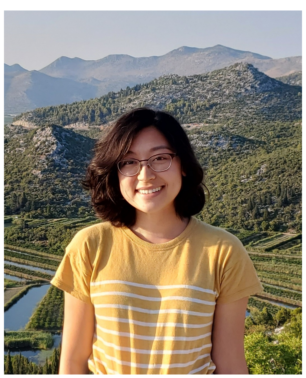
The nHz GW Background and the local population of BHs

August 2024

Executive Summary

- The expected level of the Background of GW detected by the PTAs can be robustly estimated from the local census of BHs.
- The level of the Background seems high compared to the expectations based on the current census. If BHs are missing from the local count it is unclear what mass they should have to be consistent with all constraints.
- In addition to the amplitude (and slope) there is another informative parameter that can be constrained with current observations, the typical number of sources that contribute to the background. One can use this to constrain the number and typical mass of the BHs that contribute most.



https://arxiv.org/pdf/2312.06756
https://arxiv.org/pdf/2406.17010

Work done with Gabriela Sato-Polito and Eliot Quataert

Outline

Implication of the PTA results

- Amplitude of the spectrum
- Information as a function of frequency
- Summary

Quick advertisement of results related to LIGO/VIRGO

Implications of the amplitude of the GW Background

PTA Results

Relation to local population of BHs

- Local black holes are the remanent of the mergers that produced the GW background.
- How many times did they merge? What was the mass ratio? At what redshifts?
- The relation is very robust and insensitive to number of mergers and redshift.

$$\frac{d\rho_{GW}}{d\ln f} = \int dz \frac{dn_{comoving}}{dz} \frac{1}{1+z} \frac{dE}{d\ln f}|_{emitted}$$

$$\frac{d\rho_{GW}}{d\ln f} = n_0 \frac{dE}{d\ln f}|_0 \int dz \frac{1}{n_0} \frac{dn_{comoving}}{dz} \frac{1}{(1+z)^{-1/3}}$$

$$\frac{dE}{d\ln f} = f\frac{\dot{E}}{\dot{f}} = \frac{2}{3} E \propto M_T \eta v^2 \propto \mathcal{M}_c^{5/3} f_{GW}^{2/3}$$

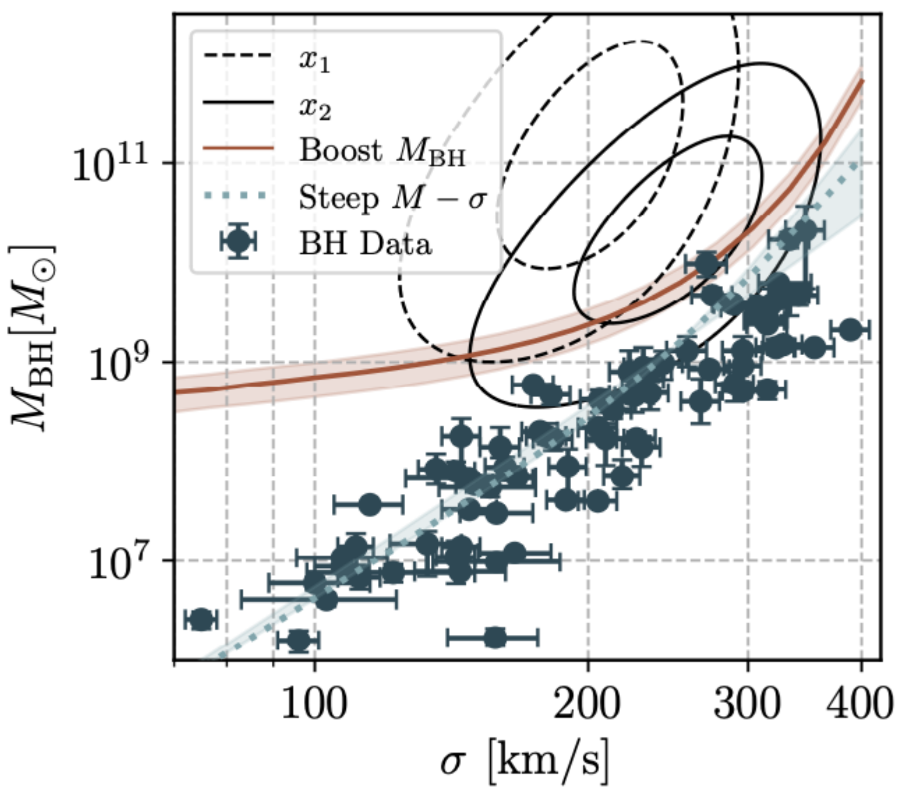
$$\frac{d\rho_{GW}}{d\ln f} = \rho_{BH} \epsilon \langle (1+z)^{-1/3} \rangle$$

$$\rho_{BH} = n_0 M_T c^2 \qquad \epsilon = \frac{1}{3}\eta(v/c)^2$$

[13] E. S. Phinney, "A Practical theorem on gravitational wave backgrounds," arXiv:astro-ph/0108028.

Local population of BHs

Scaling relations



N. J. McConnell and C.-P. Ma, "Revisiting the Scaling Relations of Black Hole Masses and Host Galaxy Properties," Astrophys. J. **764** no. 2, (Feb., 2013) 184, arXiv:1211.2816 [astro-ph.CO].

$$\log_{10} M = a_\bullet + b_\bullet \log_{10} X.$$

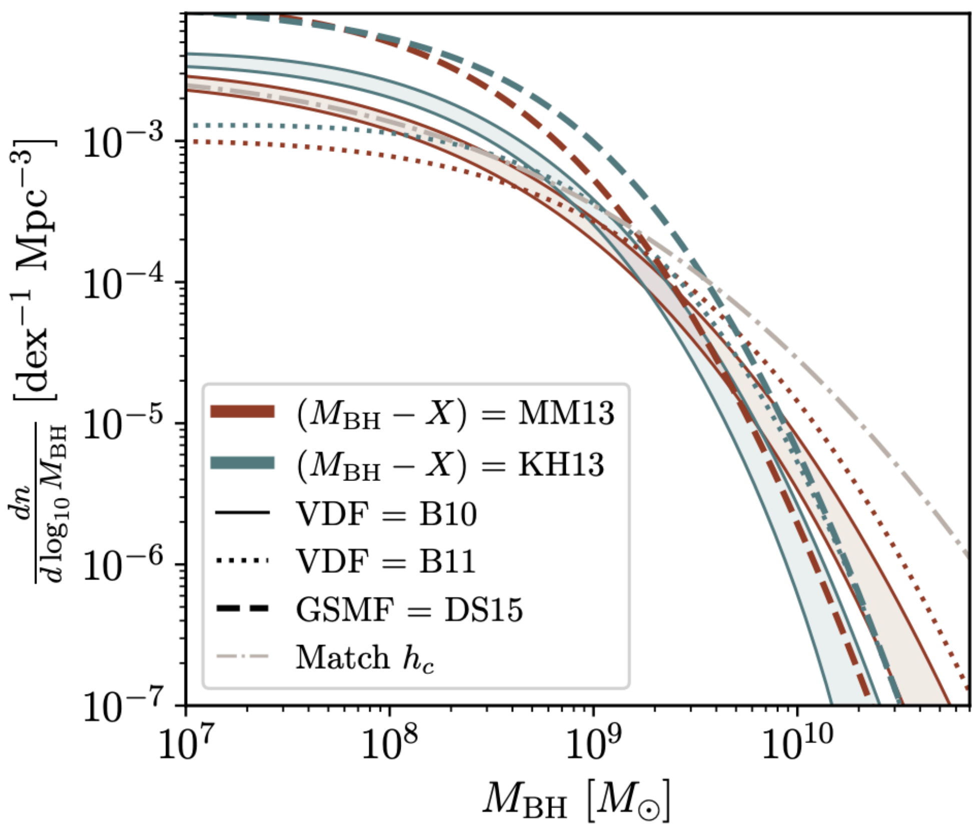
$$\phi(\sigma)d\sigma = \phi_* \left(\frac{\sigma}{\sigma_*}\right)^\alpha \frac{e^{-(\sigma/\sigma_*)^\beta}}{\Gamma(\alpha/\beta)} \beta \frac{d\sigma}{\sigma}.$$

