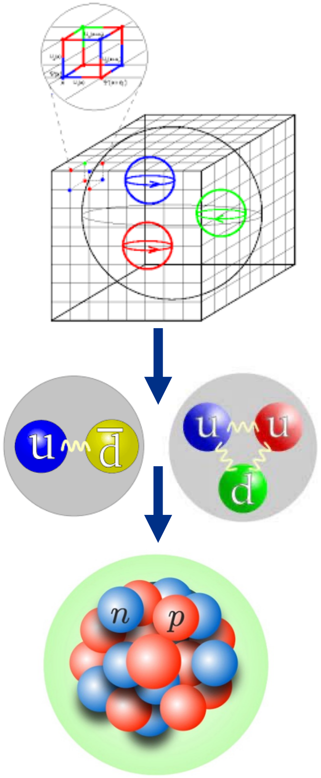
Ab-Initio methods for Neutrino-Nucleus Scattering

Noah Steinberg
Sept 7th, 2024
Neutrino Oscillation Workshop

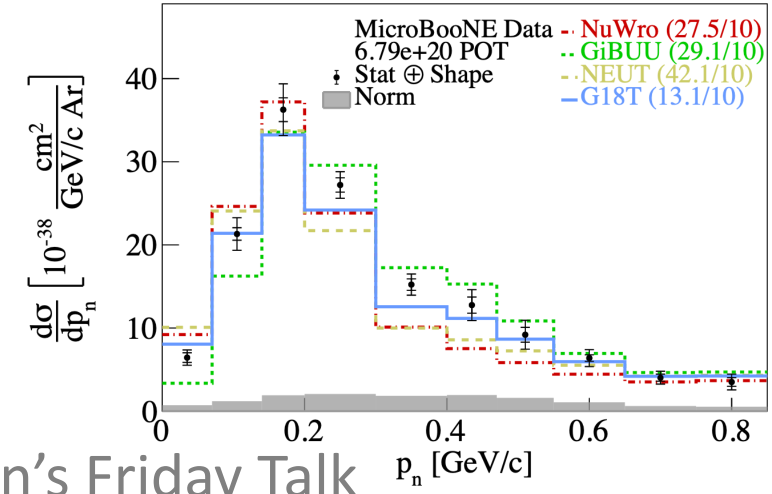
Neutrino-Nucleus Cross Sections

- Even in the *simplest* kind of cross sections are not well predicted

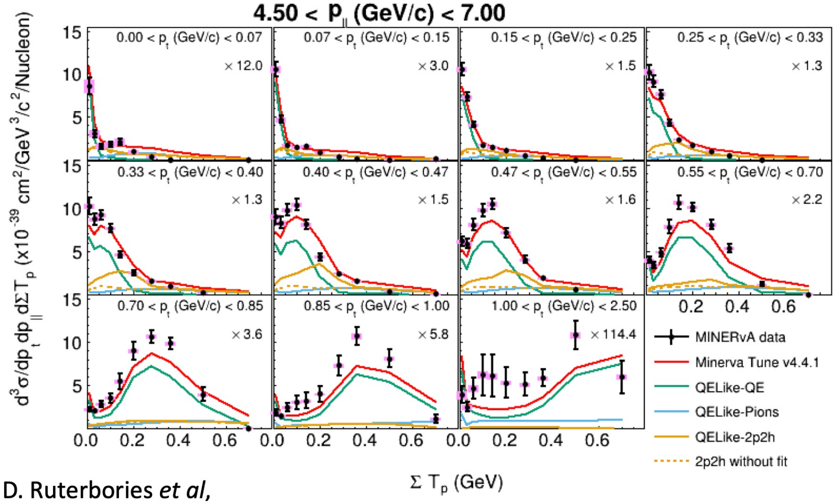
- New paradigm needed with a bottom up approach



MicroBooNE e-Print: 2310.06082 [nucl-ex]



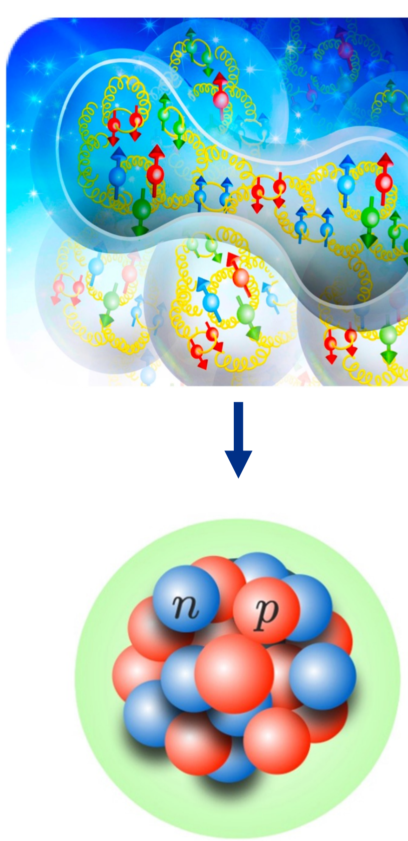
See Ben’s Friday Talk



D. Ruterbories *et al*,

D. Ruterbories *et al*, 26 October 2022 *Phys.Rev.Lett.* 129 (2022) 2, 021803

Nuclear Many Body Theory



- Nuclei are self bound systems of protons and neutrons
- Description in terms of interacting quarks and gluons is computationally intractable*
- At low energies correct effective degrees of freedom to describe nuclei are nucleons and pions
- Properties of nuclei can be computed using a non-relativistic Hamiltonian and consistent EW currents

$$H = \sum_{i}^{A} \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} V_{ijk} \quad \longleftrightarrow \quad J_A^{\mu}(q) = \sum_i j_i^{\mu}(q) + \sum_{ij} j_{ij}^{\mu}(q)$$

Nuclear Many Body Theory

$$H = \sum_{i}^{A} \frac{\mathbf{p}_i^2}{2m_N} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

