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Energy reconstruction and final-state interactions in proton knockout

Alexis Nikolakopoulos NOW 2024, 7th September 2024

Neutrino-nucleus cross section in oscillation analysis

A measurement in a neutrino experiment:

$$\frac{dN}{dX} = \int dE_{\nu} \Phi(E_{\nu}) K(E_{\nu}, X)$$

Is an integral transform with complicated kernel

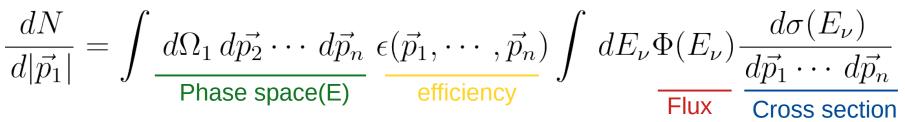
$$\frac{dN}{d|\vec{p_1}|} = \int \underbrace{d\Omega_1 \, d\vec{p_2} \cdots d\vec{p_n}}_{\text{Phase space(E)}} \underbrace{\epsilon(\vec{p_1}, \cdots, \vec{p_n})}_{\text{efficiency}} \int \frac{dE_{\nu} \Phi(E_{\nu})}{dE_{\nu} \Phi(E_{\nu})} \underbrace{\frac{d\sigma(E_{\nu})}{d\vec{p_1} \cdots d\vec{p_n}}}_{\text{Cross section}}$$

Neutrino-nucleus cross section in oscillation analysis

A measurement in a neutrino experiment:

$$\frac{dN}{dX} = \int dE_{\nu} \Phi(E_{\nu}) K(E_{\nu}, X)$$

Is an integral transform with complicated kernel



Oscillation analysis

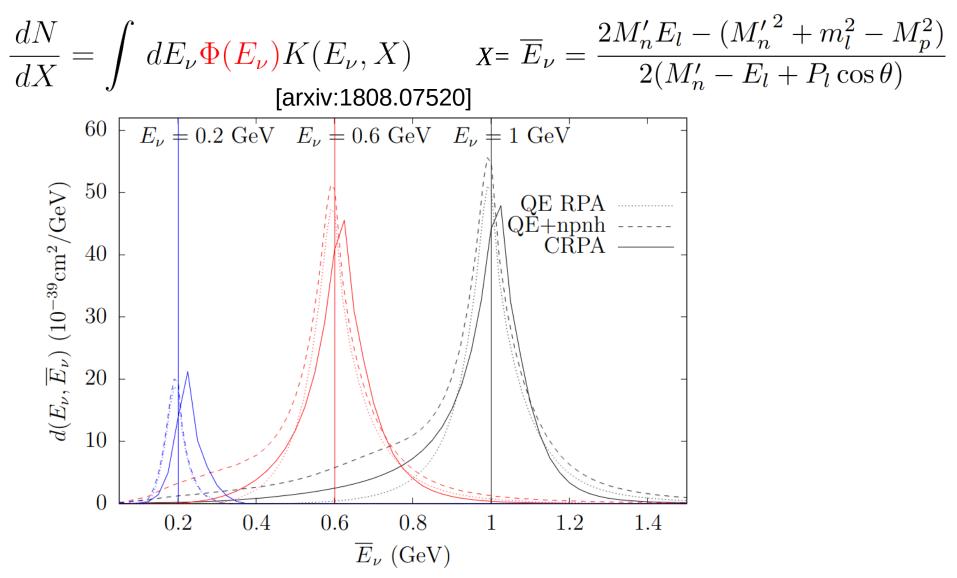
Constrain and determine uncertainty on K(E,X)

Compare predictions with different $\phi(E)$ to measurement

Uncertainty, constraints, and flexibility in K(E,X) Determine possible precision of analysis

Neutrino-nucleus cross section in oscillation analysis

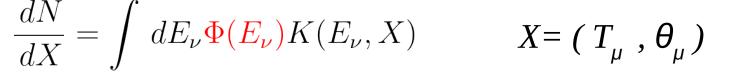
MiniBooNE use a 1-d observable

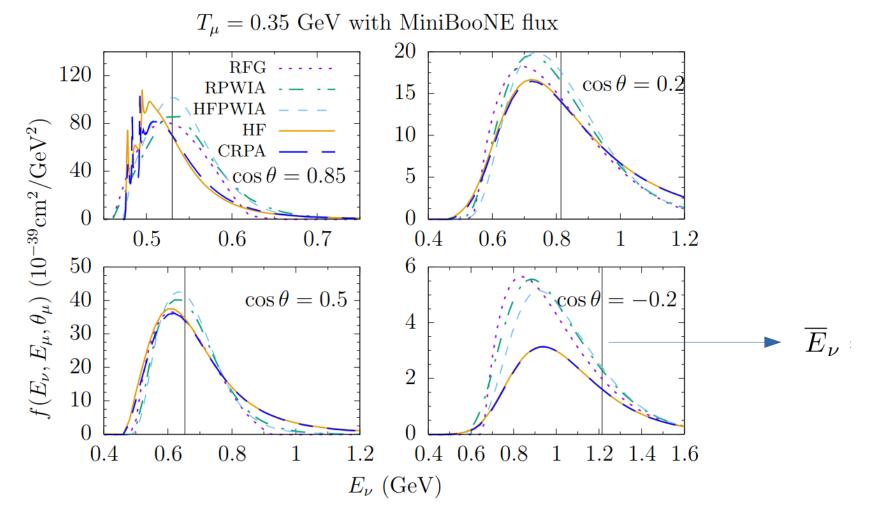


Width and model-dependence of distribution affect oscillation analyses

More observables \rightarrow more information

[arxiv:1808.07520]

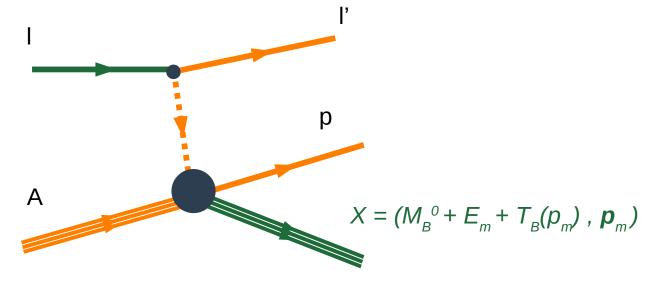




Smearing determined by inclusive double differential CS

From inclusive to semi-inclusive with LArTPCs \rightarrow 1µ1p events

$$\left\langle \frac{d^{6}\sigma}{\mathrm{d}k_{l}\mathrm{d}\Omega_{l}\mathrm{d}p_{N}\mathrm{d}\Omega_{N}}\right\rangle = \int dE\phi(E)\frac{d^{6}\sigma(E)}{\mathrm{d}k_{l}\mathrm{d}\Omega_{l}\mathrm{d}p_{N}\mathrm{d}\Omega_{N}} \quad X=(K_{\mu},K_{N})$$

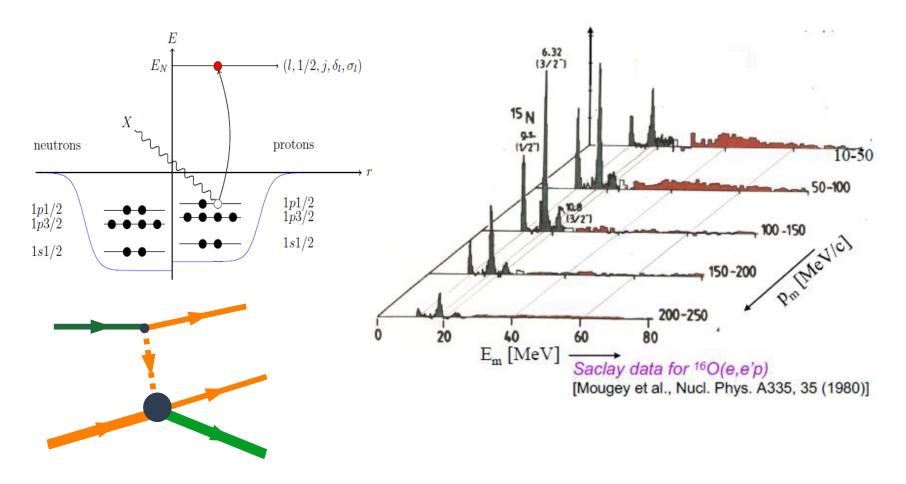


Missing energy

Missing momentum

 $E_m = E_l - E_{l'} - T_N - T_B$ $|\mathbf{p}_m| = |\mathbf{k}_l - \mathbf{k}_{l'} - \mathbf{k}_N|$