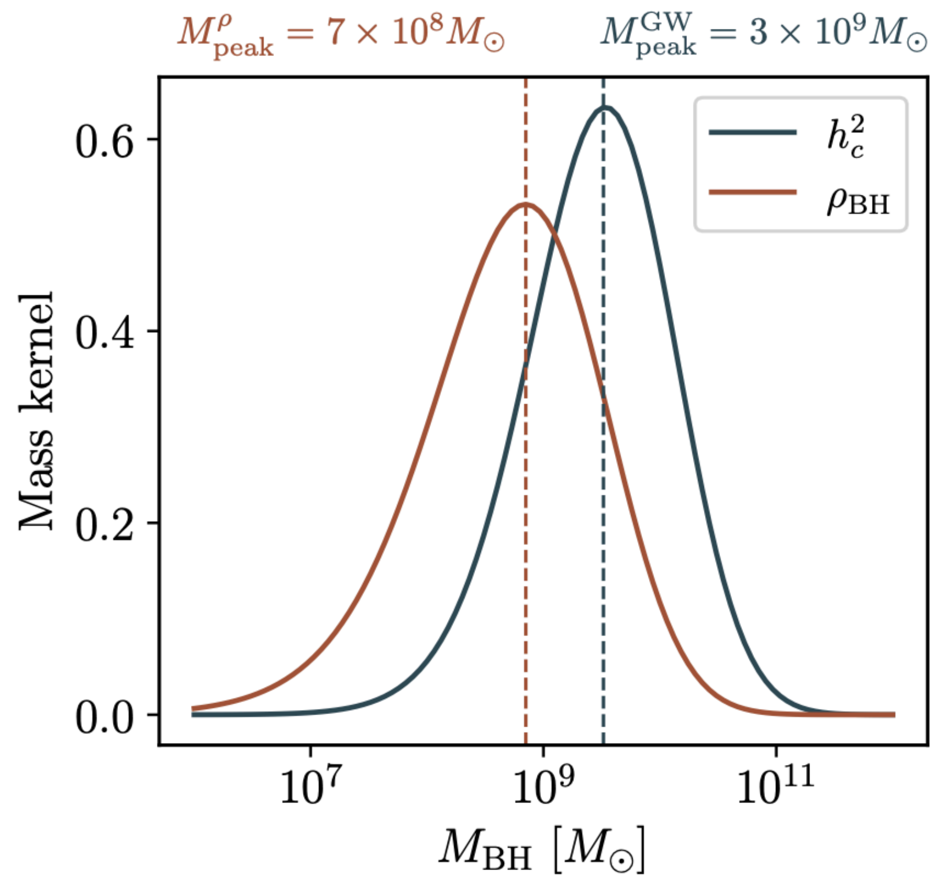
$$\phi(M) = \int d\sigma \frac{p(\log_{10} M | \log_{10}\sigma)}{M \log(10)} \phi(\sigma),$$

$$p(\log_{10} M | \log_{10}\sigma) = \frac{1}{\sqrt{2\pi}\epsilon_0} \times \exp\left[-\frac{1}{2}\left(\frac{\log_{10} M - a_\bullet - b_\bullet \log_{10}\sigma}{\epsilon_0}\right)^2\right]$$

PTA results

Which BHs contribute





$$h_c^2(f) = \frac{4G}{\pi f^2}\langle(1+z)^{-1/3}\rangle \int dM\ M\phi(M)\langle\epsilon\rangle(M),$$

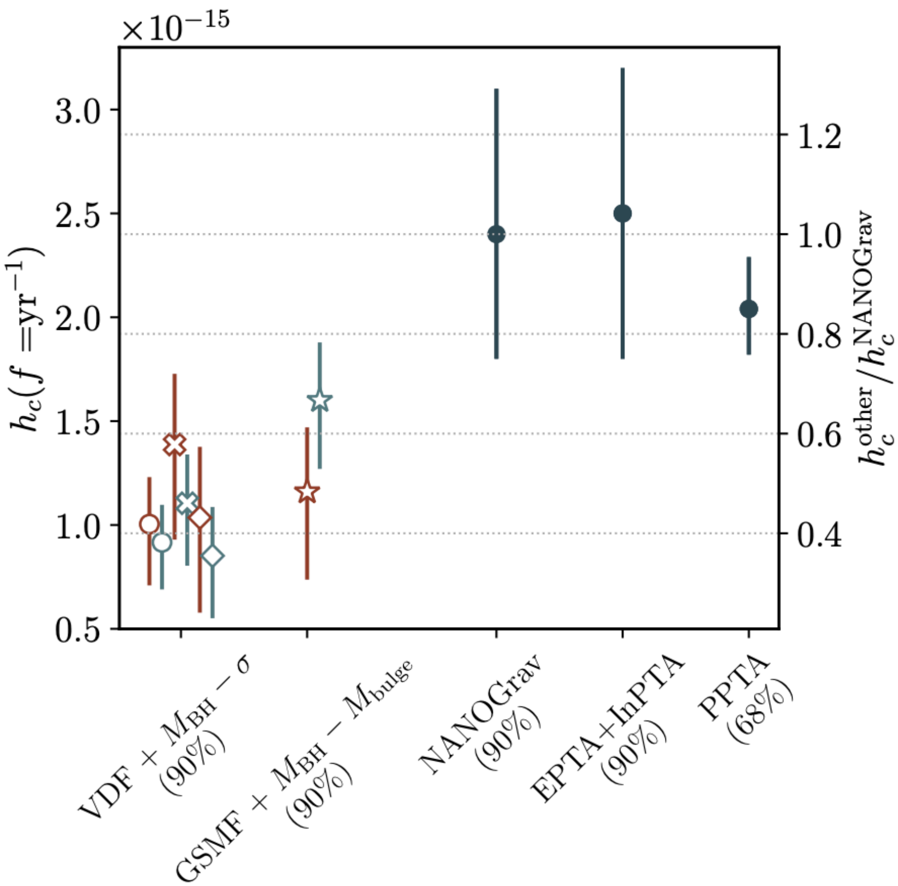
$$\epsilon \equiv \frac{\eta}{3}\frac{(GM\pi f)^{2/3}}{c^2}$$

