Inverse seesaw with flavour and CP symmetries

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NOW2024, Otranto, Italy, 02.-08.09.2024

F.P. Di Meglio, CH, 2407.19734 [hep-ph]

Introduction

- Standard Model (SM) is very successful.
  Nevertheless, several phenomena are not explained within SM.
  - Replication of fermion generations
  - Fermion masses
  - Quark and lepton mixing
  - Baryon asymmetry of the Universe (BAU)
  - Dark Matter (DM)
  - …

Introduction

- Standard Model (SM) is very successful.
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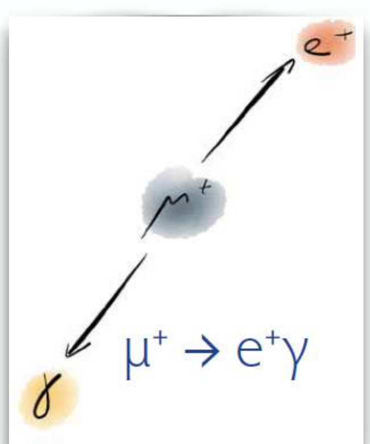
  - Fermion masses
  - Quark and lepton mixing
  - Baryon asymmetry of the Universe (BAU)
  - Dark Matter (DM)
  - …

- Additionally, beyond SM (BSM) theories can have a rich phenomenology.
  - Processes forbidden/highly suppressed in SM can be in reach
  - Flavour and CP violation needs to be kept under control
  - Possible correlations among different signals



Flavour and CP symmetries

- Let us be inspired by the success of gauge symmetries.
- Assume a **new symmetry, acting on flavour space**, e.g.

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \to \begin{pmatrix} q_2 \\ q_3 \\ q_1 \end{pmatrix}$$

with q_i being the ith quark generation.
This constrains the couplings in the flavour sector, i.e. the quark masses and mixing.

Properties of this new symmetry G_f ?

Flavour and CP symmetries

Properties of this new symmetry G_f ?

G_f could be …

- … abelian or **non-abelian** (**three generations**)
- … continuous or **discrete** (**preferred directions**)
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to **non-trivial subgroups** (**predictive**)
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

Flavour and CP symmetries

There are many options …

- Dihedral symmetries D_n as well as D'_n
- Symmetric and alternating groups, S_n and A_n
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
- Adding CP symmetries
- **Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP**
- …

Reviews

Ishimori et al. ('10), King/Luhn ('13), Feruglio/Romanino ('19); Grimus/Ludl ('11)

Inverse seesaw mechanism

- Consider a scenario of **(3,3) ISS**,
  i.e. 3 generations of LH doublets,
  3 generations of N_i and S_j, all of them gauge singlets

$$-(y_D)_{\alpha i}\,\overline{L}^c_\alpha\,H\,N^c_i - (M_{NS})_{ij}\,\overline{N}_i\,S_j - \frac{1}{2}\,(\mu_S)_{kl}\,\overline{S}^c_k\,S_l + \text{h.c.}$$

Mass matrix of neutral states

$$\mathcal{M}_{\text{Maj}} = \begin{pmatrix} \mathbb{0} & m_D & \mathbb{0} \\ m_D^T & \mathbb{0} & M_{NS} \\ \mathbb{0} & M_{NS}^T & \mu_S \end{pmatrix} \quad \text{with } m_D = y_D\,\frac{v}{\sqrt{2}}$$

- Light neutrino masses

$|\mu_S| \ll |m_D| \ll |M_{NS}|$:

$$m_\nu = m_D\left(M_{NS}^{-1}\right)^T \mu_S\, M_{NS}^{-1}\, m_D^T$$

Mohapatra / Valle ('86),
Mohapatra (´86),
Bernabeu et al. (´87),
Gonzalez-Garcia / Valle (´89)

Inverse seesaw mechanism

- Heavy sterile states form pseudo-Dirac pairs

$|\mu_S| \ll |m_D| \ll |M_{NS}|$,

$$V^T \begin{pmatrix} \mathbb{0} & M_{NS} \\ M_{NS}^T & \mu_S \end{pmatrix} V \approx \text{diag}\,(m_4, ..., m_9)$$

- Mixing matrix

$$\mathcal{U}^T \mathcal{M}_{\text{Maj}} \mathcal{U} = \mathcal{M}_{\text{Maj}}^{\text{diag}}$$

$$\mathcal{U} = \begin{pmatrix} \tilde{U}_\nu & S \\ T & V \end{pmatrix} \text{ and } \mathcal{M}_{\text{Maj}}^{\text{diag}} = \text{diag}\,(m_1, m_2, m_3, m_4, \ldots, m_9)$$

- For lepton mixing matrix we have

$$\widetilde{U}_{\text{PMNS}} = \left(\mathbb{1} - \eta\right) U_0 \quad \text{with} \quad \eta = \frac{1}{2} m_D^\star \left(M_{NS}^{-1}\right)^\dagger M_{NS}^{-1} m_D^T$$

If $U_\ell = \mathbb{1}$ then $\tilde{U}_{\text{PMNS}} = U_\ell^\dagger \tilde{U}_\nu = \tilde{U}_\nu$

Scenario

[F.P. Di Meglio, CH ('24)]

- We take

$$\alpha_R \sim 1 \qquad L_\alpha \sim 3\ , N_i \sim 3'\ , S_j \sim 3'$$

[detail: use additional Z_3 to distinguish e, μ, τ]

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$$L_\alpha \sim 3 \;, N_i \sim 3' , S_j \sim 3'$$

Charged lepton mass matrix

residual symmetry G_e

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

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$L_\alpha \sim 3$, $N_i \sim 3'$, $S_j \sim 3'$

Mass matrix of neutral states

residual symmetry G_ν

$$-(y_D)_{\alpha i}\, \overline{L}^c_\alpha\, H\, N^c_i - (M_{NS})_{ij}\, \overline{N}_i\, S_j - \frac{1}{2}\, (\mu_S)_{kl}\, \overline{S}^c_k\, S_l + \text{h.c.}$$

No symmetry breaking

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \mu_S = \mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_0 > 0 \quad \text{and} \quad \mu_0 > 0$$

Symmetry breaking

$$y_D = \Omega(\mathbf{3})^\star\, R_{ij}(\theta_L)\, \text{diag}(y_1, y_2, y_3)\, P^{ij}_{kl}\, R_{kl}(-\theta_R)\, \Omega(\mathbf{3}')^T$$

Scenario

[F.P. Di Meglio, CH ('24)]

- Parameters of the scenario (all from sector of neutral states)
  2 scales M_0 and μ_0
  5 real parameters: 3 couplings y_1, y_2, y_3 and 2 angles θ_L and θ_R
- Their role is
  M_0 mass of the 3 pseudo-Dirac pairs $150\,\mathrm{GeV} \leq M_0 \leq 10\,\mathrm{TeV}$
  μ_0 lepton number breaking parameter $100\,\mathrm{eV} \leq \mu_0 \leq 100\,\mathrm{keV}$
  y_f adjust light neutrino masses m_1, m_2, m_3 $4 \cdot 10^{-5} \lesssim y_f \lesssim 1.2$
  θ_L fitted to accommodate lepton mixing angles best
  θ_R free parameter $0 \leq \theta_R \leq 2\,\pi$
- Study of lepton mixing and charged lepton flavour violation for **Case 1)** **Case 2)** **Case 3 a)** **Case 3 b.1)**, analytically and numerically.

Results

[F.P. Di Meglio, CH ('24)]

Some analytical results

- Charged lepton flavour violating observables $\text{BR}(\ell_\beta \to \ell_\alpha \gamma)$, $\text{BR}(\ell_\beta \to 3\,\ell_\alpha)$ and $\text{CR}(\mu - e, \text{N})$ are mostly proportional to $|\eta_{\alpha\beta}|^2$
- Tri-lepton decays and mu-e conversion in nuclei are dominated by Z penguin, especially for larger $x_0 = \left(\frac{M_0}{M_W}\right)^2$
- Strong suppression of $\text{CR}(\mu - e, \text{N})$ for certain value of x_0 depending on the nucleus N, e.g. for aluminium

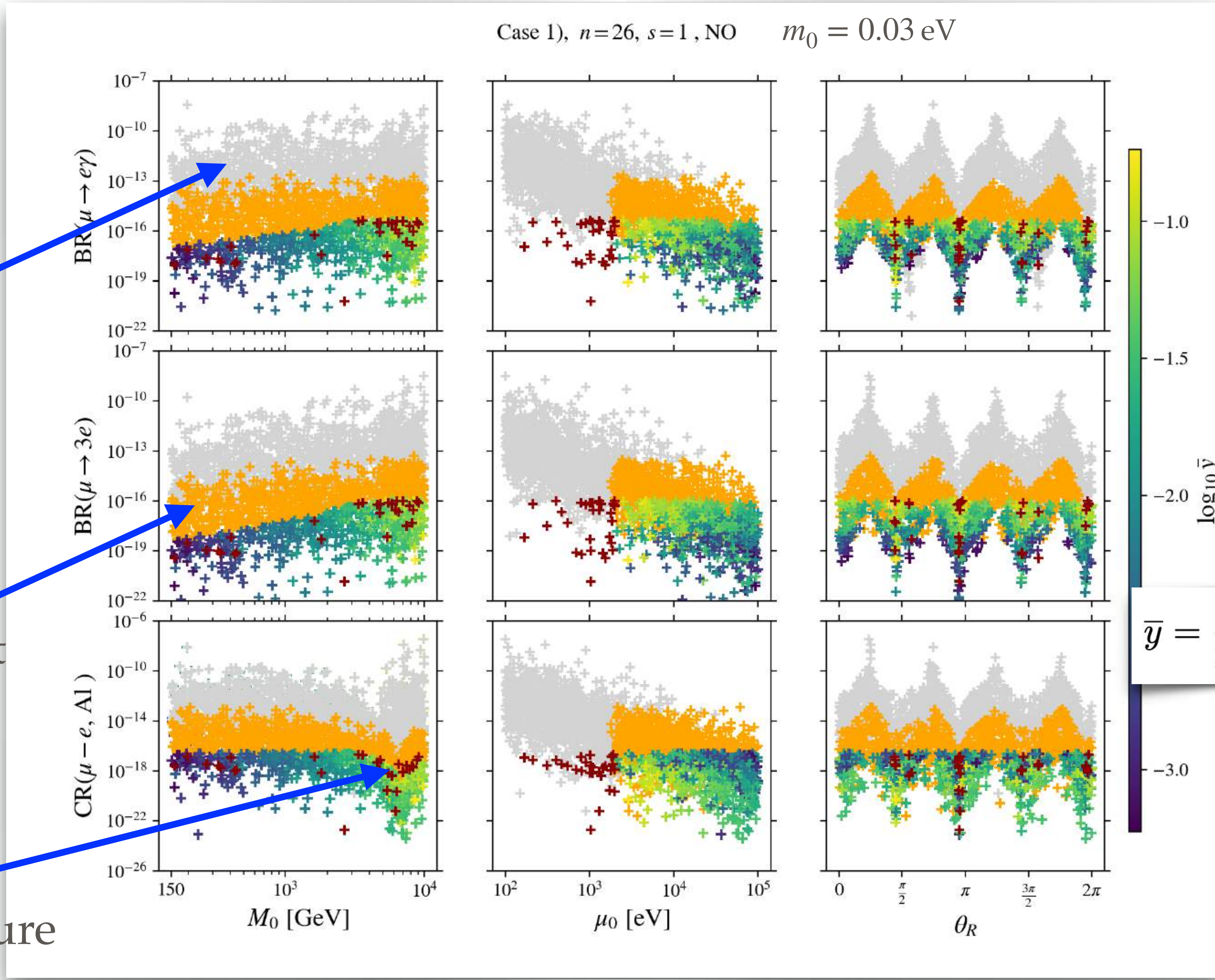
$$x_0 \approx 6470 \quad \text{corresponding to} \quad M_0 \approx 6.5\,\text{TeV}$$

see e.g. Alonso et al. ('12), Ilakovac/Pilaftsis ('95), CH et al. ('21), Abada et al. ('12), Hirsch/Staub/Vicente ('12), …

Results

Case 1)

