

# Spectral Shape of Forbidden $\beta$ -decays: Recent Results on In-115

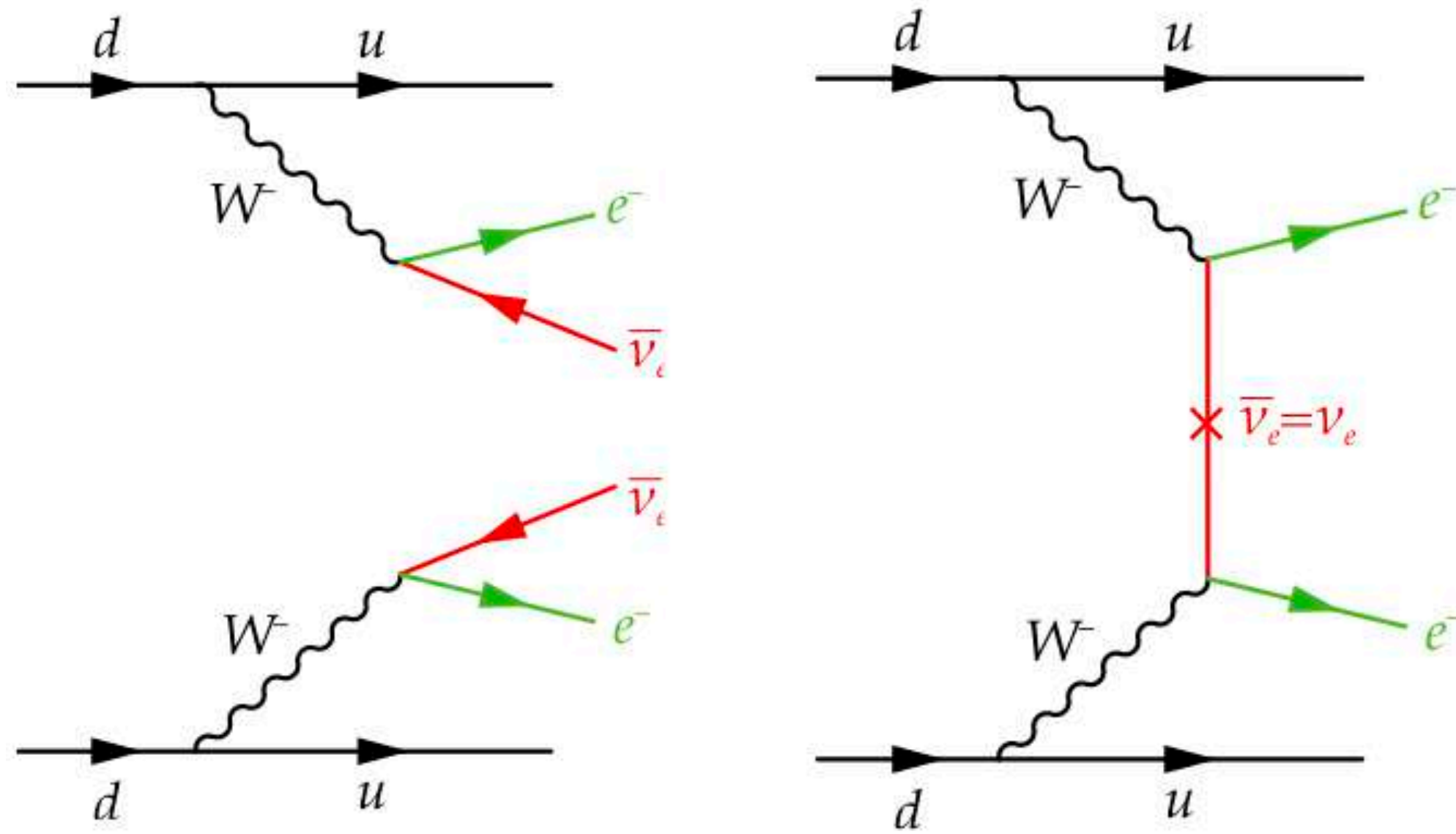


*Lorenzo Pagnanini*

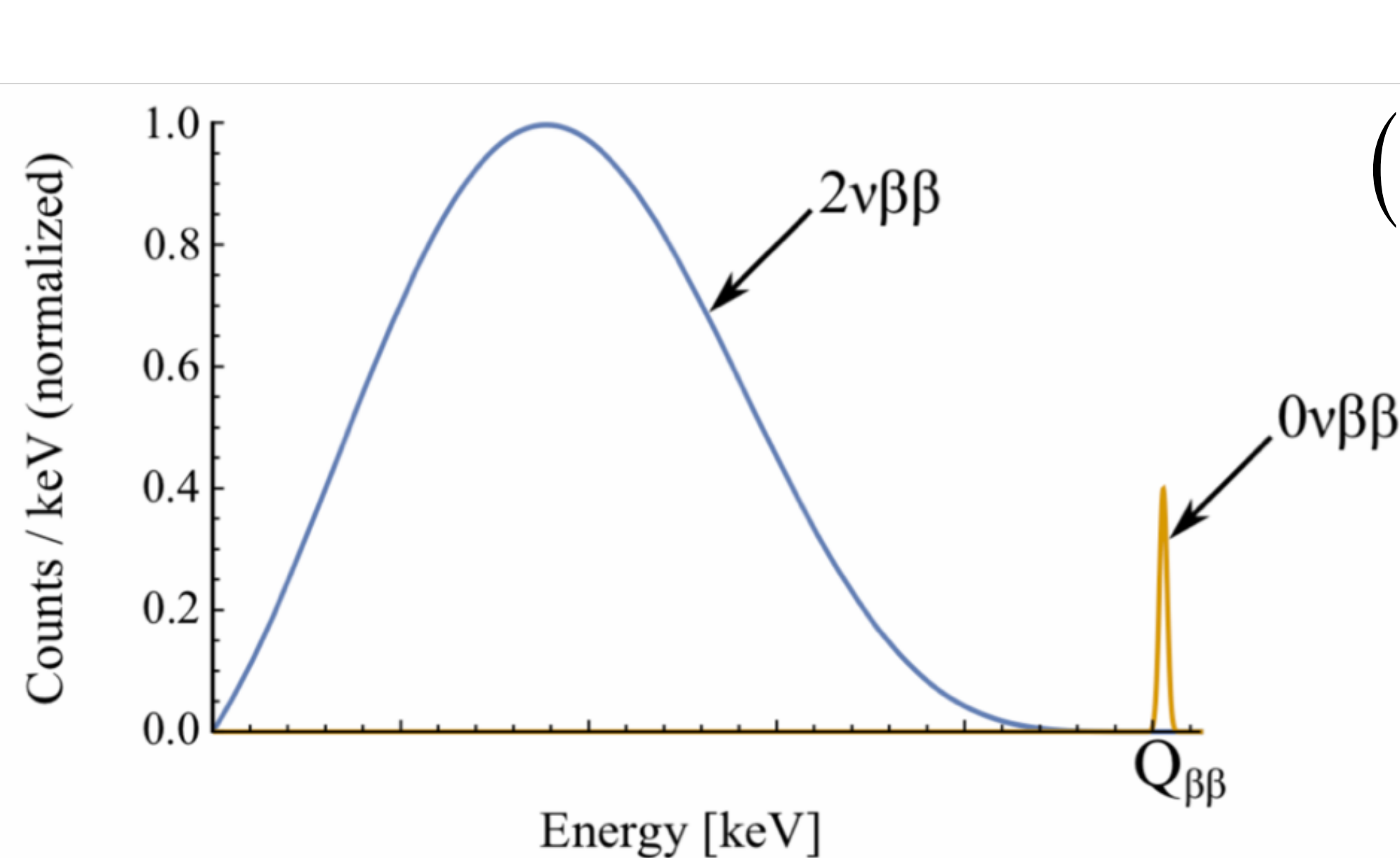
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*Gran Sasso Science Institute  
Laboratori Nazionali del Gran Sasso*

# Neutrinoless Double Beta Decay



- It only occurs **if neutrino is a Majorana** particle
- Forbidden by Standard Model: **violation of B-L**
- **Matter creation** (no anti-matter balancing)
- Insights on the **neutrino mass**



$$(T_{1/2}^{0\nu})^{-1} = g_A^4 \cdot \mathcal{G}^{0\nu}(Q_{\beta\beta}, Z) \cdot \left| \mathcal{M}^{0\nu}(A, Z) \right|^2 \cdot \left| \frac{m_{\beta\beta}}{m_e} \right|^2$$

$$m_{\beta\beta} = \left| \sum_{j=1}^3 m_j U_{ej}^2 \right| = \left| U_{e1}^2 m_1 + U_{e2}^2 e^{i\alpha} m_2 + U_{e3}^2 e^{i\beta} m_3 \right|$$

Effective Majorana Mass

# Neutrinoless Double Beta Decay

The international effort to observe neutrinoless double beta decay is increasing and new experiments are growing.

AMORE 🇰🇷 🇨🇳 🇩🇪 🇺🇦 🇮🇹 🇸🇮 🇵🇰

CUPID 🇮🇹 🇺🇸 🇫🇷 🇷🇺 🇨🇳 🇺🇦

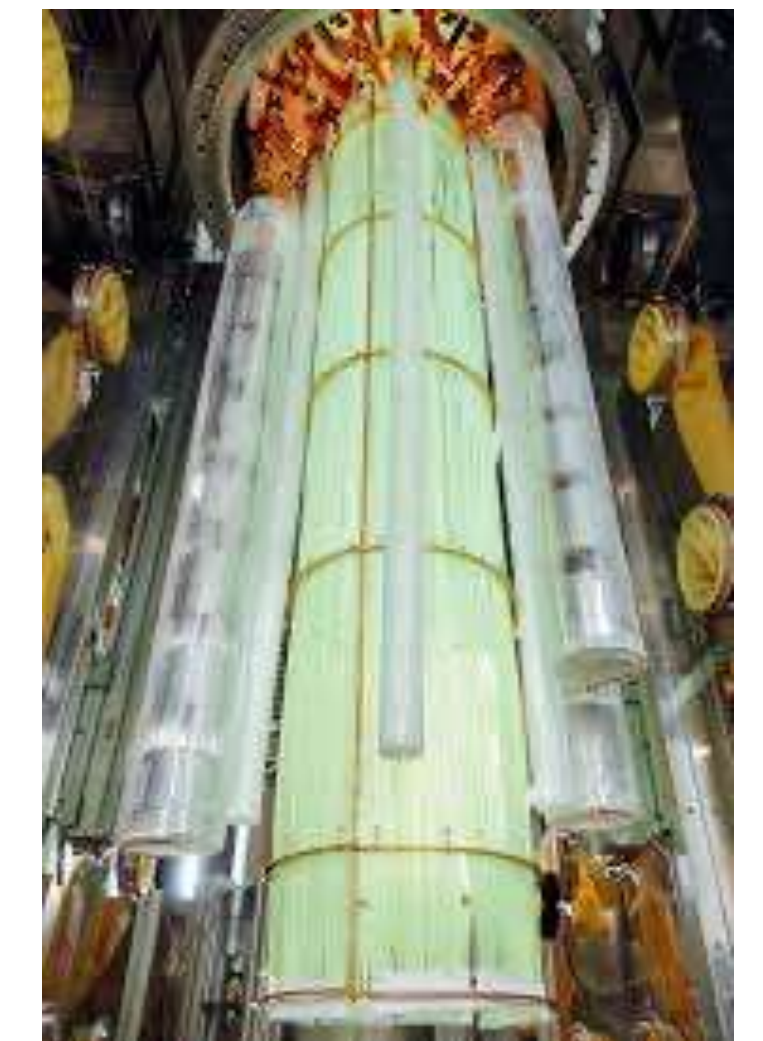
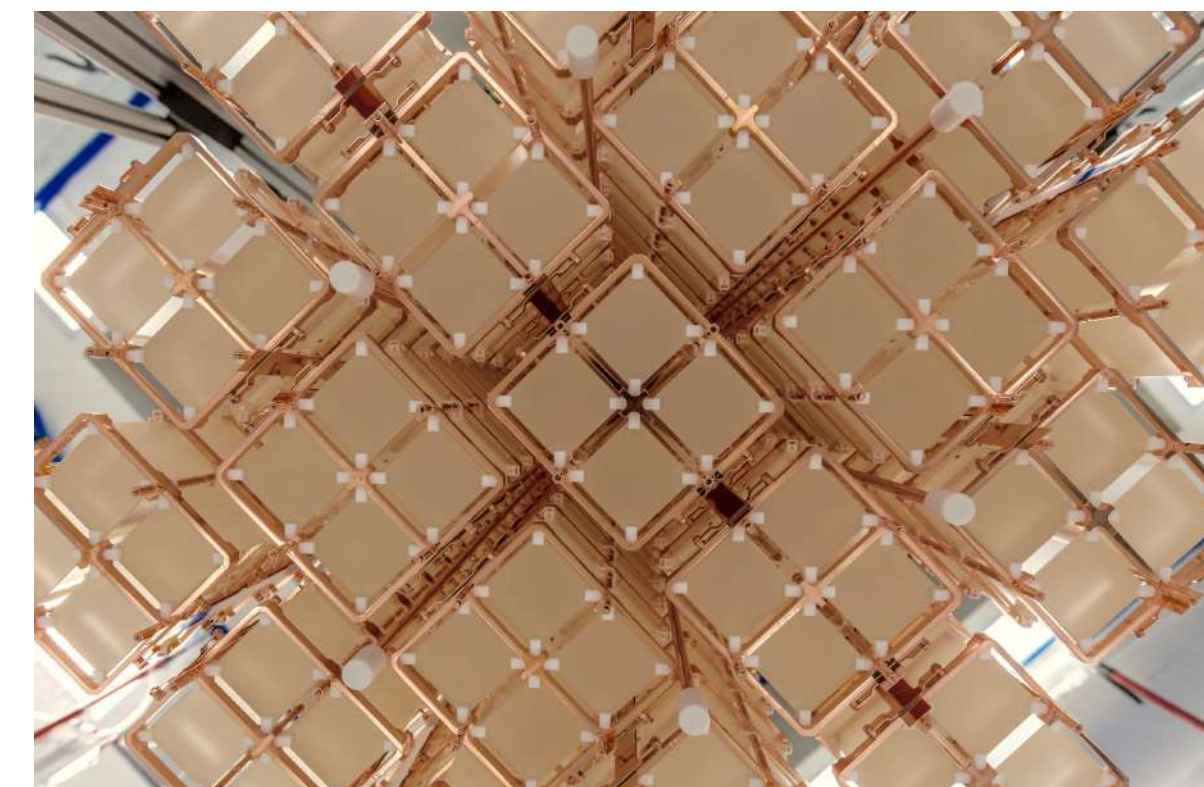
KamLAND-ZEN 🇯🇵 🇺🇸 🇮🇹