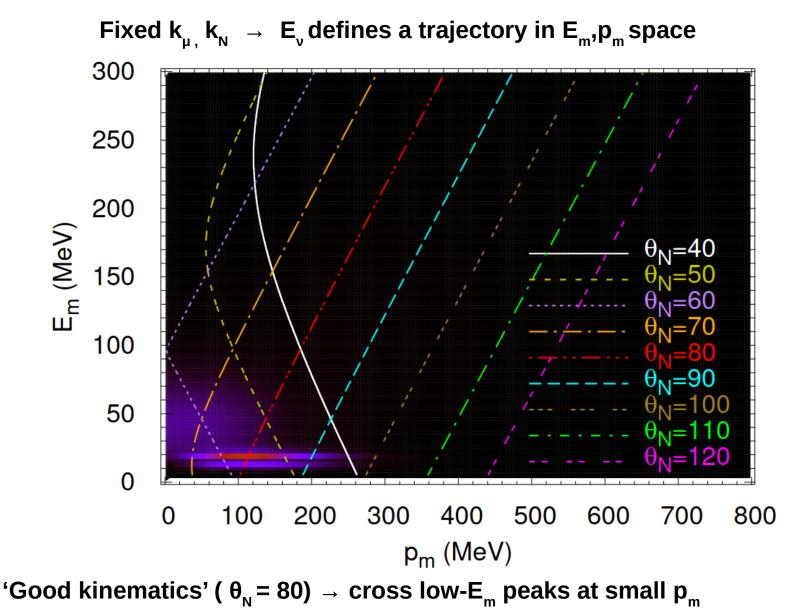
Exclusive electron scattering: Missing energy distributions

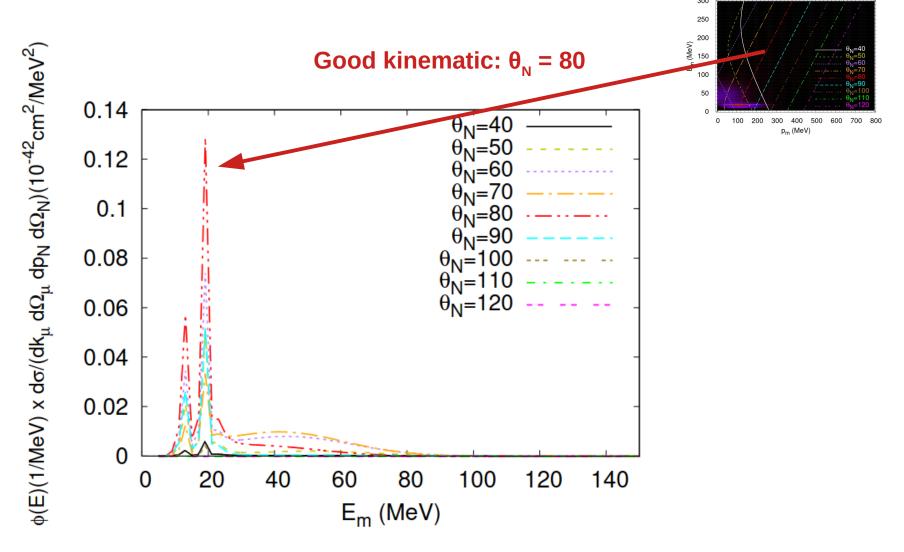


Direct nucleon knockout dominated by distinct shell-model peaks

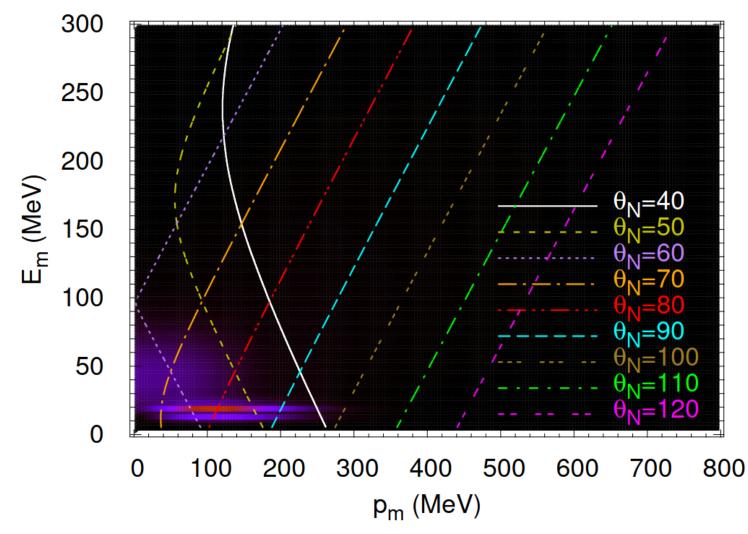
 \rightarrow Residual system with low excitation energy E_m



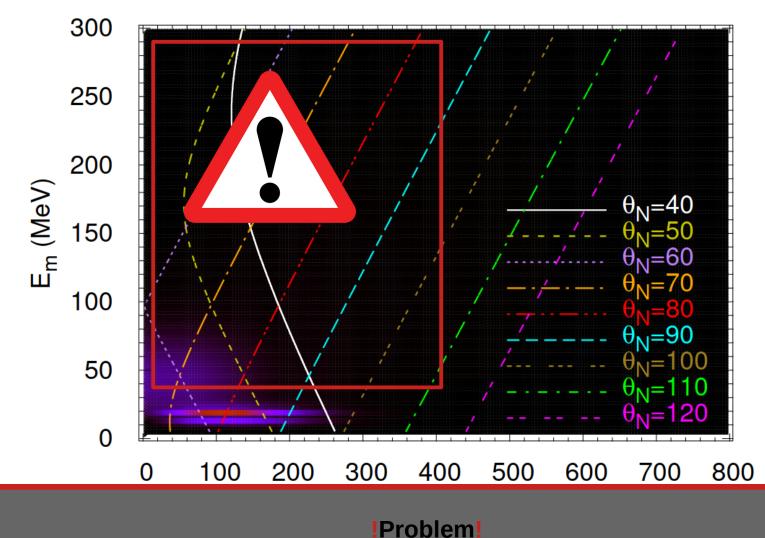




Good kinematics \rightarrow Energy estimator with narrow uncertainty! Sub-percent energy reconstruction \rightarrow [arxiv:2104.017101]



'Good kinematics' → percent-level energy reconstruction [arxiv:2104.017101]



2 – nucleon knockout, pion-production, final-state interactions, ... Populate the high- E_m region Neutrino scattering: energy reconstruction for 1u1n events

FF

This talk: Final-state interactions in nucleon knockout

Based on

[A. Nikolakopoulos , A. Ershova, R. Gonzalez-Jimenez, J. Isaacson, A.M. Kelly, K. Niewczas, N. Rocco, F. Sanchez, **arxiv:2406.09244**]

[A. Nikolakopoulos , R. Gonzalez-Jimenez, N. Jachowicz, K. Niewczas, F. Sanchez, J.M. Udias, **PRC 105, 054603**]

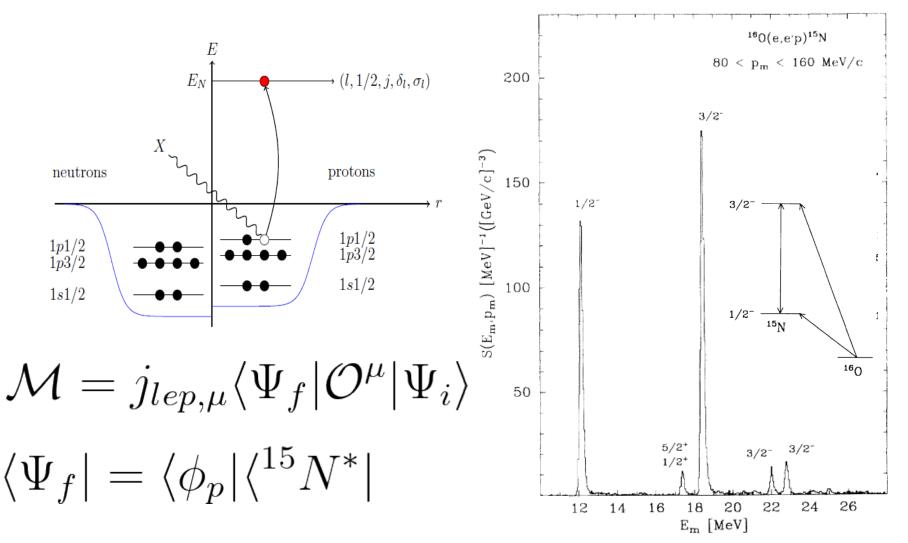
(Other efforts, e.g. electron-muon neutrino interactions in extra slides)

0 100 200 300 400 500 600 700 800

Problem

2 – nucleon knockout, pion-production, final-state interactions, ... Populate the high- E_m region