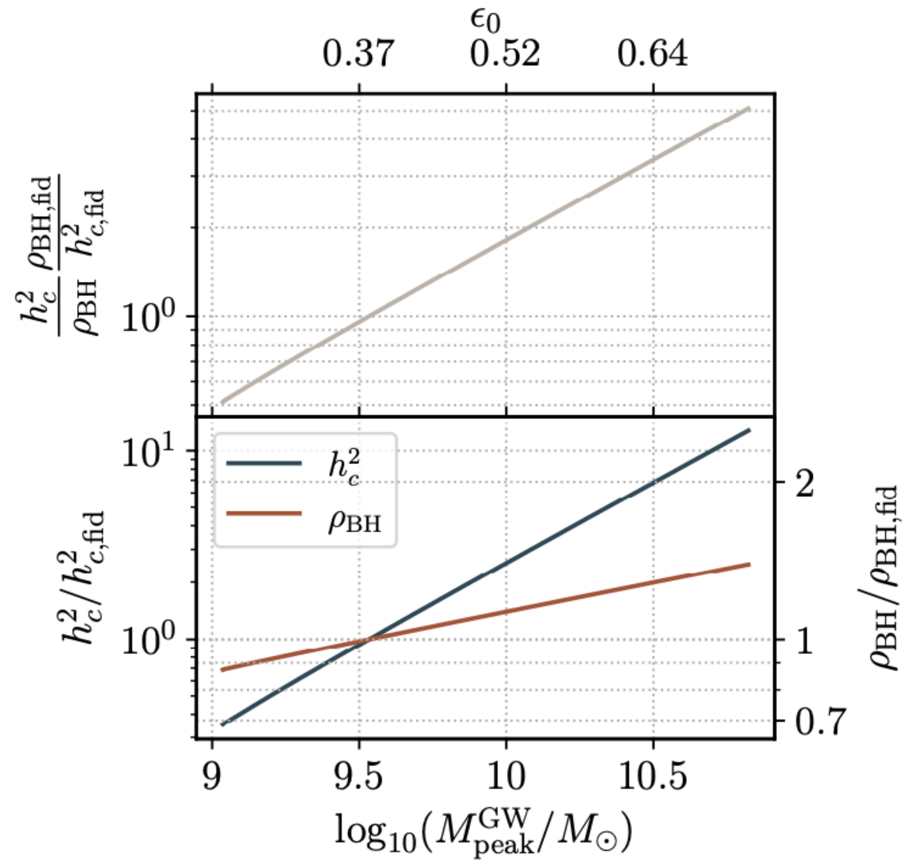
$$h_c^2(f) \propto \phi_\star M_{\rm peak}^{5/3} f^{-4/3},$$

Peak contribution determined by the brake in the mass function. Contribution to the GW background peaks at a slightly higher mass.

PTA results

Relation to local population of BHs





There are two basic parameters we might try to constrain, the overall normalization of the mass function and the mass that contributes the most. One can change these quantities by changing the scaling relations or their scatter, etc.

How to change the local population

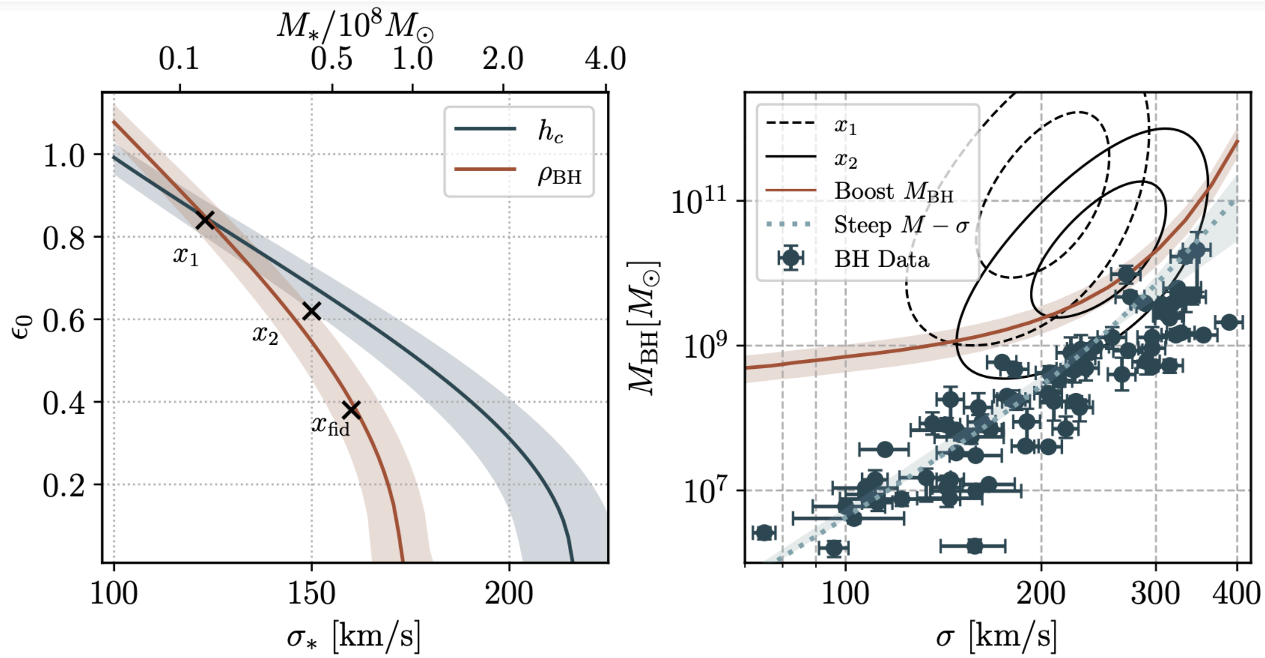


FIG. 6. Parameter values required to match both the measured characteristic strain from PTAs and the inferred black hole mass density. The panel on the left shows the curves of constant h_c (in dark blue) and $\rho_{\rm BH}$ (in red). The best-fit parameters from direct observations of SMBHs and host-galaxy properties correspond to the point $x_{\rm fid} = (159.6, 0.38)$. The points $x_1 = (123, 0.84)$ and $x_2 = (150, 0.62)$ correspond to sets of parameters where the predicted h_c^2 and $\rho_{\rm BH}$ exactly match the central fiducial value, and where they match within a 90% confidence interval, respectively. The points in the panel on the right are the BH data used in Ref. [12]. The inner (outer) contours show the regions where 50% (90%) of the contribution to the SGWB come from for the set of parameters x_1 in dashed and x_2 in solid lines. The red curve corresponds to the fiducial VDF and $M-\sigma$ relation, but with an additional population of BHs associated with galaxies of a given σ. Each point along the red curve therefore shows the required BH mass if a single BH population were to account for the h_c value measured by NANOGrav. The light blue dotted line and shaded band shows Eq. 23 for $d_\bullet = 10.5^{+1.5}_{-3.6}$.

Matching the measure background requires a population of BHs that have not been observed before.

Predictions for local surveys

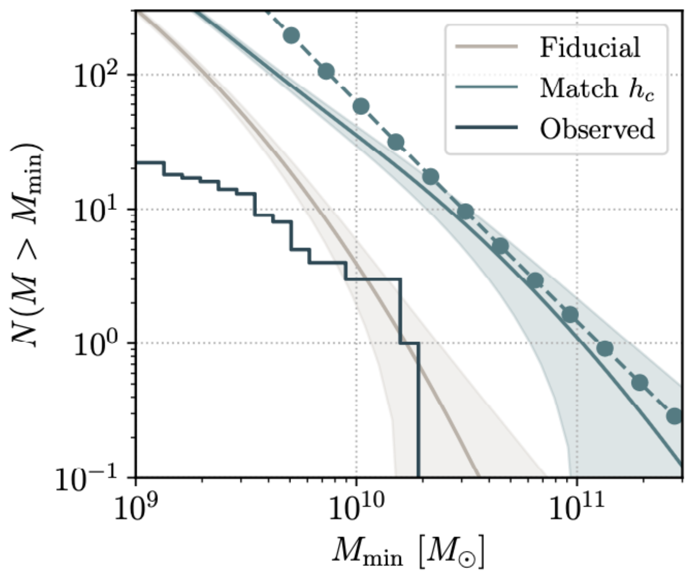


FIG. 9. Total number of SMBHs above a given minimum mass $M_{\rm min}$. Solid lines with error bands correspond to predictions using the fiducial VDF and $M-\sigma$ relation (grey) and the fiducial VDF and an $M-\sigma$ relation given by Eq. 23 with the best-fit value that matches the characteristic strain measured by PTAs (light blue). The dashed line with dots correspond to the fiducial mass function, with a Gaussian bump above $M_{\rm min}$ such that the amplitude is chosen to match the measured h_c. The dark blue curve shows the SMBHs observed so far.