- Potentials can be provided by

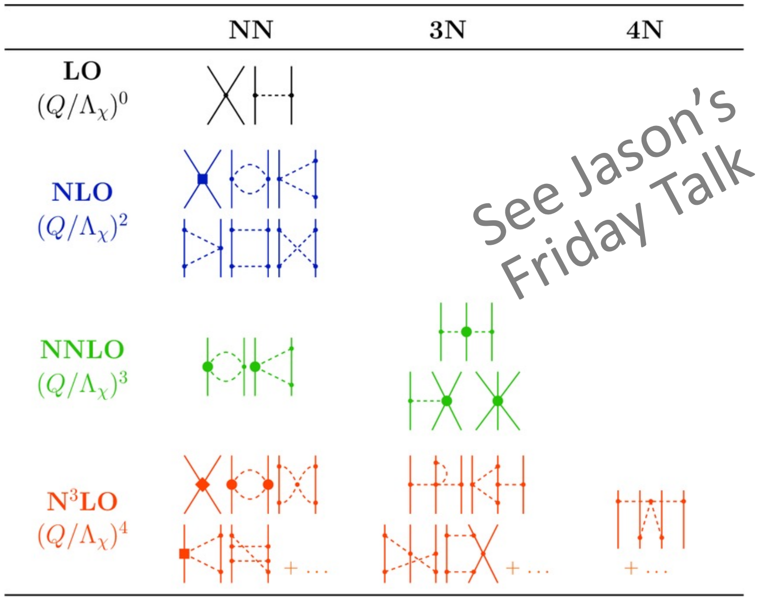
Phenomenological Parameterizations

AV18 + IL7 Potentials

$$v_{ij} = \sum_{p=1}^{18} v_p(r_{ij}) O_{ij}^p$$

See Gandolfi, S. et al *Front.in Phys.* 8 (2020) 117, *Front.Phys.* 8 (2020) 117

χEFT



See Jason's Friday Talk

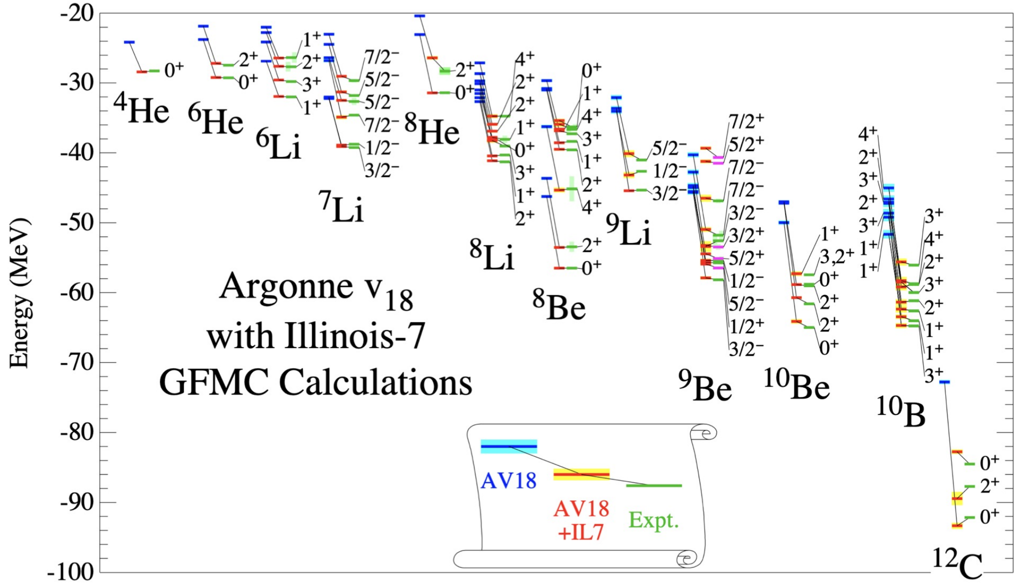
Quantum Monte Carlo – Static Properties

- Ground state energies and wave functions computed via variational principle

$$\frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} = E_V \geq E_0$$

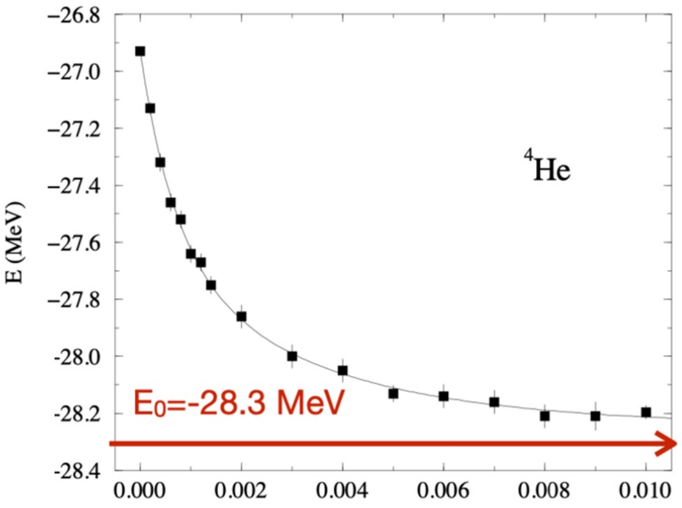
- Improved via Green’s Function Monte Carlo Techniques

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad e^{-(H-E_0)\tau} |\Psi_T\rangle \to |\Psi_0\rangle$$



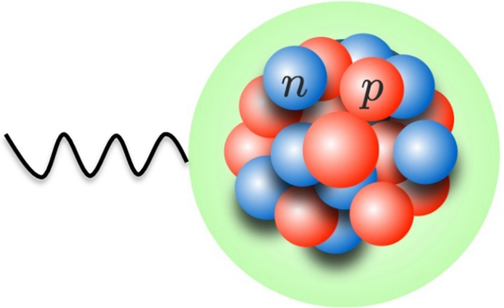
J. Carlson, et al. Rev.Mod.Phys. 87 (2015) 1067

B. Pudliner et al., PRC **56**, 1720 (1997)



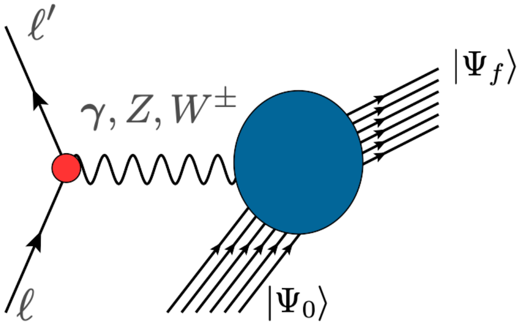
- Able to compute ground state (and excited state) energies of light nuclei with 1% precision

QMC methods in lepton nucleus-scattering



- Nuclear dependence of cross section

$$R_{\mu\nu} = \sum_f \langle \Psi_0 | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi_0 \rangle \delta(E_0 + \omega - E_f)$$



- Instead of summing over all final states, use an integral transform

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega, \mathbf{q}) = \langle \psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \psi_0 \rangle$$

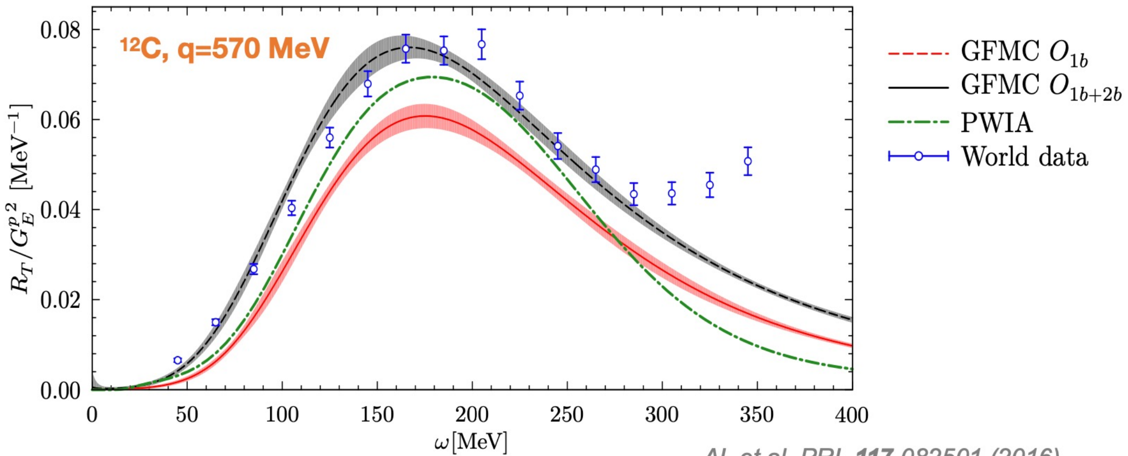
- Converts problem to computation of a ground state expectation value
  - Amenable to QMC techniques
  - Inversion takes you back to the response tensor*