LEGEND 🇩🇪 🇺🇸 🇷🇺 🇨🇳 🇮🇹 🇬🇧 🇨🇳 🇨🇭

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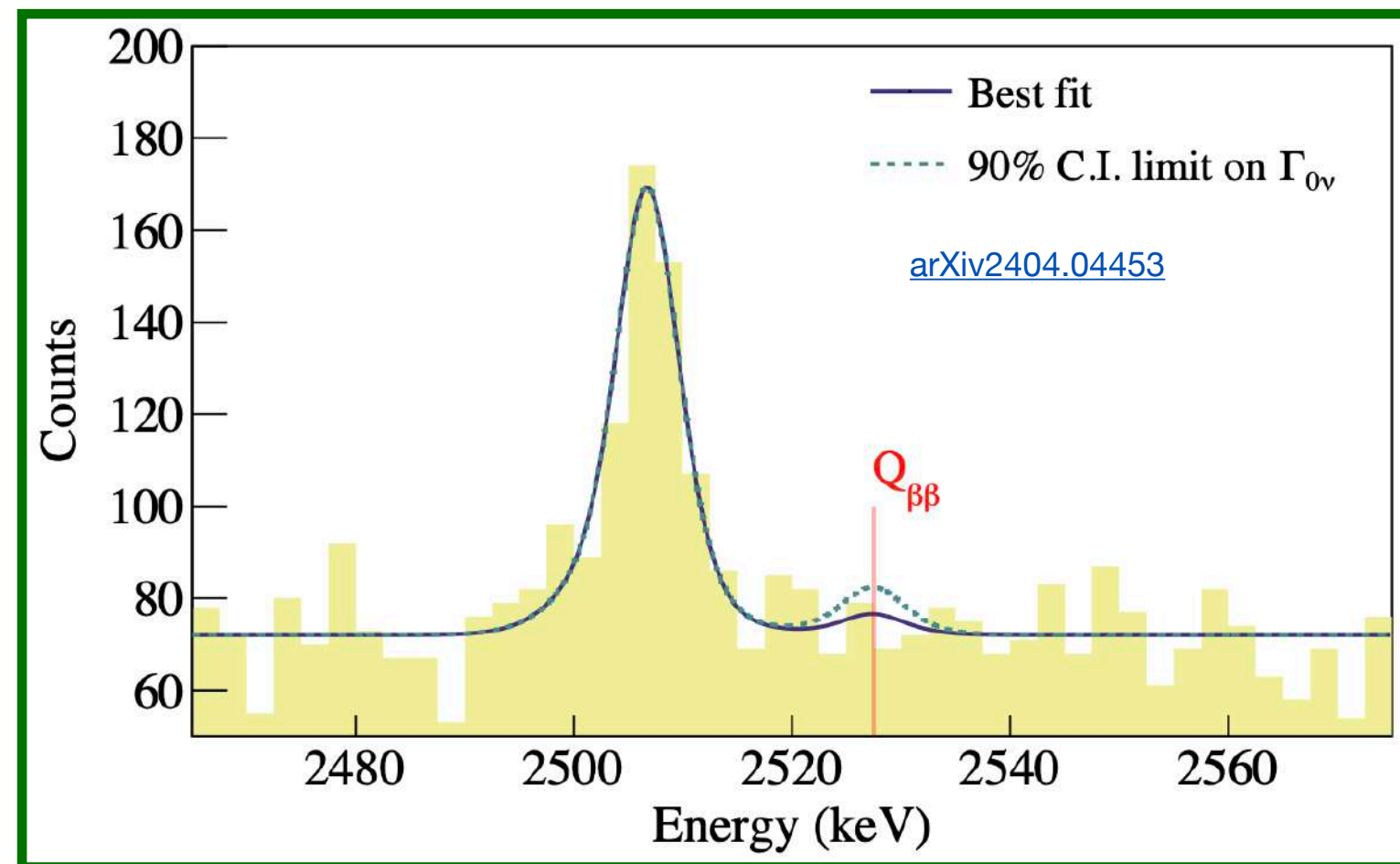
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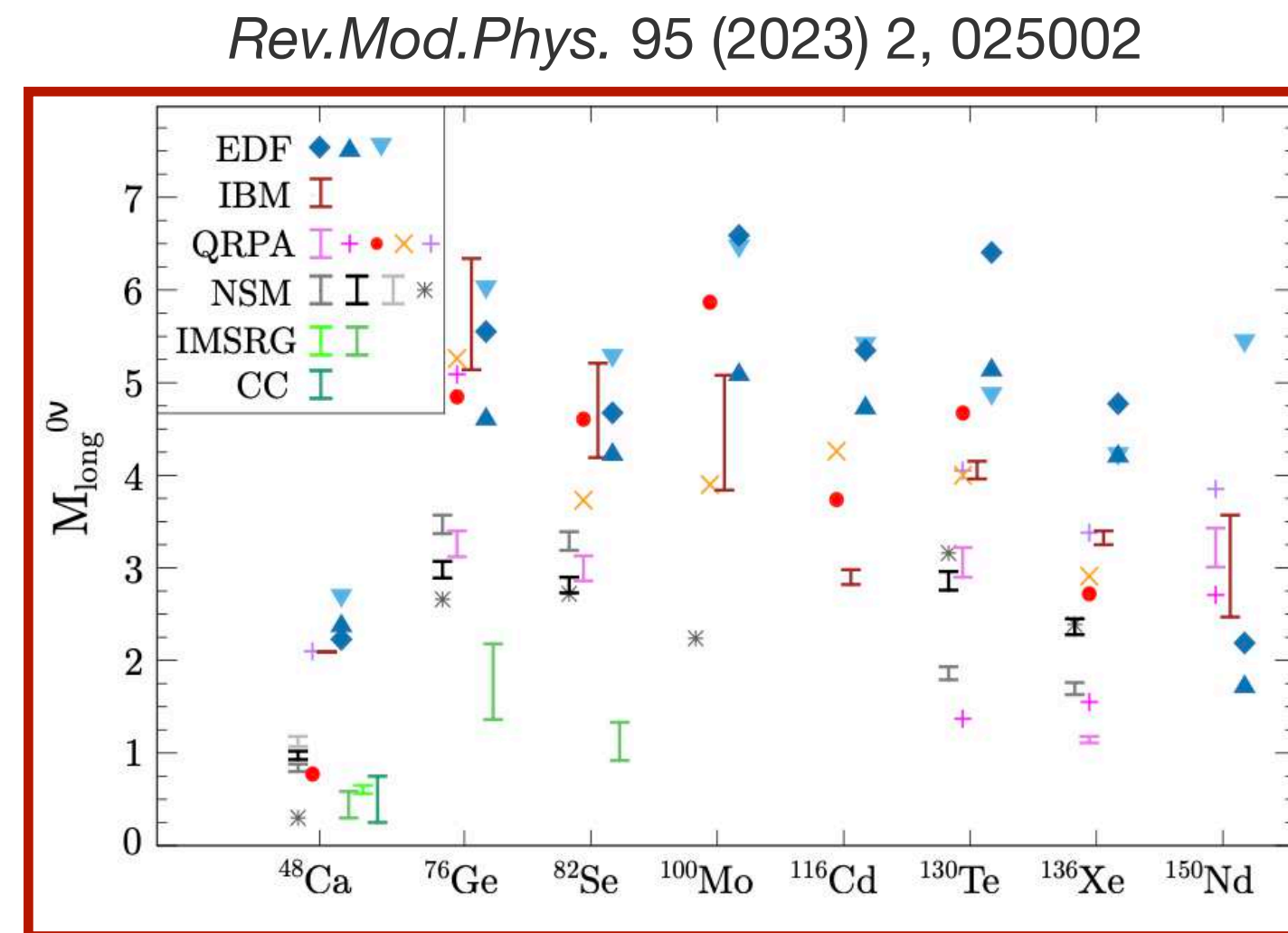


# Neutrinoless Double Beta Decay

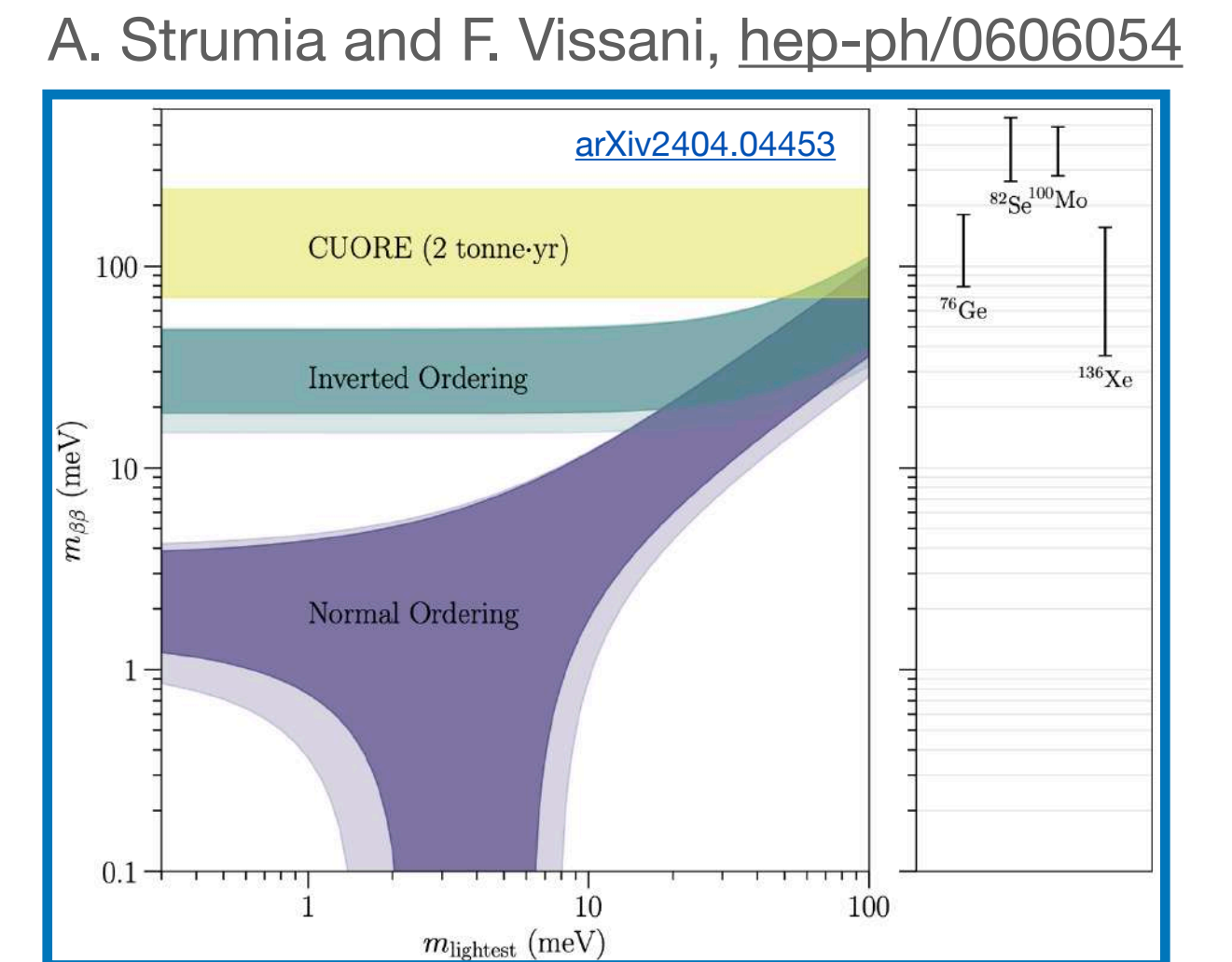
$$\left(T_{1/2}^{0\nu}\right)^{-1} = g_A^4 \cdot \mathcal{G}^{0\nu}(Q_{\beta\beta}, Z) \cdot \left|\mathcal{M}^{0\nu}(A, Z)\right|^2 \cdot \left|\frac{m_{\beta\beta}}{m_e}\right|^2$$



A single experimental limit on the half-life...



...due to the uncertainty on the NME...



...results in a wide interval!

Also the isotope down-selection is affected by this uncertainty!

<sup>76</sup>Ge (GERDA) - Phys. Rev. Lett., 125:252502, 2020

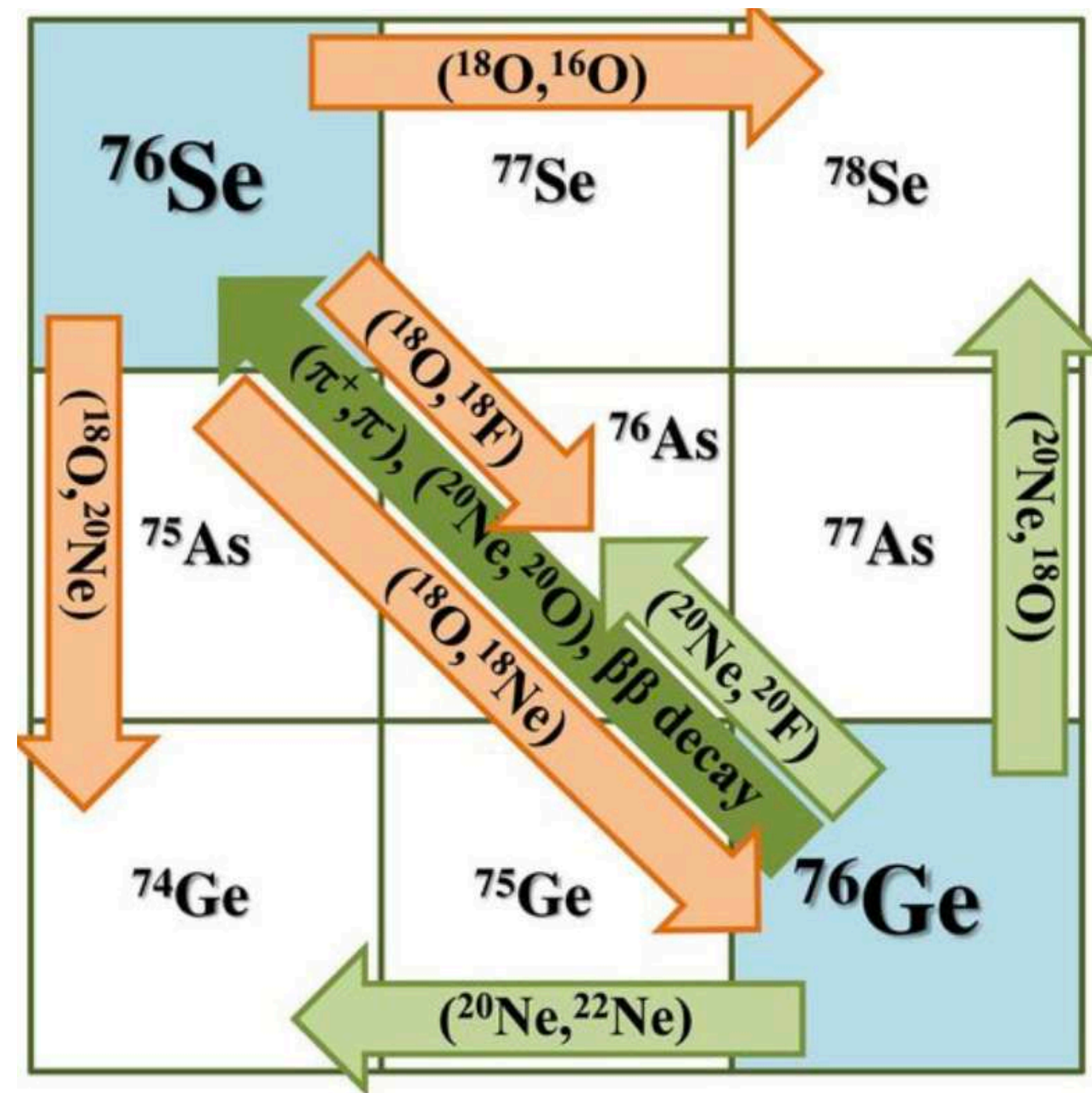
<sup>82</sup>Se (CUPID-0) - Phys. Rev. Lett., 129(11):111801, 2022