Matching the measure background would imply for example 60 BHs above 10^{10} or 2 above 10^{11} in a local survey of 100 Mpc radius.

Observed points from:

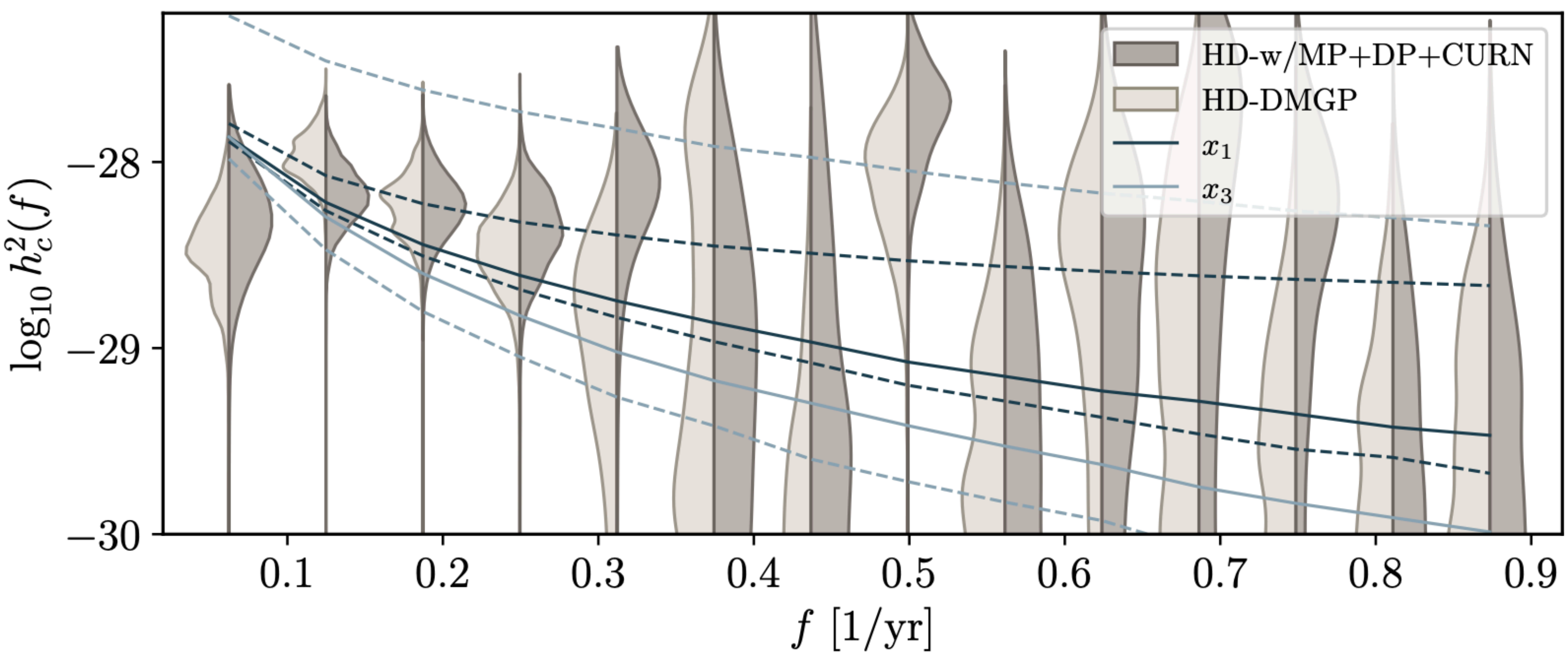
[12] N. J. McConnell and C.-P. Ma, "Revisiting the Scaling Relations of Black Hole Masses and Host Galaxy Properties," Astrophys. J. **764** no. 2, (Feb., 2013) 184, arXiv:1211.2816 [astro-ph.CO].

[45] J. Thomas, C.-P. Ma, N. J. McConnell, J. E. Greene, J. P. Blakeslee, and R. Janish, "A 17-billion-solar-mass black hole in a group galaxy with a diffuse core," Nature (London) **532** no. 7599, (Apr., 2016) 340–342, arXiv:1604.01400 [astro-ph.GA].

Implications of the measurements as a function of frequency

PTA results

Detections as a function of frequency



Modeling the distribution of power

Relation to the luminosity function

Power from a collection of sources

$$h_t^2(f) = \sum_s h_s^2(f).$$

Averages in terms of Luminosity function

$$\bar{N} = \int dh_s^2 \frac{dN}{dh_s^2}$$

$$\bar{h_t^2} = \int dh_s^2 \frac{dN}{dh_s^2} h_s^2.$$

Distribution

$$P(h_t^2) = \sum_{N_s=0}^{\infty} P_{\text{Poiss}}(N_s|\bar{N}) P(h_t^2|N_s).$$

$$P(h_t^2|N_s = 1) = \frac{1}{\bar{N}} \frac{dN}{dh_s^2},$$

Fourier transform

$$\hat{P}(\omega) = \sum_{N_s=0}^{\infty} P(N_s|\bar{N}) \hat{P}(\omega|N_s),$$

$$\hat{P}(\omega|N_s) = [\hat{P}(\omega|N_s = 1)]^{N_s}.$$

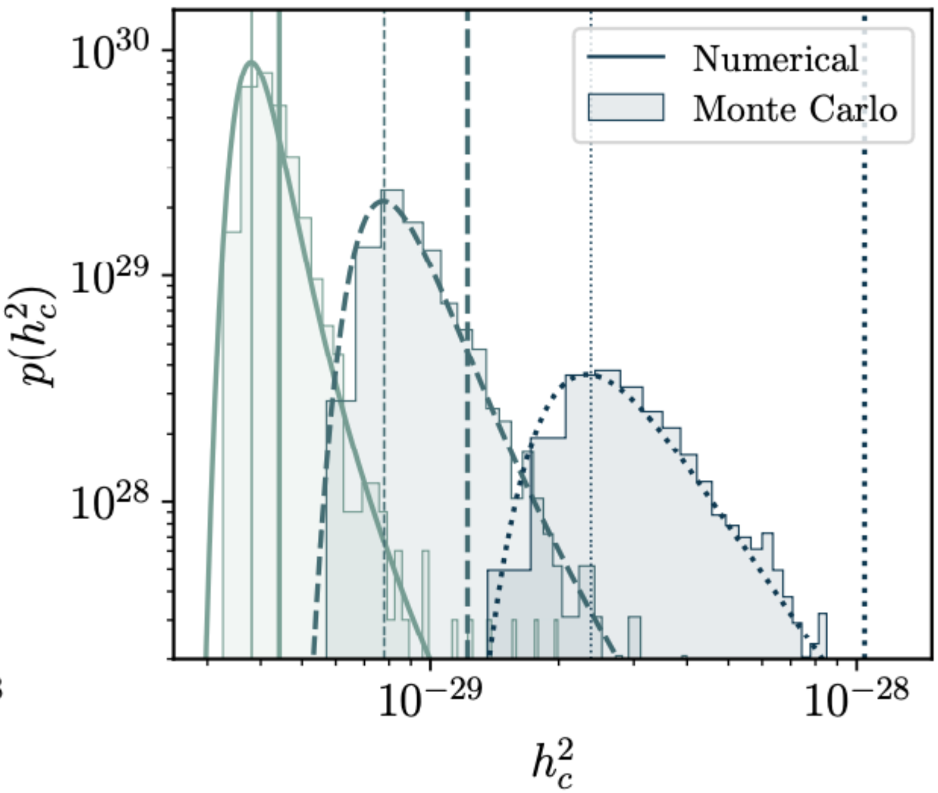
$$\begin{aligned}\hat{P}(\omega) =& e^{-\bar{N}} \sum_{N_s=0}^{\infty} \frac{[\bar{N}\hat{P}(\omega|N_s = 1)]^{N_s}}{N_s!} \\ =& \exp\left\{\bar{N}\hat{P}(\omega|N_s = 1) - \bar{N}\right\} \equiv \exp[u(\omega)]\end{aligned}$$

