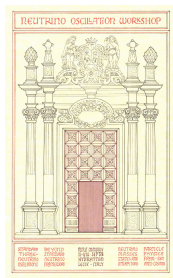


Neutrino-nucleus scattering in the SuSAv2 model including Meson Exchange Currents

Valerio Belocchi

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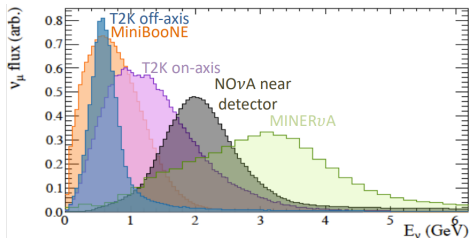
7th September 2024



Neutrinos:

- ▶ Light fermions that interact through Weak interaction only \Rightarrow Very low signals, heavy target needed: nuclei!
- ▶ Flavour Oscillation: mass eigenstate \neq flavour eigenstate

Neutrino experiments want to study the properties of this particle, and extract information on the **Oscillation Matrix**, especially on the **CP violating phase**.



Incident neutrino fluxes distribution for several experiments

Experiments measure the number of events. In a neutrino oscillation experiment:

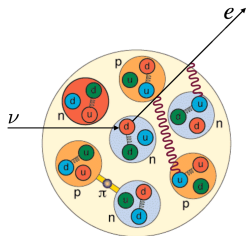
$$N_{\nu_\beta}(\overline{E}_\nu) \sim \int dE_\nu \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \rightarrow \nu_\beta}(E_\nu) \sigma(E_\nu) \epsilon_{det} d(E_\nu, \overline{E}_\nu)$$

Reconstructed energy $\overline{E}_\nu \Leftrightarrow E_\nu$ True neutrino energy

Nucleus as a Target

The nucleus is a very rich and complex target, composed by

- ▶ Nucleons: not elementary particles
- ▶ Mesons: mediators of nuclear interaction

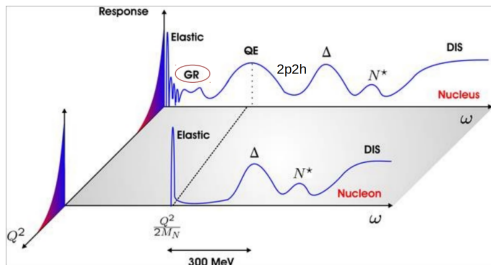


Lepton-Nucleus interaction: several processes

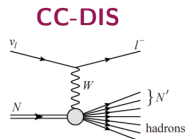
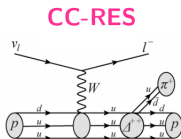
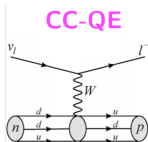
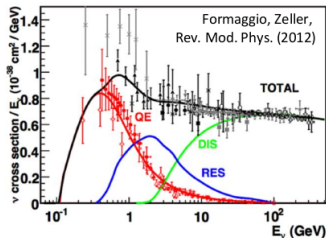
Nuclear Effects:

Free Nucleon → Nucleus

- ▶ Broadening of QE, Fermi motion → Initial hadronic state from nuclear model
- ▶ Pauli Blocking PB, Final State Interactions FSI
- ▶ Multinucleon excitations: **2p2h**



At a given E_ν , several channels are active



It is very difficult to reconstruct the interaction vertex:

- ▶ We don't know the initial hadronic state → **nuclear model**
- ▶ Incident flux wide in energy: we don't know the initial E_ν
- ▶ Outgoing hadron particles are affected by **final state interactions**

SuperScaling Approach SuSA

Suited for the QE channel, encompasses nuclear effects through the **scaling function** $f(\psi)$, extracted by **inclusive** (final lepton detected only) electron scattering data

⇒ **phenomenological model**

- ▶ Inclusive electron-nucleus scattering cross section:

$$\frac{d^2\sigma}{dE_{k'}d\Omega_{k'}} = \sigma_{Mott}[V_L R_L + V_T R_T]$$

- ▶ **Nuclear responses R_i are combinations of the nuclear hadronic tensor components**
- ▶ V_i terms are connected to the leptonic tensor

- ▶ From $e - A$ scattering data:

$$f(q, \omega; k_F) = k_F \times \frac{[d^2\sigma/d\omega d\Omega]^{(e,e')}}{\bar{\sigma}_{eN}}$$

⇓

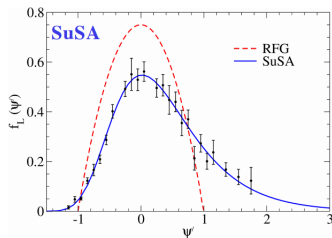
$$f(q, \omega; k_F) \Rightarrow f(\psi) \quad \psi \equiv \psi(\mathbf{q}, \omega; k_F)$$

- ▶ Pauli suppression: Rosenfelder method

$$f(\psi) = f(\psi(\omega, \mathbf{q})) - f(\psi(-\omega, \mathbf{q}))$$

- ▶ $E_{shift} = 20$ MeV

*Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990);
Donnelly and Sick, PRL82; PRC60 (1999)*



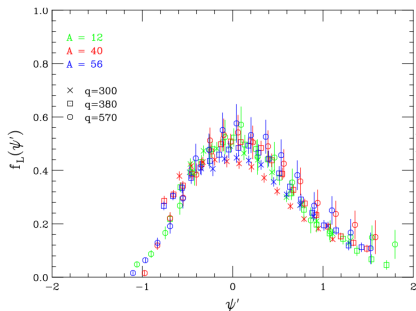
Exact and analytical results for RFG

$$f_L(\psi) = k_F \frac{R_L(q, \omega)}{G_L^{e, e'}(q, \omega)}$$

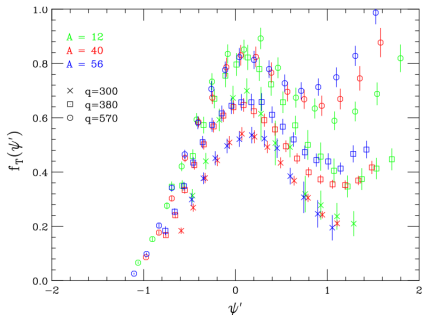
$$f_T(\psi) = k_F \frac{R_T(q, \omega)}{G_T^{e, e'}(q, \omega)}$$

SuSA assumption: $f_L = f_T$, Donnelly et al. PRC 60 (1999)

Longitudinal



Transverse

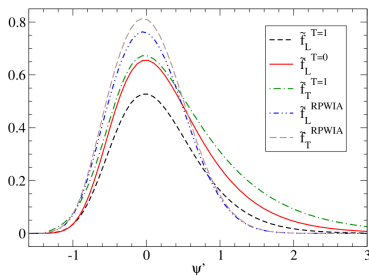


Scaling violation in f_T due to non-QE contributions and to finite nucleus effects

⇒ Microscopical nuclear description: **SuSAv2**

SuSAv2 is based on the Relativistic Mean Field RMF, but it follows the procedure of SuSA for the scaling behaviour

- ▶ Finite nucleus with scalar (attractive) and vector (repulsive) relativistic central potentials
 $\rightarrow (i\partial\!\!\!/ - M - S(x) + V(x))\psi(x) = 0$
- ▶ Channel-dependent scaling functions $\rightarrow f_T > f_L$ (relativistic effect, anti-particles)
 different isovector and isoscalar contributions \rightarrow Important for neutrino scattering!



