$\frac{1}{2}$ Fermilab $\circled{1}$ ENERGY \circledcirc Science

Energy reconstruction and final-state interactions in proton knockout

Alexis Nikolakopoulos NOW 2024, 7th September 2024

Neutrino-nucleus cross section in oscillation analysis

A measurement in a neutrino experiment:

$$
\frac{dN}{dX} = \int dE_{\nu} \Phi(E_{\nu}) K(E_{\nu}, X)
$$

Is an integral transform with complicated kernel

$$
\frac{dN}{d|\vec{p}_1|} = \int \frac{d\Omega_1 d\vec{p}_2 \cdots d\vec{p}_n}{\text{Phase space(E)}} \frac{\epsilon(\vec{p}_1, \cdots, \vec{p}_n)}{\text{efficiency}} \int dE_{\nu} \Phi(E_{\nu}) \frac{d\sigma(E_{\nu})}{d\vec{p}_1 \cdots d\vec{p}_n}
$$

Neutrino-nucleus cross section in oscillation analysis

A measurement in a neutrino experiment:

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\frac{dN}{dX} = \int dE_{\nu} \Phi(E_{\nu}) K(E_{\nu}, X)
$$

Is an integral transform with complicated kernel

Oscillation analysis

Constrain and determine uncertainty on K(E,X)

Compare predictions with different ɸ(E) to measurement

Uncertainty, constraints, and flexibility in K(E,X) Determine possible precision of analysis

Neutrino-nucleus cross section in oscillation analysis

MiniBooNE use a 1-d observable

Width and model-dependence of distribution affect oscillation analyses

More observables → more information

[arxiv:1808.07520]

Smearing determined by inclusive double differential CS

From inclusive to semi-inclusive with LArTPCs → 1μ1p events

$$
\langle \frac{d^6 \sigma}{\mathrm{d} k_l \mathrm{d} \Omega_l \mathrm{d} p_N \mathrm{d} \Omega_N} \rangle = \int dE \phi(E) \frac{d^6 \sigma(E)}{\mathrm{d} k_l \mathrm{d} \Omega_l \mathrm{d} p_N \mathrm{d} \Omega_N} \quad X = (K_\mu, K_\nu)
$$

Missing energy

Missing momentum *| p^m*

 $T_m = E_i - E_i - T_N - T_B$ $|p_m| = |k_r - k_{l'} - k_{N'}|$

Exclusive electron scattering: Missing energy distributions

Direct nucleon knockout dominated by distinct shell-model peaks

 \rightarrow Residual system with low excitation energy E_m

'Good kinematics' ($\theta_{\sf N}$ = 80) $\;\rightarrow$ cross low-E_m peaks at small ${\sf p}_{\sf m}$

Good kinematics → Energy estimator with narrow uncertainty! Sub-percent energy reconstruction → [arxiv:2104.017101]

'Good kinematics'→ percent-level energy reconstruction [arxiv:2104.017101]

!Problem! 2 – nucleon knockout, pion-production, final-state interactions, … Populate the high-E^m region

Neutrino scattering: energy reconstruction for 1μ1p events

[F**] This talk: Final-state interactions in nucleon knockout**

Based on

[A. Nikolakopoulos , A. Ershova, R. Gonzalez-Jimenez, J. Isaacson, A.M. Kelly, K. Niewczas, N. Rocco, F. Sanchez, **arxiv:2406.09244**]

[A. Nikolakopoulos , R. Gonzalez-Jimenez, N. Jachowicz, K. Niewczas, F. Sanchez, J.M. Udias, **PRC 105, 054603**]

(Other efforts, e.g. electron-muon neutrino interactions in extra slides)

200 300 100 400 500 600 700 0 800

!Problem! 2 – nucleon knockout, pion-production, final-state interactions, … Populate the high-E^m region