Final-state interactions in exclusive (e,e'p) : optical potential

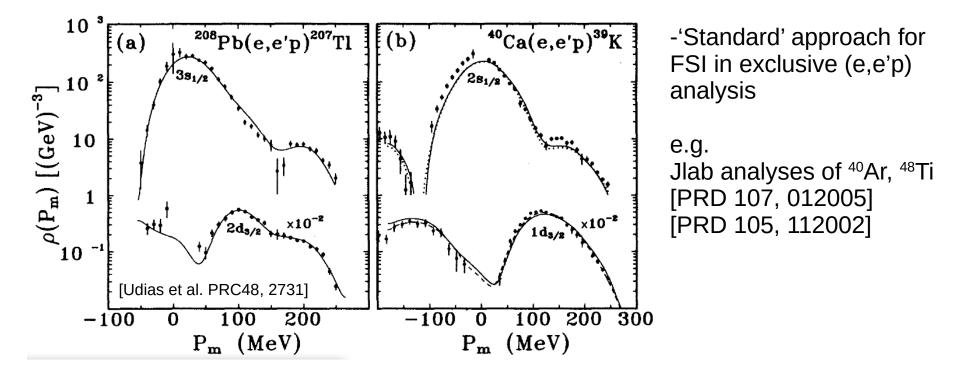


 \rightarrow Outgoing proton does not exchange energy with residual system



Final-state interactions in exclusive (e,e'p) : optical potential $\mathcal{J}_{\kappa}^{m_{j}}(Q, P_{N}) = \int d\mathbf{p} \ \overline{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_{N}, s_{N}) \ \mathcal{O}^{\mu} \ \psi_{\kappa}^{m_{j}}(\mathbf{p})$

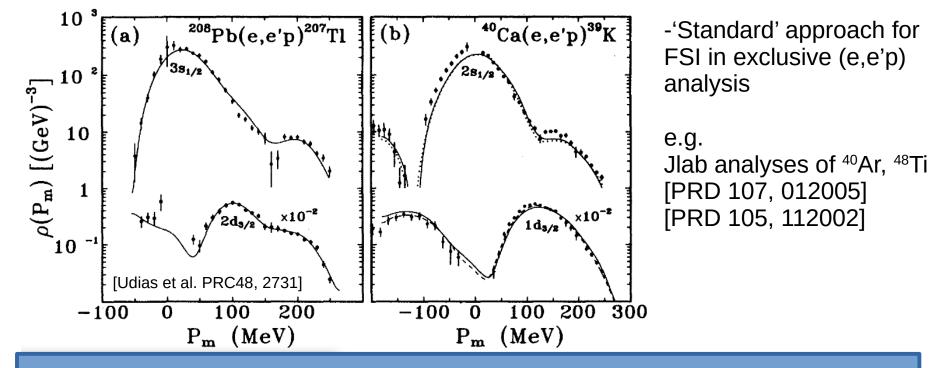
Final-state : solution of Dirac equation in optical potential





Final-state interactions in exclusive (e,e'p) : optical potential $\mathcal{J}_{\kappa}^{m_{j}}(Q, P_{N}) = \int d\mathbf{p} \ \overline{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_{N}, s_{N}) \ \mathcal{O}^{\mu} \ \psi_{\kappa}^{m_{j}}(\mathbf{p})$

Final-state : solution of Dirac equation in optical potential



The optical potential removes nucleons that undergo inelastic FSI

In neutrino experiments want to know where the nucleon goes

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Where do the protons go ?: Intranuclear Cascade model (INC)

- ED-RMF FSI in inclusive interactions -INC FSI for (semi-)exclusive channels - ROP FSI in single exclusive channel

Production of final-state $|X\rangle = |p\rangle|^{39} \text{Ar}^*\rangle$

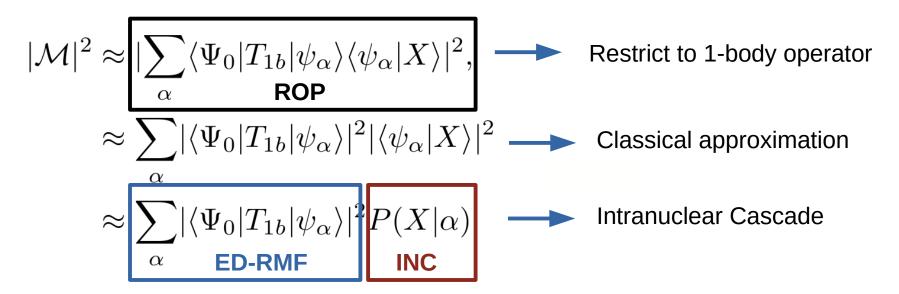
$$\begin{split} |\mathcal{M}|^2 &\approx |\sum_{\alpha} \langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle \langle \psi_{\alpha} | X \rangle |^2, \quad \text{Restrict to 1-body operator} \\ &\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle |^2 |\langle \psi_{\alpha} | X \rangle |^2 \quad \text{Classical approximation} \\ &\approx \sum_{\alpha} |\langle \Psi_0 | T_{1b} | \psi_{\alpha} \rangle |^2 P(X | \alpha). \quad \text{Intranuclear Cascade} \end{split}$$



Where do the protons go ?: Intranuclear Cascade model (INC)

- ED-RMF -INC FSI for relevant (semi-)exclusive channels FSI in single FSI in inclusive exclusive channel

Production of final-state $|X\rangle = |p\rangle|^{39} \text{Ar}^*\rangle$



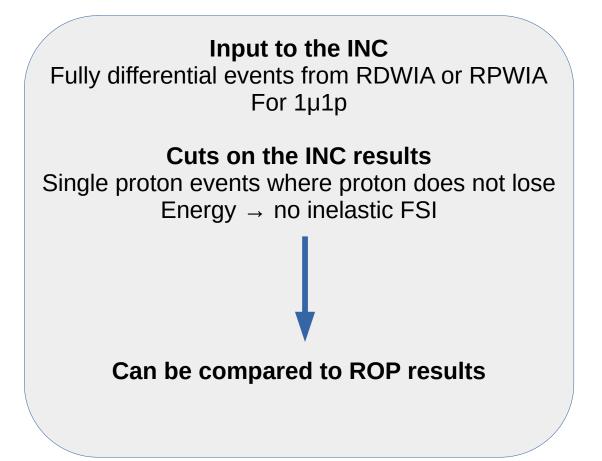
Can benchmark the INC with Optical potential with same nuclear model For direct proton knockout



- ROP

Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

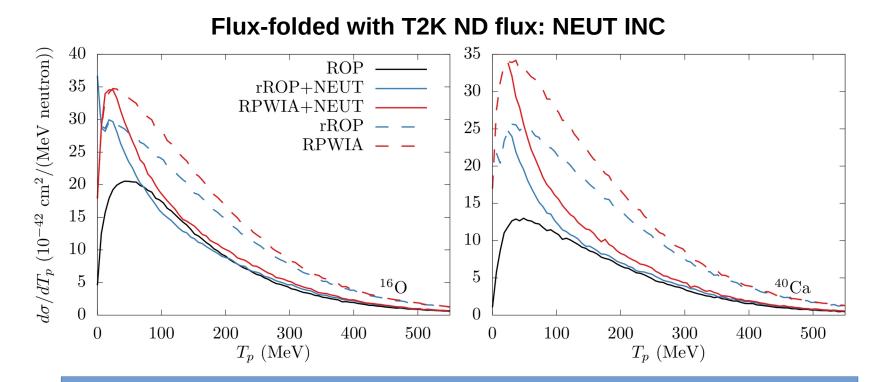
A. Nikolakopoulos⁽⁰⁾,^{1,2,*} R. González-Jiménez⁽⁰⁾,³ N. Jachowicz,¹ K. Niewczas,^{1,4} F. Sánchez⁽⁰⁾,⁵ and J. M. Udías⁽⁰⁾





Benchmarking intranuclear cascade models for neutrino scattering with relativistic optical potentials

A. Nikolakopoulos^(a),^{1,2,*} R. González-Jiménez^(a),³ N. Jachowicz,¹ K. Niewczas,^{1,4} F. Sánchez^(a),⁵ and J. M. Udías^(a)



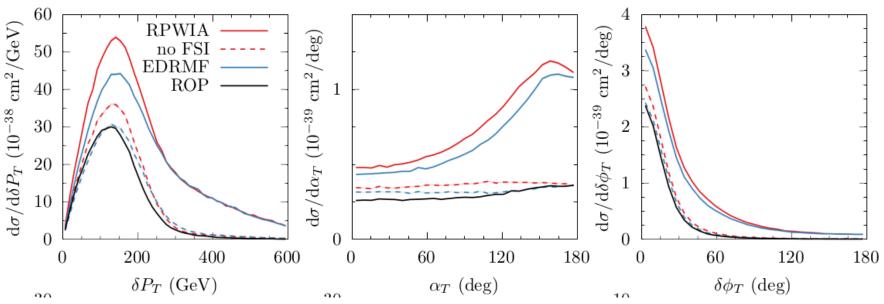
ROP and INC agree at large T_p but large disagreement for small T_p



Neutrino-induced proton knockout from argon in MicroBooNE

[Arxiv:2406.09244]

- Flux-folded results for MicroBooNE
- ACHILLES, INCL, NEUT, and NuWro INC models
- Large set of kinematic distributions for comparison
- Detailed comparisons in backup slides



