k_*} \right) \right]$$

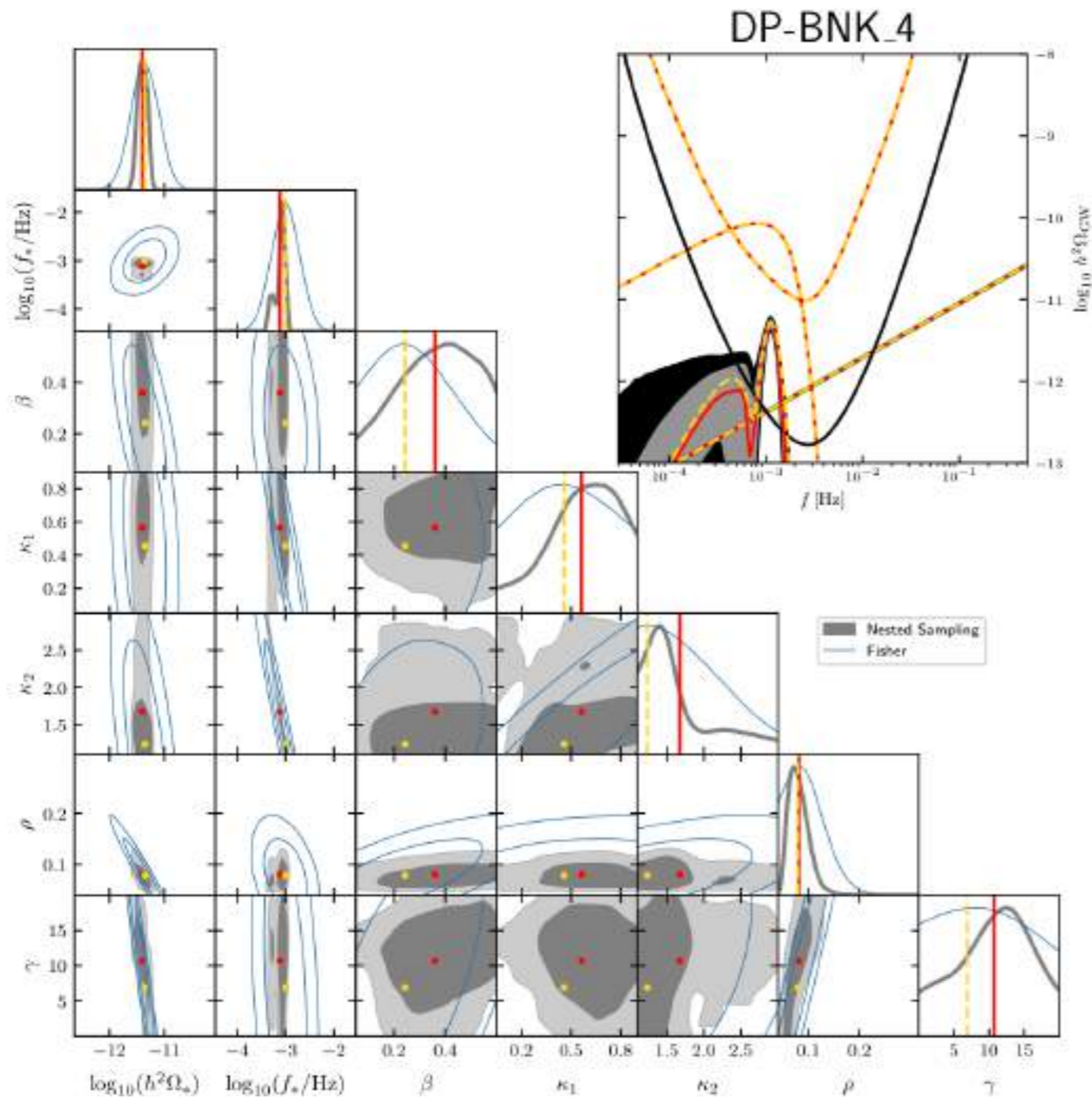


**Very loud signal:  
tight constraints on all  
parameters.**

[Ozsoy & Tasinato 2301.03600]



# Double Peak (faint) signal



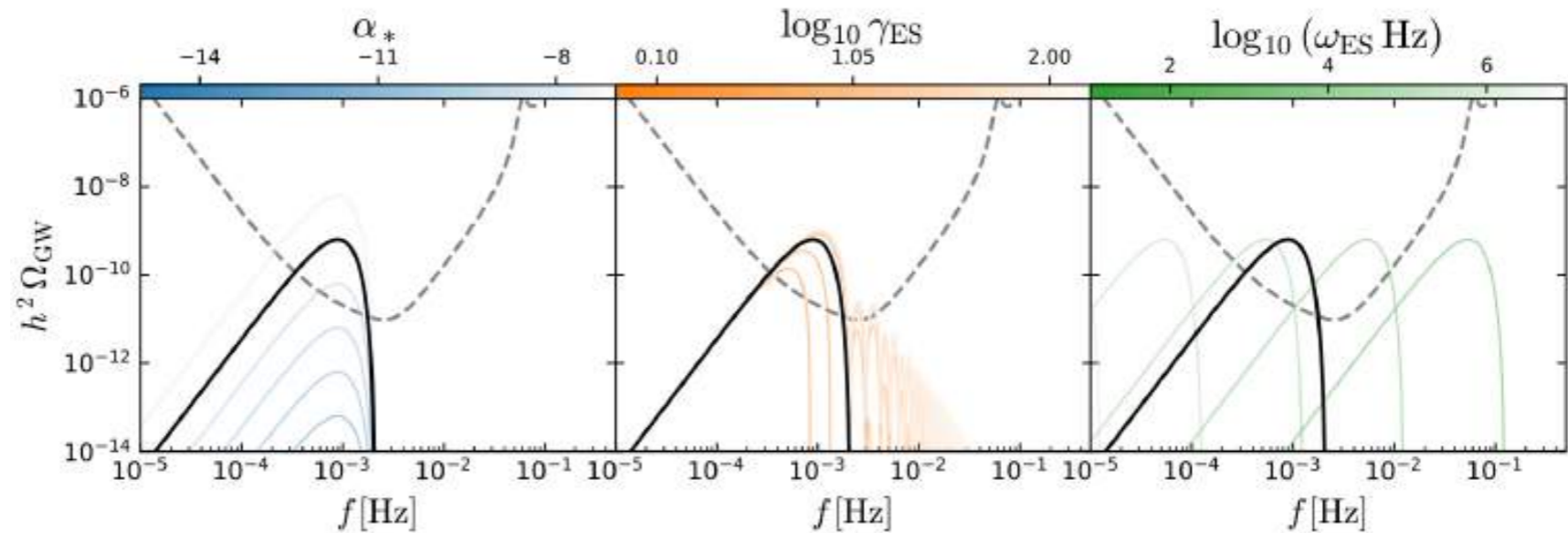
**Faint signal:** only certain features of the signal are constrained.

# Other Templates

## Excited States

Model:  
Scalar- induced GWs during  
inflation

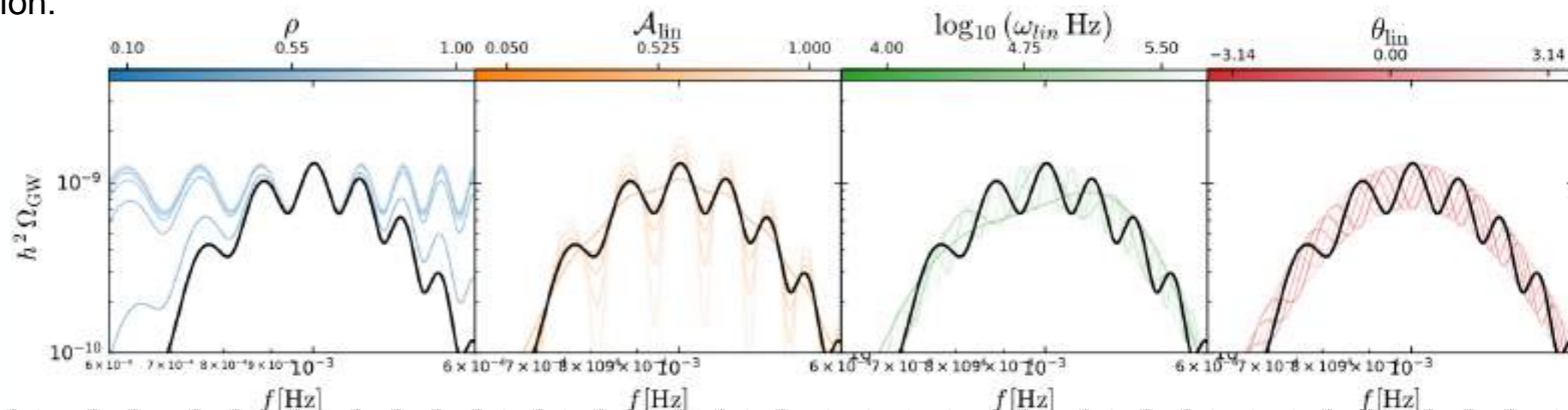
$$h^2 \Omega_{\text{GW}}^{\text{ES}}(f, \vec{p}) = \frac{10^{\alpha_*}}{0.052} \frac{1}{x^3} \left[ 1 - \frac{x^2}{4\gamma_{\text{ES}}^2} \right]^2 \left[ \sin(x) - 2 \frac{1 - \cos(x)}{x} \right]^2 \Theta(x_{\text{cut}} - x)$$



## Linear Oscillation

Model:  
Sharp features during inflation:

$$h^2 \Omega_{\text{GW}}^{\text{LO}}(f, \vec{p}) = \left[ 1 + \mathcal{A}_{\text{lin}} \cos(\omega_{\text{lin}} f + \theta_{\text{lin}}) \right] h^2 \Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$

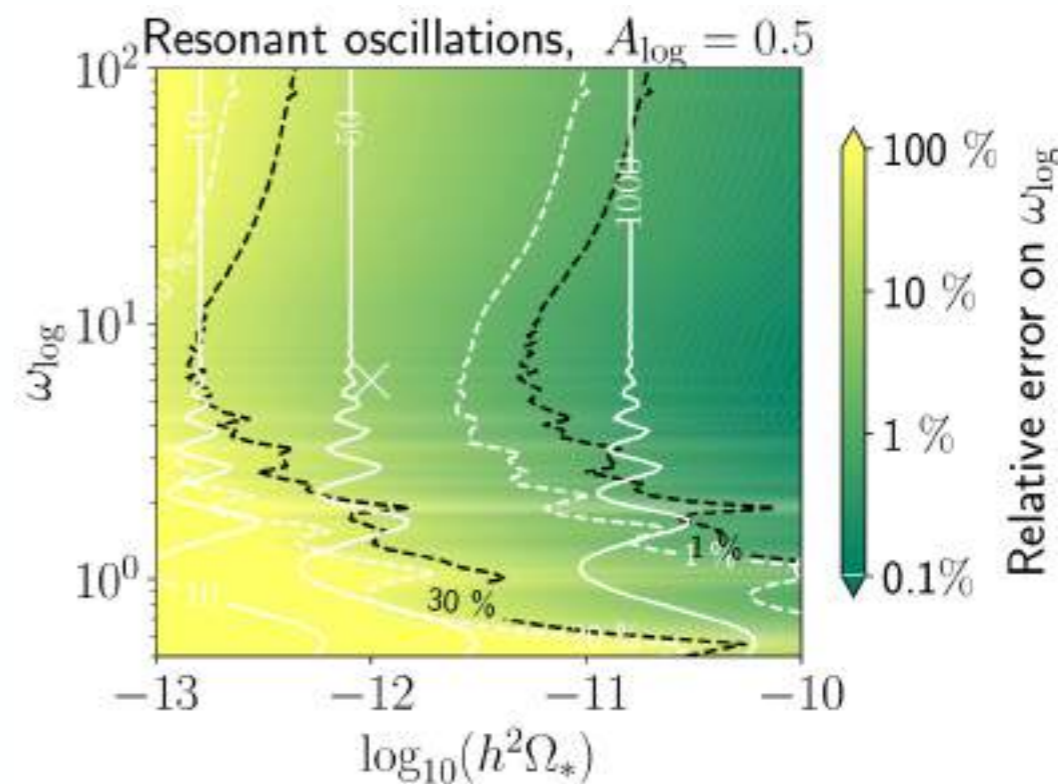
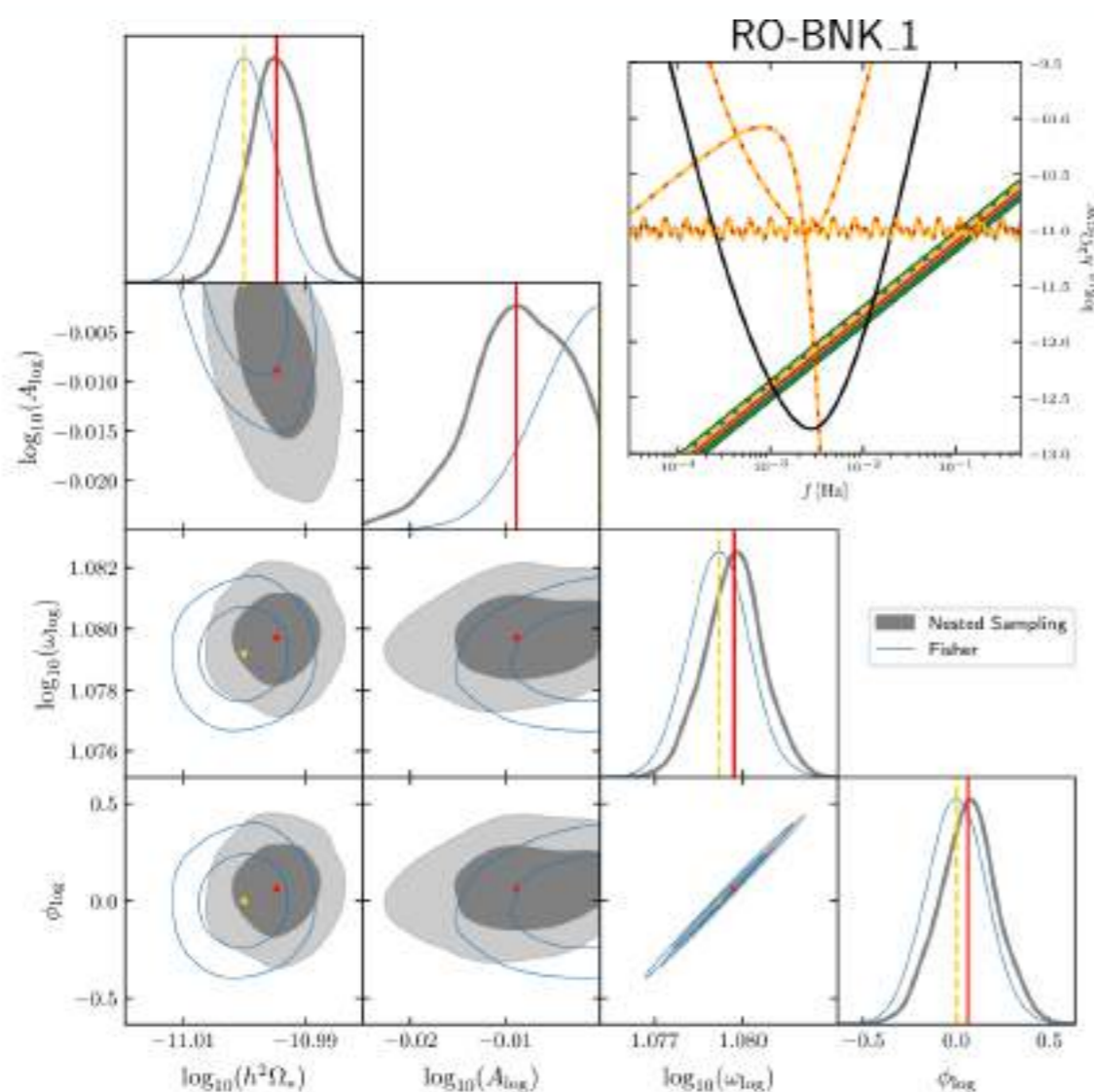
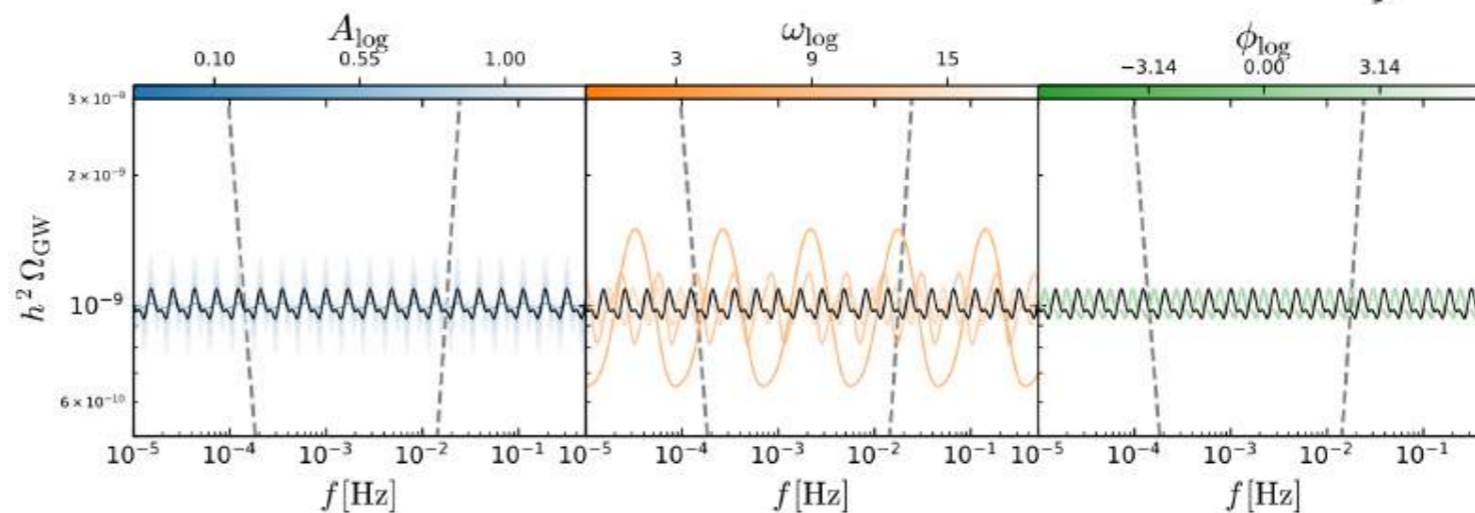


# Resonant Oscillations

$$h^2 \Omega_{\text{GW}}^{\text{RO}}(f, \vec{p}) = \left\{ 1 + \mathcal{A}_1(A_{\log}, \omega_{\log}) \cos [\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,1}] \right.$$