Green’s Function Monte Carlo

- Compute Euclidean Response, imaginary time evolve

$$E_{\alpha\beta}(\mathbf{q},\tau) = \int_{\omega_{\rm th}}^{\infty} d\omega e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q},\omega)$$

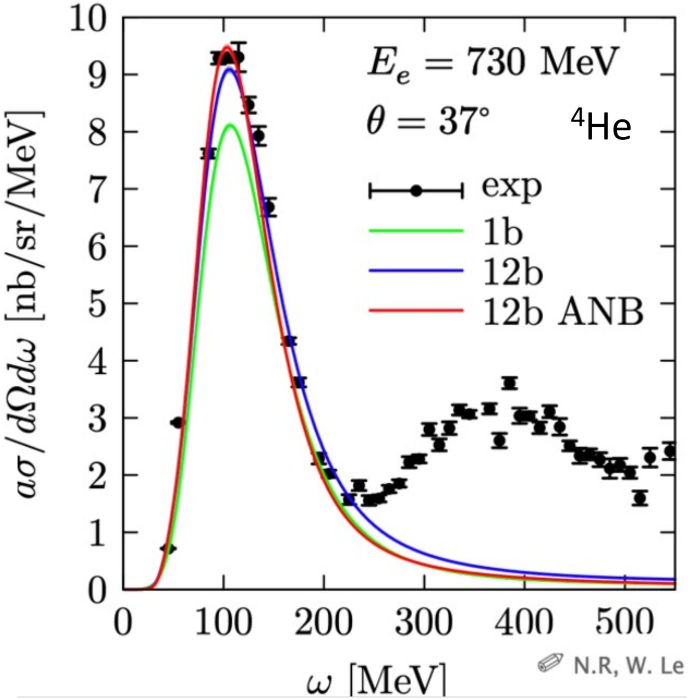
  - Inversion needed to obtain response



AL et al. PRL **117** 082501 (2016)

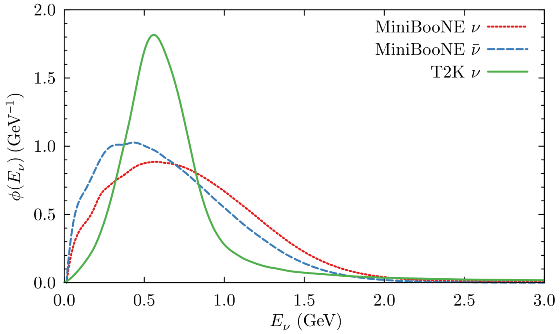
- Virtually exact results in the region of the quasi-elastic peak

N.R. et al. PRC 97 (2018) no.5, 055501

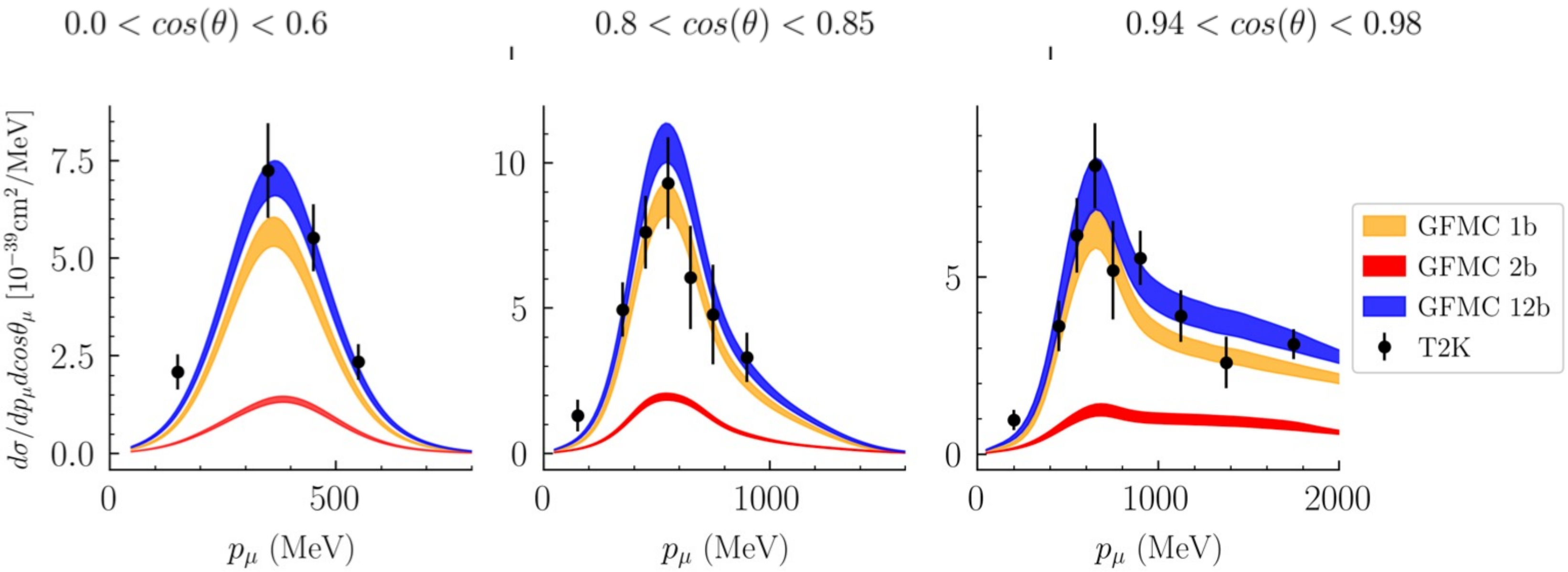


- Retains initial and final state correlations
- Limitations
  - Non relativistic
  - Static Delta
  - Inclusive observables
  - A <= 12

Green’s Function Monte Carlo Results for T2K



- Extension to neutrino CC inclusive cross sections is straight forward once axial currents are included