[F.P. Di Meglio, CH ('24)]



cLFV future and η not OK

cLFV future not OK, but η OK

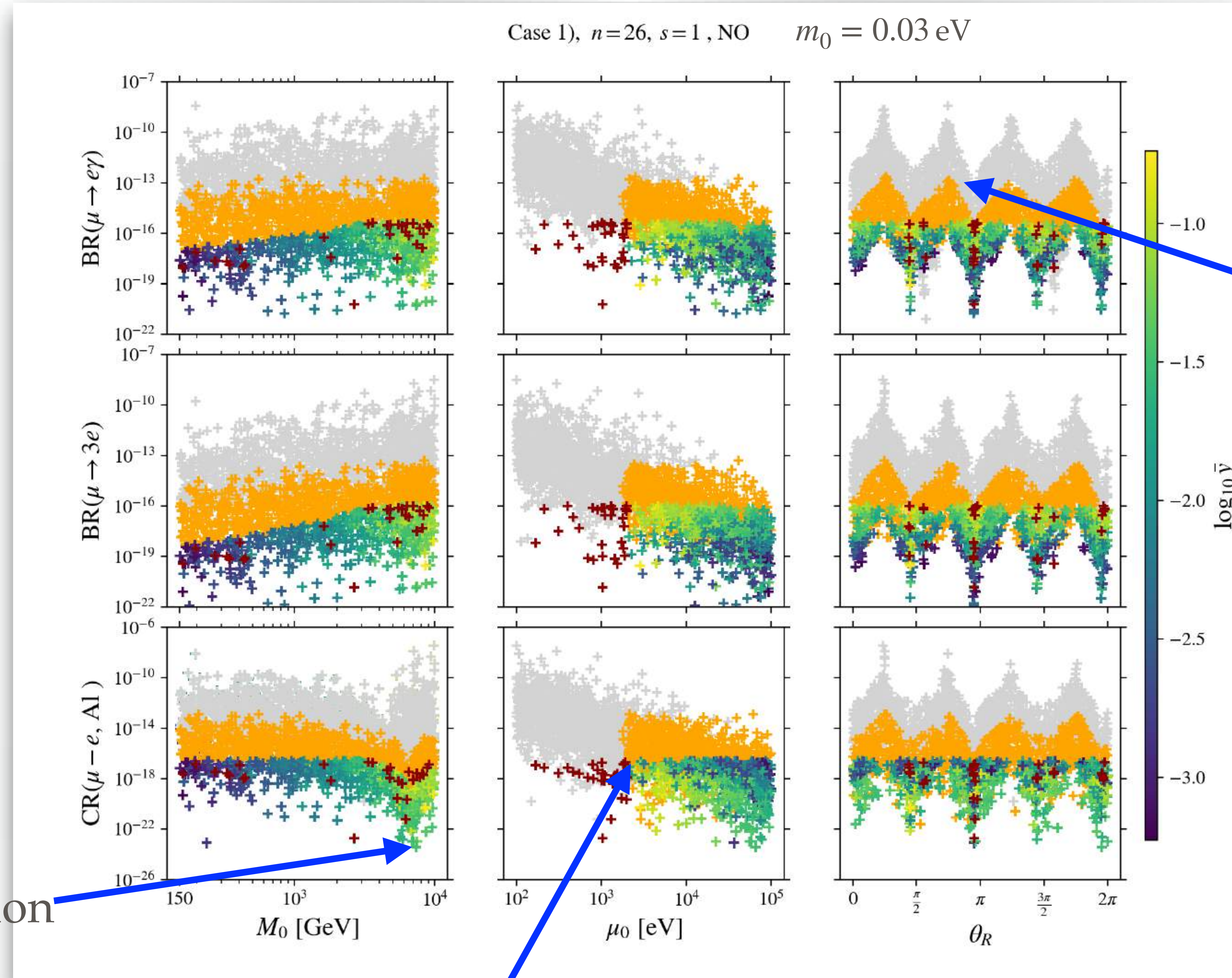
cLFV future OK, but η not OK

$$\overline{y} = \frac{1}{3}\,(y_1 + y_2 + y_3)$$

Results

Case 1)

[F.P. Di Meglio, CH ('24)]



θ_R dependence

$\cos 2\theta_R \approx 0$ enhanced BRs, CR;

for $|\cos 2\theta_R|$ large, suppressed

cancellation

$\mu_0 \gtrsim 2\,\mathrm{keV}$

Results

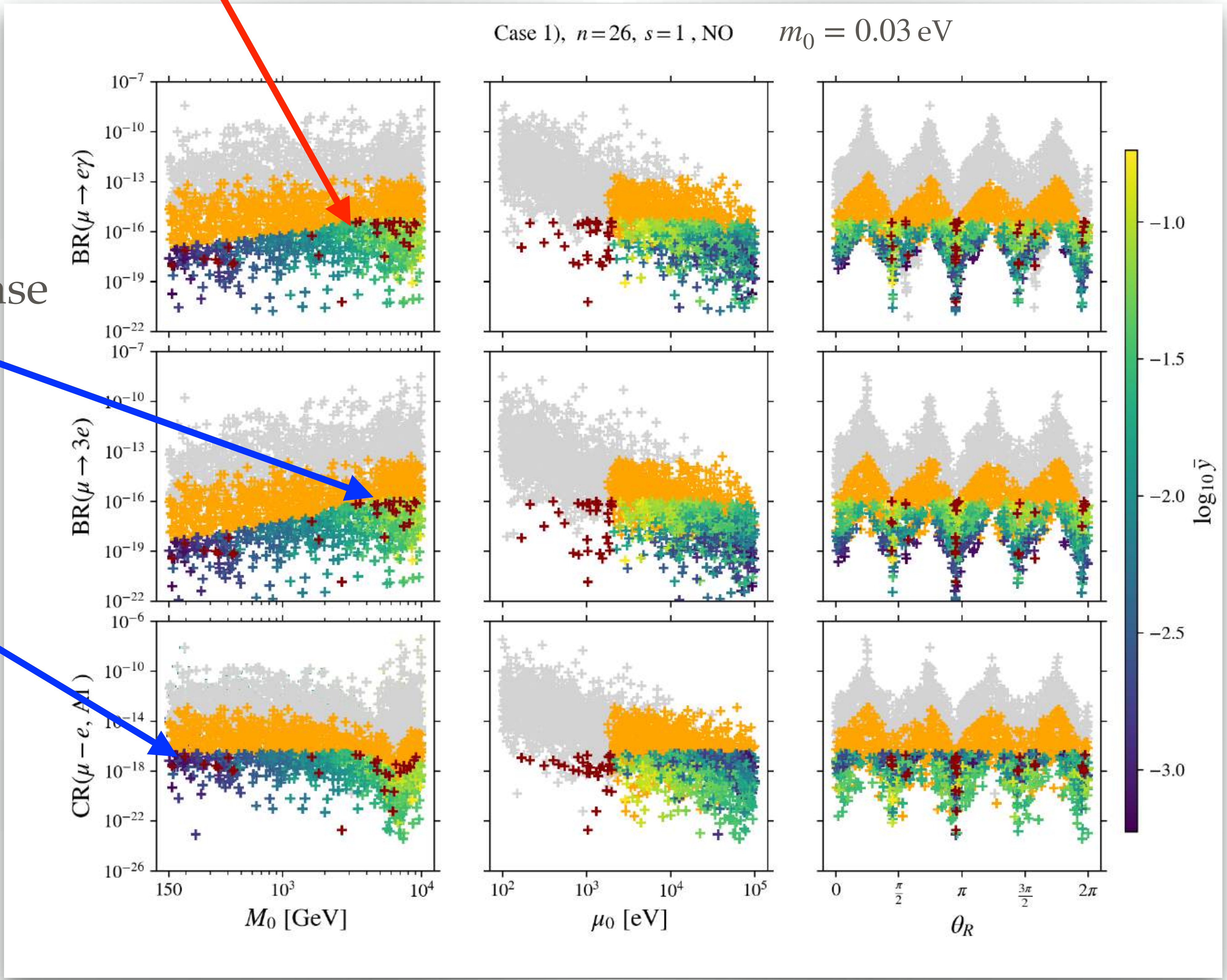
$\mathrm{BR}(\mu \to e\gamma) \lesssim 6 \times 10^{-16}$

Case 1)

[F.P. Di Meglio, CH ('24)]

Mu3E Phase 2 bound reached

COMET bound reached

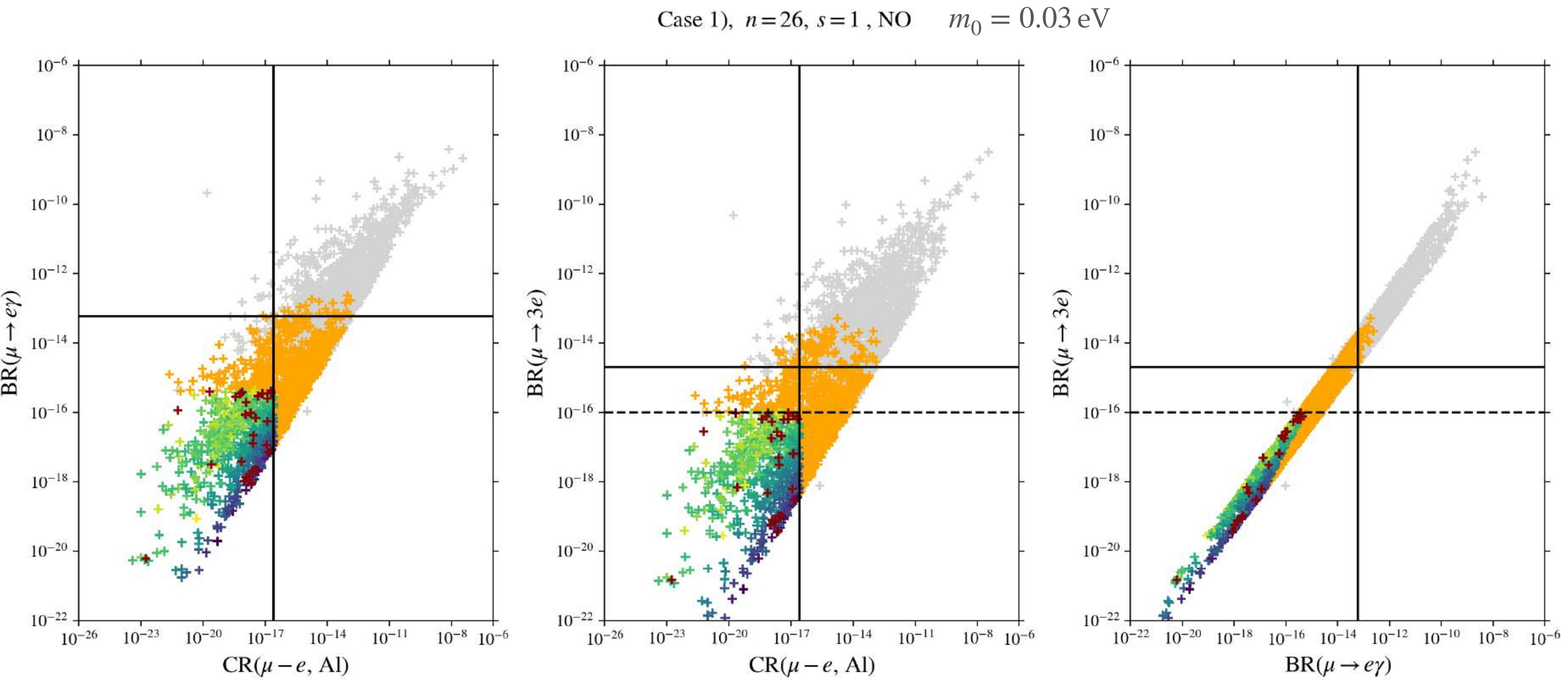


Results for IO and small m_0 also studied.

Results

Case 1)

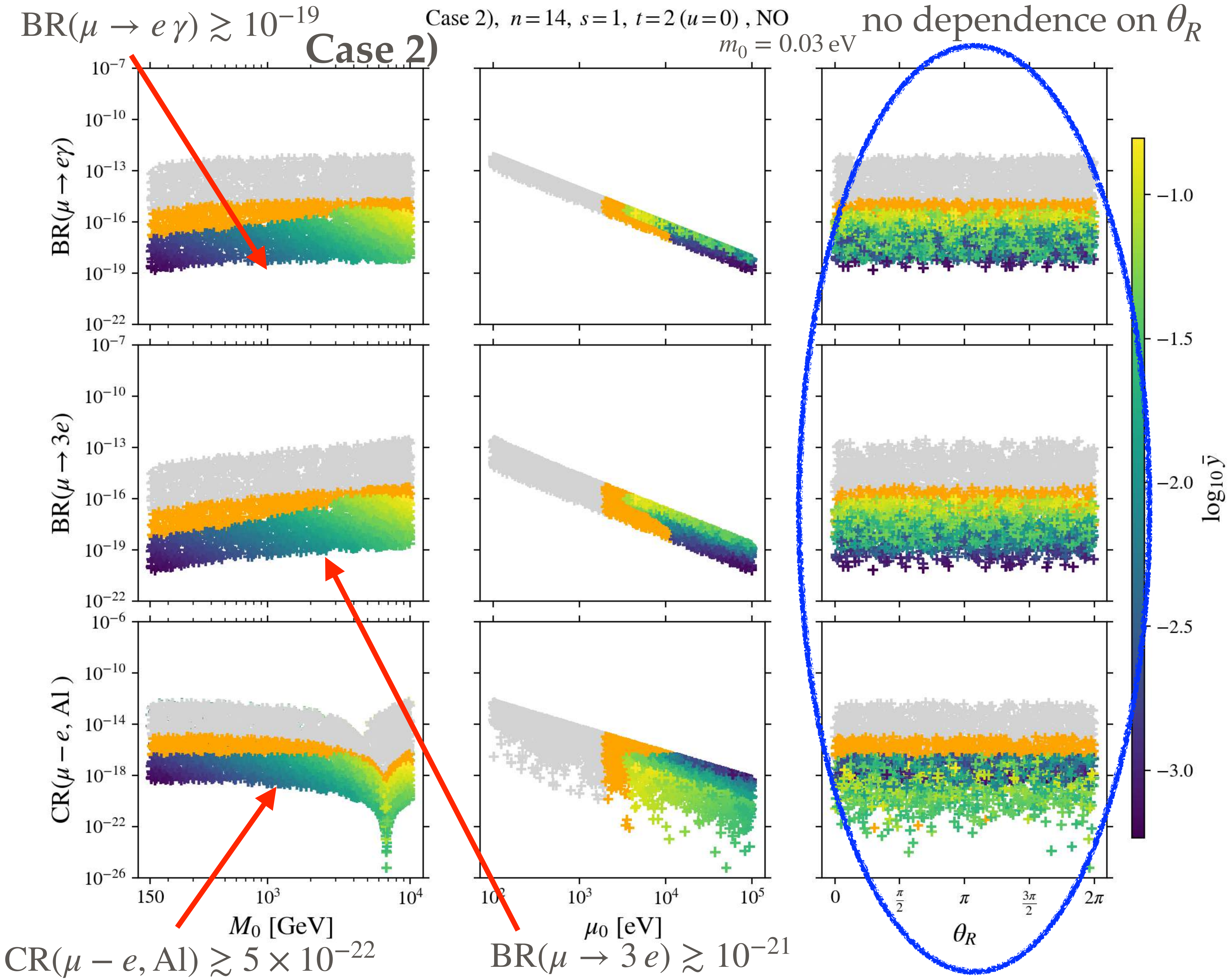
[F.P. Di Meglio, CH ('24)]