Model:  
Resonant features during inflation

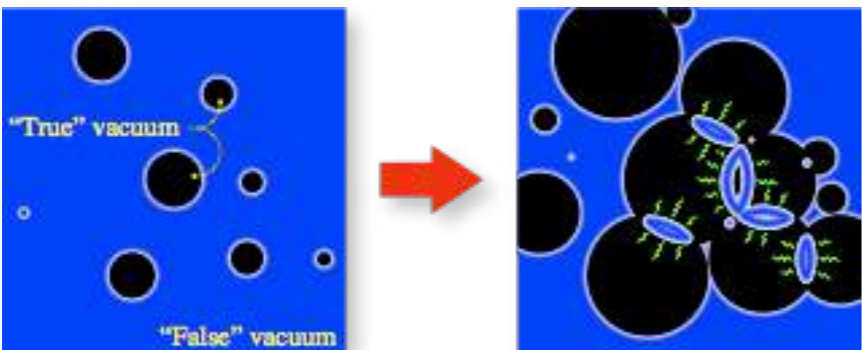
$$\left. + \mathcal{A}_2(A_{\log}, \omega_{\log}) \cos [2\omega_{\log} \ln(f/\text{Hz}) + \theta_{\log,2}] \right\} h^2 \Omega_{\text{GW}}^{\text{env}}(f, \vec{p}_{\text{env}})$$



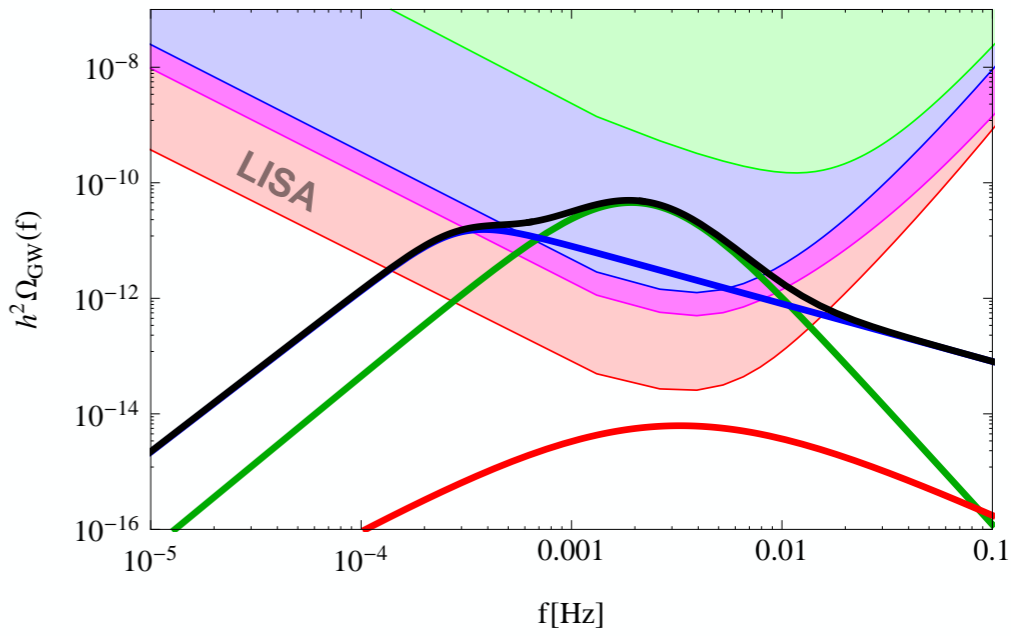
# Phase Transition in the Early Universe

As the temperature in the very early universe decreases, there can be several **PTs: QCD, EW...Beyond Standard Model?**

If the **PT is first order**, the **SGWB** signal could be detectable by **LISA**



- Processes**
- Bubble collisions
  - MHD Turbulence
  - Sound Waves



● peaked spectrum with

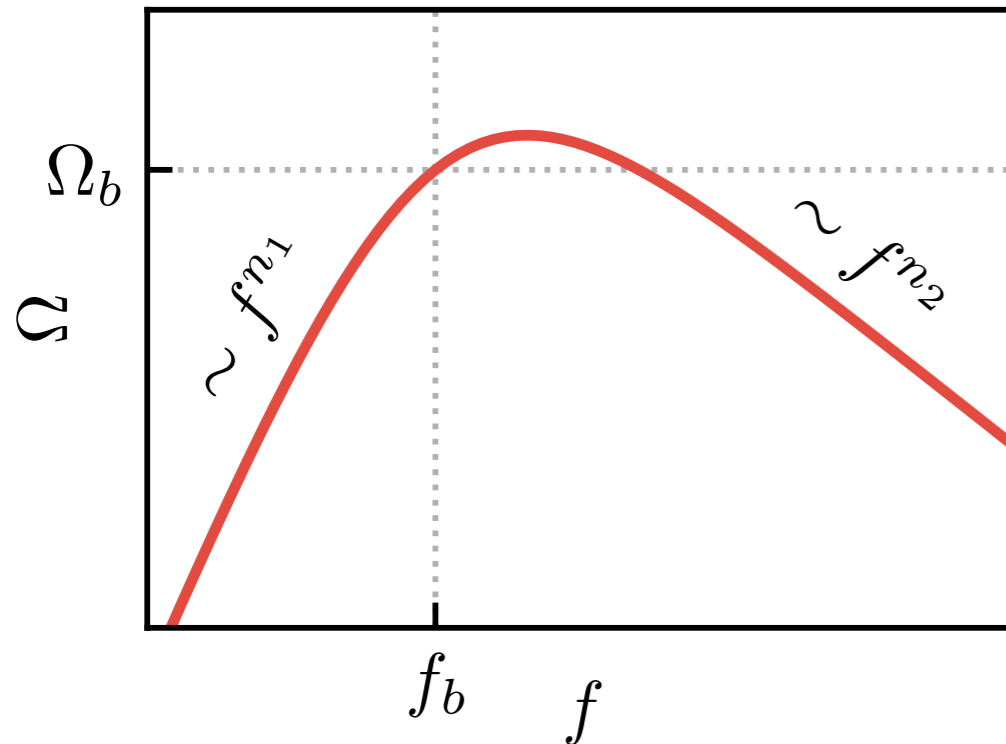
$$f_{\text{peak}} \sim 10^{-3} \text{ Hz} \frac{T}{100 \text{ GeV}}$$

[Caprini C., et al '16, '19- LISA CosWG paper]

# Template for Phase Transition

broken power-law

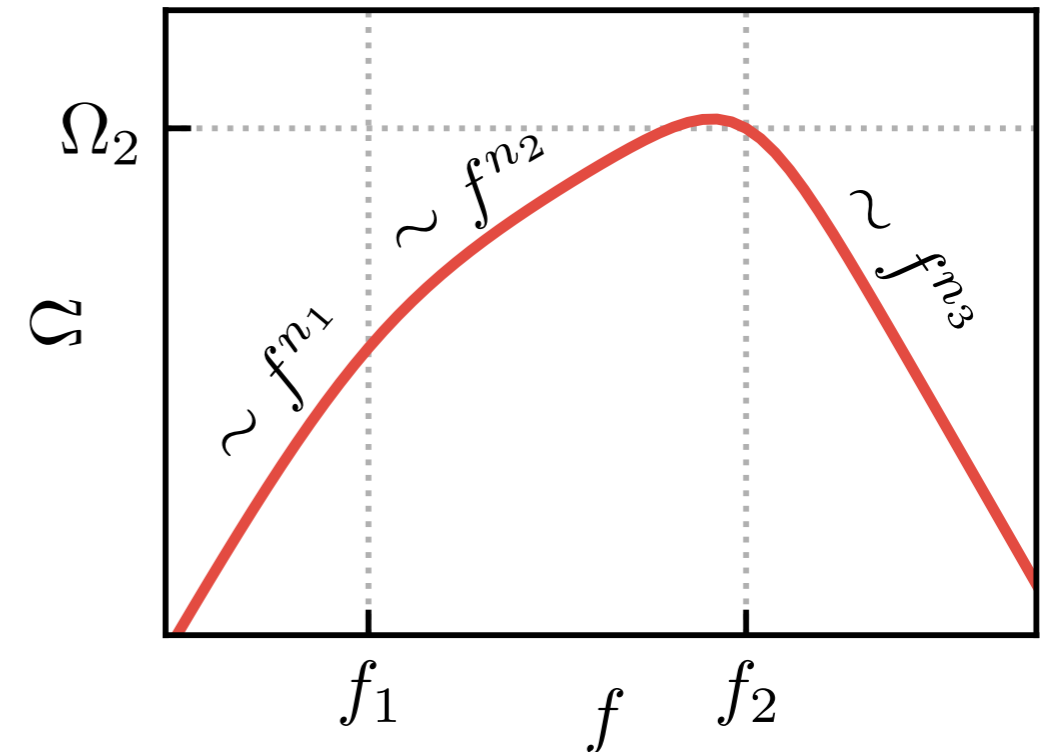
$$\Omega_b \mathcal{N} \left( \frac{f}{f_b} \right)^{n_1} \left[ 1 + \left( \frac{f}{f_b} \right)^{a_1} \right]^{\frac{n_2 - n_1}{a_1}}$$



bubble collisions:  $(n_1, n_2, a_1) = (2.4, -2.4, 4)$

double broken power-law

$$\Omega_2 \mathcal{N} \left( \frac{f}{f_1} \right)^{n_1} \left[ 1 + \left( \frac{f}{f_1} \right)^{a_1} \right]^{\frac{n_2 - n_1}{a_1}} \left[ 1 + \left( \frac{f}{f_2} \right)^{a_2} \right]^{\frac{n_3 - n_2}{a_2}}$$



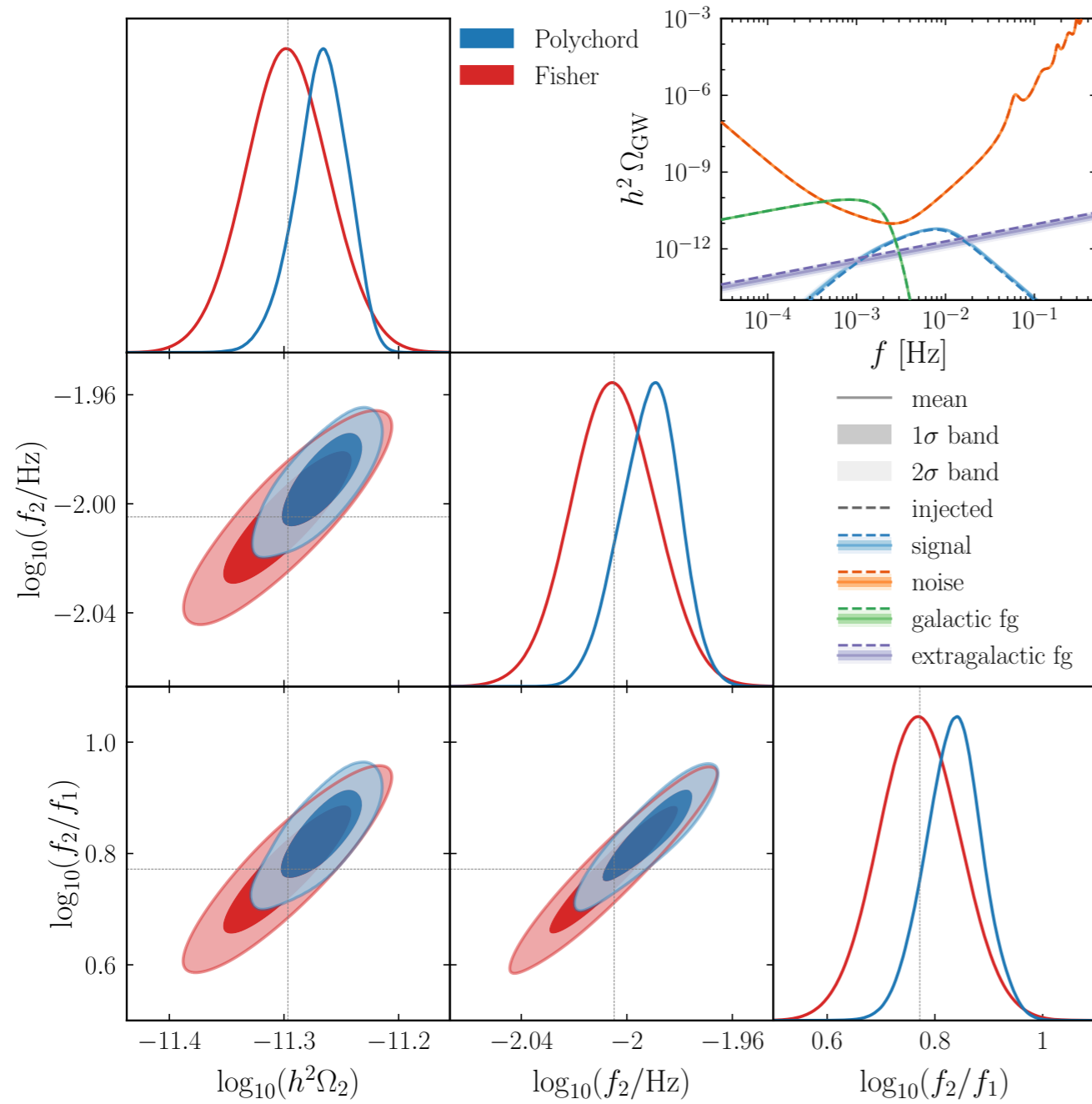
Sound waves:  $(n_1, n_2, n_3, a_1, a_2) = (3, 1, -3, 2, 4)$

MHD turbulence:  $(n_1, n_2, n_3, a_1, a_2) = (3, 1, -\frac{8}{3}, 4, 2.15)$

[Lewicki & Vaskonen, 2023]

LISA CosGW project Arxiv: 2407.04356 (phase transition)

# Forecast: Nested Sampling vs Fisher



estimation of parameter reconstruction based on Polychord vs Fisher

$$h^2\Omega_2 = 5 \times 10^{-12}, \quad f_2 = 10 \text{ mHz}, \quad \frac{f_2}{f_1} \approx 6$$

# Reconstructing thermodynamics parameters

3 geom. params.:  $\Omega_2, f_2, f_1$   
 4 therm. params.:  $K, H_*R_*, \xi_w, T_*$

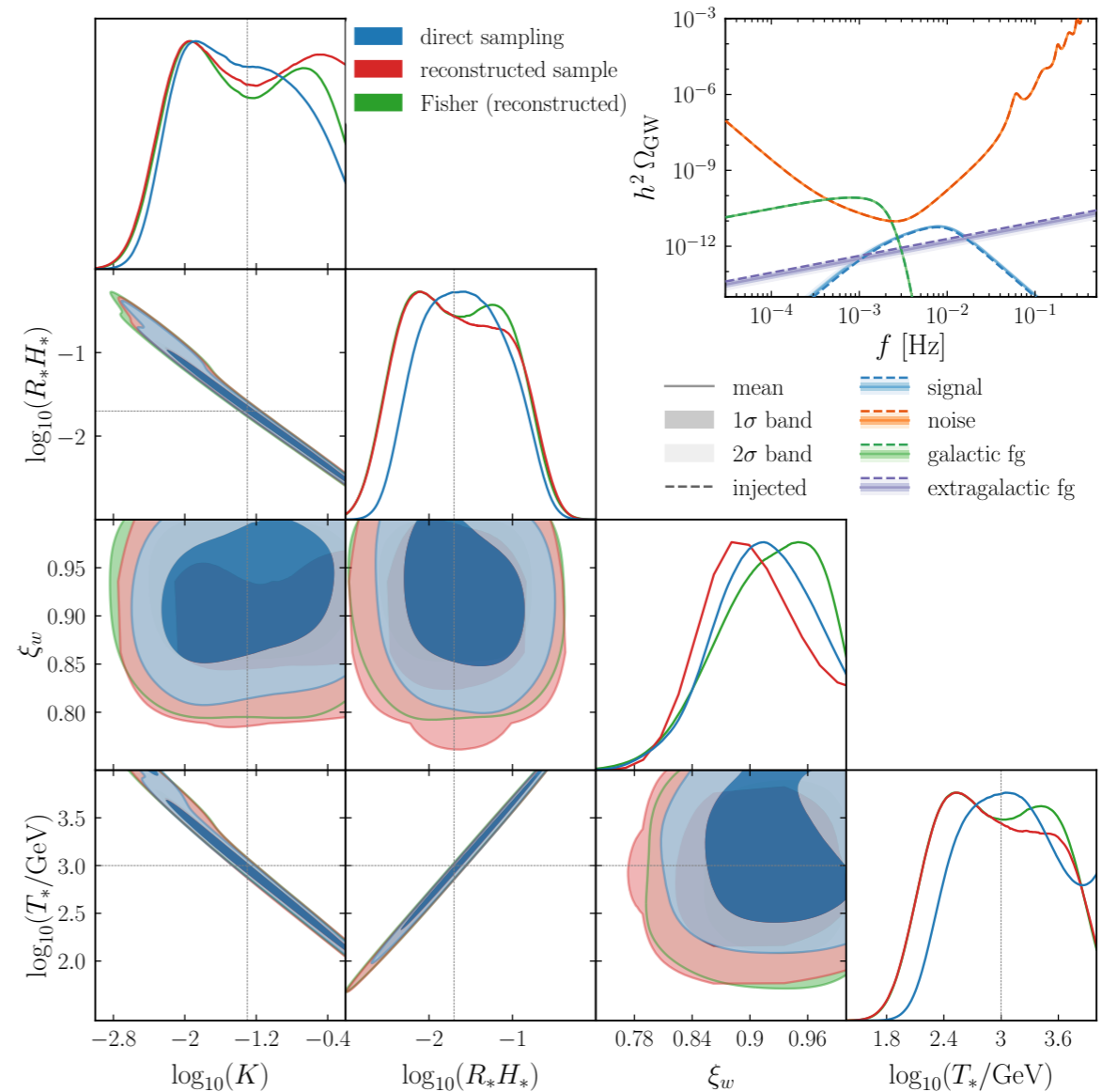
$\implies$  degenerate

consider fixed  $T_*$ :

$$\Omega_2 \propto \frac{\xi_{\text{shell}}}{\xi_w} \begin{cases} K^2 H_* R_* & \text{if } H_* \tau > 1 \\ K^{3/2} (H_* R_*)^2 & \text{if } H_* \tau < 1 \end{cases}$$