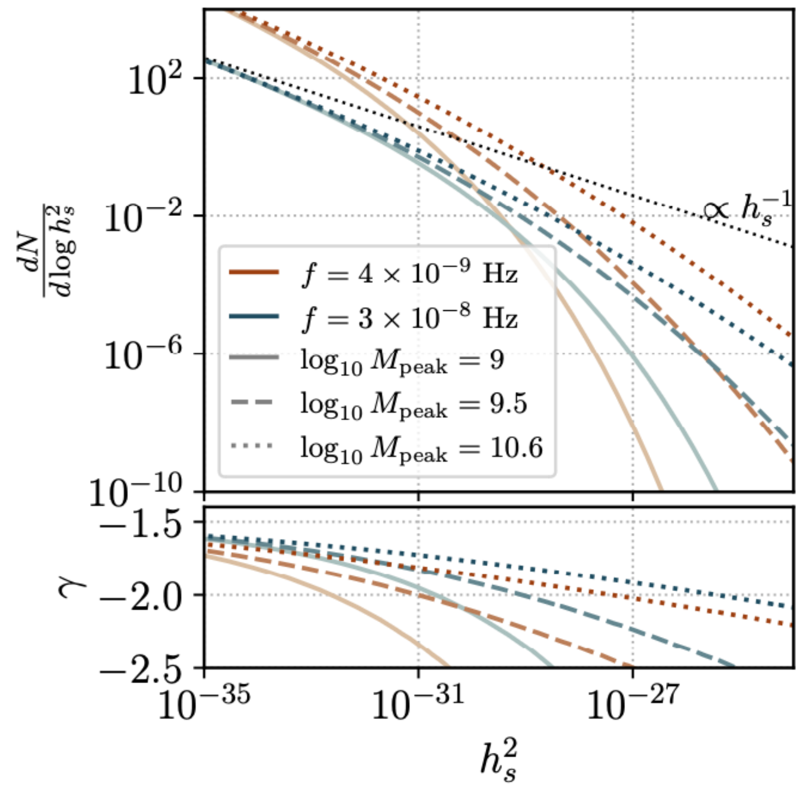
$$\text{Re}[u(\omega)] = \int dh_s^2 \frac{dN}{dh_s^2} (\cos(\omega h_s^2) - 1)$$

$$\text{Im}[u(\omega)] = \int dh_s^2 \frac{dN}{dh_s^2} \sin(\omega h_s^2).$$

Modeling the distribution of power

Relation to the luminosity function





The distribution of power has a Gaussian like core and an tail. The mean of the distribution and the location of the peak can be very different.

Things change rapidly with model parameters and frequency but that evolution can be understood analytically.

Although the Luminosity function is not a power law, most of the aspects of the distribution of power can be understood using that case as a reference. In that case there are analytic expressions.

The luminosity function

General expression

$$\frac{dN}{d\log h_s^2 d\log f} = \int d\log M \int dz \int dq\ \delta^D(\log h_s^2 - \log \tilde{h}_s^2) \times \frac{dN}{d\log M dq dz d\log f}$$
$$= \int d\log M \int dz \int dq\ \delta^D(\log h_s^2 - \log \tilde{h}_s^2) \times \frac{dn}{d\log M} p_z(z) p_q(q) \frac{dt_r}{d\log f_r}\frac{dz}{dt_r}\frac{dV_c}{dz},$$

In terms of chirp mass

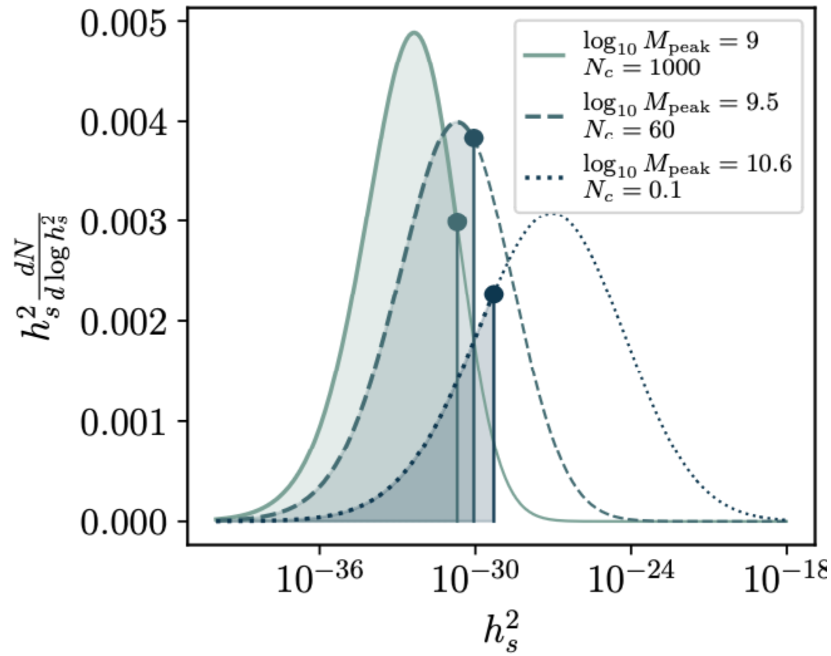
$$\frac{dN}{d\log h_s^2} = \int dz \left|\frac{\partial \log M_c}{\partial \log h_s^2}\right| \frac{dn}{d\log M_c} p_z(z) \times \frac{dt_r}{d\log f_r}\frac{dz}{dt_r}\frac{dV_c}{dz}\frac{\Delta f}{f}.$$

Volume factor

$$\frac{dV_c}{d\log f} = \frac{dt_r}{d\log f_r}\frac{dz}{dt_r}\frac{dV_c}{dz} \propto M_c^{-5/3} f^{-8/3}.$$

Mass vs strain

$$\log h_s^2 = \frac{10}{3}\log M_c + \log\frac{(1+z)^{4/3}}{\chi^2(z)} + \text{constant}.$$