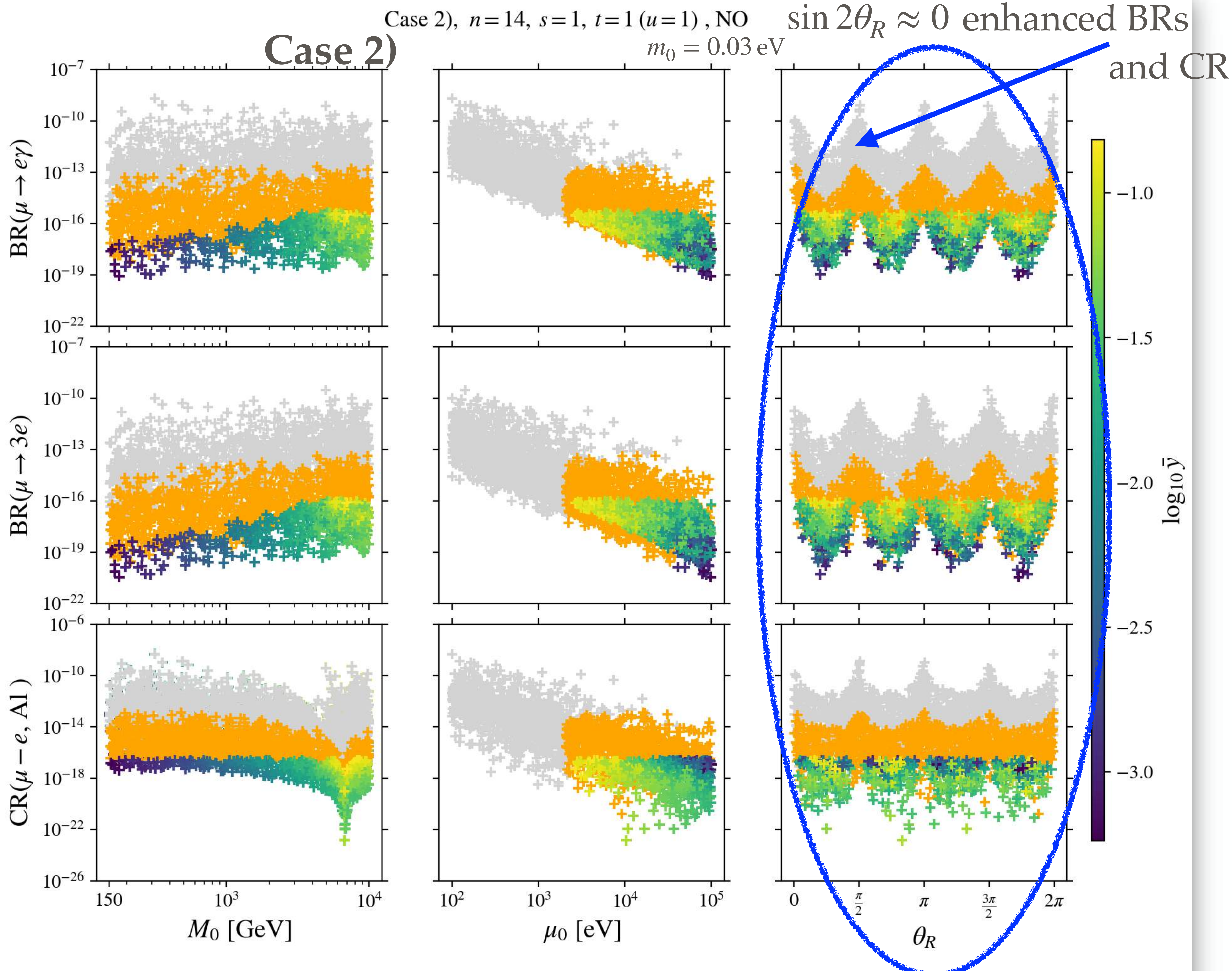
bound on $BR(\mu \to e\,\gamma)$ only mild constraint

COMET and Mu2e have large potential

Mu3E (Phase 2) limit reduces parameter space

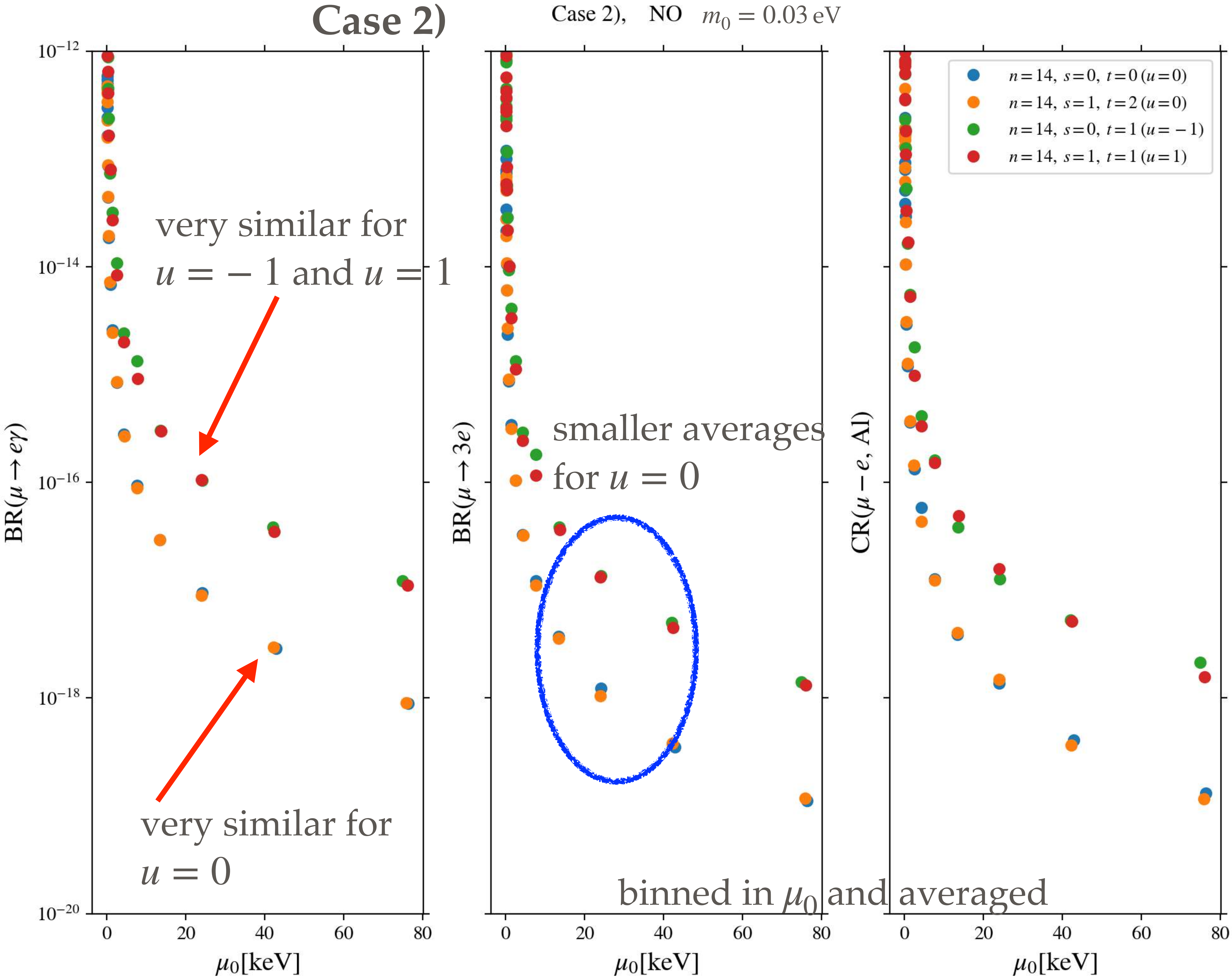


BR(μ → e γ) ≳ 10^{-19}
Case 2)
Case 2), n = 14, s = 1, t = 2 (u = 0), NO
m_0 = 0.03 eV
no dependence on θ_R
BR(μ → eγ)
BR(μ → 3e)
CR(μ − e, Al)
M_0 [GeV]
μ_0 [eV]
θ_R
log_10 ȳ
CR(μ − e, Al) ≳ 5 × 10^{-22}
BR(μ → 3 e) ≳ 10^{-21}



Case 2), $n=14$, $s=1$, $t=1$ $(u=1)$, NO
$m_0 = 0.03$ eV
Case 2)
$\sin 2\theta_R \approx 0$ enhanced BRs and CR
BR($\mu \to e\gamma$)
BR($\mu \to 3e$)
CR($\mu - e$, Al)
M_0 [GeV]
μ_0 [eV]
θ_R
$\log_{10}\bar{y}$

Case 2)



Case 2), NO $m_0 = 0.03\,\mathrm{eV}$
$n=14,\ s=0,\ t=0\ (u=0)$
$n=14,\ s=1,\ t=2\ (u=0)$
$n=14,\ s=0,\ t=1\ (u=-1)$
$n=14,\ s=1,\ t=1\ (u=1)$
$\mathrm{BR}(\mu \to e\gamma)$
$\mathrm{BR}(\mu \to 3e)$
$\mathrm{CR}(\mu - e,\ \mathrm{Al})$
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
0
20
40
60
80
$\mu_0[\mathrm{keV}]$
very similar for $u = -1$ and $u = 1$
very similar for $u = 0$
smaller averages for $u = 0$
binned in μ_0 and averaged

Results

[F.P. Di Meglio, CH ('24)]

Results for light neutrino masses with IO as well as different values of m_0 also studied.

Results for **Case 3 a)** **Case 3 b.1)** also available.

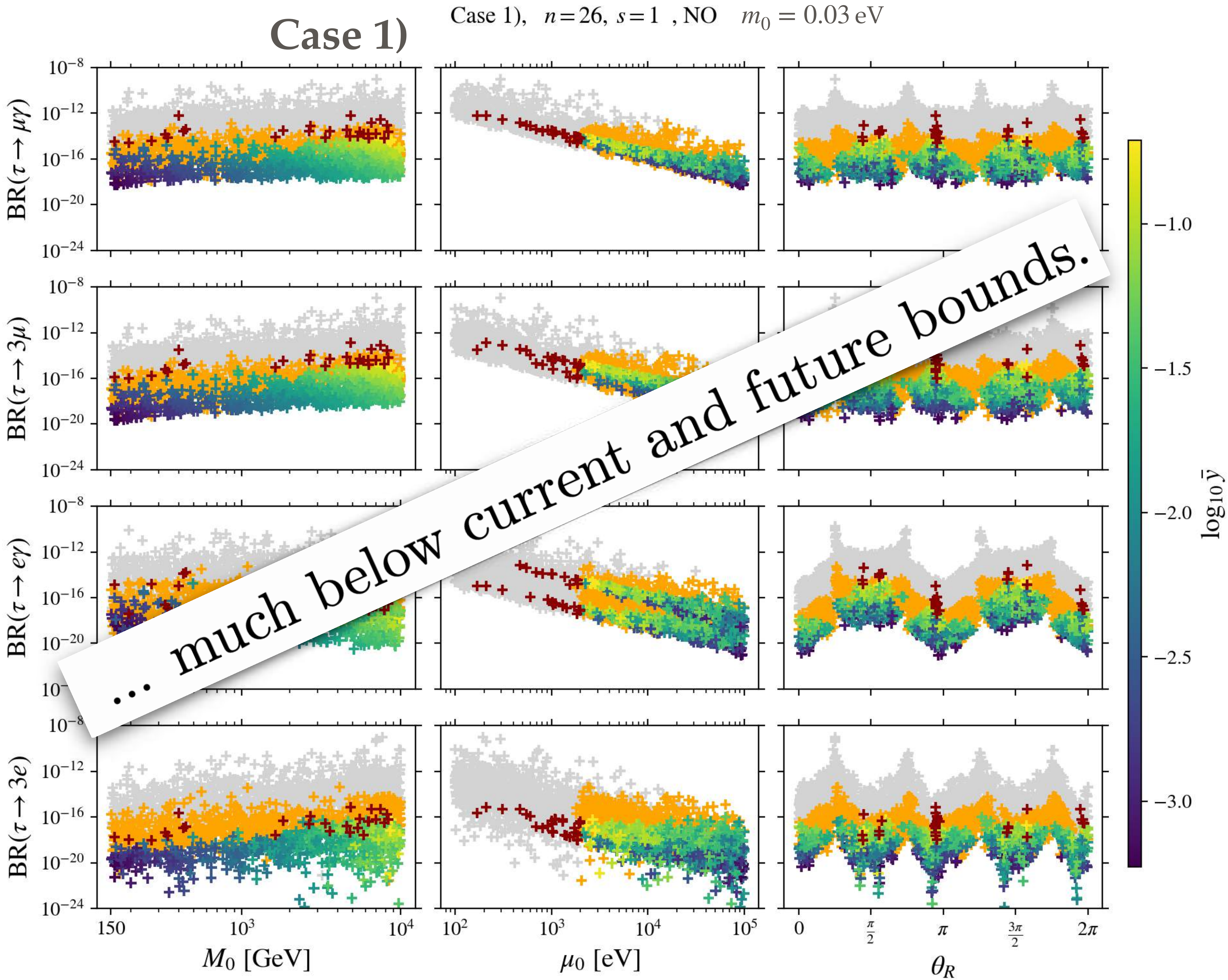
Quick look at charged lepton flavour violating tau decays

$$\tau \to \mu \gamma \, , \, \tau \to e \gamma \, , \, \tau \to 3 \mu \, , \, \tau \to 3 e$$

