GEMMA2 workshop, September 19th, 2024

Ultralight dark matter (ULDM) search with KAGRA

-O3GK result and what's next?-

Università degli Studi di Padova

INFN Padova
Istituto Nazionale di Fisica Nucleare
Sezione di Padova

RESCEU
THE UNIVERSITY OF TOKYO

KAGRA

ダークマターの正体は何か？
広大なディスカバリースペースの網羅的研究
What is dark matter? - Comprehensive study of the huge discovery space in dark matter

Jun'ya Kume (Univ. of Padova, INFN, RESCEU)

on behalf of the LIGO-Virgo-KAGRA collaboration

Collaborators:

M. Ando, T. Fujimori, H. Fujimoto, T. Fujita,

K. Komori, Y. Manita, Y. Michimura, S. Morisaki,

A. Nishizawa, I. Obata Y. Oshima & H. Takidera

Acknowledgement:
This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation.

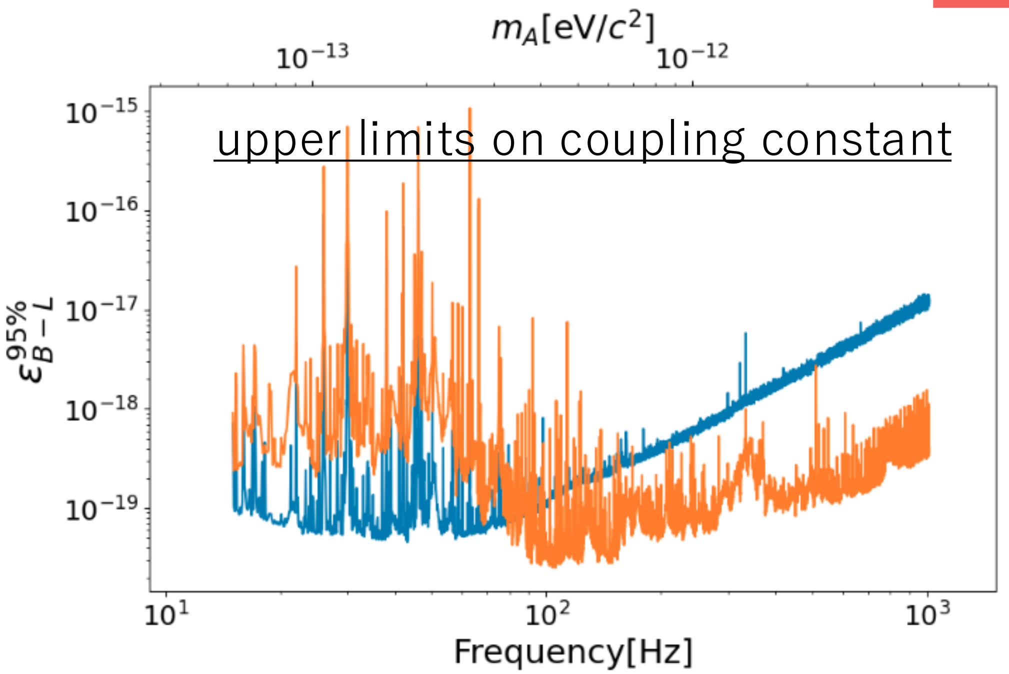
The paper just came out!

PHYSICAL REVIEW D **110**, 042001 (2024)

Ultralight vector dark matter search using data from the KAGRA O3GK run

A. G. Abac *et al.**
(LIGO Scientific, Virgo, and KAGRA Collaborations)

※data available at GWOSC: https://gwosc.org/O3/kagradm/



- Topical study on "Direct DM search with GW interferometers" (Talk by A. Miller, J. Carlton)

  → **KAGRA has a unique feature as a vector DM detector**!! (Y. Michimura+ 2020)

- Dedicated analysis pipeline to address **ULDM stochasticity** (Nakatsuka, Morisaki, Fujita, **JK**+ 2022)

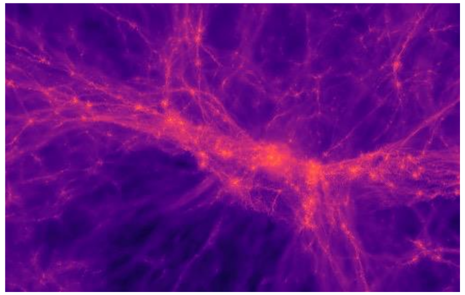
  → to be improved and extended for O4 data analysis (→ S. Morisaki's talk)

Contents

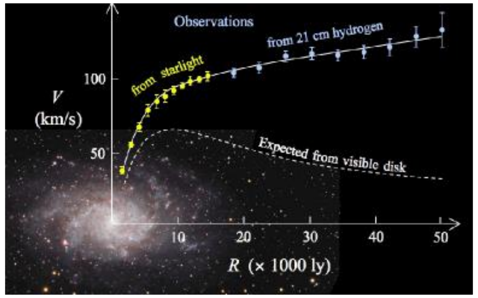
- ➢Ultralight vector DM and KAGRA as its detector
- ➢Pipeline construction based on stochasticity study
- ➢O3GK data analysis as a demonstration
- ➢Summary & Discussion

- A window for ultralight DM

Lots of indirect evidences!!

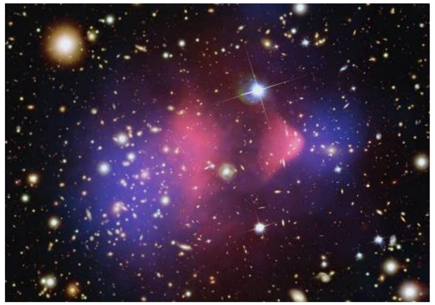


Structure formation



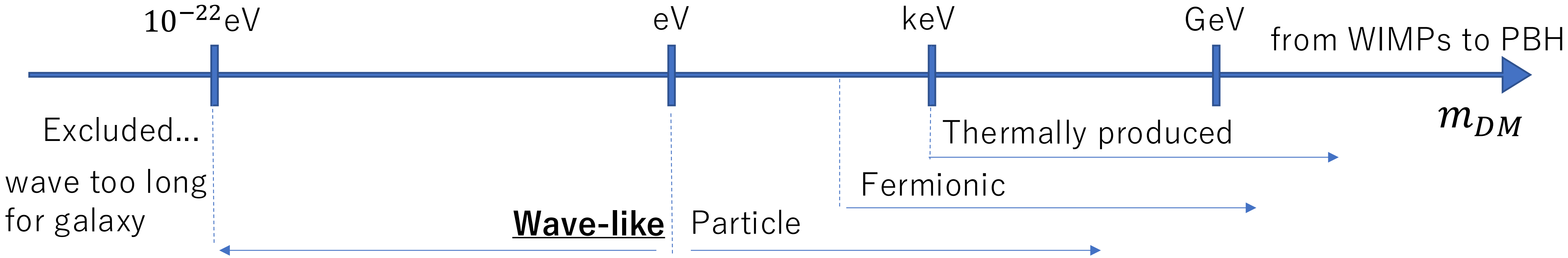
Galaxy rotation curve

Gravitational lensing



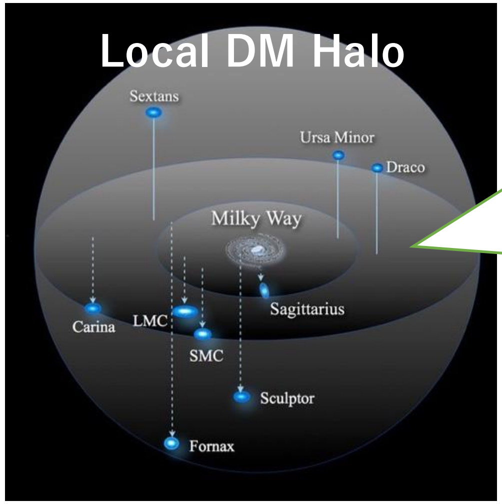
Bullet cluster

Vast discovery space (10^{-22}eV ~10^{67}eV) for the DM: <u>90 orders of magnitude</u>!!



If non-thermally produced, $m_{DM} \lesssim$ eV is allowed <u>for bosons</u>.

- How does ULDM behave?



$\rho_{\rm DM} = 0.4\ {\rm GeV/cm^3}$

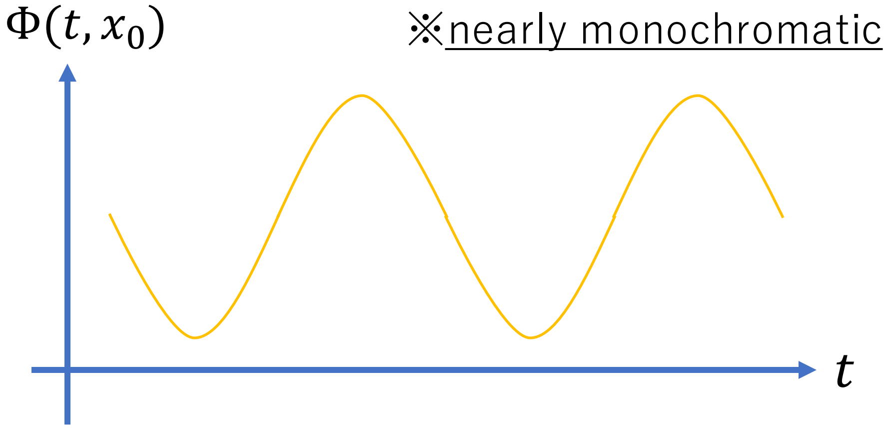
$\gg \bar{\rho}_{\rm DM} \approx 10^{-6}\ {\rm GeV/cm^3}$

Higher density in DM halo!

w/ large de Broglie wavelength

→ occupation number ≫ 1

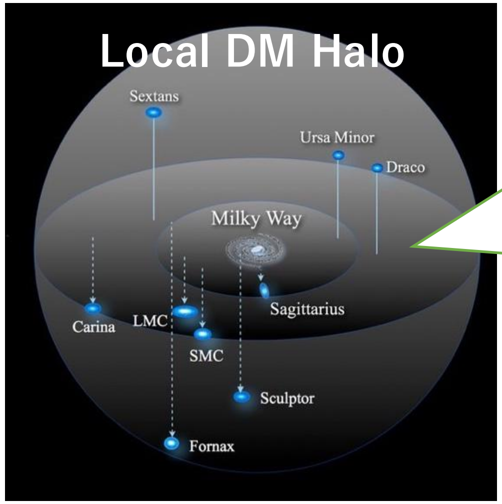
※nearly monochromatic



behave as classical waves!

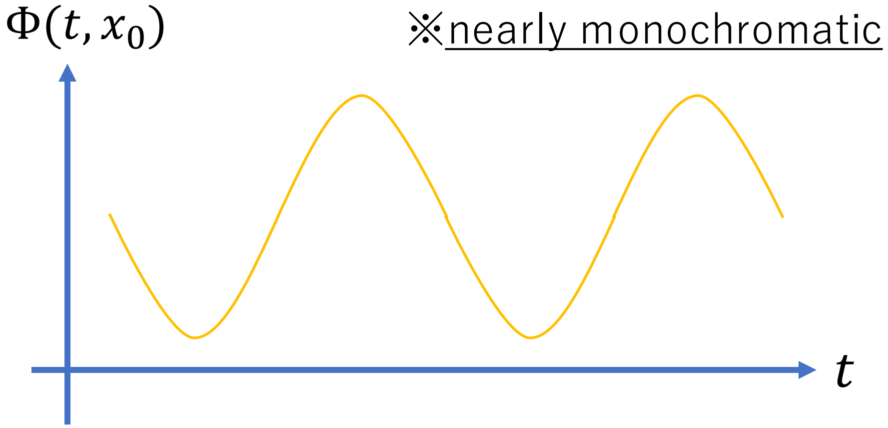
→ solving small scale structure issue?

- How does ULDM behave?



$\rho_{\rm DM} = 0.4\ \mathrm{GeV/cm^3}$
$\gg \bar{\rho}_{\rm DM} \approx 10^{-6}\ \mathrm{GeV/cm^3}$
Higher density in DM halo!

w/ large de Broglie wavelength
→ occupation number ≫ 1



behave as classical waves!
→ solving small scale structure issue?