General conclusions

$$\frac{dN}{d\log h_s^2} \propto M_c^{-5/3}\frac{dn}{d\log M_c} \quad ; \quad M_c \propto h_s^{3/5}.$$

$$\frac{dN}{d\log h_s^2} \propto \phi_\star M_{\rm peak}^{-5/3} f^{-11/3}.$$

$$h_s^2|_{\rm peak} \propto M_{\rm peak}^{10/3} f^{7/3}.$$

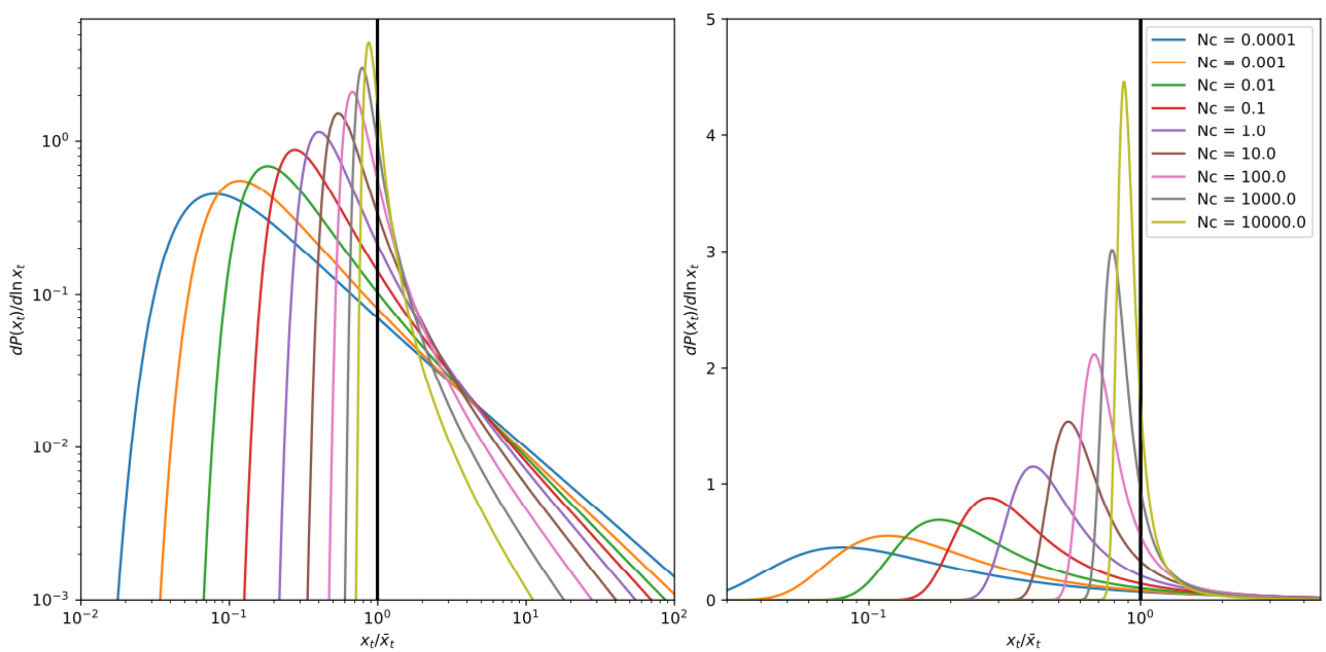
Characteristic number of sources

The shape of the probability distribution

$$N_c(f) \equiv \frac{h_c^2(f)}{h_s^2|_{\rm peak}(f)},$$

$$N_c \approx 0.2 \times \left(\frac{\phi_\star}{2.6\ 10^{-2}\ {\rm Mpc}^{-3}}\right)\left(\frac{M_{\rm peak}}{10^{10}M_\odot}\right)^{-5/3} \times \left(\frac{f}{1/3\ {\rm yr}^{-1}}\right)^{-11/3}.$$

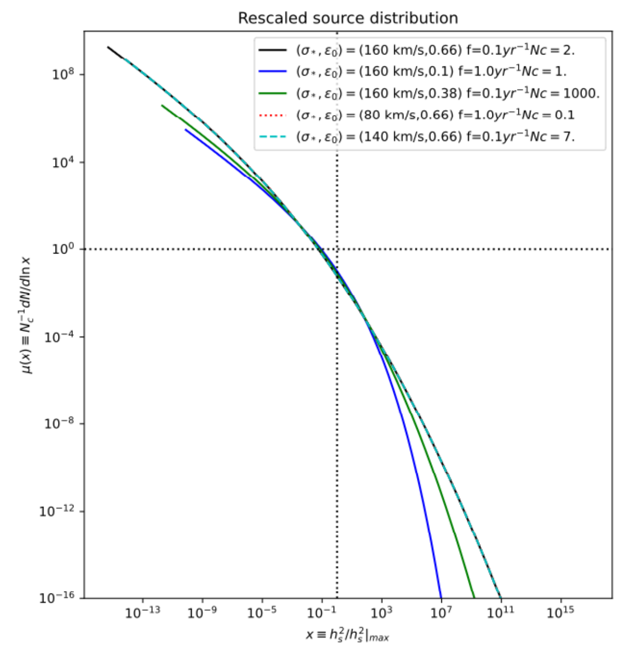
$$N_c \approx 1\times \left(\frac{h_c(f={\rm yr}^{-1})}{2.4\times 10^{-15}}\right)^2 \left(\frac{M_{\rm peak}}{10^{10}M_\odot}\right)^{-10/3} \times \left(\frac{f}{1/3\ {\rm yr}^{-1}}\right)^{-11/3}.$$



Characteristic number of sources

The shape of the probability distribution

$$x = \frac{h_s^2}{h_s^2|_{\text{peak}}}. \qquad \frac{dN}{d\log x} = N_c \mu(x),$$



In the rescaled variables the shape of the luminosity function is fairly Universal. So the data depends only on one parameter, the characteristic number of sources.

Furthermore results are very close to the one of a power law, with the slope at the place where the typical number of sources is one.

The distribution of power looks like a Gaussian core with a power law tail.

The power law tail is directly given by the luminosity function of sources, the easiest way to obtain a very high value of the total power is to have one very bright source.

The Gaussian-like core is determined by the numerous and faint sources. As their importance grows as the shape of the distribution is progressively closer to a Gaussian.

