

Rapid prediction of kilonova light curves from gravitational wave signals for observed binary neutron star coalescences

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(Credits: University of Warwick/Mark Garlick, wiki)

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BACKGROUND

- What is kilonova
- Science with kilonova
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- Kilonova model
- Normalising flow

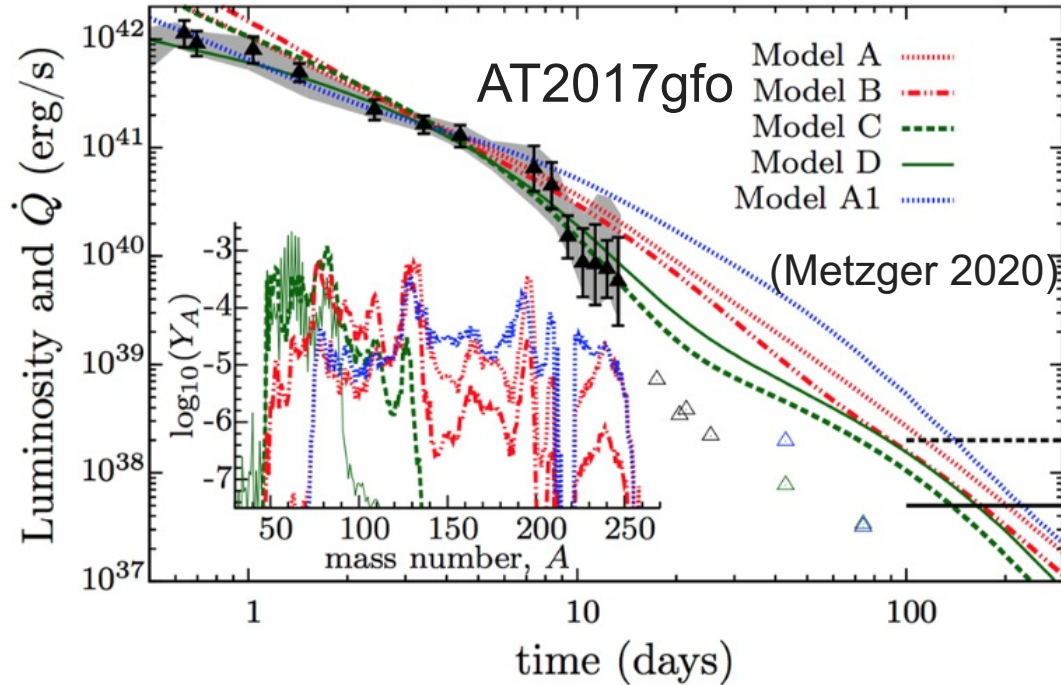


RESULTS

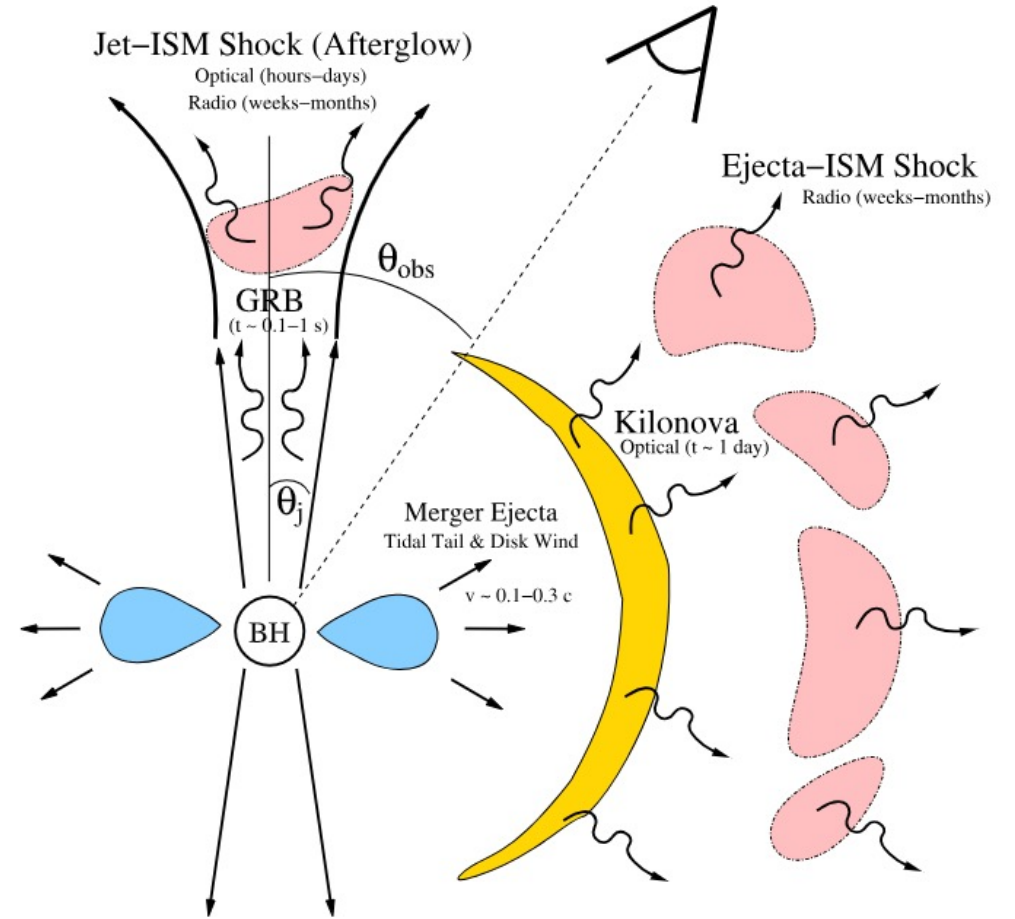


OUTLOOK

Background: Kilonovae

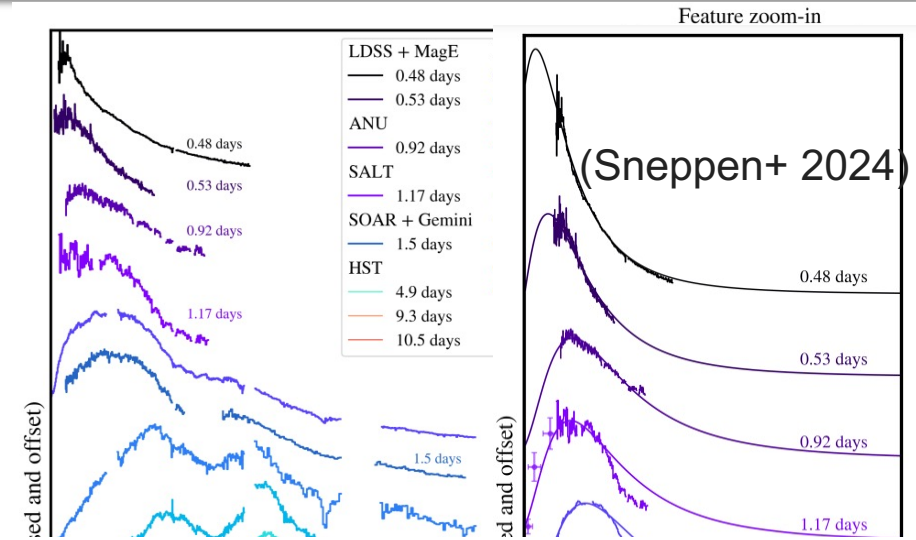
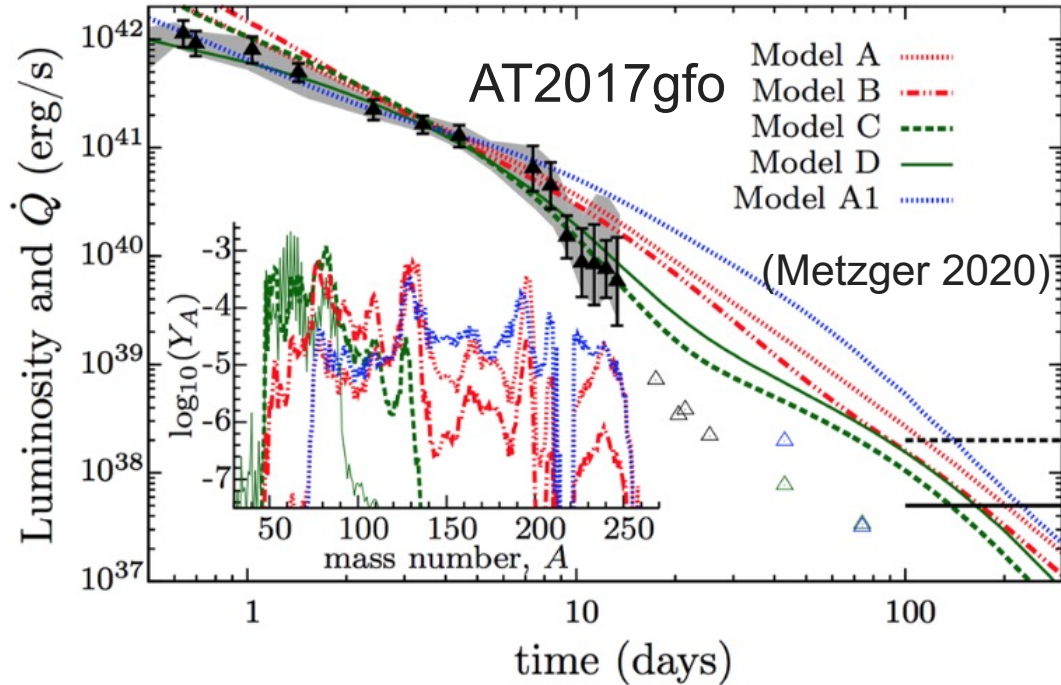