<sup>100</sup>Mo (CUPID-Mo) - Eur. Phys. J. C, 82(11):1033, 2022

<sup>136</sup>Xe (KamLAND-Zen) - Phys. Rev. Lett., 130(5):051801, 2023

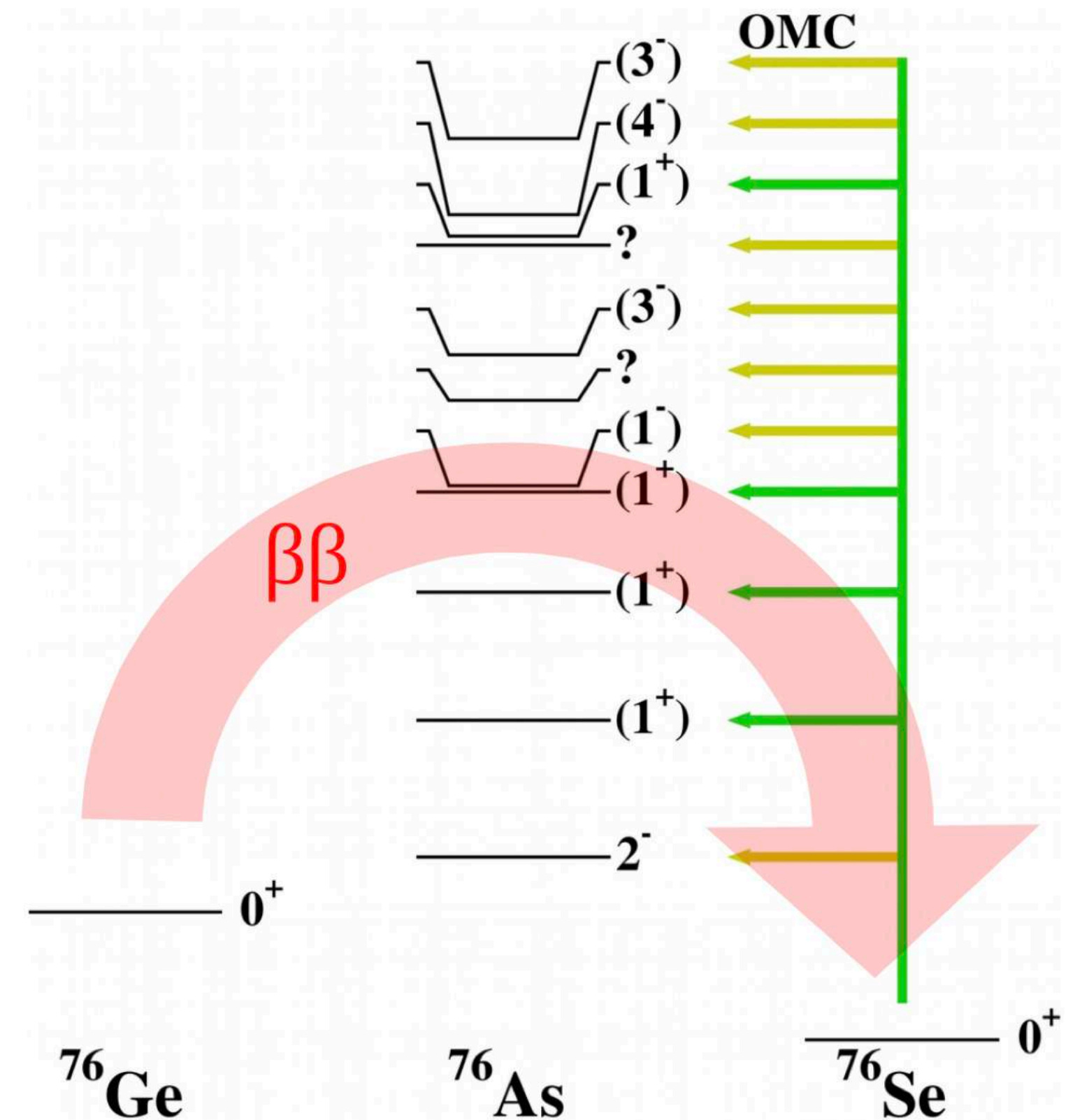
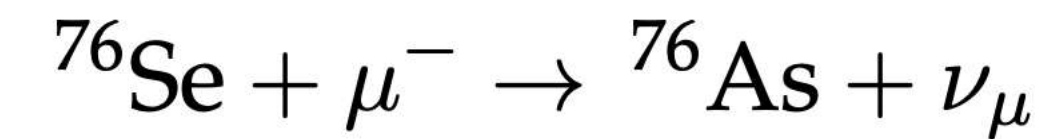
# Data-driven improvements of Nuclear Models

## Double Charge Exchange (DCE)

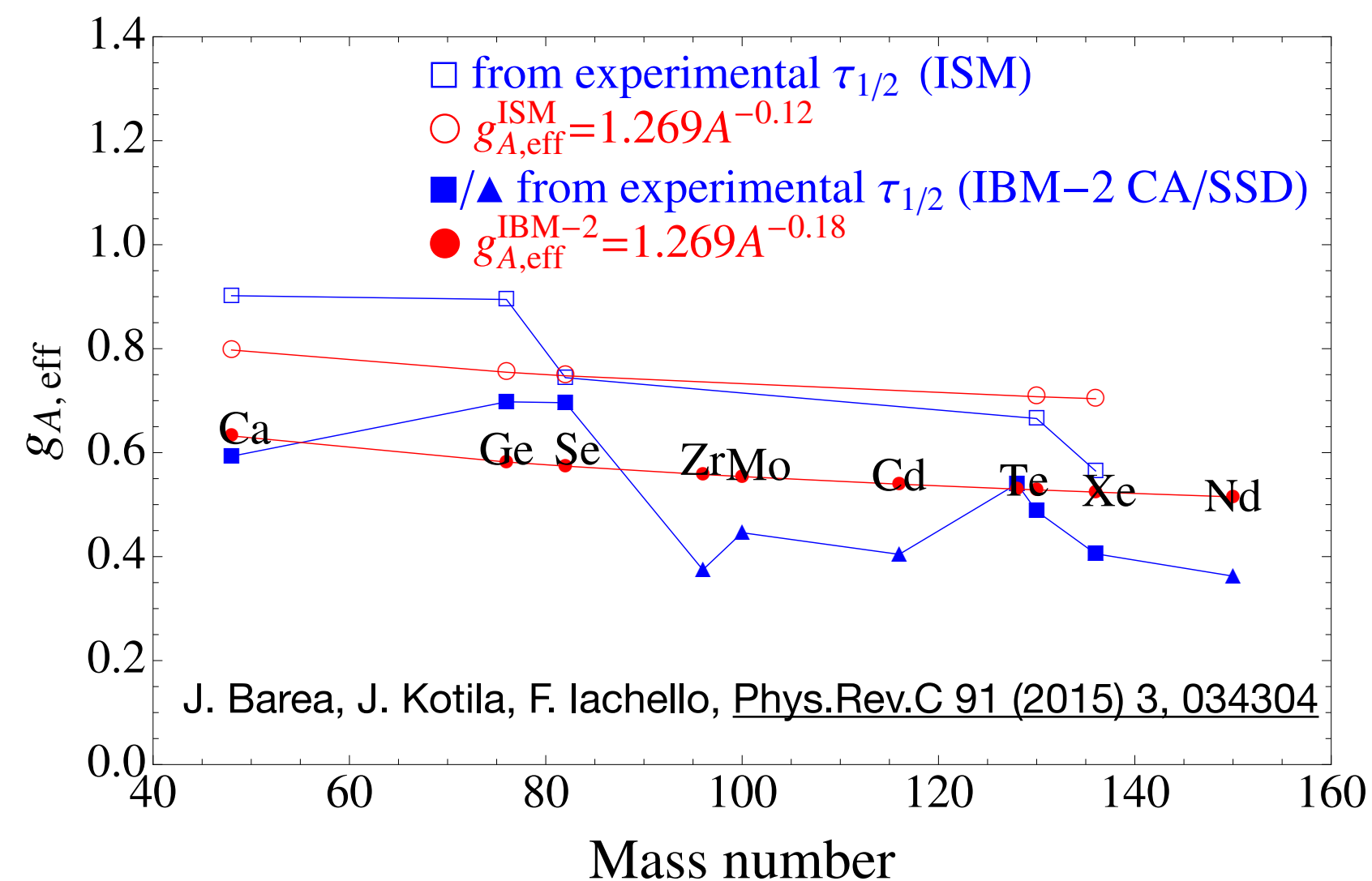
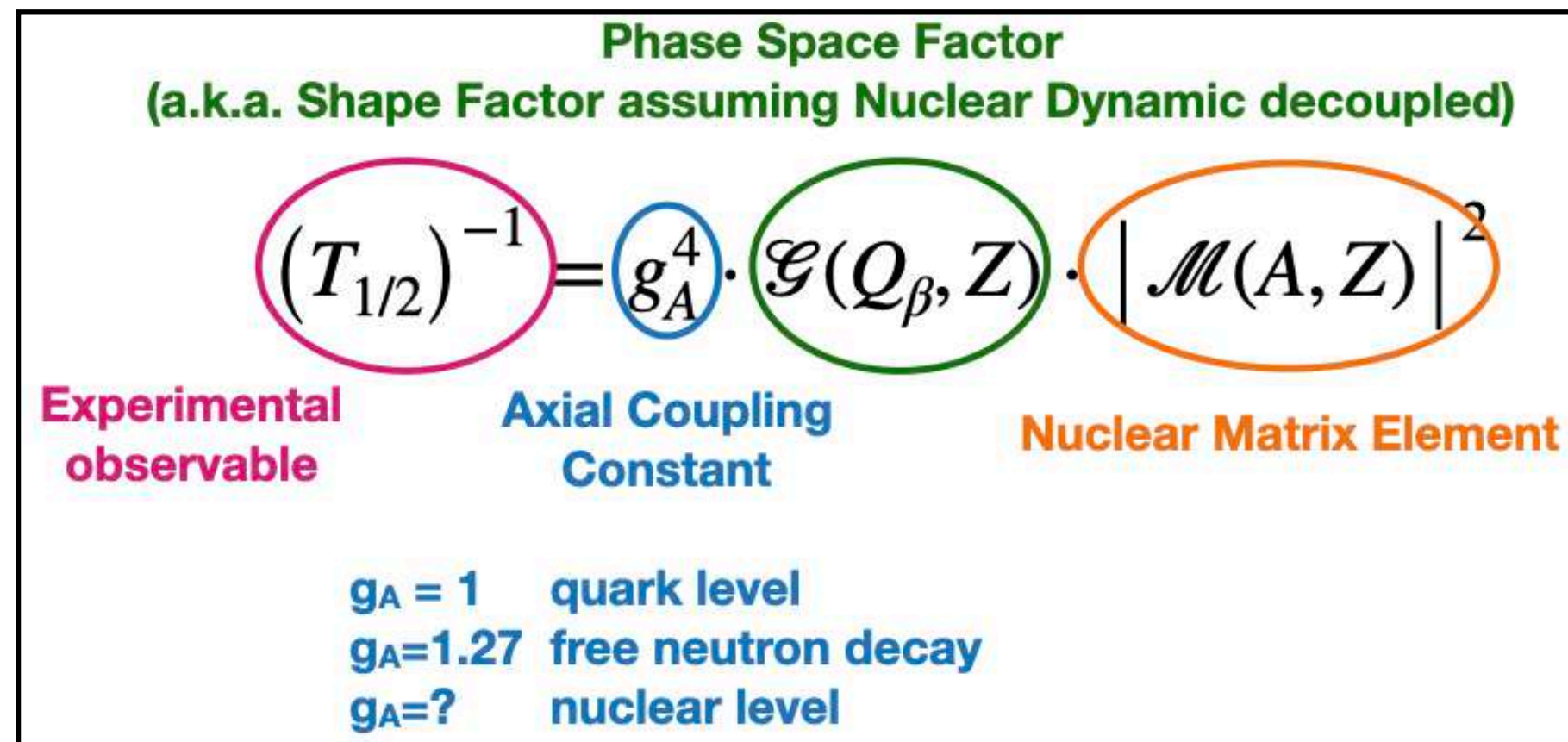


See Clementina Agodi's talk on NUMEN

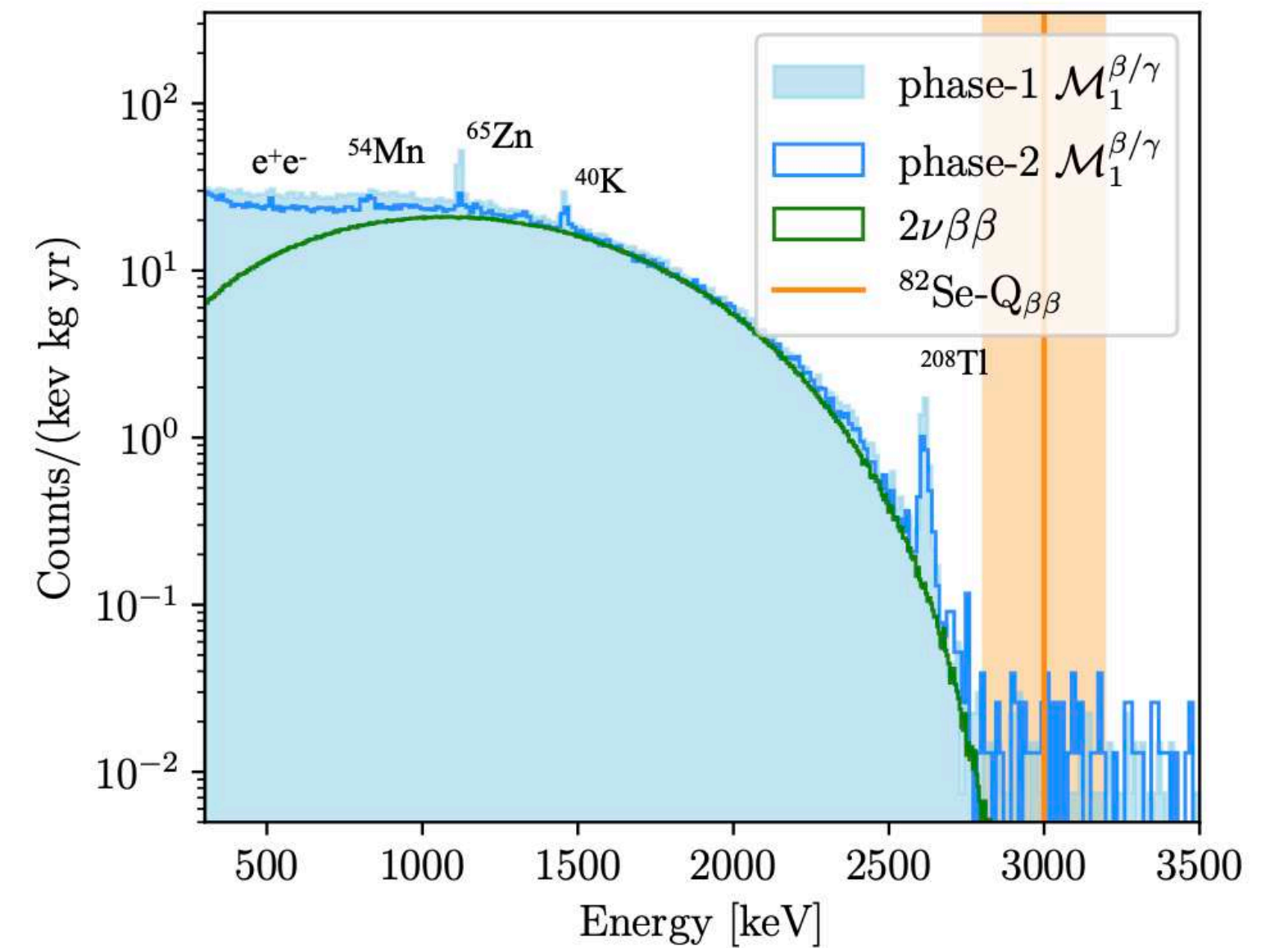
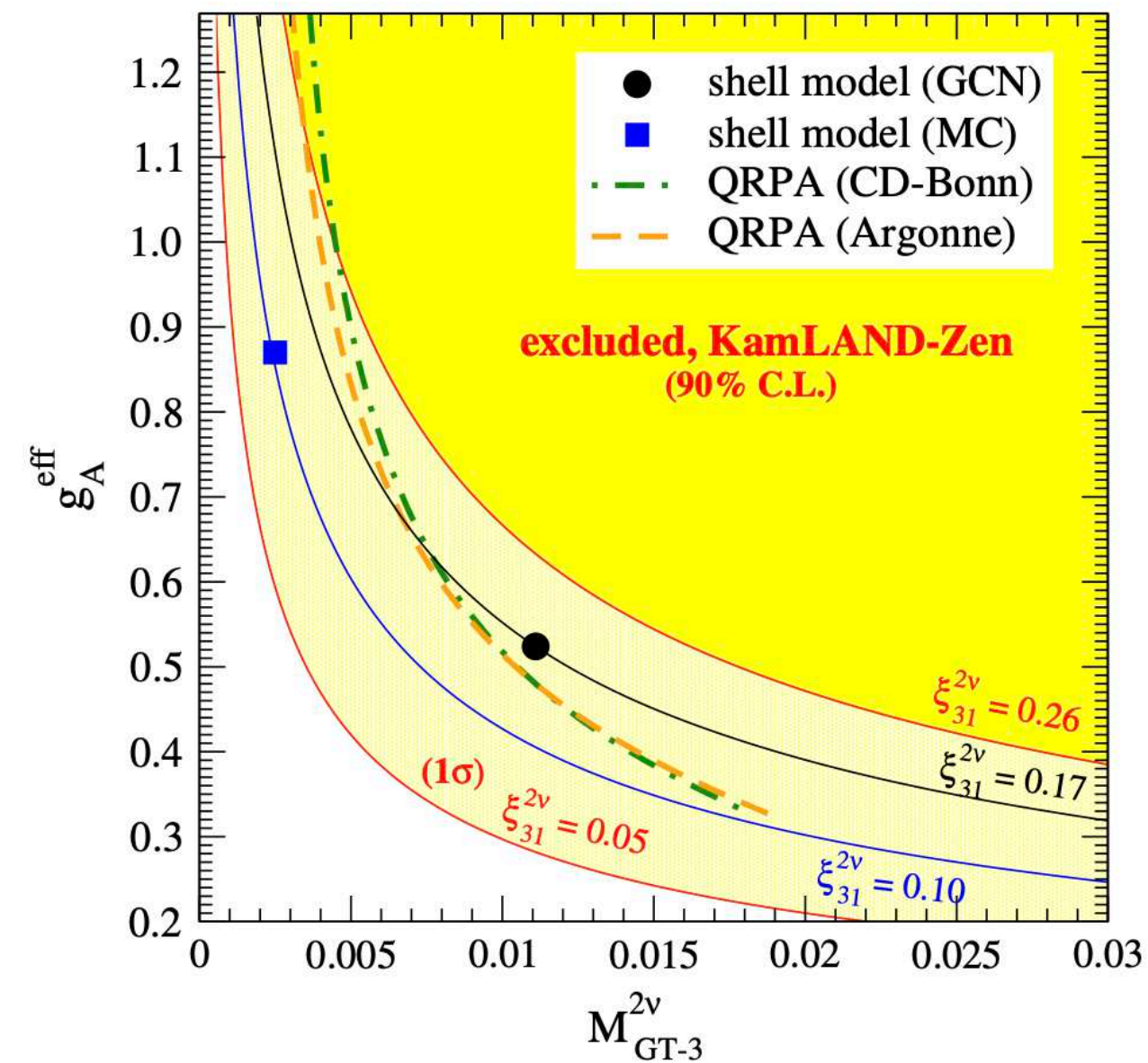
## Ordinary Muon Capture (OMC)



