Constraining Microscopic Dynamics in Dense Matter with Multimessenger Observations

Omar Benhar
INFN, Sezione di Roma. I-00185 Rome, Italy

GEMMA2
Gravitational-wave, Electromagnetic, and Dark Matter Physics
September 18, 2024

Outline

- ⋆ The paradigm of nuclear theory
- ⋆ Phenomenological nuclear Hamiltonian
  - nucleon-nucleon (NN) potential
  - irreducible three-nucleon (NNN) interactions
  - relativistic corrections
- ⋆ Impact of NNN interactions on neutron star properties
- ⋆ Constraining NNN potential models with astrophysical data
  - results obtained using available data
  - potential of future gravitational wave observatories
- ⋆ Summary & outlook

The Paradigm of Nuclear Theory

- ⋆ To a remarkable extent, atomic nuclei behave as a collection of point-like protons and neutrons, that can be described within the non-relativistic approximation

- ⋆ Ideally, nuclear theory should be based on a dynamical model capable to describe interactions at all scales relevant to nuclear systems, from deuteron to neutron stars

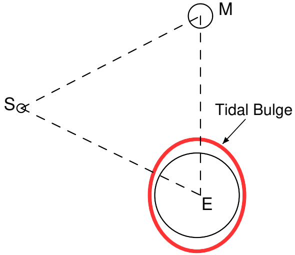
- ⋆ This philosophy has been applied extensively using phenomenological models of the nuclear Hamiltonian, constrained by the observed properties of *exactly solvable* two- and three-nucleon systems—in both bound and scattering states—and the equilibrium density of isospin-symmetric nuclear matter inferred from nuclear data

THE NUCLEAR HAMILTONIAN

- The nuclear Hamiltonian consists of a non relativistic kinetic energy term and the potentials v_{ij} and V_{ijk}, accounting for two- and three-nucleon interactions

$$H = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

- The inclusion of three-body forces is the price to pay to describe the interactions of composite objects neglecting their internal structure
- Note that the archetypal three-body force appears in the context of gravitational Physics



- The NNN potential V_{ijk} is needed to explain the observed properties of the few nucleon systems, ^{3}He and ^{4}He

Phenomenological Models of the NN potential

- ⋆ Phenomenological potentials describing the full NN interaction consist of two components

$$v = v_R + \widetilde{v}_\pi$$

where $\widetilde{v}_\pi$ is Yukawa's one-pion exchange (OPE) potential

- ⋆ The spin-isospin dependence and the non central nature of NN interactions, clearly emerging from observations, can be written in fhe form

$$v_{ij} = \sum_p v^p(r_{ij}) O^p_{ij}$$

where

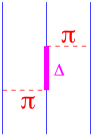
$$O^{p\leq 6}_{ij} = [\mathbb{1}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes [\mathbb{1}, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)]$$

- ⋆ State-of-the art models of v_{ij}, such as the Argonne v_{18} (AV18) [PRC **51**, 38 (1995)], include additional terms, taking into account non-static interactions and small violations of charge symmetry.
- ⋆ Phenomenological NN potentials—designed designed to explain all properties of the NN system, in both bound and scattering states—reduce to the OPE potential at large distances

Phenomenological Models of the NNN potential

- ⋆ The full nuclear Hamiltonian is obtained combining phenomenological NN and NNN potentials
- ⋆ Urbana IX NNN potentiall: Fujita-Miyazawa two-pion exchange + phenomenological repulsive term

$$V_{ijk} = V^{2\pi}_{ijk} + V^{R}_{ijk} \quad , \quad V^{2\pi}_{ijk} = A_{2\pi} \times$$

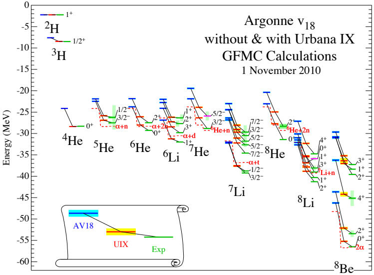


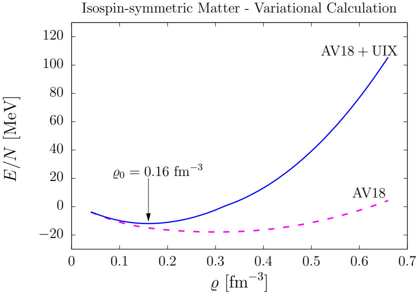
$$V^{R}_{ijk} = U_0 \times \sum_{\text{cycl}} T^2(r_{ij}) T^2(r_{ij}) \quad , \quad T(r) = \left(1 - e^{-cr^2}\right)^2 \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x}$$