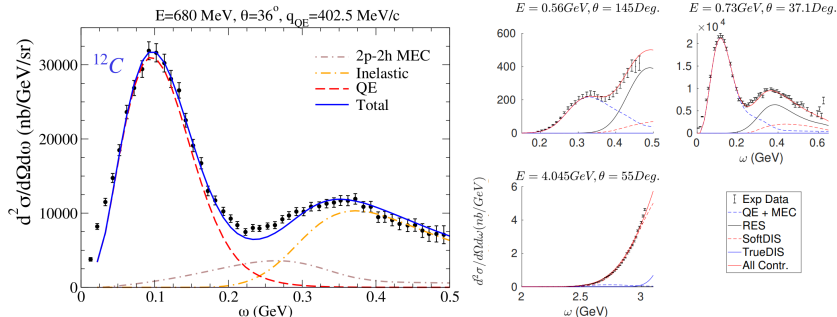
*R. González-Jiménez et al,
 Phys. Rev. C88, 025502 (2013)*

- ▶ Pauli blocking through use of orthogonal set of Hamiltonian eigenfunctions
- ▶ FSI too strong at high q values due to energy-independent potentials:
 - blended with Relativistic Plane Wave Impulse Approximation RPWIA \leftarrow SuSAv2
 - Energy-Dependent potentials

SuSAv2 describes inclusive QE, but it is possible to extend it to the full spectrum

- ▶ 2p2h through MEC formalism (see later)
- ▶ Resonances, defining a **generalized scaling variable** for each invariant mass
- ▶ Deep inelastic scattering, folding the elementary inelastic response with the SuSA scaling function *Barbaro et al., PRC69 (2004), Gonzalez-Rosa et al., PRD108 (2023)*

Model validation and fitting: **Electron Scattering**



Megias et al., PRD94 (2016), Gonzalez-Rosa et al., PRD108 (2023) Data: *Barreau, NPA402 (1983)*

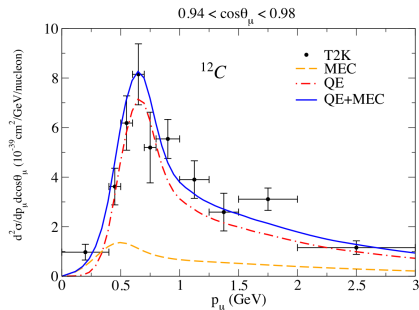
Scaling functions for the Electro-Weak interaction are the same extracted from electron-scattering:

- ▶ From 2 EM- to 5 EW-non vanishing inclusive responses

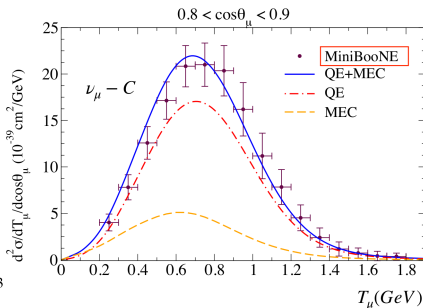
- ▶ $f_T^{VV} = f_T^{AA} = f_{T'}^{VA} = \tilde{f}_T$ $f_L^{VV} = \tilde{f}_L^{T=1}$

- ▶ Assumptions: $f_T^{T=0EM} = f_{CC,CL,LL}^{AA} = \tilde{f}_L^{T=1}$

$\nu_\mu - {}^{12}\text{C}$ inclusive scattering data CC0 π

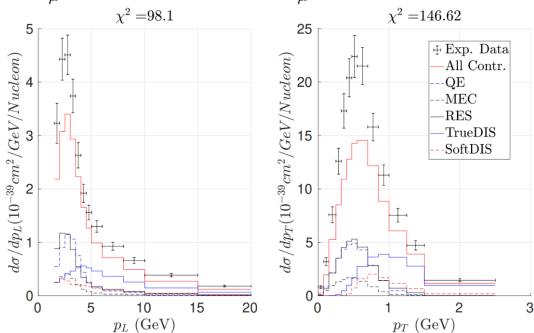
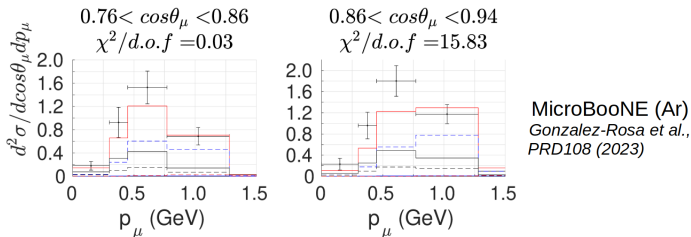


Megias et al., JPG46 (2019)



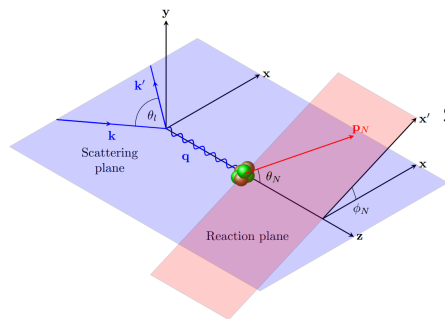
Megias et al., PRD94 (2016)

Neutrino-scattering inclusive data



MINERVA, Gonzalez-Rosa et al., PRD108 (2023)

Outgoing lepton and one ejected nucleon are detected in coincidence

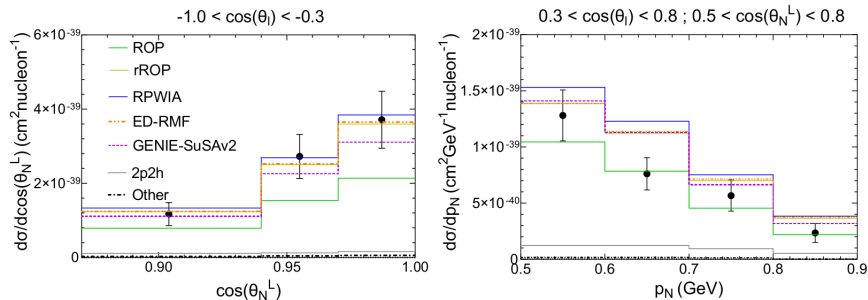


SuSA and SuSAv2 are **intrinsically inclusive**

↓
do not predict hadronic observables

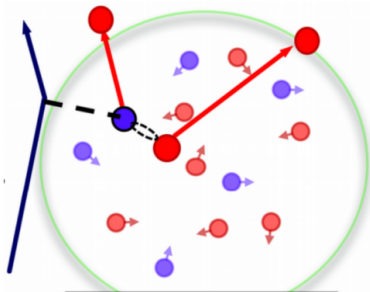
- ▶ RMF allows for hadronic observables predictions → ED-RMF *Franco-Patiño et al., PRD104 (2021)*
- ▶ GENIE SuSAv2 implementation *S. Dolan et al., PRD101 (2022)*