Final-state interactions in exclusive (e,e'p) : optical potential

→ Outgoing proton does not exchange energy with residual system

Final-state interactions in exclusive (e,e'p) : optical potential $\mathcal{J}_{\kappa}^{m_j}(Q,P_N)=\int\mathrm{d}\mathbf{p}\;\overline{\psi}(\mathbf{p}+\mathbf{q},\mathbf{k}_N,s_N)\; \mathcal{O}^{\mu}\;\psi_{\kappa}^{m_j}(\mathbf{p})$

Final-state : solution of Dirac equation in optical potential

Final-state interactions in exclusive (e,e'p) : optical potential $\mathcal{J}_{\kappa}^{m_j}(Q,P_N)=\int\mathrm{d}\mathbf{p}\;\overline{\psi}(\mathbf{p}+\mathbf{q},\mathbf{k}_N,s_N)\; \mathcal{O}^{\mu}\;\psi_{\kappa}^{m_j}(\mathbf{p})$

Final-state : solution of Dirac equation in optical potential

The optical potential **removes nucleons that undergo inelastic FSI ↔**

In neutrino experiments want to know **where the nucleon goes**

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Where do the protons go ?: Intranuclear Cascade model (INC)

- ED-RMF FSI in inclusive interactions

-INC FSI for (semi-)exclusive channels

- ROP FSI in single exclusive channel

Production of final-state $|X\rangle = |p\rangle|^{39}Ar^*\rangle$

$$
|\mathcal{M}|^2 \approx |\sum_{\alpha} \langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle \langle \psi_{\alpha} | X \rangle|^2, \qquad \text{Restrict to 1-body operator}
$$

$$
\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 |\langle \psi_{\alpha} | X \rangle|^2 \longrightarrow \text{ Classical approximation}
$$

$$
\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle|^2 P(X|\alpha). \qquad \text{Intranuclear Cascade}
$$

Where do the protons go ?: Intranuclear Cascade model (INC)

FSI in single exclusive channel - ED-RMF FSI in inclusive -INC FSI for relevant (semi-)exclusive channels

Production of final-state $|X\rangle = |p\rangle|^{39}Ar^*\rangle$

Can benchmark the INC with Optical potential with same nuclear model For direct proton knockout

- ROP

Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

A. Nikolakopoulos \bullet , ^{1,2,*} R. González-Jiménez \bullet ,³ N. Jachowicz,¹ K. Niewczas, ^{1,4} F. Sánchez \bullet ,⁵ and J. M. Udías \bullet ³

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ROP and INC **agree at large T^p** but **large disagreement for small Tp**

Neutrino-induced proton knockout from argon in MicroBooNE

[Arxiv:2406.09244]

- **Flux-folded results for MicroBooNE**
- **ACHILLES, INCL, NEUT, and NuWro INC models**
- **Large set of kinematic distributions for comparison**
- **Detailed comparisons in backup slides**

60 ${\rm d}\sigma/{\rm d}\delta P_T$ (10⁻³⁸ cm²/GeV) **RPWI** $\rm cm^2/deg)$ $(10^{-39}$ cm²/deg) 50 no FS EDRMF **ROP** 40 $(10^{-39}$ 30 $\overline{2}$ 20 $d\sigma/d\delta\phi_T$ $d\sigma/d\alpha_T$ 10 0 0 200 400 600 60 120 180 θ 60 120 180 Ω Ω δP_T (GeV) α_T (deg) $\delta \phi_T$ (deg) Ω Ω 1Ω

ACHILLES

Some findings:

- **Agreement depends on input calculation (ED-RMF ↔ RPWIA)**
- **Large differences between INCs (low Tp & treatment of correlations)**
- **No full agreement between any INC and ROP**

RDWIA calculations with realistic spectral functions for MicroBooNE

See: [J. M. Franco-Patino et al. PRD 109, 013004] & [R. Gonzalez-Jimenez et al. PRC 105, 025502]