ACHILLES

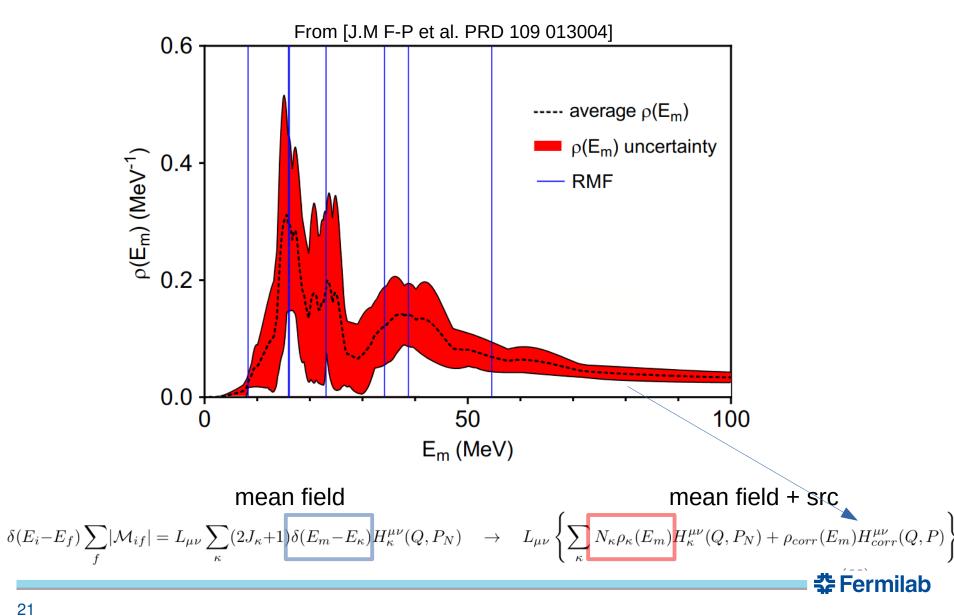
Some findings:

- Agreement depends on input calculation (ED-RMF \leftrightarrow RPWIA)
- Large differences between INCs (low T_n & treatment of correlations)
- No full agreement between any INC and ROP



RDWIA calculations with realistic spectral functions for MicroBooNE

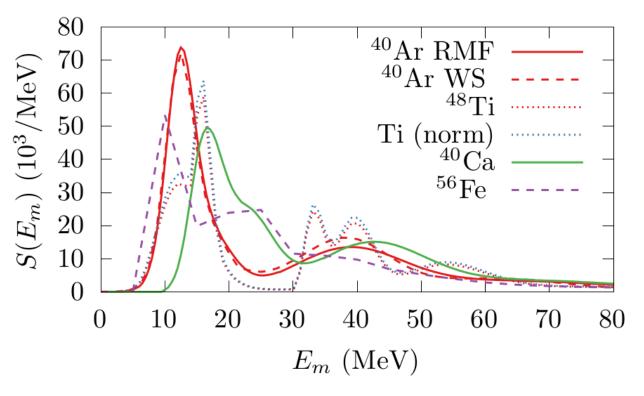
See: [J. M. Franco-Patino et al. PRD 109, 013004] & [R. Gonzalez-Jimenez et al. PRC 105, 025502]



RDWIA calculations with spectral functions for MicroBooNE

$$L_{\mu\nu}\left\{\sum_{\kappa}N_{\kappa}\rho_{\kappa}(E_m)H_{\kappa}^{\mu\nu}(Q,P_N)+\rho_{corr}(E_m)H_{corr}^{\mu\nu}(Q,P)\right\}$$

Choices of N_{κ} and $\varrho(E_m)$



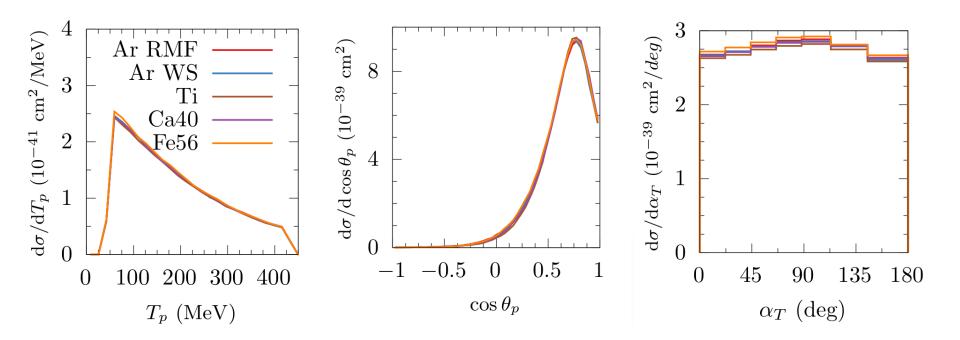
- ⁴⁰Ar spectral functions [Butkevich PRC 85, 065501] & [Jlab, PRD 107, 012005]
- ⁴⁸Ti from Jlab [PRD 107, 012005]
- ⁵⁶Fe [Benhar et al. NPA 579, 493]
- ⁴⁰Ca [Butkevich PRC 85, 065501]

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Large variation in E_m profiles to check sensitivity of observables

Sensitivity to variations in the spectral functions: PWIA calculations [Arxiv:2406.09244]

Observables for MicroBooNE flux-averaged signal



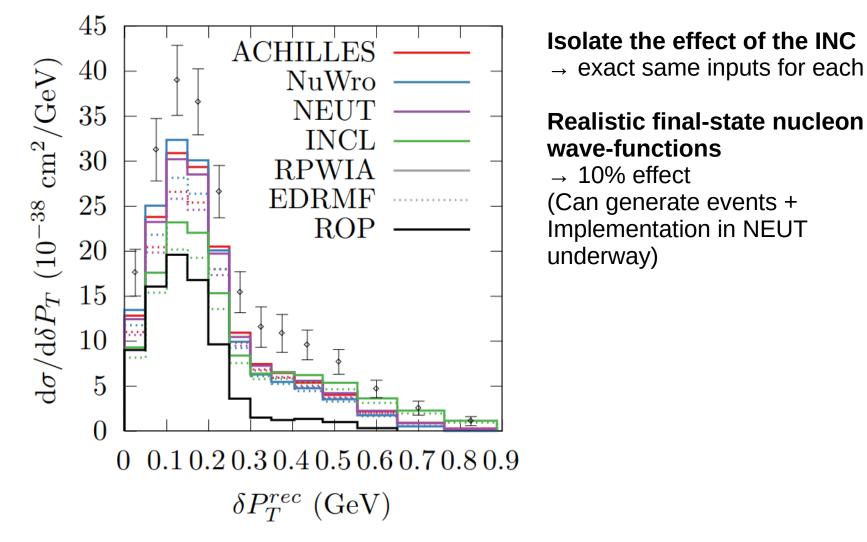
-Negligible differences between different spectral-functions for observables that do not correlate p_p and p_μ -Mild sensitivity only to p_m distributions

- Current MicroBooNE data not sensitive to missing energy distributions



Comparisons to MicroBooNE data

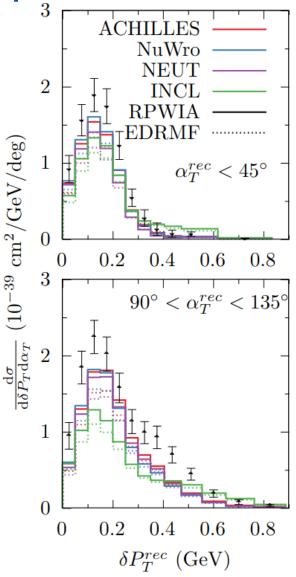
[Arxiv:2406.09244]





Comparisons to MicroBooNE data

[Arxiv:2406.09244]



Isolate the effect of the INC

 \rightarrow exact same inputs for each

Realistic final-state nucleon wavefunctions

 \rightarrow 10% effect

Cut on α_{T}

- $\rightarrow\,$ reduce rescattering for small $\alpha_{_{\rm T}}$
- \rightarrow Still sizeable contribution of resc
- \rightarrow ROP alone cannot reproduce data

Low $\alpha_{\tau} dP_{\tau}$ underpredicted

- \rightarrow Increase axial coupling ??
- \rightarrow Need to include the interference with 2-body currents!



- 1-proton knockout could provide excellent energy-resolution
 - → Backgrounds non-trivial !
- **RDWIA + realistic spectral function + INC**
 - = most comprehensive description of single-nucleon knockout
 - \rightarrow No full agreement with recent MicroBooNE data

Where to go from here?

Theory

* Interference with 2-body currents needs to be included !

See e.g. [T. Franco Munoz et al. PRC108, 064608] [Lovato et al. , 2312.12545]

- ***** Two-nucleon and single pion production (SPP) contributions
- → ACHILLES: will soon include 2-body interference + SPP with full FSI
- → NEUT: will include RDWIA calculations with SF
- \rightarrow NuWro: new SPP [2405.0512] and 2-N [K. Niewczas Phd] implementations

Experiment

- * High statistics in SBND can test 'good event' selection in 1µ1p in LarTPC
- * New electron scattering data in non-trivial kinematic regions
 - \rightarrow A1 at MAMI, e4v at Jlab take electron data for v program (+ more facilities ?!)