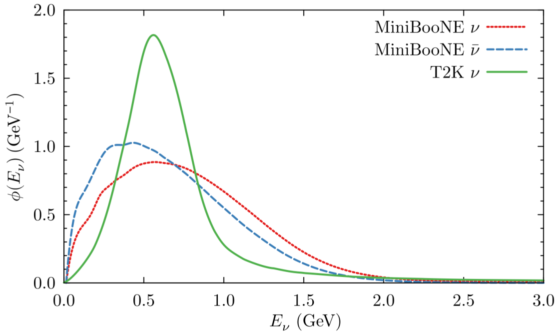


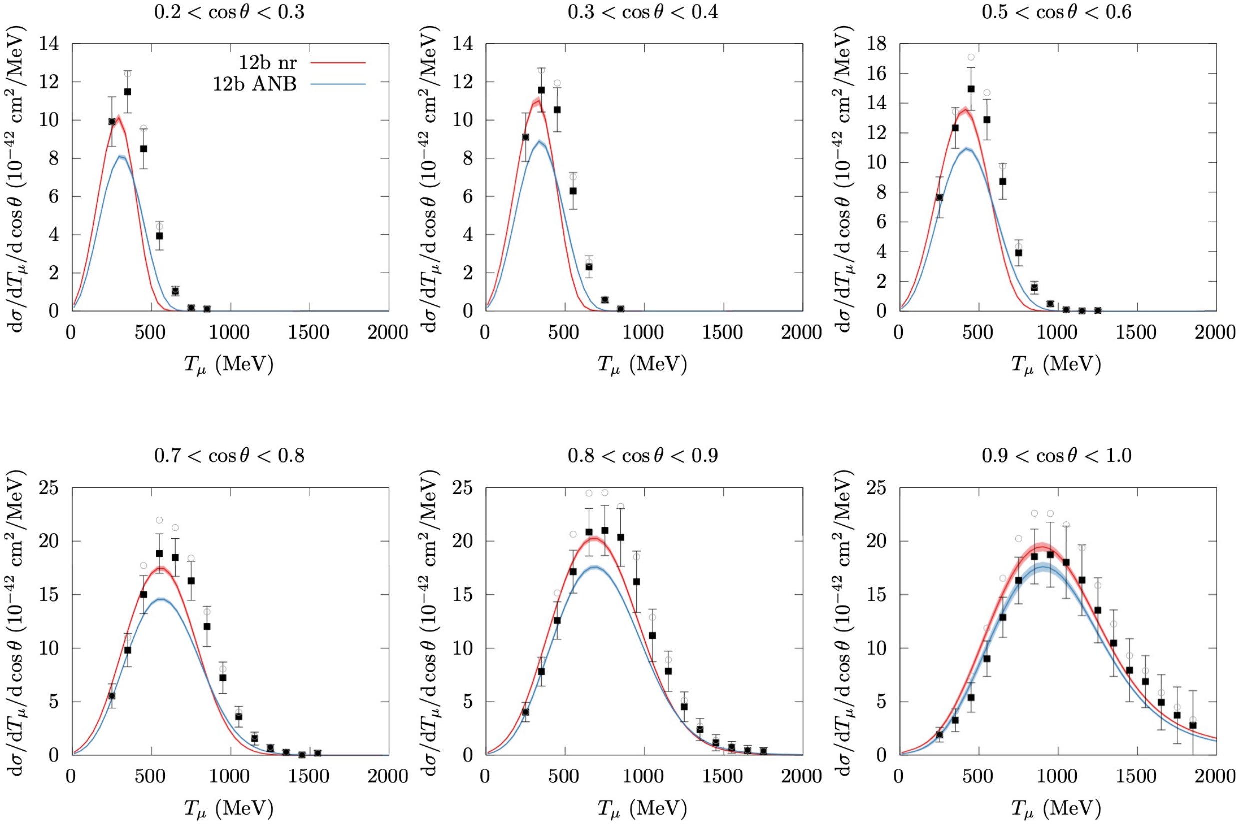
D. Simons, et al, arXiv:2210.02455

- Large contribution of two body currents includes interference effects with one body currents

Green’s Function Monte Carlo Results for MiniBooNE

- At higher energy experiments inclusion of relativistic effects is necessary



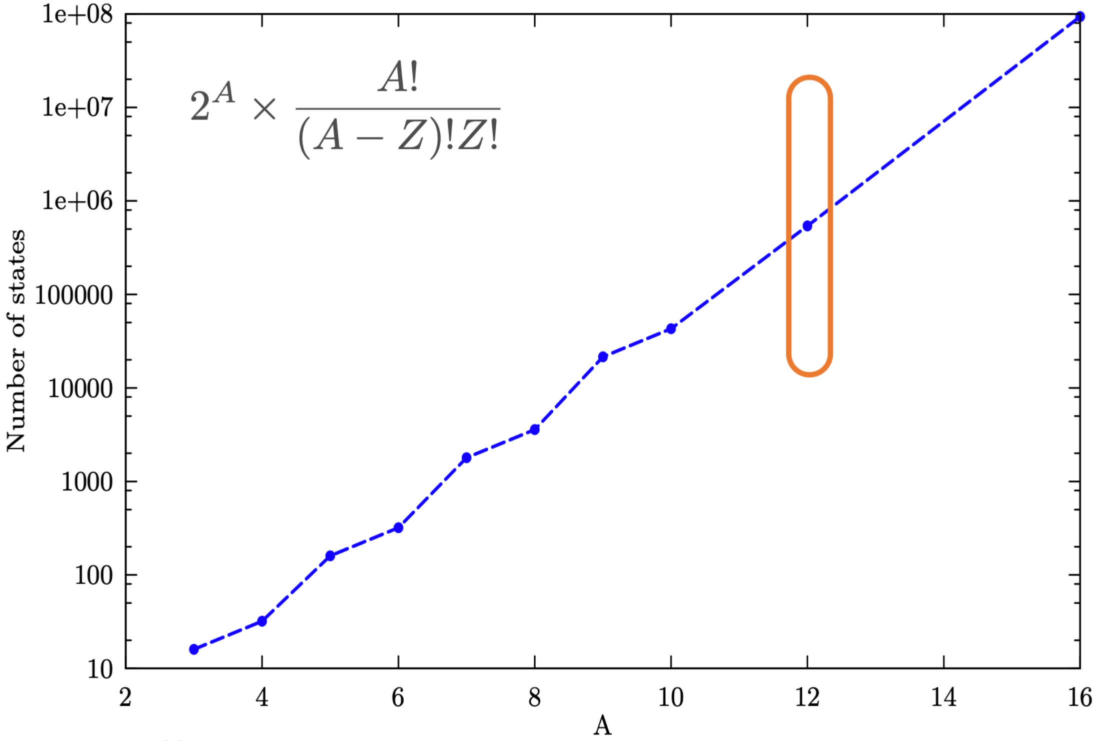


- MB model dependent pion background subtraction makes quantitative comparisons difficult

A. N. et al, 2304.11772 [nucl-th]

Auxiliary Field Diffusion Monte Carlo (AFDMC): Beyond Carbon

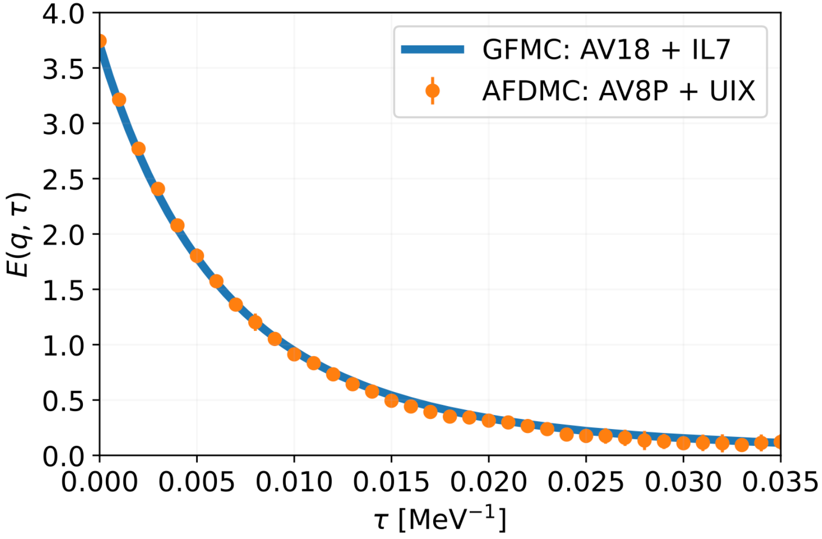
- GFMC techniques are limited to A ~ 14 because of exponential scaling of the state of the system