RDWIA calculations with spectral functions for MicroBooNE

$$
L_{\mu\nu}\left\{\sum_{\kappa} N_{\kappa}\rho_{\kappa}(E_m) H_{\kappa}^{\mu\nu}(Q, P_N) + \rho_{corr}(E_m) H_{corr}^{\mu\nu}(Q, P)\right\}
$$

Choices of **N κ** and **ϱ(E^m)**

- ⁴⁰Ar spectral functions [Butkevich PRC 85, 065501] & [Jlab, PRD 107, 012005]
- 48 Ti from Jlab [PRD 107, 012005]
- \bullet 56Fe [Benhar et al. NPA 579, 493]
- $40Ca$ [Butkevich PRC 85, 065501]

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Large variation in E^m profiles to check sensitivity of observables

Sensitivity to variations in the spectral functions: PWIA calculations [Arxiv:2406.09244]

Observables for MicroBooNE flux-averaged signal

-Negligible differences between different spectral-functions **for observables i** that do not correlate $\bm{{\mathsf{p}}}_{{}_{\mathsf{p}}}$ and $\bm{{\mathsf{p}}}_{{}_{\mathsf{p}}}$ **-Mild sensitivity only to p^m distributions**

- Current MicroBooNE data not sensitive to missing energy distributions

Comparisons to MicroBooNE data

[Arxiv:2406.09244]

Isolate the effect of the INC

 \rightarrow exact same inputs for each

Realistic final-state nucleon wave-functions

 \rightarrow 10% effect (Can generate events + Implementation in NEUT

Comparisons to MicroBooNE data

[Arxiv:2406.09244]

Isolate the effect of the INC

 \rightarrow exact same inputs for each

Realistic final-state nucleon wavefunctions

 \rightarrow 10% effect

Cut on α

- \rightarrow reduce rescattering for small α_{τ}
- \rightarrow Still sizeable contribution of resc
- \rightarrow ROP alone cannot reproduce data

Low α^T dPT underpredicted

- \rightarrow Increase axial coupling ??
- → **Need to include the interference with 2-body currents!**

- **1-proton knockout could provide excellent energy-resolution → Backgrounds non-trivial !**
- **RDWIA + realistic spectral function + INC**
	- **= most comprehensive description of single-nucleon knockout**
	- **→ No full agreement with recent MicroBooNE data**

Where to go from here?

Theory

* **Interference with 2-body currents needs to be included !**

See e.g. [T. Franco Munoz et al. PRC108, 064608] [Lovato et al. , 2312.12545]

- * **Two-nucleon and single pion production (SPP) contributions**
- **→ ACHILLES: will soon include 2-body interference + SPP with full FSI**
- → **NEUT: will include RDWIA calculations with SF**
- **→ NuWro: new SPP [2405.0512] and 2-N [K. Niewczas Phd] implementations**

Experiment

- *** High statistics in SBND can test 'good event' selection in 1μ1p in LarTPC**
- *** New electron scattering data in non-trivial kinematic regions**
	- **→ A1 at MAMI, e4ν at Jlab take electron data for ν program (+ more facilities ?!)**

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Other stuff

Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

-Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$
\mathcal{J}_{\kappa}^{m_j}(Q, P_N) = \int \mathrm{d} \mathbf{p} \; \overline{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_N, s_N) \; \mathcal{O}^{\mu} \; \psi_{\kappa}^{m_j}(\mathbf{p})
$$

- Relativistic Plane Wave Impulse Approximation (RPWIA)

$$
\mathcal{J} = (2\pi)^{3/2} \; \overline{u}(\mathbf{k}_N, s_N) \; \mathcal{O}^{\mu} \; \psi_{\kappa}^{m_j}(\mathbf{k}_N - \mathbf{q})
$$

- Plane-Wave Impulse Approximation (PWIA)