- ► The strength of $V^{2\pi}$ $(A_{2\pi})$ is adjusted to reproduce the observed ground state energies of ^{3}He and ^{4}He
- ► the strength of the isoscalar repulsive term V^R (U_0) is adjusted to reproduce the empirical equilibrium density of isospin-symmetric matter (SNM), inferred fom nuclear data

AV18 + UIX HAMILTONIAN

Spectra of light nuclei [PRC **64**, 014001 (2001)] and binding energy of SNM [PRC **58**, 1804 (1998)] obtained from the AV18 + UIX Hamiltonian





NNN interactions, provide a small negative correction to the binding energies of light nuclei. In SNM their contribution is positive, and becomes large at supranuclear densities

RELATIVISTIC CORRECTIONS TO THE NN POTENTIAL

- ⋆ The effects of relativistic corrections to the AV18 + UIX Hamiltonian on the properties of the three- and four-nucleon systems have been analysed by Forest *et al.* [PRC **60**, 014002 (1999)] using Monte Carlo techniques.

- ⋆ The results of these studies show that only the boost correction to the NN potential—needed to take into account the motion of the total momentum of the interacting pair—provides a significant contribution to the energy.

- ⋆ Leading boost correction to v_{ij}, derived by Friar [PRC **12**, 695 (1975)] and Forest *et al.* [PRC **52**, 568 (1995)]

$$v_{ij}(\mathbf{r}) \to v_{ij}(\mathbf{r}) + \delta v_{\mathbf{ij}}(\boldsymbol{P}, \boldsymbol{r}) \ ,$$

$$\delta v_{ij}(\boldsymbol{P}, \boldsymbol{r}) = -\frac{P^2}{8m^2} v^s_{ij}(\boldsymbol{r}) + \frac{(\boldsymbol{P}\cdot\boldsymbol{r})}{8m^2} \boldsymbol{P}\cdot\boldsymbol{\nabla} v^s_{ij}(\boldsymbol{r}) \ ,$$

where $\mathbf{P} = \mathbf{p}_i + \mathbf{p}_j$, and v^s_{ij} denotes the static part of the NN potential.

Boost Corrections to the Energy

- Ground-state energies are obtained combining the boost-corrected NN potential and a modified NNN potential

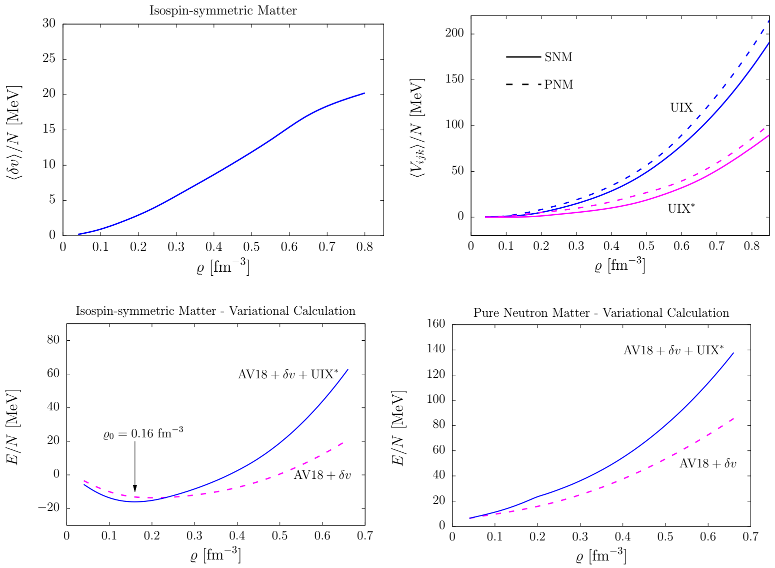
$$H \to H_R = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} [v_{ij} + \delta v_{ij}] + \sum_{k>j>i} V^*_{ijk} \ .$$

- The boost interaction, δv_{ij} provides a positive contribution of $\sim$0.9 and $\sim$1.9 MeV in ^{3}He and ^{4}He, respectively, which entails a corresponding softening of the repulsive NNN potential V^R. The attractive $V^{2\pi}$ is left unchanged.
- The full correction to $\langle H \rangle$ is

$$\delta E_R = \langle \delta v \rangle - \gamma \langle V^R \rangle \quad , \quad \gamma = 0.37 \ .$$