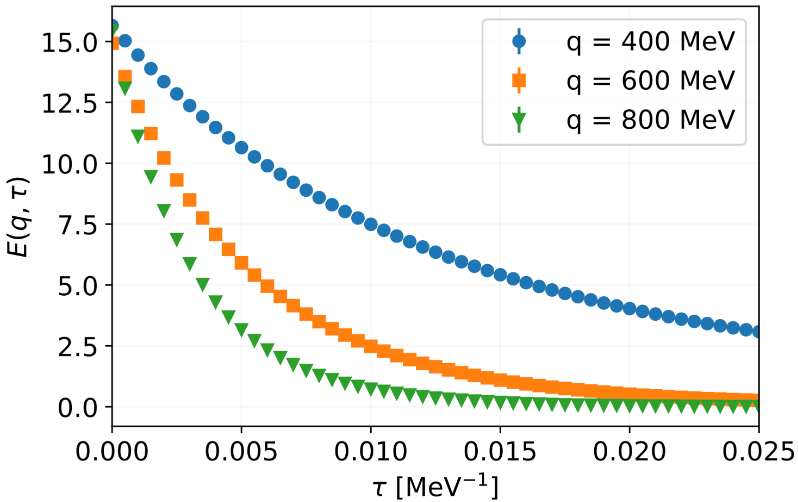
- AFDMC scales polynomialy by using a different representation of the wave function and sampling spin-isospin states

Auxiliary Field Diffusion Monte Carlo (AFDMC): Beyond Carbon

- Comparison with GFMC calculations of responses for light nuclear systems are encouraging
  - ^{4}He isoscalar density response
    - Very similar to EM longitudinal response



- Response of ^{16}O computed for the first time
  - Obvious relevance for Hyper-K
  - EM and CC responses in the future



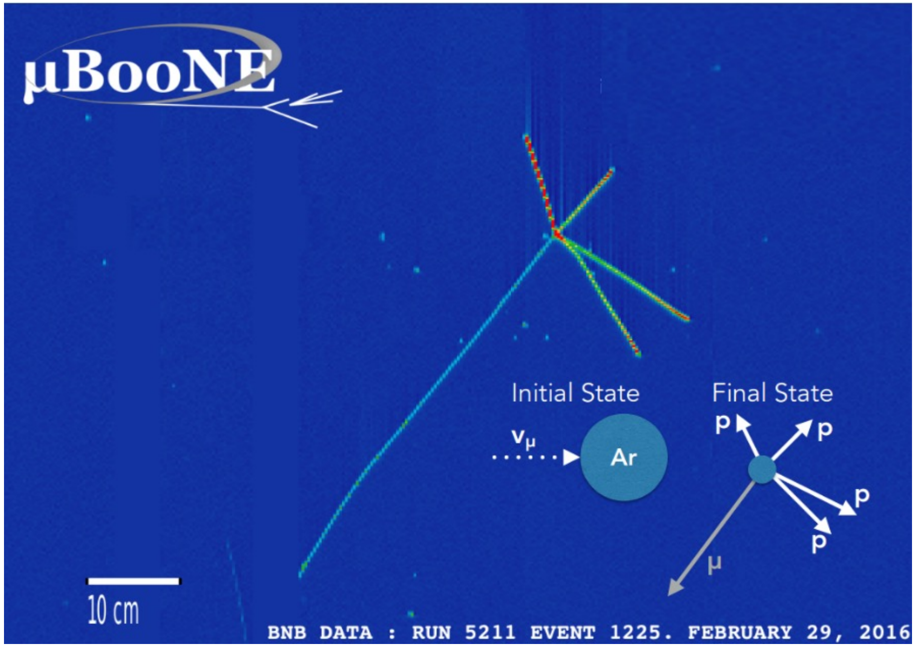
A. G. et al., 2405.14916 [nucl-th]

Inclusive is not enough

- SBN (and DUNE) program will reconstruct hadronic system with incredible precision

- Need exclusive predictions to predict calorimetric estimates of Neutrino Energy

- Generators need exclusive predictions!



Courtesy of Afroditi Papadopoulou

Many body methods: Spectral Function Approach

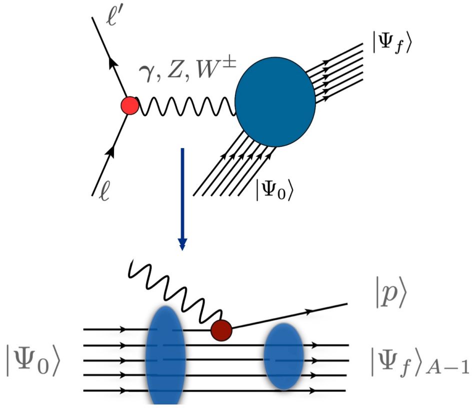
- For sufficient $|\mathbf{q}|$, scattering factorizes

$$d\sigma = \int \boxed{(d\sigma)}_{nucleon} \boxed{P(\mathbf{p}, E)} d^3k dE$$

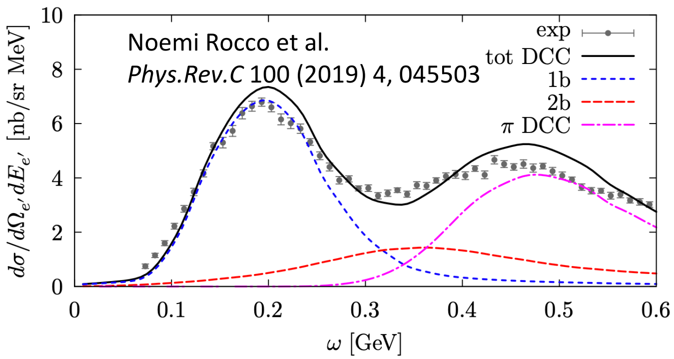
  - Single nucleon knockout (QE)
    $|f\rangle = |\mathbf{p}'\rangle \otimes |\Psi_f^{A-1}, \mathbf{p}_{A-1}\rangle$
  - Multi nucleon knockout (MEC)
    $|f\rangle = |\mathbf{p}_1'\mathbf{p}_2'\rangle \otimes |\Psi_f^{A-2}, \mathbf{p}_{A-2}\rangle$
  - Single Pion Production
    $|f\rangle = |\mathbf{p}'\mathbf{p}_\pi\rangle \otimes |\Psi_f^{A-1}, \mathbf{p}_{A-1}\rangle$

- Transition from nuclear ground state captured by the Spectral Function

$$P_h(\mathbf{k}, E) = \sum_f |\langle\psi_0^A|[|k\rangle \otimes |\psi_f^{A-1}\rangle]|^2 \times \delta(E + E_f^{A-1} - E_0^A)$$