Example **Case 1)**

Results for other cases **Case 2)** **Case 3 a)** **Case 3 b.1)** are similar.

Case 1)



... much below current and future bounds.

Summary

- Flavour and CP symmetries can be the key to understand fermion mixing and also fermion masses
- Inverse seesaw mechanism is an interesting way to generate neutrino masses with potentially rich phenomenology
- Different realisations of residual symmetry among neutral states lead to distinct phenomenology — here: option 2
- Option 2
  - effect on lepton mixing small, but more general than for option 1
  - **signals of cLFV processes ($\mu - e$ transitions) can be sizeable**
- More options and variants of the scenario possible

Many thanks for your attention!

Back-up slides

Flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(3\,n^2)$

Luhn / Nasri / Ramond ('07)

$$a^3 = e \;,\quad c^n = e \;,\quad d^n = e \;,$$
$$c\,d = d\,c \;,\quad a\,c\,a^{-1} = c^{-1}d^{-1} \;,\quad a\,d\,a^{-1} = c$$

$$g = a^\alpha c^\gamma d^\delta \quad \text{with} \quad \alpha = 0,1,2\,,\;\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group A_4

Flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(6\,n^2)$ Add to relations of $\Delta(3\,n^2)$ Escobar / Luhn ('08)

$$b^2 = e \;\;,\;\; (a\,b)^2 = e \;\;,\;\; b\,c\,b^{-1} = d^{-1} \;\;,\;\; b\,d\,b^{-1} = c^{-1}$$

$$g = a^{\alpha} b^{\beta} c^{\gamma} d^{\delta} \quad \text{with} \quad \alpha = 0, 1, 2\,, \;\; \beta = 0, 1\,, \;\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group S_4

Flavour and CP symmetries

Add CP as further symmetry

Grimus/Rebelo ('95),

Ecker/Grimus/Neufeld (´84,'87,'88)

- Motivation:
  For more than one generation of certain particle species, define CP that also acts on generations of particles, e.g.

$$\Phi_i(x) \quad \rightarrow \quad X_{ij}\, \Phi_j^\dagger(x_P) \;\; \text{with} \;\; (x_P)_\mu = x^\mu$$

  with

$$X X^\dagger = X X^\star = 1$$

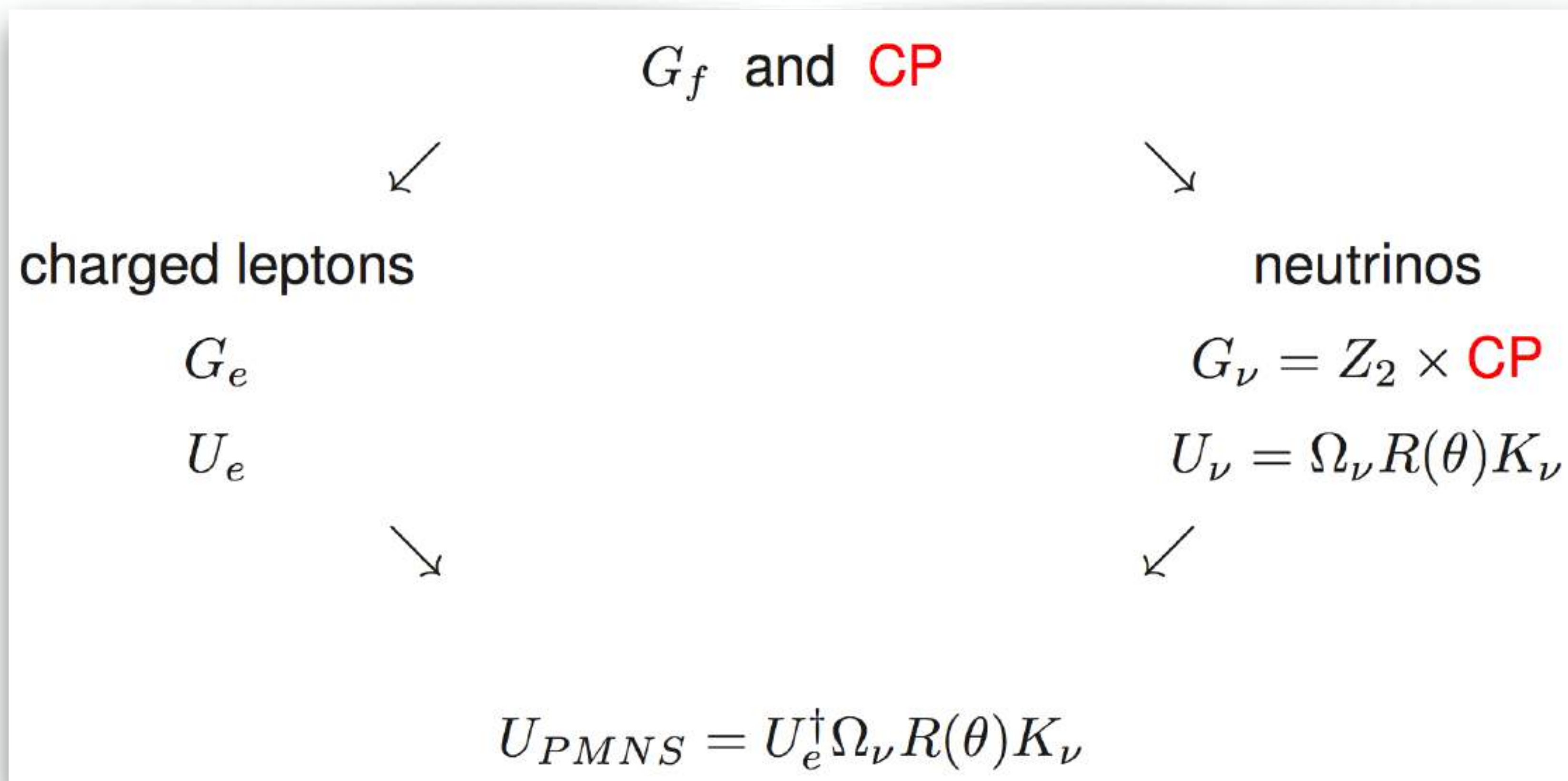
- CP is involution and corresponds to automorphism of flavour symmetry

Feruglio/CH/Ziegler ('12)
Holthausen/Lindner/Schmidt ('12), Chen et al. ('14)

Flavour and CP symmetries

Breaking of symmetries Feruglio/CH/Ziegler ('12)

Idea: Keep some residual symmetry among charged leptons and neutrinos, G_e and G_ν , with $G_e \neq G_\nu$
Mismatch of symmetries corresponds to lepton mixing



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Mismatch of symmetries corresponds to lepton mixing

G_f and CP

↙ ↘

Minimal choice Z_3	charged leptons	neutrinos	Choice Z_2 and CP
	G_e	$G_\nu = Z_2 \times$ CP	
3 different masses	U_e	$U_\nu = \Omega_\nu R(\theta) K_\nu$	Majorana masses

↘ ↙

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Free parameter

Size of masses not explained

Flavour and CP symmetries

Breaking of symmetries

Feruglio/CH/Ziegler ('12)

G_f and CP

↙ ↘

charged leptons	neutrinos
G_e	$G_\nu = Z_2 \times$ CP
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↘ ↙

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Minimal choice Z_3

3 different masses

Choice Z_2 and CP

Majorana masses

Free parameter

Size of masses not explained

CH/Meroni/Molinaro ('14)

Result: four different types of mixing patterns with different properties

Case 1) **Case 2)** **Case 3 a)** **Case 3 b.1)**

Flavour and CP symmetries

Case 1)

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta_L$$
$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta_L}$$
$$\sin^2\theta_{23} = \frac{1}{2}\left(1+\frac{\sqrt{3}\sin 2\theta_L}{2+\cos 2\theta_L}\right)$$

$$\sin\delta = 0$$
$$\sin\beta = 0$$

s fixed by CP symmetry

$$|\sin\alpha| = \left|\sin\left(\frac{6\,\pi\, s}{n}\right)\right|$$

Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

$$\begin{aligned}\sin^2\theta_{13} &\approx 0.0220\,(0.0222)\\ \sin^2\theta_{12} &\approx 0.341\\ \sin^2\theta_{23} &\approx 0.605\,(0.606)\end{aligned}$$

$$\begin{aligned}\sin\delta &= 0\\ \sin\beta &= 0\end{aligned}$$

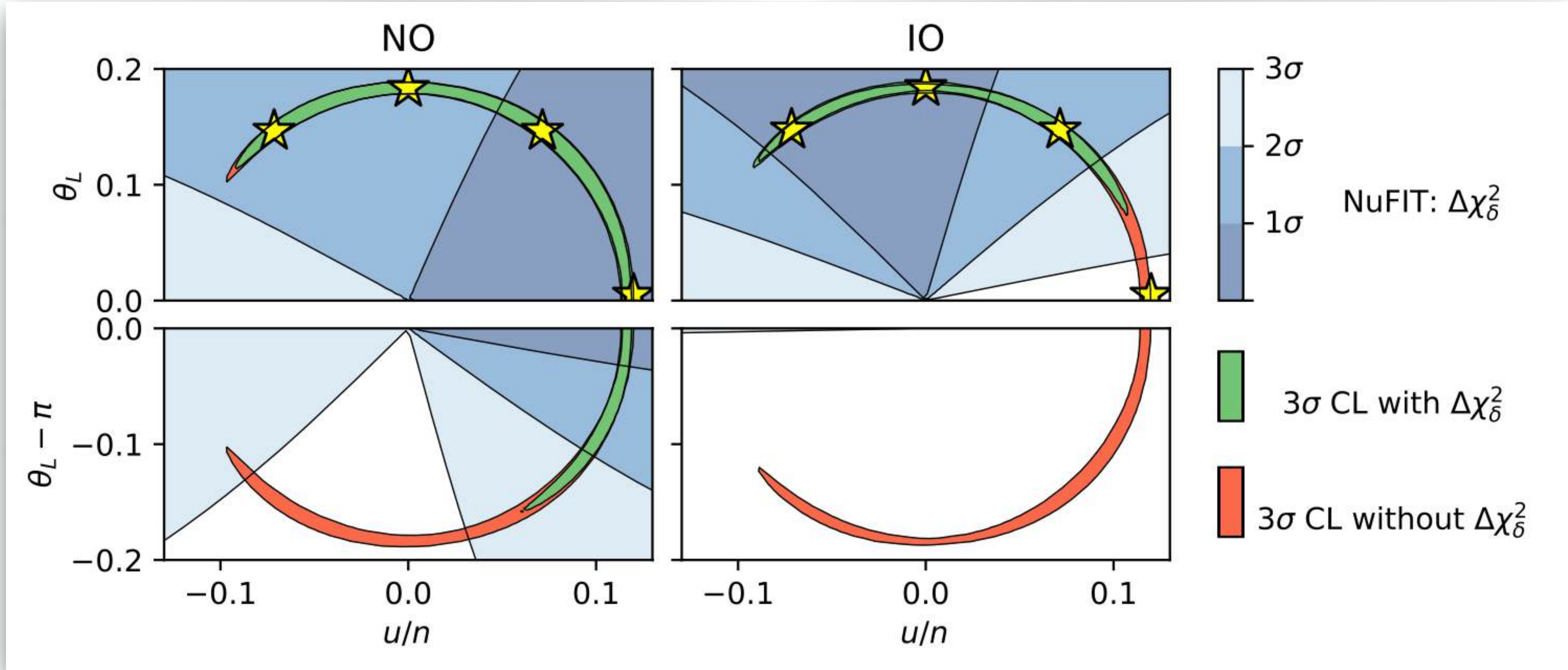
s fixed by CP symmetry

$$|\sin\alpha| = \left|\sin\left(\frac{6\,\pi\,s}{n}\right)\right|$$

Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)