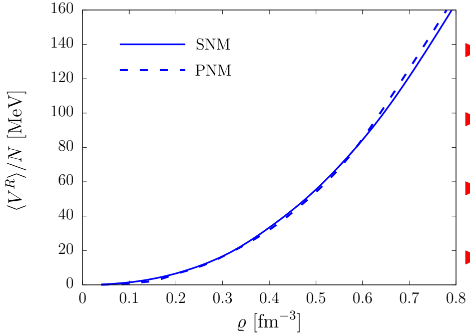
- The above relativistic corrections are included in the energies of pure neutron matter (PNM) and isospin-symmetric matter (SNM) computed by Akmal Pandharipande & Ravenhall [PRC **58**, 1804 (1988)].

Boost Corrections in Nuclear Matter



NNN Repulsion in Nuclear Matter

- Contribution of repulsive NNN interactions to the energy of SNM and PNM, obtained using the AV18 + δv + UIX* Hamiltonian



- Repulsive NNN interactions are isoscalar
- Provide large contributions to nuclear matter energy
- Largely determine the stiffness of the EOS of neutron star matter
- Are totally unconstrained at supranuclear density

- Can astrophysical data constrain the strength of NNN interactions in dense matter?

IMPACT OF V^R ON NEUTRON STAR PROPERTIES

- We have generated a set of EOS using the parametrisation of the EOS of Akmal *et al.* [PRC **58**, 1804 (1998)]

$$\varrho \frac{E}{N} = \epsilon(\varrho, x_p) = \epsilon_K(\varrho, x_p) + \epsilon_I(\varrho, x_p)$$

and replacing

$$\langle V^R \rangle \to \alpha \langle V^R \rangle \Longrightarrow \epsilon_I(\varrho, x_p, \alpha) \to \epsilon_I(\varrho, x_p) + (\alpha - 1) \frac{\varrho}{N} \langle V^R \rangle$$

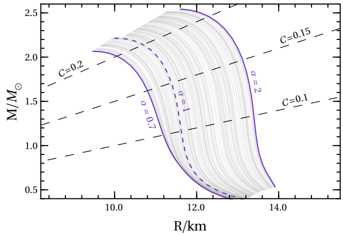
- The case $\alpha = 1$ corresponds to the EOS of Akmal *et al.*, providing the baseline for our analysis. The range of α has been chosen in such a way as to limit to $\sim 15\%$ the displacement of the equilibrium density of SNM from its empirical value
- Using the above parametrisation, we have obtained the EOSs of β-stable matter needed to perform calculations of neutron star properties for any given value of α

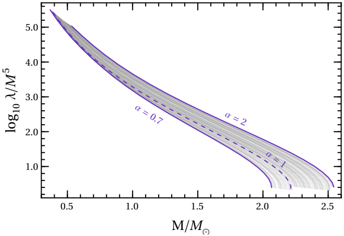
Constraining α through Bayesian Inference

- We have considered a family of neutron star configurations specified by the value of α, employed to obtain the EOS, and the central pressure

$$\{\alpha, p_c\} \rightarrow \{M, R, \Lambda\}$$

- Mass-radius and mass-tidal deformability for $0.7 \leq \alpha \leq 2.0$





Bayesian Inference Framework

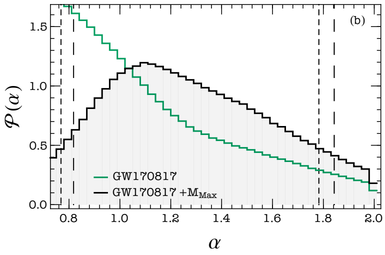
- Given a set of observations O^i of m neutron stars, Bayes' theorem can be used to infer the distribution of $\{\alpha, \vec{p}_c\} = \{\alpha, p_c^1, \ldots, p_c^m\}$
- We have sampled the posterior distribution

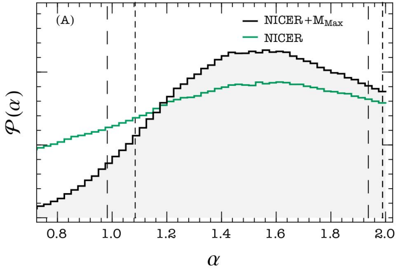
$$\mathcal{P}(\alpha, \vec{p}_c|\vec{O}) \propto \mathcal{P}_0(\alpha, \vec{p}_c) \prod_{i=1}^{m} \mathcal{L}(O^i|\alpha, p_c^i)$$