- $L_{\text{peak}} \sim 10^{40-42} \text{ erg s}^{-1}$; optical and near-infrared
- Fast evolving: \sim day to \sim weeks
- Radioactive decay of the r -process elements
- From the merger of compact objects, involving at least one neutron star
- Associated with gravitational waves (GWs) and Gamma-ray bursts (GRBs)



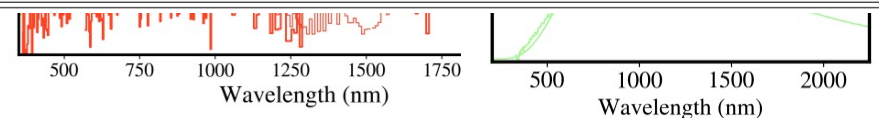
(Metzger 2020)

Background: Kilonovae

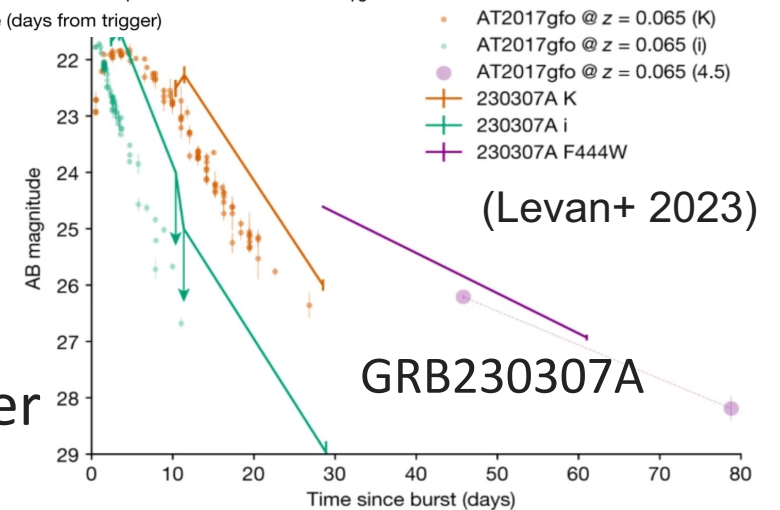
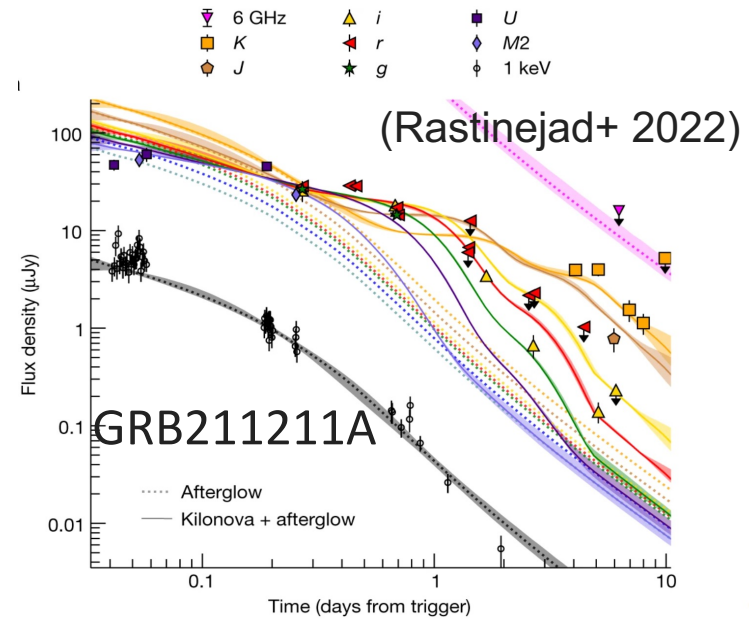
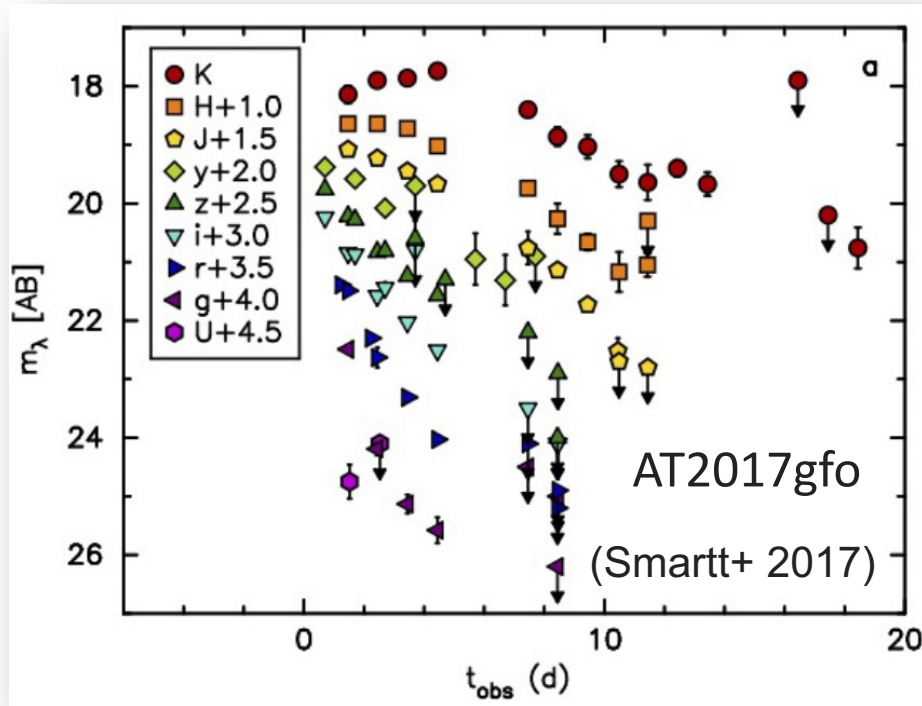


Time (days)	Telescope/Instrument	T_{obs} (K)	Publication
0.48	Magellan/LDSS	$11\,000^{+3400}_{-900}$ [1]	Shappee et al. (2017)
0.53	Magellan/MagE	9300 ± 300 [1]	Shappee et al. (2017)
0.92	ANU/WiFeS	6800 ± 200 [2]	Andreoni et al. (2017)
1.17	SALT/RSS	6400 ± 110	Buckley et al. (2018)
1.43	VLT/X-shooter	5440 ± 60	Pian et al. (2017)
1.45	VLT/X-shooter	5380 ± 60	Sneppen et al. (2023a)
1.47	SOAR/GHTS, Gemini/FLAMINGOS-2	5330 ± 60	Nicholl et al. (2017), Chornock et al. (2017)
2.42	VLT/X-shooter	3940 ± 50	Smartt et al. (2017)
3.41	VLT/X-shooter	3420 ± 40	Pian et al. (2017)
4.40	VLT/X-shooter	3330 ± 40	Smartt et al. (2017)
5.40	VLT/X-shooter	3070 ± 40	Pian et al. (2017)