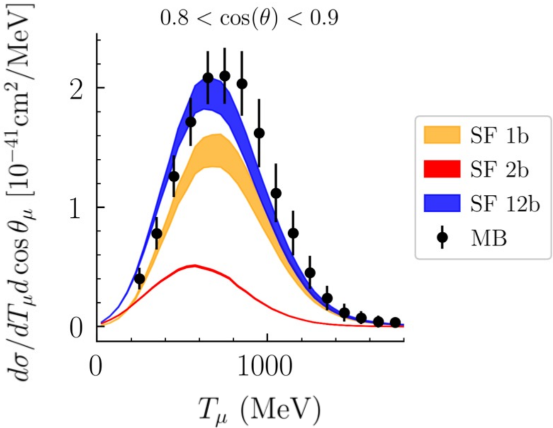


E_e=961 MeV, θ_e=37.5°



Noemi Rocco et al.
Phys.Rev.C 100 (2019) 4, 045503

$0.8 < \cos(\theta) < 0.9$

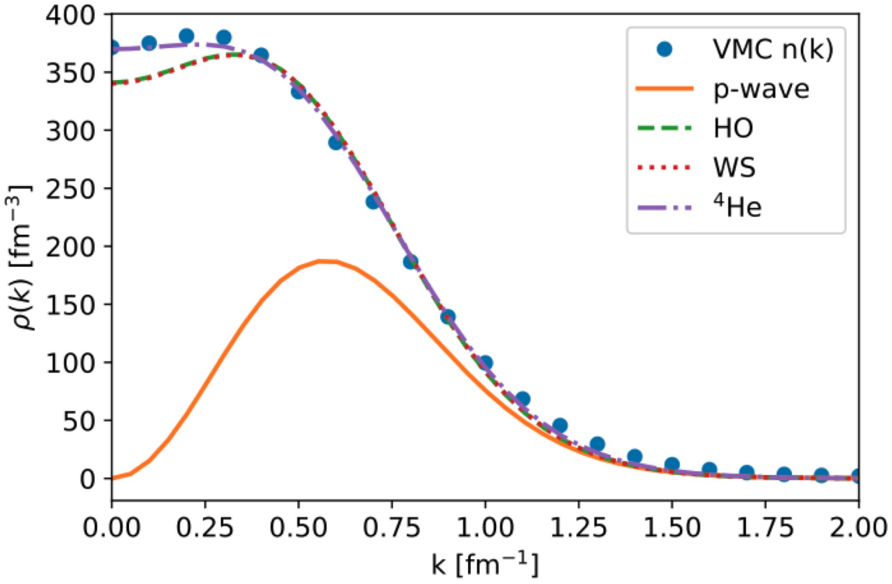


QMC Spectral Function

- One nucleon Spectral Function

$$P_{p,n}(\mathbf{k},E) = \sum_n |\langle\Psi_0^A|[|k\rangle|\Psi_n^{A-1}\rangle]|^2 \times \delta(E+E_0^A-E_n^{A-1})$$
$$= P^{MF}(\mathbf{k},E) + P^{\text{corr}}(\mathbf{k},E)$$

  - Mean Field (A-1 Bound States)
  - Correlation component from continuum
  - Momentum space overlaps obtained from VMC calculations of wavefunctions



CLAS Phys.Rev.C 107 (2023) 6, L061301

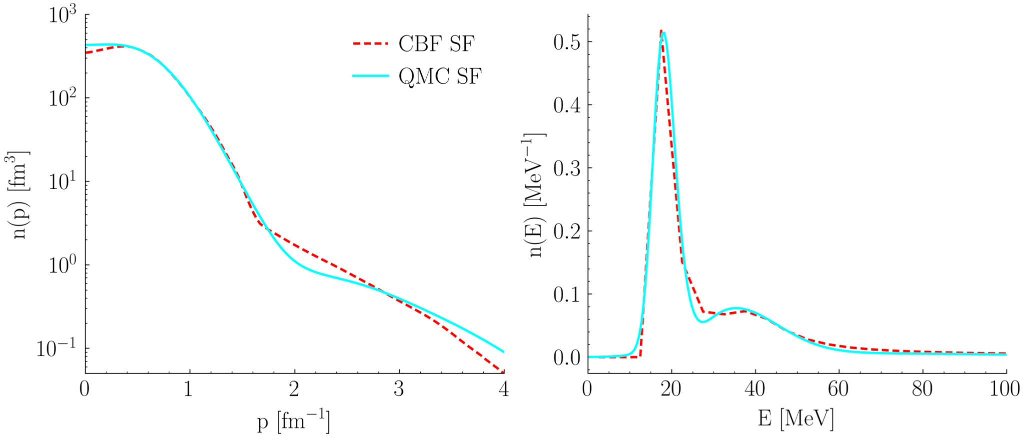
- Two nucleon spectral function
  - Mean field only

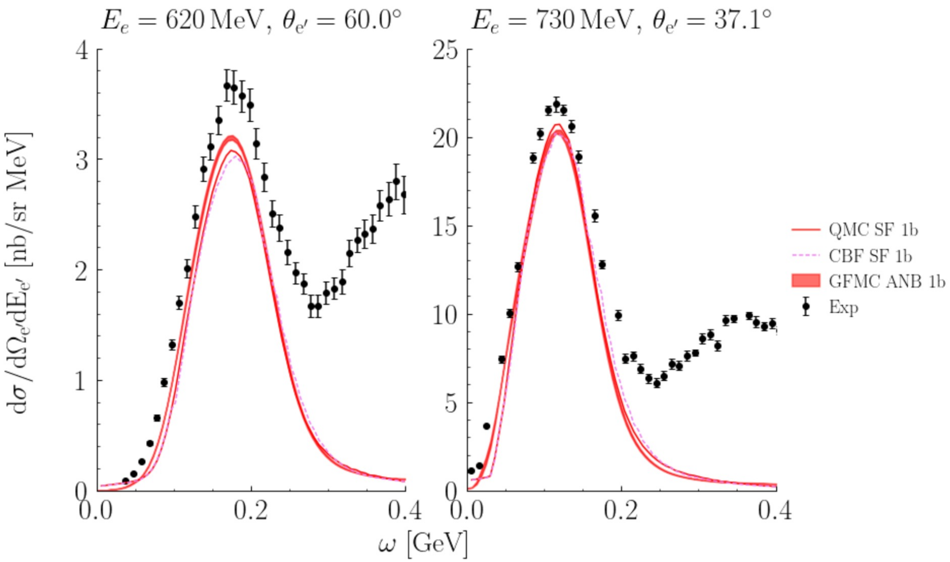
$$P^{\text{MF}}_{\tau_k,\tau_k'}(\mathbf{k},\mathbf{k}',E) = n_{\tau_k,\tau_{k'}}(\mathbf{k},\mathbf{k}')$$
$$\times\,\delta\Big(E - B_0 + \bar{B}_{A-2} - \frac{\mathbf{K}^2}{2m_{A-2}}\Big)$$

Comparison of Spectral Functions

A.L. et al, Universe 9 (2023) 8, 367

- QMC vs. Benhar CBF Spectral Function
    - Benhar derived from (e,e’p) + LDA for correlations
- QMC SF derived from same underlying Hamiltonian as GFMC time evolution operator
- Separate form factors and multinucleon effects from many body method
- Systematic on use of factorization