# Two-neutrinos Double Beta Decay



$g_A$  modeling based on experimental  $2\nu\beta\beta$  half-lives only



## Precision Spectral Shape Studies of $2\nu\beta\beta$

$^{136}\text{Xe}$  (KamLAND-Zen) - [Phys.Rev.Lett. 122 \(2019\) 19, 192501](#)

$^{82}\text{Se}$  (CUPID-0) - [Phys.Rev.Lett. 131 \(2023\) 22, 222501](#)

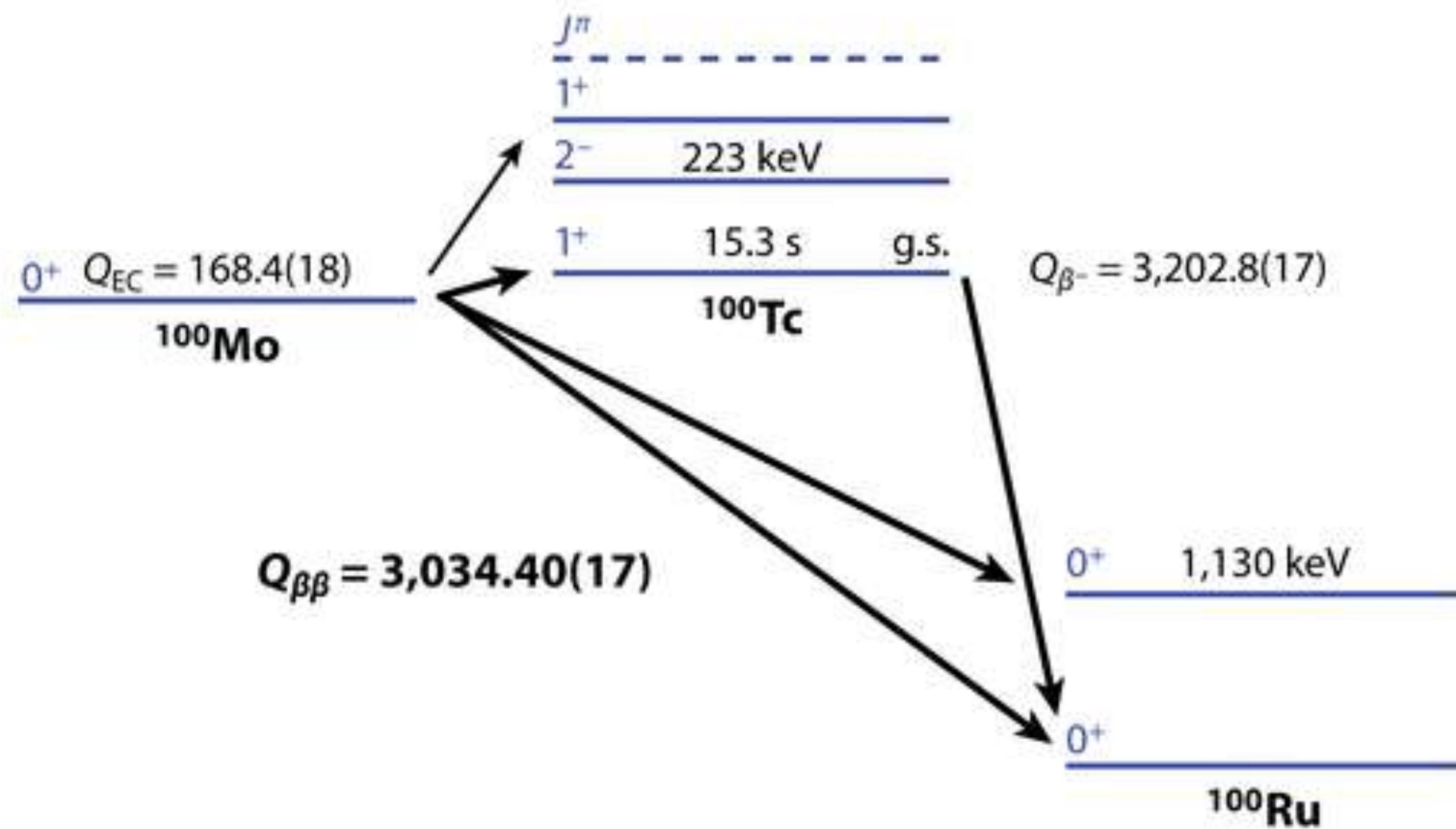
$^{100}\text{Mo}$  (CUPID-Mo) - [Phys.Rev.Lett. 131 \(2023\) 16, 162501](#)

Using the **improved description** of  $2\nu\beta\beta$  presented in  
 F. Šimković *et al.* - [Phys.Rev.C 97 \(2018\) 3, 034315](#)

Deeper comparison with theory

6 Effective nuclear matrix elements as a function of  $g_A$

# Forbidden transitions in NLDBD



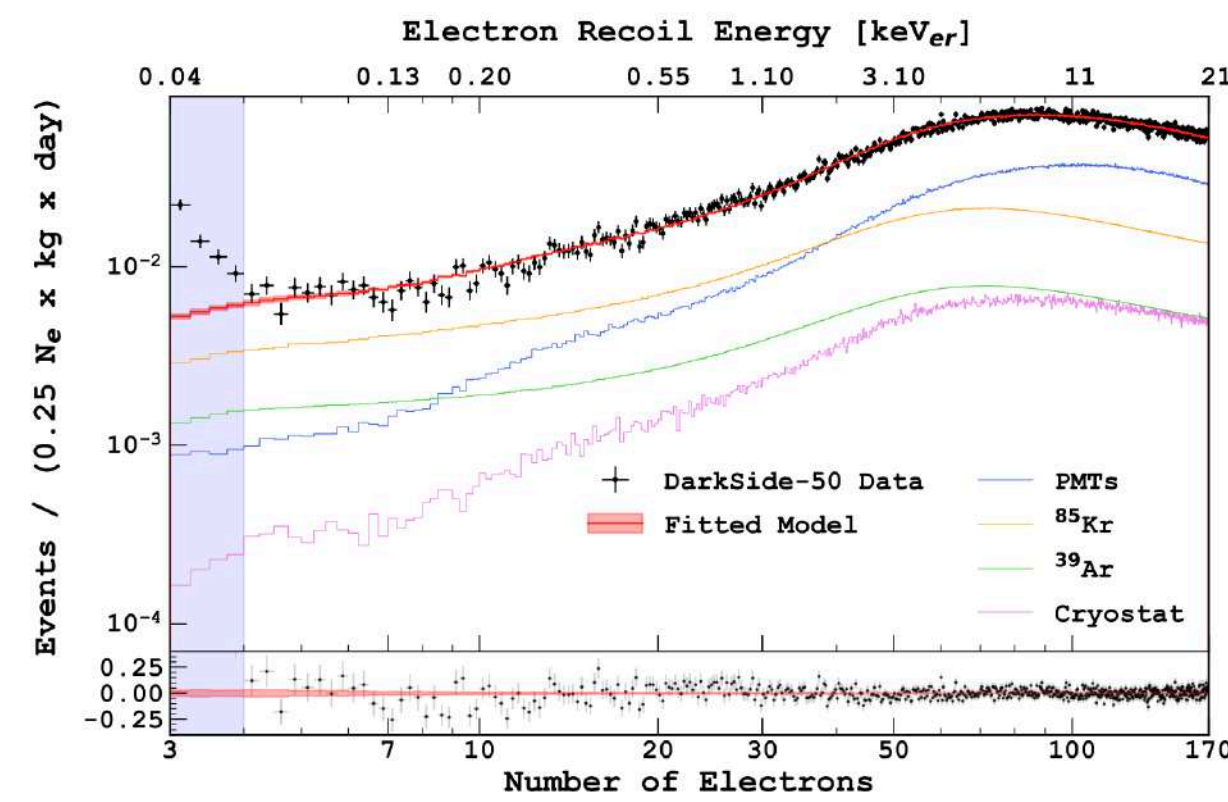
Forbidden  $\beta$ -decays are interesting for NLDBD since it **proceeds through forbidden virtual  $\beta$ -transitions** involving the excited states in the intermediate nucleus with **high multi-polarities**.

Caveat for extrapolation to NLDBD:

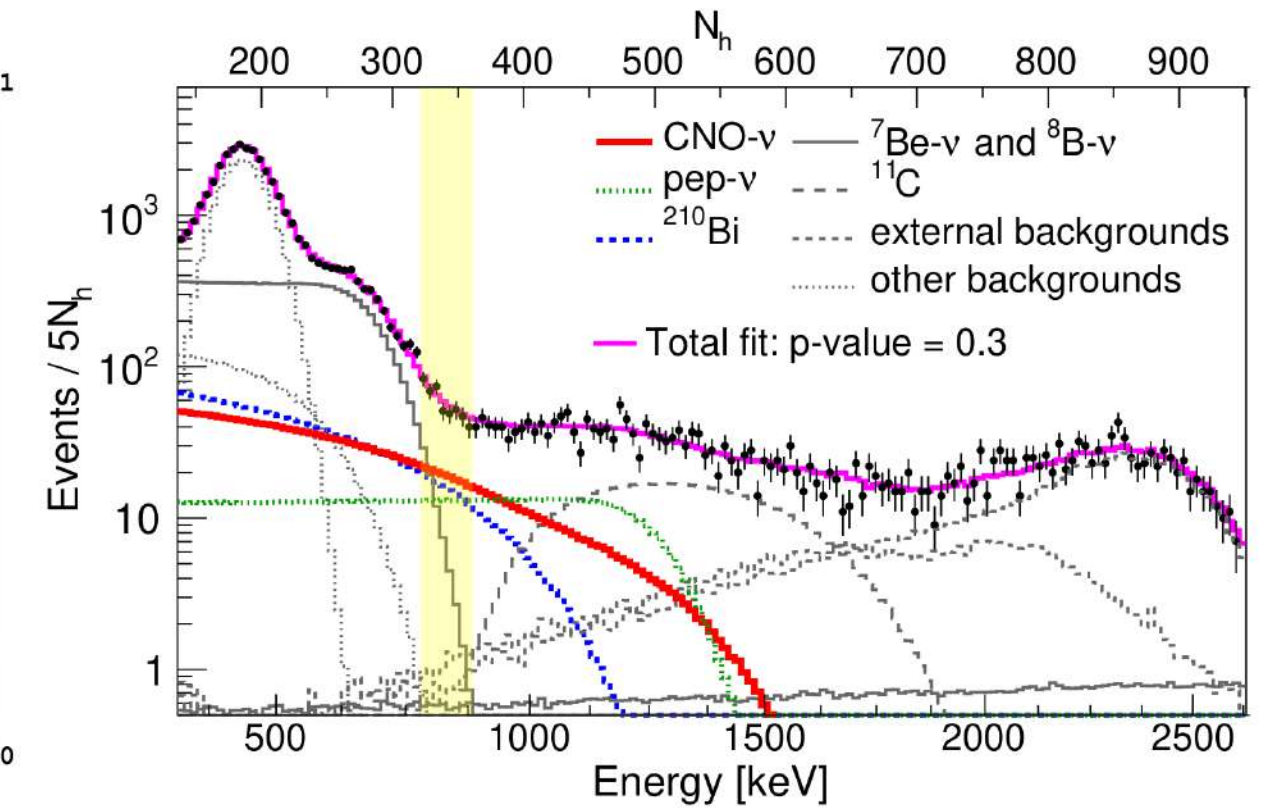
- only  $1^+$  states of the intermediate nucleus participate in the  $2\nu\beta\beta$  (apparently only the first - Single State Dominance)
- $\beta$ -decays and  $2\nu\beta\beta$  feature a lower transferred momentum with respect to NLDBD

# Further Motivations

- **Background in rare event search**
  - Background source in dark matter search
    - $^{40}\text{K}$ ,  $^{42}\text{Ar}$ ,  $^{39}\text{Ar}$ , Pb isotopes
  - Ingredients in NLDBD background modeling
    - $^{90}\text{Sr}/^{90}\text{Y}$ ,  $^{210}\text{Bi}$ ,  $^{40}\text{K}$
  - Background in Neutrino experiment
    - $^{210}\text{Bi}$