$$f_2 \propto T_* / (H_* R_*)$$

$$\frac{f_2}{f_1} \propto \xi_w / \xi_{\text{shell}}$$



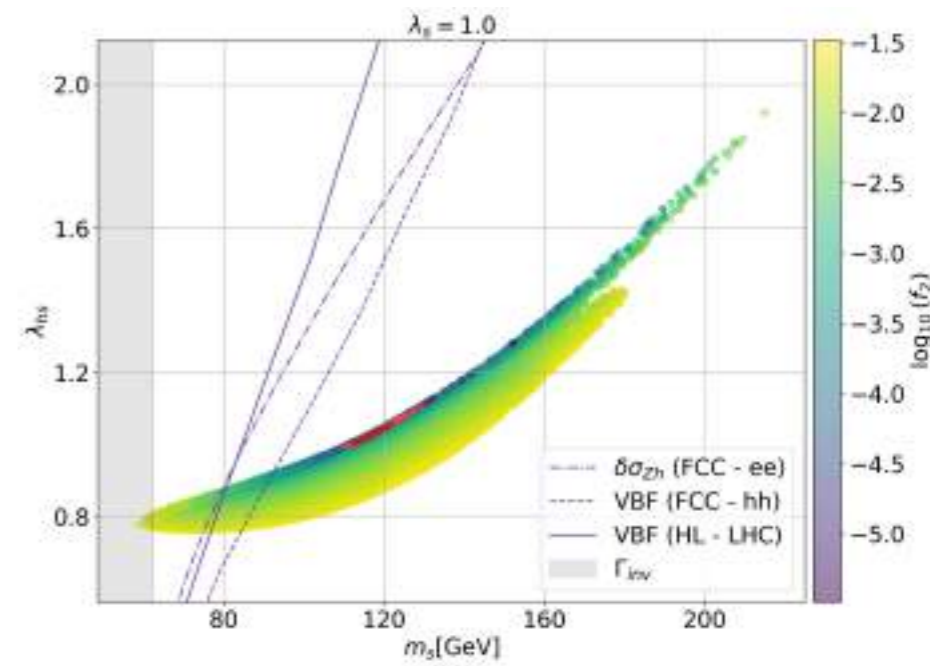
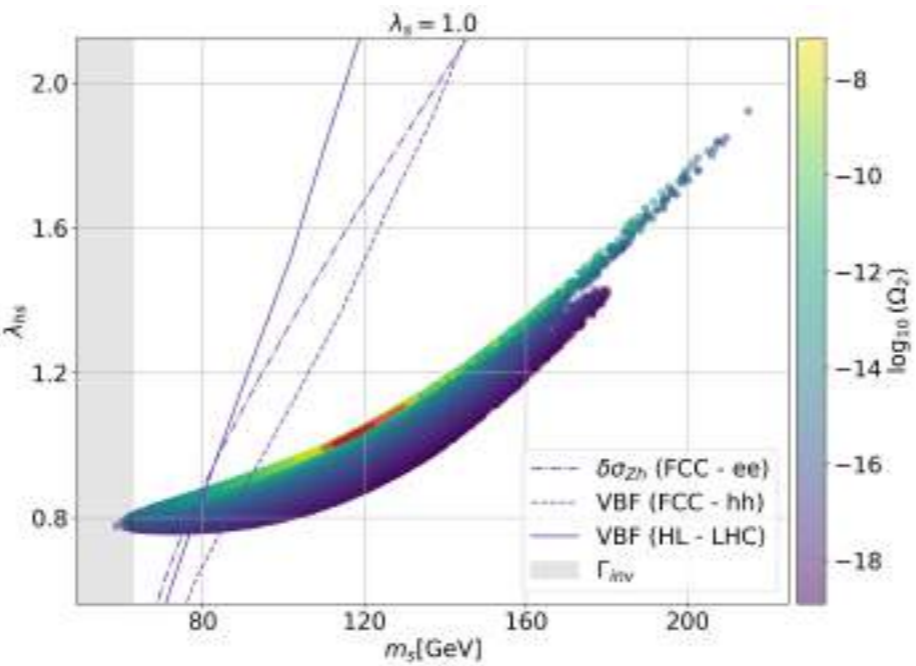
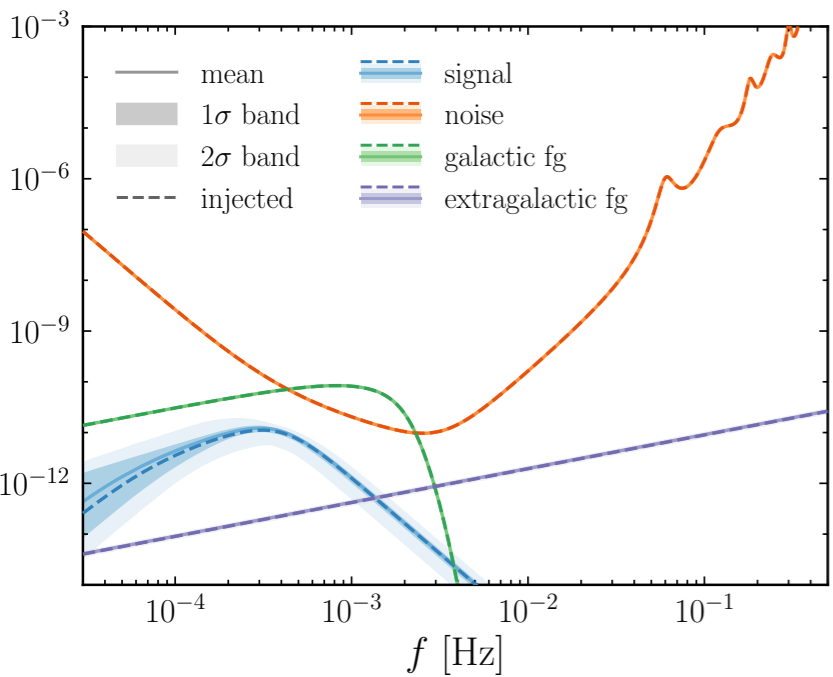
$$K = 0.05, R_* H_* = 0.02, \xi_w = 1, T_* = 1 \text{ TeV}$$

# What we can learn from Phase Transition?

- LISA could act as a **probe of Beyond Standard Model physics, complementary to colliders**

Simplest extensions of the SM 
$$V_{\text{tree}}(\Phi, s) = -\mu_h^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2 + \mu_s^2 \frac{s^2}{2} + \frac{\lambda_s}{4} s^4 + \frac{\lambda_{hs}}{2} s^2 \Phi^\dagger \Phi,$$

Extra scalar singlet under the SM gauge group endowed with a  $Z_2$  symmetry



- LISA could act as a **probe of Beyond Standard Model physics, complementary to colliders**
- In some BSM scenarios possible **joint detection at LISA and LHC/FCC**

$\lambda_{hs}$  Singlet coupling  
 $m_s$  Singlet mass

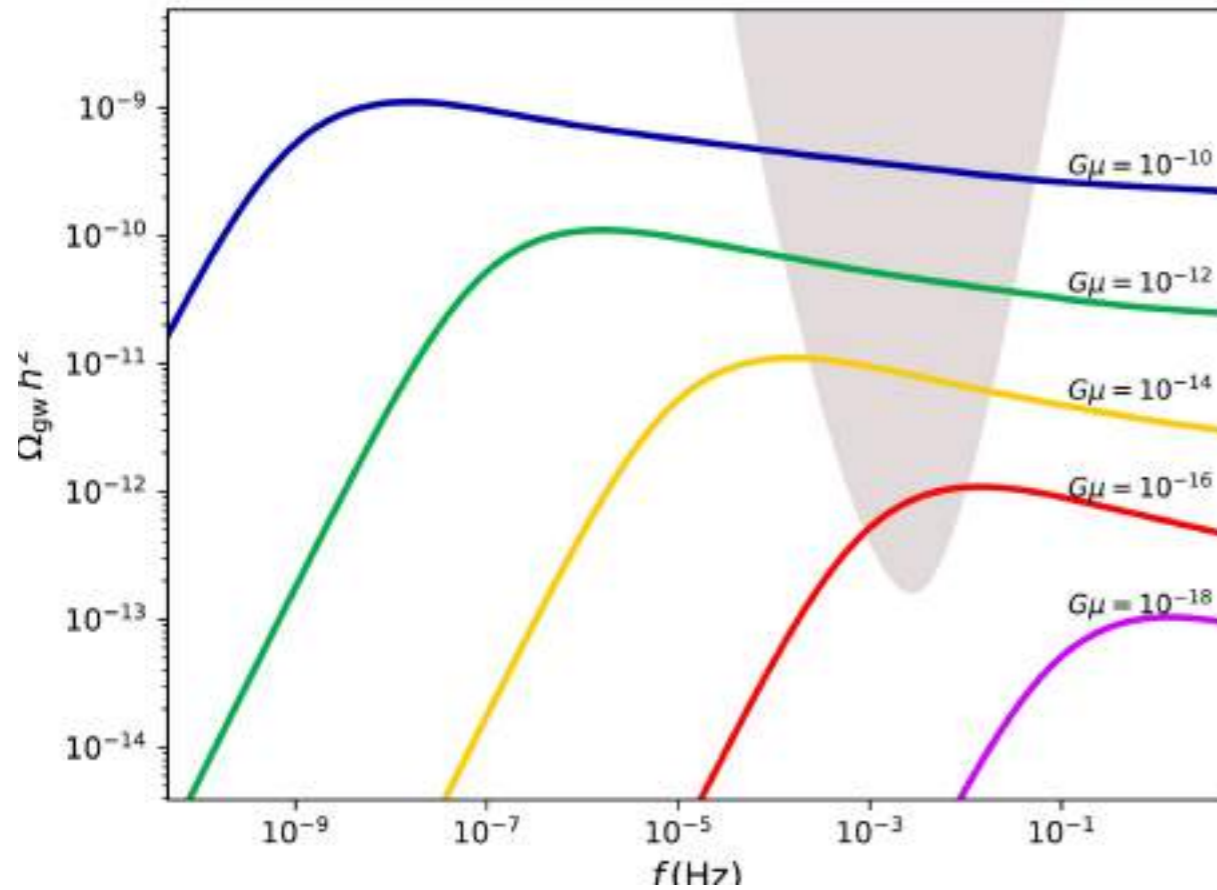
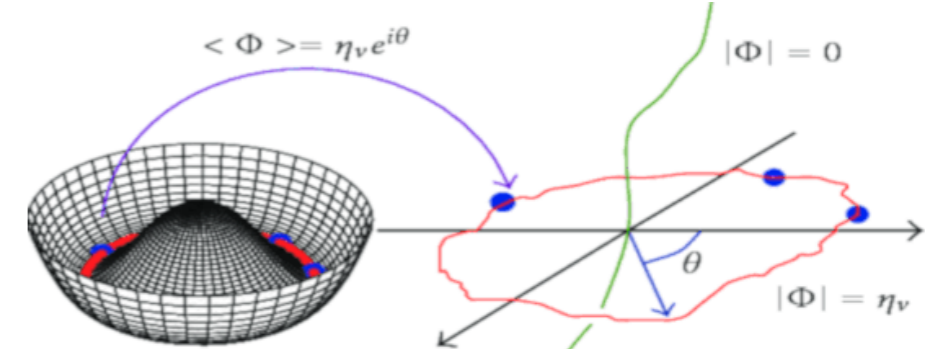


# Cosmic Strings in the Early Universe

Cosmic Strings (or other kind of topological defects) are non-trivial field configurations left-over after the phase transition has completed

A network of cosmic strings emits GWB

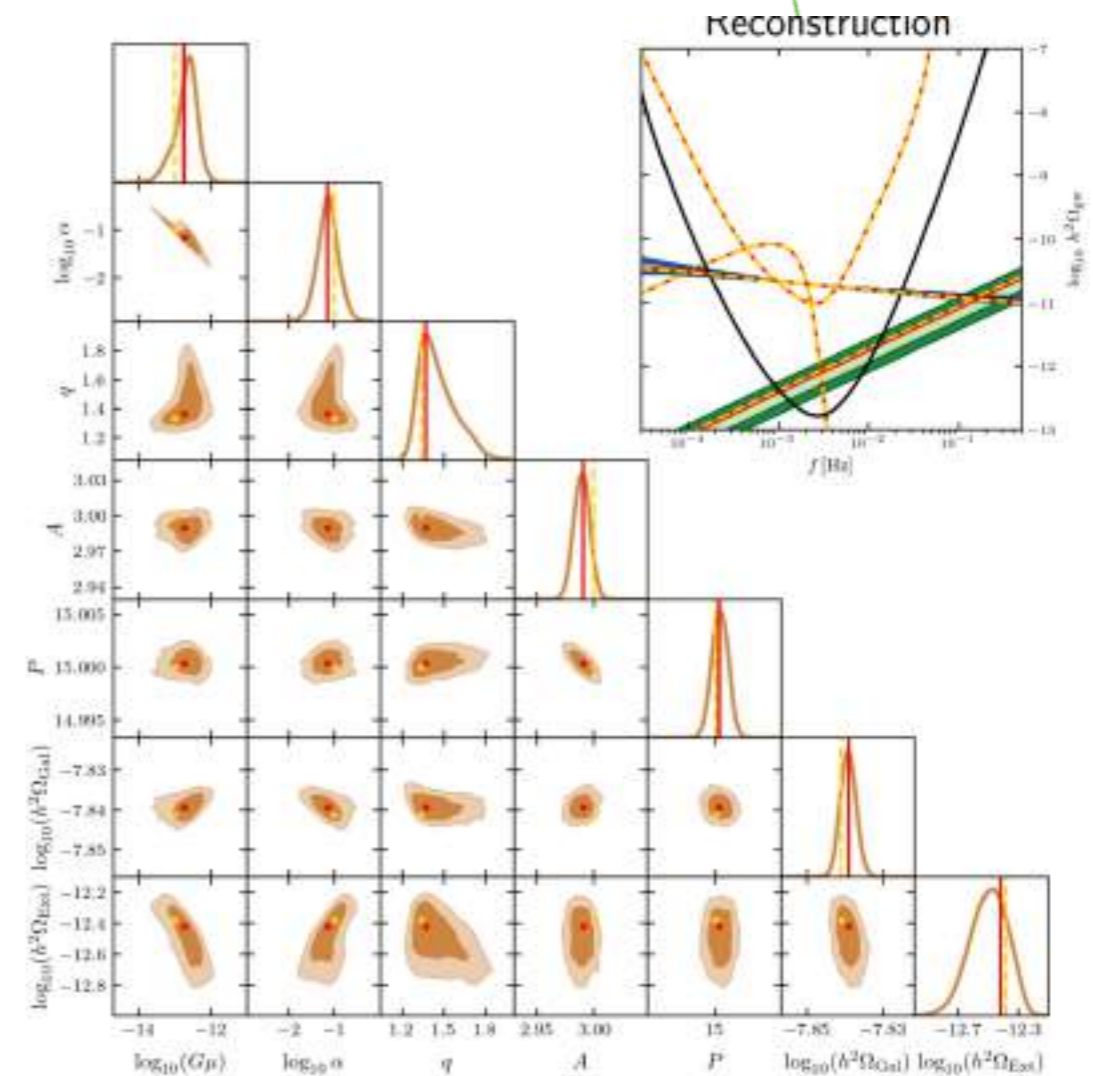
CS might form in the early Universe



String parameters:  $q$  - spectral index

$\alpha$  - loops size

$G\mu$  - tension



LISA CosGW project Arxiv: 2407.04356 (cosmic strings)

# Non-Gaussianity with LISA through SIGW

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}^{lm} \mathcal{S}_{lm}$$

[Matarrese et al; '93,'94]

[Ananda et al; '06]

[Baumann et al; '07]

[Domenech '21 review] ...

Source term

$$\mathcal{S}_{ij}(\mathbf{x}, \eta) \equiv 4\Phi\partial_i\partial_j\Phi + \frac{2(1+3w)}{3(1+w)}\partial_i\Phi\partial_j\Phi - \frac{4}{3(1+w)\mathcal{H}^2} [\partial_i\Phi'\partial_j\Phi' + \mathcal{H}\partial_i\Phi\partial_j\Phi' + \mathcal{H}\partial_i\Phi'\partial_j\Phi]$$

We can include the presence of **primordial non-Gaussianity**

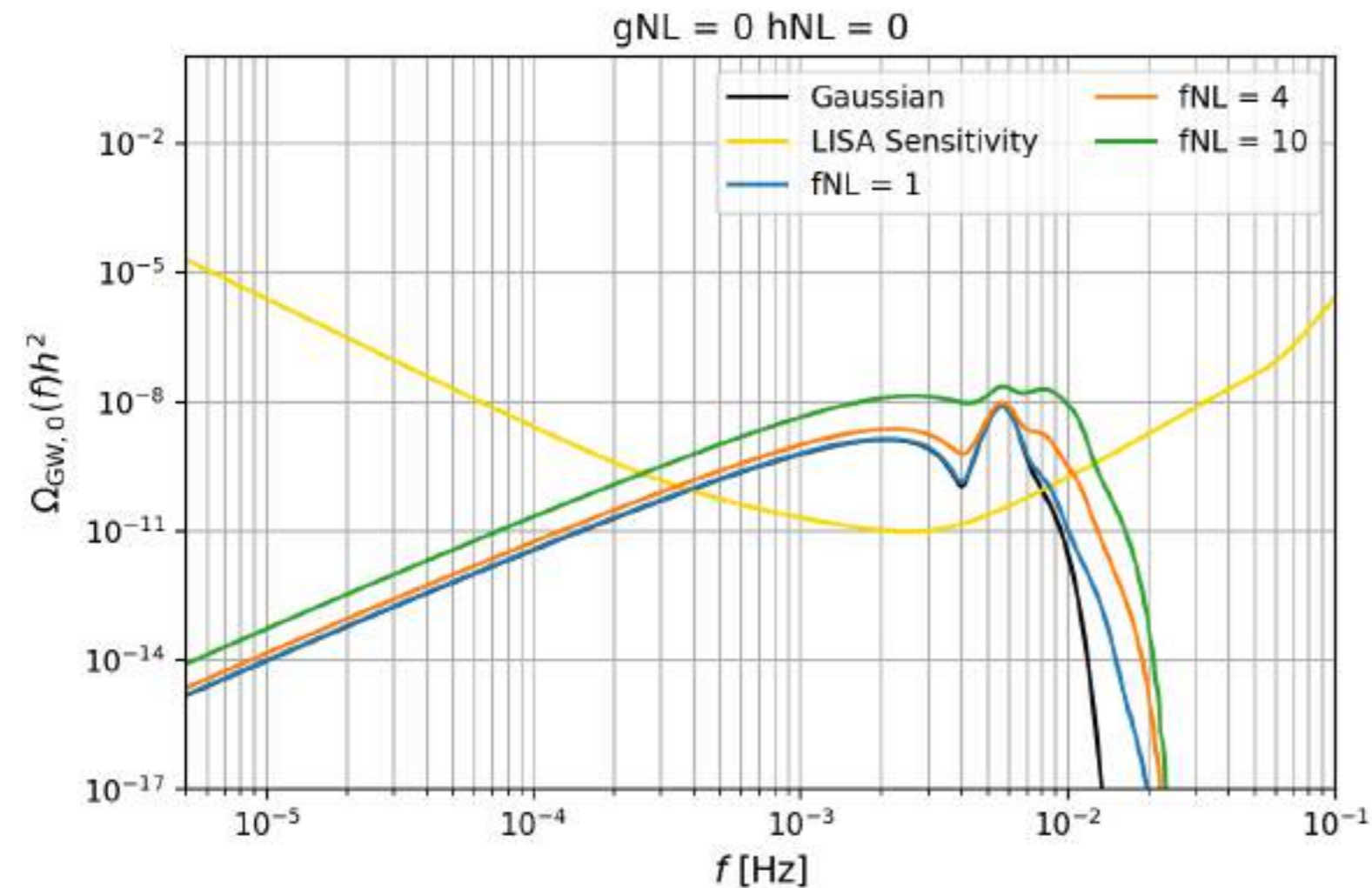
$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + f_{\text{NL}}(\mathcal{R}_g^2(\mathbf{x}) - \langle \mathcal{R}_g^2 \rangle).$$

$$\Phi(\mathbf{k}, \eta) = \frac{3+3w}{5+3w}\phi(k\eta)\mathcal{R}(\mathbf{k})$$

We focus on local non-Gaussianity

$$\Omega_{\text{GW}}(k, \eta) = \Omega_{\text{GW}}^G(k, \eta) + f_{\text{NL}}^2 \Omega_{\text{GW}}^{(2)}(k, \eta) + f_{\text{NL}}^4 \Omega_{\text{GW}}^{(4)}(k, \eta)$$