- $L_{\text{peak}} \sim 10^{40-42} \text{ erg s}^{-1}$; optical and near-infrared
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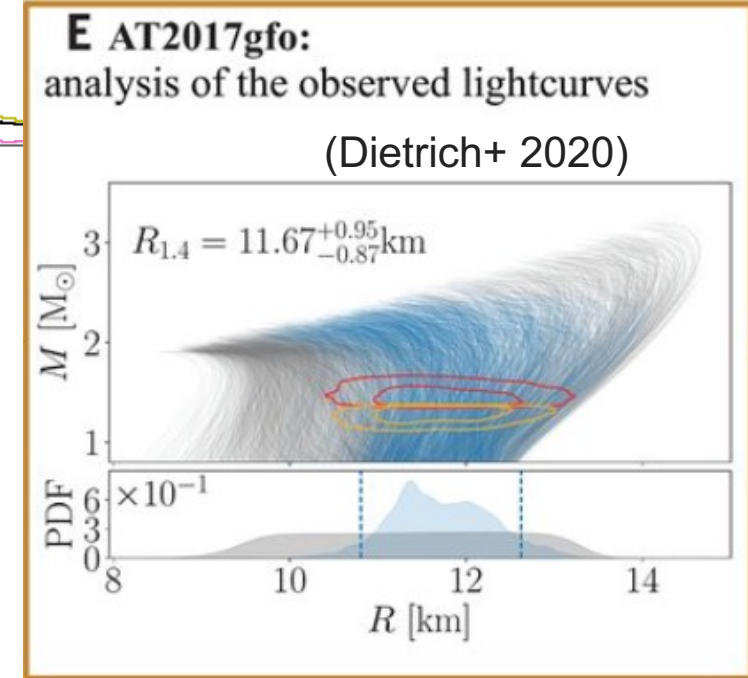
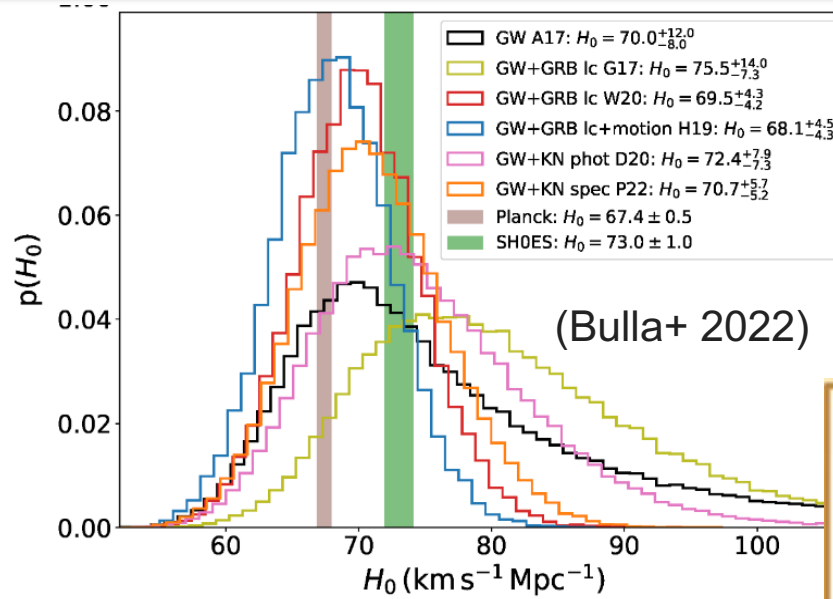
Background: Kilonovae



- AT2017gfo
- A few kilonova candidates associated with GRBs
- More evidence that they originate from the same BNS merger

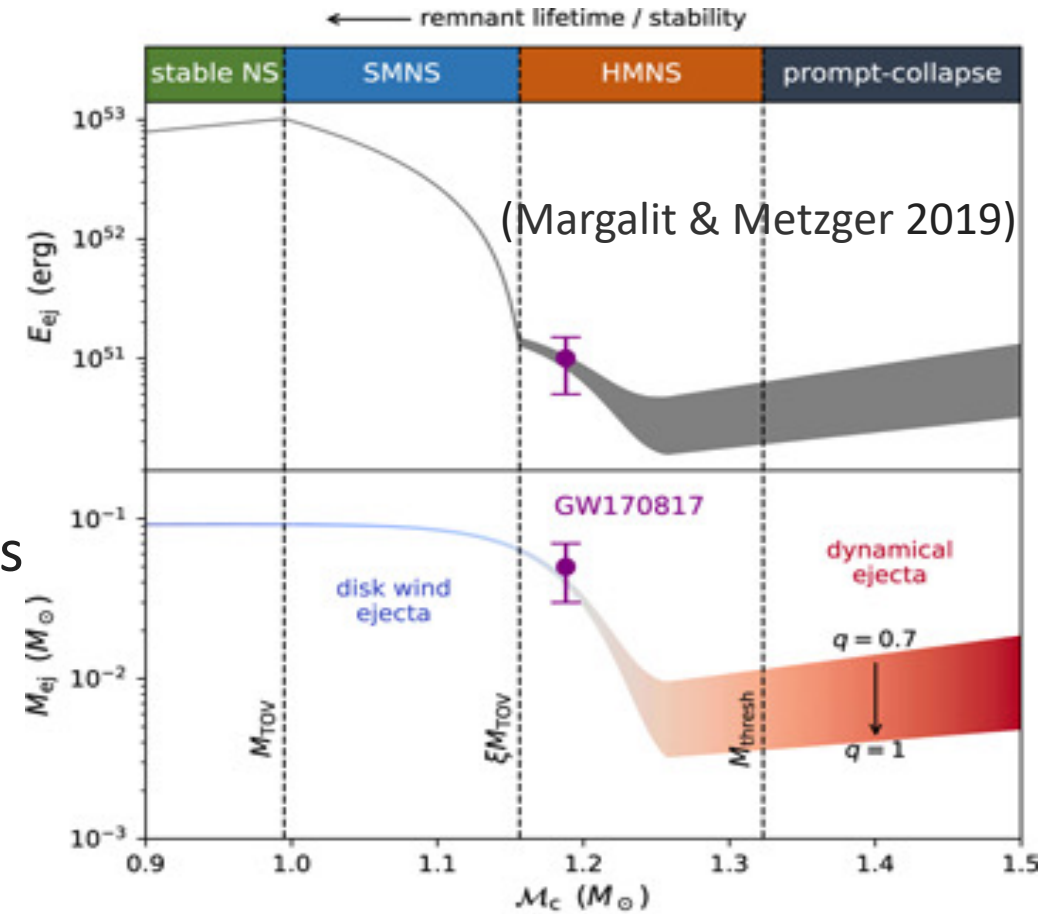
Background: Science with kilonovae

- Cosmology
 - Bright siren, H_0
- Nuclear matter physics
 - Equation of State
- Nucleosynthesis in the Universe
 - Enrichment of heavy elements from r -process
- Compact objects evolution
 - Supermassive/Hypermassive neutron stars
 - Black holes
- ...



Background: Science with kilonovae

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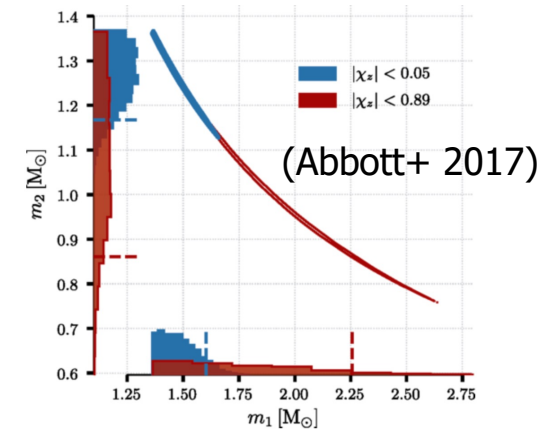
Goal

- Only one confirmed kilonova from the binary neutron star (BNS) merger
- Numerical simulations are usually time-consuming
- **Our goal: Rapidly predict** kilonova light curves from gravitational wave signals, while **considering model uncertainties**
- Inform EM follow-up strategy

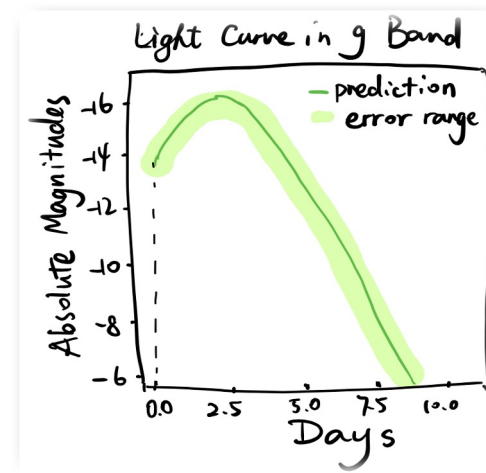
Method

- A **machine learning (ML) tool** to rapidly produce kilonova light curves based on **GW posteriors**: mass (m) and tidal deformability (Λ)
- Factoring in **uncertainties of theoretical kilonova models**.
- Instruct EM follow-up strategy

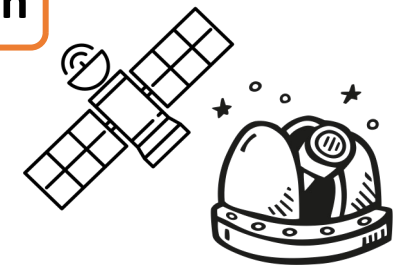
GW Observations
 $[m_1, m_2, \Lambda_1, \Lambda_2]$