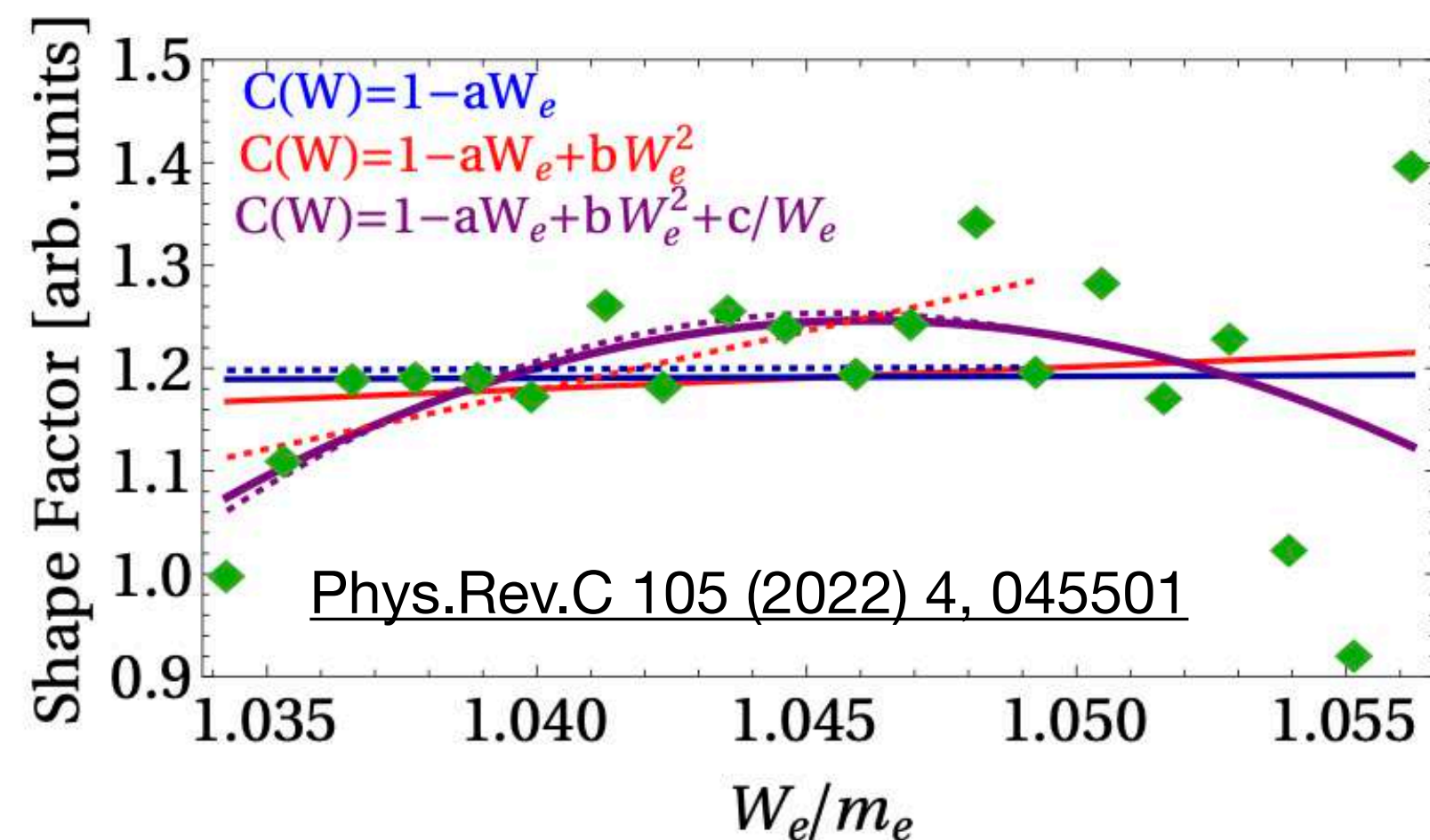
$^{39}\text{Ar}$  in Ar-based detector



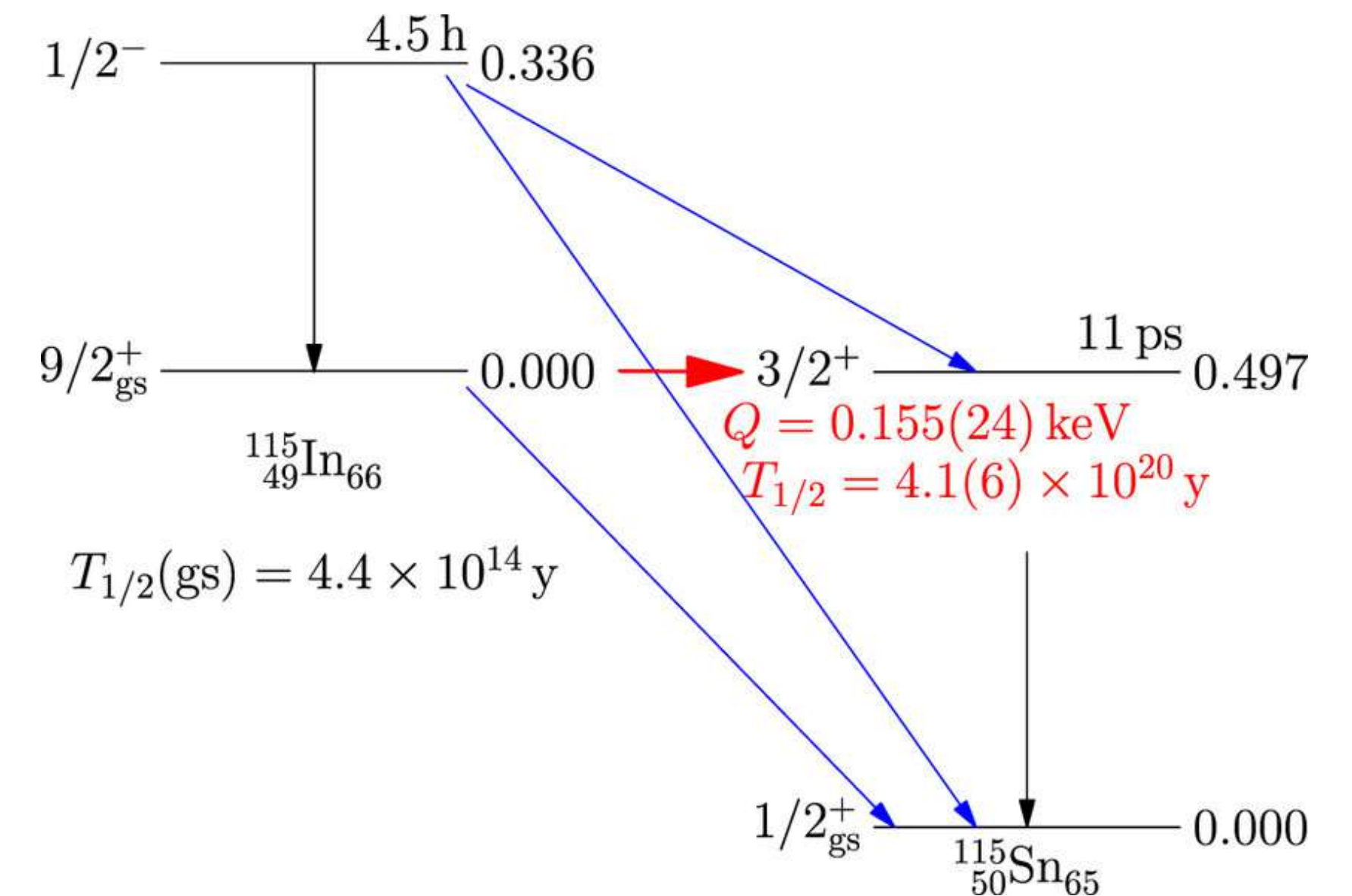
$^{210}\text{Bi}$  in Liquid Scintillators

- **Low Q-value decays**

- Cosmic Neutrino Background detection =>  $^{151}\text{Sm}$ ,  $^{171}\text{Tm}$
- Neutrino mass =>  $^{115}\text{In}$  decay on  $^{115}\text{Sn}^*$



$^{171}\text{Tm}$  shape factor





# Forbidden Beta Decays: Indium-115

# Indium-115

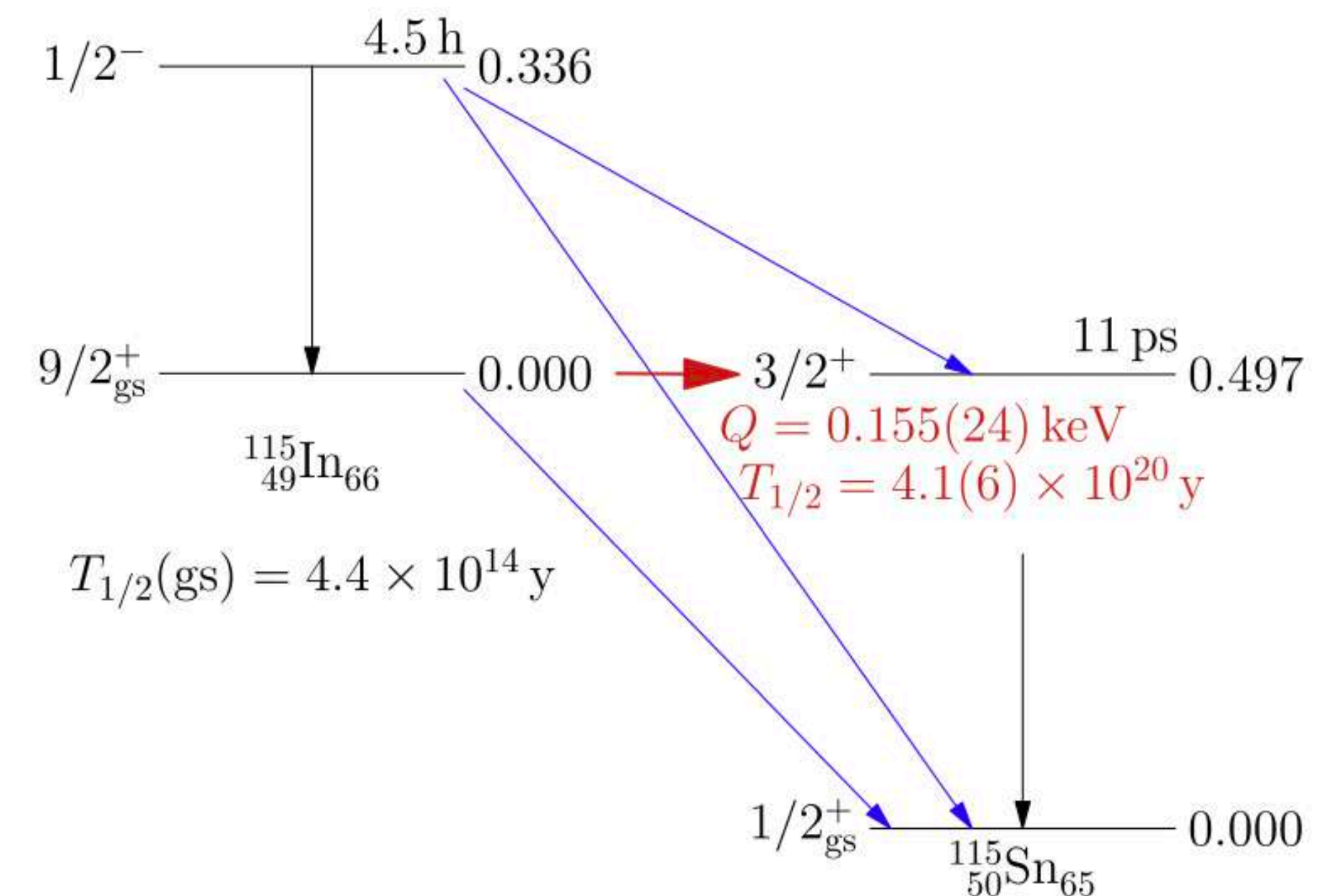
## Situation in \*2022\*

Only three *historical* measurements:

- *G.B. Beard and W. H. Kelly, PR 122 (1961) 1576*
  - $T_{1/2} = (6.9 \pm 1.5) \times 10^{14}$  yr
  - Threshold 50 keV
  - **No spectral shape**
- *D. E. Watt, R. N. Glover, Phil.Mag 7, 105 (1962)*
  - $T_{1/2} = (5.1 \pm 0.4) \times 10^{14}$  yr
  - **No spectral shape**
- *L. Pfeiffer et al., PRC 19 (1979) 1035*
  - $T_{1/2} = (4.41 \pm 0.25) \times 10^{14}$  yr
  - Spectral shape but with not clear background subtraction
  - Threshold not clear

**New low-background measurements needed!**

Q-value	Half-life	Classification
496 keV	$4.41 \times 10^{14}$ yr	$\frac{9^+}{2} \rightarrow \frac{1^+}{2} \quad \Delta J^{\Delta\pi} = 4^+$