PTA results

Poisson Fluctuations

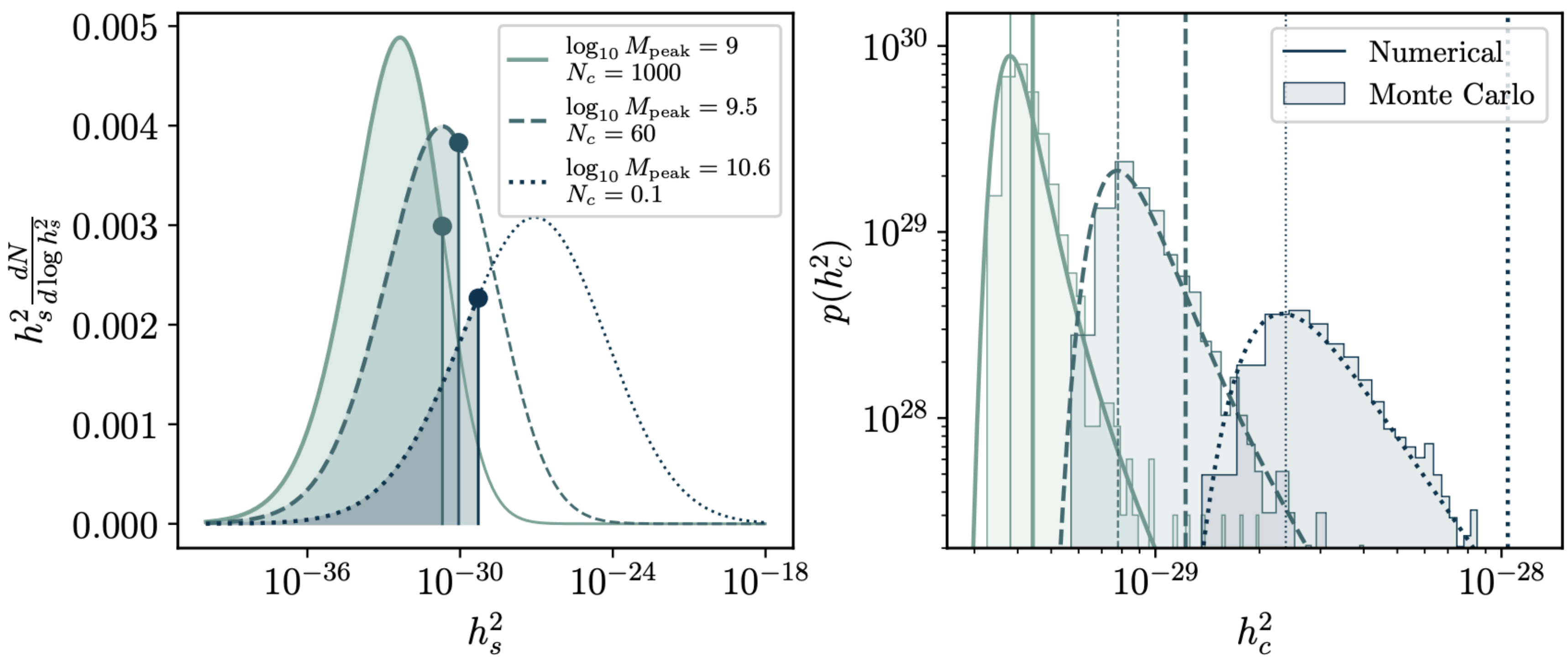
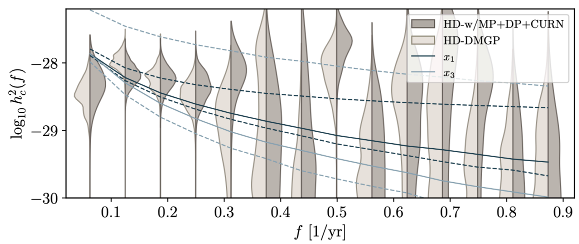


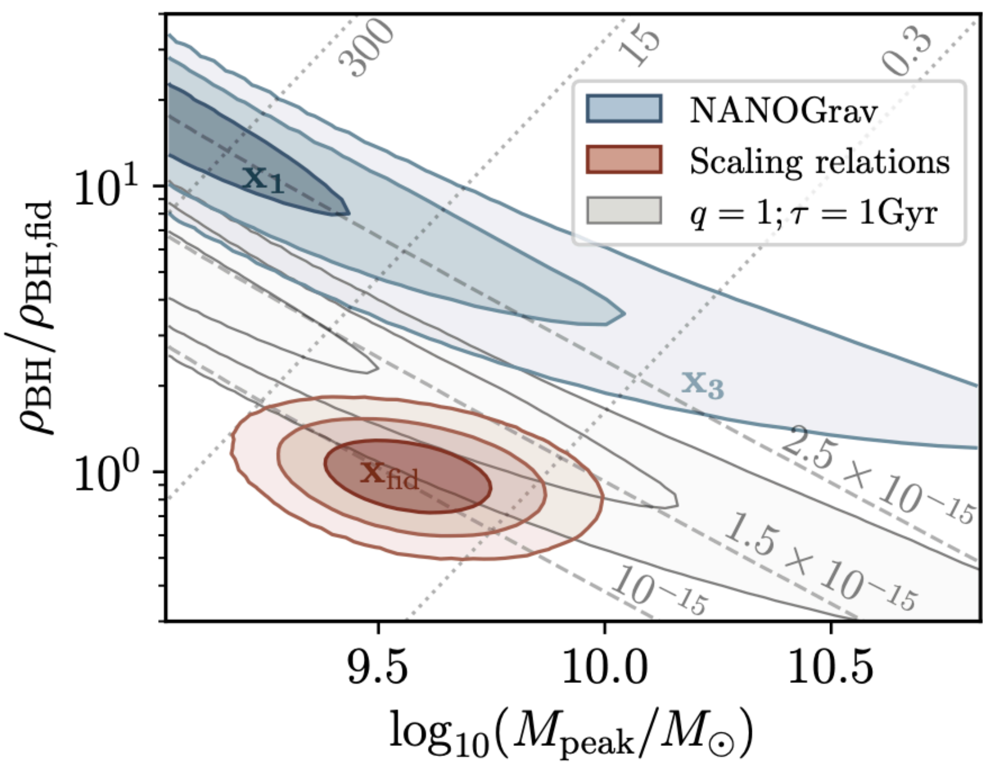
FIG. 3. Kernel of the mean characteristic strain for different SMBH models on the left panel and their distributions on the right, all shown for a frequency of $f = 4 \times 10^{-9}$Hz. The dot on the left panel shows the point $h_{s,1}^2$, defined in Eq. 33. The peak of the distribution on the right panel can be estimated via Eq. 34 (shown in the thin vertical lines), and the integral of the shaded region on the left panel corresponds exactly to the value of the peak. The thick vertical lines show the mean, computed via the full integral of the kernel, showing that the smaller the number of sources that contribute to the background, the more the mean is dominated by the bright (rare) tail of the distribution and differs from the peak of the distribution. We also compare the numerical results presented in this work with the standard Monte Carlo approach, finding excellent agreement.

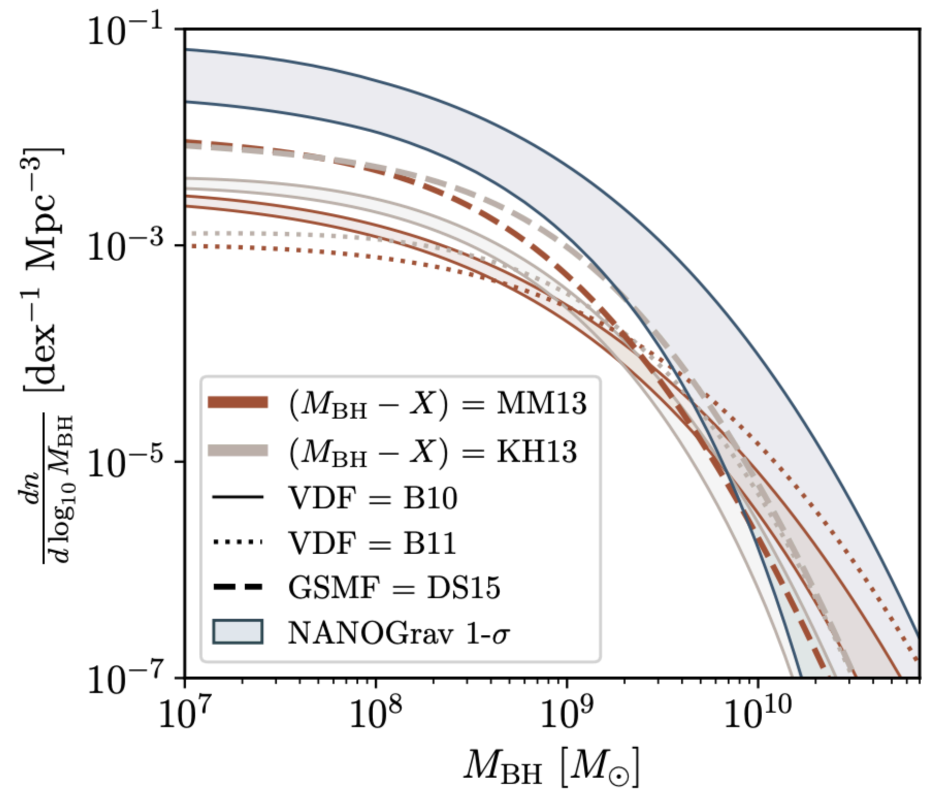
PTA results

Implication of the frequency measurements



Nc can already be constrained with current data and provides interesting information about the local population of BHs