Rapid prediction using a
machine learning model



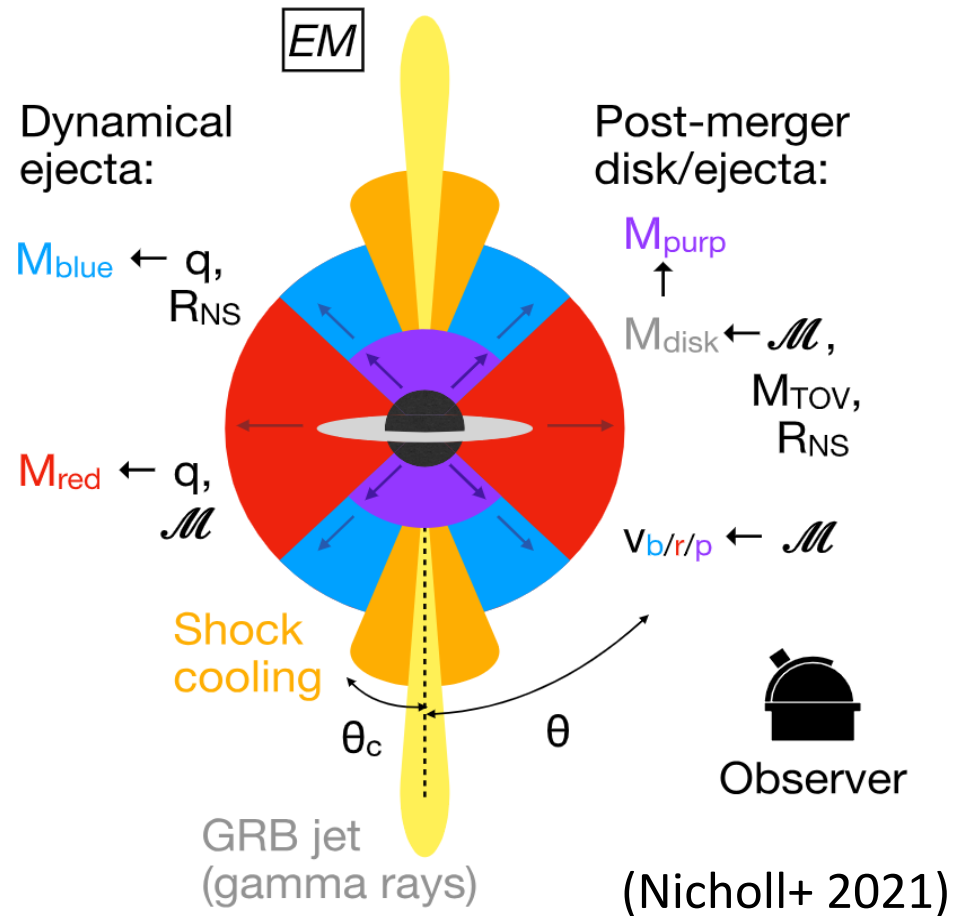
Instruction



EM follow-up

Kilonova model

- BNS model (Nicholl+ 2021)



Intrinsic

Mass and velocity of the dynamical and disk ejecta

$$[m_{\text{ej,red}}, v_{\text{ej,red}}]$$

$$[m_{\text{ej,blue}}, v_{\text{ej,blue}}]$$

Extrinsic

Opacity κ (electron fraction Y_e)

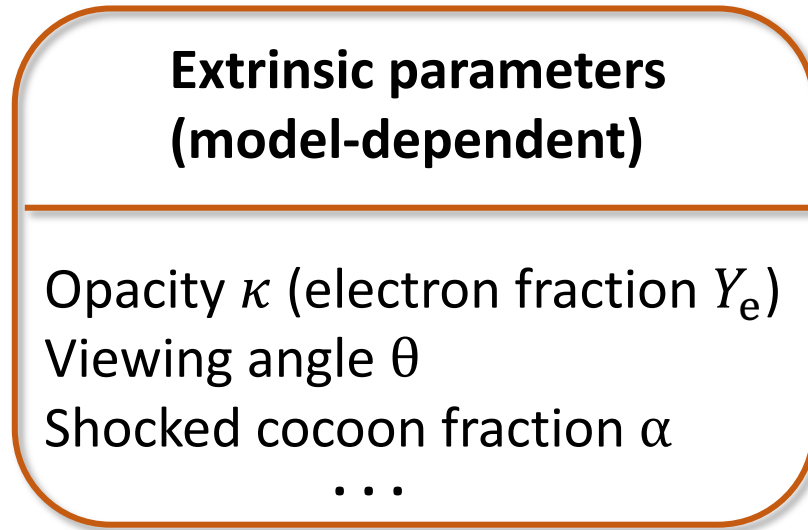
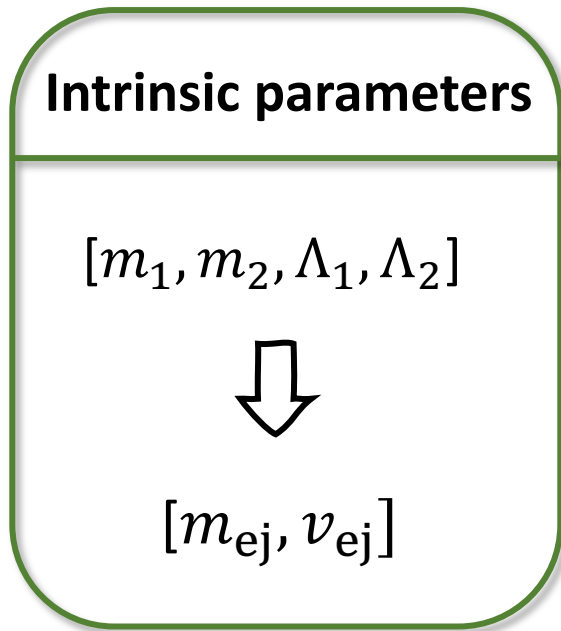
Viewing angle θ

Shocked cocoon fraction α

Kilonova light curves

Generating kilonova light curves from the theoretical KN model

- BNS model (Nicholl+ 2021)

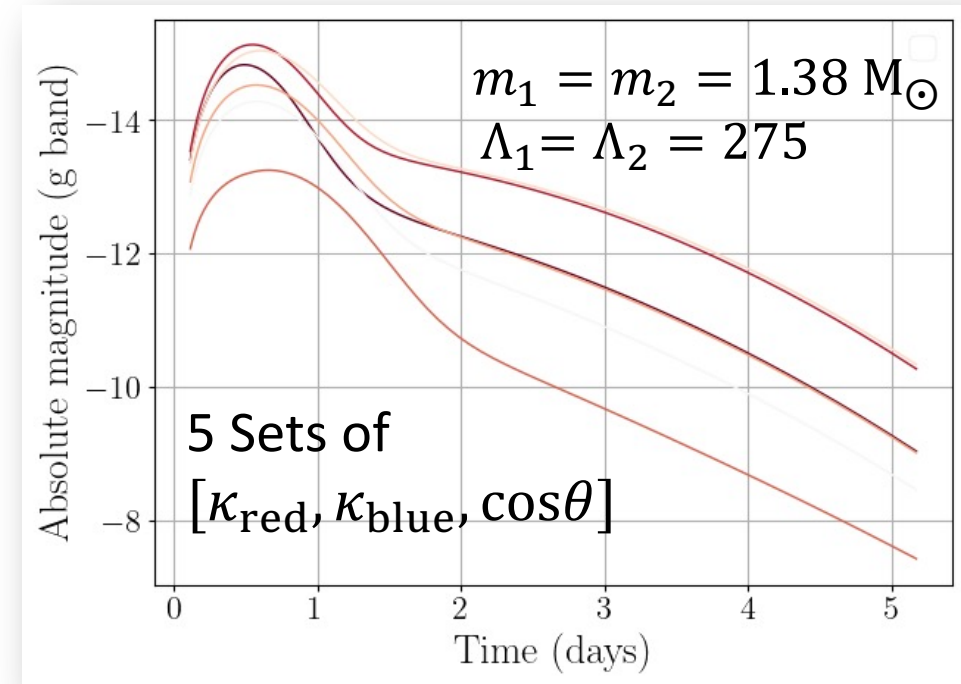


- $\kappa_{\text{red}} \sim U(1.0, 20)$
- $\kappa_{\text{blue}} \sim U(0.01, 1.0)$
- $\cos\theta \sim U(0.0, 1.0)$

Uncertainties!



Kilonova light curves generated using **REDBACK** (Sarin+ 2024)



Embedded into light curves

Method: What is a normalising flow?

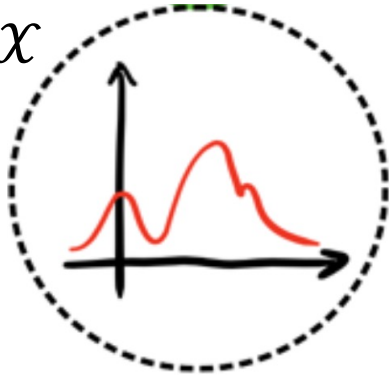
The normalising flow machine learning method

Goal: minimise the 'distance' between the learned distribution and the distribution of input data.