- Mass & frequency relation:

$$m = 4.1 \times 10^{-13}\ \mathrm{eV} \left(\frac{f_{\rm DM}}{10^2 \mathrm{Hz}} \right)$$

- Coherence time:

$$\tau \equiv \frac{2\pi}{m\bar{v}^2} \simeq 0.3\ \mathrm{day}\ \frac{10^{-13}\ \mathrm{eV}}{m}$$

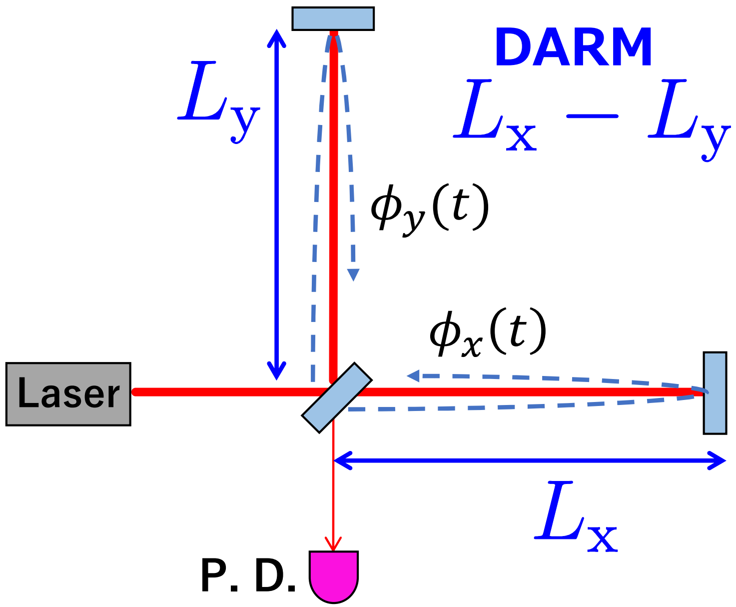
For $\Delta t \ll \tau$
≃const. amplitude & phase

For $m_{DM} \sim 10^{-14}$~10^{-11} eV, (GW-)laser interferometers can be powerful probe!

- ULDM search with GW interferometers (see S. Morisaki's talk for more details!)

We can search for DM interaction with **GW detector as it is**!

Schematics of Michelson interferometer



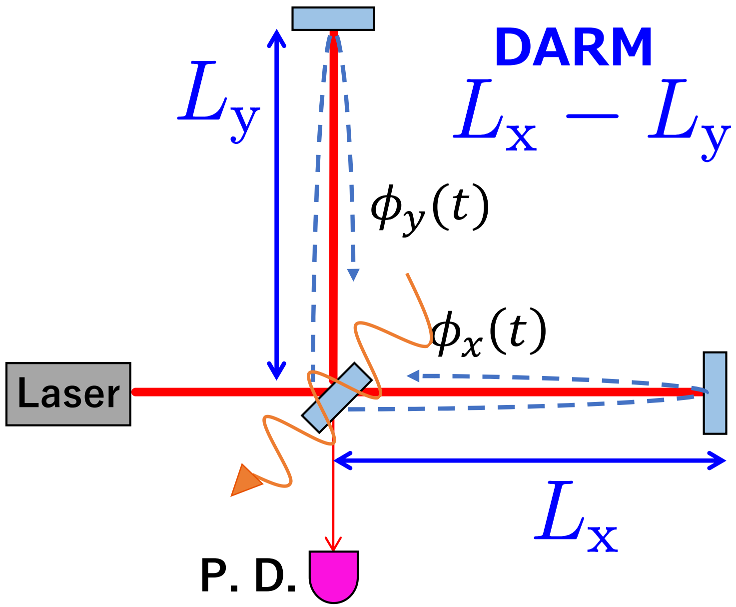
readout: $\phi_x(t) - \phi_y(t)$

→ **phase modulation due to ULDMs**

- ULDM search with GW interferometers (see S. Morisaki's talk for more details!)

We can search for DM interaction with **GW detector as it is**!

Schematics of Michelson interferometer



readout: $\phi_x(t) - \phi_y(t)$

→ **phase modulation due to ULDMs**

spin-0 candidates:

- **dilaton ϕ** changes size and refraction index

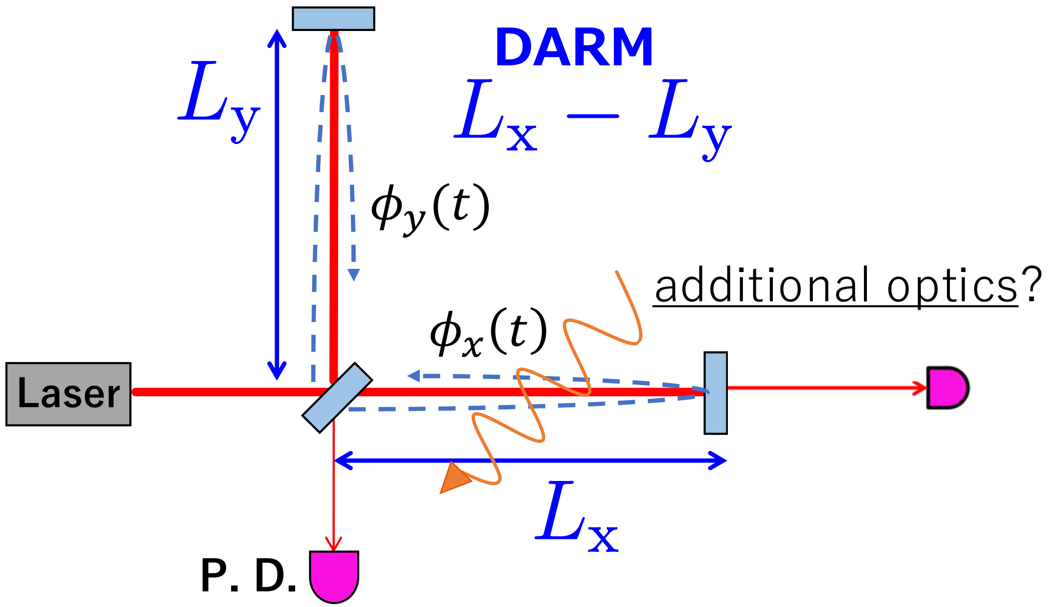
$\frac{\phi}{\Lambda_\gamma} F_{\mu\nu}F^{\mu\nu}, \frac{\phi}{\Lambda_e} m_e \bar{\psi}_e \psi_e$ → GEO600 put strong bounds

S. M. Vermeulen+ Nature 600, 424–428 (2021).

- ULDM search with GW interferometers (see S. Morisaki's talk for more details!)

We can search for DM interaction with **GW detector as it is**!

Schematics of Michelson interferometer



readout: $\phi_x(t) - \phi_y(t)$

→ **phase modulation due to ULDMs**

spin-0 candidates:

- **dilaton** ϕ changes size and refraction index

$\frac{\phi}{\Lambda_\gamma} F_{\mu\nu}F^{\mu\nu}, \frac{\phi}{\Lambda_e} m_e \bar{\psi}_e \psi_e$ → GEO600 put strong bounds

S. M. Vermeulen+ Nature 600, 424–428 (2021).

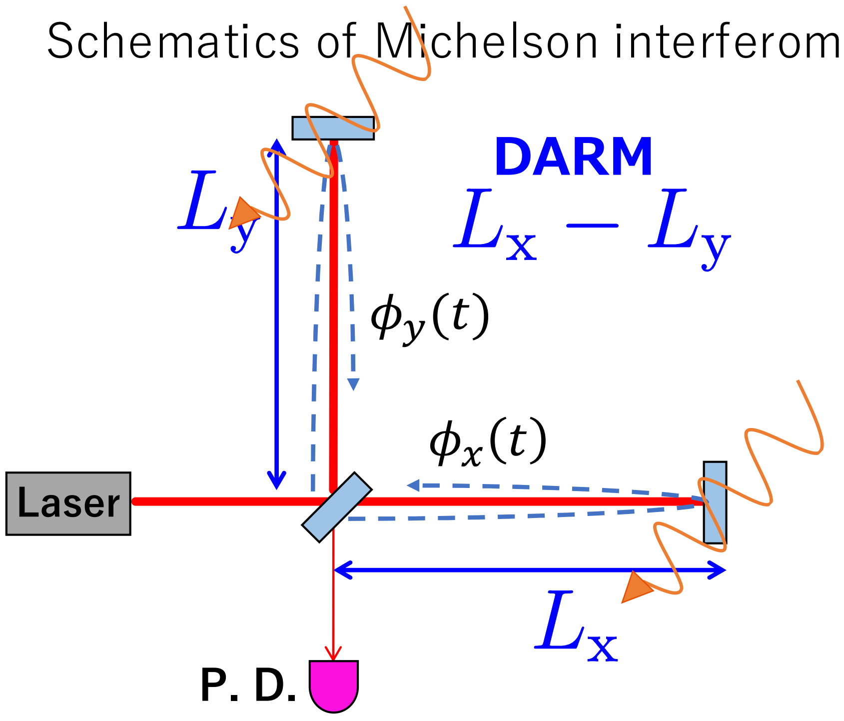
- **axion-like particle(ALP)** a (K. Nagano+ 2019,2021)

$g_{a\gamma} a \tilde{F}_{\mu\nu} F^{\mu\nu}$ → photon polarization rotation

- ULDM search with GW interferometers (see S. Morisaki's talk for more details!)

We can search for DM interaction with **GW detector as it is**!

Schematics of Michelson interferometer



readout: $\phi_x(t) - \phi_y(t)$

→ **phase modulation due to ULDMs**

spin-0 candidates:

- **dilaton** ϕ changes size and refraction index

$\frac{\phi}{\Lambda_\gamma} F_{\mu\nu}F^{\mu\nu}, \frac{\phi}{\Lambda_e} m_e \bar{\psi}_e \psi_e$ → GEO600 put strong bounds

S. M. Vermeulen+ Nature 600, 424–428 (2021).

- **axion-like particle (ALP)** a (K. Nagano+ 2019,2021)

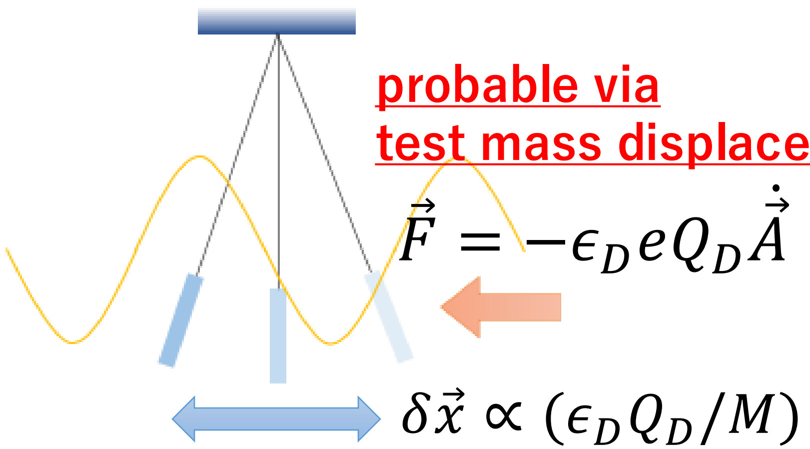
$g_{a\gamma} a \tilde{F}_{\mu\nu} F^{\mu\nu}$ → photon polarization rotation

spin-1 candidate:

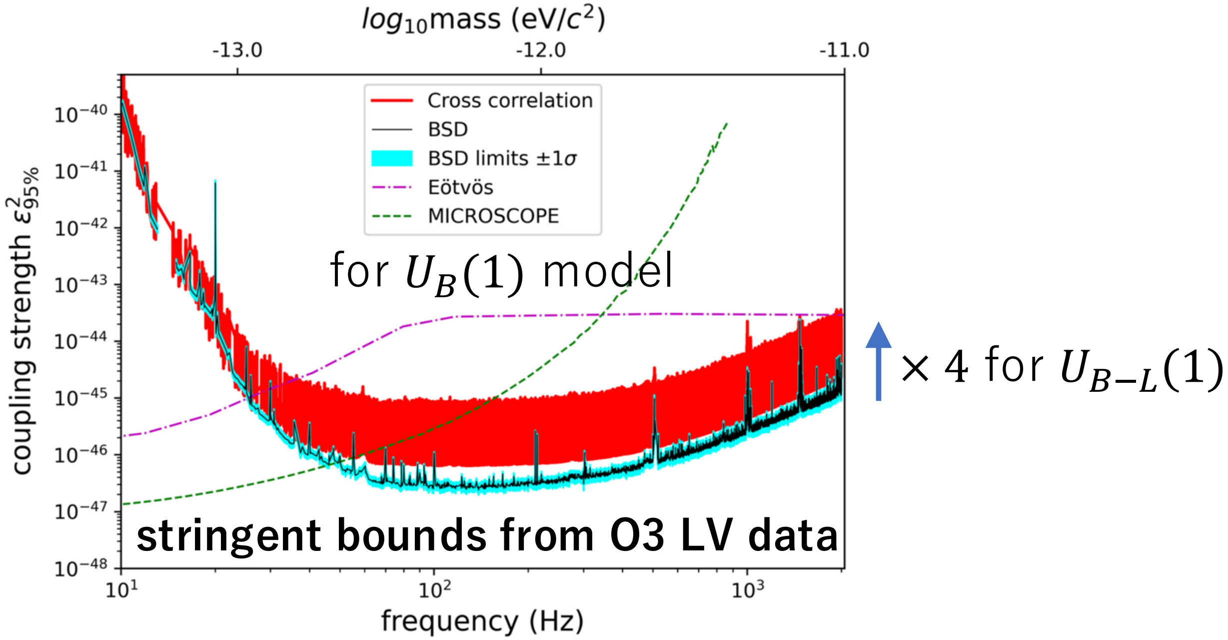
- **dark photon** A_μ (from gauged $U_D(1)$)