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Other stuff



Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

-Relativistic Distorted Wave Impulse Approximation (RDWIA)

$$\mathcal{J}_{\kappa}^{m_{j}}(Q, P_{N}) = \int \mathrm{d}\mathbf{p} \ \overline{\psi}(\mathbf{p} + \mathbf{q}, \mathbf{k}_{N}, s_{N}) \ \mathcal{O}^{\mu} \ \psi_{\kappa}^{m_{j}}(\mathbf{p})$$

- Relativistic Plane Wave Impulse Approximation (RPWIA)

$$\mathcal{J} = (2\pi)^{3/2} \ \overline{u}(\mathbf{k}_N, s_N) \ \mathcal{O}^{\mu} \ \psi_{\kappa}^{m_j}(\mathbf{k}_N - \mathbf{q})$$

- Plane-Wave Impulse Approximation (PWIA)

The initial state is assumed proportional to a positive-energy spinor:

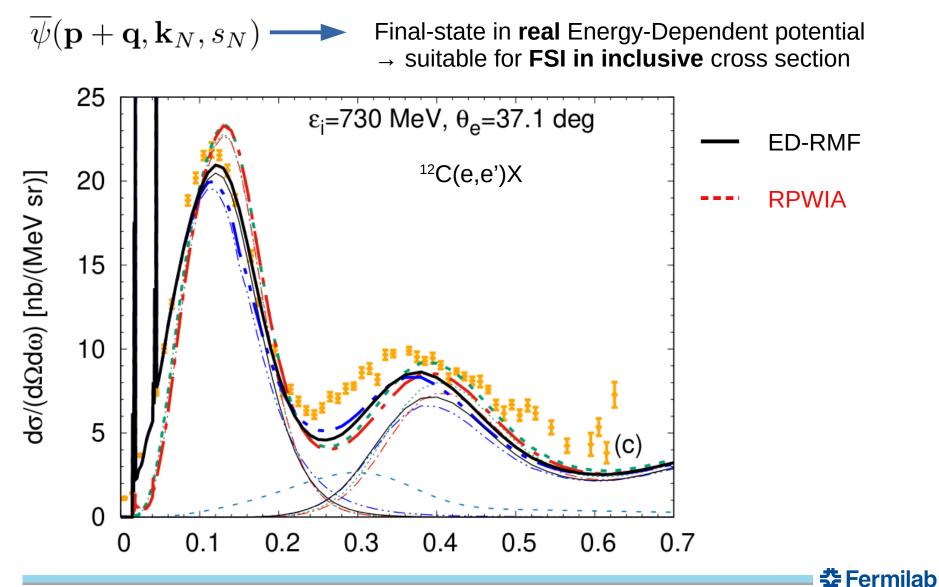
$$\psi_{\kappa}^{m_j}(\mathbf{p}) \propto f(|\mathbf{p}|)u(\mathbf{p})$$

One obtains a factorized expression ('spectral function approach')

$$\frac{d\sigma(E_{\nu})}{dp_{\mu}d\Omega_{\mu}d\Omega_{p}dp_{N}} = \frac{G_{F}^{2}\cos^{2}\theta_{c}}{(2\pi)^{2}} \frac{p_{\mu}^{2}p_{N}^{2}}{E_{\nu}E_{\mu}} \frac{M_{N}^{2}}{E_{N}\overline{E}} L_{\mu\nu}h_{s.n.}^{\mu\nu}S(E_{m},p_{m})$$

Terminology : RDWIA, RPWIA and PWIA & ED-RMF and ROP

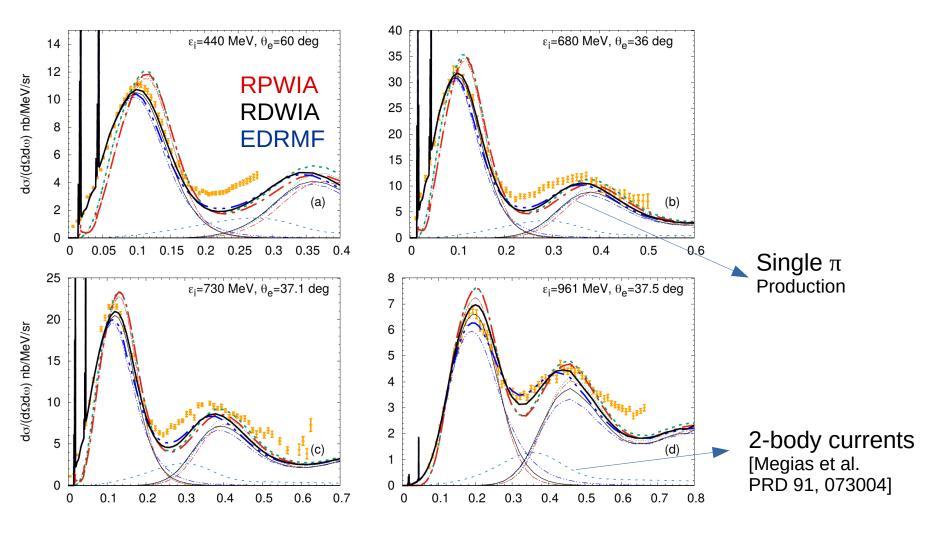
- Energy-Dependent Relativistic Mean-Field (ED-RMF)



[R. Gonzalez-Jimenez et al Phys. Rev. C 100, 045501 (2019)]

Inclusive electron scattering off a RMF nucleus

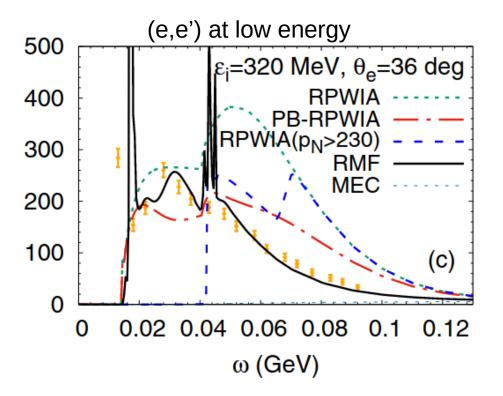
[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]





Nucleon knockout from a RMF nucleus: consistent initial & final-states

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]



- Consistent states are crucial at low energies
 - \rightarrow provided by RMF
 - \rightarrow ED-RMF = RMF

by construction at low-energy

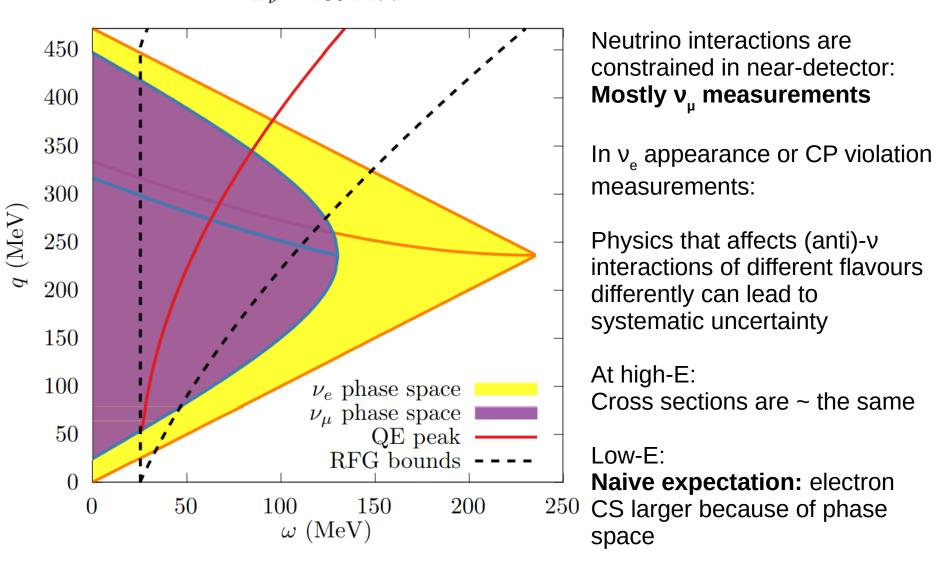
 Consistent states are orthogonal => Pauli-Blocking

'Pauli-Blocked' RPWIA (PB-RPWIA): orthogonalize with respect to bound-states

$$|\Psi^{s_N}(\mathbf{p}_N)\rangle = |\psi^{s_N}_{pw}(\mathbf{p}_N)\rangle - \sum_{\kappa,m_j} \left[C^{m_j,s_N}_{\kappa}(\mathbf{p}_N) \right]^{\dagger} |\psi^{m_j}_{\kappa}\rangle \qquad C^{m_j,s_N}_{\kappa}(\mathbf{p}_N) \equiv \left\langle \psi^{s_N}_{pw}(\mathbf{p}_N) \right| \psi^{m_j}_{\kappa}\rangle.$$