The initial state is assumed proportional to a positive-energy spinor:

$$
\psi_{\kappa}^{m_j}(\mathbf{p}) \propto f(|\mathbf{p}|)u(\mathbf{p})
$$

One obtains a factorized expression ('spectral function approach')

$$
\frac{d\sigma(E_{\nu})}{dp_{\mu}d\Omega_{\mu}d\Omega_{p}dp_{N}} = \frac{G_{F}^{2}\cos^{2}\theta_{c}}{(2\pi)^{2}}\frac{p_{\mu}^{2}p_{N}^{2}}{E_{\nu}E_{\mu}}\frac{M_{N}^{2}}{E_{N}\overline{E}}L_{\mu\nu}h_{s.n.}^{\mu\nu}S(E_{m},p_{m})
$$

Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

- **Energy-Dependent Relativistic Mean-Field (ED-RMF)**
- $\psi(\mathbf{p}+\mathbf{q},\mathbf{k}_N,s_N)$ Final-state in **real** Energy-Dependent potential → suitable for **FSI in inclusive** cross section 25 ϵ _i=730 MeV, θ _e=37.1 deg ED-RMF ${}^{12}C(e,e')X$ 20 dσ/(dΩdω) [nb/(MeV sr)] RPWIA 15 10 5 $\mathbf 0$ 0.1 0.2 0.3 0.4 0 0.5 0.6 0.7 춮 Fermilab

[R. Gonzalez-Jimenez et al Phys. Rev. C 100, 045501 (2019)]

Inclusive electron scattering off a RMF nucleus

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

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Nucleon knockout from a RMF nucleus: consistent initial & final-states

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

- Consistent states are crucial at low energies
	- \rightarrow provided by RMF \rightarrow ED-RMF = RMF

by construction at low-energy

• Consistent states are orthogonal => Pauli-Blocking

'Pauli-Blocked' RPWIA (PB-RPWIA): orthogonalize with respect to bound-states

$$
|\Psi^{s_N}(\mathbf{p}_N)\rangle = \left|\psi^{s_N}_{pw}(\mathbf{p}_N)\right\rangle - \sum_{\kappa,m_j} \left[C^{m_j,s_N}_{\kappa}(\mathbf{p}_N)\right]^\dagger \left|\psi^{m_j}_{\kappa}\right\rangle \qquad C^{m_j,s_N}_{\kappa}(\mathbf{p}_N) \equiv \left|\psi^{s_N}_{pw}(\mathbf{p}_N)\right|\psi^{m_j}_{\kappa}\rangle.
$$

Electron- and muon neutrino interactions

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 $E_{\nu} = 236$ MeV

Electron- and muon neutrino interactions at low energy

At low energy-momentum transfer $\mathsf{v}_{_{\mathsf{p}}}$ cross sections are larger than $\mathsf{v}_{_{\mathsf{e}}}$

$$
q = \sqrt{E_{\nu}^2 + P_l^2 - 2\cos\theta_l E_{\nu} P_l} \approx E_{\nu} - \sqrt{(E_{\nu} - \omega)^2 - m_l^2}
$$

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Electron- and muon neutrino interactions at low energy
 $E_{\nu} = 236 \text{ MeV}$

Electron- and muon neutrino interactions at low energy

[A. Nikolakopoulos et al. PRL 123, 123, 052501 (2019)]

Can be understood from the orthogonality with bound states

Uncertainties on the ω_{e}/ω_{μ} and $\nu_{e}/\bar{\nu}_{e}$ cross-section ratio from the modelling of nuclear effects at 0.2 to 1.2 GeV neutrino energies and their impact on neutrino oscillation experiments

> T. Dieminger, 1, * S. Dolan, 2, † D. Sgalaberna, 1, ‡ A. Nikolakopoulos, 3 T. Dealtry,⁴ S. Bolognesi,⁵ L. Pickering,⁶ and A. Rubbia¹ arXiv:2301.08065v3 [hep-ph] 5 Apr 2023

Assessing model-dependence: double ratios

$$
RR_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 1/Model 2}}(E_{\nu}, \theta) = \frac{R_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 1}}(E_{\nu}, \theta)}{R_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 2}}(E_{\nu}, \theta)}
$$

High-angle region in T2K is most relevant

Assessing model-dependence

Uncertainty from averaging model-differences *in Ratio* over predicted event rate

Quite robust when varying parameters within one model

Two 'sets': LFG and PWIA+SF and mean-field based models (or NEUT generator vs. other calculations ?)