– **coupling to fermions** $\epsilon_D e J_D^\mu A_\mu$

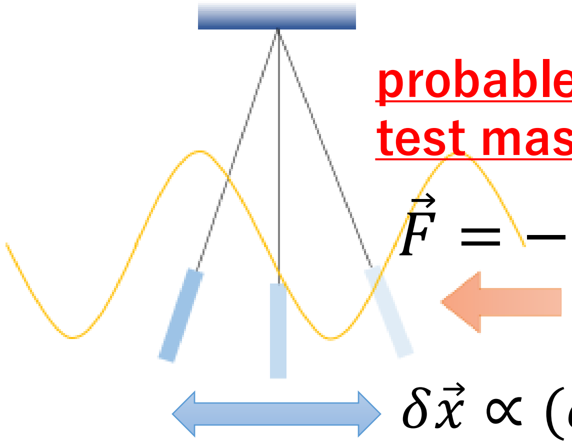
- Vector DM search with KAGRA (Y. Michimura+ 2020)



(LVK collaboration 2022)



- Vector DM search with KAGRA (Y. Michimura+ 2020)

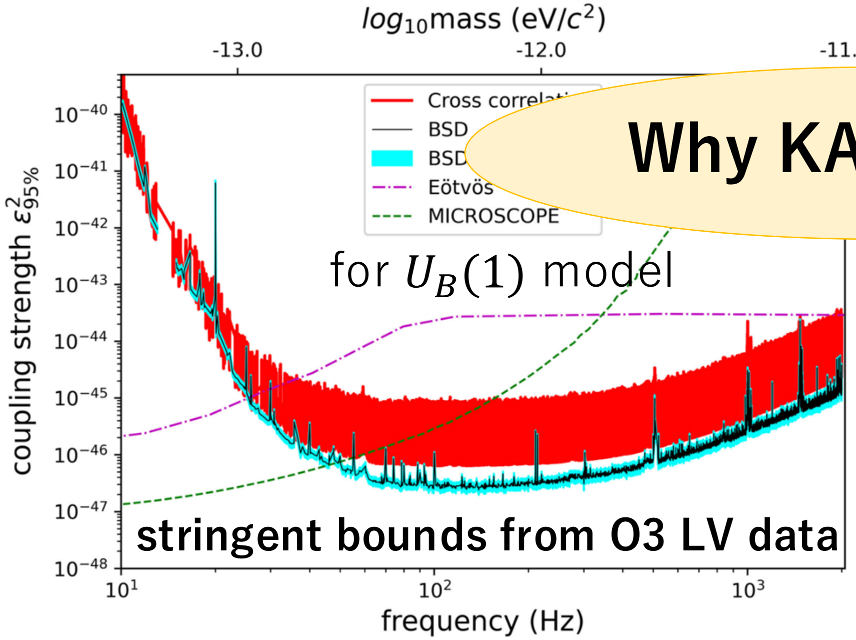


probable via test mass displacements!

$$\vec{F} = -\epsilon_D e Q_D \dot{\vec{A}}$$

$$\delta\vec{x} \propto (\epsilon_D Q_D / M)$$

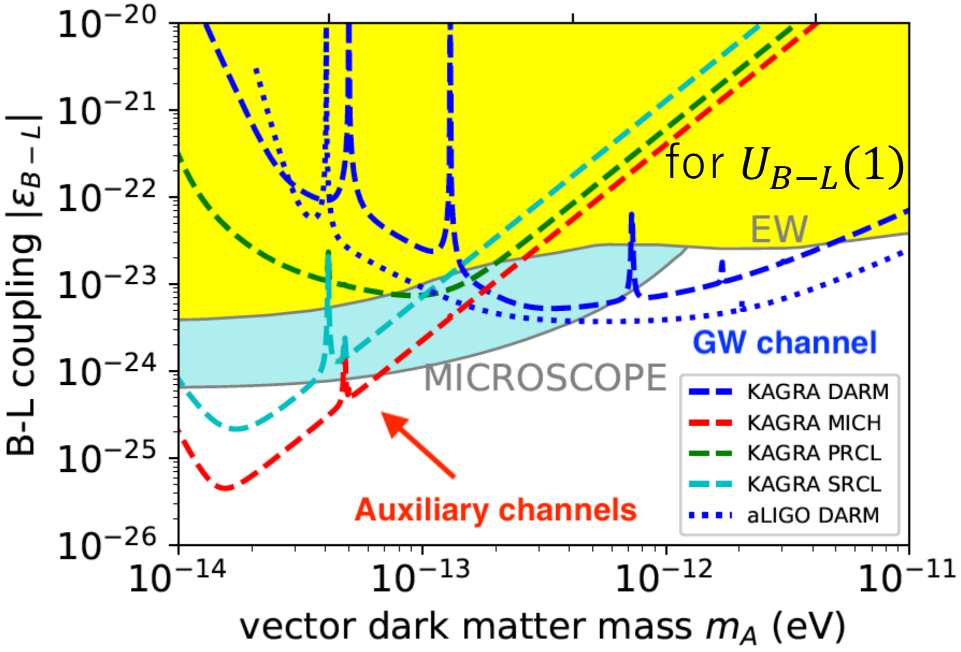
(LVK collaboration 2022)

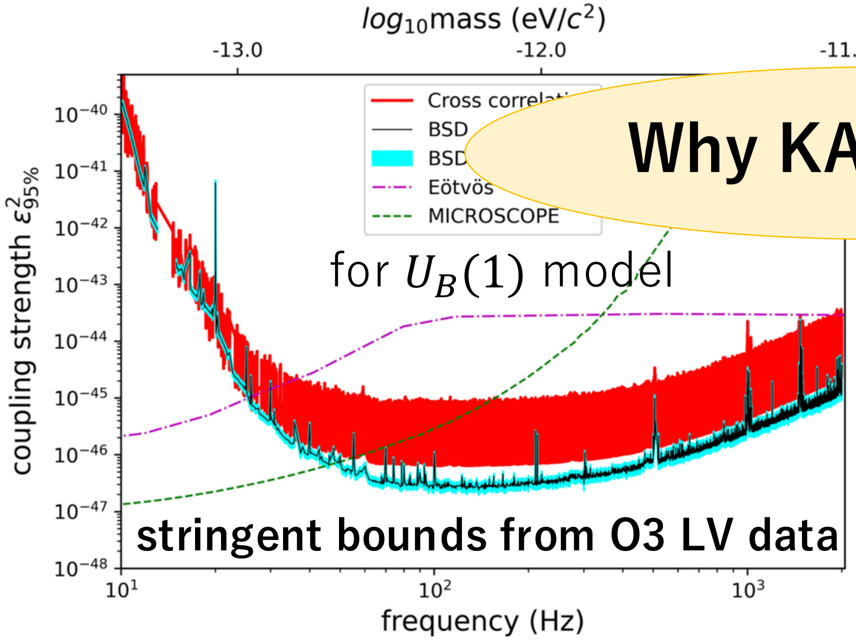


Why KAGRA...??

$\times$ 4 for $U_{B-L}(1)$

- Vector DM search with KAGRA (Y. Michimura+ 2020) (LVK collaboration 2022)





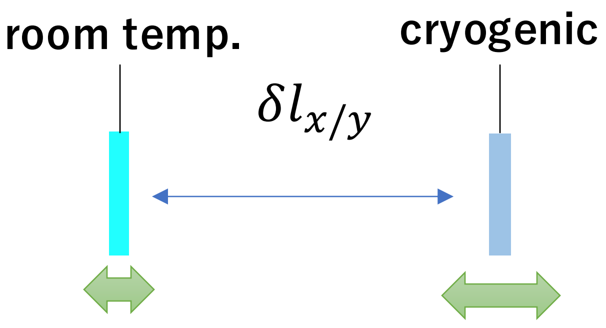
Why KAGRA…??

$\times$ 4 for $U_{B-L}(1)$

employs **sapphire/fused silica** mirrors

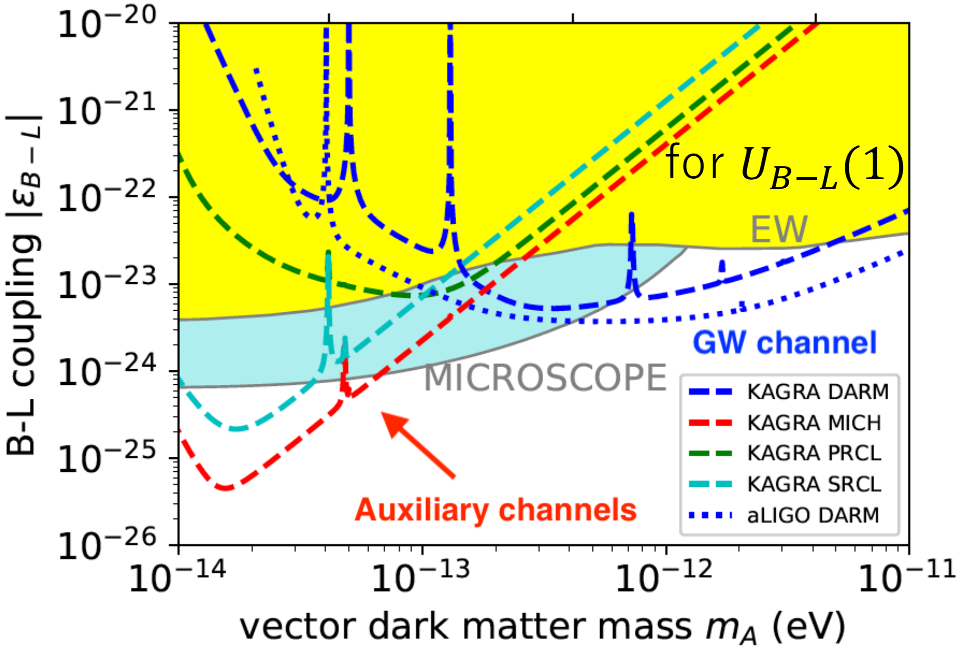
→ **difference in Q_{B-L}/M** enhances the sensitivity to VDM!

KAGRA can be the best detector for specific masses!

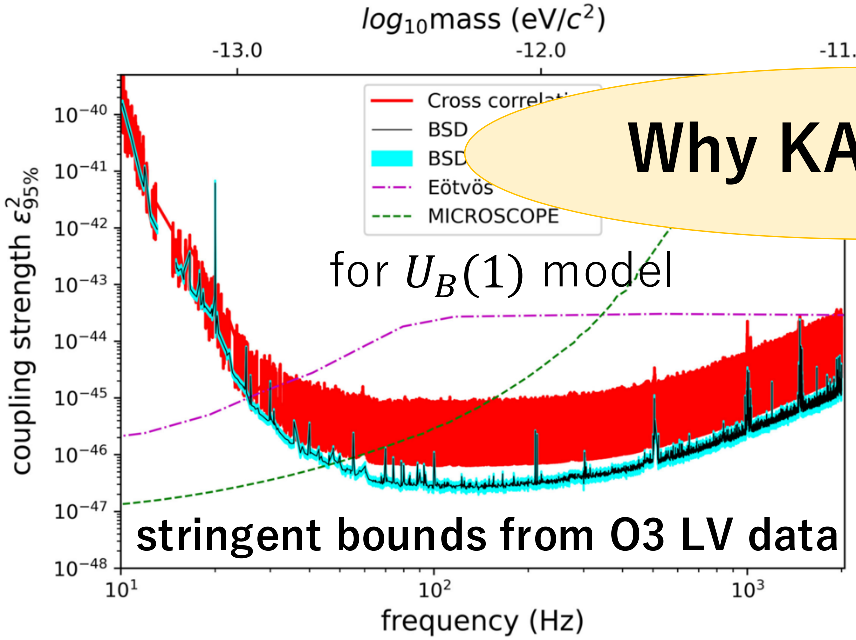


signal in length change enhanced!

- Vector DM search with KAGRA (Y. Michimura+ 2020)



(LVK collaboration 2022)



Why KAGRA…??

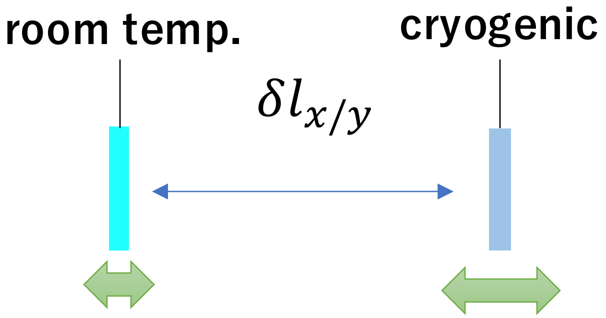
↑ × 4 for $U_{B-L}(1)$

employs **sapphire/fused silica** mirrors

→ **difference in Q_{B-L}/M** enhances the sensitivity to VDM!

KAGRA can be the best detector for specific masses!

– **We built an analysis pipeline and applied it to real data!**



signal in length change enhanced!