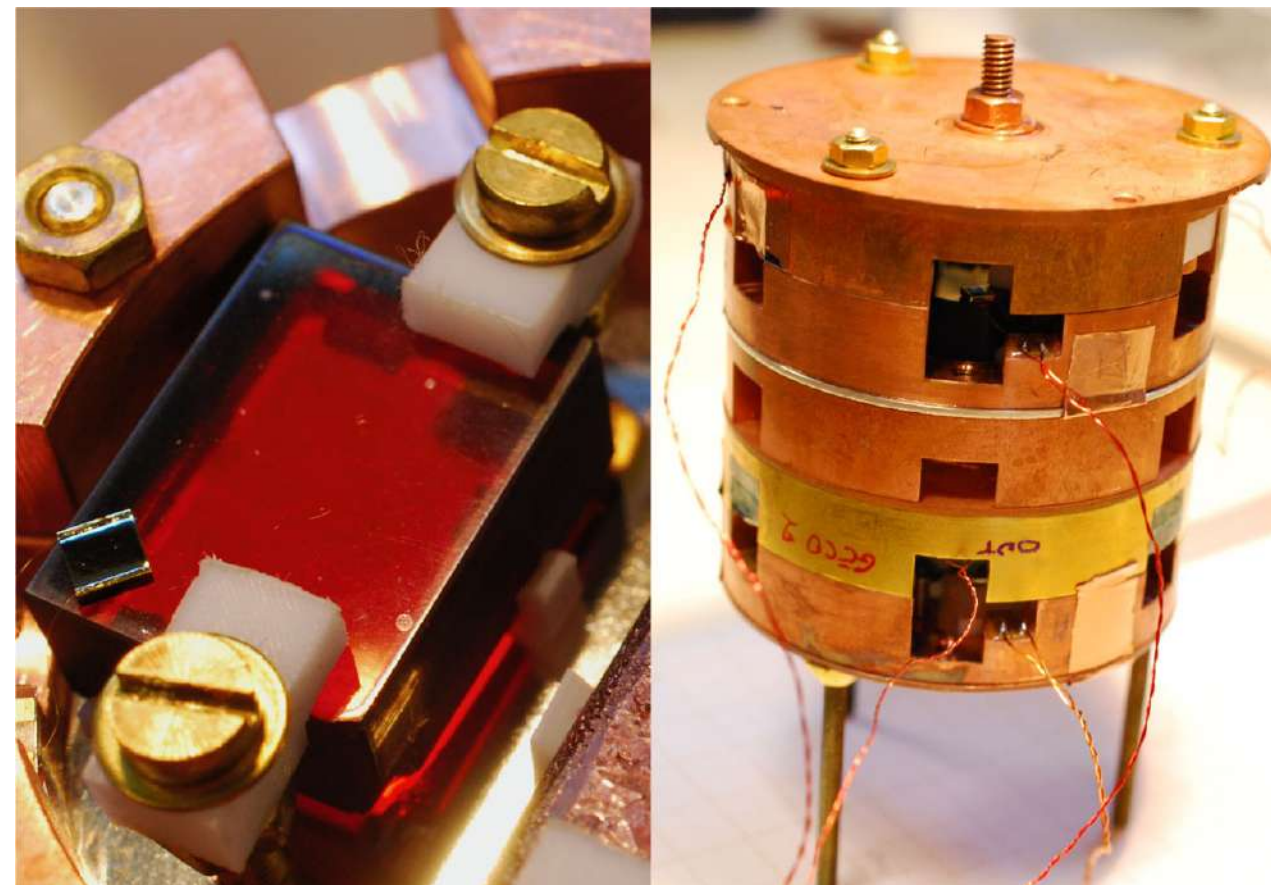


## Very good experimental conditions:

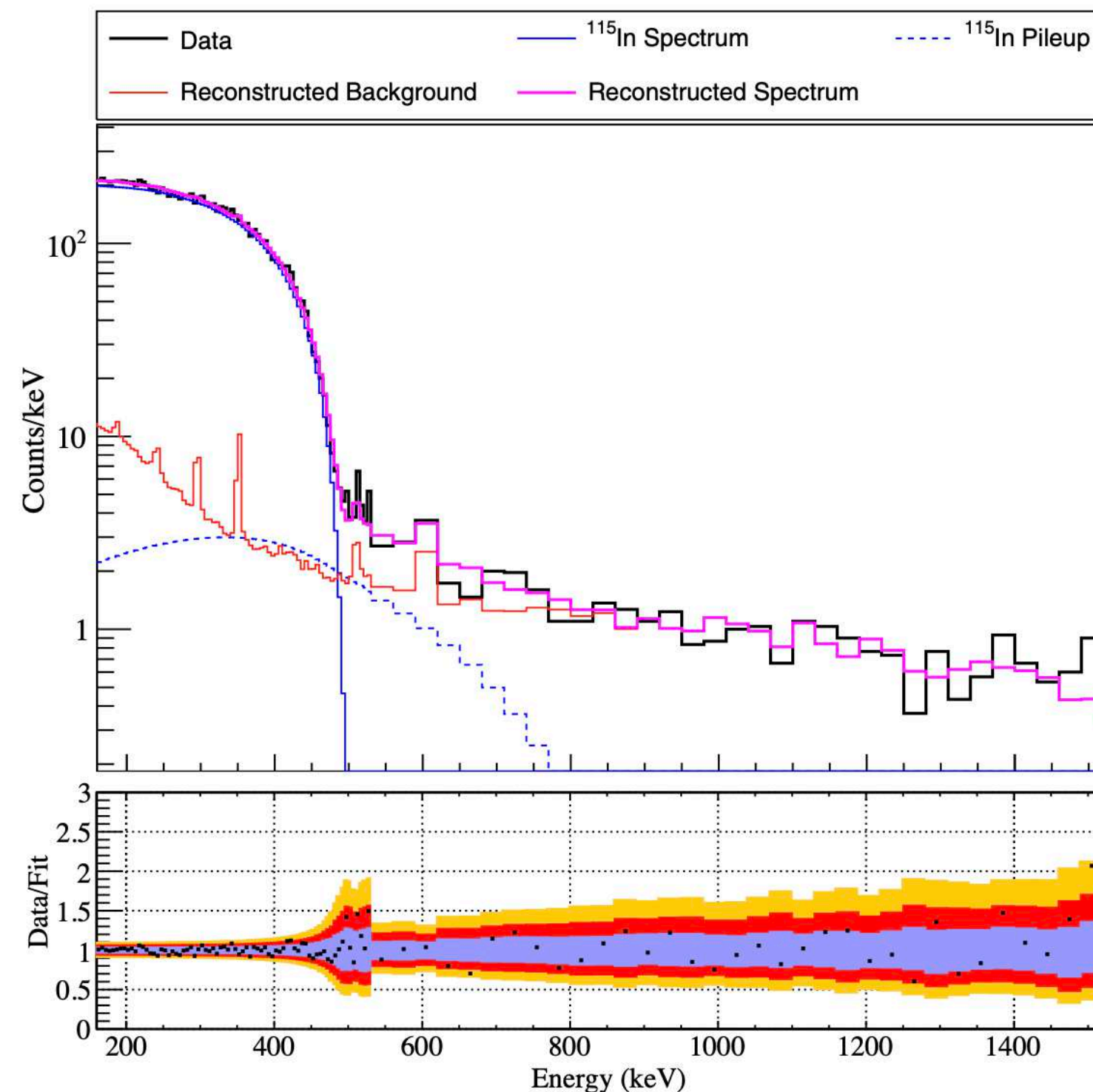
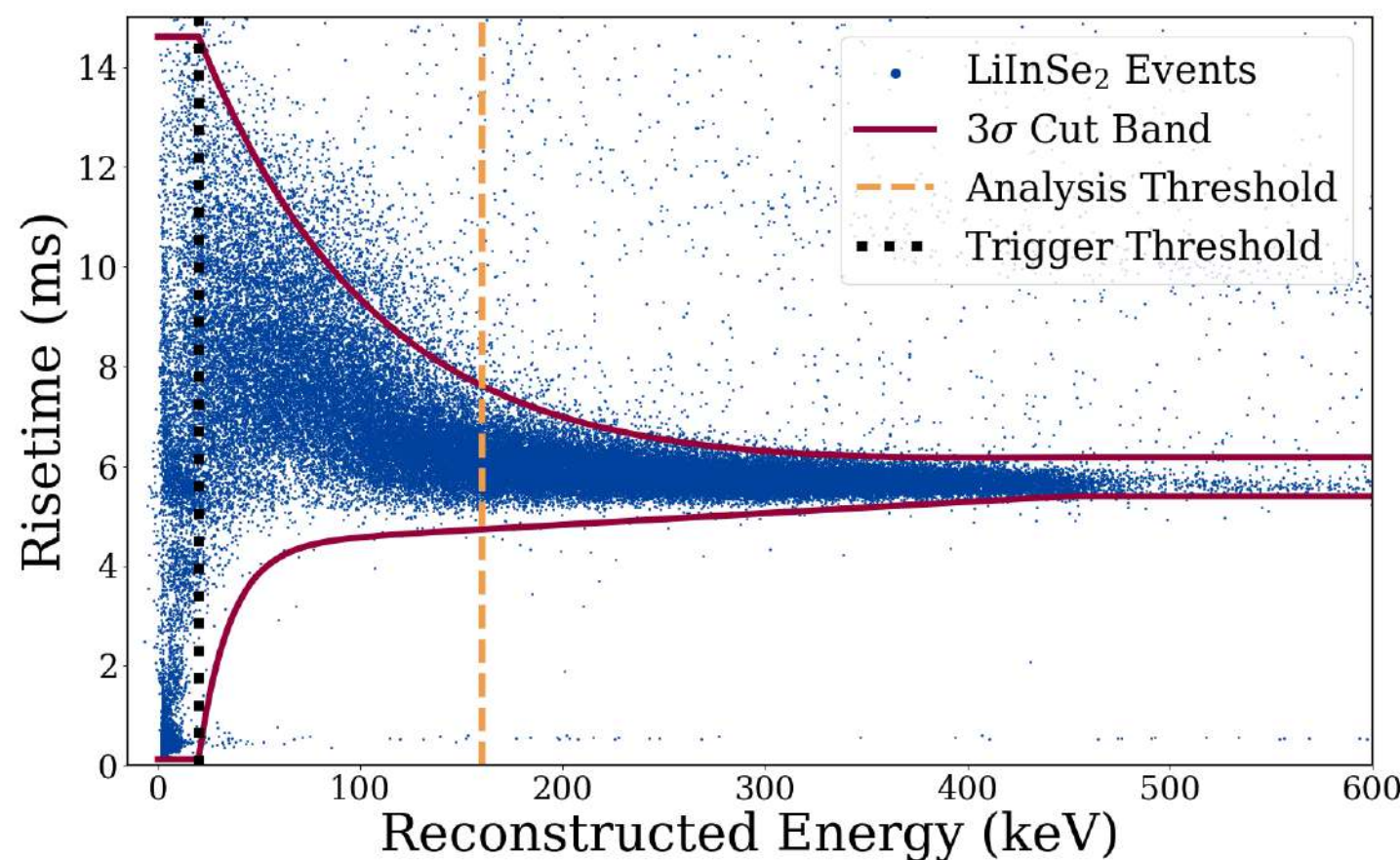
- High natural abundance i.a. = 95.71%
- Embedded in crystal as InI, InO, LiInSe<sub>2</sub>
- Excellent radiopurity levels

# A new measurement (MIT/Berkeley/CNRS)

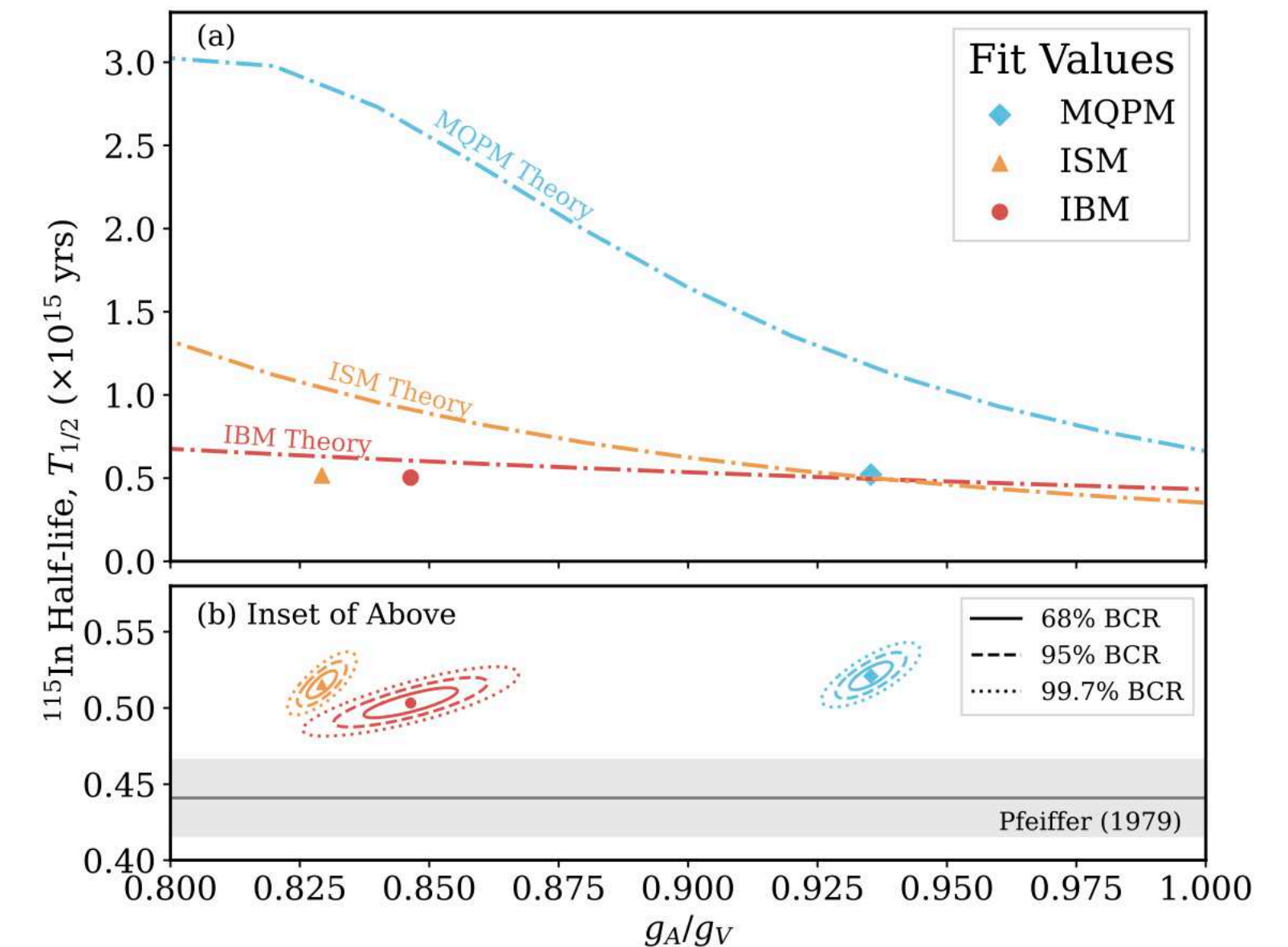
- LiInSe<sub>2</sub> operated as cryogenic calorimeter
- Excellent performance but high rate at low energy
- High analysis threshold (160 keV)



*Phys. Rev. Lett.* 129, 232502 (2022)

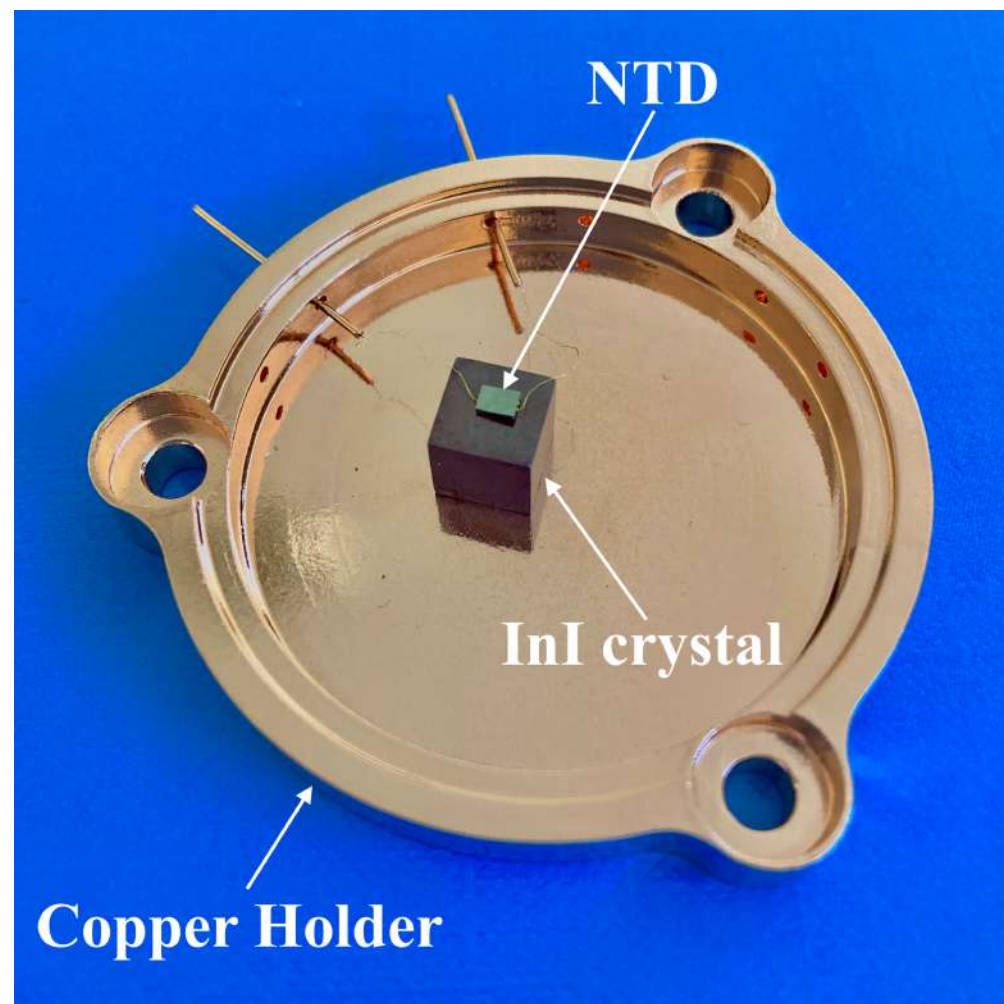


Model	$g_A/g_V$	$T_{1/2}^{115\text{In}}$ ( $10^{14}$ yr)	Reduced $\chi^2$
ISM	$0.830 \pm 0.002$	$5.177 \pm 0.060$	1.58
IBM	$0.845 \pm 0.006$	$5.031 \pm 0.065$	1.50
MQPM	$0.936 \pm 0.003$	$5.222 \pm 0.061$	1.60
Pfeiffer <i>et al.</i> [42]		$4.41 \pm 0.25$	
Watt and Glover [70]		$5.1 \pm 0.4$	
Beard and Kelly [71]		$6.9 \pm 1.5$	