T2K data CC0 π with at least one ν proton in the final state with momentum p_N above 0.5 GeV

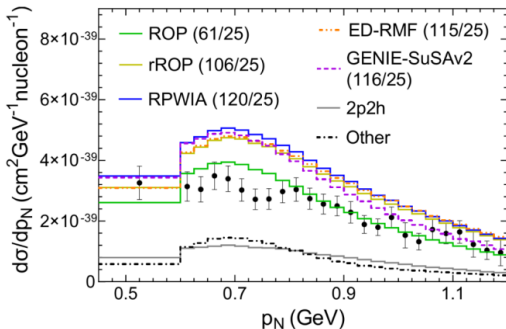


- ▶ Relativistic Optical Potentials can be added to improve reliability in FSI description
- ▶ Discrepancies between models and large uncertainties due to FSI, a defined trend does not emerge

Increasing the transferred energy, two nucleons can be emitted: **2p2h**



Semi-inclusive $2p2h$ contribution is very important: **a reliable model is needed**



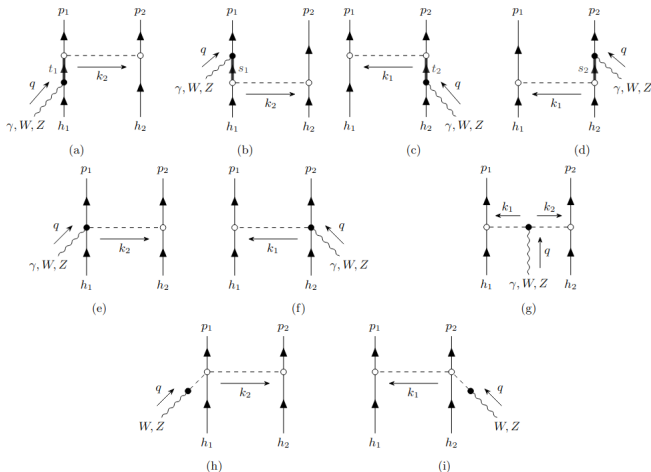
MINERνA	k' (GeV)	$\cos \theta_t$	p_N (GeV)	$\cos \theta_N^L$	ϕ_N^L (°)
	1.5-10	> 0.939	0.45-1.2	> 0.342	-

Current CC $2p2h$ predictions are **based on inclusive calculation**, thus their implementation for semi-inclusive processes is questionable and **strongly affect comparison of one-body semi-inclusive models to the experimental data**

Meson Exchange Currents formalism

Effective Field Theory EFT (pions and nucleons) + Δ description

De Pace et al., NPA 726 (2003), E. Hernandez et al., Phys. Rev. D 76, 033005 (2007)

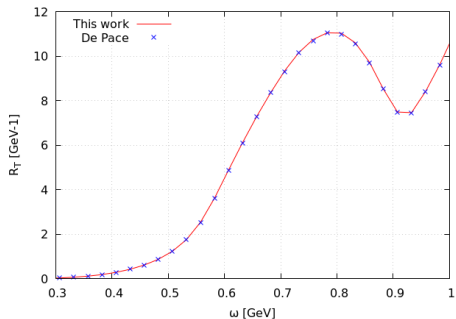


This work: RFG+MEC

We tested our model with previous results:

$$e - {}^{56}\text{Fe}$$

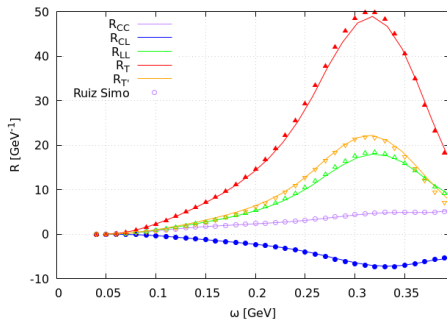
$q=1140$ MeV



De Pace et al., NPA 741, 249 (2004)

$$CC \nu_{\mu} - {}^{12}\text{C}$$

MEC Nuclear Responses at $q=400$ MeV



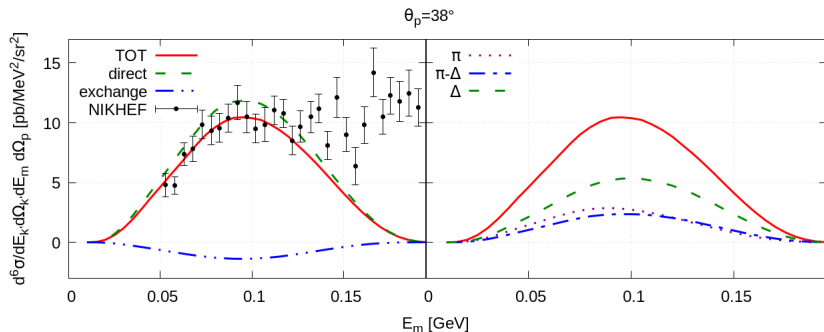
Ruiz Simo, Amaro et al., JPhG 44, 065105 (2017)

Semi-inclusive MEC results - electron scattering

- ▶ Our model is among the few that allow for a microscopic and fully relativistic computation of 2p2h semi-inclusive contributions

Electron scattering validation *Belocchi, Barbaro, De Pace, Martini, PRC109 (2024)*

Data: *Ryckebusch et al, PLB 333, 310 (1994)* Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV



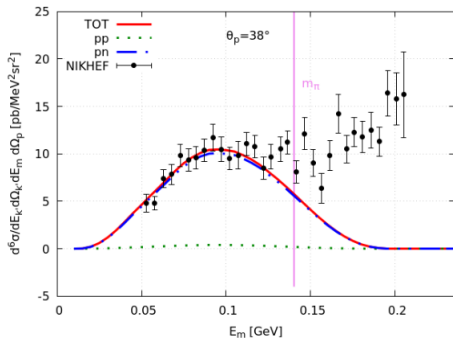
- ▶ Direct and exchange contribution included (exchange $\sim 12\%$ of direct, $\sim 13\%$ of total)
- ▶ Δ most important contribution ($\sim 50\%$), π and $\pi - \Delta$ similar
- ▶ **Very good agreement with data below $E_m \simeq 130$ MeV**
- ▶ For $E_m > 130$ MeV other processes start to contribute: π production via Δ excitation

Isospin separation

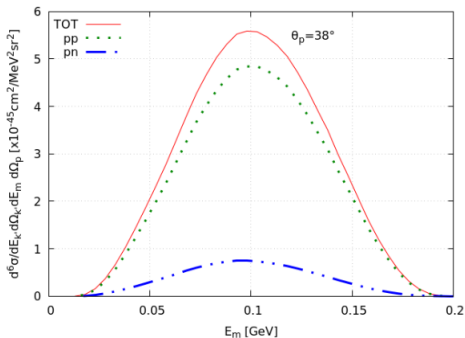
Isospin separation of the **2-nucleons final state**

Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV

Electron scattering



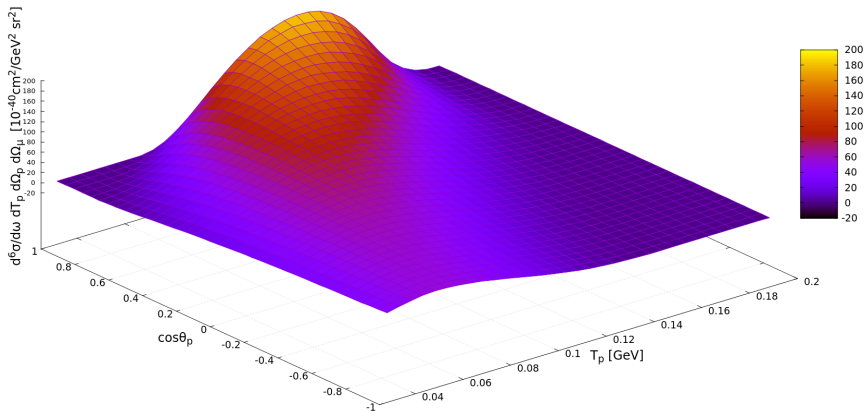
Neutrino CC scattering



- ▶ **Final** pn states for EM scattering and pp for CC EW dominate
- ▶ **Initial** pn states give the major contribution

Fixing incident and four-momentum transfer we can span over the **detected particle phasespace**

$$^{12}\text{C} \quad E_{\nu_{\mu}}=750 \text{ MeV}, \quad \omega=200 \text{ MeV}, \quad \theta_{\mu}=15, \quad \phi_p=0$$



Predictions for the same kinematical conditions as in *T. Van Cuyck et al., PRC 95 (2017)* and in *K. Niewczas, PhD thesis (2023)* → Similar results

Summary

- ▶ SuSAv2 is an excellent tool to reproduce inclusive data
- ▶ Semi-inclusive predictions are provided by ED-RMF (QE) and the present model ($2p2h$)
- ▶ Semi-inclusive EM $2p2h$ theoretical predictions are in a very good agreement with available data, providing a proof of the importance of this process and of the MEC model validity
- ▶ Semi-inclusive EW $2p2h$ theoretical predictions are among the few available on the market, not extracted in an effective way from inclusive computations

Work in Progress for $2p2h$ channel:

- ▶ Further investigation of the other TL and L nuclear responses
- ▶ Evaluate semi-inclusive cross-section vs the TKI variables, that mix leptonic and hadronic momentum variables
- ▶ Provide EW ν -flux folded predictions and compare with semi-inclusive data towards which the experimental community is moving to (T2K upgrade, Fermilab argon program)
- ▶ Implementation in Monte Carlo event generators

Thanks for the attention!

Backup

E_ν Reconstruction

We only know, in a reliable way, outgoing lepton kinematics. Modelling the nuclear initial state, it's possible to reconstruct the $\overline{E_\nu}$, using the inclusive CCQE formula:

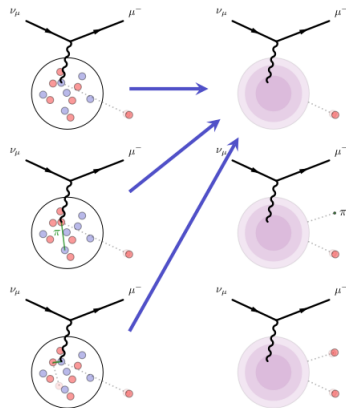
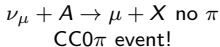
$$\overline{E_\nu} = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2(m_n - E_b - E_\mu + p_\mu \cos \theta)}$$

- ▶ There are biases due to Fermi motion inside the nucleus
- ▶ This formula doesn't work for other channels

But what do we see? Event topology



Classification based on detected final particles



The general EM electron-nucleus cross-section formula is:

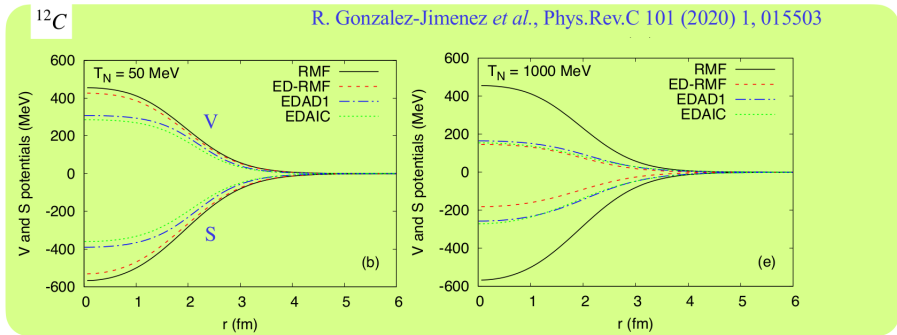
$$\frac{d\sigma}{dE_{k'} d\Omega_{k'}} = \underbrace{\frac{\alpha^2 |\mathbf{k}'|}{Q^2 E_k}}_{\sigma_{Mott}} \nu_0 (2\pi)^3 \frac{L_{\mu\nu}}{\nu_0} W_A^{\mu\nu}$$

Nuclear Hadronic Tensor: described using a Nuclear Model

$$W_A^{\mu\nu} := \sum_X \langle A | (J_A^\mu)^\dagger | X \rangle \langle X | J_A^\nu | A \rangle \delta^4(M_A + q - P_X)$$

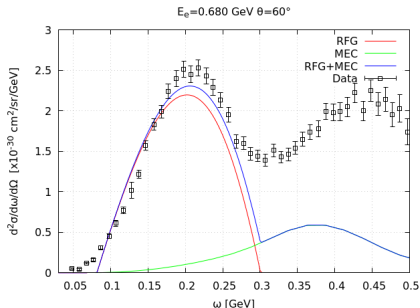
If the sum is performed over every possible final state X , W_A describes an inclusive process

FSI can be described in several ways, introducing an outgoing nucleon energy dependency



- ▶ Optical Potentials: energy dependent A-(in)dependent **EDAD1 (EDAI)**
- ▶ RMF: energy independent potentials
- ▶ ED-RMF: energy dependent potentials

Increasing the transferred energy, two nucleons can be emitted: **2p-2h**



Features:

- ▶ Leptonic probe interacts with a nucleon in a correlated pair of nucleons exchanging mediator (III order process, not FSI)
- ▶ Leptonic probe interacts directly with the exchanged virtual meson between two nucleons, exciting them both



Meson Exchange Current: two-body current

Effective Field Theory to describe nucleons-mesons interaction:

E. Hernandez et al, Mod.Phys.Lett.A 23 (2008)

Non-Linear σ -model

Non-Linear: chirality is not like other simmetries, i.e. isospin

Lagrangian invariant under chiral transformation, constructed starting from pionic fields and isospin operators instead γ_5 and isospin operators

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - M)\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 + \mathcal{L}_{\text{int}}^\sigma$$

$$\begin{aligned} \mathcal{L}_{\text{int}}^\sigma = & \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi - \frac{1}{6f_\pi^2}\left[\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})\right] \\ & + \frac{m_\pi^2}{24f_\pi^2}(\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\phi}^2\frac{\vec{\tau}}{2}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{2}\vec{\phi}\right]\Psi + \mathcal{O}\left(\frac{1}{f_\pi^4}\right) \end{aligned}$$

Effective Field Theory to describe nucleons-mesons interaction:

E. Hernandez et al, Mod.Phys.Lett.A 23 (2008)

Non-Linear σ -model

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - M)\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 + \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\vec{\tau}(\partial_\mu\vec{\phi})\Psi + \mathcal{O}(1/f_\pi^2)$$

- ▶ Provides a description of a system composed by nucleons and pions
- ▶ Nucleon-pion vertex is dominated by pseudo-vector interaction
- ▶ πNN coupling is $\frac{g_{\pi NN}}{2m_N} = \frac{g_A}{2f_\pi}$ using the Goldberger-Treiman relation
- ▶ Ψ : nucleon, isospin doublet
- ▶ $\vec{\phi}$: pionic field, scalar isospin triplet, $\pi^\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm i\phi_2)$
- ▶ $\vec{\tau}$: isospin Pauli matrices, $\tau^\pm = \tau_1 \pm i\tau_2$

Physical pions in the Lagrangian:

$$\vec{\tau}\vec{\phi} = \frac{1}{\sqrt{2}}(\tau^+\pi^- + \tau^-\pi^+) + \tau_3\pi^0$$

To 'switch on' the EW interaction in the previous Lagrangian, the standard $SU(2)_L \times U(1)_Y$ local symmetry procedure is performed, with associated gauge bosons \vec{W}^μ and B^μ , followed by the physical separation between weak and EM interactions

Covariant derivatives

$$\text{Fermions: } D^\mu = \partial^\mu + ig \frac{\vec{\tau}}{2} \vec{W}^\mu + ig' Y B^\mu$$

$$\text{Pions: } \mathcal{D}^\mu \phi_i = \partial^\mu \phi_i - g \epsilon_{ijk} \phi_j W_k^\mu$$

with

$$\frac{\vec{\tau}}{2} \vec{W}_\mu = \frac{1}{\sqrt{2}} (\tau^+ W_\mu^- + \tau^- W_\mu^+) + \frac{\tau_3}{2} W_{3\mu} \quad W_\mu^\pm = \frac{1}{\sqrt{2}} (W_{1\mu} \pm iW_{2\mu})$$

EM interaction

$$ig \frac{\vec{\tau}}{2} \vec{W}^\mu + ig' Y B^\mu \rightarrow ie A^\mu$$

$$g \epsilon_{ij3} W_3^\mu \rightarrow e \epsilon_{ij3} A^\mu$$

$W^\mu/A^\mu N \Delta$ transition

$$\mathcal{L} = g \bar{\Psi}_\mu \vec{T}^\dagger \vec{W}^\mu \Psi + h.c. \quad \rightarrow \quad \mathcal{L}_{EM} = e \bar{\Psi}_\mu T_3^\dagger A^\mu \Psi + h.c.$$

$N \Delta \pi$ transition

$$\mathcal{L} = \sqrt{\frac{3}{2}} \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger \partial^\mu \vec{\phi} \Psi + h.c.$$

- ▶ Ψ_μ : Rarita-Schwinger $\frac{3}{2}$ -spinor, with four isospin indices (μ) and four Dirac indices -omitted-
- ▶ T^\dagger : $\frac{1}{2} \rightarrow \frac{3}{2}$ isospin transition 4×2 operator

$$T_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & 0 & 1 & 0 \\ 0 & -1 & 0 & \sqrt{3} \end{pmatrix} \quad T_2 = -\frac{i}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 0 & 1 & 0 \\ 0 & 1 & 0 & \sqrt{3} \end{pmatrix} \quad T_3 = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$T_i T_j^\dagger = \frac{2}{3} \delta_{ij} - \frac{i}{3} \epsilon_{ijk} T_k$$

In the MEC currents, Δ appears always as a virtual particle

Δ propagator

$$G_{\alpha\beta}(p) = \frac{\mathcal{P}_{\alpha\beta}(p)}{p^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}$$

Where $\mathcal{P}_{\alpha\beta}$ is the projector over the physical states

$$\sum_{spin} u_\alpha(p)\bar{u}_\beta(p) = \mathcal{P}_{\alpha\beta}(p) = -(\not{p} + M_\Delta) \left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p_\alpha p_\beta}{M_\Delta} + \frac{p_\alpha\gamma_\beta - p_\beta\gamma_\alpha}{3M_\Delta} \right]$$

In the following the imaginary contribution to the responses arising from the Δ propagator is not included

Diagram 1: A nucleon (N) emits a pion (π) with momentum k and interacts with a photon (γ). The vertex factor is $-\frac{g_A}{2f_\pi}(k^2)\vec{\tau}k^\mu$.

Diagram 2: A nucleon (N) interacts with a photon (γ) and emits a pion (π). The vertex factor is $\frac{e}{\sqrt{2}}\frac{g_A}{2f_\pi}F_1^V(q^2)\gamma^\mu\gamma_5(\tau_-\pi^+ - \tau_+\pi^-)$.

Diagram 3: A pion (π) interacts with a photon (γ) and emits another pion (π). The vertex factor is $-ieF_1^V(q^2)(k_1^\mu - k_2^\mu)(\pi^+\pi^- - \pi^-\pi^+)$.

Diagram 4: A nucleon (N) interacts with a photon (γ) and emits a delta baryon (Δ_α). The vertex factor is $ieT_3^\dagger\Gamma^{\alpha\mu}$.

Diagram 5: A delta baryon (Δ_α) interacts with a photon (γ) and emits a nucleon (N). The vertex factor is $\sqrt{\frac{3}{2}}\frac{f^*}{m_\pi}\vec{T}k^\alpha$.

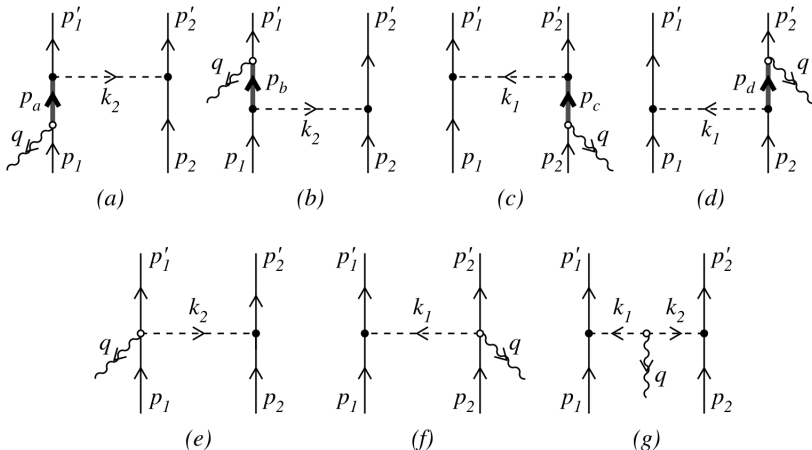
$\Gamma^{\alpha\mu}$ contains vector and axial form factors. EM case:

$$\Gamma_V^{\alpha\mu} = \left[\overbrace{\frac{C_{3V}}{M}(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu)}^{\text{dominant FF}} + \frac{C_{4V}}{M^2}(g^{\alpha\mu} q p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_{5V}}{M^2}(g^{\alpha\mu} q p - q^\alpha p^\mu) \right] \gamma_5$$