PTA results

Individual Sources

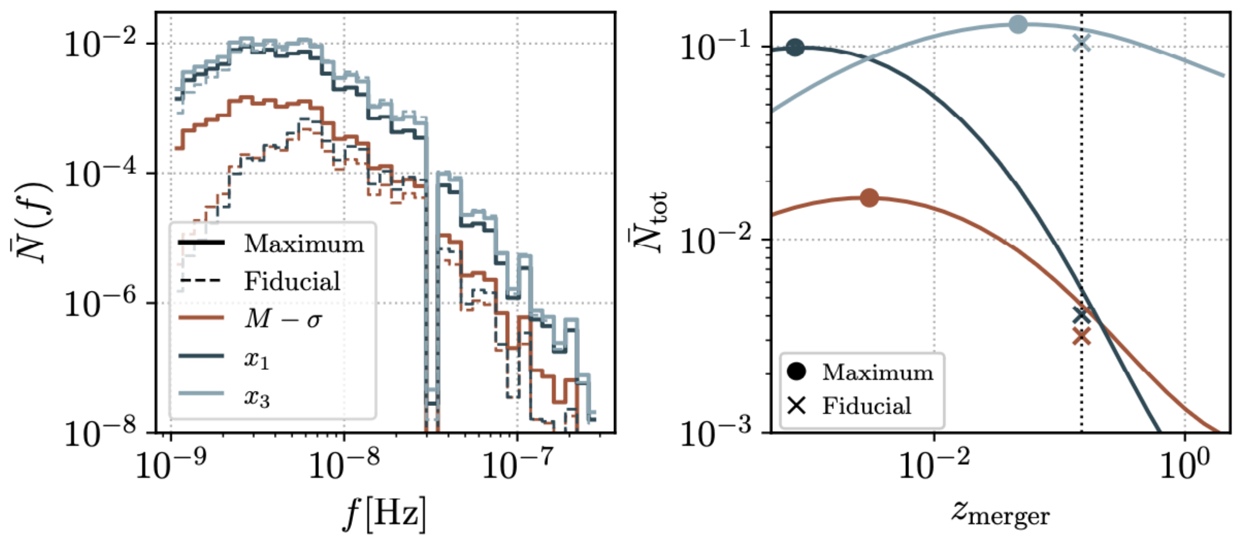


FIG. 7. Mean number of detectable sources given the current sensitivity of the individual source search from NANOGrav [33], for different models consistent with the isotropic background. The curves labelled $M-\sigma$, x_1, and x_3 correspond to mass functions consistent with the local mass function estimate and points along the degeneracy of the contour in Fig. 5, with $(\rho/\rho_{\mathrm{fid}}, \log_{10} M_{\mathrm{peak}}) = (1, 9.5), (10, 9.25), (2, 10.3)$, respectively. The points x_1, and x_3 are within the 1- and 3-σ confidence levels of the NANOGrav posterior. The solid curves on the left and dots on the right correspond to the optimal redshift distribution that leads to the maximum number of detectable sources, while the dashed (left) and crosses (right) show the values for the fiducial redshift distribution given in Eq. 4 and adopted throughout this work. The panel on the right shows the total number of detectable sources (across all frequencies) assuming that all black holes merge at a single redshift z_{merge}, while the dotted vertical line shows the redshift from which most of the characteristic strain signal is sourced in the fiducial redshift distribution. Notice that the typical redshift for which the maximum number of detectable sources is achieved is unphysically small and only corresponds to a mathematical upper limit.

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Big Galaxies and Big Black Holes: The Massive Ends of the Local Stellar and Black Hole Mass Functions and the Implications for Nanohertz Gravitational Waves

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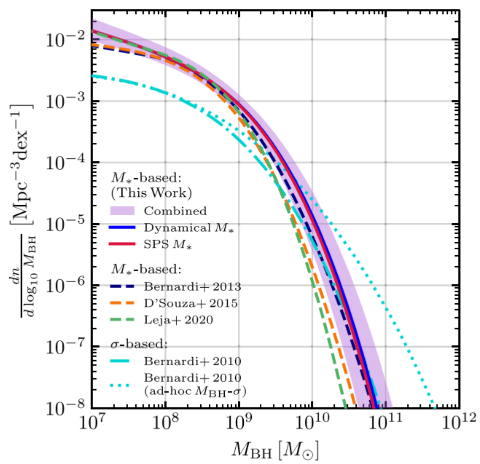

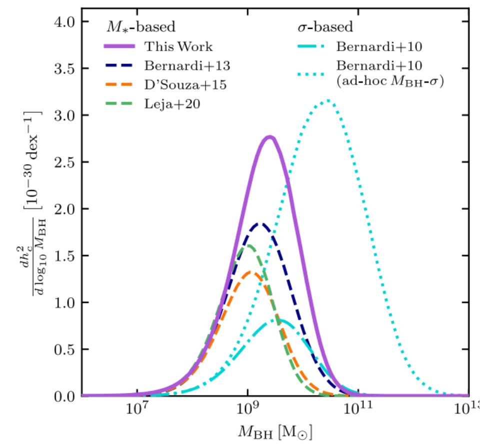

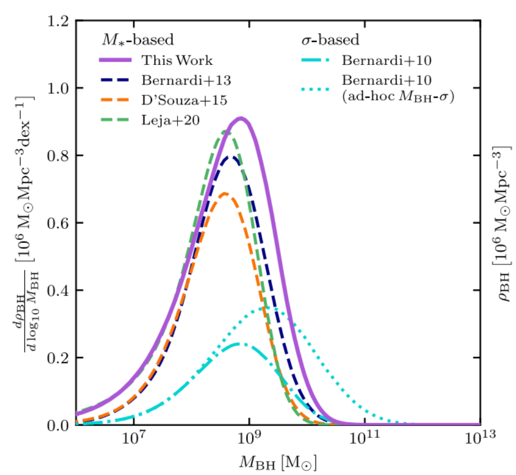

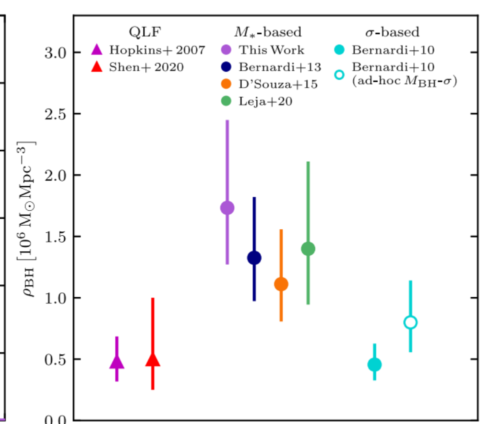

Summary

- Mergers of SMBHs are the expected source of this background. Its amplitude can be related. The amplitude of this background can be calculated based on the local population of Black Holes
- The measured amplitude is large compared to baseline predictions.
- Adding a small number of heavy black holes would increase the background but would lead to fluctuations in the power that are not seen
- Another solution if to increase the overall size of the population of BHs but the increase is substantial.
- Current observations constrain a second parameter in addition to the amplitude and thus both the number density and typical mass of the BHs that most contribute can be constrained.