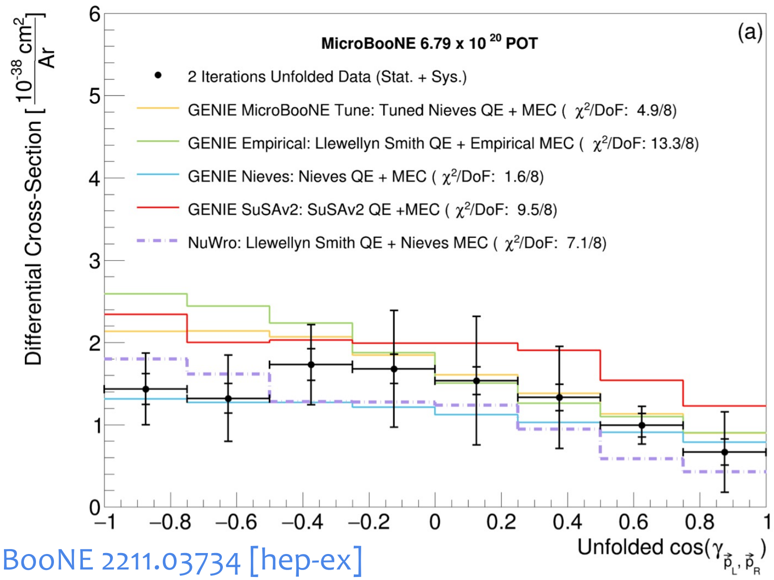




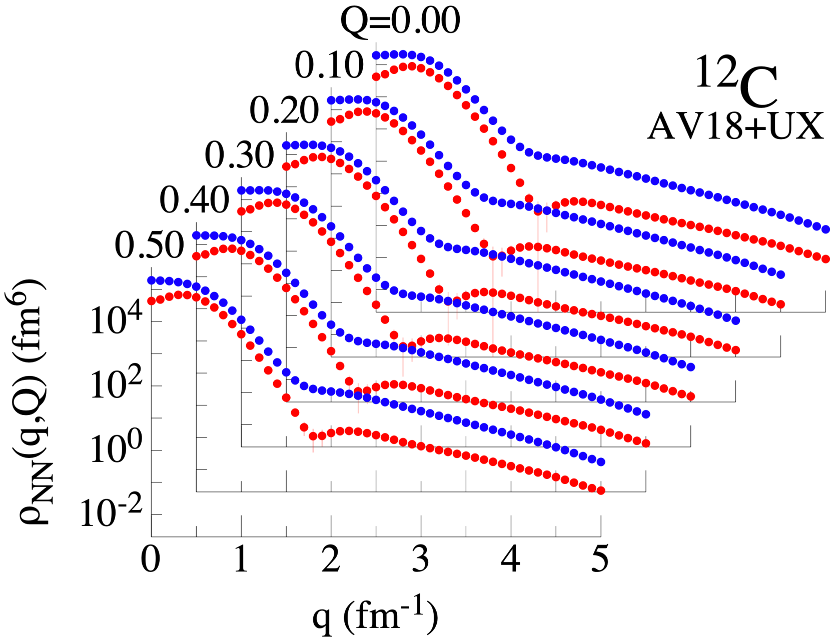
Two nucleon physics – QMC Inputs

- Two nucleon momentum distributions
  - Neutron-proton pairs (blue) dominate at large relative momentum
  - Distribution evolves as a function of CM momentum

- Highly relevant for LArTPC detectors
  - Understand evolution of NP vs PP final states vs nucleon energy



MicroBooNE 2211.03734 [hep-ex]

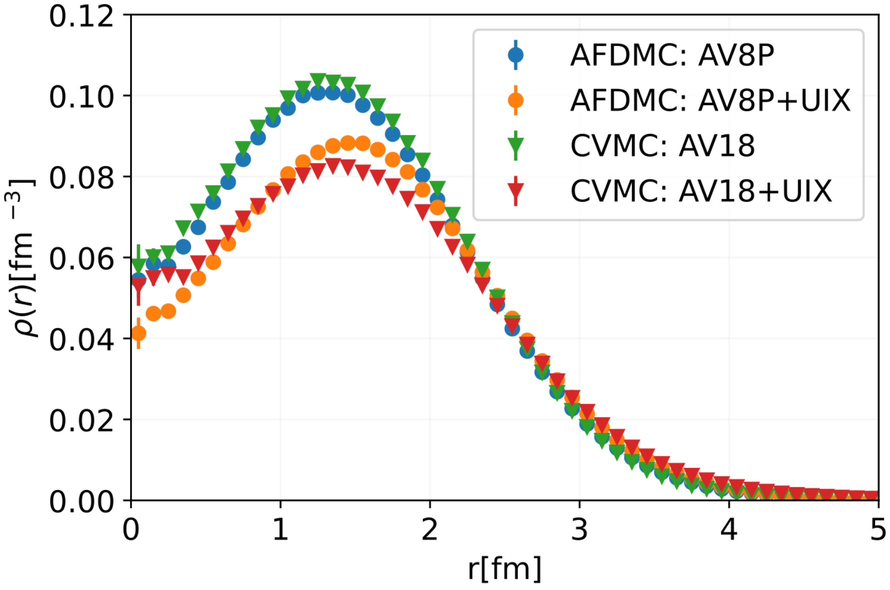


Wiringa, *et al.*, Phys. Rev. C **89**, 024305 (2014)

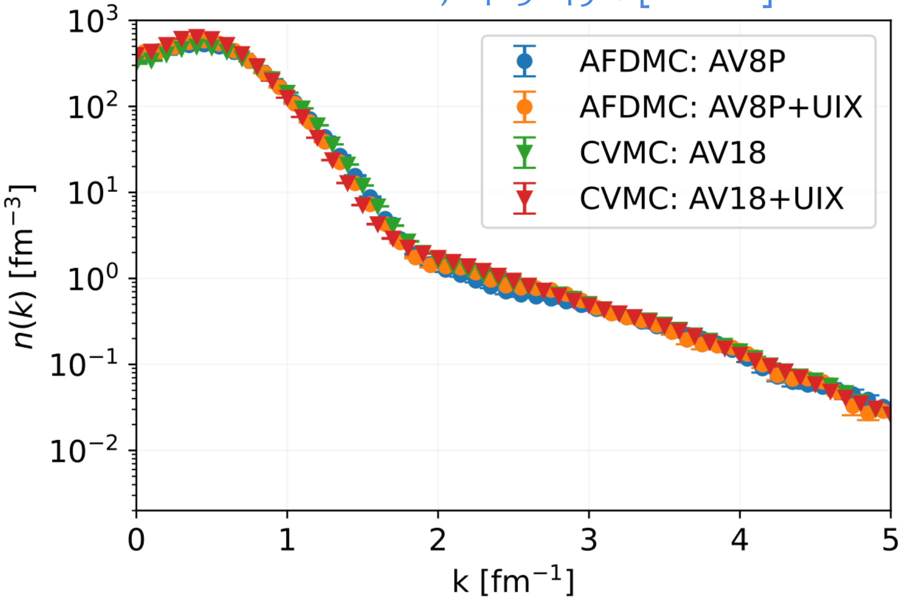
Momentum and density distributions for A > 12

- Inputs to generators needed are Spectral functions, momentum distributions, density distributions, etc.

- VMC calculations become intractable for A > 12, rely on AFDMC and other novel calculations



A. G. et al., 2405.14916 [nucl-th]



- Can be used as inputs in generators (hard scattering, cascade, etc.)