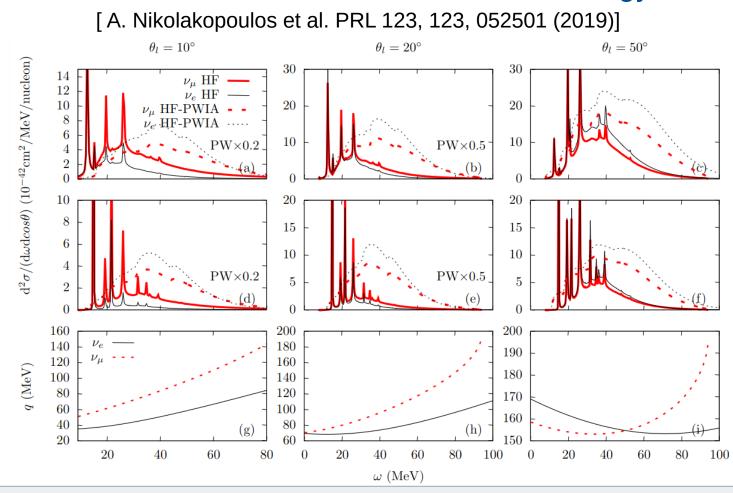


Electron- and muon neutrino interactions



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 $E_{\nu} = 236 \text{ MeV}$

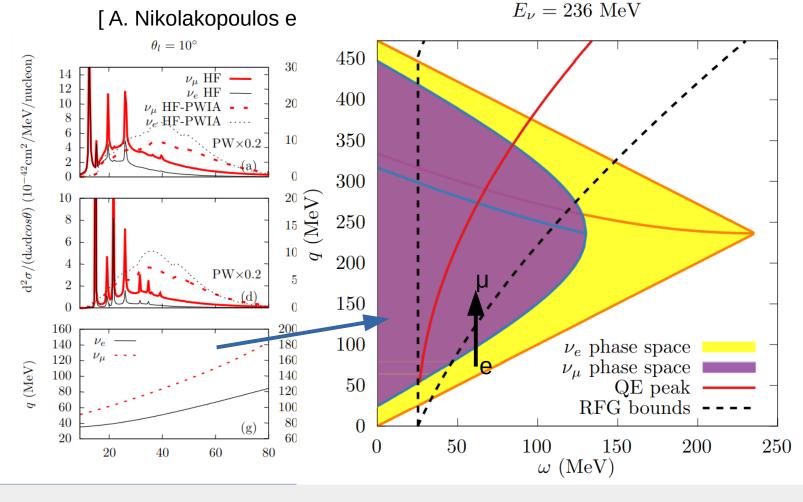


Electron- and muon neutrino interactions at low energy

$$= \sqrt{E_{\nu}^{2} + P_{l}^{2} - 2\cos\theta_{l}E_{\nu}P_{l}} \approx E_{\nu} - \sqrt{(E_{\nu} - \omega)^{2} - m_{l}^{2}}$$

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q



Electron- and muon neutrino interactions at low energy

At low energy-momentum transfer v_{μ} cross sections are larger than v_{e}

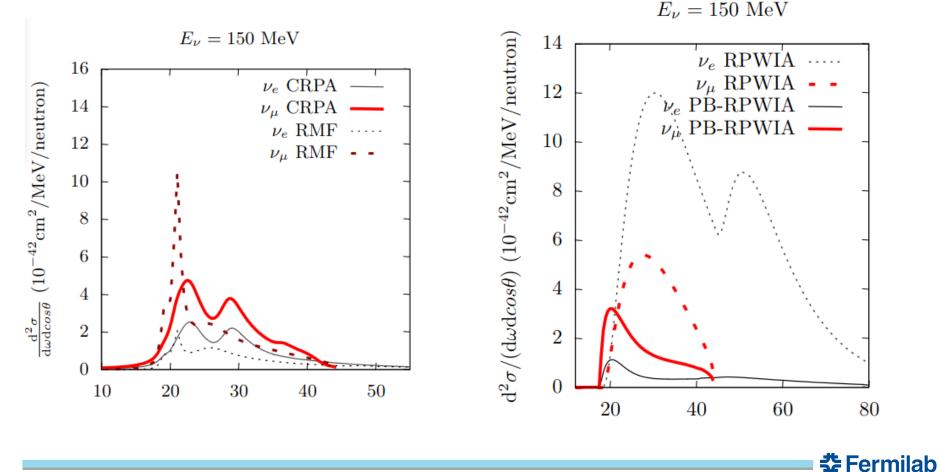
$$q = \sqrt{E_{\nu}^{2} + P_{l}^{2} - 2\cos\theta_{l}E_{\nu}P_{l}} \approx E_{\nu} - \sqrt{(E_{\nu} - \omega)^{2} - m_{l}^{2}}$$

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Electron- and muon neutrino interactions at low energy

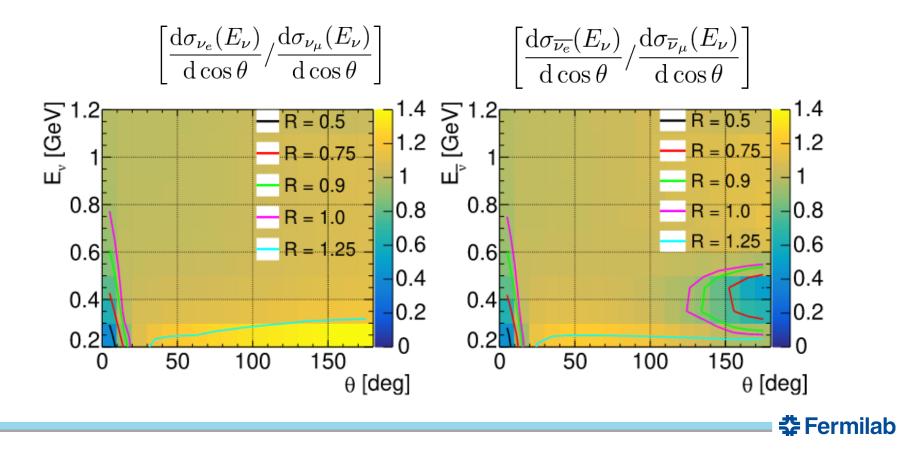
[A. Nikolakopoulos et al. PRL 123, 123, 052501 (2019)]

Can be understood from the orthogonality with bound states



Uncertainties on the $(\bar{\nu}_{e})^{(\bar{\nu}_{\mu})}_{\mu}$ and $\nu_{e}/\bar{\nu}_{e}$ cross-section ratio from the modelling of nuclear effects at 0.2 to 1.2 GeV neutrino energies and their impact on neutrino oscillation experiments

T. Dieminger,^{1,*} S. Dolan,^{2,†} D. Sgalaberna,^{1,‡} A. Nikolakopoulos,³
T. Dealtry,⁴ S. Bolognesi,⁵ L. Pickering,⁶ and A. Rubbia¹
arXiv:2301.08065v3 [hep-ph] 5 Apr 2023

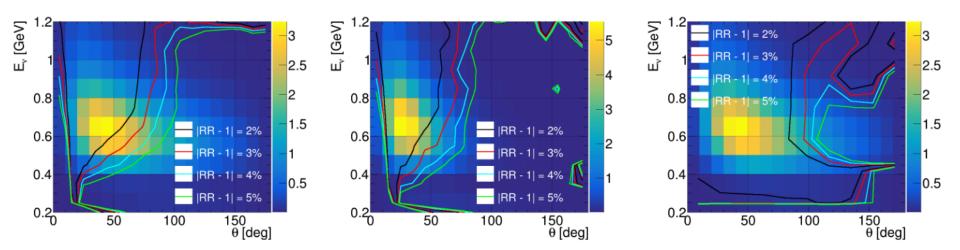


Assessing model-dependence: double ratios

$$RR_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 1/Model 2}}(E_{\nu},\theta) = \frac{R_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 1}}(E_{\nu},\theta)}{R_{\nu_{\alpha}/\nu_{\beta}}^{\text{Model 2}}(E_{\nu},\theta)}$$