Meson Exchange Current

Describe the possibility that a probe excites two holes into two particles: *De Pace et al, Nucl.Phys.A 726 (2003)*

- Two-body ElectroMagnetic Meson Exchange Currents J_{2p2h}^{μ}



RFG 2p2h

Inclusive hadronic tensor, no hadronic particles detected:

$$W_{2p2h}^{\mu\nu} = (2\pi)^3 \frac{V}{4} \int_{|\mathbf{p}| \leq p_F} \frac{m_N d\mathbf{p}_1}{(2\pi)^3 E_{p1}} \frac{m_N d\mathbf{p}_2}{(2\pi)^3 E_{p2}} \frac{m_N d\mathbf{p}'_1}{(2\pi)^3 E_{p1'}} \frac{m_N d\mathbf{p}'_2}{(2\pi)^3 E_{p2'}} \tilde{W}_{2p2h}^{\mu\nu} \delta^4\{\theta_{PB}\}$$

Semi-inclusive hadronic tensor, one final proton detected (p'_1):

$$W_{2p2h}^{\mu\nu}(N'_1) = (2\pi)^3 \frac{V}{4} \int_{|\mathbf{p}| \leq p_F} \frac{m_N d\mathbf{p}_1}{(2\pi)^3 E_{p1}} \frac{m_N d\mathbf{p}_2}{(2\pi)^3 E_{p2}} \frac{m_N d\mathbf{p}'_2}{(2\pi)^3 E_{p2'}} \tilde{W}_{2p2h}^{\mu\nu} \delta^4\{\theta_{PB}\}$$

$$\tilde{W}_{2p2h}^{\mu\nu} = \sum_{\substack{\text{spin} \\ \text{isospin}}} \langle 2p2h | J_{2p2h}^\mu | F \rangle \langle F | J_{2p2h}^{\nu\dagger} | 2p2h \rangle \quad | 2p2h \rangle = b_{p'_2}^\dagger b_{p'_1}^\dagger b_{p1} b_{p2} | F \rangle$$

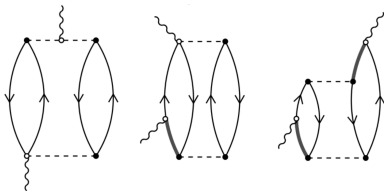
- ▶ J_{2p2h}^μ is a two-body operator, acting on the spin, isospin and momentum space
- ▶ Is possible to invert the two particles, obtaining another current that must be included

↓

$$J_{2p2h}^\mu = J_{2p2h}^\mu(p_1, p_2, p'_1, p'_2) - J_{2p2h}^\mu(p_1, p_2, p'_2, p'_1)$$

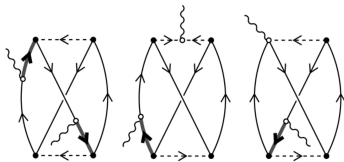
Combining two meson exchange currents \rightarrow **Two possibilities!**

- ▶ Direct term: $J^\mu J^{\nu\dagger}(p_1, p_2, p_{1'}, p_{2'}) + J^\mu J^{\nu\dagger}(p_1, p_2, p_{2'}, p_{1'})$



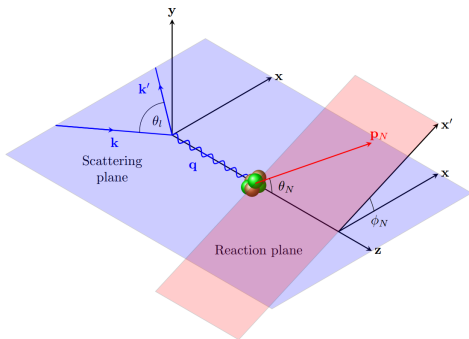
3 examples of the 16 EM many body direct diagrams

- ▶ Exchange term: $J^\mu(p_1, p_2, p_{1'}, p_{2'}) J^{\nu\dagger}(p_1, p_2, p_{2'}, p_{1'}) + \mu \leftrightarrow \nu$



3 examples of the 12 EM many body exchange diagrams

Inclusive calculation, fixing ω , q



- ▶ Integration over two particles and two holes momenta
- ▶ Four-momentum conservation
- ▶ q-system: nucleus symmetry \rightarrow azimuthal invariance

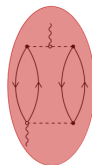
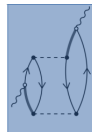
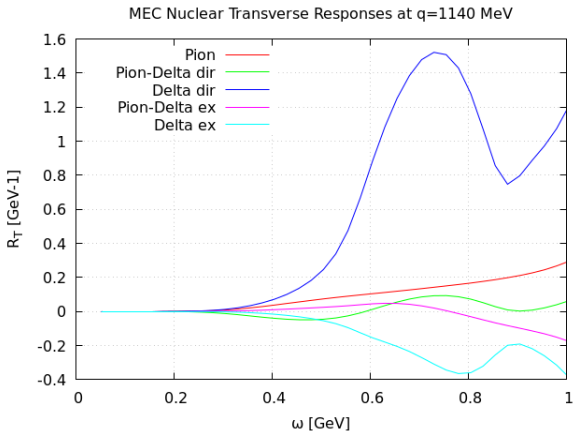
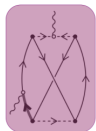
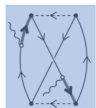
Non-vanishing responses

$$R_L = W_{2p2h}^{00} \quad R_T = W_{2p2h}^{11} + W_{2p2h}^{22}$$

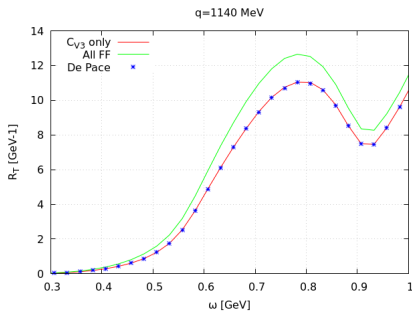
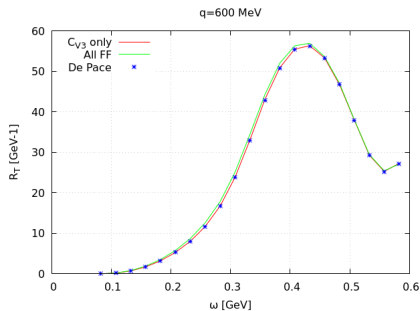
Electric charge conservation:

R_L includes contribution from W_{2p2h}^{00} , W_{2p2h}^{03} , W_{2p2h}^{33}

- ▶ RFG model in nuclear matter for Carbon target, $p_F = 228$ MeV
- ▶ Energy shift $E_s = 20$ MeV for each particle $\rightarrow E_s^{2p2h} = 2E_s$