Theory  $\neq$  Experiment

# In-115 by ACCESS

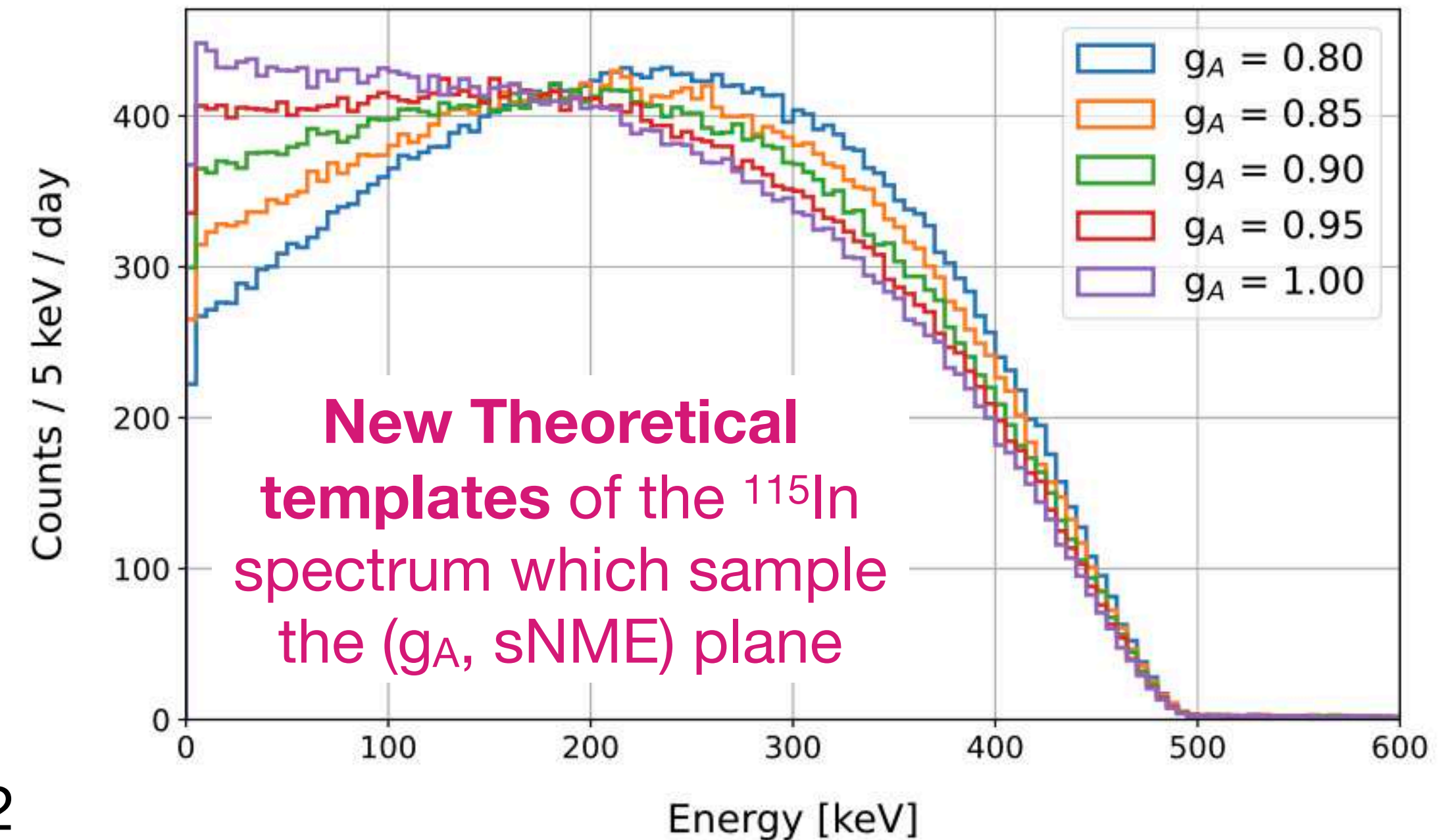
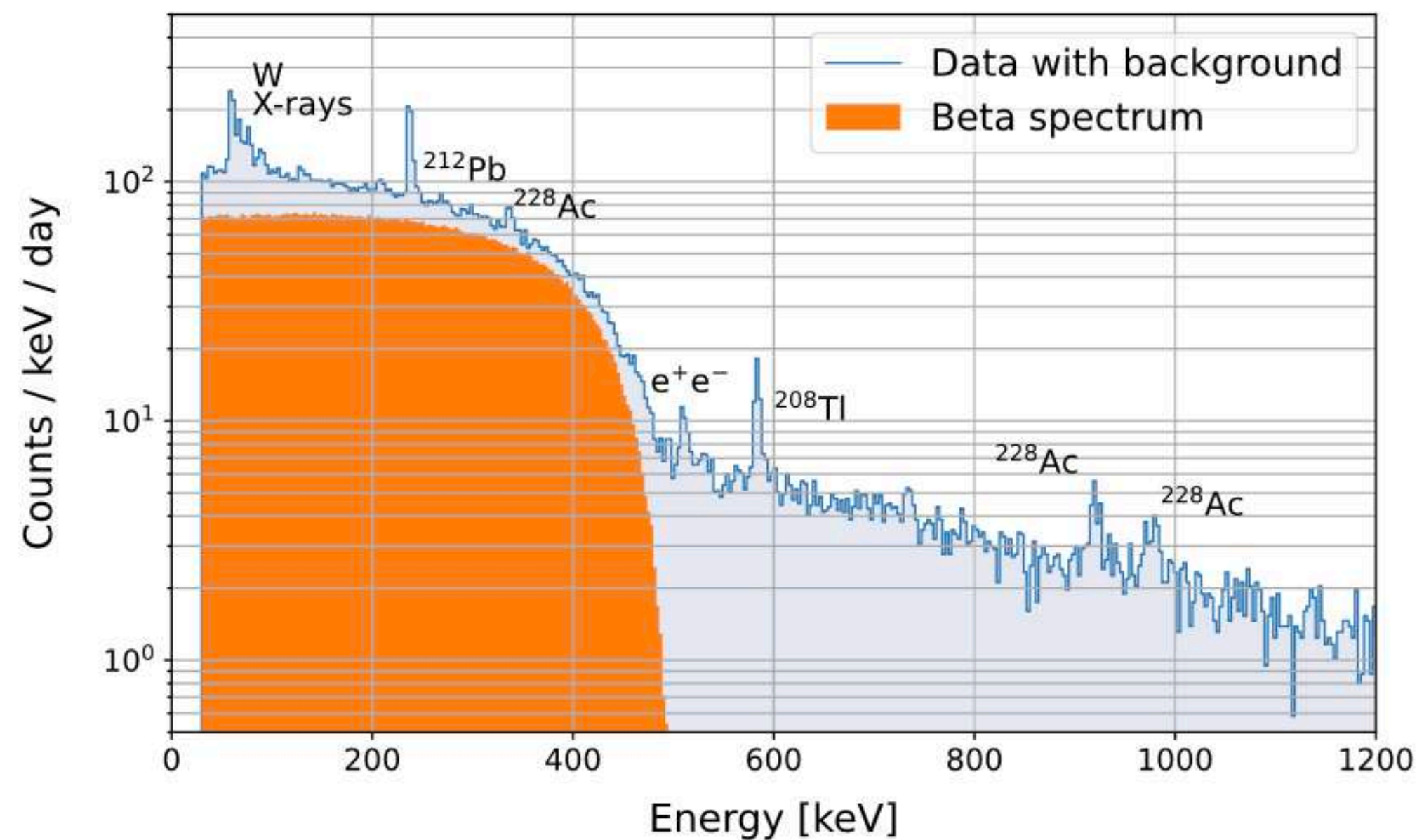


- **Cryogenic calorimeter**
  - Indium Iodine (InI) crystal -  $m = 1.91 \text{ g} - 7 \times 7 \times 7 \text{ mm}^3$
  - Semiconductor sensor (CUPID-0 like)
  - Calibration source with  $^{232}\text{Th}$
  
- **Very good performance**
  - 138 hours of stable data taking
  - Energy threshold of 3.4 keV
  - Energy resolution of 3.9 keV FWHM @ 238.6 keV

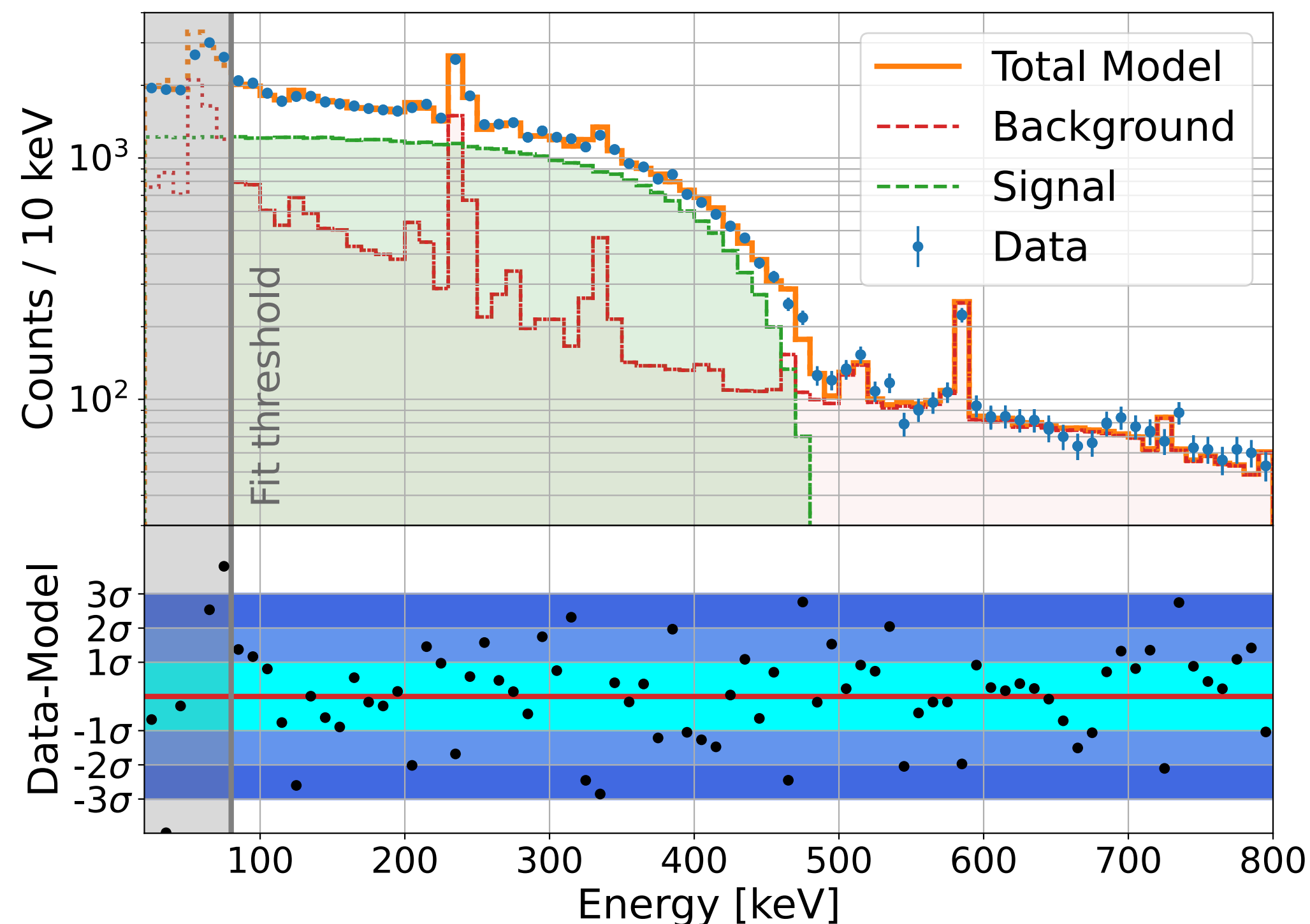


[ACCESS webpage](#)

*Eur.Phys.J.Plus* 138 (2023) 5, 445



# In-115 by ACCESS - Best Fit



- Bayesian fit based on BAT with 5 free parameters:
- 2 bkg components due to the calibration source
  - half-life of  $^{115}\text{In}$
  - $g_A$  in the range [0.60, 1.39]
  - sNME in the range [-5.9, 5.9]

<https://arxiv.org/abs/2401.16059>  
soon on PRL

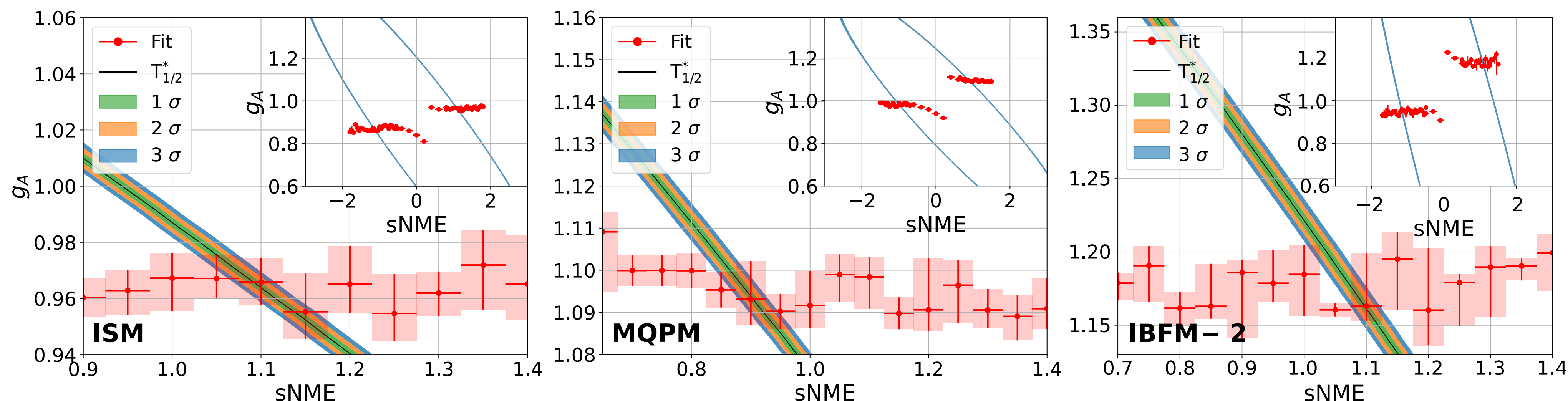
Model	$g_A$	sNME	$T_{1/2} [\times 10^{14} \text{ yr}]$	$\chi_{red}^2$
<i>Best fit</i>				
ISM	$0.964^{+0.010}_{-0.006}$	$1.75^{+0.13}_{-0.08}$	$5.26 \pm 0.06$	1.55
MQPM	$1.104^{+0.019}_{-0.017}$	$2.88^{+0.49}_{-0.71}$	$5.26 \pm 0.07$	1.65
IBFM-2	$1.172^{+0.022}_{-0.017}$	$0.81^{+0.52}_{-0.24}$	$5.25 \pm 0.07$	1.66

- Different values of  $g_A$  for different models
- Lower quenching ( $g_A \approx 1$ )
- sNME not fixed
- Stable and precise evaluation of  $T_{1/2}$
- Theory  $\neq$  Experiment for  $T_{1/2}$

# In-115 by ACCESS - Matched Fit

For each value of sNME, we run the fit to choose the best value of  $g_A$  (red points).