$u = 2\,s - t$ fixed by CP symmetry

$v = 3\,t$ relevant mainly for Majorana phase α

Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)

$n = 14$

u	$u=-1$	$u=0$	$u=+1$
θ_L	0.146 (0.148)	0.184	0.146 (0.148)
$\sin^2\theta_{12}$	0.341	0.341	0.341
$\sin^2\theta_{13}$	0.0222 (0.0224)	0.0222 (0.0224)	0.0222 (0.0224)
$\sin^2\theta_{23}$	0.437	0.5	0.563
$\Delta\chi^2$	9.25 (11.2)	10.8 (12.5)	8.27 (8.62)

$$\sin\delta = -1 \text{ for } u = 0$$

$$\sin\delta \approx -0.811\,(-0.813) \text{ for } u = \pm 1$$

several choices for v admitted

Option 1

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- We take

$$\alpha_R \sim 1 \qquad L_\alpha \sim 3 \;, N_i \sim 3 \;, S_j \sim 3$$

[detail: use additional Z_3 to distinguish e, μ, τ]

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Charged lepton mass matrix

residual symmetry G_e

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

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No symmetry breaking

$$m_D = y_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{v}{\sqrt{2}} \quad \text{with} \;\; y_0 > 0$$

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \;\; M_0 > 0$$

Symmetry breaking

$$U_S^T\, \mu_S\, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

Option 1

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- We take

$\alpha_R \sim 1$

[detail: use additional Z_3 to distinguish e, μ, τ]

$L_\alpha \sim 3 \,, N_i \sim 3 \,, S_j \sim 3$

Light neutrino mass matrix

$$m_\nu = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_S = \frac{y_0^2\, v^2}{2\, M_0^2}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Neutrino masses

$$m_i = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_i \quad \text{for} \quad i = 1, 2, 3$$

Lepton mixing

$$\widetilde{U}_{\rm PMNS} = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

at leading order

Option 1

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Charged lepton flavour violation

Relevant points

- Lepton number and flavour breaking are **both** encoded in the matrix

$$U_S^T\, \mu_S\, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

- Non-unitarity effects are **flavour-diagonal and flavour-universal**

$$\eta = \frac{y_0^2\, v^2}{4\, M_0^2}\, \mathbb{1} \equiv \eta_0\, \mathbb{1}$$

- Mass spectrum of heavy states is peculiar:
  they form **pseudo-Dirac pairs** with very small mass splitting
  and all three such pairs have a **common mass scale**

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \;\; \text{and} \;\; M_{h,i+3} = M_0 + \frac{\mu_i}{2} \;\; \text{with} \;\; i = 1, 2, 3\,.$$

Option 1

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Charged lepton flavour violation

Relevant points

- Lepton number and flavour breaking are bot…

$$U_S^T\, \mu_S\, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \dots \end{pmatrix}$$

Conclusion

Rates of charged lepton flavour violating processes

$\ell_\beta \to \ell_\alpha \gamma$ $\quad$ $\ell_\beta \to 3\,\ell_\alpha$ $\quad$ $\mu - e$ conversion

are very suppressed!

for general formulae see Alonso et al. ('12), Ilakovac/Pilaftsis ('95)

…ular:

… **pairs** with very small mass splitting

…ch pairs have a **common mass scale**

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \quad \text{and} \quad M_{h,i+3} = M_0 + \frac{\mu_i}{2} \quad \text{with} \quad i = 1, 2, 3\,.$$

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 1)**

- No dependence on parameter s, thus set $s = 1$

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 1)**

- No dependence on parameter *s*, because

$$\eta = \frac{1}{2}\, m_D^\star \left(M_{NS}^{-1}\right)^\dagger M_{NS}^{-1}\, m_D^T$$

reads

$$\eta = \eta_0'\, U_0(\theta_L)\, \mathrm{diag}(y_1^2, y_2^2, y_3^2)\, U_0(\theta_L)^\dagger$$

with

$$U_0(\theta) = \Omega(\mathbf{3})\, R_{ij}(\theta)$$

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 1)**

- No dependence on parameter *s*, because

$$\eta = \eta_0' \, U_0(\theta_L) \, \mathrm{diag}(y_1^2, y_2^2, y_3^2) \, U_0(\theta_L)^\dagger$$

with

$$U_0(\theta) = \Omega(\mathbf{3}) \, R_{ij}(\theta)$$

and we have

$$U_0(\theta) = U_0(\theta, s=0) \, \mathrm{diag}(e^{i\,\phi_s}, e^{-2\,i\,\phi_s}, e^{i\,\phi_s})$$

since

$$\Omega(s)(\mathbf{3}) = e^{i\,\phi_s} \, U_{\mathrm{TB}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-3\,i\,\phi_s} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ and $$R_{13}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 1)**

- No dependence on parameter s, thus set $s = 1$
- Inspect dependence on θ_R

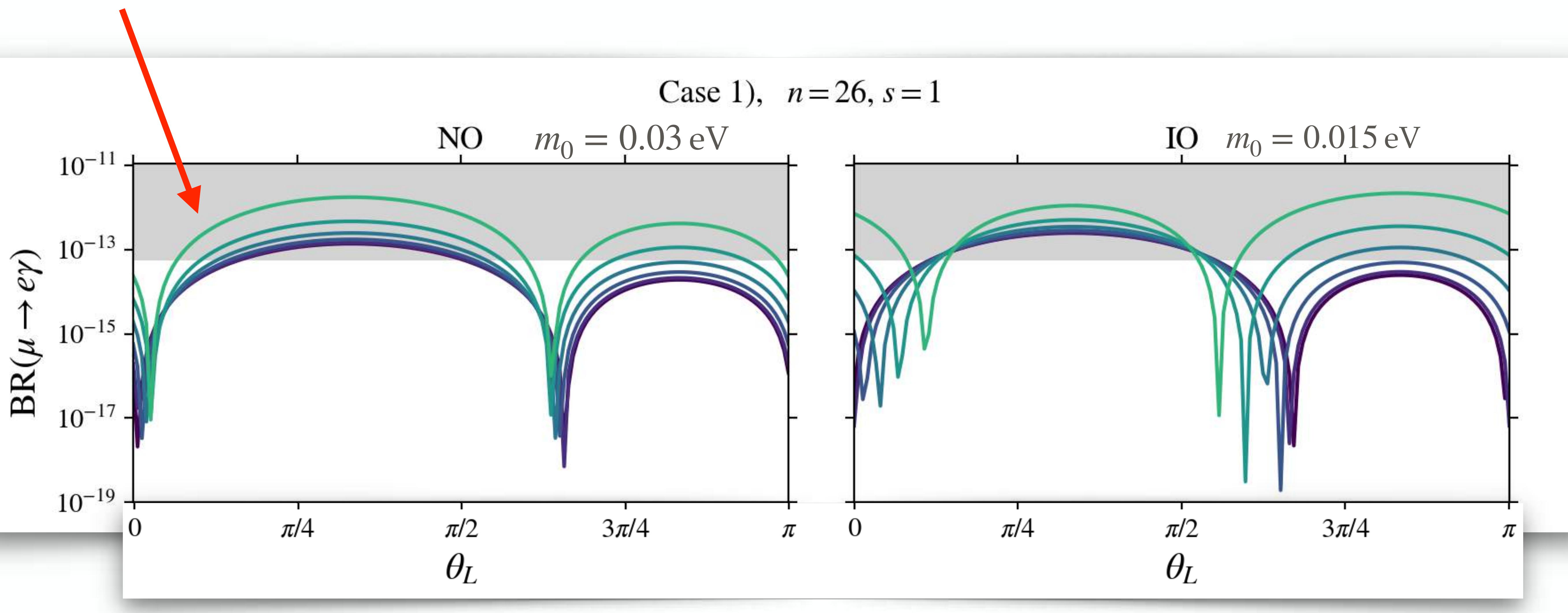
Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0

$$\mu_0 = 1\,\text{keV} \text{ and } M_0 = 3\,\text{TeV}.$$

MEG II future bound



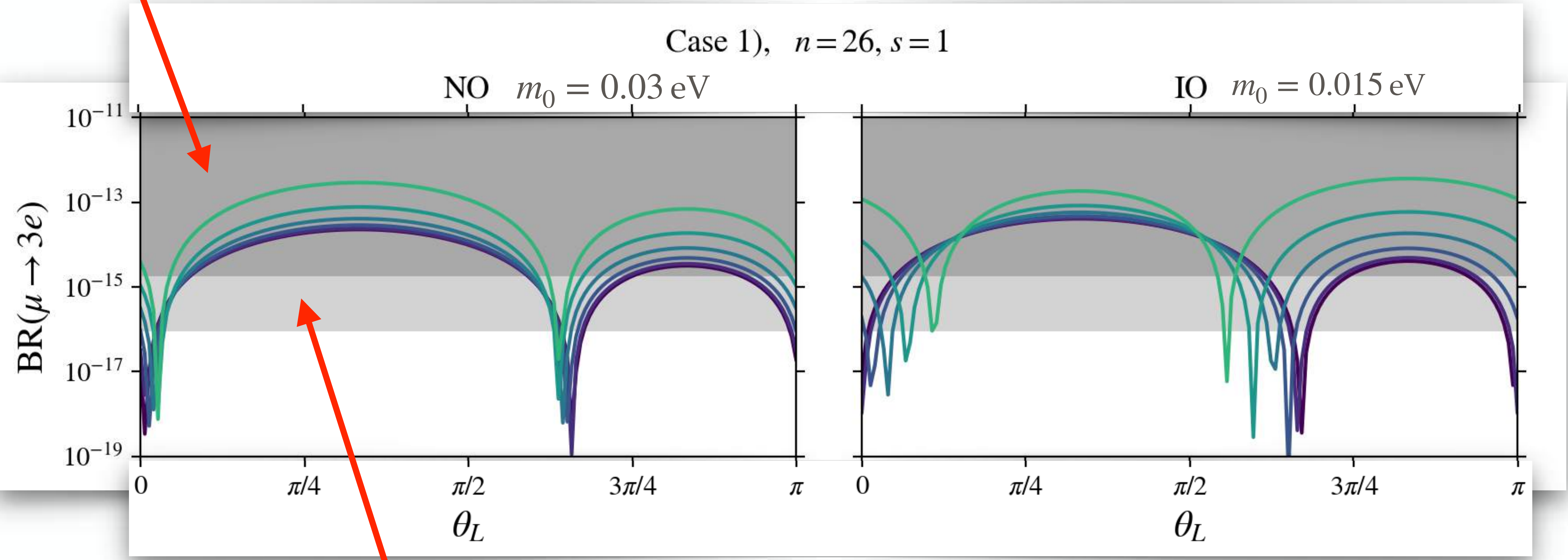
Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0

$$\mu_0 = 1\,\text{keV} \text{ and } M_0 = 3\,\text{TeV}.$$

Mu3E Phase 1 bound



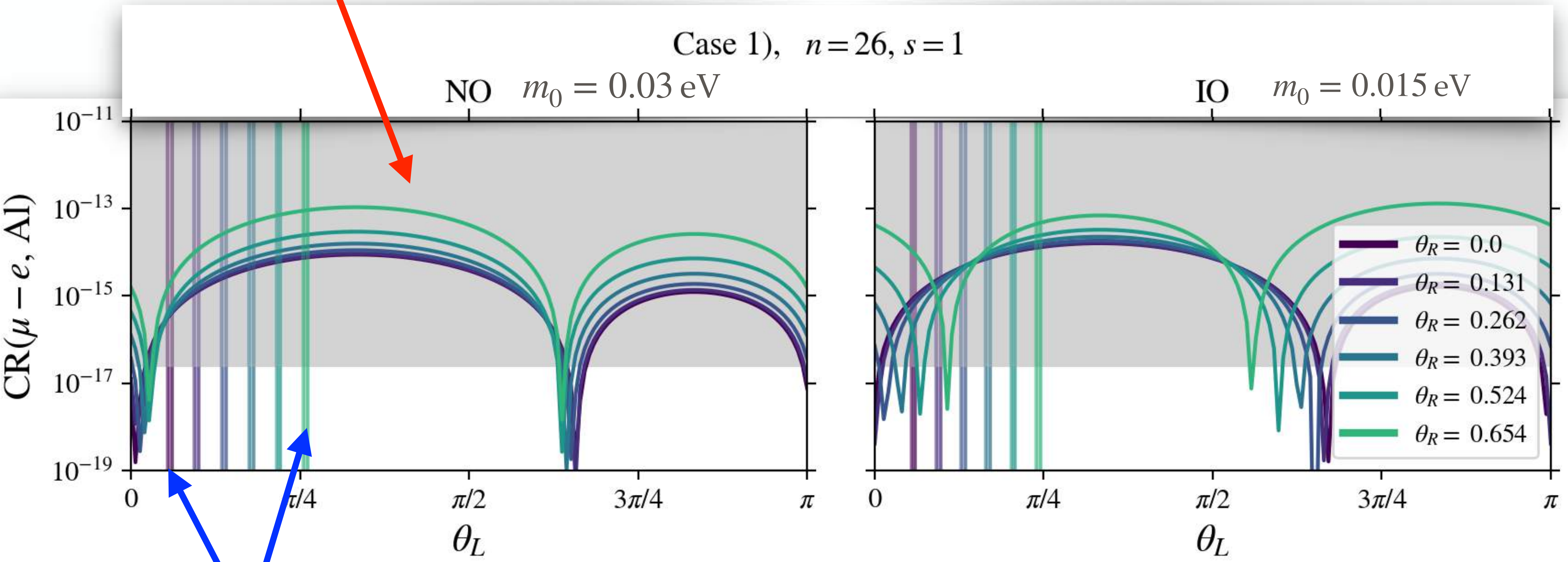
Mu3E Phase 2 bound

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\mu_0 = 1\,\text{keV}$ and $M_0 = 3\,\text{TeV}$.