Conditional space \mathcal{Y}



Data space \mathcal{X}



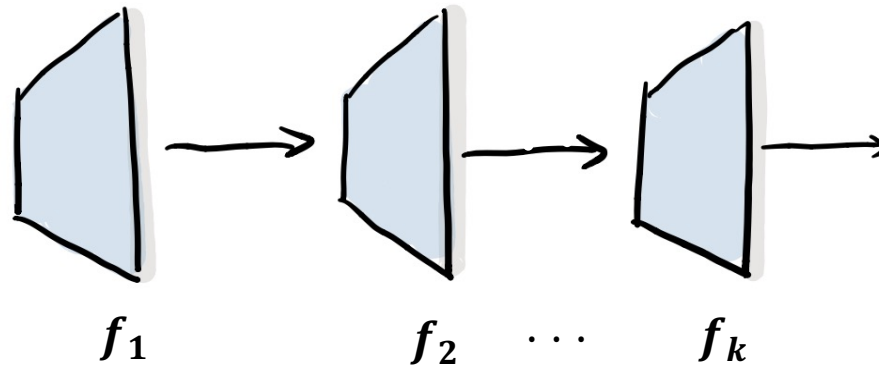
A complex distribution

Training direction

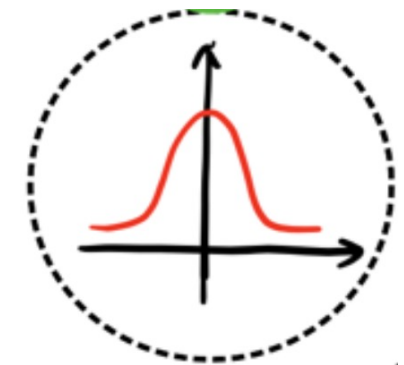


Invertible transforms $f(x; \Phi)$ → Trainable parameters

$$f = f_1 \circ f_2 \circ \dots \circ f_k$$



Latent space \mathcal{Z}



A standard normal distribution

Generative direction



Method: What is a normalising flow?

The normalising flow machine learning method

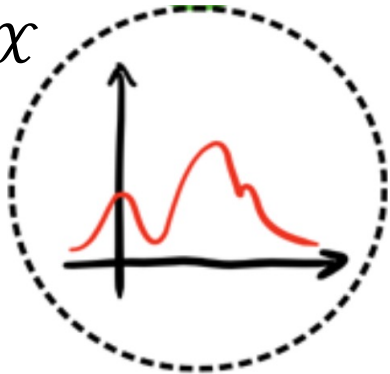
'Distance' : KL divergence

$$\text{Loss} \approx -\frac{1}{K} \sum_{k=1}^K \ln p_z(f(x|y; \phi)) + \ln \left| \det \frac{\partial f(x|y; \phi)}{\partial x} \right|$$

Conditional space \mathcal{Y}

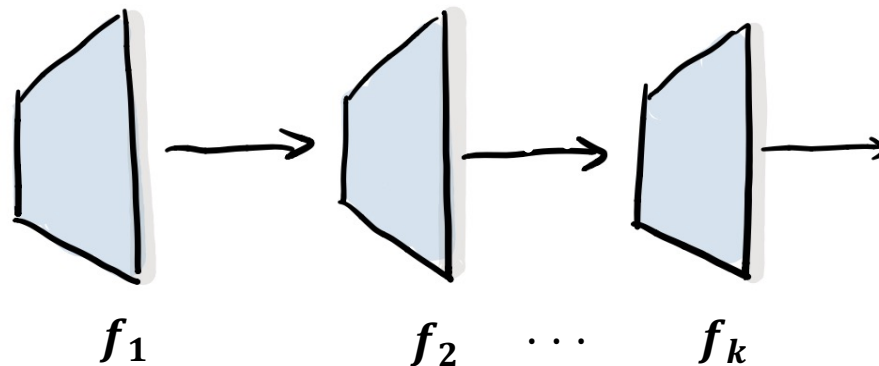


Data space \mathcal{X}

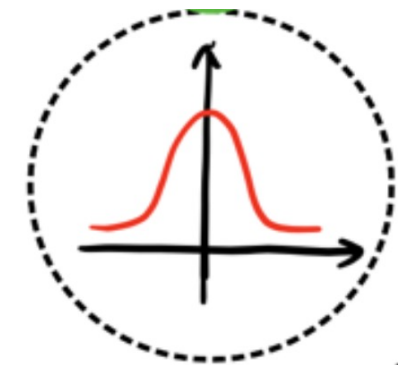


A complex distribution

Training direction



Latent space \mathcal{Z}



A standard normal distribution

Generative direction

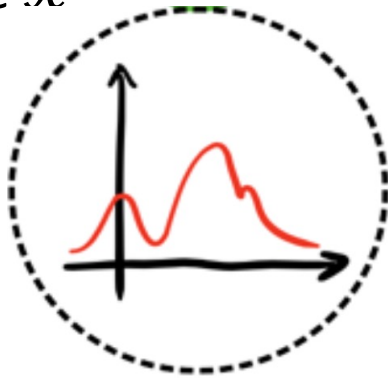


Method: What is a normalising flow?

Training

Conditional space \mathcal{Y}

Data space \mathcal{X}



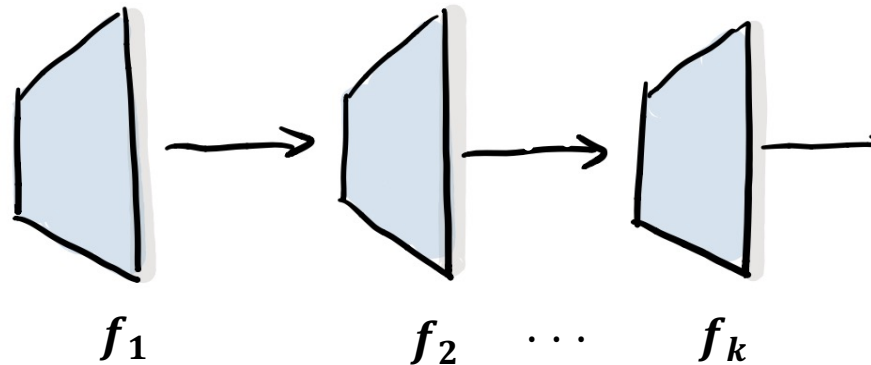
A target distribution

Training direction

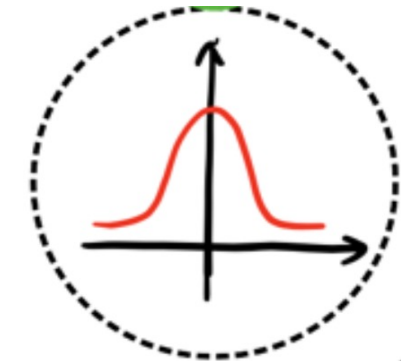
Invertible transforms $f(x; \Phi)$

$$f = f_1 \circ f_2 \circ \dots \circ f_k$$

Trainable parameters



Latent space \mathcal{Z}



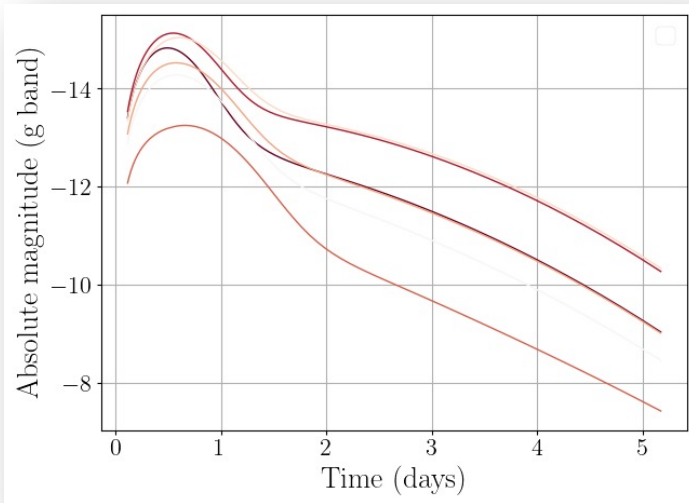
A standard normal distribution

Generative direction

Method: Training process

Training

Training data space \mathcal{X} :
 $\sim 10^6$ light curves, each
contains 30 data points