# Non-Gaussianity with LISA through SIGW

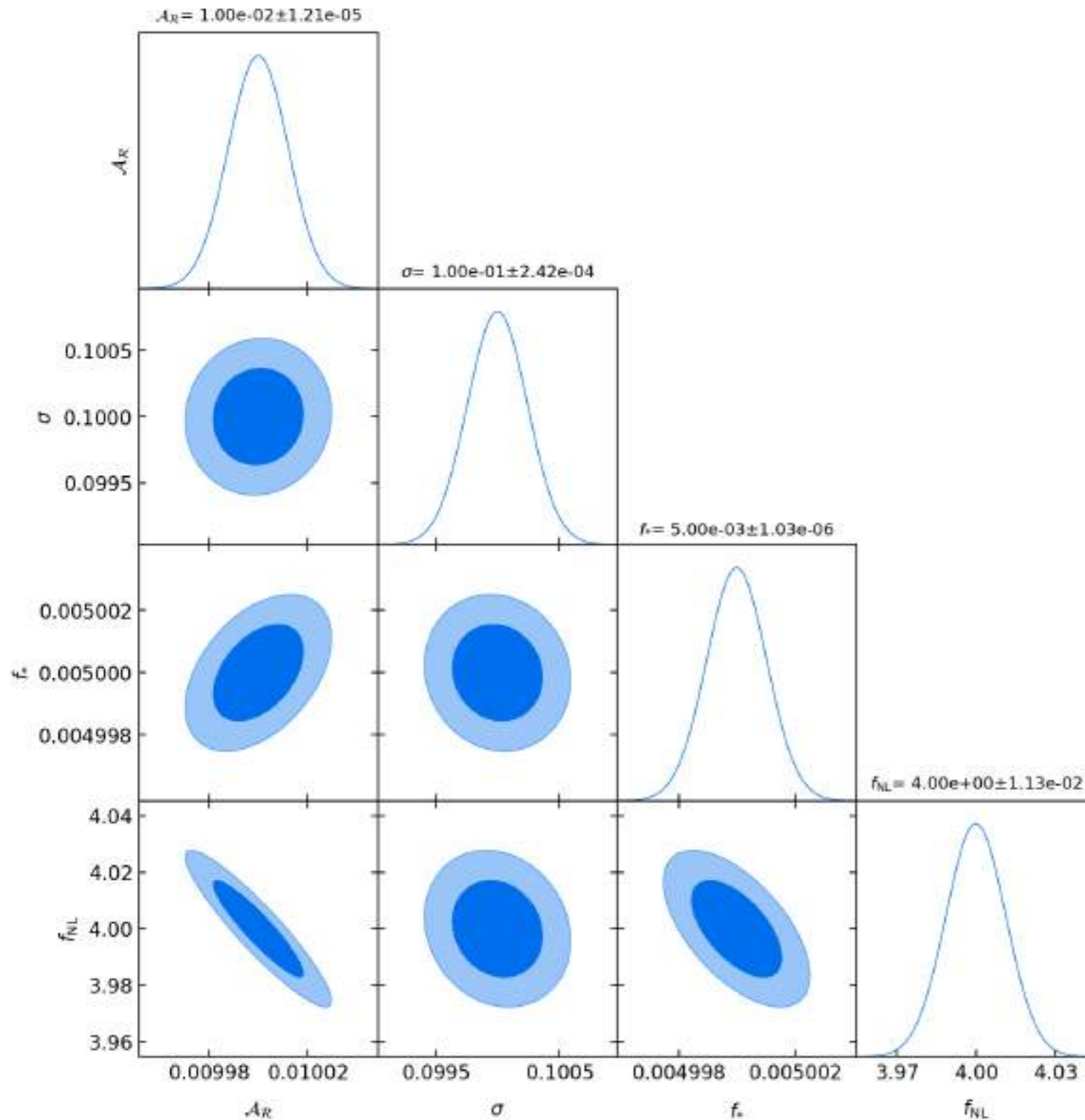


- Radiation domination
- LogNormal scalar power spectrum

$$\Delta_g^2(k) = \frac{\mathcal{A}_{\mathcal{R}}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(k/k_*)}{2\sigma^2}\right)$$

Non-Gaussianity has an impact on the amplitude and on the shape

# Probing non-Gaussianity with LISA through SIGW



- Radiation domination
- LogNormal scalar power spectrum

$$\Delta_g^2(k) = \frac{A_{\mathcal{R}}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(k/k_*)}{2\sigma^2}\right)$$

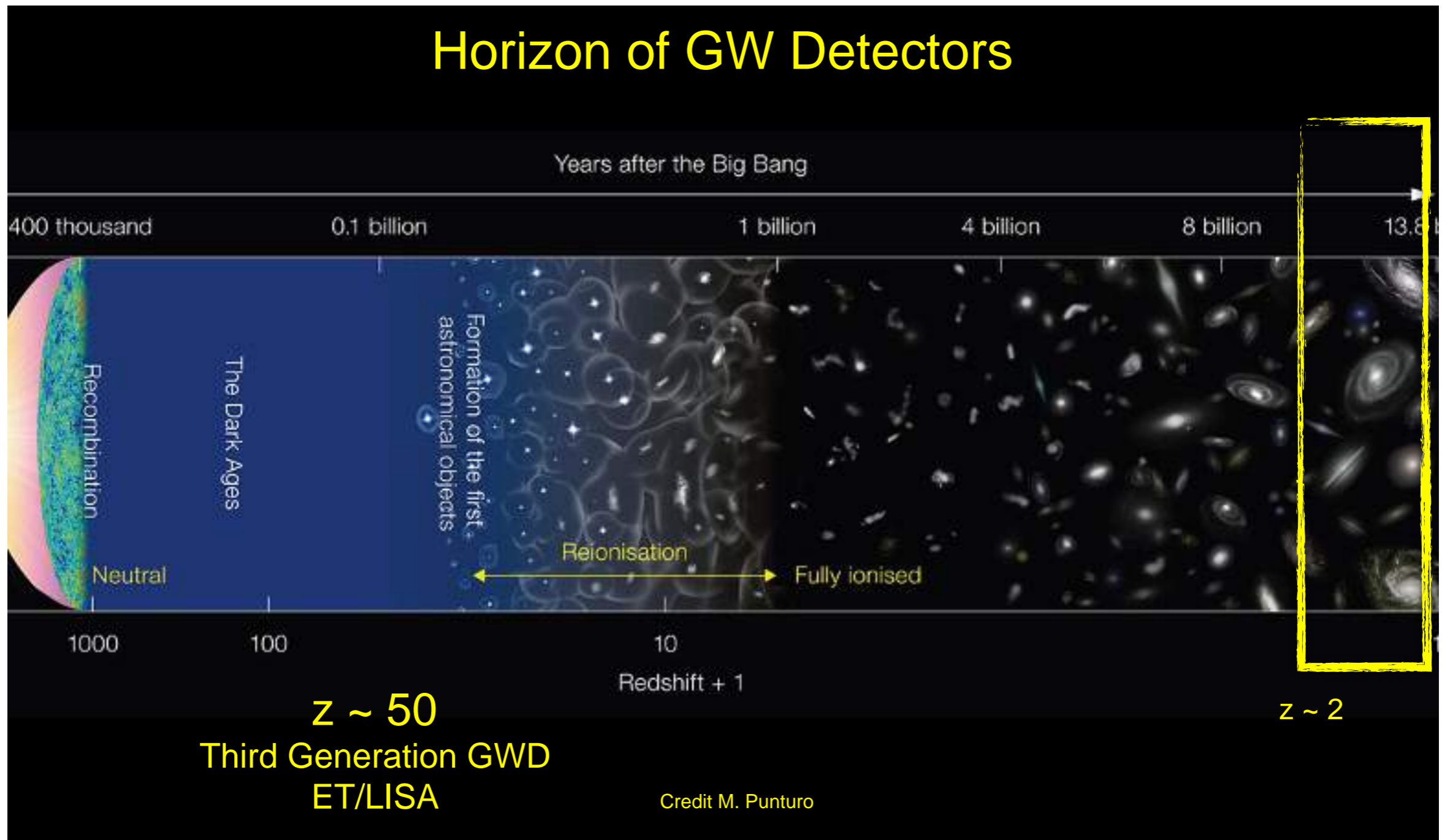
Fiducial values

$$A_{\mathcal{R}} = 10^{-2}$$

$$\sigma = 0.1$$

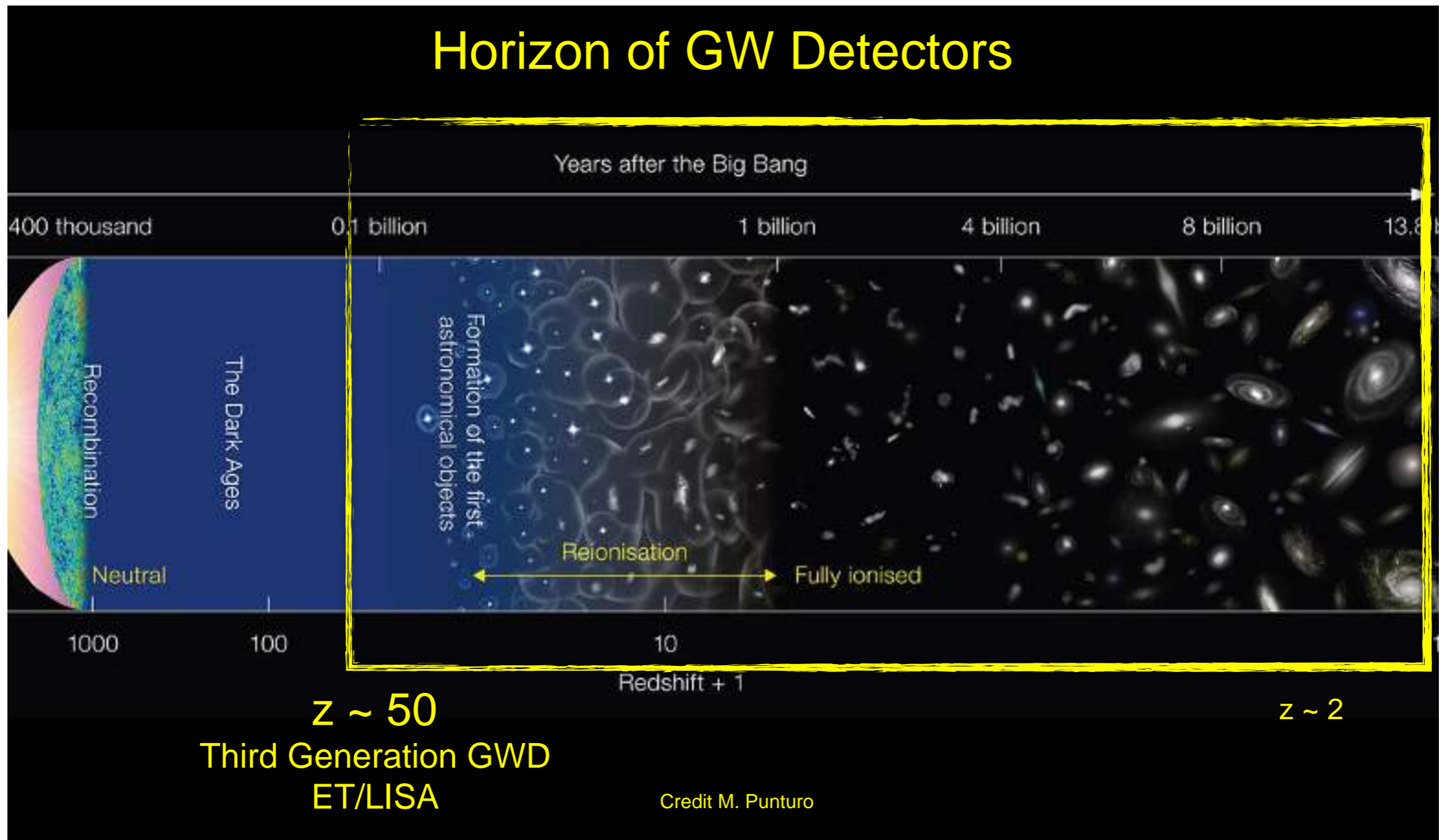
$$f_* = 0.005 \text{ Hz}$$

# Where is the horizon for 3G detectors?



**Einstein Telescope:**  
BBHs up to cosmic Dark Ages ( $z > 30$ )  
BNSs up to cosmic Noon ( $z \sim 2$ )

# Where is the horizon for 3G detectors?

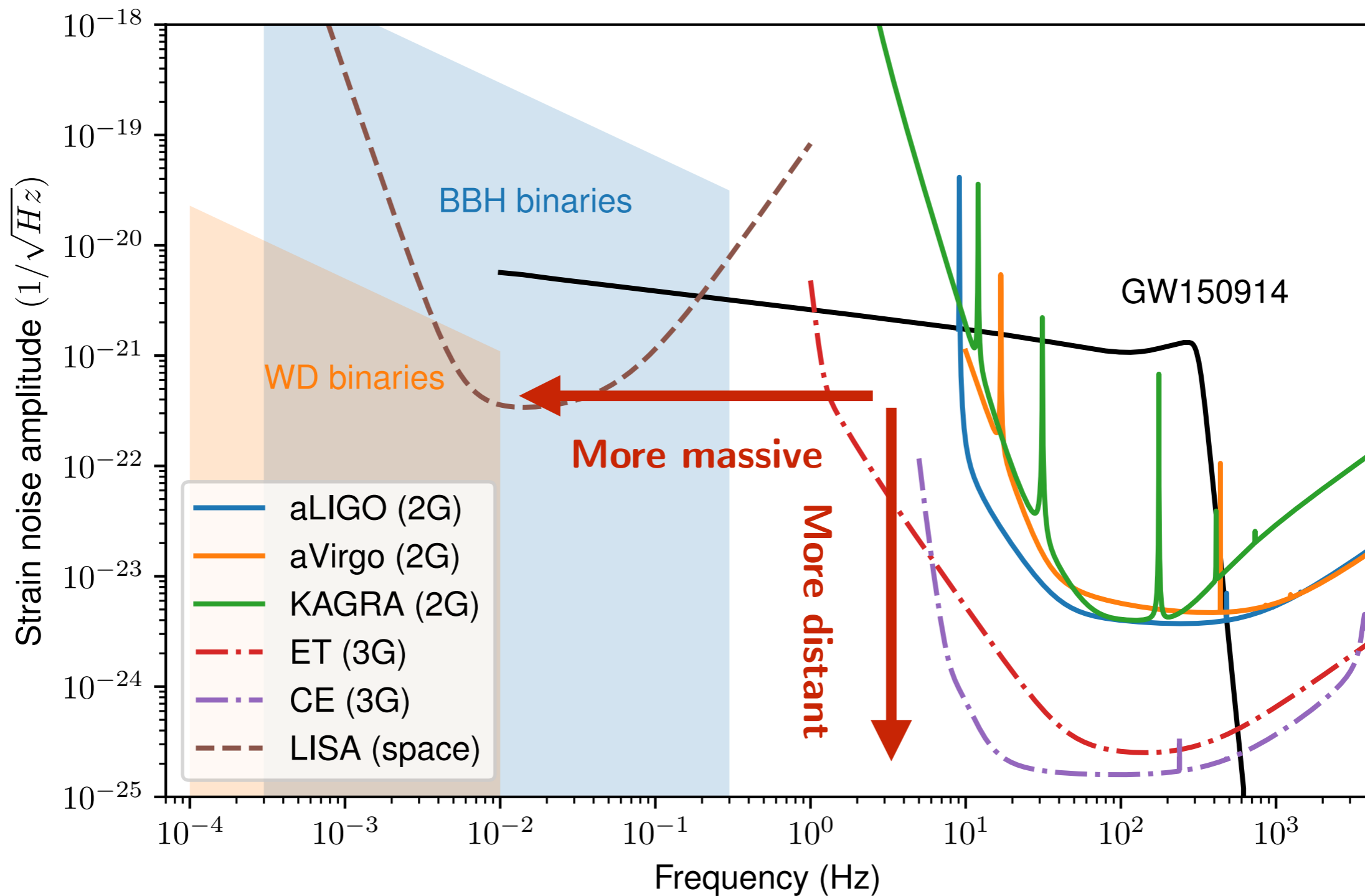


### Einstein Telescope:

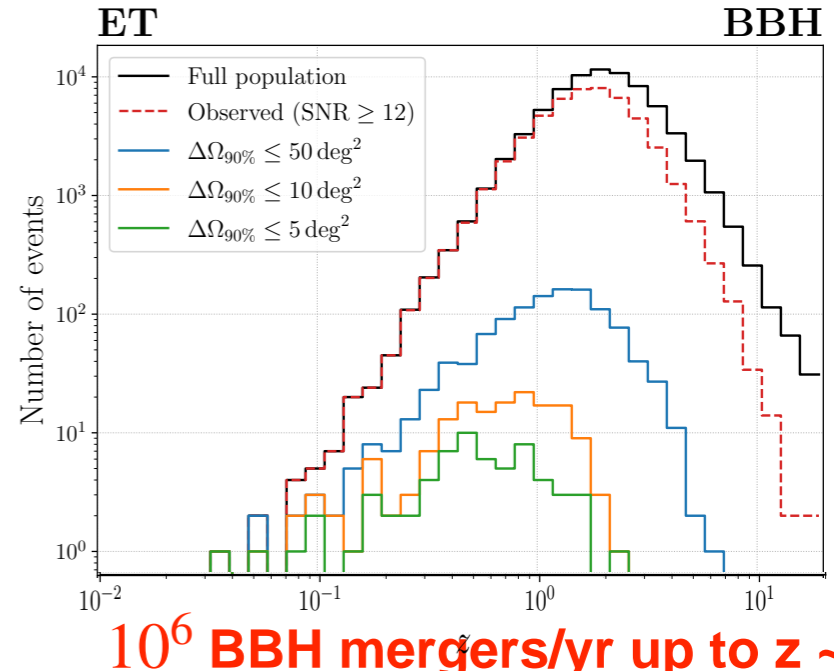
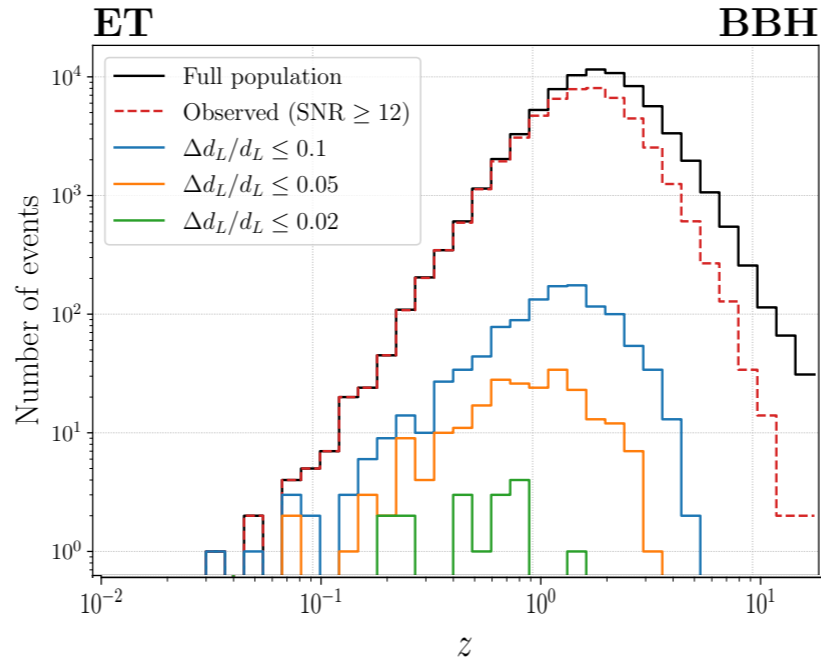
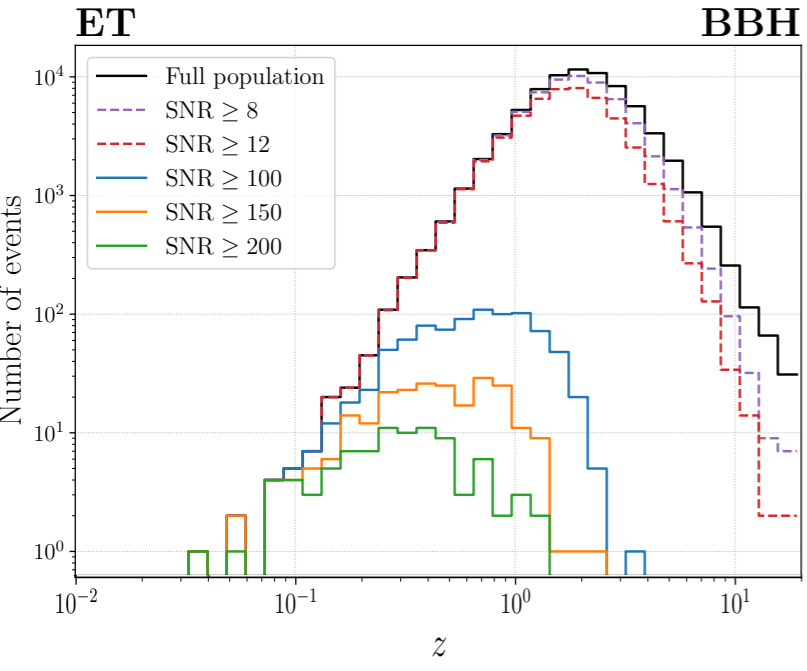
BBHs up to cosmic Dark Ages ( $z > 30$ )  
BNSs up to cosmic Noon ( $z \sim 2$ )