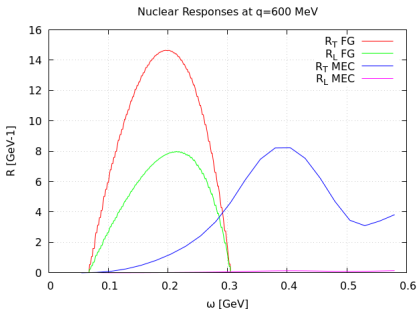
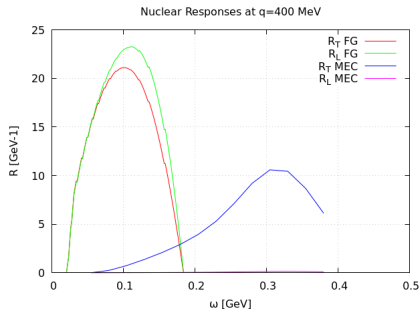
We tested the results with previous work, for iron *De Pace et al, Nucl.Phys.A 726 (2003)*



Delta Form Factors impact

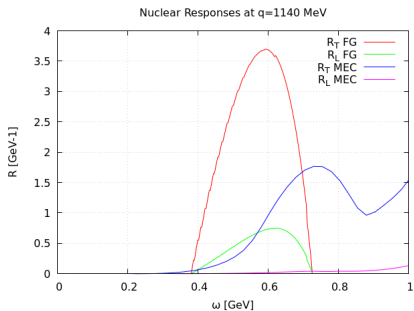
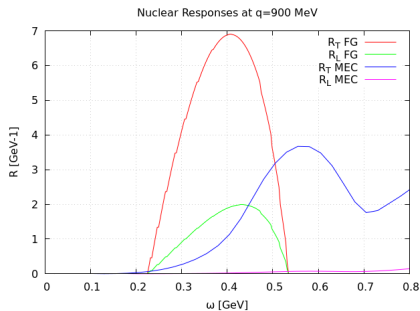
- In *De Pace et al* only C_{3V} was included. Here all Δ form factors are included: responses increase, more relevant at high q -values

MEC Transverse and Longitudinal Nuclear Responses at several q -values:



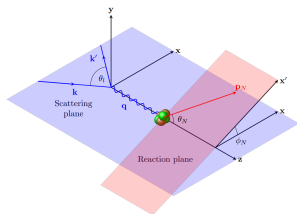
- ▶ EM MEC contribution is almost totally transverse with respect to q^μ
- ▶ MEC responses show a wide but defined peak, due to Δ role. Strength is about a half of the transverse QE responses
- ▶ At low q -values QE peak is well separated from MEC contributions. As q increases the two peaks overlap
- ▶ MEC responses are truncated when the exchanged q^μ becomes time-like

Higher q -values:



- ▶ Same considerations as before
- ▶ Ratio between RFG and MEC transverse responses is still the same

Semi-inclusive calculation, fixing ω , q , $\mathbf{p}'_1(p'_1, \theta_{p'_1}, \phi_{p'_1})$



- Integration over one particle and two holes momenta
- Four-momentum conservation
- q-system: nucleus symmetry
→ NO MORE azimuthal invariance

⇒ 5 dimension integration

Non-vanishing responses

$$R_L = W_{2p2h}^{00}$$

$$R_T = W_{2p2h}^{11} + W_{2p2h}^{22}$$

$$R_{TT} = W_{2p2h}^{22} - W_{2p2h}^{11}$$

$$R_{TL} = \frac{1}{2}(W_{2p2h}^{10} + W_{2p2h}^{01})$$

Electric charge conservation:

R_{TL} includes contributions from W_{2p2h}^{10} , W_{2p2h}^{13}

We tested our models with data showed in *J. Ryckebusch et al, Phys. Lett. B 333, 310 (1994), arXiv:nucl-th/9406015*.

Experimental settings, q -system

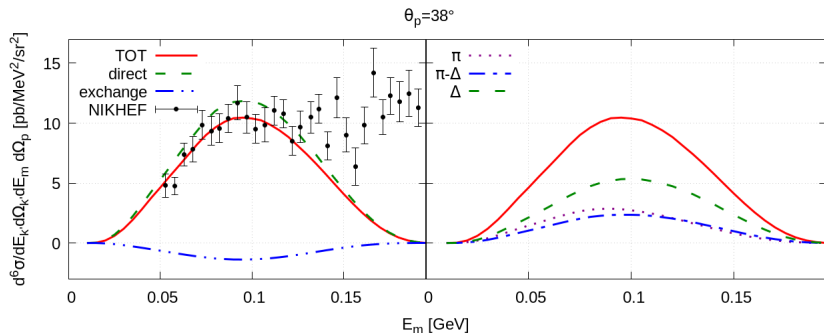
- ▶ Fixed incident and final lepton energy $E_k, E_{k'}$, scattering angle and \mathbf{p}'_1
- ▶ Scattering plane $x - z \rightarrow$ no contribution from $W^{\mu 2}, \mu \neq 2$
- ▶ Proton detected in the scattering-plane $\rightarrow \phi_{p_{1'}} = 0, \pi$
Note that $\phi_{p_{1'}}$ value affects the sign of R_{TL} contribution
- ▶ 6th differential cross-section

$$\frac{d\sigma}{d\omega d\Omega_{k'} dE_m d\Omega_{p_{1'}}}$$
$$E_m = \omega - T_{p_{1'}} \quad T_{p_{1'}} = E_{p_{1'}} - m_N$$

Semi-Inclusive Results

R_T and R_{TT} contributions included only

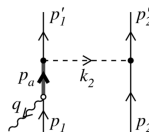
Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV



- ▶ Defined peak
- ▶ Direct and exchange contribution included (exchange $\sim 12\%$ of direct, $\sim 13\%$ of total)
- ▶ Δ most important contribution ($\sim 50\%$), π and $\pi - \Delta$ similar
- ▶ **Very good agreement with data below $E_m \simeq 130$ MeV**
- ▶ For $E_m > 130$ MeV other process starts to contribute: π production via Δ excitation

Isospin channel separation

Example: Δ forward current



- Isospin operator:

$$I_{\Delta F} = 2\tau_3^{(1)}\mathbb{1}^{(2)} - I_{V_3}$$

$$I_{V_3} = \frac{1}{2}(\tau_-^{(1)}\tau_+^{(2)} - \tau_+^{(1)}\tau_-^{(2)}) \quad I_{V_3}^\dagger |pp\rangle = 0$$

- Δ current is the only term contributing to pp channel (pionic current has I_{V_3} only)

$$I_{\Delta F}^\dagger |pp\rangle = 2|pp\rangle \quad I_{\Delta F}^\dagger |pn\rangle = 2|pn\rangle - 2|np\rangle$$