Wigner Distributions as inputs for Cascades

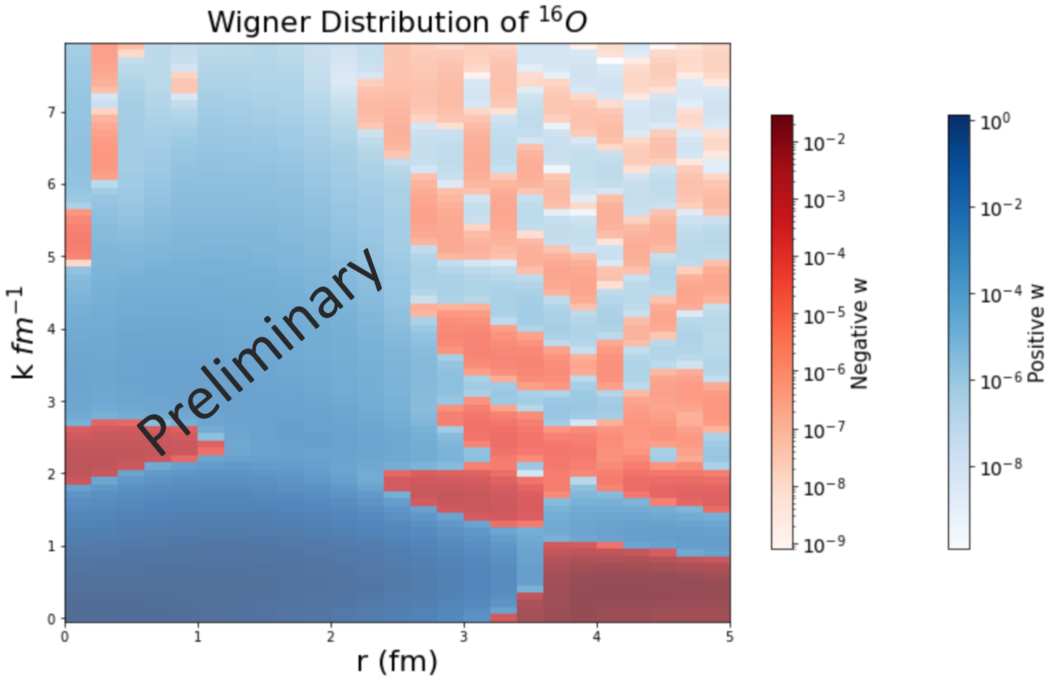
- Wigner Distributions are Quasi-Probability distributions

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \varphi^*(p+q)\varphi(p-q)e^{-2ixq/\hbar}\,dq$$

- Represents information on both the position and momentum of nucleons within the nucleus
  - Negative values reflect [p,q] != 0

- Affect the INCs used in generators

- Implementation into Achilles forthcoming!



Theoretical Uncertainties

- “All model’s are bad, but some are useful” especially those with fully fleshed errors

- Ab-initio methods promise an estimate of theoretical uncertainty from
  - Many Body Method
  - Input Hamiltonian
  - EFT truncation

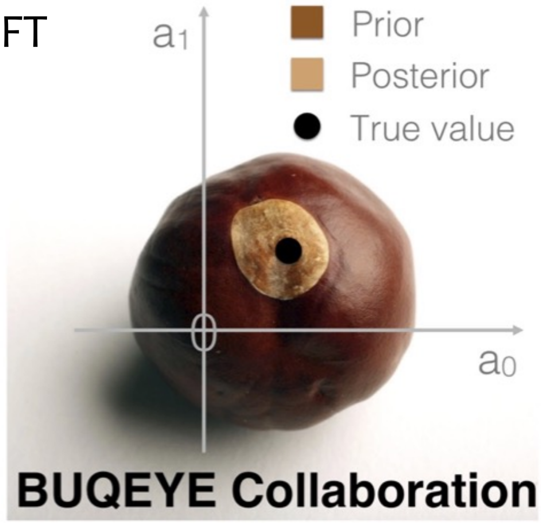
- Multiple groups working on this front
  - Bayesian Uncertainty Quantification: Errors in your EFT

Rigorous constraints on three-nucleon forces in chiral effective field theory from fast and accurate calculations of few-body observables

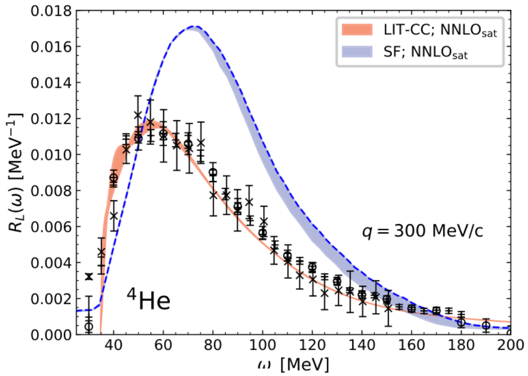
S. Wesolowski,[1,*] I. Svensson,[2,†] A. Ekström,[2,‡] C. Forssén,[2,§] R. J. Furnstahl,[3,‖]
J. A. Melendez,[3,¶] and D. R. Phillips[4,5,6,#]

Bayesian Analysis of Nuclear Dynamics

https://bandframework.github.io/

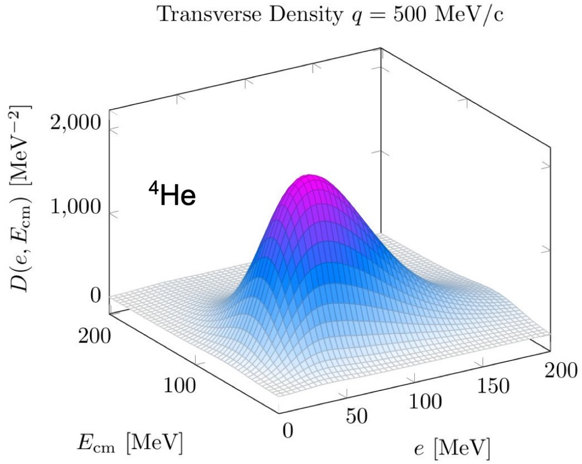


What we didn’t cover



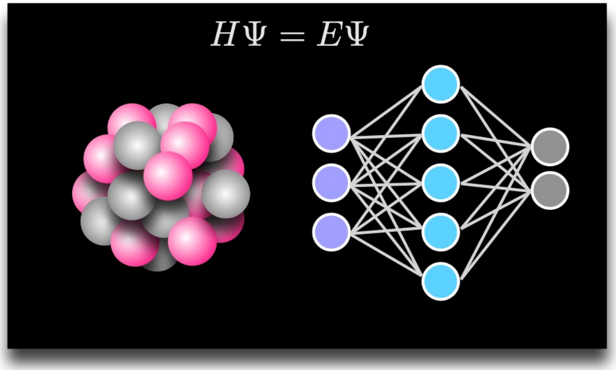
- Numerous other ab-initio methods being worked on
  - Coupled Cluster theory
    - Polynomial scaling A >> 12

    J. Sobczyk Few Body Syst. 65 (2024) 2, 48



  - Short time approximation (STA)
    - A > 12, Exclusive predictions

    SP *et al.* PRC101(2020)044612



  - Neural Network Quantum States
    - A > 12, large speed ups

    A. L. et al. Phys.Rev.Res. 4 (2022) 4, 043178

Conclusion

- Ab-initio methods start with a correct identification of the degrees of freedom appropriate for the problem and a Hamiltonian which describes their interactions

- Many body solvers, using as input these Hamiltonians, can compute ground state and dynamical responses of nuclei with tremendous precision

- Exclusive predictions can be accommodated in QMC approaches by factorizing the problem and computing momentum distributions, Spectral functions, densities within QMC

- Extending the reach of QMC methods to $A > 12$ has been achieved and is an ongoing goal with many branches

- Thank you!