# From 2G to 3G detectors: ET and LISA

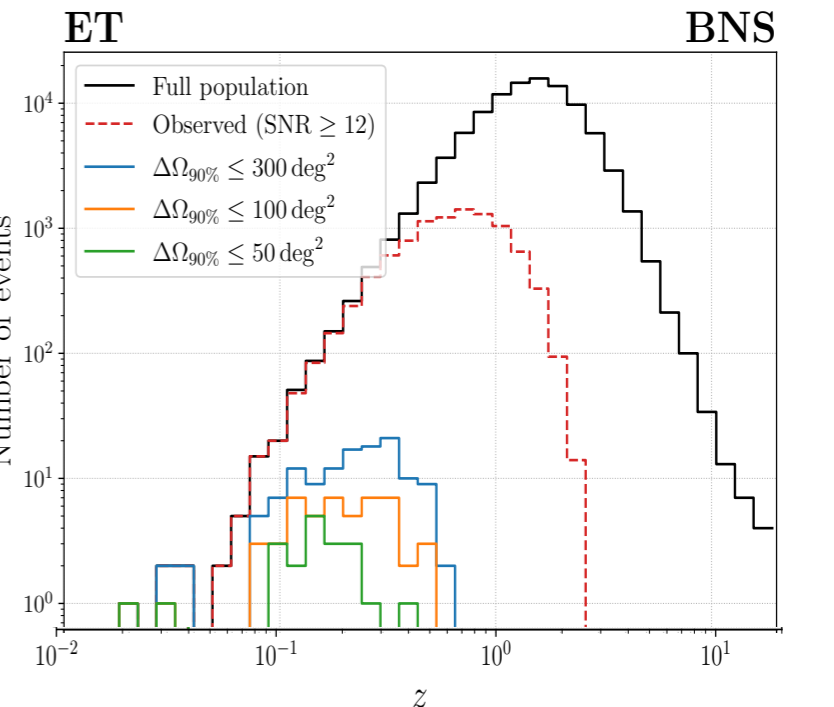
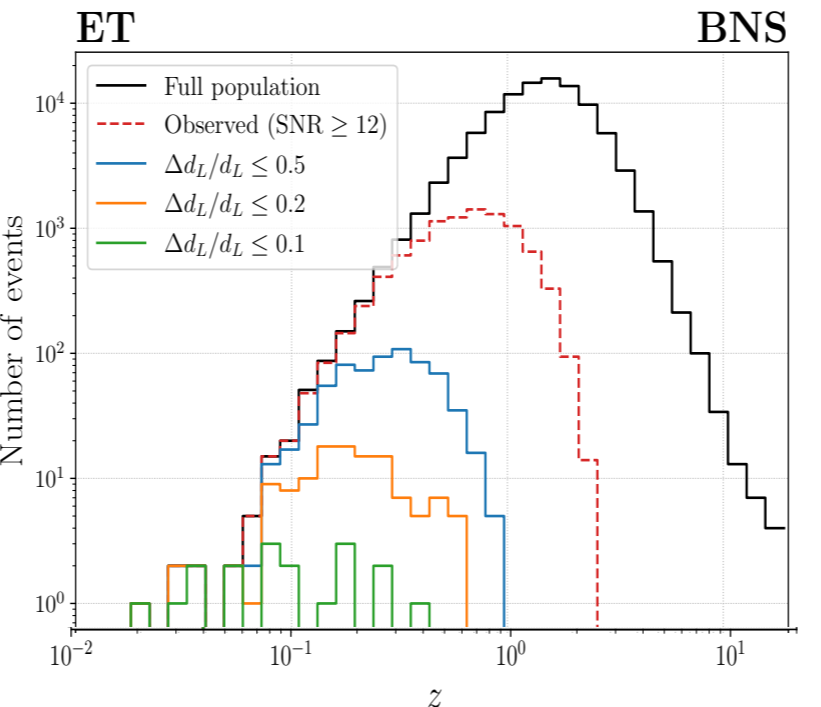
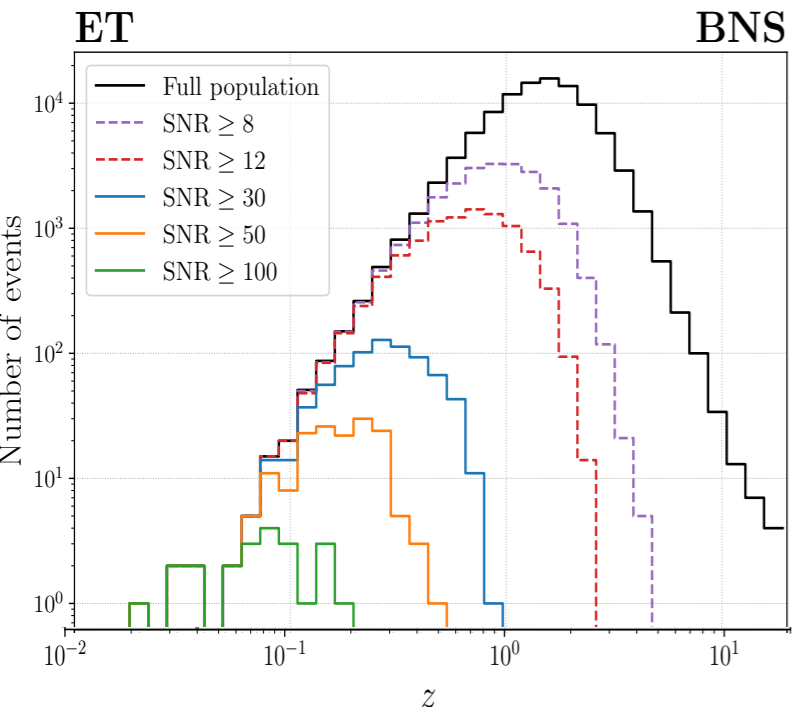
$$h_{\text{gw}} \sim 1/d_L \qquad f_{\text{gw}} \sim 1\text{kHz} \left( \frac{10M_\odot}{M} \right)$$



# Probing the Late Universe with ET



**10<sup>6</sup> BBH mergers/yr up to z ~ 50**



**10<sup>5</sup> BNS mergers/yr up to z ~ 2**

**Many Golden Events**

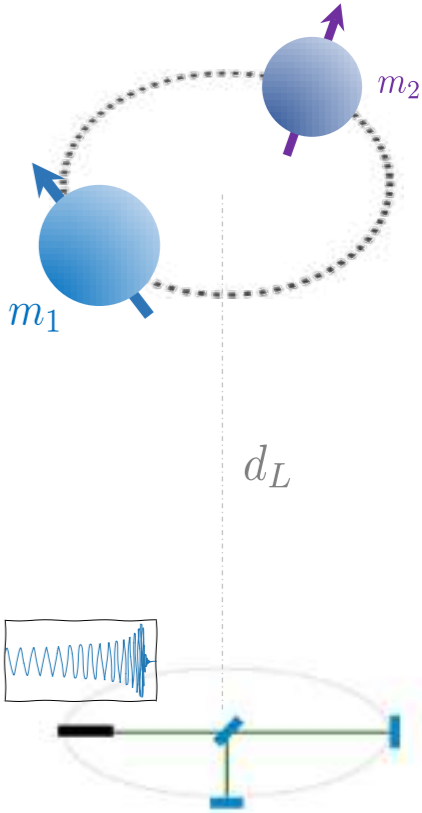


# Using GWs as Standard Sirens

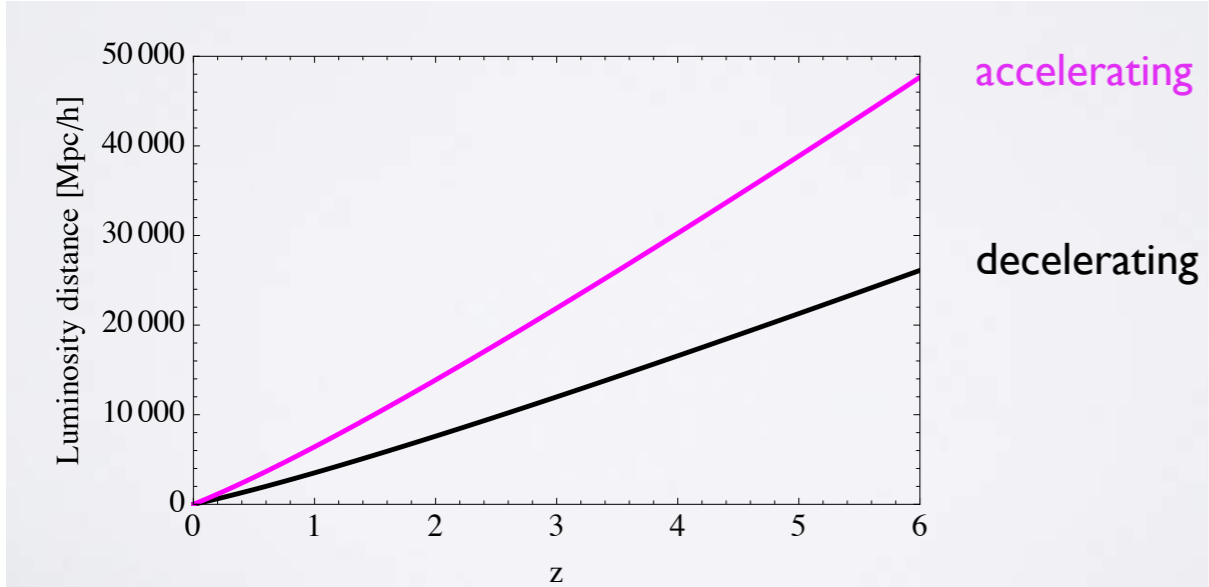
Gravitational waves from **individual sources at cosmological distances** (e.g. binary black holes, binary neutron stars...) have the potential to give a totally independent measurement of  $H_0$

[B.Schultz, Nature, 1986] **standard sirens**

- Detect GWs emitted by coalescing binaries
- From the waveform, measure directly the luminosity distance  $d_L(z)$



- If, in addition, can determine the redshift  $z$  of the source, then have a point on curve  $d_L(z)$



Direct probe of cosmology

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{[\Omega_M(1+z')^3 + \Omega_\Lambda(1+z')^{3(1+w)}]^{1/2}}$$

For  $z \ll 1$ , the relationship reduces simply to the Hubble law:

$$cz = H_0 \times d_L$$

redshift
Hubble constant
Luminosity distance

# Using GWs as Standard Sirens

$$c z = H_0 \times d_L$$

Diagram illustrating the equation  $c z = H_0 \times d_L$ . Arrows point from the labels below to the corresponding variables in the equation:

- redshift points to  $z$
- Hubble constant points to  $H_0$
- Luminosity distance points to  $d_L$

- three quantities: pick any two and infer the third.
- With standard sirens:

$d_L$  from GW measurements;

$z$  from, e.g. electromagnetic measurements (if have an optical counterpart, and know the host galaxy, can determine  $z$ ).

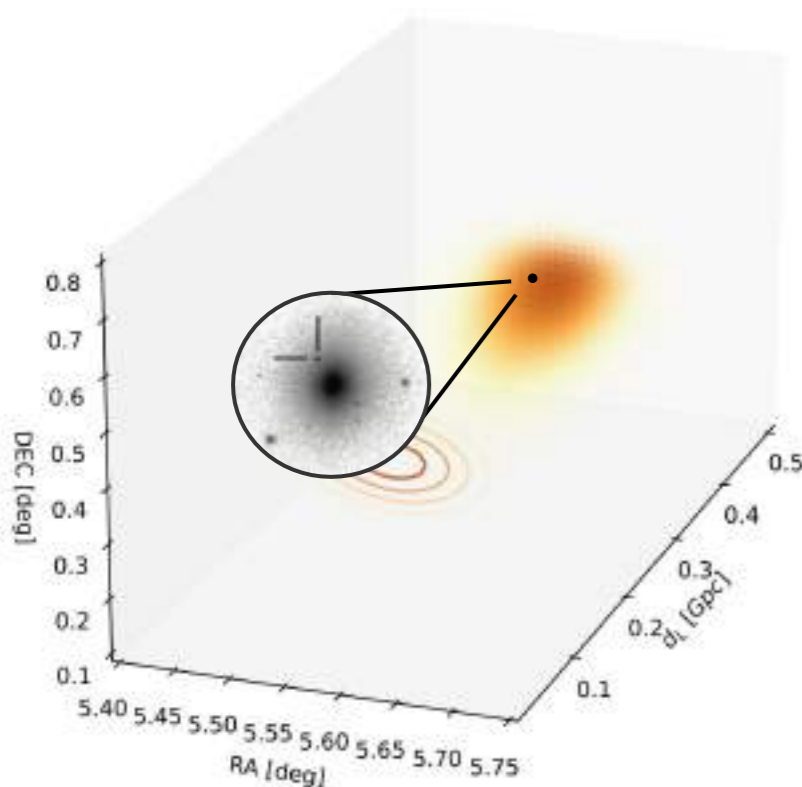
=> independent measure of  $H_0$

**Cosmology via the distance-redshift relation**  
**... but no **redshift** measurement with GW data alone**  
**(Degeneracy with masses)**

# Using GWs as Standard Sirens - Redshift Information

## Bright Sirens

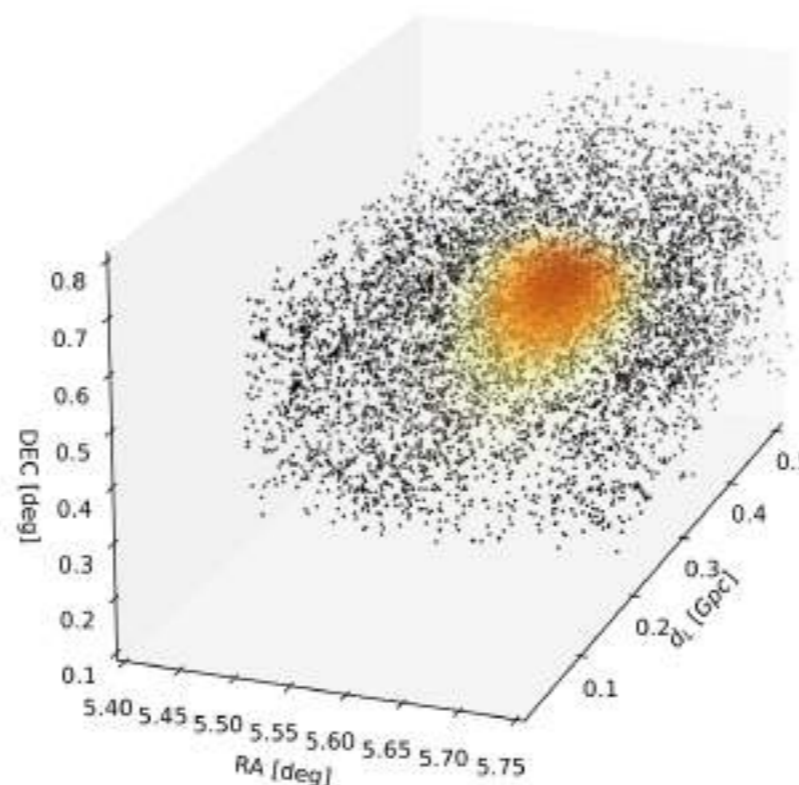
An **EM counterpart** is observed and used to obtain the host galaxy redshift.



(Holz & Hughes 2005)

## Dark Sirens

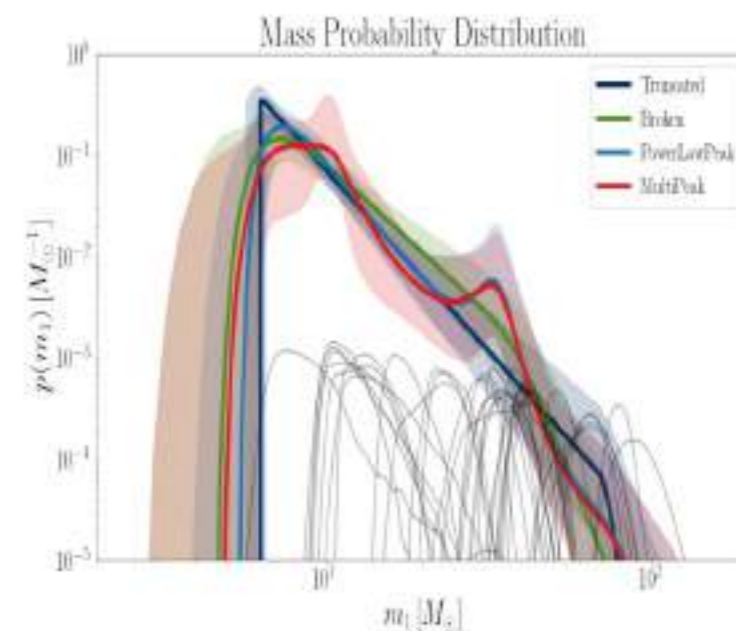
No EM counterpart observed. **Galaxy surveys** are used to provide redshift estimates for potential host galaxies.



(Schutz 1986, Del Pozzo 2012)

## Spectral sirens

No EM counterpart or galaxy survey is used. Features in the **mass distribution** of the GW population break the mass-redshift degeneracy.