In the semi-inclusive channel, a proton is detected in the final state

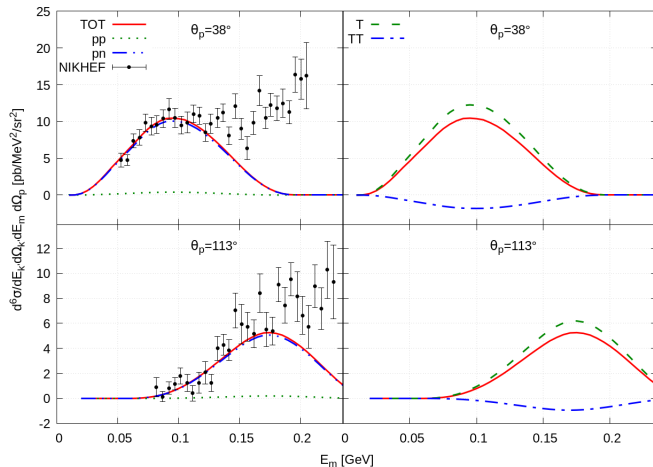
⇓

$|pn\rangle, |np\rangle$ both contribute

⇓

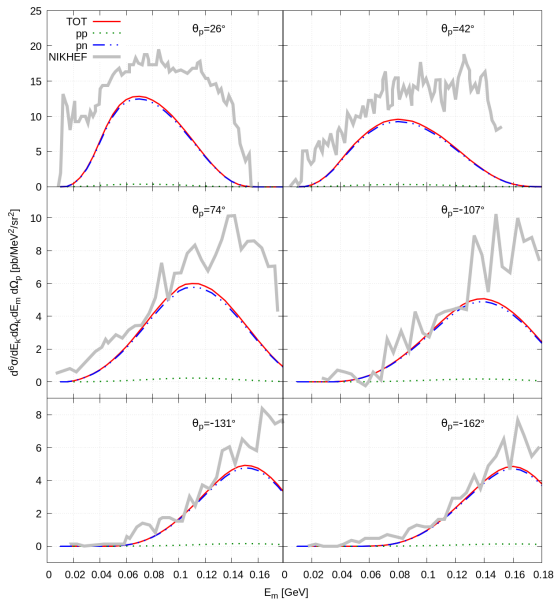
pn channel four times bigger than pp

Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV

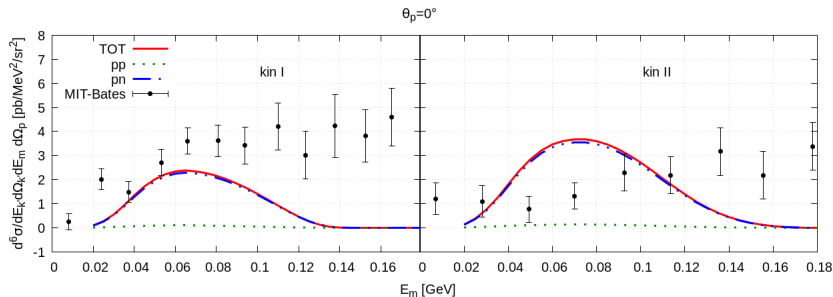


- ▶ pn dominates with respect to pp channel
- ▶ T contribution is the most important, TT reduces the strength for $\simeq 14\%$ (of T)

Kinematics: $E_k = 475$ MeV, $\omega = 212$ MeV, $q = 270$ MeV
 data from *L. J. H. M. Kester et al., Phys. Lett. B 344, 79 (1995)*



Higher Q^2 values, parallel kinematics ($\theta_p = 0$)
 data from *H. Baghaei et al., Phys. Rev. C 39, 177 (1989)*.



► kinI: $E_k = 460$ MeV, $\omega = 275$ MeV, $q = 401$ MeV

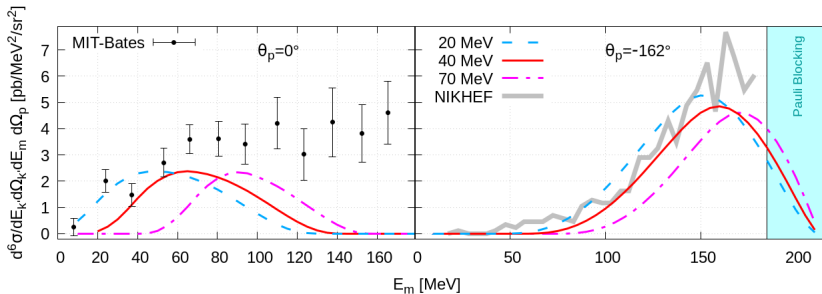
► kinII: $E_k = 647$ MeV, $\omega = 382$ MeV, $q = 473$ MeV

Discrepancies:

► Higher Q^2 → RFG unable to describe very off-shell probe interaction

► Higher q values → $E_s^{2p2h} = 40$ MeV probably not enough, FSI effects reduced

E_s^{2p2h} dependence and Pauli Blocking effect



- ▶ Increasing E_s^{2p2h} , shift toward higher E_m
- ▶ Effect more relevant in parallel kinematics, response accumulated and localized at lower E_m
- ▶ Best agreement with data in the range $E_s^{2p2h} = 20 - 40$ MeV
- ▶ Pauli Blocking, included via step function, truncates the responses at $E_m = \omega - T_F$

To extend the formalism to electronweak sector several changes must be made

Neutrino-nucleus CC cross-section formula

$$\frac{d^2\sigma}{dE_{k'} d\Omega_{k'}} = \underbrace{\frac{G_F^2 \cos^2 \theta_C}{8\pi^2} \frac{|\mathbf{k}'|}{E_k}}_{\sigma_0} \nu_0 (2\pi)^3 \frac{L_{\mu\nu}}{\nu_0} W_A^{\mu\nu}$$

Inclusive non-vanishing responses

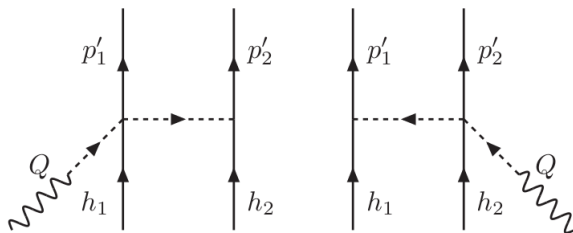
$$R_{CC} = W_{2p2h}^{00} \quad R_{CL} = -\frac{1}{2} (W_{2p2h}^{03} + W_{2p2h}^{30}) \quad R_{LL} = W_{2p2h}^{33}$$

$$R_T = W_{2p2h}^{11} + W_{2p2h}^{22} \quad R_{T'} = -\frac{i}{2} (W_{2p2h}^{12} - W_{2p2h}^{21})$$

No electric charge conservation, so no grouped responses (due to axial contributions)

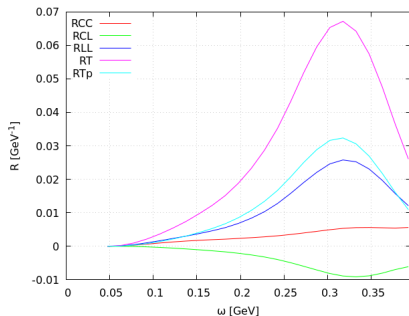
MEC are also modified, and new interactions are active

- ▶ Axial part and vector-axial interference
- ▶ New kind of interaction: pion-pole

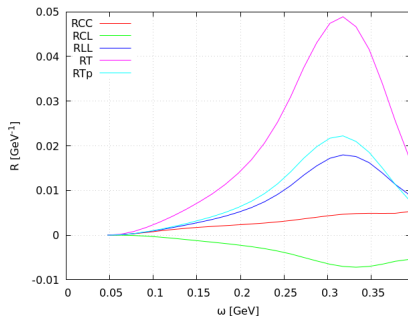


- ▶ Different isospin operators, especially important when separating different isospin channel in the final state

EW MEC responses for $q = 400$ MeV, $p_F = 225$ MeV.



Left Panel: direct responses



Right Panel: direct + exchange responses