Contents

・DM signal in KAGRA's auxiliary length channels

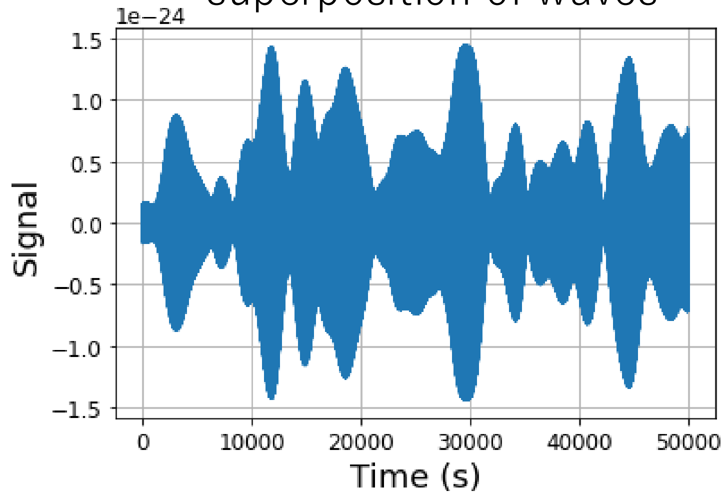
※neither coherent nor monochromatic!!

(Nakatsuka, Morisaki, Fujita, **JK**+ 2022)

$$\tilde{h}_X(f;t_0) = \mathrm{i}\frac{\epsilon_D e}{2\pi f}\Delta\left(\frac{Q}{M}\right) d^i_X(t_0)\tilde{A}_i(f;t_0)$$

field amplitude

superposition of waves



・Statistical fluctuation:

superposition of waves → Gaussian dist.

$\langle \tilde{A}^*_i(f,t_0)\tilde{A}_i(f,t_1)\rangle$ characterized by $\tau \equiv 2\pi m_A/v^2_{vir}$

➡ Special care in deriving upper bound on ϵ_{B-L}

・DM signal in KAGRA's auxiliary length channels

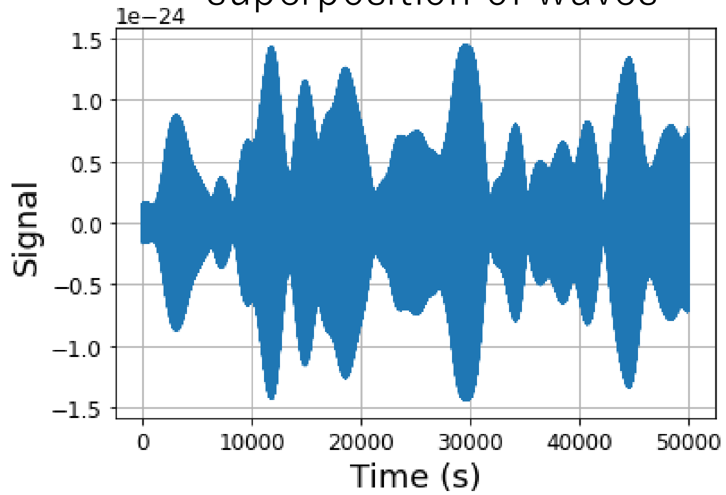
(Nakatsuka, Morisaki, Fujita, **JK**+ 2022)

$$\tilde{h}_X(f;t_0) = \mathrm{i}\frac{\epsilon_D e}{2\pi f}\Delta\left(\frac{Q}{M}\right) d^i_X(t_0)\tilde{A}_i(f;t_0)$$

field amplitude

※neither coherent nor monochromatic!!

superposition of waves



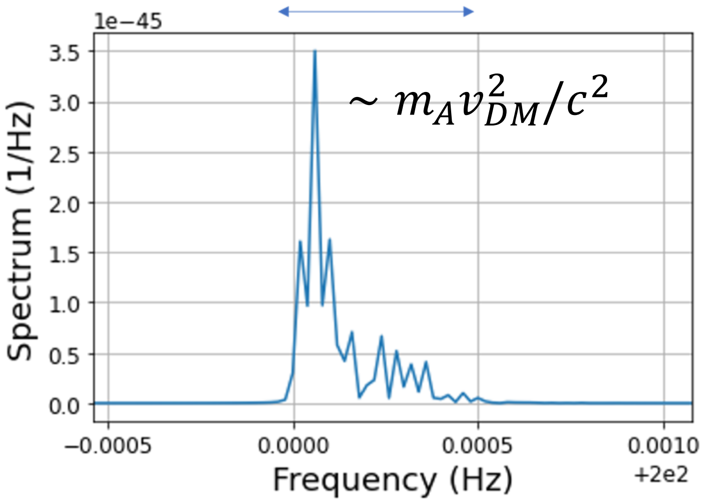
・Statistical fluctuation:

superposition of waves → Gaussian dist.

$\langle \tilde{A}_i^*(f,t_0)\tilde{A}_i(f,t_1)\rangle$ characterized by $\tau \equiv 2\pi m_A/v_{vir}^2$

⇨ Special care in deriving upper bound on ϵ_{B-L}

Spectrum of DM signal



・Non-relativistic dispersion:

peak of spectrum: $f_c = m_A/2\pi$

narrow width: $\Delta f \sim f_c v_{DM}^2/c^2 \sim 10^{-6} f_c \simeq$ **CW search**

⇨ Incoherently collecting spectra in Fourier space

・Detection statistics inspired by Continuous Wave search

inputs of pipeline:

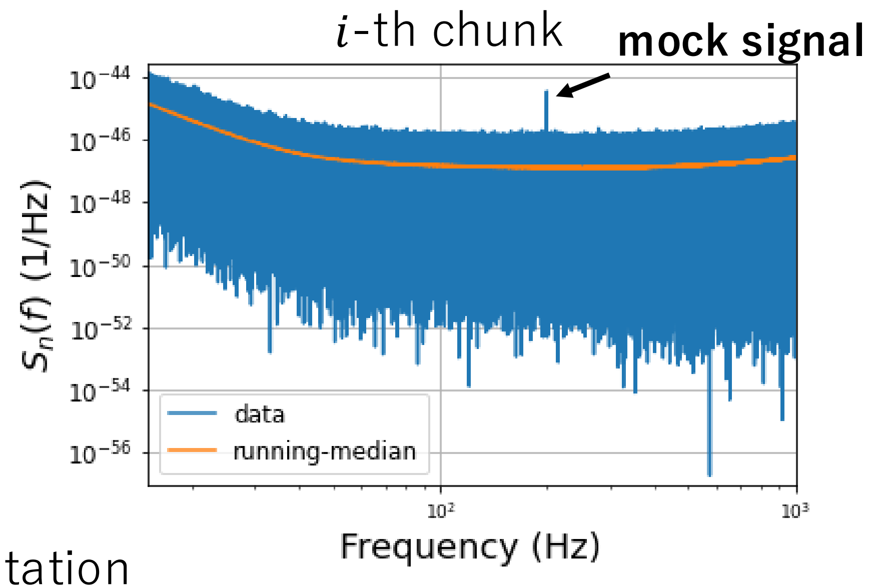
✓ Fourier Transform of data segment: $\tilde{d}(f_n; t_i)$

✓ Noise PSD: $S(f_n; t_i)$ → estimated by running median

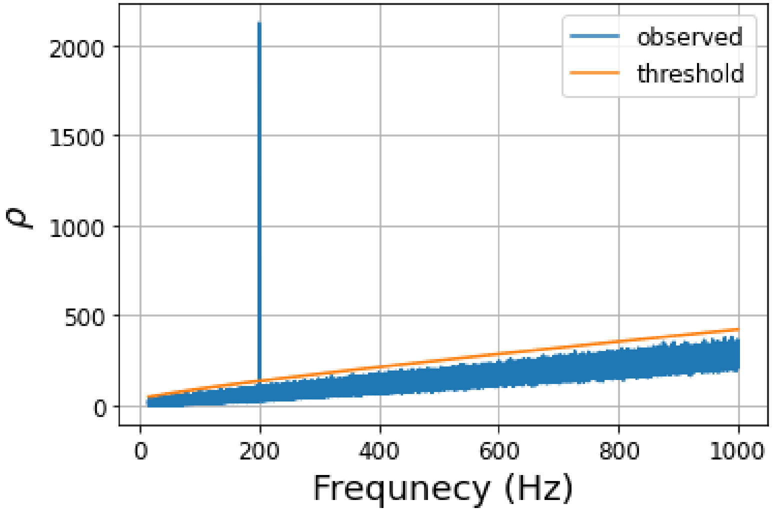
$$\rho(f_c) \equiv \frac{4}{T} \sum_{i=1}^{N_{ch}} \sum_{n=1}^{N_{bin}} \frac{|\tilde{d}(f_n;t_i)|^2}{S(f_n;t_i)}$$

Incoherent sum of spectrum

→ non-stationarity/detector rotation



summing up



・Summation within the narrow band: $f_c \leq f_n \leq f_c(1 + \kappa v_{vir}^2/c^2)$

$\kappa = 3.17$ → 1% loss w.r.t. expected total signal power

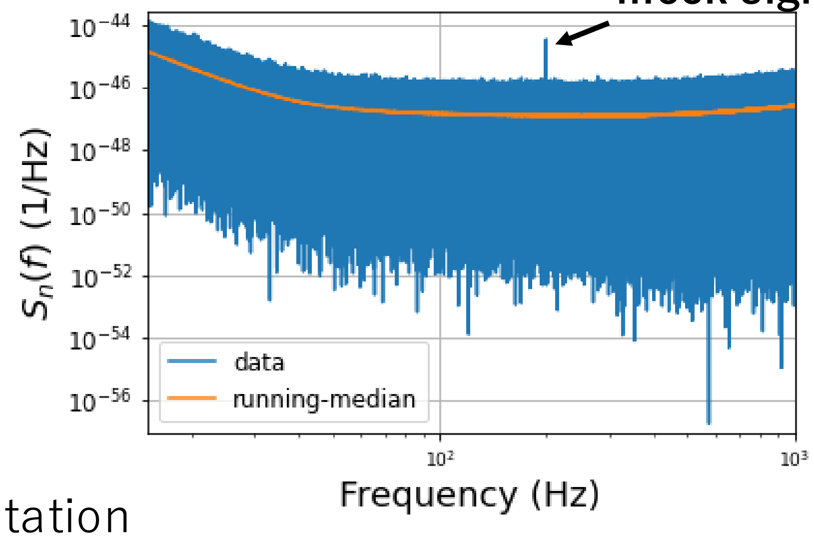
・$T = 30$ min. ($\ll 1$ day, to avoid the effect of Earth's rotation)

・Detection statistics inspired by Continuous Wave search

inputs of pipeline:

✓ Fourier Transform of data segment: $\tilde{d}(f_n; t_i)$

✓ Noise PSD: $S(f_n; t_i)$ → estimated by running median

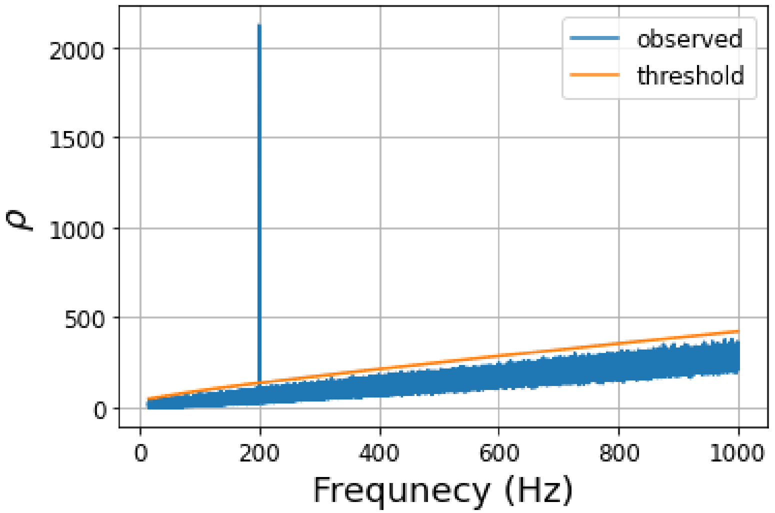


$$\rho(f_c) \equiv \frac{4}{T} \sum_{i=1}^{N_{ch}} \sum_{n=1}^{N_{bin}} \frac{|\tilde{d}(f_n; t_i)|^2}{S(f_n; t_i)}$$

Incoherent sum of spectrum

→ non-stationarity/detector rotation

summing up



・Summation within the narrow band: $f_c \le f_n \le f_c(1 + \kappa v_{vir}^2/c^2)$

$\kappa = 3.17$ → 1% loss w.r.t. expected total signal power

・$T = 30$ min. ($\ll 1$ day, to avoid the effect of Earth's rotation)

✓ Gaussian noise only: χ^2 dist. with $2N_{bin}N_{ch}$ DoFs

⇨ 95% percentile as a threshold for "detection"