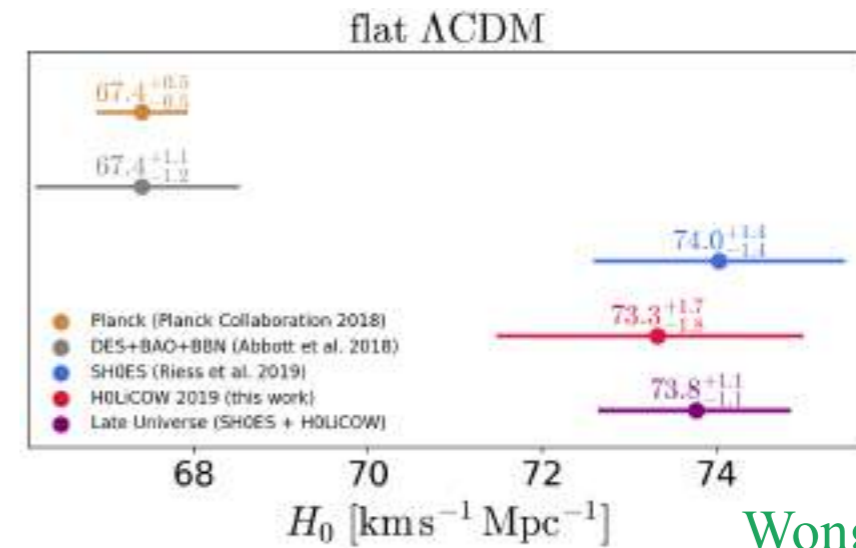
(Chernoff & Finn 1993)

# $H_0$ - where we are

~ 4 \sigma tension between low and high redshift measurements of the Hubble parameter

$$H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ (Early Universe)}^1$$

$$H_0 = 73.30 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ (Late Universe)}^2$$



Wong et al.,  
H0LiCOW 2019

<sup>a</sup>Abbott et al., 2017 [arXiv:1710.05835]

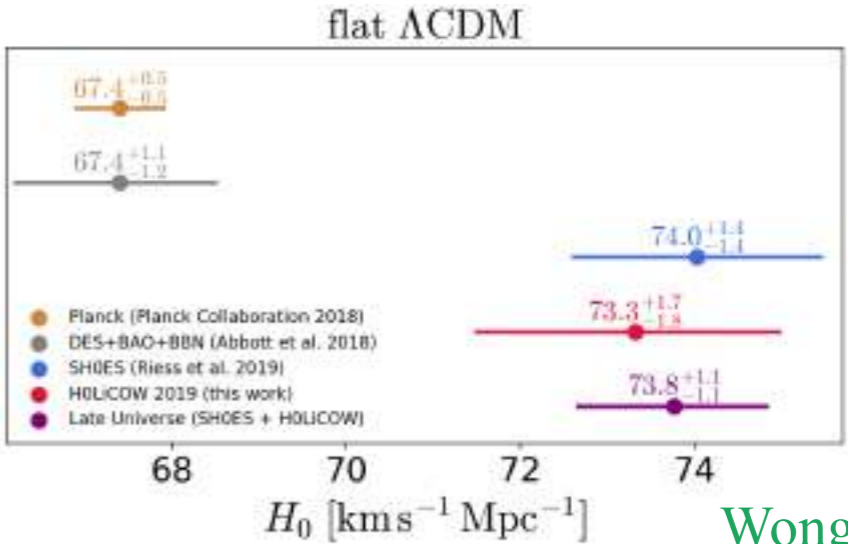
<sup>b</sup>Abbott et al., 2021 [arXiv:2111.03604]

# $H_0$ - where we are

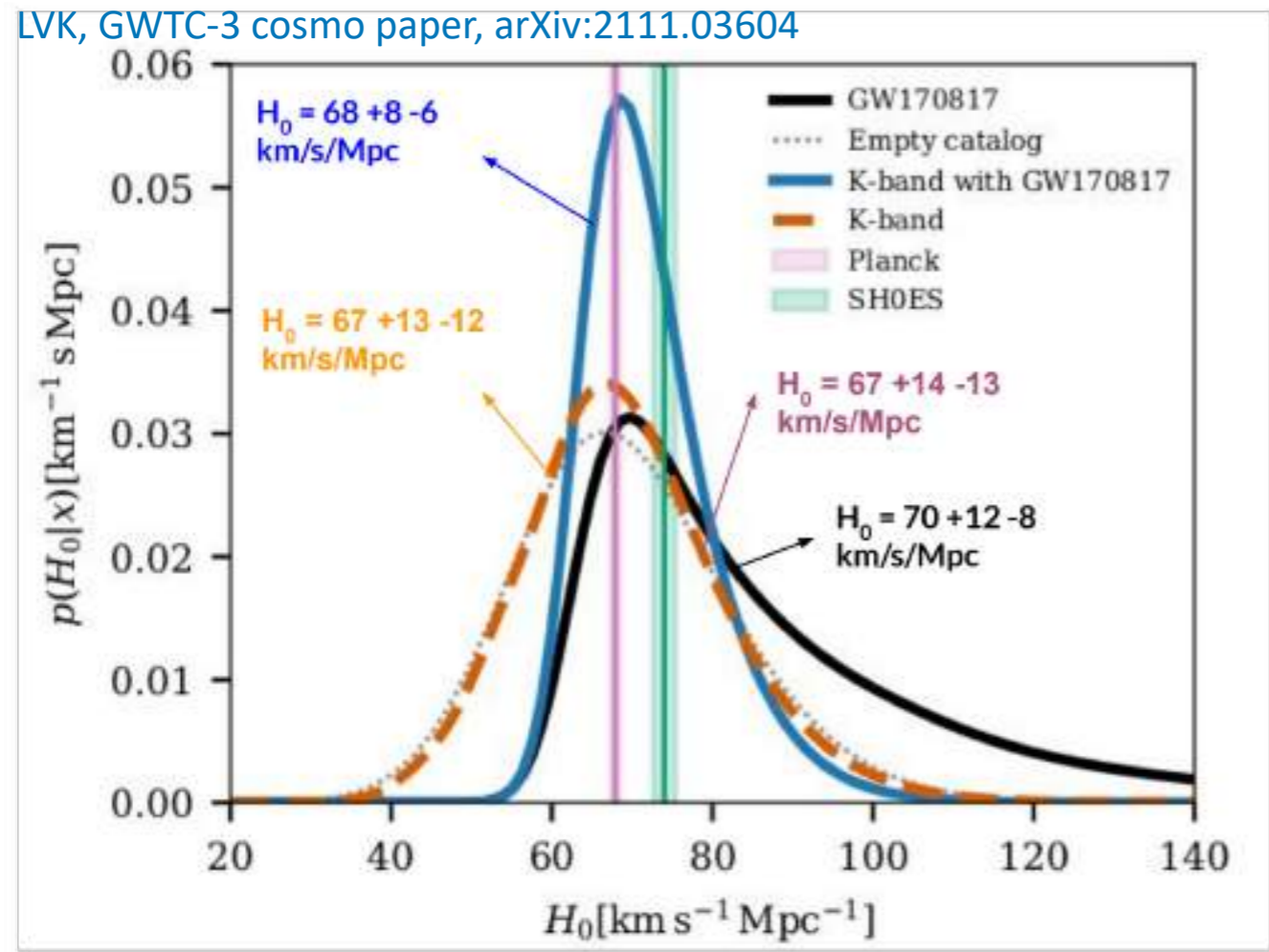
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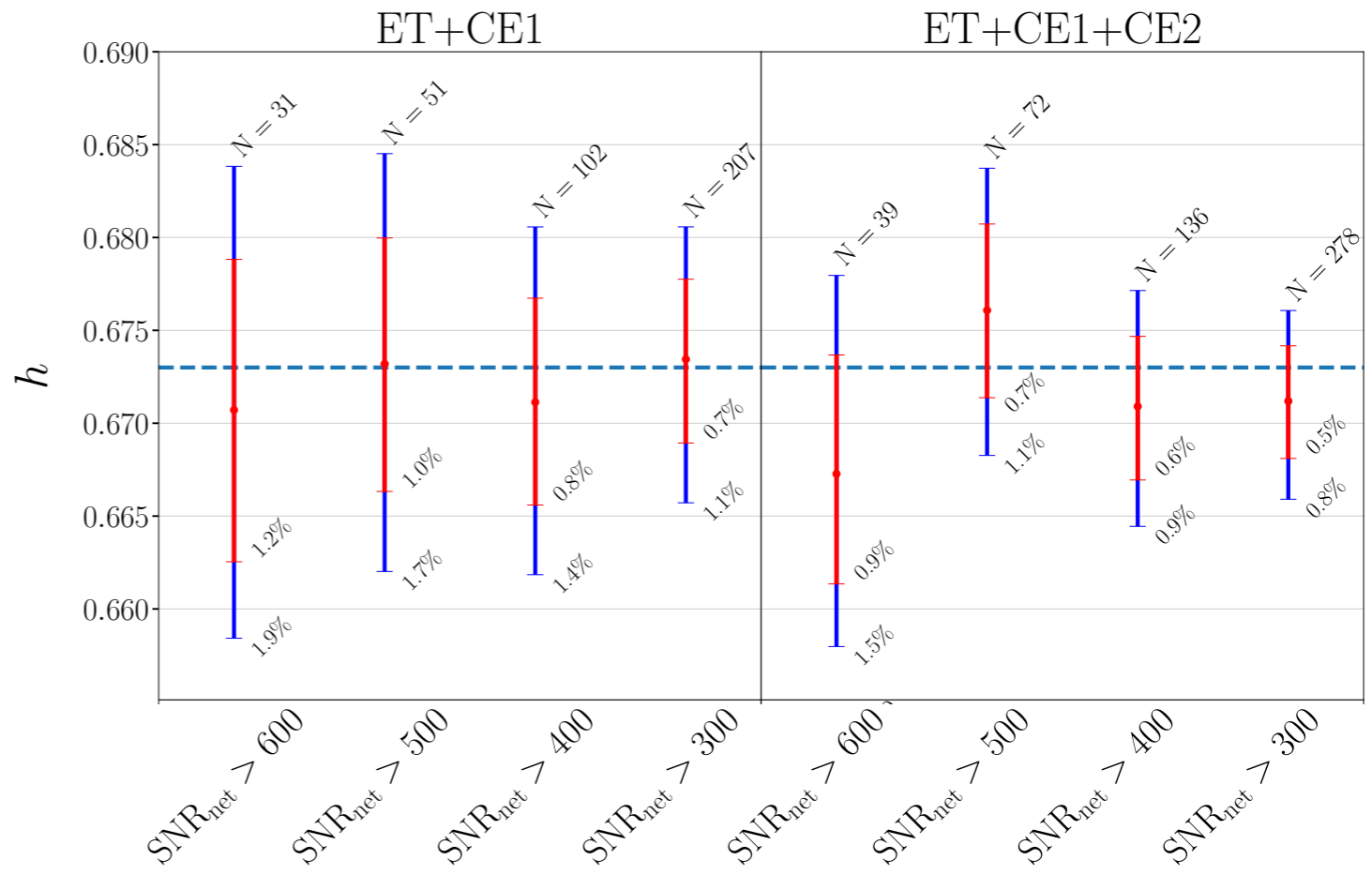
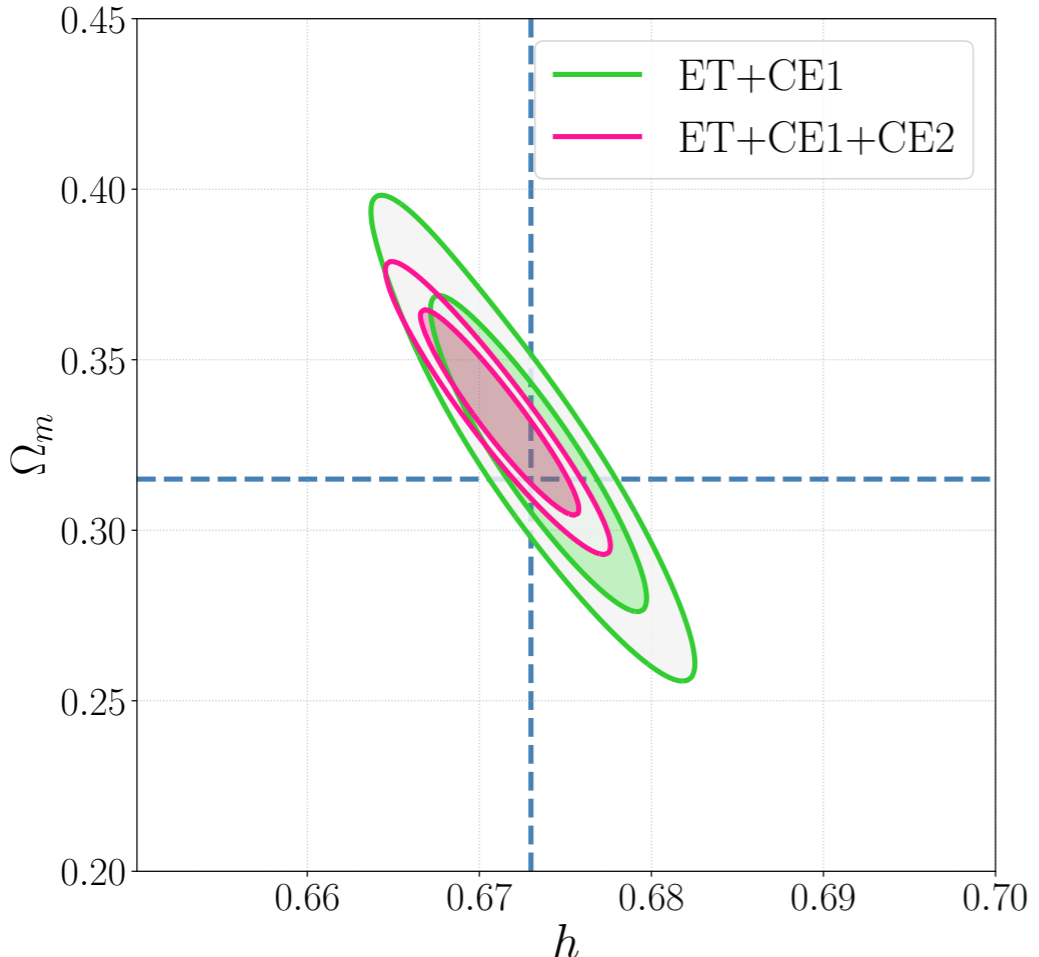
Wong et al.,  
H0LiCOW 2019



- $H_0 = 72.2^{+13.9}_{-7.5}$  km/s/Mpc  
[Finke et al., JCAP (2021) with GWTC-2 catalog]
- $H_0 = 72.8^{+11.0}_{-7.55}$  km/s/Mpc  
[Palmese et al., ApJ (2023) with GWTC-3 catalog]

<sup>a</sup>Abbott et al., 2017 [arXiv:1710.05835]  
<sup>b</sup>Abbott et al., 2021 [arXiv:2111.03604]