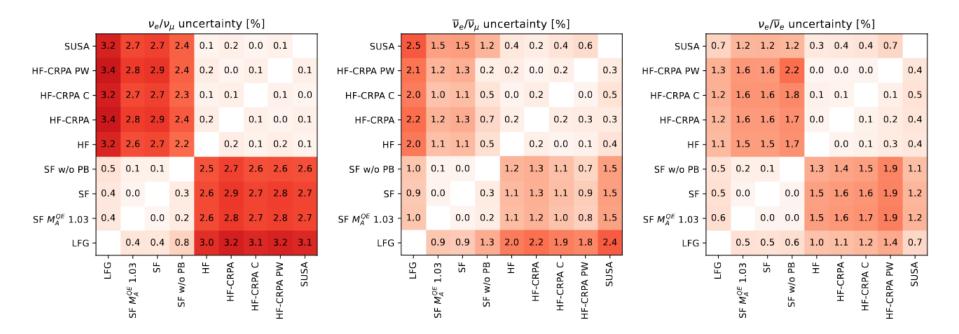


High-angle region in T2K is most relevant



Assessing model-dependence

Uncertainty from averaging model-differences in Ratio over predicted event rate

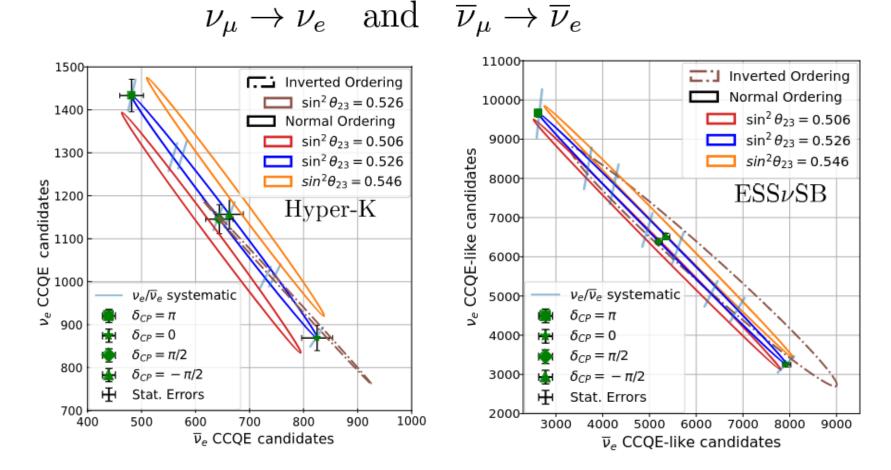


Quite robust when varying parameters within one model

Two 'sets': LFG and PWIA+SF and mean-field based models (or NEUT generator vs. other calculations ?)



Assessing impact on appearance experiment: bi-event plots



1) Not significant for $sin\delta_{CP}$

2) Increases degeneracy in $sin^2\theta_{23}$

Magnitude 2-4% percent: uncertainty in line with values used in experiment

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Article

From inclusive to semi-inclusive one-nucleon knockout in neutrino event generators

Alexis Nikolakopoulos^{1,*}, Steven Gardiner¹, Afroditi Papadopoulou², Stephen Dolan³ and Raúl González-Jiménez⁴

 $P(E_l, \theta_l, T_N, \Omega_N)$

Replace by 'factorized approach'

We get the GENIE version based on **the same** inclusive cross section!

We generate events for (e,e'p) in RDWIA with **real potential**

- Full consistent description of exclusive kinematics 1e1p
- Integrate over the proton → get the correct inclusive cross section (=includes 'elastic' FSI!)
- For every event we **replace** the nucleon kinematics by the GENIE prediction (SuSAv2 implementation)



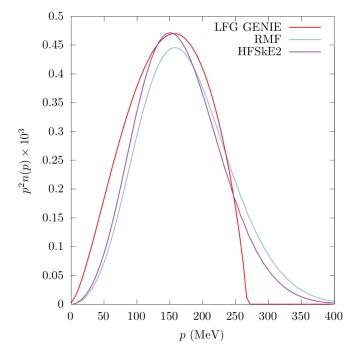
$$\frac{\mathrm{d}\sigma(E_{\nu})}{\mathrm{d}E_{l}\mathrm{d}\cos\theta_{l}} = G^{2}\frac{k_{l}}{E_{\nu}}L_{\mu\nu}\int\mathrm{d}\Omega_{N}\sum_{n,\kappa}H_{n,\kappa}^{\mu\nu}(\omega,q,\Omega_{N},E_{n,\kappa})$$

Lost nucleon information \rightarrow Need to generate it in GENIE

1. Draw initial nucleon \mathbf{p}_{m} from p^{2} n(p) (e.g. LFG)

!! 2. Compute
$$E_m^2 = p_m^2 + M_N^2$$

3. $E_{N} = E_{m} + \omega - E_{b}(q)$

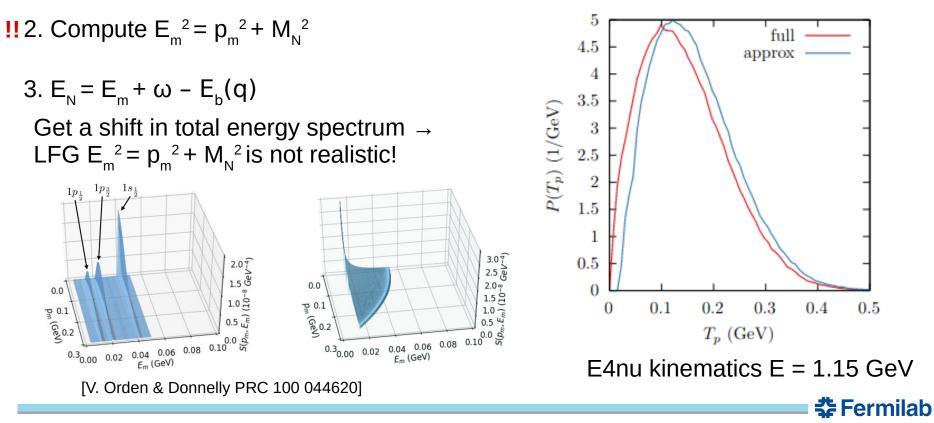




$$\frac{\mathrm{d}\sigma(E_{\nu})}{\mathrm{d}E_{l}\mathrm{d}\cos\theta_{l}} = G^{2}\frac{k_{l}}{E_{\nu}}L_{\mu\nu}\int\mathrm{d}\Omega_{N}\sum_{n,\kappa}H_{n,\kappa}^{\mu\nu}(\omega,q,\Omega_{N},E_{n,\kappa})$$

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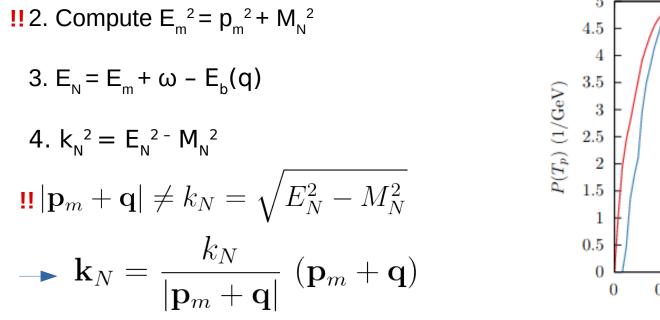
1. Draw initial nucleon \mathbf{p}_{m} from $p^{2} n(p)$ (e.g. LFG)

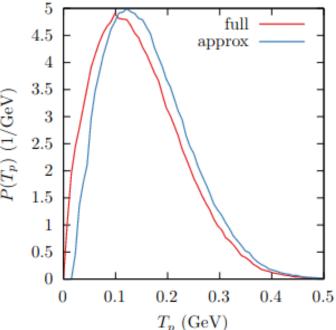


$$\frac{\mathrm{d}\sigma(E_{\nu})}{\mathrm{d}E_{l}\mathrm{d}\cos\theta_{l}} = G^{2}\frac{k_{l}}{E_{\nu}}L_{\mu\nu}\int\mathrm{d}\Omega_{N}\sum_{n,\kappa}H_{n,\kappa}^{\mu\nu}(\omega,q,\Omega_{N},E_{n,\kappa})$$

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1. Draw initial nucleon \mathbf{p}_{m} from $p^{2} n(p)$ (e.g. LFG)







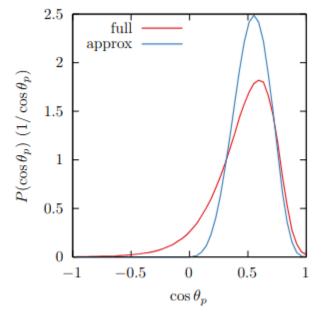
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Lost nucleon information \rightarrow Need to generate it in GENIE

1. Draw initial nucleon \mathbf{p}_{m} from p^{2} n(p) (e.g. LFG)

1. 2. Compute
$$E_m^2 = p_m^2 + M_N^2$$

3. $E_N = E_m + \omega - E_b(q)$
4. $k_N^2 = E_N^2 - M_N^2$
1. $|\mathbf{p}_m + \mathbf{q}| \neq k_N = \sqrt{E_N^2 - M_N^2}$
 $- \mathbf{k}_N = \frac{k_N}{|\mathbf{p}_m + \mathbf{q}|} (\mathbf{p}_m + \mathbf{q})$

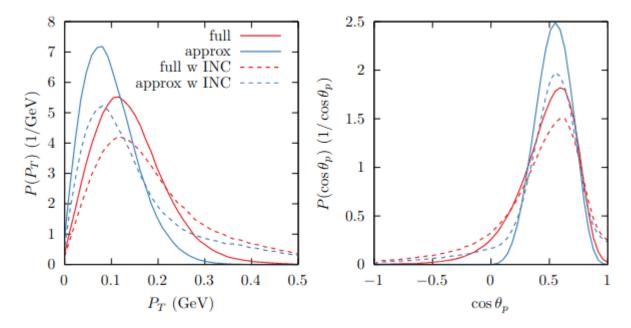


Serious differences in angular distributions!