  - $\mathcal{P}_0(\alpha, \vec{p}_c)$ prior distribution
  - $\mathcal{L}(O^i|\alpha, p_c^i)$ likelihood of the i-th observation

  using the Markov Chain Monte Carlo technique
- The distribution $\mathcal{P}(\alpha)$ has been then obtained marginalising over $\vec{p}_c$
- Data set
  - GW observation of the binary system GW170817, made by the LIGO/Virgo Collaboration (masses and tidal deformabilities)
  - Observation of the millisecond pulsars PSR J0030+0451 made by the NICER satellite (mass and radius)
  - Precise determination of the maximum neutron star mass observed so far, $M = 2.14^{+0.1}_{-0.09}\ M_\odot$ [ApJ Lett. **918**, L29 (2021)]

GW170817 & NICER + M_{max}

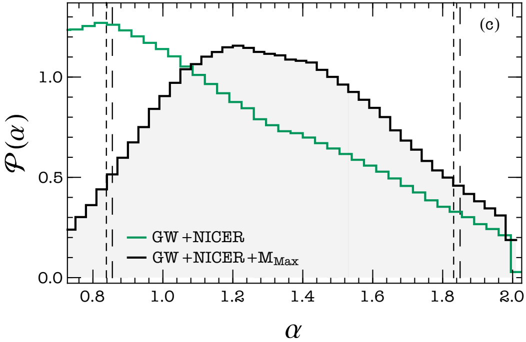




- GW170817 data alone not very constraining
- NICER looks somewhat more informative
- The maximum mass turns out to be the strongest constraint
- The inferred values of α are

$$\alpha_{GW} = 1.25^{+0.48}_{-0.53} \quad , \quad \alpha_{EM} = 1.52^{+0.43}_{-0.47}$$

GW170817 + NICER + M_{max}



- GW170817 dominates if taken alone with NICER
- Full dataset still mainly affected by the maximum mass
- The analysis, yielding

$$\alpha_{GW} = 1.32^{+0.48}_{-0.51}$$

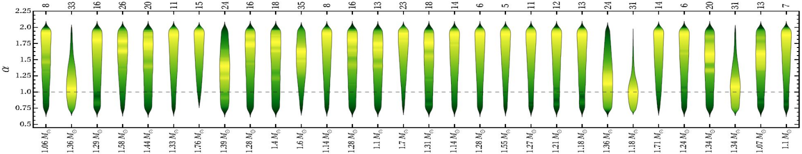
indicates that observations are sensitive to the strength of repulsive NNN interations

Potential of Future GW Observations

- ⋆ The study based on the available data has been extended using a set of *simulated GW observations* that will be feasible in the future using both upgraded and new interferometers
- ⋆ The analysis includes observations of 30 binary neutron star events made by
  - the LIGO Hanford, LIGO Livingston, and Virgo interferometers at design sensitivity
  - The future third-generation interferometer Einstein Telescope
- ⋆ For each observatory, two sets of events have been generated using EOSs corresponding to different α
  - the strength of NNN interactions was set to $\alpha = 1$ and $\alpha = 1.3$
  - the sky location and inclination were assumed to be uniformly distributed over the sky
  - the chirp mass of each event, $\mathcal{M} = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$, was assumed to be known with infinitesimal precision

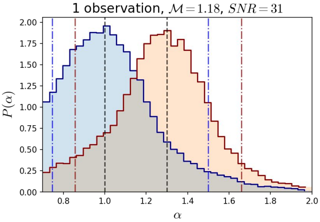
Mock Data: LIGO/Virgo

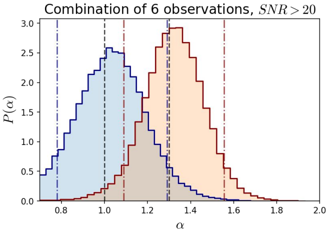
- Posterior densities inferred from simulated GW data, assuming $\alpha = 1$. Top and bottom axes give SNR and chirp mass