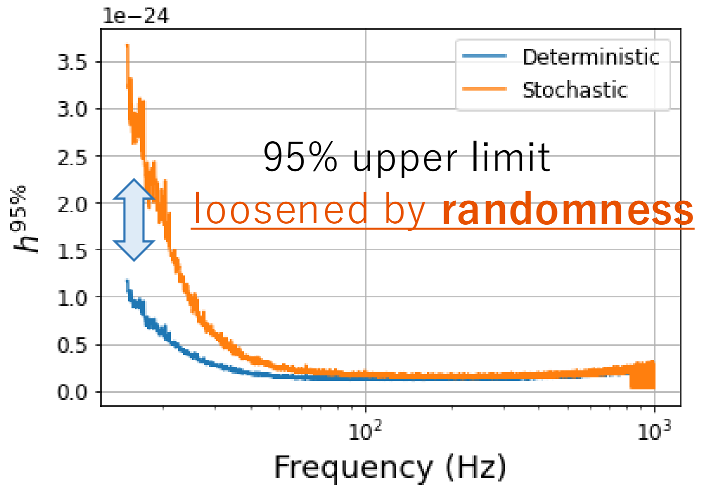
・Upper limit estimation including stochasticity

Frequentist: $\int_0^{\rho_{obs}(f_c)} d\rho \ \mathcal{L}(\rho(f_c)|\epsilon_{B-L}^{95\%}) = 0.05$

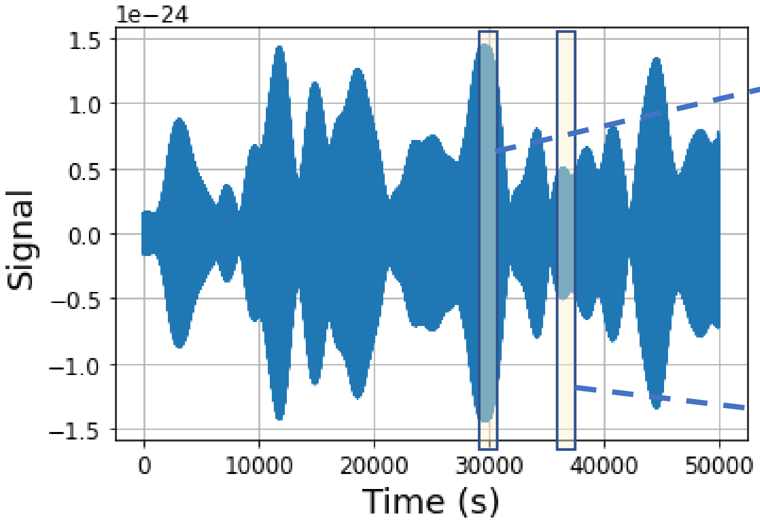
※**random amp. of ULDM matters!**

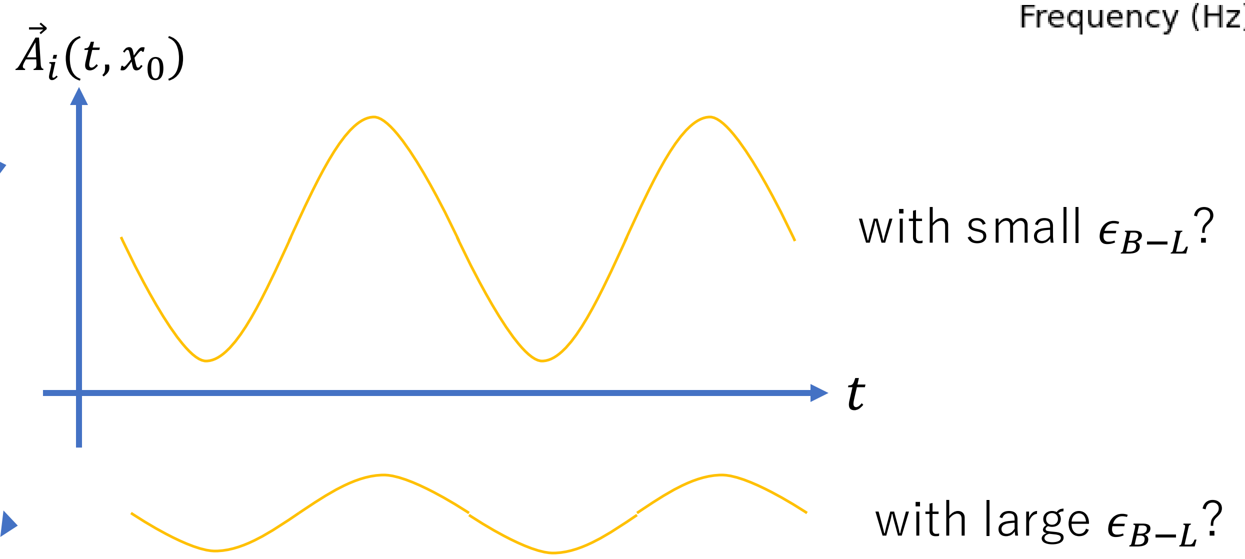
(cf. deterministic → too strong bound)

But analytical expression suffers numerical instability...



low freq: obs. time < coh. time





high freq: sufficient number of realizations → asymptotic to the deterministic case

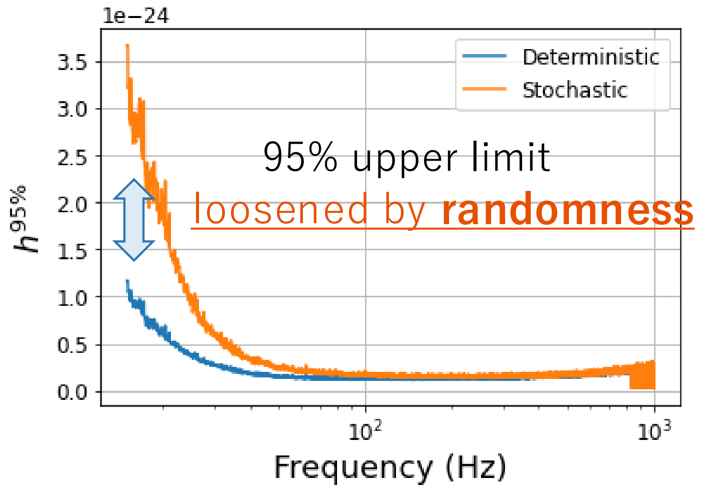
・Upper limit estimation including stochasticity

Frequentist: $\int_0^{\rho_{obs}(f_c)} d\rho \; \mathcal{L}(\rho(f_c)|\epsilon_{B-L}^{95\%}) = 0.05$

※**random amp. of ULDM matters!**

(cf. deterministic → too strong bound)

But analytical expression suffers numerical instability...



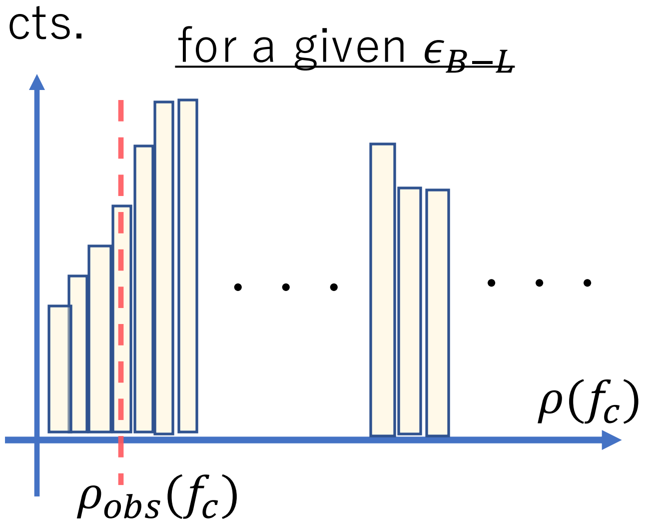
Simulation-based evaluation: (LVK collaboration (incl. **JK**) 2024)

$$\rho(f_c;\epsilon_D) = \mathcal{N}^2 + \epsilon_D \mathcal{N}\cdot\mathcal{S} + \epsilon_D^2 \mathcal{S}^2$$

※ϵ_D factorized

$\mathcal{N}$: $2N_{bin}N_{ch}$ unit Gaussian, $\mathcal{S}$: $\langle \tilde{h}_X^* \tilde{h}_X \rangle$ and noise PSD

- With analytically given $\langle \tilde{h}_X^*(f,t_0)\tilde{h}_X(f,t_1)\rangle$ ($\neq 0$ for $|t_1 - t_0| < \tau$), 10^5 realizations of $\mathcal{N}^2$, $\mathcal{N}\cdot\mathcal{S}$, $\mathcal{S}^2$ are simulated for each f_c.
- varying ϵ_{B-L} to find $\rho_{obs}(f_c;\epsilon_{B-L}^{95\%}) = 5\%$ percentile.



Contents

・Data from KAGRA O3GK run

・MICH/PRCL length change

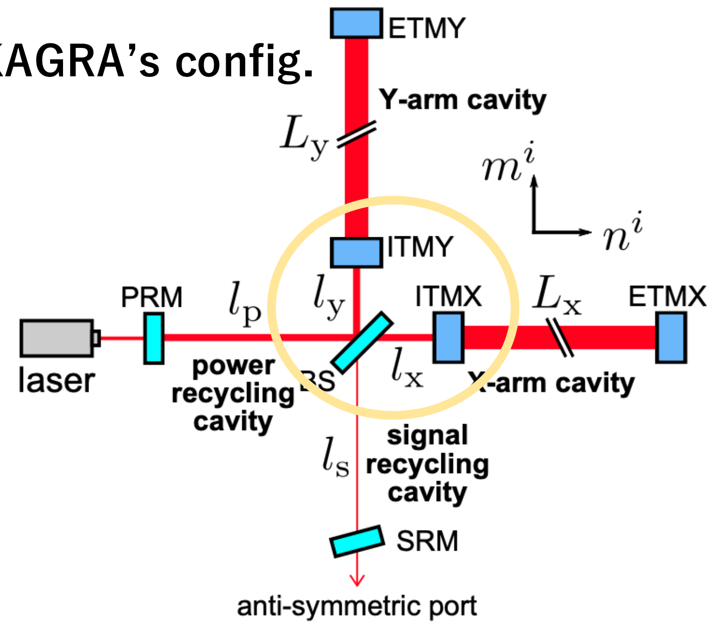
$\delta L_{\mathrm{MICH}} = \delta(l_x - l_y) \quad \delta L_{\mathrm{PRCL}} = \delta[(l_x + l_y)/2 + l_p]$

- calibration uncertainty $20\% - 30\%$

※less reliable for lower freq.

→ **Lower limit of frequency to be analyzed: 15Hz**

KAGRA's config.

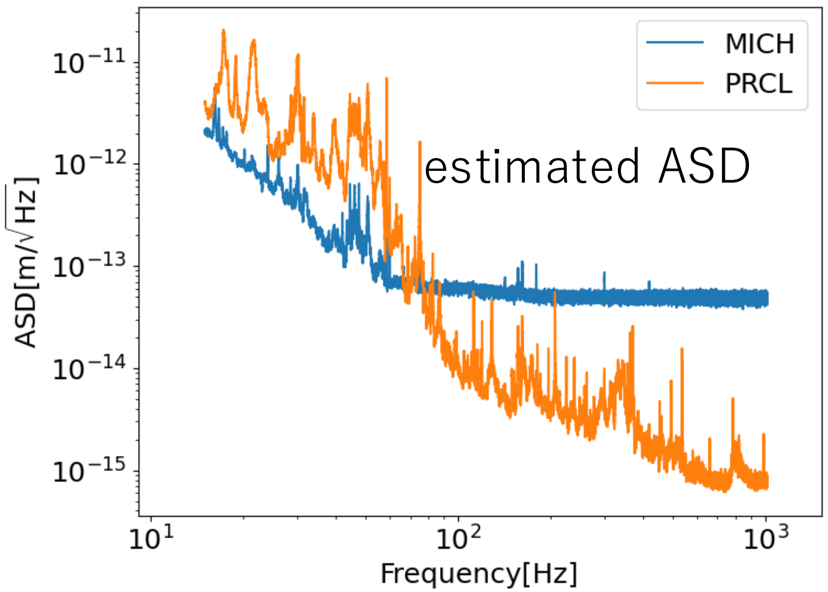


・O3GK: from 2020 April 7th to 21st

- KAGRA detector in science mode ~ 53%

※unusable segments (last 2~3 days)
due to injections that produce lots of sidebands

→ **number of 30 min. chunks: 217** ~ 4days (≪ 1yr)



- Data from KAGRA O3GK run

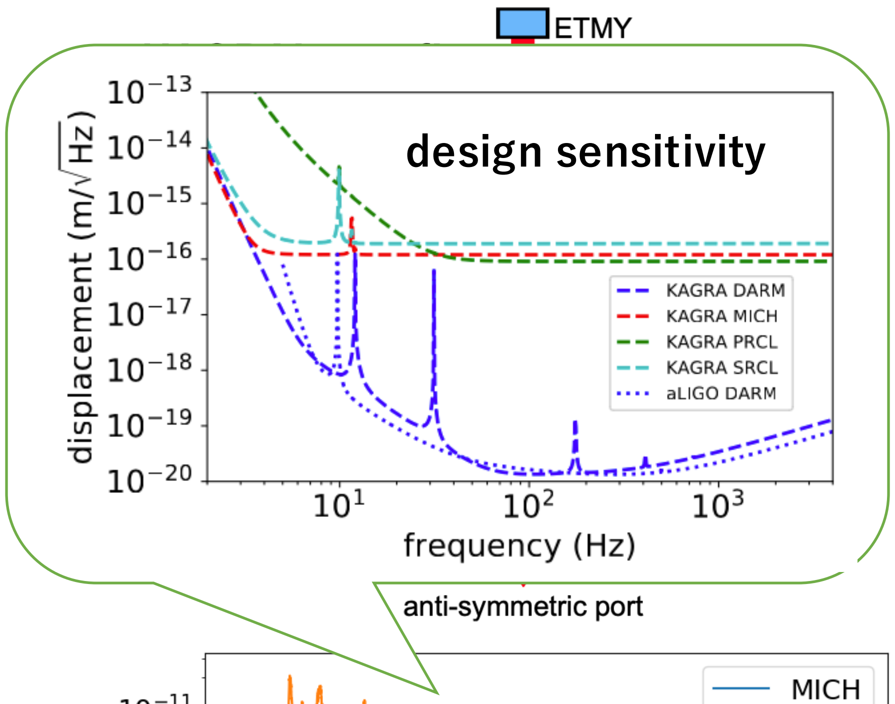
- MICH/PRCL length change

$$\delta L_{\rm MICH} = \delta(l_x - l_y) \qquad \delta L_{\rm PRCL} = \delta[(l_x + l_y)/2 + l_p]$$

- calibration uncertainty $20\% - 30\%$

※less reliable for lower freq.