# $H_0$ - where we will be with ET? DARK SIRENS method



**Fiducial scenario**

1 year of observation

Complete galaxy catalogue

Full duty cycle

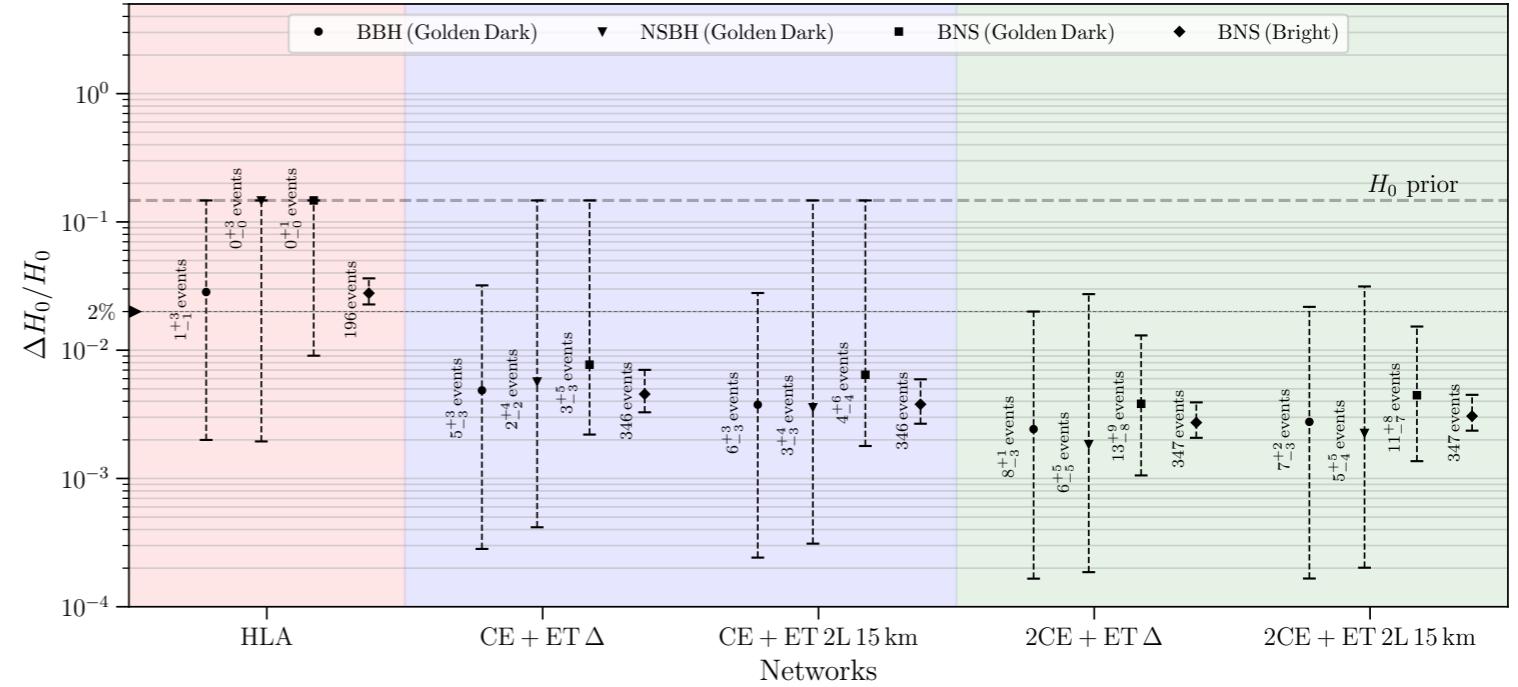
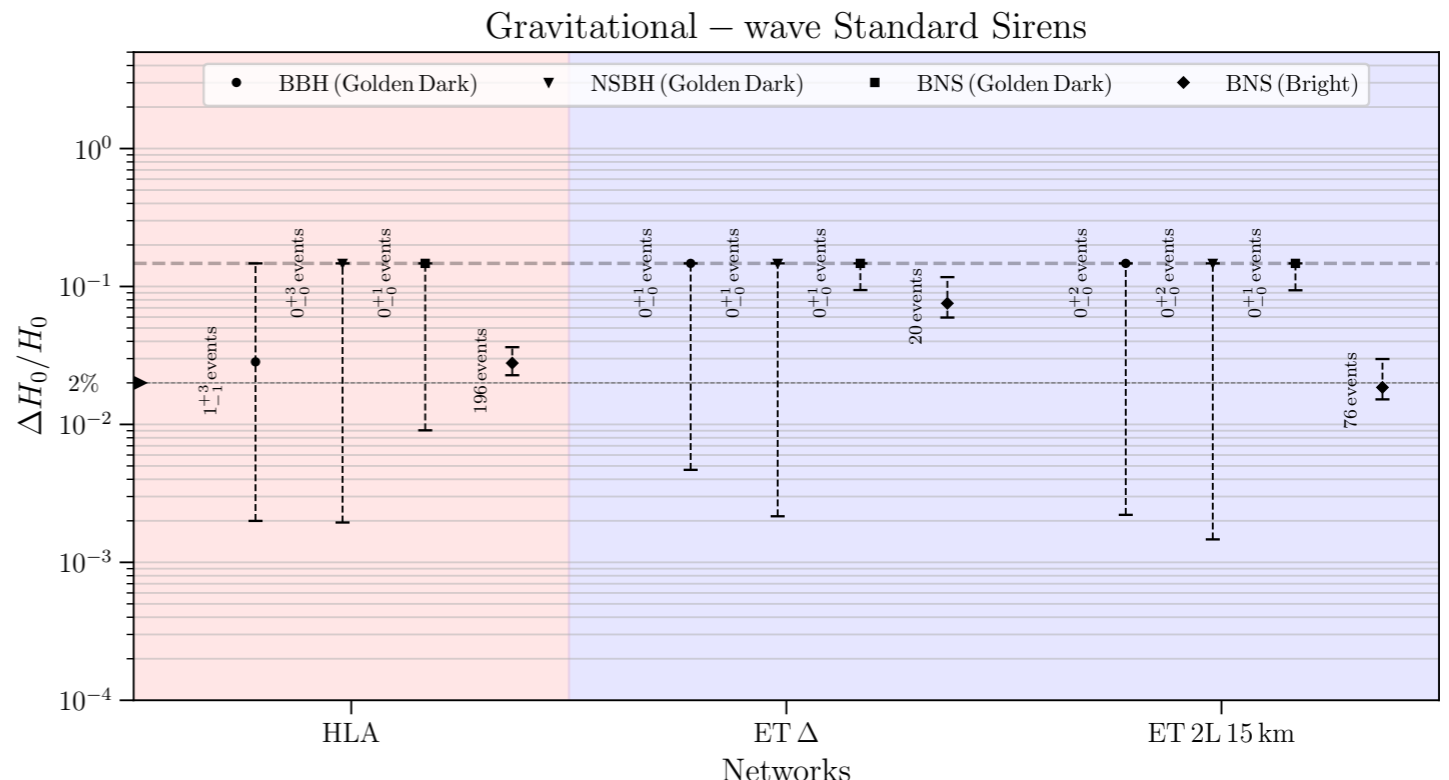
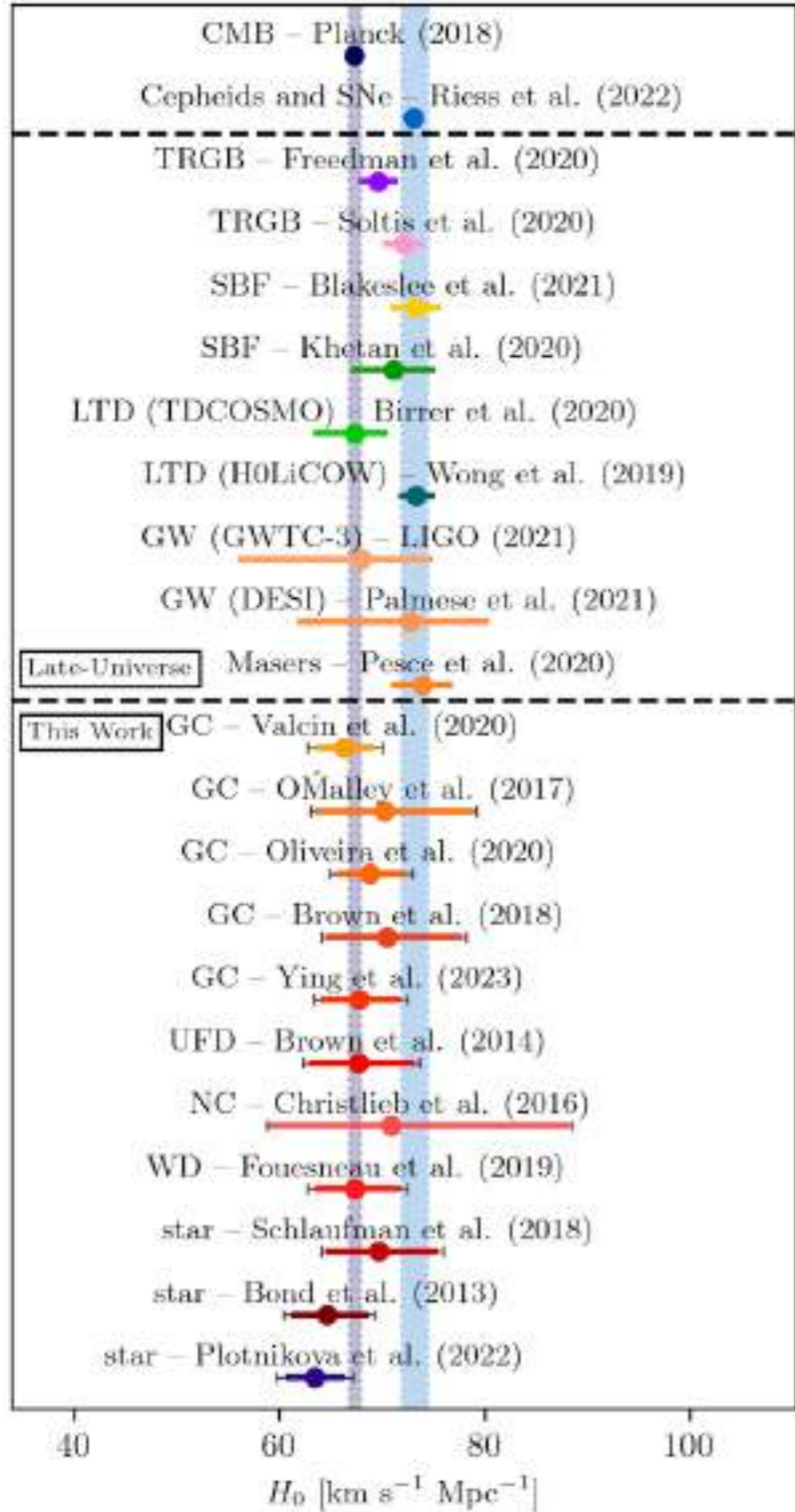
**precision for the inference of h with dark sirens with a galaxy catalog complete up to  $z < 1$**

Network	N		$\Delta h/h$ (%)				$\Delta \Omega_m / \Omega_m$ (%)		
	$z < 1$	$z < 3$	$z < 1$	$z < 1$ fixed $\Omega_m$	$z < 1$ single-host	$z < 3$	$z < 1$	$z < 1$ single-host	$z < 3$
ET+CE1	207	248	0.6 (1.1)	0.2 (0.4)	3.3-7.1 (5.6-11.2)	0.7 (1.1)	8.8 (14.4)	-	8.8 (14.6)
ET+CE1+CE2	278	348	0.5 (0.8)	0.2 (0.3)	1.7-2.1 (2.7-3.3)	0.4 (0.7)	6.1 (10.0)	-	5.3 (8.7)

Expected cosmological constraints at the 68% (90%) CI for multiyear 3G observations estimated from the 1 year fiducial results

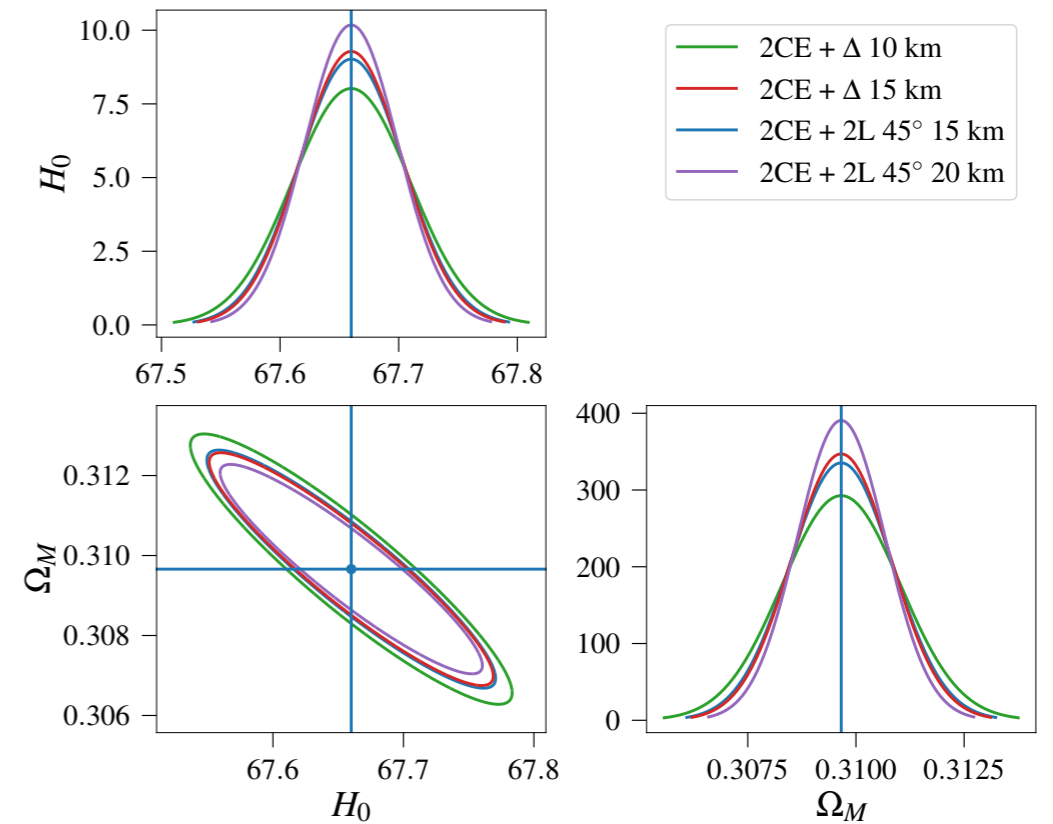
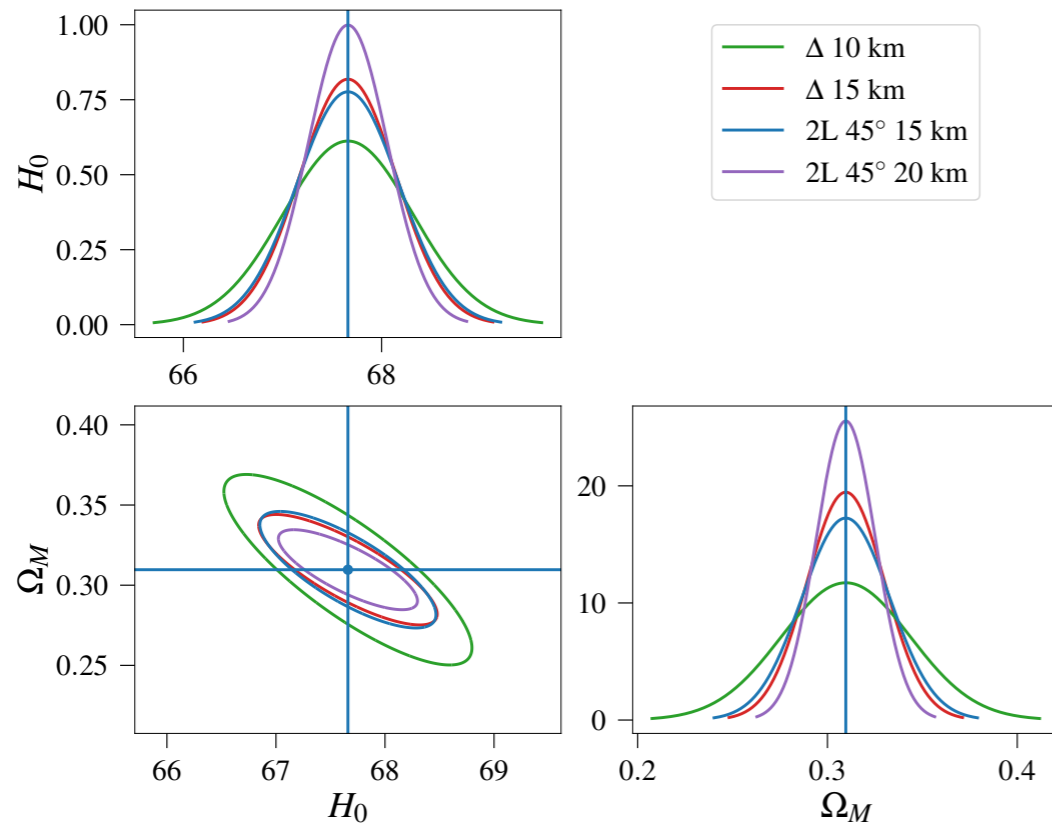
Nicolo Muttoni et al. 2023

# $H_0$ - shedding light on the $H_0$ tension



Work in progress

# $H_0$ - where we will be with ET? EM counterpart (BNS)



Configuration	$\Delta H_0/H_0$	$\Delta \Omega_M/\Omega_M$
$\Delta$ -10km	$9.63 \times 10^{-3}$	$1.10 \times 10^{-1}$
$\Delta$ -15km	$7.20 \times 10^{-3}$	$6.62 \times 10^{-2}$
2L-15km-45°	$7.59 \times 10^{-3}$	$7.47 \times 10^{-2}$
2L-20km-45°	$5.90 \times 10^{-3}$	$5.04 \times 10^{-2}$

Configuration	$\Delta H_0/H_0$	$\Delta \Omega_M/\Omega_M$
$\Delta$ -10km +2CE	$7.35 \times 10^{-4}$	$4.40 \times 10^{-3}$
$\Delta$ -15km +2CE	$6.35 \times 10^{-4}$	$3.71 \times 10^{-3}$
2L-15km-45° +2CE	$6.54 \times 10^{-4}$	$3.84 \times 10^{-3}$
2L-20km-45° +2CE	$5.79 \times 10^{-4}$	$3.30 \times 10^{-3}$

**Constraints on the parameters  $H_0$  and  $\Omega_M$  in  $\Lambda$ CDM model using one year GW observations from BNS alone for different ET geometries.**



# Conclusions

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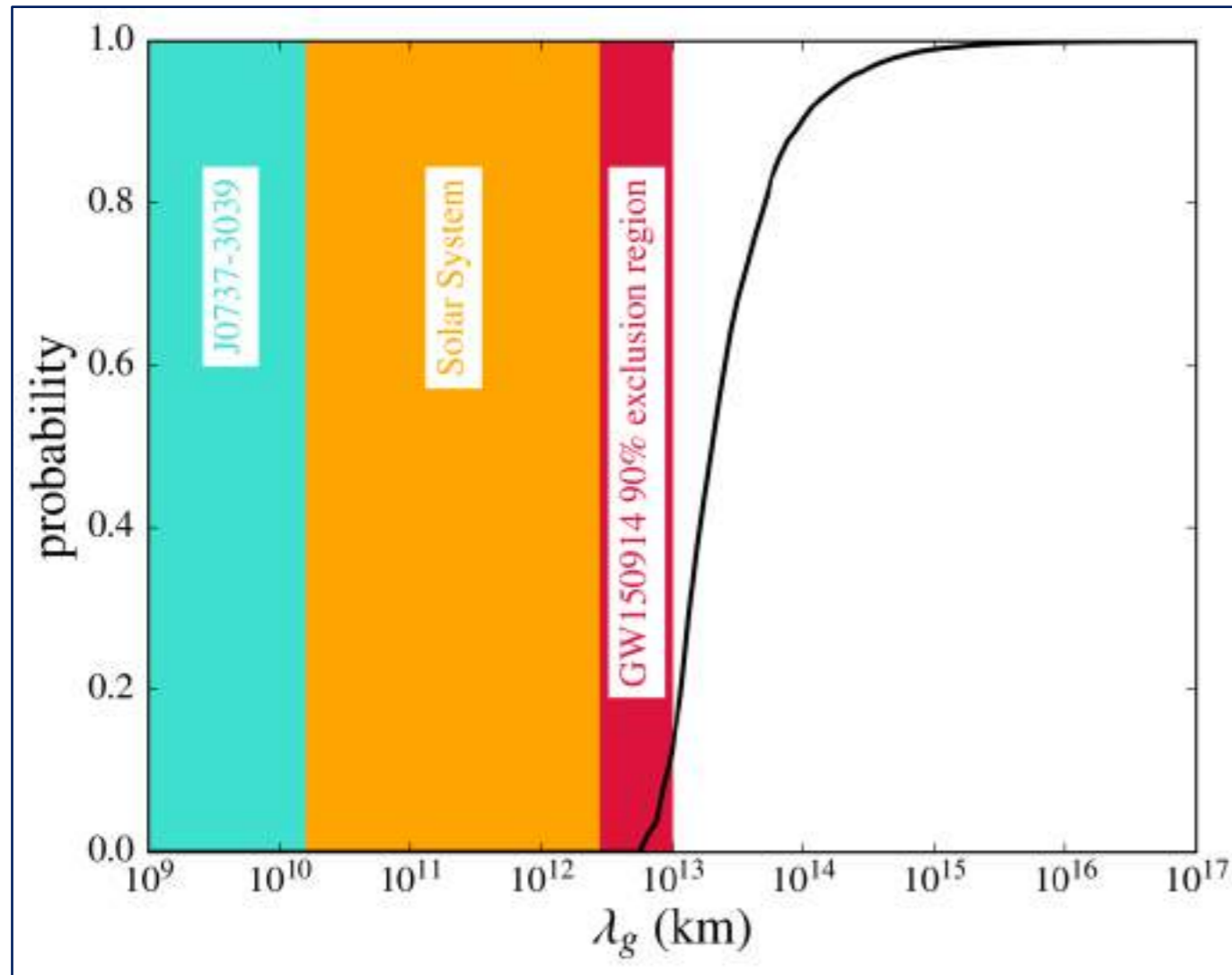
- All **GW interferometers** have been initially **conceived as GW astrophysics observatories**, they have **not been designed to do cosmology**
- **However**, they can provide new information on a variety of scales: from the Galaxy to Hubble scales, from the present time to the **very early universe** -> therefore they can be used as a **cosmological observatory** as well
- **We can have access to energy scales not accessible in any collider**
- We can test the ***late-time universe*** through the observation of the GW emission from compact binaries, and **measure cosmological parameters.**
- **A lot of expertise here in the GW community to push forward on these topics (astro+cosmo+fundamental physics+noise characterization)**

**Thank you!**



# Limit on the mass of the graviton

Bounds on the Compton wavelength  $\lambda_g = h/m_g c$  of the graviton compared to Solar System or double pulsar tests. Some cosmological tests are stronger (but make assumptions about dark matter)



$$\delta\Phi(f) = -\frac{\pi Dc}{\lambda_g^2(1+z)} f^{-1}$$

Will, Phys. Rev. D 57, 2061 (1998)

Massive-graviton theory dispersion relation  $E^2 = p^2 c^2 + m_g^2 c^4$

We have  $\lambda_g = h/(m_g c)$

Thus frequency dependent speed

$$\frac{v_g^2}{c^2} \equiv \frac{c^2 p^2}{E^2} \cong 1 - h^2 c^2 / (\lambda_g^2 E^2)$$