COMET bound



lepton mixing angles are fitted well

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 1)**

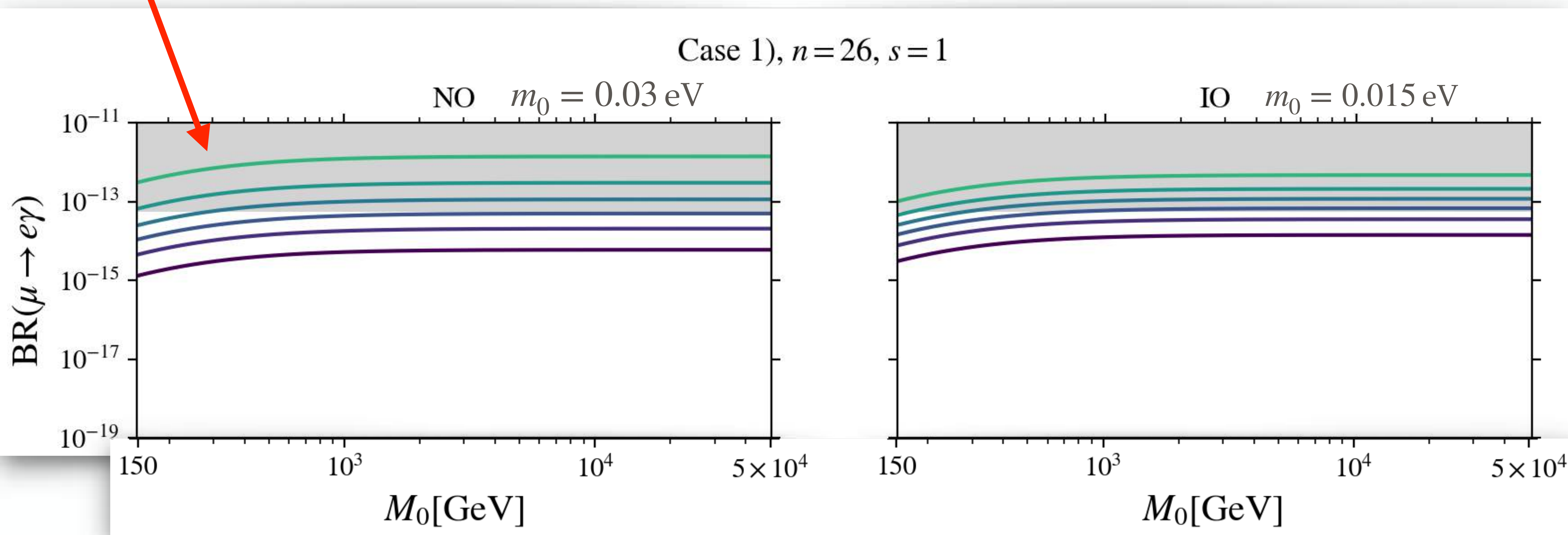
- No dependence on parameter s, thus set $s = 1$
- Inspect dependence on θ_R
- Vary M_0 and θ_R with μ_0 still fixed and θ_L fitting lepton mixing

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 $\qquad$ $\mu_0 = 1\,\text{keV}$

MEG II future bound



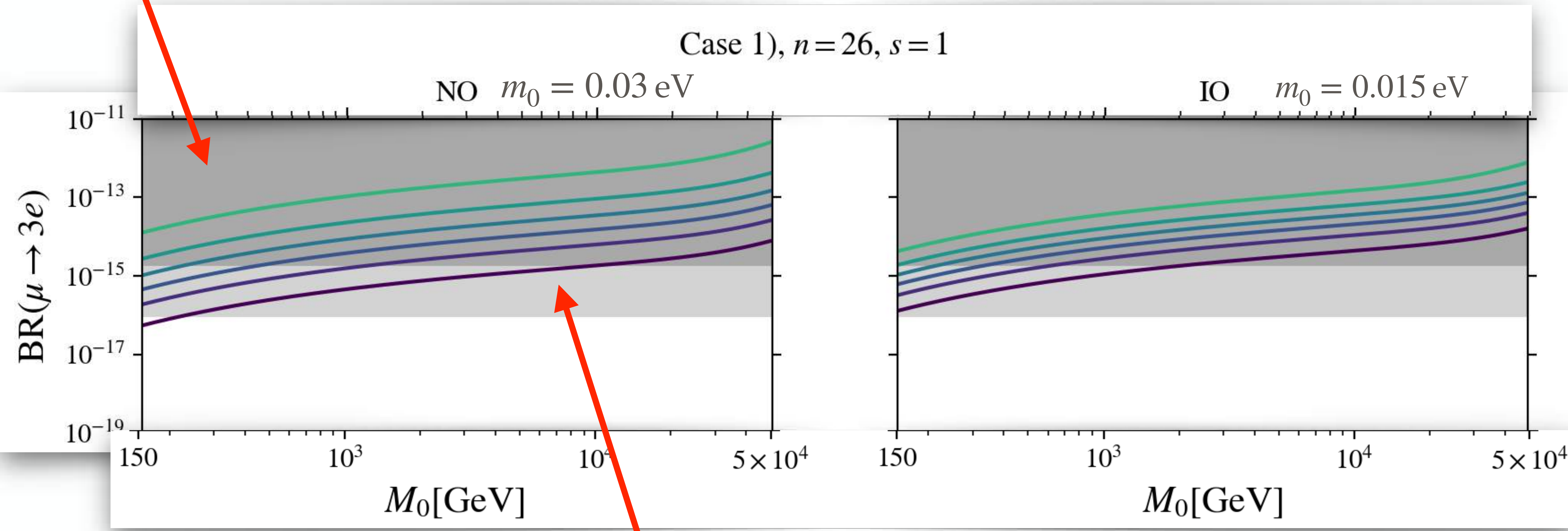
C. Hagedorn $\qquad$ NOW2024

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 $\mu_0 = 1\,\text{keV}$

Mu3E Phase 1 bound



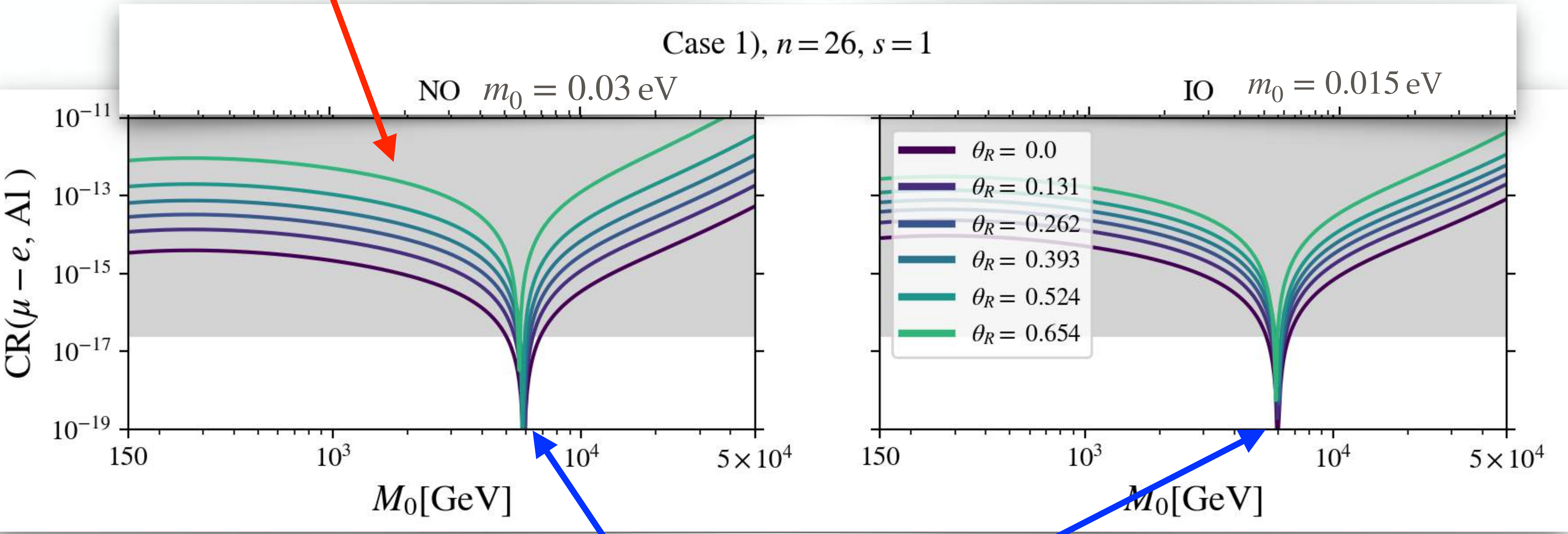
Mu3E Phase 2 bound

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 $\quad\mu_0 = 1\,\text{keV}$

COMET bound



cancellation

Results

[F.P. Di Meglio, CH ('24)]

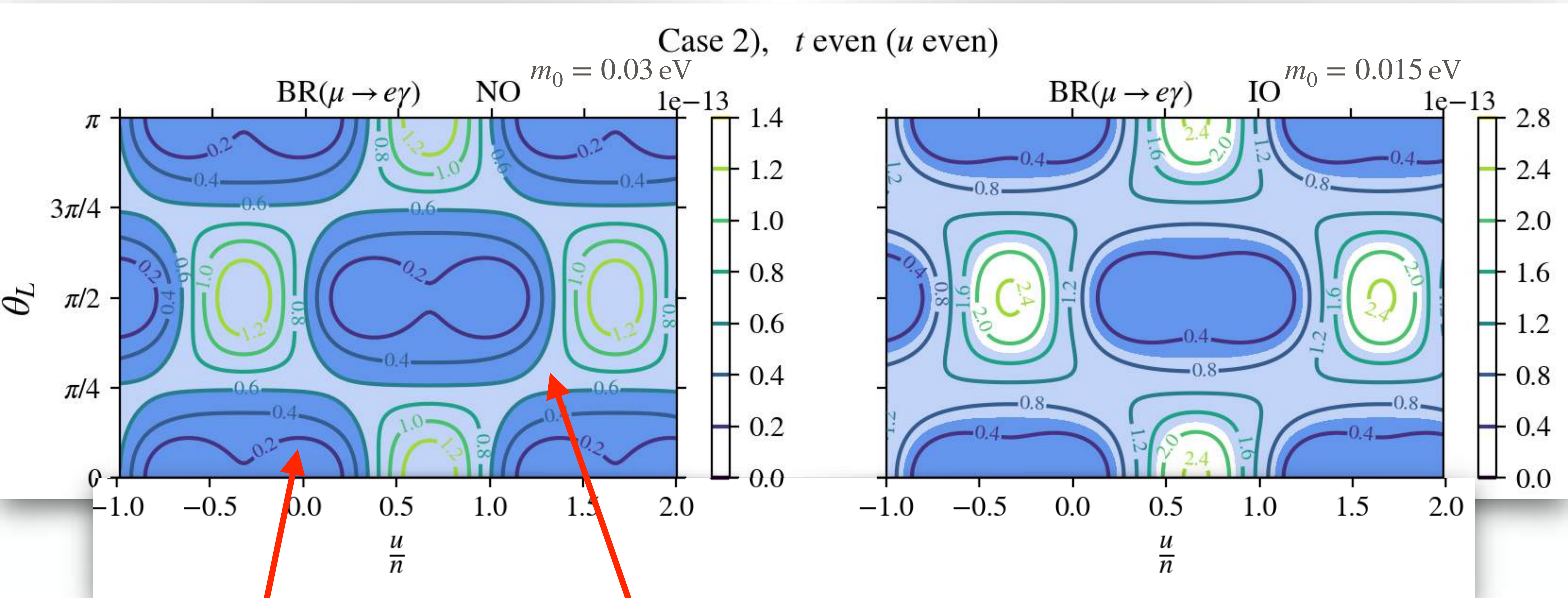
Numerical results for **Case 2)**

- Distinguish whether parameter t (also u) is even or odd, since this determines dependence on θ_R
- Whether parameter s is even or odd is irrelevant
- No dependence on parameter v
- Inspect viable parameter space in $\frac{u}{n} - \theta_L$-plane

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\quad \mu_0 = 1\,\text{keV}$ and $M_0 = 3\,\text{TeV}$.