→ **Lower limit of frequency to be analyzed: 15Hz**

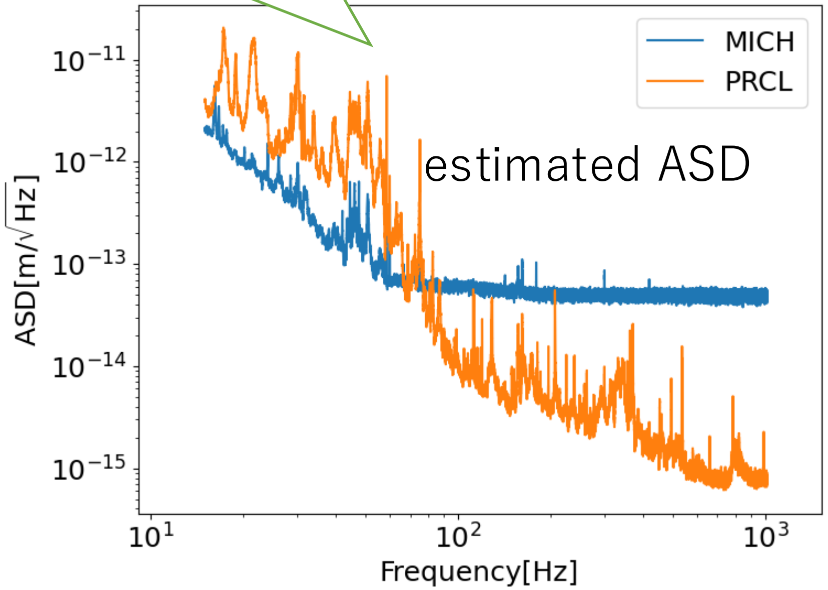


- O3GK: from 2020 April 7th to 21st

- KAGRA detector in science mode ~ 53%

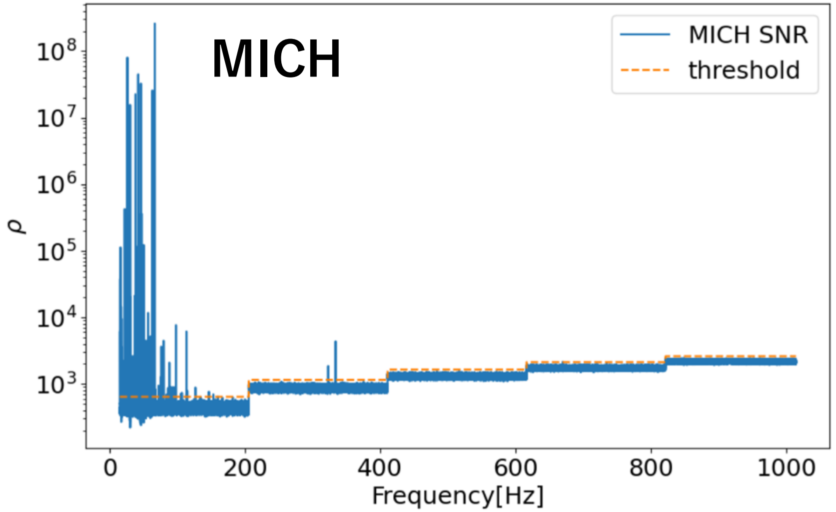
※unusable segments (last 2~3 days) due to injections that produce lots of sidebands

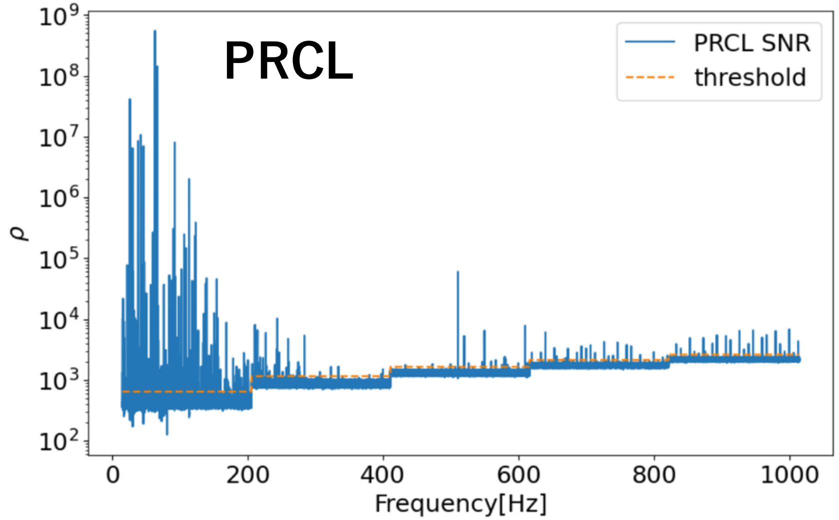
→ **number of 30 min. chunks: 217** ~ 4days (≪ 1yr)



Demonstration of our pipeline with real data

・Candidates and Veto procedure





After running the pipeline...

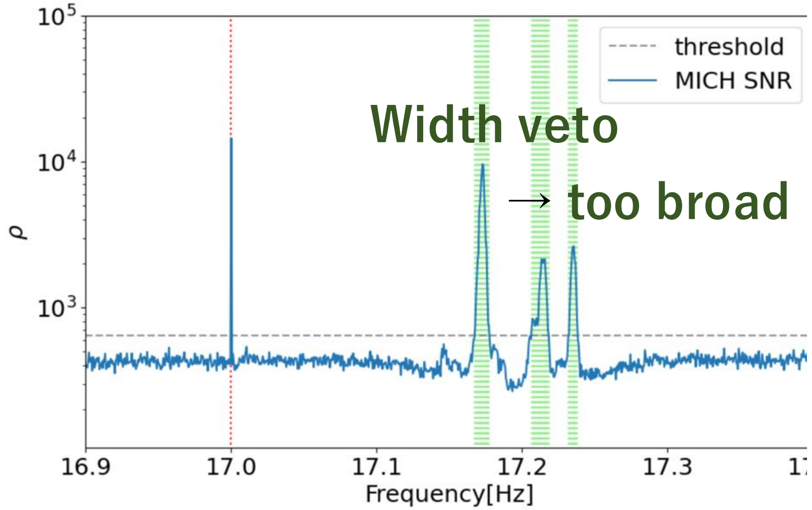
MICH: 1944

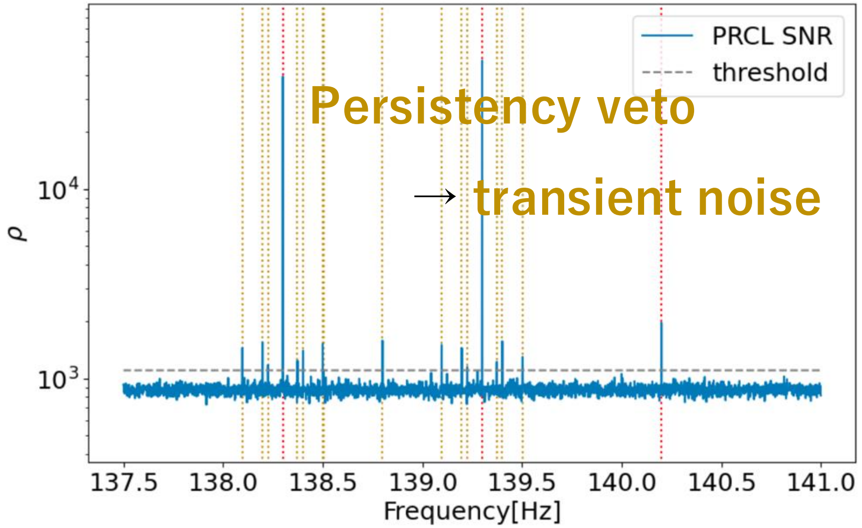
PRCL: 4133

candidates with FAP <5%.

after veto analysis

+ referring to line study





MICH: 20

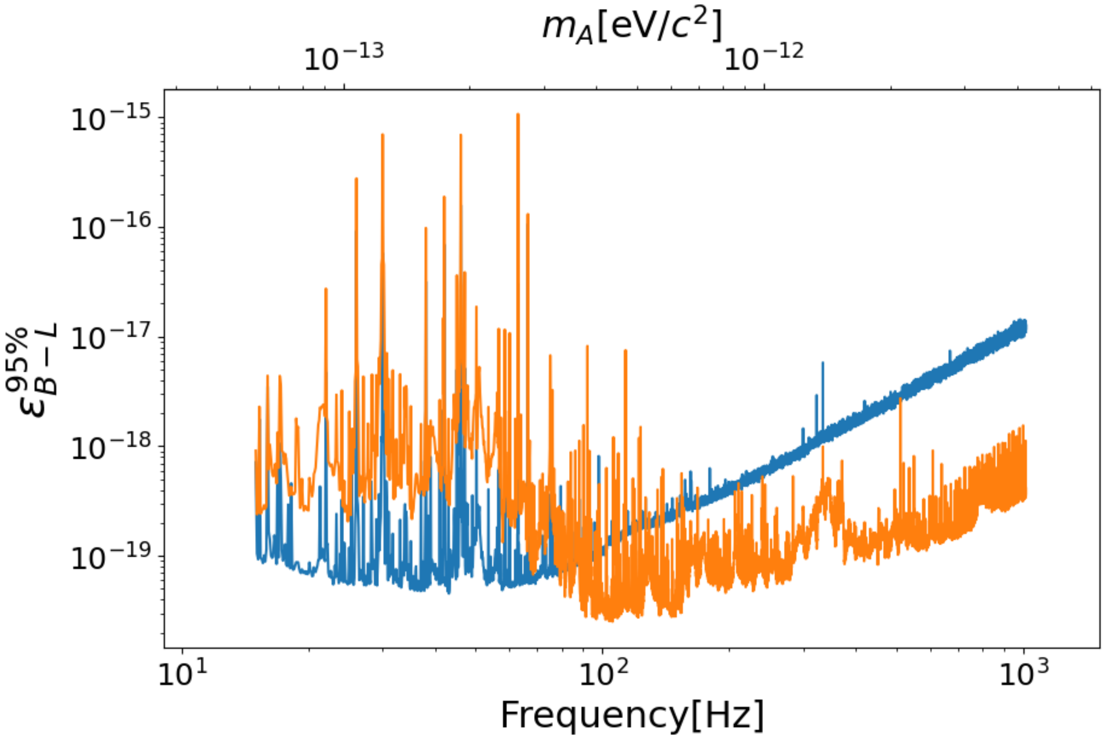
PRCL: 148

still remain...

list of lines:
LIGO-L2300092 (JK, 2023)
for our future reference

・Upper limits on the coupling constant

- For simplicity, upper limits are derived for all bins (including the “detected” ones)
- Smoothed over $\Delta\mathrm{f} = 0.1$Hz by collecting the maximum value of $\rho(f_c)$



Remarks:

✓ $\langle \tilde{h}^*_X(f,t_0)\tilde{h}_X(f,t_1)\rangle \neq 0$ matters for $m_A \lesssim 10^{-12}$eV

→ Over-constraint is avoided in lower freq.!

✓ Consistent with $\propto T^{1/4}$ scaling at high mass

(in comparison with the prediction in Michimura+ 2020)

→ another manifestation of ULDM “stochasticity”

Simulation-based pipeline: proper upper limits regardless of data length, masses of DM

Contents

・Summary of O3GK analysis

- **A new pipeline** was developed for ULDM search:
  - detection statistic inspired by CW searches
  - simulation-based upper limits – properly includes ULDM covariance
  - veto using generic feature of ULDM

- **The first-time a search for vector ULDM was performed using KAGRA data!**
  - Demonstrating the pipeline through end-to-end analysis

- Many lines (even after veto), less stringent bound on coupling constant…

→ With future upgrades, KAGRA could appreciate its uniqueness as ULDM detector.