Conditional space \mathcal{Y} :
 $[m_1, m_2, \Lambda_1, \Lambda_2]$

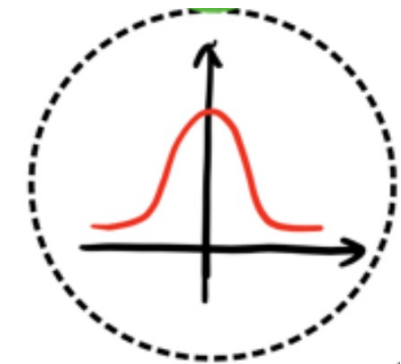
Training direction

Invertible transforms $f(x; \Phi)$

$$f = f_1 \circ f_2 \circ \dots \circ f_k$$

Trainable parameters

Latent space \mathcal{Z}



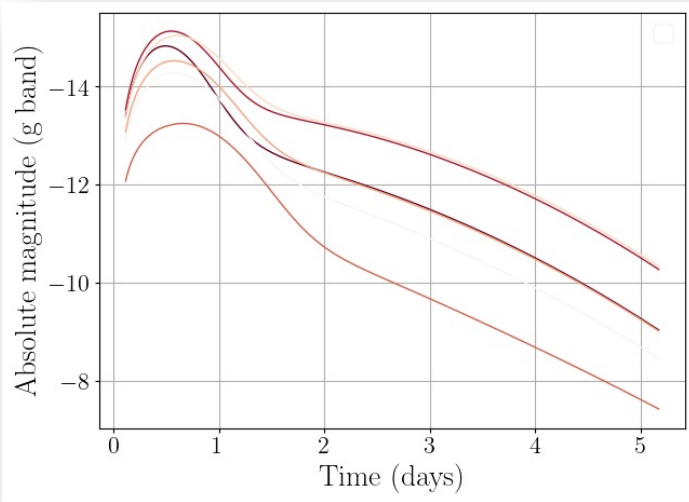
A standard normal
distribution

Generative direction

Method: Training process

Training

Training data space \mathcal{X} :
 $\sim 10^6$ light curves, each
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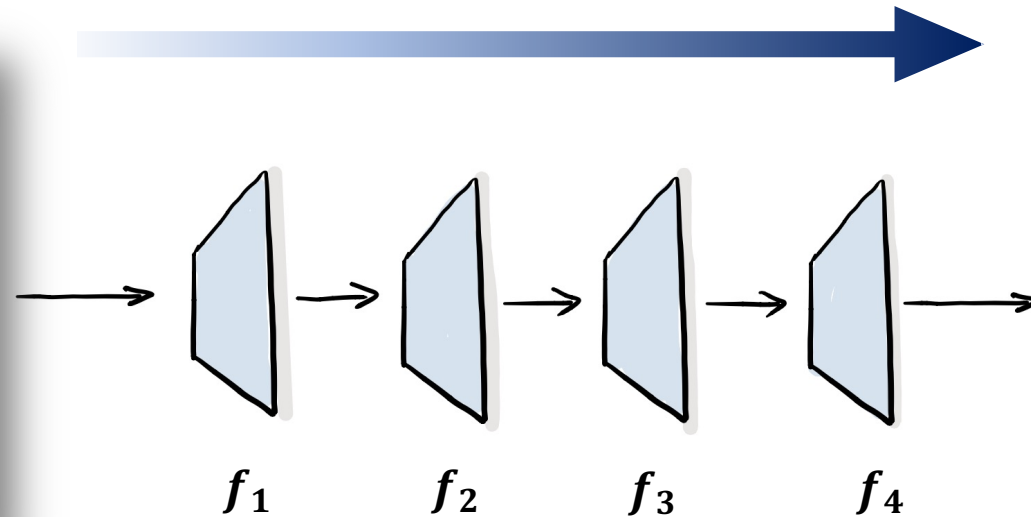
Conditional space \mathcal{Y} :
 $[m_1, m_2, \Lambda_1, \Lambda_2]$

~ 4 hours training on a GPU

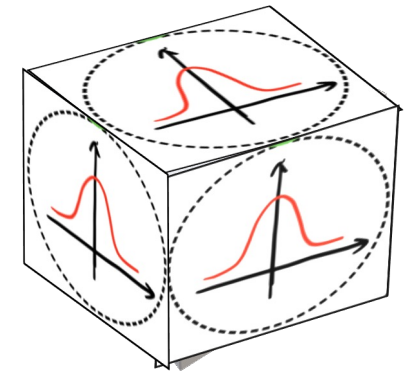
We use GLASFLOW, a normalising flow collection (Williams+, 2024).

<https://github.com/uofgravity/glasflow>

Forward mapping through $f: p(x|y) \rightarrow p(z|y)$



Latent space \mathcal{Z}

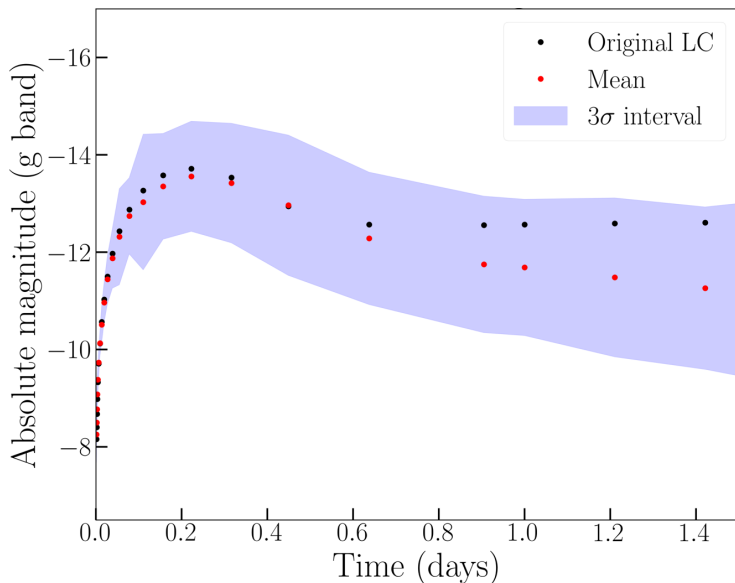


Multi-dimensional
standard normal
distribution

4 transforms: $f = f_1 \circ f_2 \circ f_3 \circ f_4$

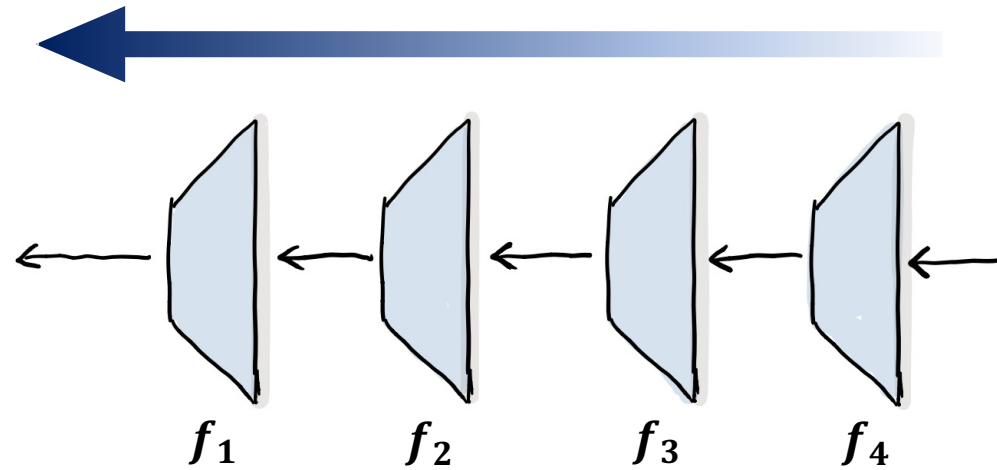
Method: Predicting process

Data space \mathcal{X} :
KN light curves
with uncertainties



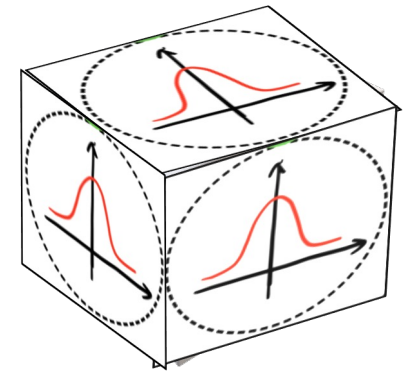
~ seconds

Apply inverse mapping through $f^{-1}: p(z|y) \rightarrow p(x|y)$



$$4 \text{ transforms: } f^{-1} = f_4^{-1} \circ f_3^{-1} \circ f_2^{-1} \circ f_1^{-1}$$