Only few, low-mass and high-SNR, events provide a meaningful constraint on α

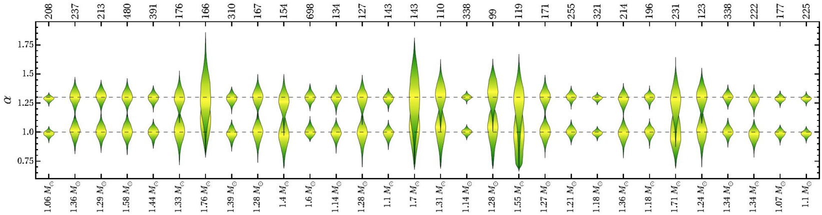
- Probability distributions of α





Mock Data: Einstein Telescope

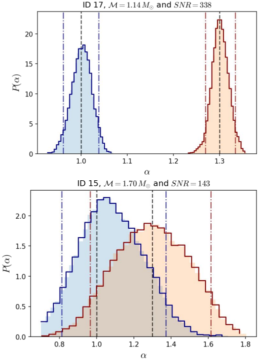
- Posterior densities inferred from simulated GW data, assuming $\alpha = 1$ and $\alpha = 1.3$ Top and bottom axes give SNR and chirp mass

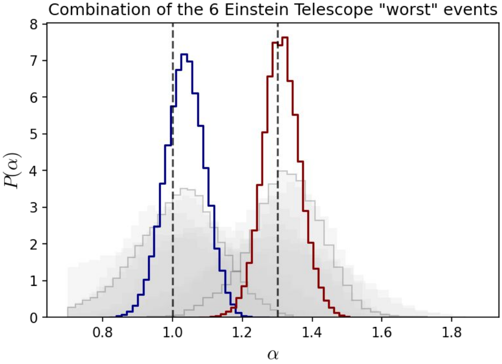


- In most of cases, the large SNRs allow the posteriors corresponding to the injected values of α to be clearly separated
- It appears that even a single observation made by the Einstein Telescope may allow to constrain the strength of NNN interactions

Mock data: Einstein Telescope

- In the few cases in which posterior distributions overlap, stacking of few observations still allows to clearly resolve the peaks corresponding to $\alpha = 1$ and 1.3





SUMMARY & OUTLOOK

- ⋆ The long anticipated observation of GWs and the ensuing developments of multimessenger astrophysics are providing unprecedented access to neutron star properties
- ⋆ The available data are being extensively employed to constrain the EOS of dense nuclear matter. The potential for pushing these studies to a deeper level, in which observations are used to infer information on the underlying model of microscopic dynamics appears to be high
- ⋆ Stronger constraints on repulsive NNN interactions will allow to improve an accurate determination of the nuclear EOS at high densities, and clarify the importance of relativistic boost interactions
- ⋆ The availability of more accurate models of the nuclear Hamiltonian will also allow to perform reliable studies of *dynamical* properties of dense nuclear matter relevant to GW emission from neutron stars, such as, e.g., the *viscosity*

Credits & References

- The analysis discussed in this talk is the result of the work of my collaborators
  - Andrea Sabatucci (INFN Pisa)
  - Andrea Maselli (GSSI)
  - Costantino Pacilio (Milano Bicocca)
  - Alessandro Lovato (ANL)

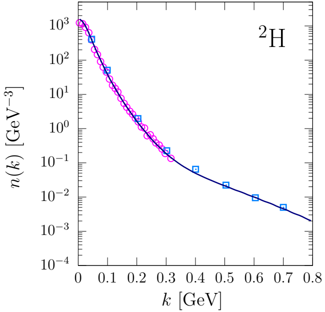
- References

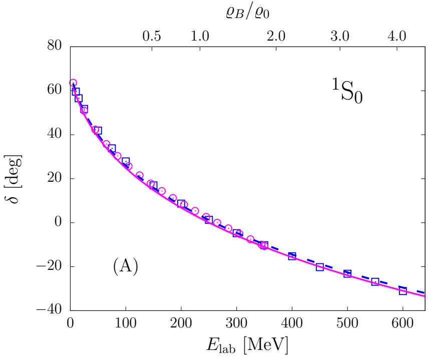
  - A. Sabatucci & OB, Phys. Rev. C **101**, 045807 (2020)
  - A. Maselli, A. Sabatucci, & OB, Phys. Rev. C **103**, 065804 (2021)
  - A. Sabatucci, OB, A. Maselli, & C. Pacilio, Phys. Rev. D **106**, 083010 (2022)
  - A. Sabatucci, OB, & A. Lovato, arXiv:2406.05732 [nucl-th], Phys. Rev. C, in press (2024)