・Toward O4 data analysis

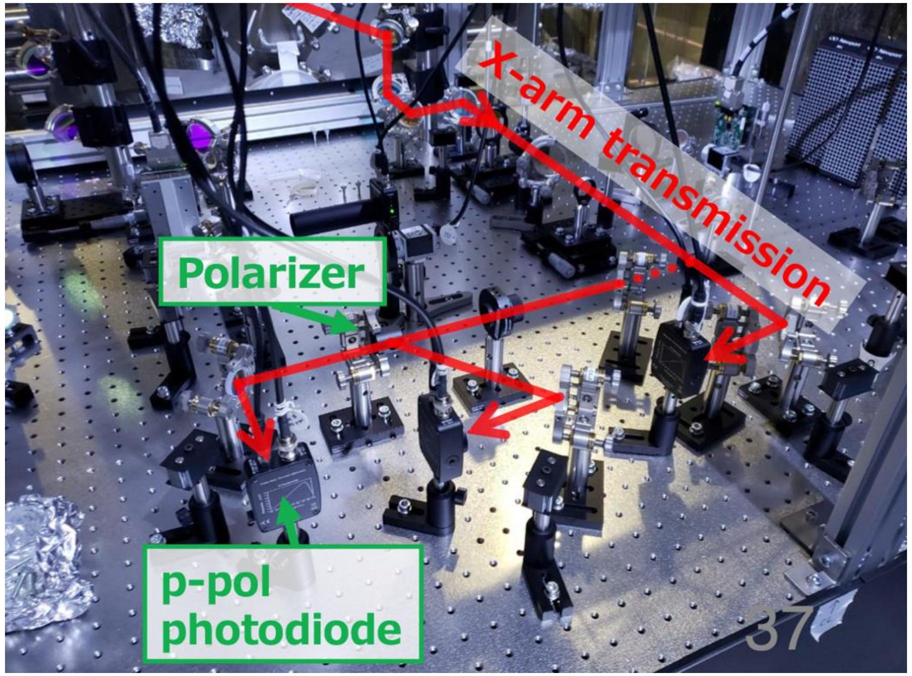
vector and axion DM simultaneous search:

・Polarization optics installed at X/Y arm transmission

- does not affect GW/VDM observation

→ axion DM search with KAGRA!!

※planned to start collecting data from O4b



Pipeline development:

・extension to ALP & spin-2 search, inclusion of DARM channel

- ALP search pipeline already applied to DANCE act1 (→ Y. Oshima, H. Fujimoto, **JK**+ 2023)

・Implementation of optimal search method (S. Morisaki, **JK**, T. Fujimori, on-going)

- Similarity with PTA data analysis (ULDM correlation over segments ↔ HD correlation)

Backup

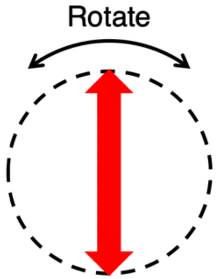
DM interactions with laser interferometers

- Axion-like-DM & photon interaction

coherently oscillating axion

$$a(t) = a_0 \cos(m_a t + \phi) \quad \text{w/} \quad \mathcal{L} \supset \frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} \simeq \underline{\textbf{birefringent medium}}$$

⇨ phase velocity: $c_{R/L} \simeq 1 \pm g_{a\gamma} \dot{a}(t)/2k$



Oscillatory rotation of polarization angle:

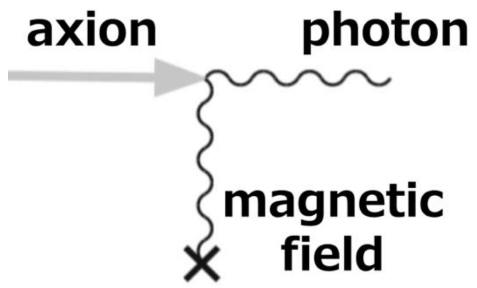
$$\delta\theta = k \int_{t-L}^{t} dt\,(c_L - c_R) \propto g_{a\gamma} a_0 m_a L \sin(m_a(t - L/2))$$

$$(m_a L \ll 1)$$

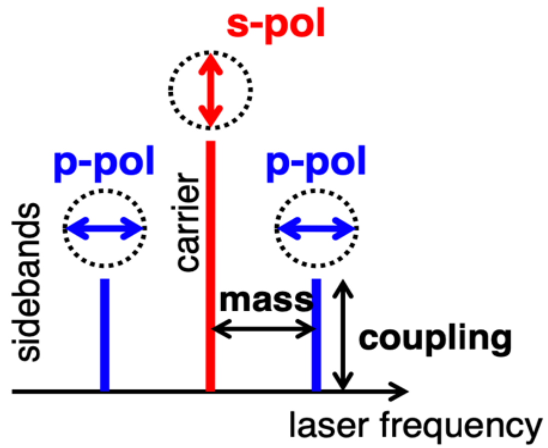
We need a longer path length!

→ **GW interferometer**...?

typical experiment:

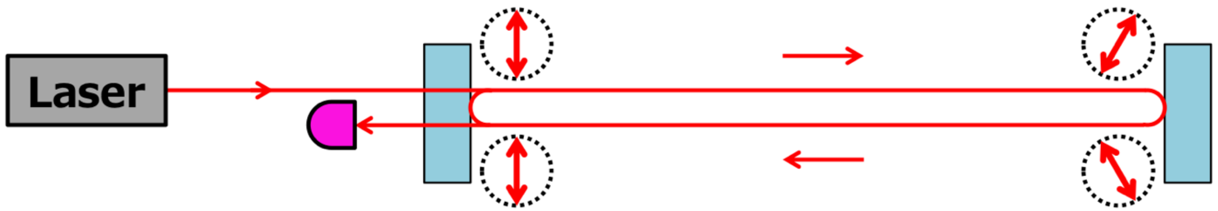


In our case:



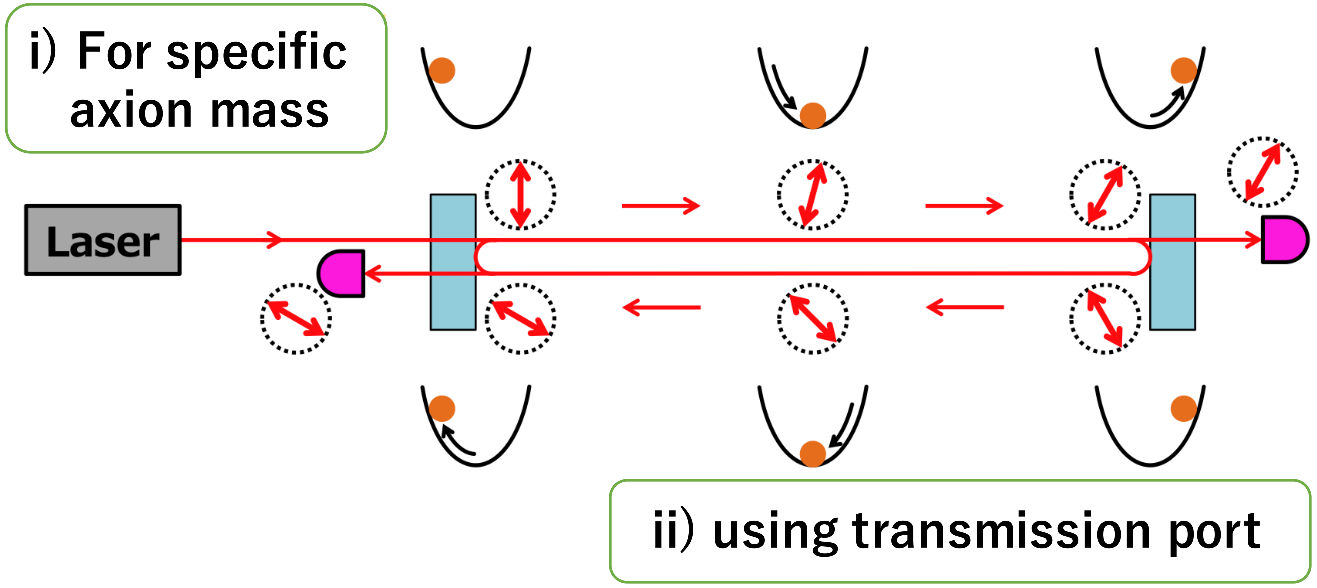
- Observing polarization rotation with optical cavity (K. Nagano+ 2021)

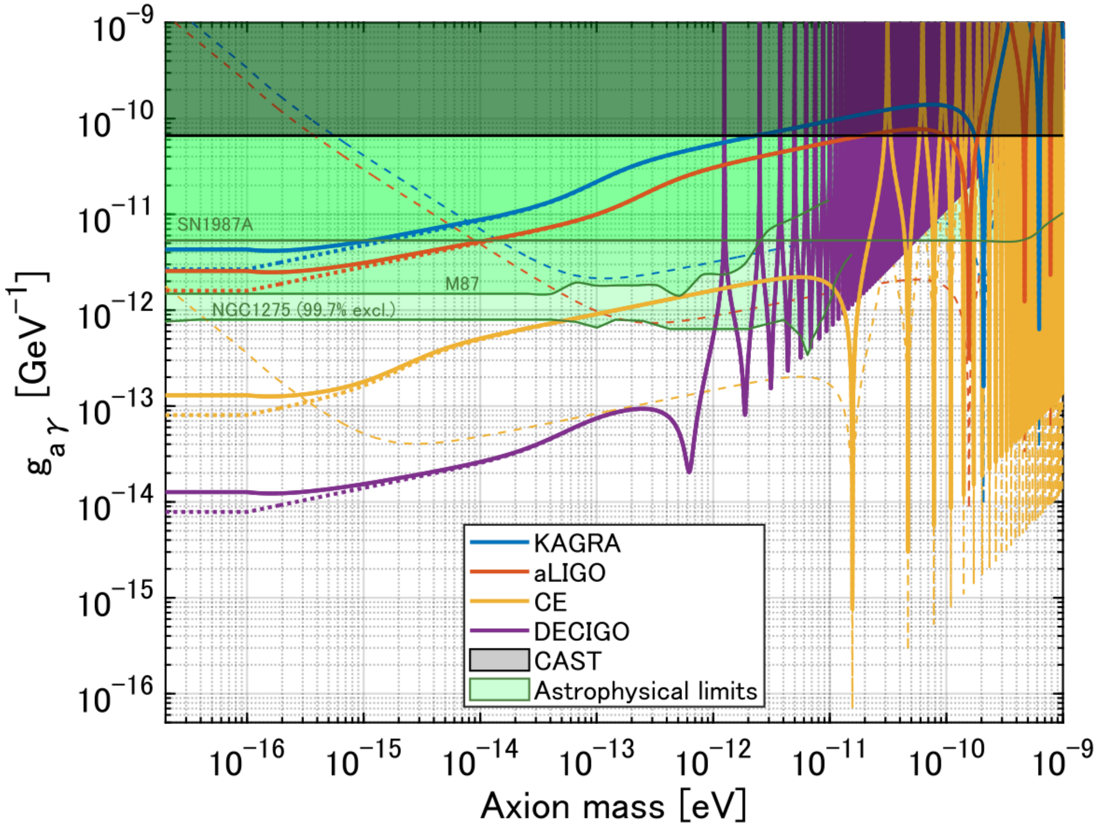
Naïvely, it's not efficient with cavity...



flipped at the reflection

But we can have more chance:





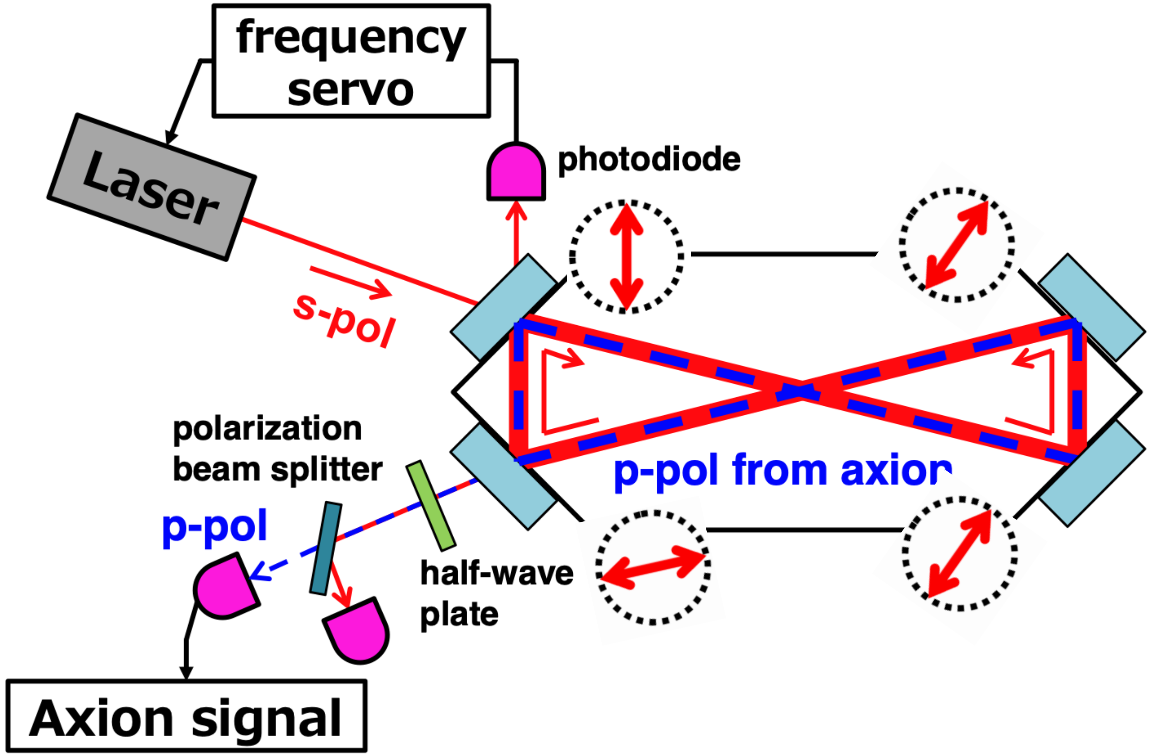
These can be implemented at

i)GW detection port (solid-line)

ii)X/Y-end (dashed-line)