Predicting

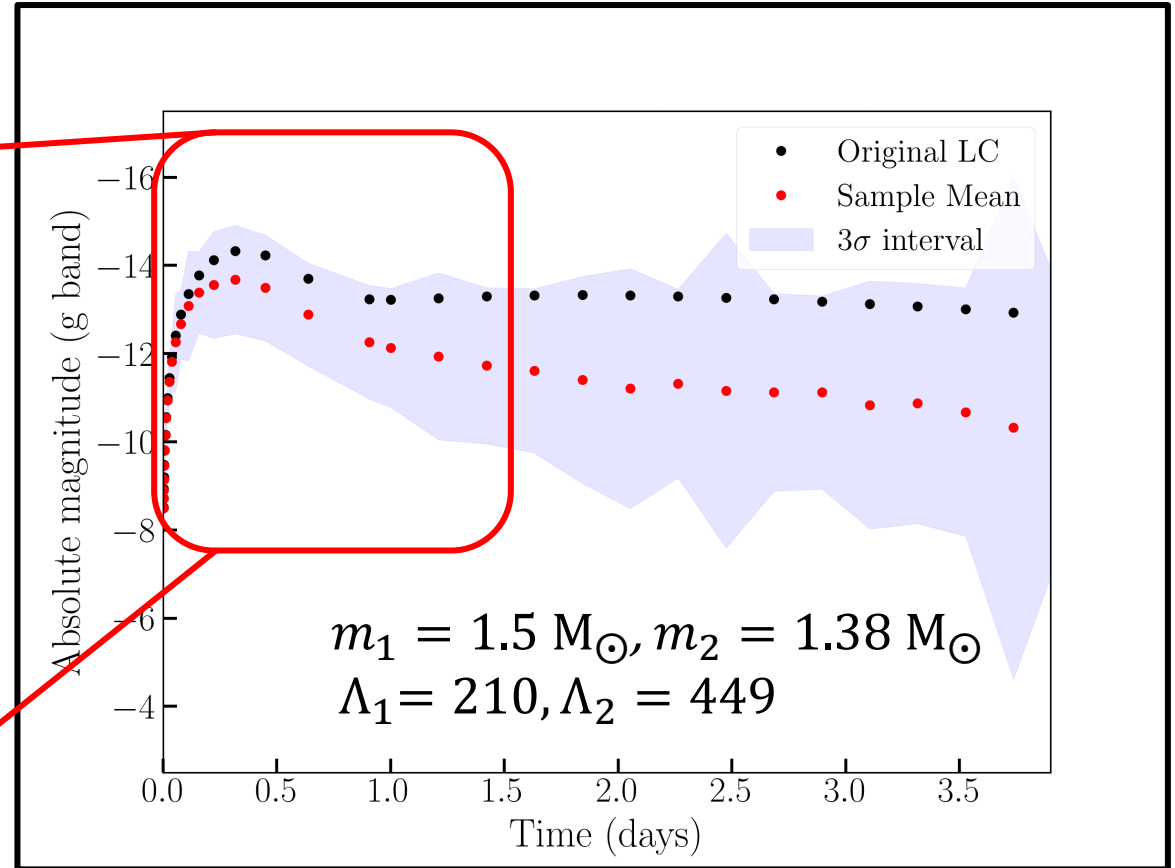
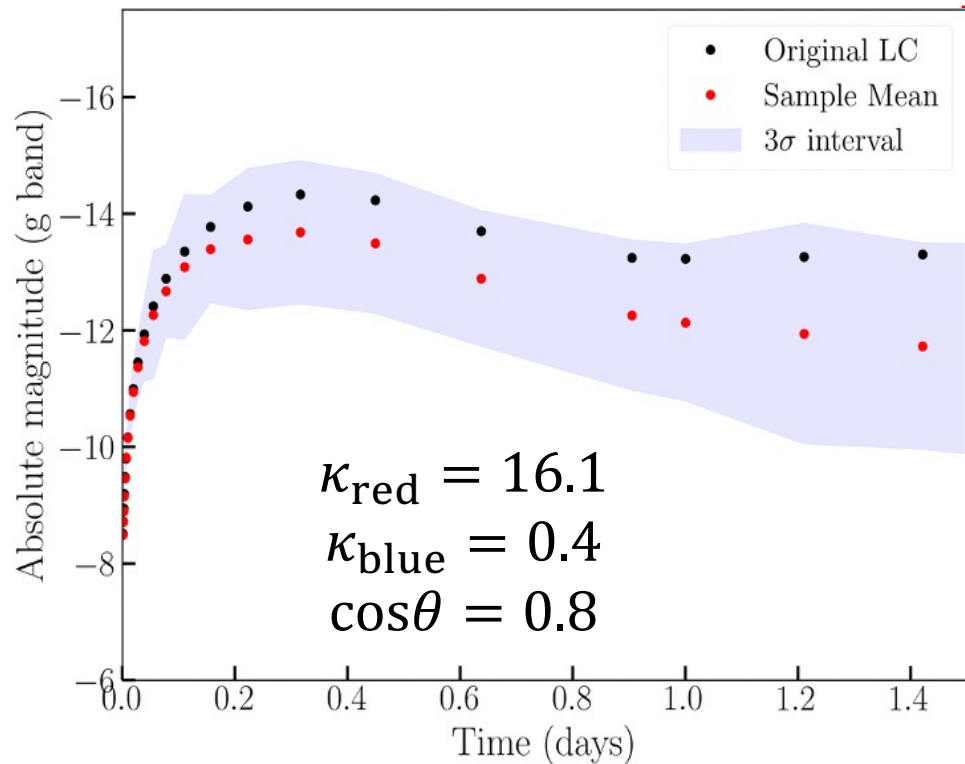


Latent space \mathcal{Z}

Conditional labels \mathcal{Y} :
 $[m_1, m_2, \Lambda_1, \Lambda_2]$

Results

Training LC in g band

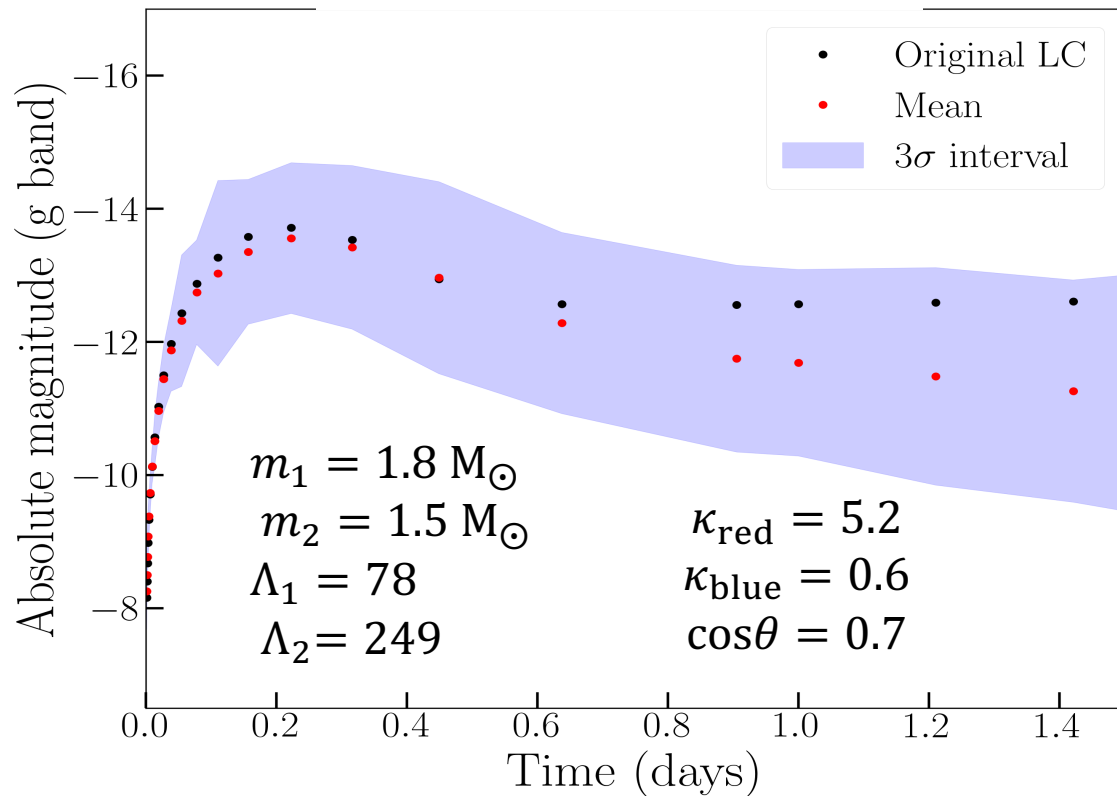


- The predicted light curve for training data falls in the error range.

The mean light curve (red dots) of 1000 predictions (the translucent purple range) in the g band.

Results

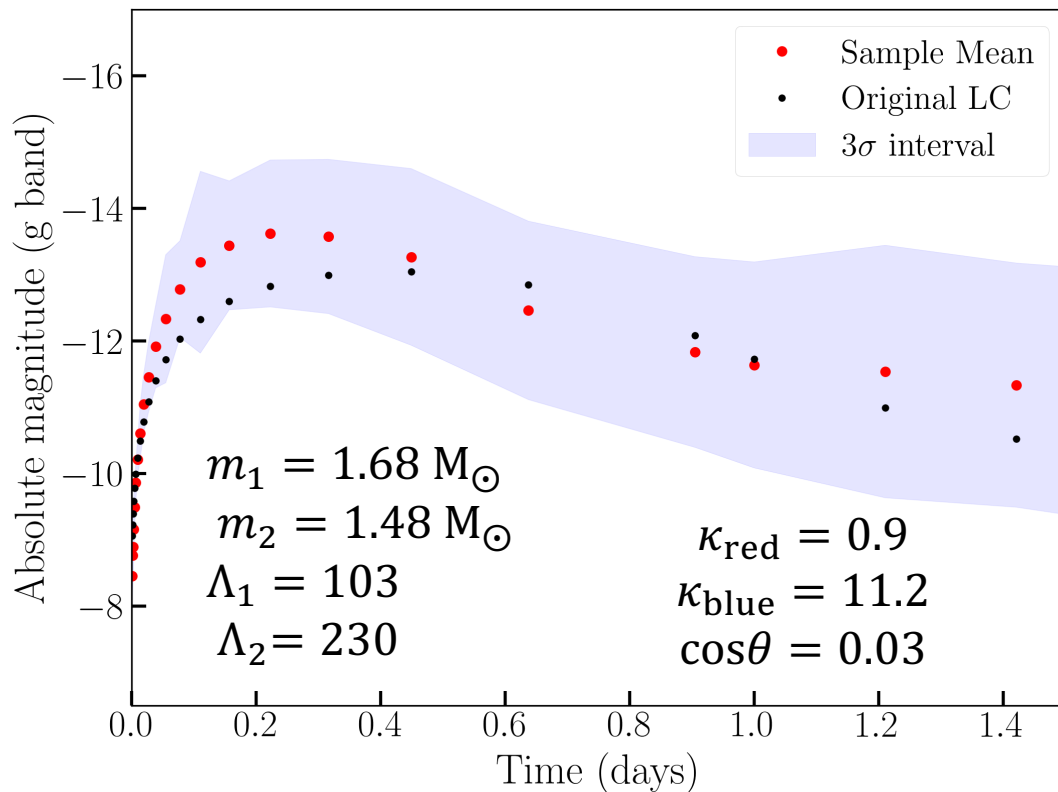
Testing curve in g band



- Predictions on a set of **unseen** intrinsic parameters $[m_1, m_2, \Lambda_1, \Lambda_2]$.
- The uncertainties from extrinsic parameters are embedded in the error range
- The predicted magnitude span is ~ 2 mag during the peak for the BNS model

Results

Prediction in g band



- Predictions on a set of **unseen** intrinsic parameters from a **different EoS**
- Reasonable predictions for intrinsic parameter sets generated within the same prior range as the training data
- Include more EoSs...