Assessing impact on appearance experiment: bi-event plots

1) Not significant for $sin\delta_{\text{CB}}$

2) Increases degeneracy in $sin^2θ$ ₂₃

Magnitude 2-4% percent: uncertainty in line with values used in experiment

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Article

From inclusive to semi-inclusive one-nucleon knockout in neutrino event generators

Alexis Nikolakopoulos ^{1,*}, Steven Gardiner ¹, Afroditi Papadopoulou ², Stephen Dolan ³ and Raúl González-Jiménez ⁴

 $P(E_l, \theta_l, T_N, \Omega_N)$

Replace by 'factorized approach'

We get the GENIE version based on **the same** inclusive cross section!

We generate events for (e,e'p) in RDWIA with **real potential**

- Full consistent description of exclusive kinematics 1e1p
- Integrate over the proton \rightarrow get the correct inclusive cross section (=includes 'elastic' FSI!)
- For every event we **replace** the nucleon kinematics by the GENIE prediction (SuSAv2 implementation)

$$
\frac{d\sigma(E_{\nu})}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_{\nu}} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})
$$

Lost nucleon information \rightarrow Need to generate it in GENIE

1. Draw initial nucleon $\bm{p}_{\scriptscriptstyle{m}}$ from $p^{\scriptscriptstyle{2}}$ n(p) (e.g. LFG)

11.2. Compute
$$
E_m^2 = p_m^2 + M_N^2
$$

3. $E_{N} = E_{m} + \omega - E_{b}(q)$

$$
\frac{d\sigma(E_{\nu})}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_{\nu}} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})
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$$

Lost nucleon information \rightarrow Need to generate it in GENIE

1. Draw initial nucleon $\bm{p}_{\scriptscriptstyle{m}}$ from $p^{\scriptscriptstyle{2}}$ n(p) (e.g. LFG)

 0.5

$$
\frac{d\sigma(E_{\nu})}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_{\nu}} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})
$$

Lost nucleon information \rightarrow Need to generate it in GENIE

1. Draw initial nucleon $\bm{p}_{\scriptscriptstyle{m}}$ from $p^{\scriptscriptstyle{2}}$ n(p) (e.g. LFG)

19.2 Compute
$$
E_m^2 = p_m^2 + M_N^2
$$

\n3. $E_N = E_m + \omega - E_b(q)$

\n4. $k_N^2 = E_N^2 - M_N^2$

\n19.1 $|P_m + q| \neq k_N = \sqrt{E_N^2 - M_N^2}$

\n10.1 $|P_m + q| \neq k_N = \sqrt{E_N^2 - M_N^2}$

\n11.1 $|P_m + q| \neq k_N = \sqrt{E_N^2 - M_N^2}$

Serious differences in angular distributions!

$$
\frac{\mathrm{d}\sigma(E_{\nu})}{\mathrm{d}E_{l}\mathrm{d}\cos\theta_{l}}=G^{2}\frac{k_{l}}{E_{\nu}}L_{\mu\nu}\int\mathrm{d}\Omega_{N}\sum_{n,\kappa}H_{n,\kappa}^{\mu\nu}(\omega,q,\Omega_{N},E_{n,\kappa})
$$

Lost nucleon information \rightarrow Need to generate it in GENIE

Results for e4nu kinematics E=1.159 including the GENIE cascade!

Shape differences biggest in P_T and angular distributions

NuWro with SRC effect in Mean-free path

NuWro without SRC effect in Mean-free path

NuWro

ACHILLES with Formation time

ACHILLES without Formation time

NEUT

 NEUT