※Axion-like-DM "targeted" interferometer

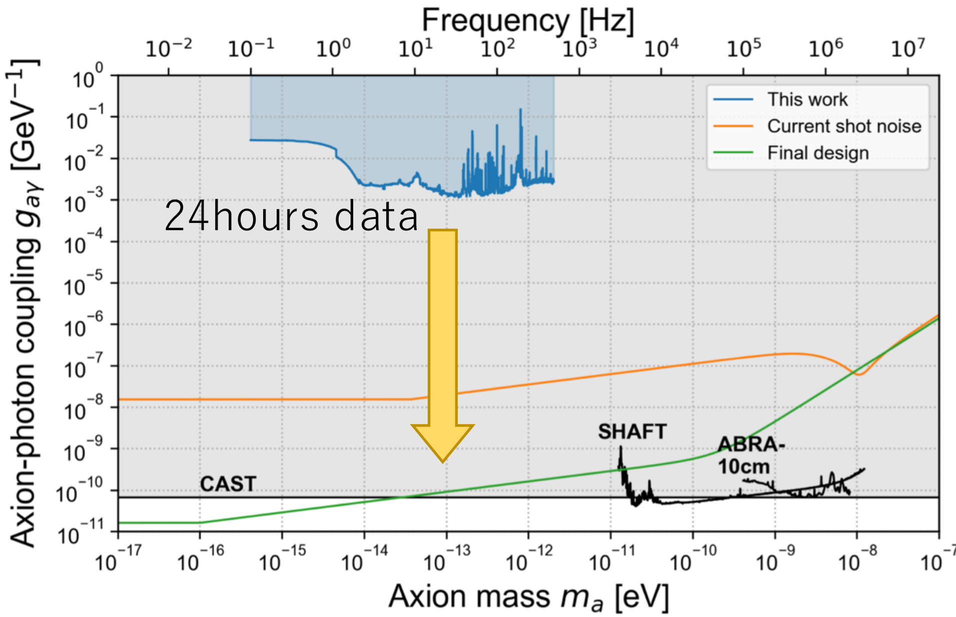
Bow-tie cavity can accumulate the rotation!!



Dark matter Axion search with riNg Cavity Experiment

(I. Obata+ 2018)

First constraint from the test run



(Oshima, Fujimoto, **JK**+ 2023)

※Stochastic amplitude of ULDM is taken into account

To be upgraded further!

finesse, longer round-trip, laser power, ...

Analytical expressions:

$$\delta\vec{x}(t,\vec{x}) = -\epsilon e\frac{Q}{M}\vec{B}(t,\vec{x}), \quad \vec{B}(t,\vec{x}) \equiv \int^t dt'\vec{A}(t',\vec{x})$$

$$\left\langle \tilde{A}_i^*(f;t_0)\tilde{A}_j(f;t_1)\right\rangle = \frac{A^2T^2v_{\rm vir}^3}{8\sqrt{\pi}V^3}\mathrm{e}^{-V^2/v_{\rm vir}^2+2\pi\mathrm{i}f_{\rm DM}(t_1-t_0)}\left(I(x_+)-I(x_-)\right)\delta_{ij}.$$

$$I(x) \equiv \int dx\,x\sinh(x)\mathrm{e}^{-x^2/X^2}$$

$$= \frac{X^2}{8}\left[\sqrt{\pi}X\mathrm{e}^{X^2/4}\left\{\mathrm{erf}\left(\frac{x}{X}-\frac{X}{2}\right)+\mathrm{erf}\left(\frac{x}{X}+\frac{X}{2}\right)\right\}-4\mathrm{e}^{-x^2/X^2}\sinh(x)\right]$$

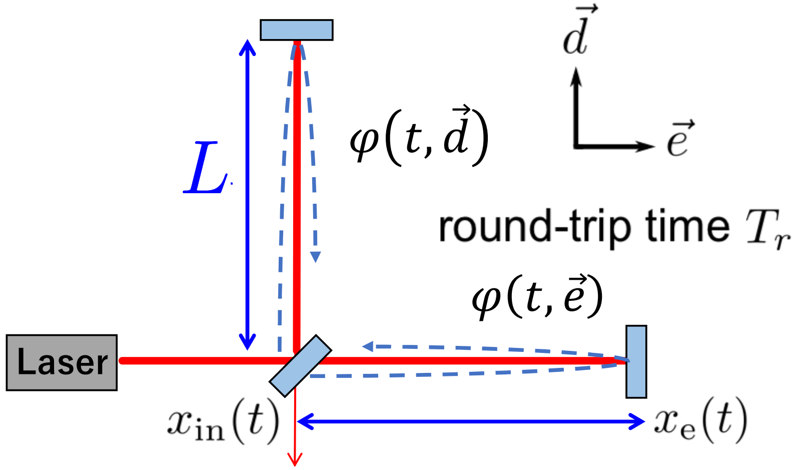
$$x = \frac{2V}{v_{\rm vir}^2}v, \quad X = \frac{2V}{v_{\rm vir}\sqrt{1-\mathrm{i}\pi v_{\rm vir}^2 f_{\rm DM}(t_1-t_0)}} \qquad v_\pm \equiv \sqrt{2\left(\frac{f\pm 1/(2T)}{f_{\rm DM}}-1\right)}, \qquad x_\pm \equiv \frac{2V}{v_{\rm vir}^2}v_\pm$$

single chunk likelihood with random DM amplitude being marginalized:

$$\mathcal{L}(\rho|\{\lambda_n\}) = \sum_n^{N_{\rm bin}}\frac{w_n}{2(1+\lambda_n^2)}\exp\left(-\frac{\rho}{2(1+\lambda_n^2)}\right) \qquad w_n \equiv \prod_{n'(\neq n)}^{N_{\rm bin}}\frac{1+\lambda_n^2}{\lambda_n^2-\lambda_{n'}^2}$$

λ_n: normalized signal amplitude at f_n

- Signals from vector DM (Nakatsuka+ 2022)

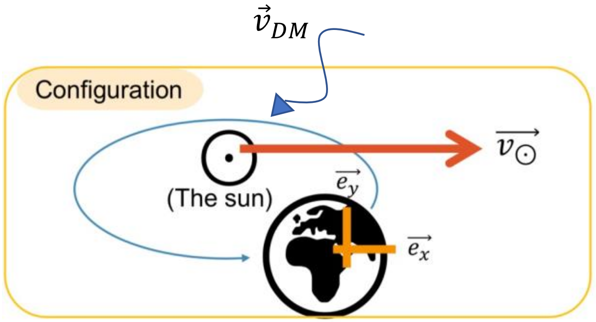


output: $h(t) = \dfrac{\varphi(t,\vec{e}) - \varphi(t,\vec{d})}{4\pi\nu L}$

phase:

$$\varphi(t,\vec{e}) = \varphi_0 + 2\pi\nu(t-2L)$$
$$- 2\pi\nu(\delta L_{\rm time} + \delta L_{\rm space} + \delta L_{\rm charge})$$

→ 3 contributions from vDM!!



- Spatial variation of DM field:

$$\delta L_{\rm space} \simeq \frac{2e\epsilon_D (Q/M)_{\rm in}}{m^2} L \frac{\partial}{\partial t} \sum_{k,j} e_k e_j \nabla_j A_k(t-L,\vec{0})$$

→ dependence on the solar velocity direction

- Light travels finite time: (Morisaki+ 2021)

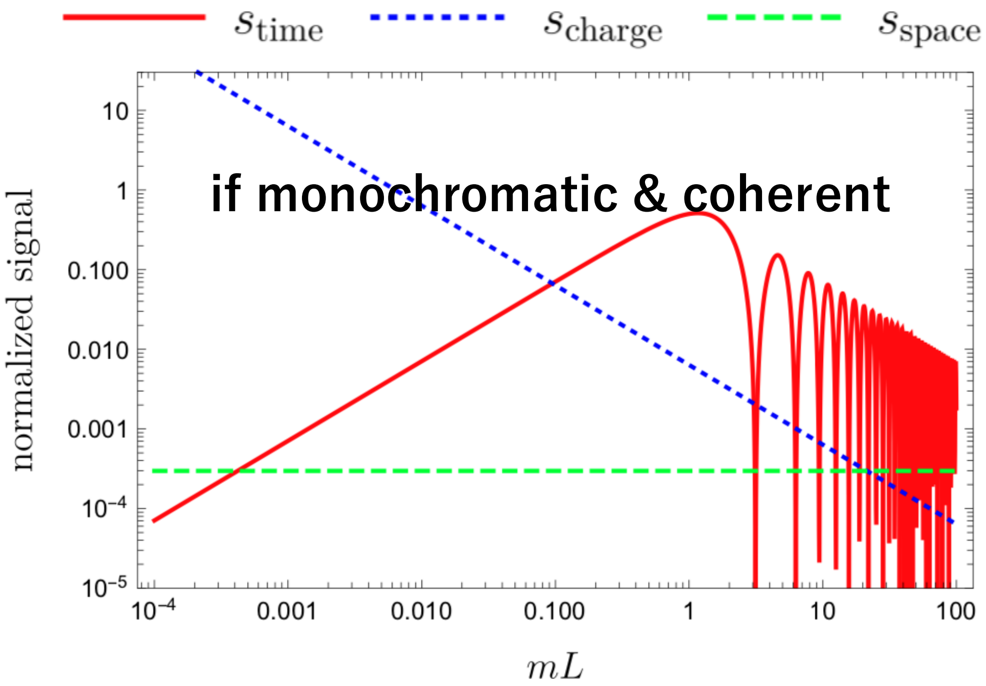
$$\delta L_{\rm time} \simeq \frac{4e\epsilon_D (Q/M)_{\rm in}}{m^2} \sin^2\left(\frac{mL}{2}\right) \frac{\partial}{\partial t} \sum_k e_k A_k(t-L,\vec{0})$$

- Asymmetry in charge-to-mass ratio: ← as KAGRA

$$\delta L_{\rm charge} \simeq \frac{2e\epsilon_D ((Q/M)_e - (Q/M)_{\rm in})}{m^2} \frac{\partial}{\partial t} \sum_k e_k A_k(t-L, L\vec{e})$$

→ dominant for lower frequency

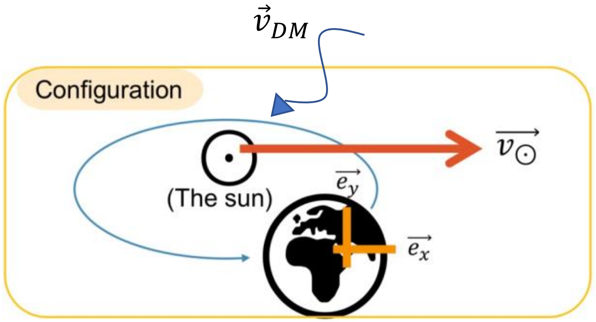
- Signals from vector DM (Nakatsuka+ 2022)



phase:

$$\varphi(t, \vec{e}) = \varphi_0 + 2\pi\nu(t - 2L) - 2\pi\nu(\delta L_{\rm time} + \delta L_{\rm space} + \delta L_{\rm charge})$$

→ 3 contributions from vDM!!



- Spatial variation of DM field:

$$\delta L_{\rm space} \simeq \frac{2e\epsilon_D (Q/M)_{\rm in}}{m^2} L \frac{\partial}{\partial t} \sum_{k,j} e_k e_j \nabla_j A_k(t - L, \vec{0})$$

→ dependence on the solar velocity direction

- Light travels finite time: (Morisaki+ 2021)

$$\delta L_{\rm time} \simeq \frac{4e\epsilon_D (Q/M)_{\rm in}}{m^2} \sin^2\left(\frac{mL}{2}\right) \frac{\partial}{\partial t} \sum_k e_k A_k(t - L, \vec{0})$$

- Asymmetry in charge-to-mass ratio: ← as KAGRA

$$\delta L_{\rm charge} \simeq \frac{2e\epsilon_D ((Q/M)_e - (Q/M)_{\rm in})}{m^2} \frac{\partial}{\partial t} \sum_k e_k A_k(t - L, L\vec{e})$$

→ dominant for lower frequency