Backup slides

Comparison to Two-Nucleon Data

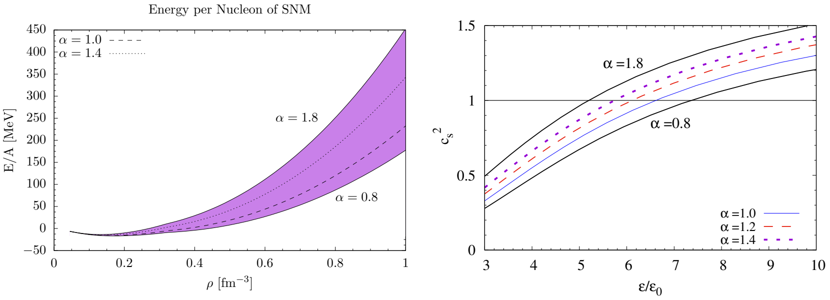
- ⋆ **Left**: momentum distribution in ^{2}H compared to the electron scattering data [M. Bernheim *et al.* NPA **365**, 349 (1981); H. Arenhövel, NPA **384** (1982); C. Ciofi degli Atti *et al.* PRC **36**, 1208 (1987).]
- ⋆ **Right**: nucleon-nucleon scattering phase shifts in the $^1S_0$ channel





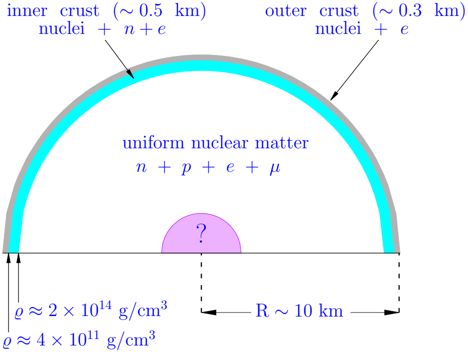
IMPACT OF V^R ON NUCLEAR MATTER PROPERTIES

Density depependence of the binding energy per nucleon of SNM (left) and the squared speed of sound in β-stable matter (right) corresponding to different values of α



One-slide Introduction to Neutron Stars

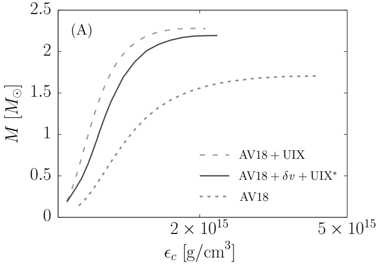
- Overview of NS structure (Recall: $T \sim 10^9$ K $\ll T_F \sim 10^{12}$ K)

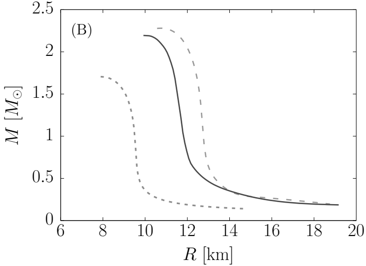


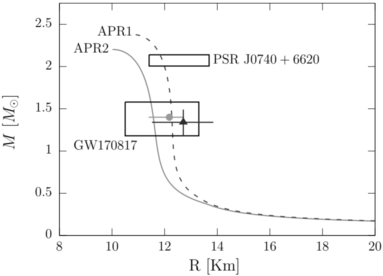
- NS properties such as mass, radius and tidal deformability are largely determined by the equation of state (EOS) of matter in its interior,

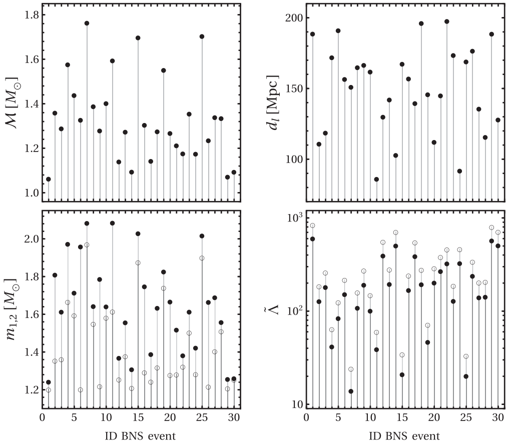
$$\begin{cases} \text{energy density}: \epsilon(\varrho) = (E(\varrho) + Nm)/V \\ \text{pressure}: P(\varrho) = -\partial E(\varrho)/\partial V \end{cases} \Rightarrow P(\epsilon)$$

Impact of Boost Corrections on NS properties









⋆ Component masses, luminosity distance, chirp mass, and tidal parameter for the catalogue of NS binaries

Comparison Between Present and Future Constraints

- ⋆ Neutron star mass-radius relations, obtained from EOSs corresponding to the distributions $\mathcal{P}(\alpha)$ resulting from our analysis
  - Left panel: available observations
  - Right two panel: simulated observations with the Einstein Telescope