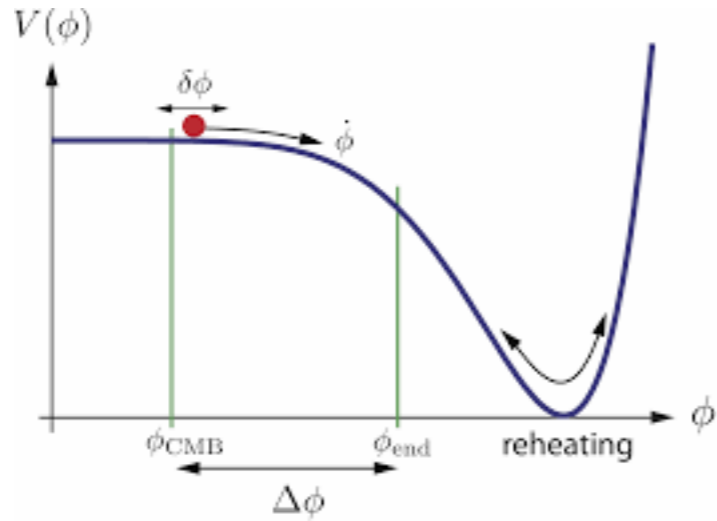
$$\lambda_g > 10^{13} \text{ km}$$

$$m_g \leq 5 \times 10^{-23} \text{ eV}/c^2$$

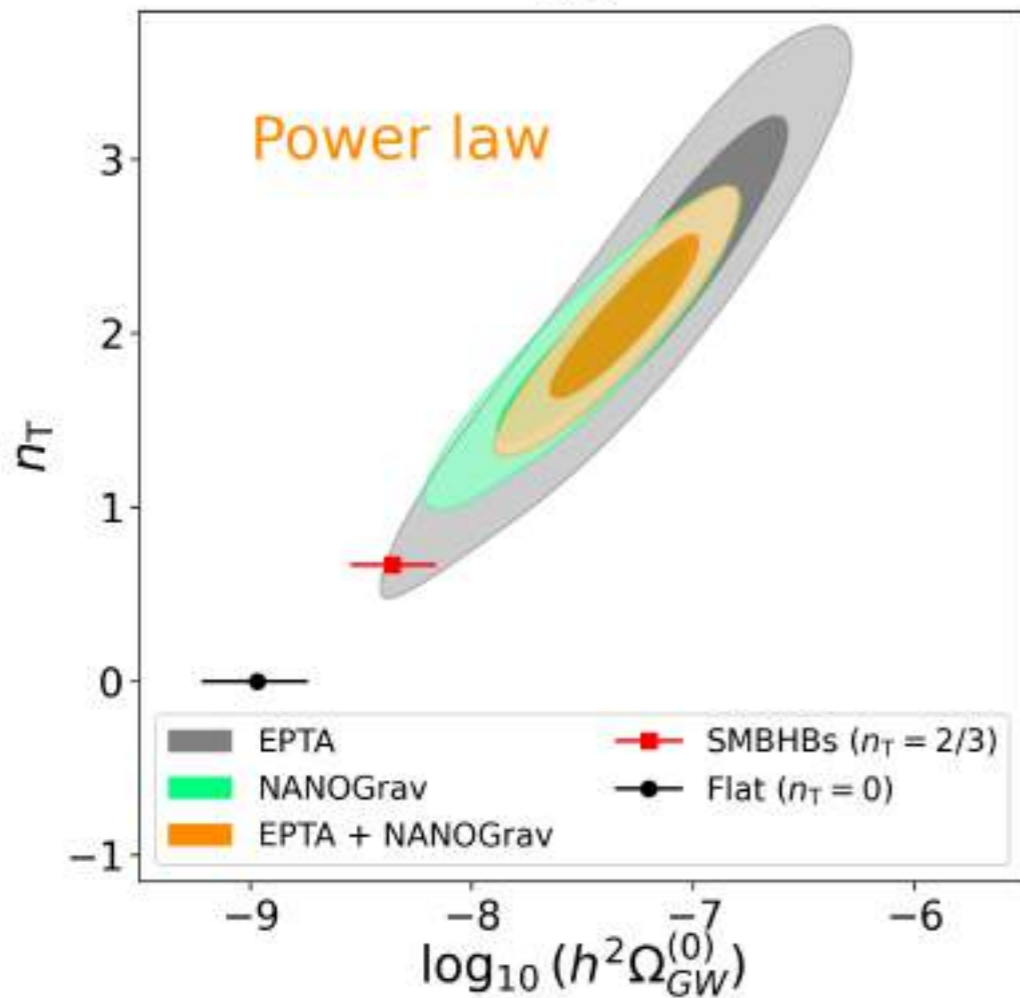
See “Tests of general relativity with GW150914”

<http://arxiv.org/abs/1602.03841>

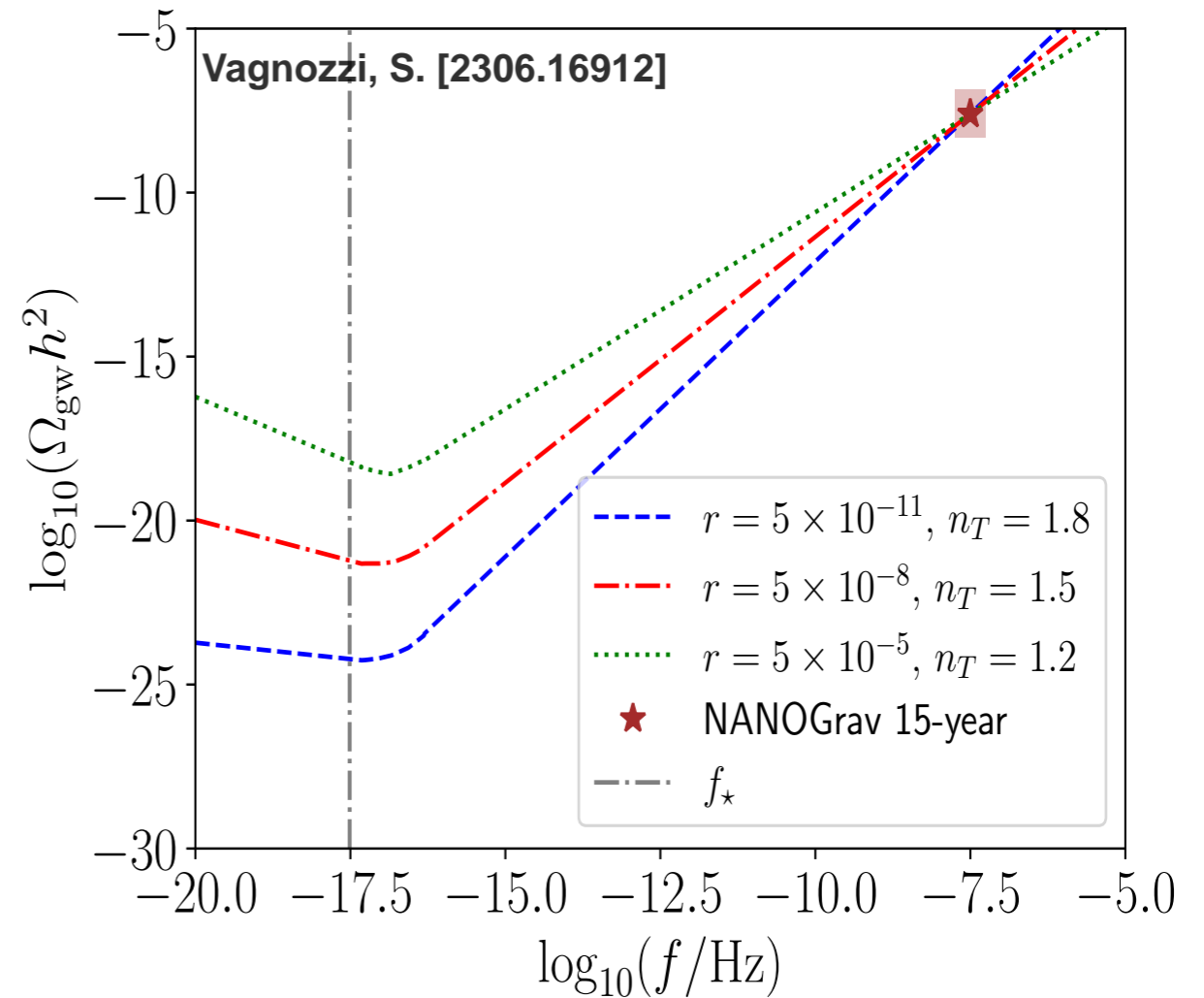
# Inflation



(B)



$$\Omega_{\text{GW}}^{(0)}(f) = \mathcal{A}_{\text{inf}}^{(*)} (f/f_*)^{n_t}$$



# LISA noise model

Two analytical approximations for **acceleration** and **interferometric** noise:

$$P_{acc}(f, A) = A^2 \cdot 10^{-30} \cdot \left[ 1 + \left( \frac{4 \cdot 10^{-4}}{f} \right)^2 \right] \left[ 1 + \left( \frac{f}{8 \cdot 10^{-3}} \right)^4 \right] \left( \frac{1}{2\pi f} \right)^4 \left( \frac{2\pi f}{c} \right)^2 ,$$

$$P_{IMS}(f, P) = P^2 \cdot 10^{-24} \cdot \left[ 1 + \left( \frac{2 \cdot 10^{-3}}{f} \right)^4 \right] \left( \frac{2\pi f}{c} \right)^2 .$$

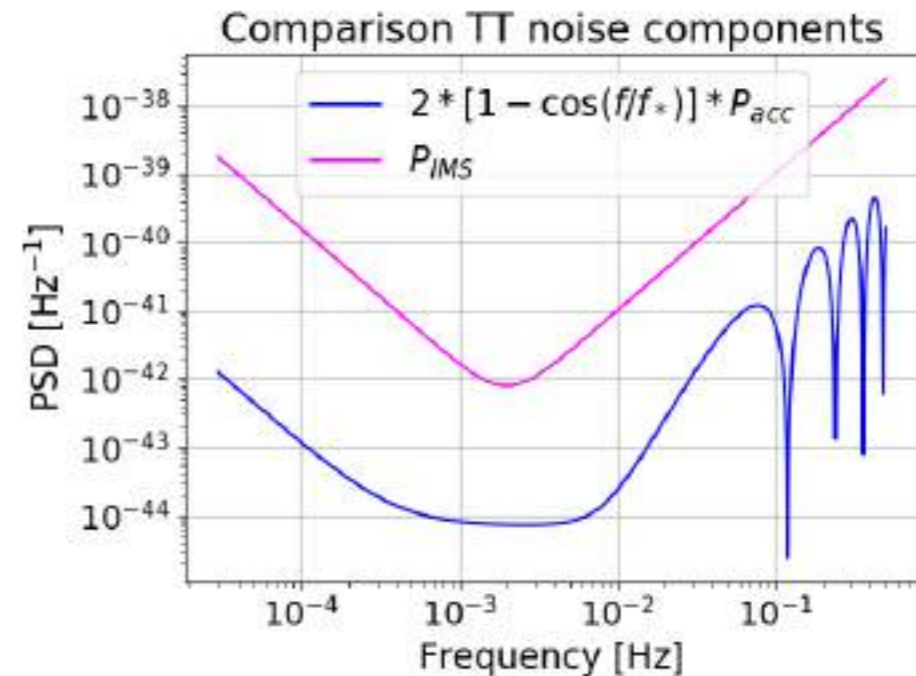
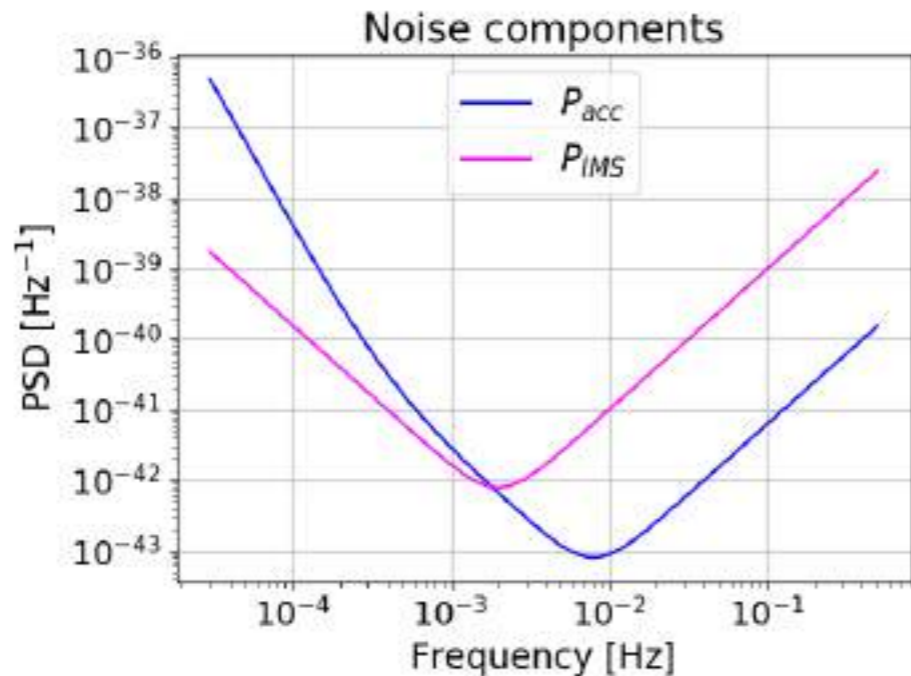
The **power spectral densities** are ( $L = 2.5 \times 10^9$  m is the arm length):

$$P_{PSD}^{XX}(f) = 16 \sin^2 \left( \frac{2\pi fL}{c} \right) \left\{ P_{IMS}(f, P) + \left[ 3 + \cos \left( \frac{4\pi fL}{c} \right) \right] P_{acc}(f, A) \right\} ,$$

$$P_{PSD}^{XY}(f) = -8 \sin^2 \left( \frac{2\pi fL}{c} \right) \cos \left( \frac{2\pi fL}{c} \right) \left\{ P_{IMS}(f, P) + 4P_{acc}(f, A) \right\} ,$$

which for the TT combination gives:

$$P_{PSD}^{TT}(f, A, P) = 16 \sin^2 \left( \frac{2\pi fL}{c} \right) \left\{ 2 \left[ 1 - \cos \left( \frac{2\pi fL}{c} \right) \right]^2 P_{acc}(f, A) + \left[ 1 - \cos \left( \frac{2\pi fL}{c} \right) \right] P_{IMS}(f, P) \right\} .$$



# LISA data generation

Assume **signal and noise** ( $\Omega$  units) to be **Gaussian distributed**  
The spectra ( $\Omega_{\text{GW}}$  and  $\Omega_n$ ) quantify the **variance of fluctuations**

$$\tilde{s}_c(f_i) = \left| \frac{G(0, \sqrt{\Omega_{\text{GW}}(f_i)}) + i G(0, \sqrt{\Omega_{\text{GW}}(f_i)})}{\sqrt{2}} \right|$$
$$\tilde{n}_c(f_i) = \left| \frac{G(0, \sqrt{\Omega_n(f_i)}) + i G(0, \sqrt{\Omega_n(f_i)})}{\sqrt{2}} \right|$$

For **each data segment and frequency** we generate a **gaussian realization**.

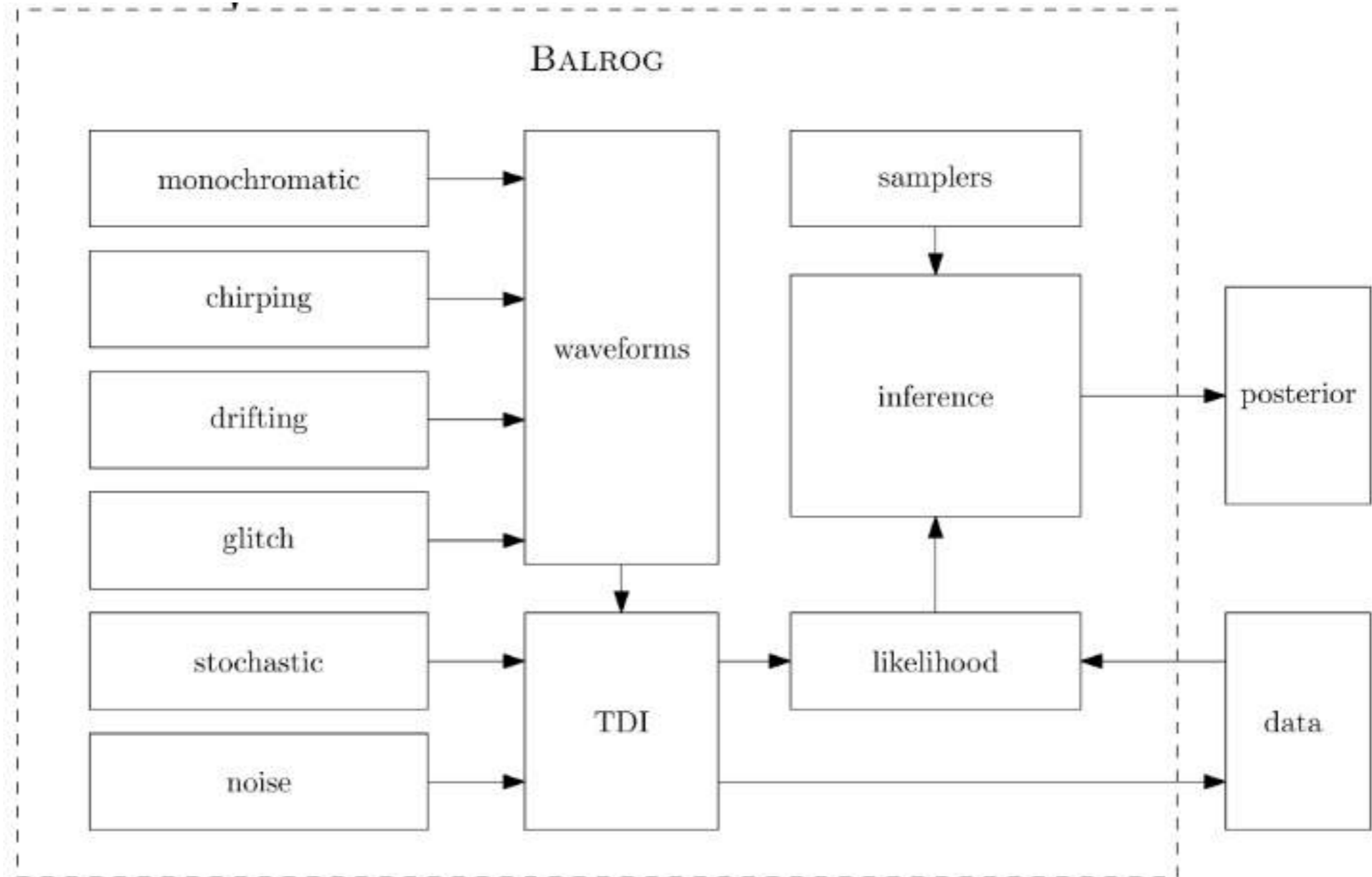
Given that:

- LISA will be operating for **4yrs (75% efficiency)**
- We choose **data segments** of **roughly 12 days**

we conclude that:

- Roughly **95** independent **measurements at each frequency**.
- The **resolution** of the detector is **roughly  $10^{-6}$  Hz**

# Possible prototype



Buscicchio et al.