MEG II future bound at 1 σ

MEG II future bound at 3 σ

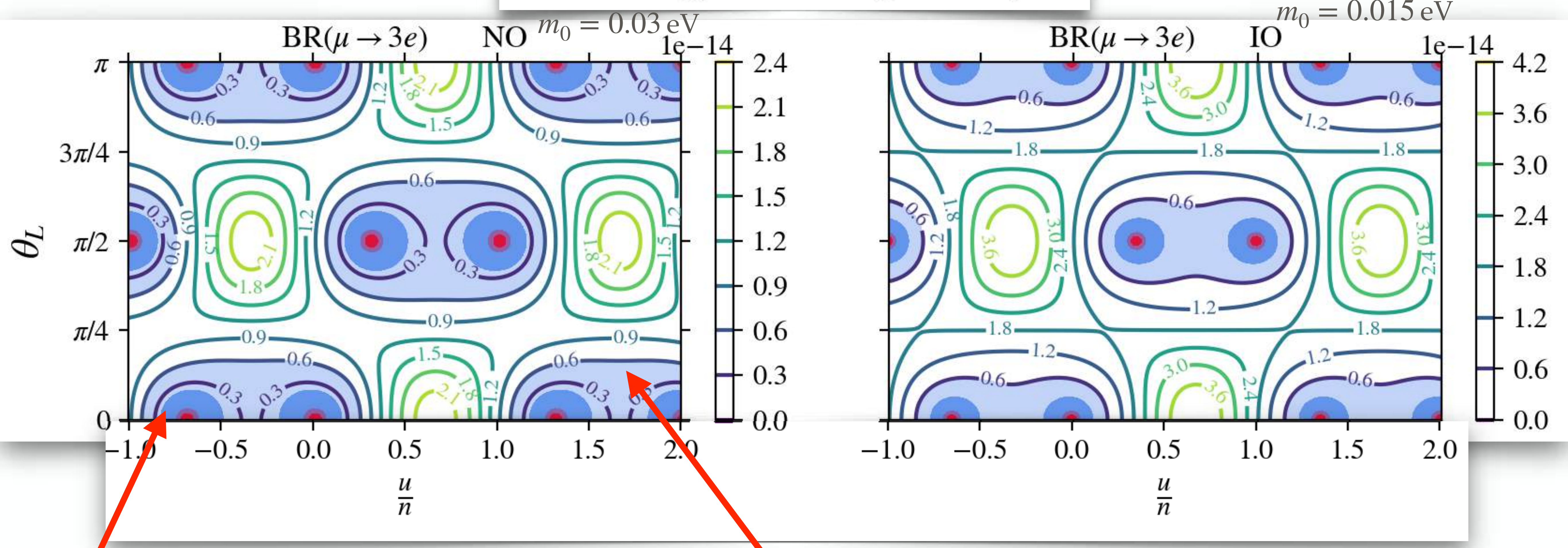
… mild constraint

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\quad \mu_0 = 1\,\mathrm{keV}$ and $M_0 = 3\,\mathrm{TeV}$.

Case 2), t even (u even)



Mu3E Phase 1 bound at 1 σ

Mu3E Phase 1 bound at 3 σ

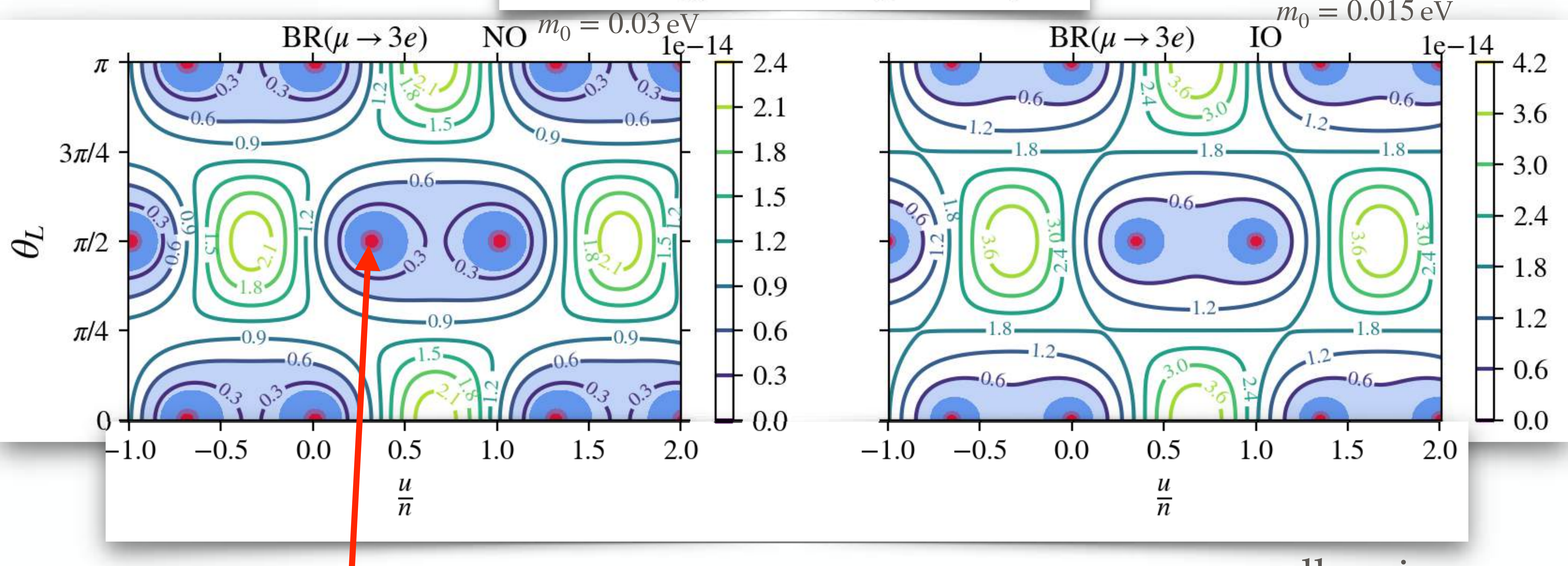
… 1/2 of parameter space disfavoured

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\quad \mu_0 = 1\,\mathrm{keV}$ and $M_0 = 3\,\mathrm{TeV}$.

Case 2), t even (u even)



Mu3E Phase 2 bound at 1 (3) σ

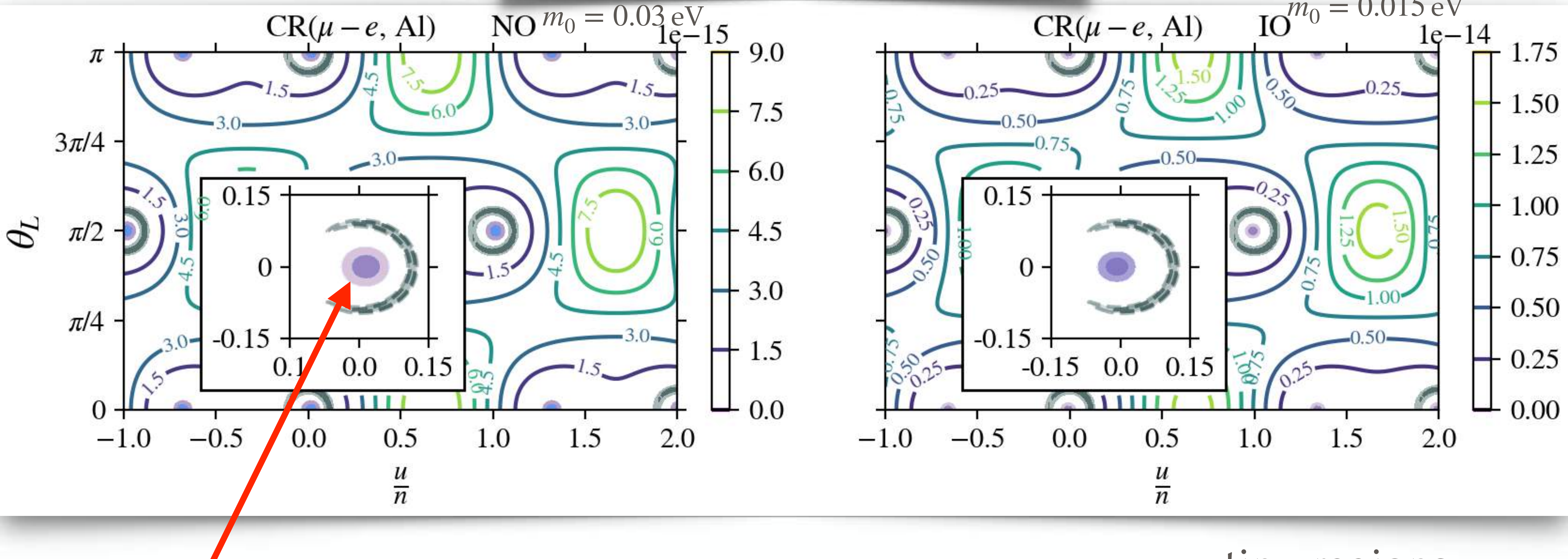
… small regions remain

Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\quad \mu_0 = 1\,\mathrm{keV}$ and $M_0 = 3\,\mathrm{TeV}$.

Case 2), $\;t$ even (u even)



dark (light) blue COMET limit at 1 (3) σ
dark (light) red Mu2e limit at 1 (3) σ

… tiny regions remain

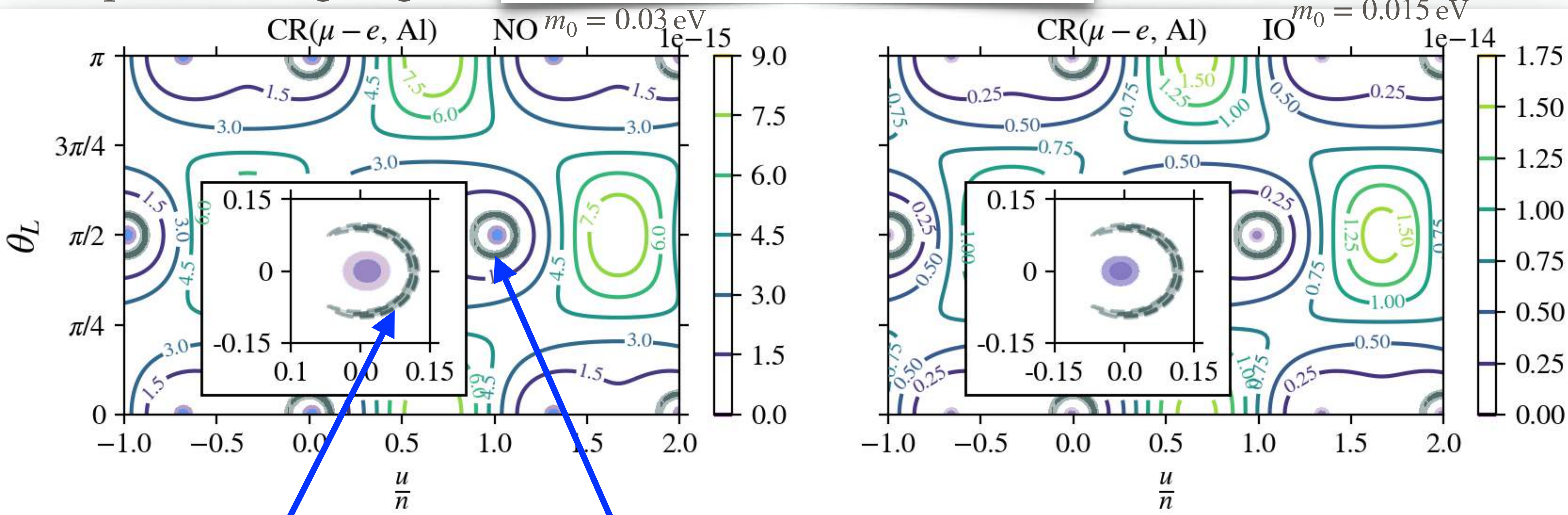
Results

[F.P. Di Meglio, CH ('24)]

For fixed μ_0 and M_0 $\mu_0 = 1\,\mathrm{keV}$ and $M_0 = 3\,\mathrm{TeV}$.