$$\frac{\mathrm{d}\sigma(E_{\nu})}{\mathrm{d}E_{l}\mathrm{d}\cos\theta_{l}} = G^{2}\frac{k_{l}}{E_{\nu}}L_{\mu\nu}\int\mathrm{d}\Omega_{N}\sum_{n,\kappa}H_{n,\kappa}^{\mu\nu}(\omega,q,\Omega_{N},E_{n,\kappa})$$

Lost nucleon information \rightarrow Need to generate it in GENIE

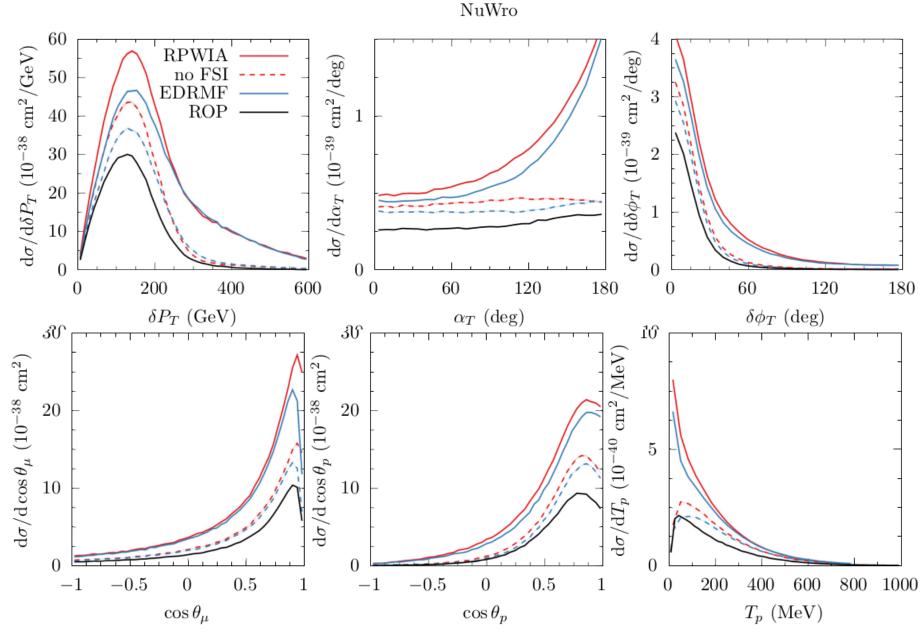


Results for e4nu kinematics E=1.159 including the GENIE cascade!

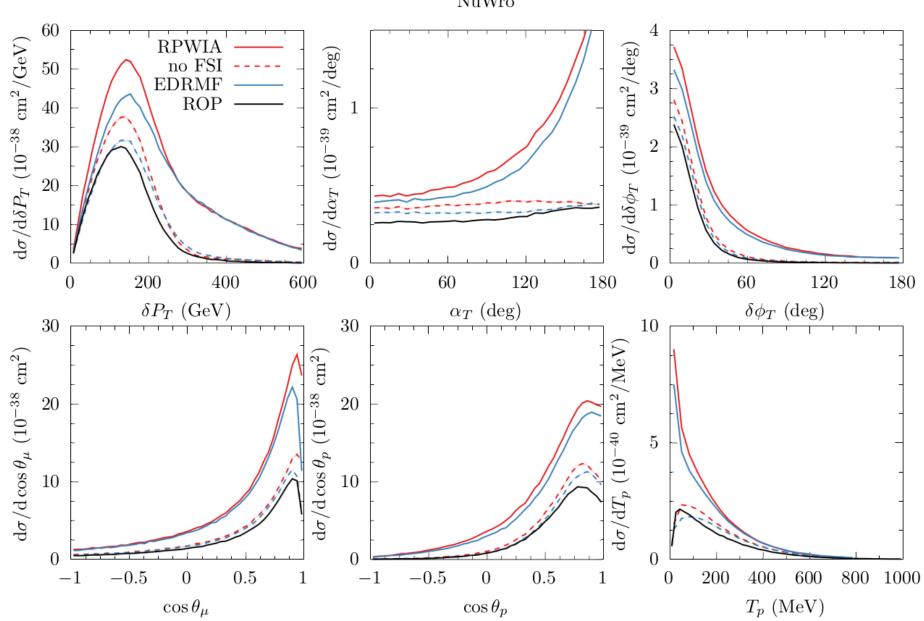
Shape differences biggest in $\mathsf{P}_{_{\!\mathrm{T}}}$ and angular distributions



NuWro with SRC effect in Mean-free path

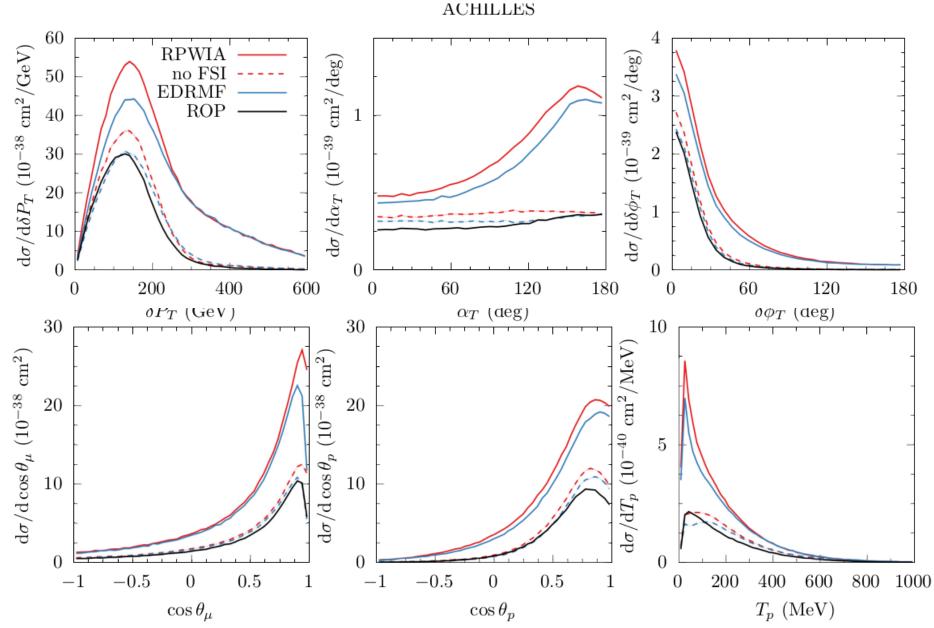


NuWro without SRC effect in Mean-free path

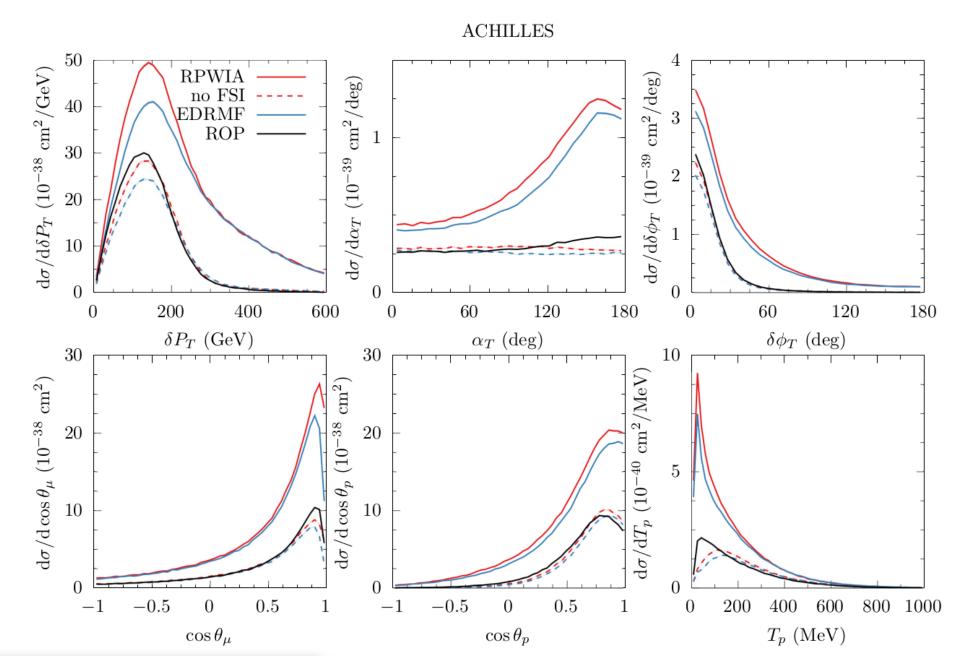


NuWro

ACHILLES with Formation time



ACHILLES without Formation time



NEUT

NEUT