Summary and outlook

- ❖ These initial results indicate the promising potential of normalising flow as an alternative method for **rapidly predicting** kilonova light curves
- ❖ Taking into account model **uncertainties**, more flexible
- ❖ Model-independent: can be applied to any kilonova model

Future focuses

- Increase complexities: more numerical models (e.g., Bulla+ 2023), multiple EoSs...
- Test the model on multi-messenger observations of GW170817
- Explore collaborations with other ML tools, such as [DINGO](#) (Maximilian+ 2021), for rapid mass-lambda posteriors
- Long-term goal: possible implementation in LVK public alerts

Backup 1: GW posteriors to ejecta properties

$$q = M_2/M_1$$

$$\mathcal{M} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

$$C_{\text{NS},i} = 0.36 - 0.0355 \ln(\Lambda_i) + 0.000705 \ln(\Lambda_i)^2.$$

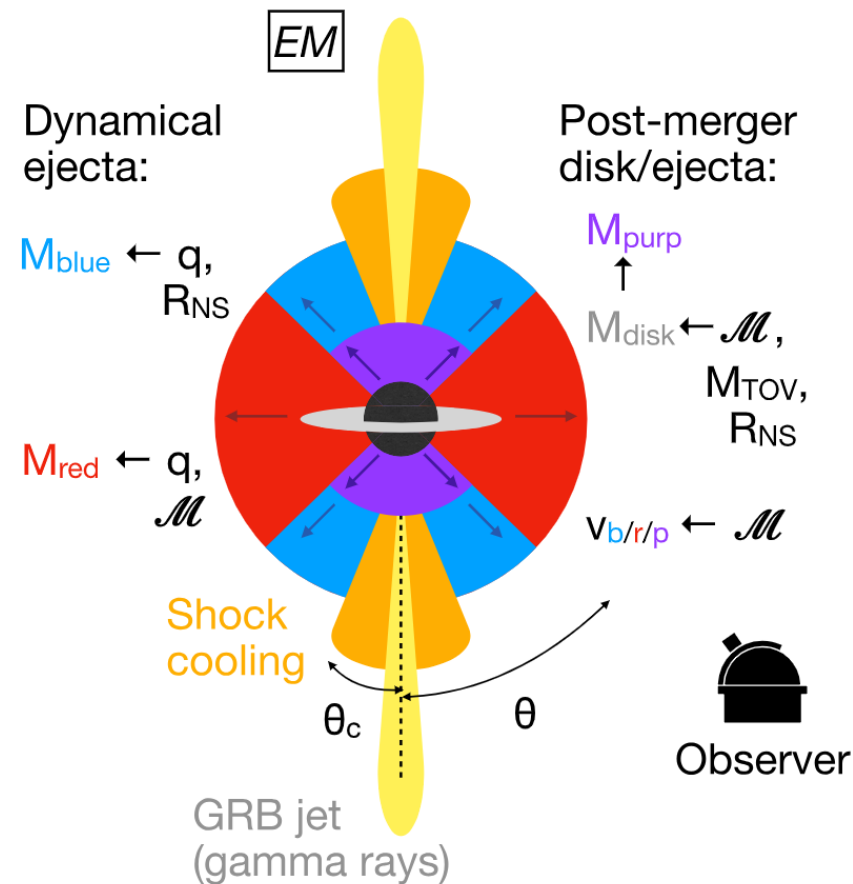
$$\frac{M_{\text{ej}}^{\text{fit}}}{10^{-3} M_{\odot}} = \left[a \left(\frac{M_2}{M_1} \right)^{1/3} \left(\frac{1 - 2C_{\text{NS},1}}{C_{\text{NS},1}} \right) + b \left(\frac{M_2}{M_1} \right)^n + c \left(1 - \frac{M_1}{M_1^*} \right) \right] M_1^* + (1 \leftrightarrow 2) + d$$

$M_i^* = M_i + 0.08 M_i^2.$

$$M_{\text{thr}} = (2.38 - 3.606 \cdot M_{\text{TOV}}/R_{\text{max}}) \times M_{\text{TOV}}$$

$$R_{\text{rem}} = 11.2 \mathcal{M} \cdot (\tilde{\Lambda}/800)^{1/6}$$

$$\log_{10}(m_{\text{disc}}[M_{\text{tot}}/M_{\text{thr}}]) = \max \left(-3, a \left(1 + b \tanh \left[\frac{c - M_{\text{tot}}/M_{\text{thr}}}{d} \right] \right) \right)$$



(Nicholl+ 2021)

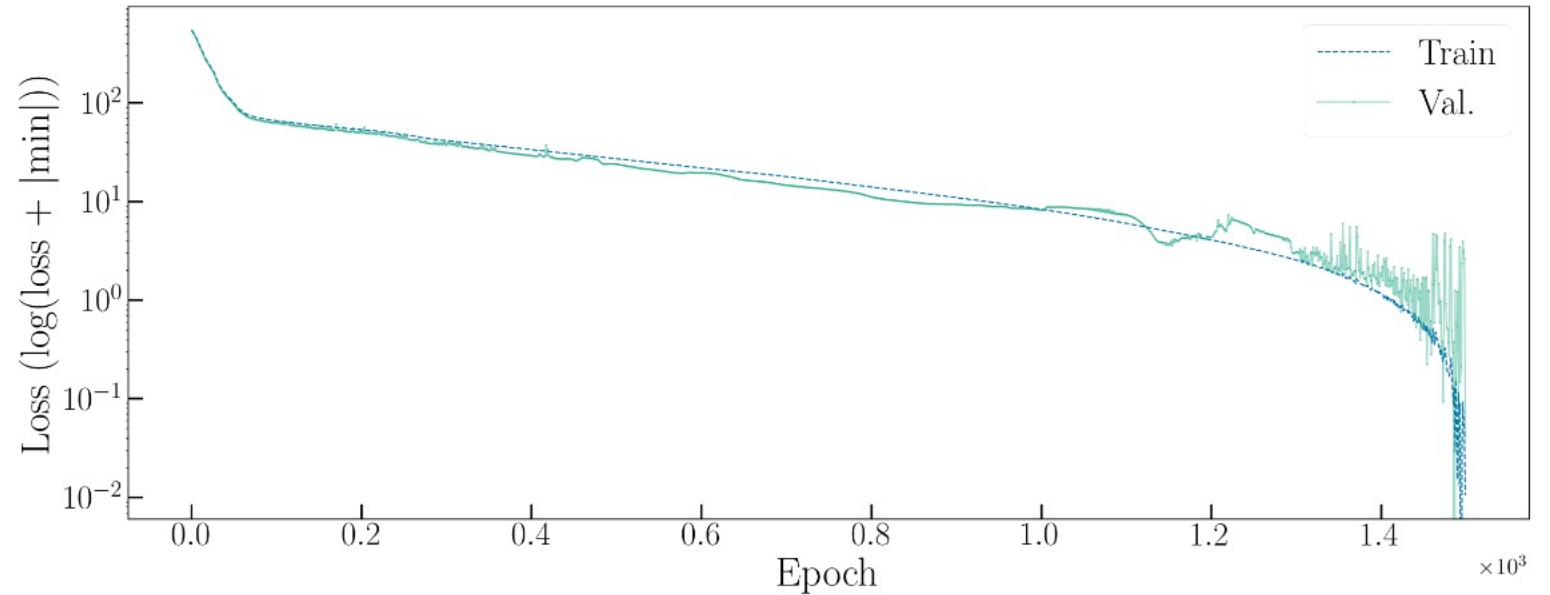
Backup 2: Maths of normalising flow

KL Divergence

- a non-symmetric distance measure between two distributions.

If conditions A in the conditional space \mathcal{Y}

$$p_x^*(x) \longrightarrow p_x^*(x|A)$$



$$\begin{aligned} D_{\text{KL}}[p_x^*(x) \| p_x(x)] &= \int_{-\infty}^{+\infty} p_x^*(x) \ln \left(\frac{p_x^*(x)}{p_x(x)} \right) dx \\ &= -\mathbb{E}_{p_x^*(x)} \left[\ln p_z(f(x; \phi)) + \ln \left| \det \frac{\partial f(x; \phi)}{\partial x} \right| \right] + \text{const.} \end{aligned}$$