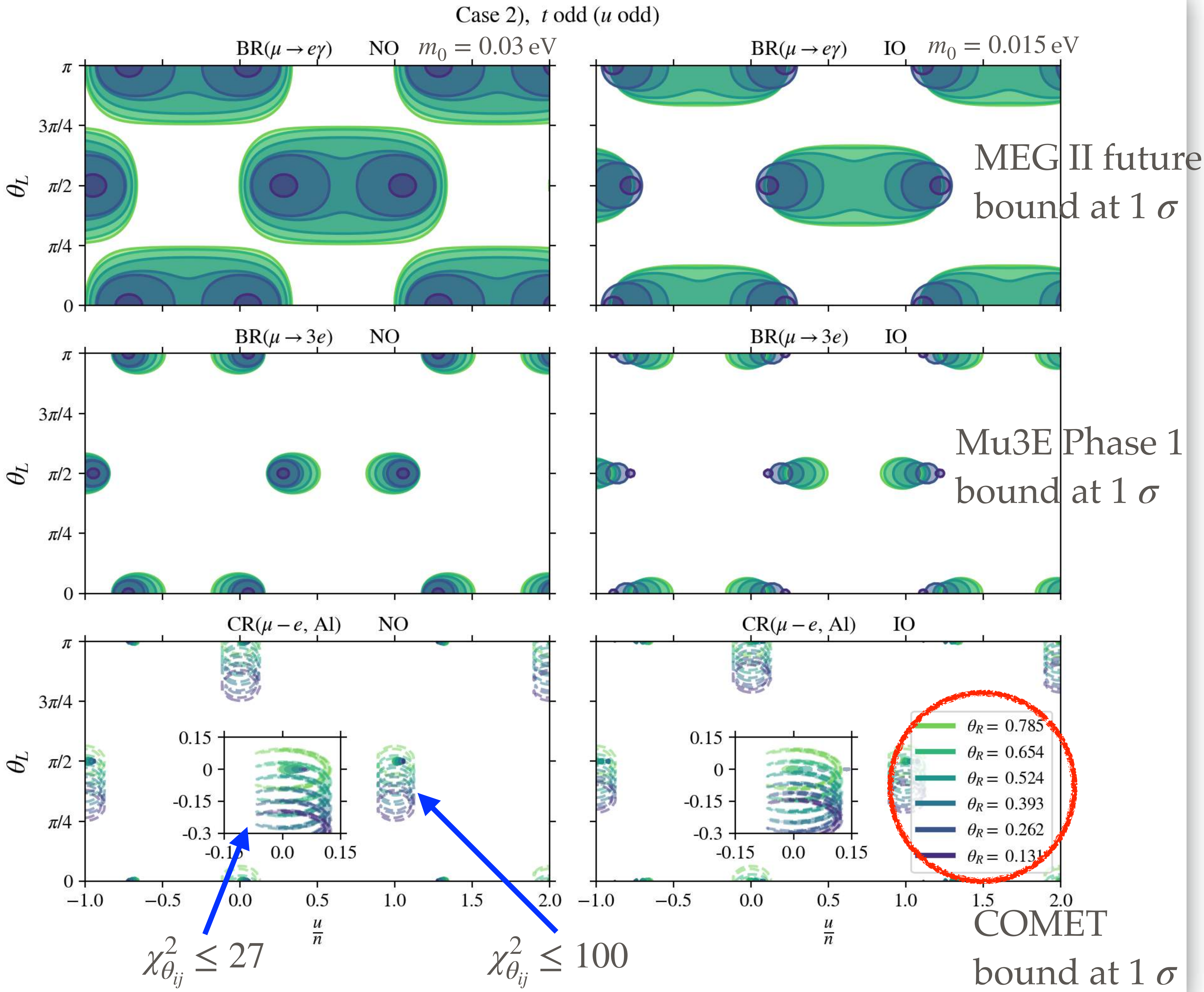
… add information on lepton mixing angles

Case 2), t even (u even)



$\chi^2_{\theta_{ij}} \leq 27$

dark (light) grey corresponds to $\chi^2_{\theta_{ij}} \leq 100\,(300)$



Case 2), t odd (u odd)
BR(μ → eγ) NO $m_0 = 0.03$ eV
BR(μ → eγ) IO $m_0 = 0.015$ eV
MEG II future bound at 1 σ
BR(μ → 3e) NO
BR(μ → 3e) IO
Mu3E Phase 1 bound at 1 σ
CR(μ − e, Al) NO
CR(μ − e, Al) IO
$\theta_R = 0.785$
$\theta_R = 0.654$
$\theta_R = 0.524$
$\theta_R = 0.393$
$\theta_R = 0.262$
$\theta_R = 0.131$
COMET bound at 1 σ
$\chi^2_{\theta_{ij}} \leq 27$
$\chi^2_{\theta_{ij}} \leq 100$
θ_L
$\frac{u}{n}$

Results

[F.P. Di Meglio, CH ('24)]

Numerical results for **Case 2)**

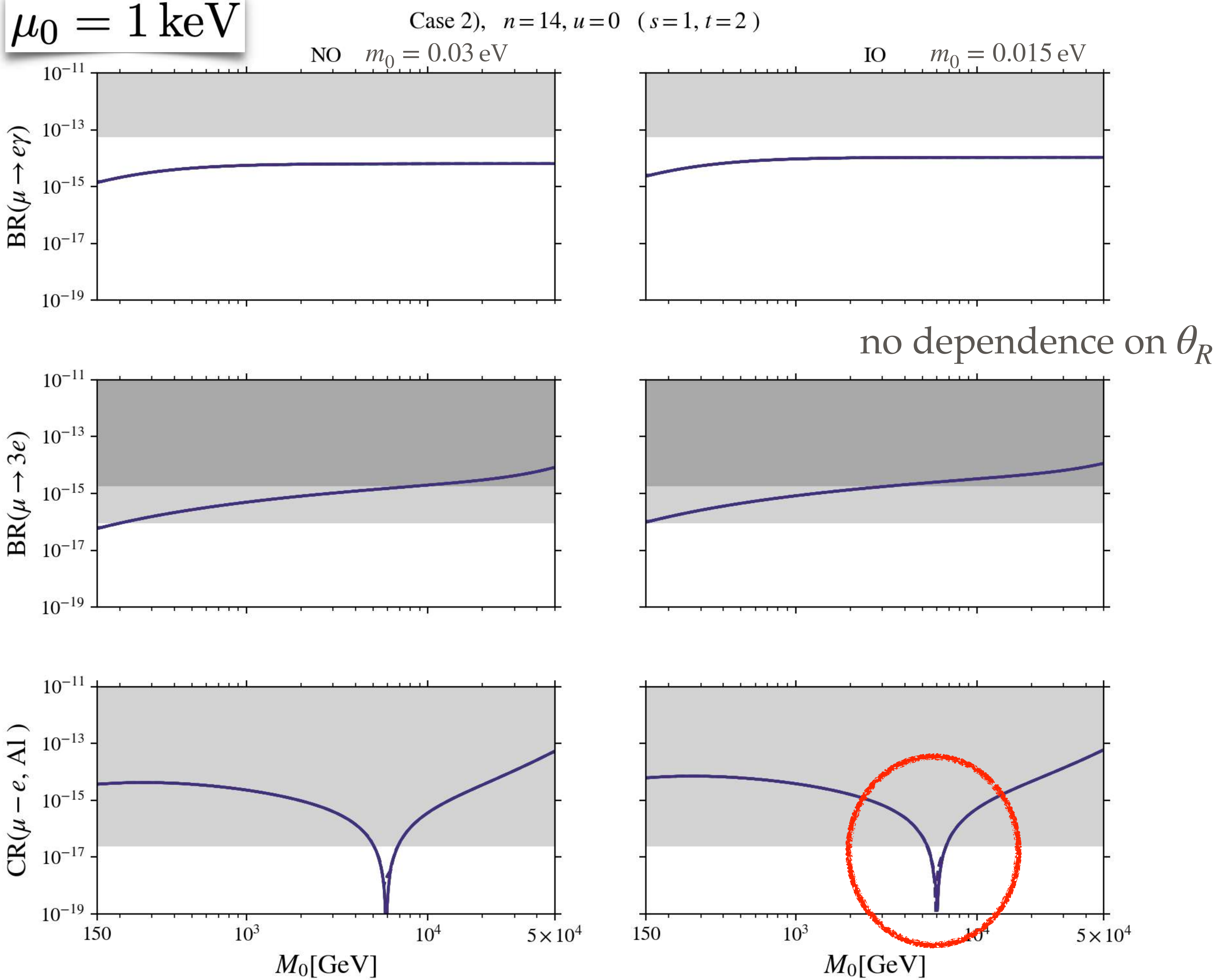
- Distinguish whether parameter t (also u) is even or odd, since this determines dependence on θ_R
- Whether parameter s is even or odd is irrelevant
- No dependence on parameter v
- Inspect viable parameter space in $\frac{u}{n} - \theta_L$-plane
- Take $n = 14$ together with $u = -1, 0, 1$

  and vary M_0 and θ_R with μ_0 still fixed and θ_L fitting lepton mixing (2 possible values)

Examples of s and t

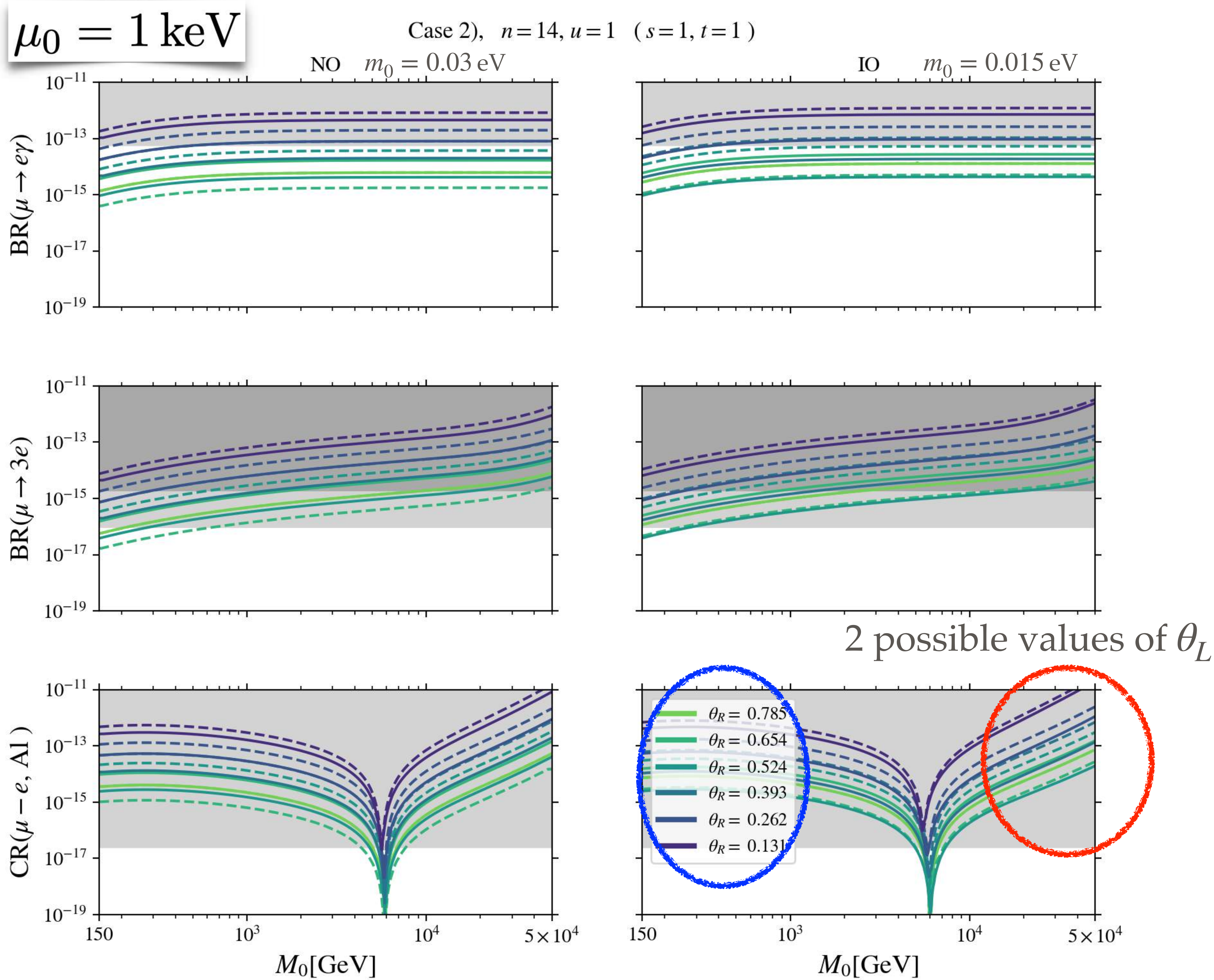
$$u = 0 \ : \ s = 0, \ t = 0 \ \text{ and } \ s = 1, \ t = 2$$
$$u = -1 \ : \ s = 0, \ t = 1$$
$$u = 1 \ : \ s = 1, \ t = 1$$

$\mu_0 = 1\,\mathrm{keV}$



no dependence on θ_R

2 possible values of θ_L



$\mu_0 = 1\,\mathrm{keV}$
Case 2), $n=14$, $u=1$ ($s=1$, $t=1$)
NO $m_0 = 0.03\,\mathrm{eV}$
IO $m_0 = 0.015\,\mathrm{eV}$
BR($\mu \to e\gamma$)
BR($\mu \to 3e$)
CR($\mu - e$, Al)
2 possible values of θ_L
$\theta_R = 0.785$
$\theta_R = 0.654$
$\theta_R = 0.524$
$\theta_R = 0.393$
$\theta_R = 0.262$
$\theta_R = 0.131$
M_0[GeV]