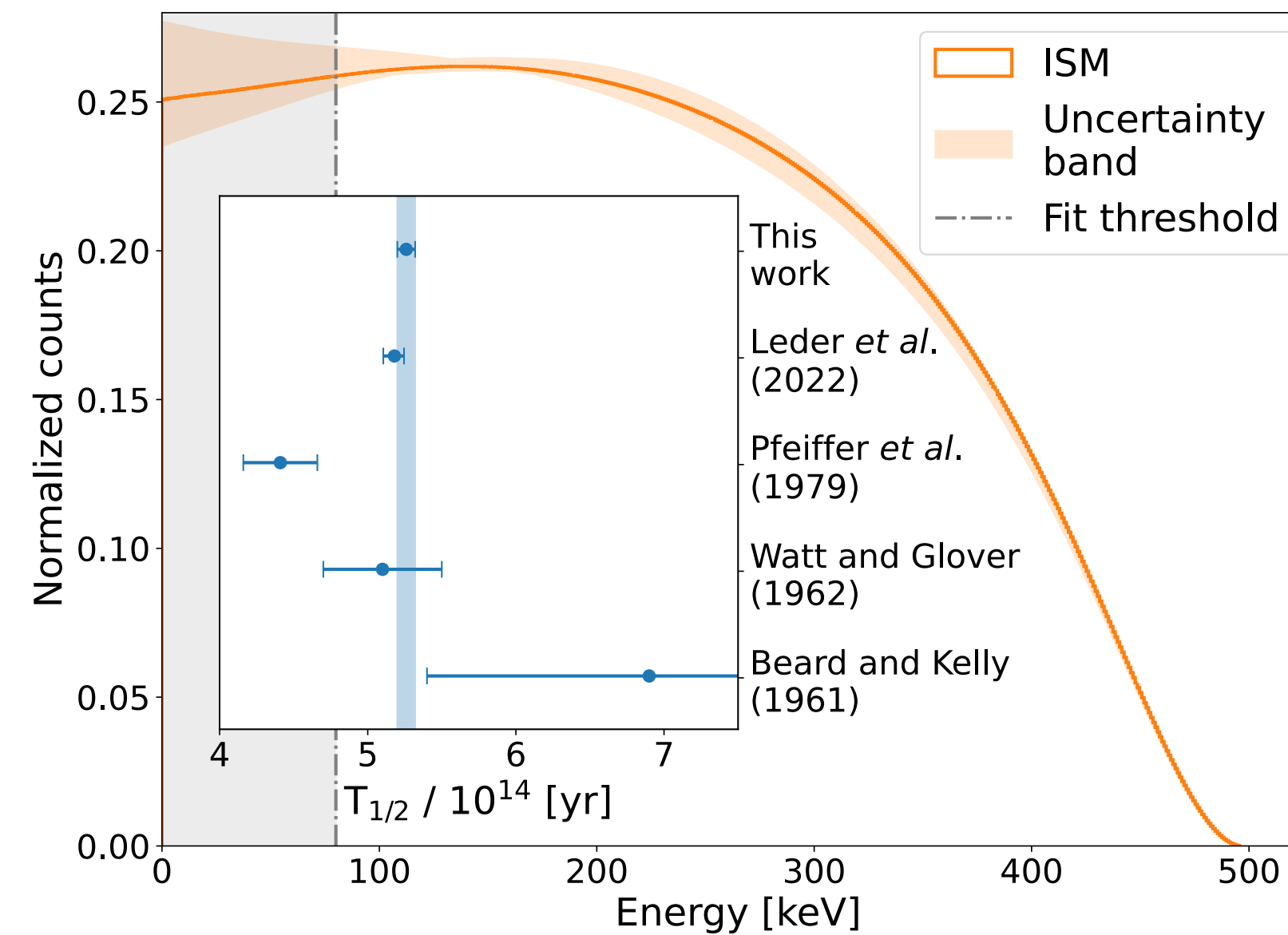
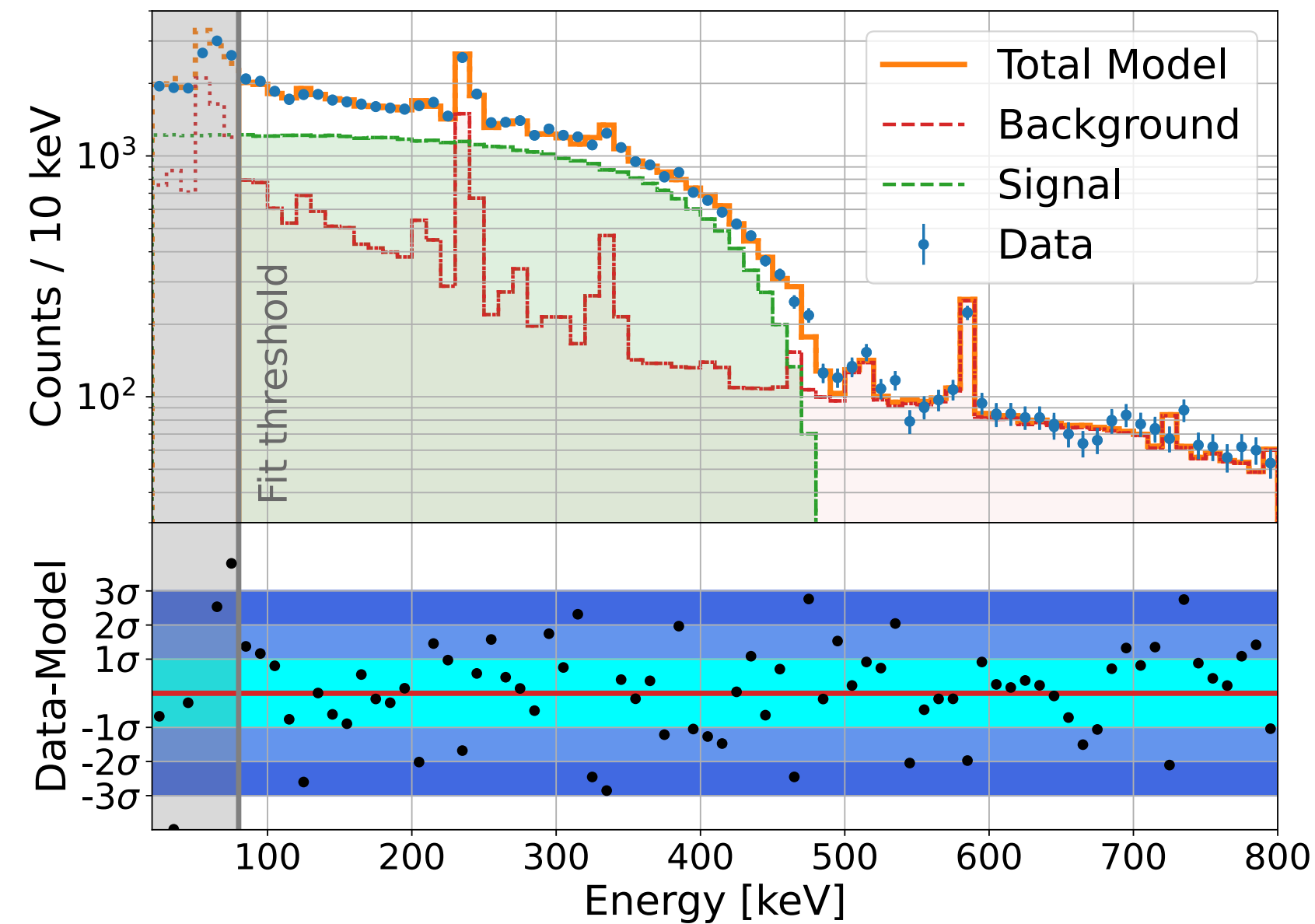
The solution in the ( $g_A$ , sNME) plane is given by the interception of the red line (exp.) and the half-life ellipse (theory).



Model	$g_A$	sNME	$T_{1/2} [\times 10^{14} \text{ yr}]$	$\chi_{red}^2$
<i>Matched half-life</i>				
ISM	$0.965^{+0.013}_{-0.010}$	$1.10 \pm 0.03$	$5.20 \pm 0.07$	1.78
MQPM	$1.093^{+0.009}_{-0.007}$	$0.90 \pm 0.03$	$5.05 \pm 0.06$	2.32
IBFM-2	$1.163^{+0.036}_{-0.010}$	$1.10 \pm 0.03$	$5.28 \pm 0.06$	1.67

- $g_A$  values compatible with the best fit
- sNME fixed by the ellipse interception to similar values
- Theory = Experiment for  $T_{1/2}$
- Bias from previous  $T_{1/2}$  in the ellipse

# In-115 by ACCESS

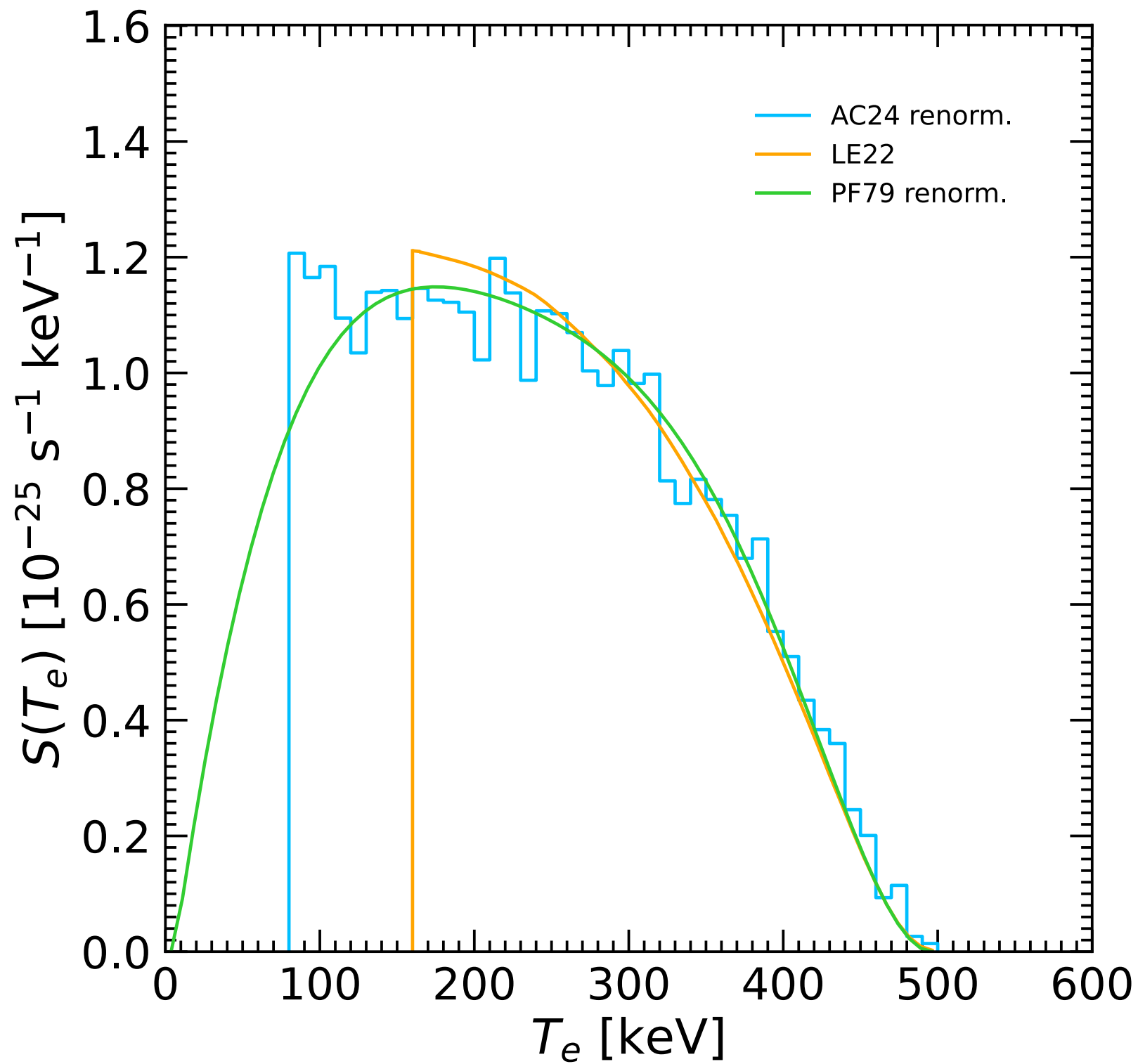
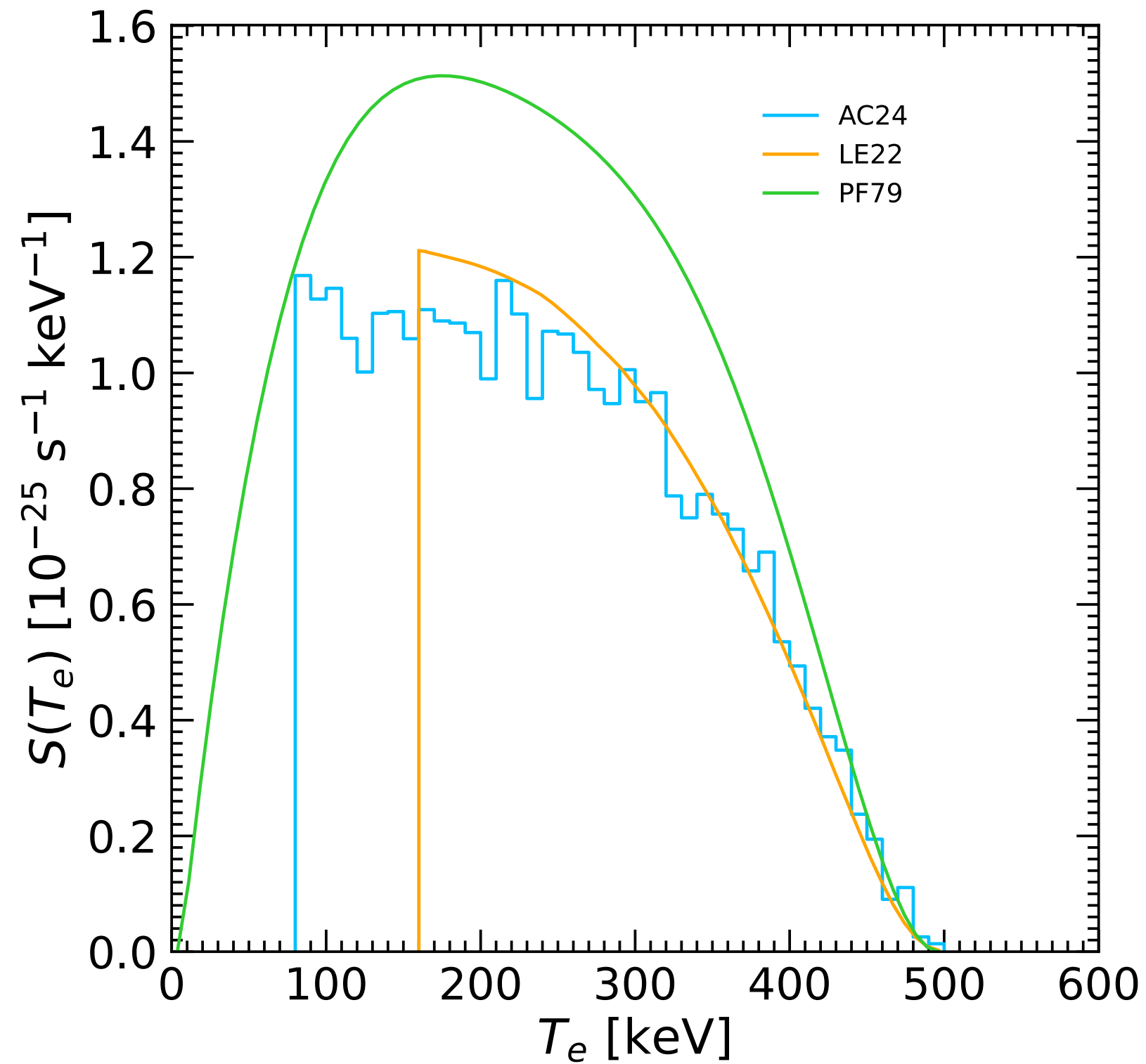


Model	$g_A$	sNME	$T_{1/2} [\times 10^{14} \text{ yr}]$	$\chi_{red}^2$
<i>Best fit</i>				
ISM	$0.964^{+0.010}_{-0.006}$	$1.75^{+0.13}_{-0.08}$	$5.26 \pm 0.06$	1.55
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## Outcomes:

- Stable and precise estimate of  $T_{1/2}$
- Evaluation of the spectral shape
- $g_A$  quenching still needed but reduced (wrt slide 11)
- Positive solutions (sNME>0) are always favored
- sNME fixed only with the  $T_{1/2}$  match
- Theoretical  $T_{1/2}$  compatible with experimental one

# In-115: measurement comparison



Very useful comparison:

- AC24 and LE22 fully compatible
- PF79 very different (possible issues in the background subtraction)

See details in J. Kostensalo, E. Lisi, A. Marrone and J. Suhonen, [arXiv 2405.11920](https://arxiv.org/abs/2405.11920)



# Theory progress: an example

G. De Gregorio, R. Mancino, L. Coraggio, N. Itaco, [Phys.Rev.C 110 \(2024\) 1, 014324](#)

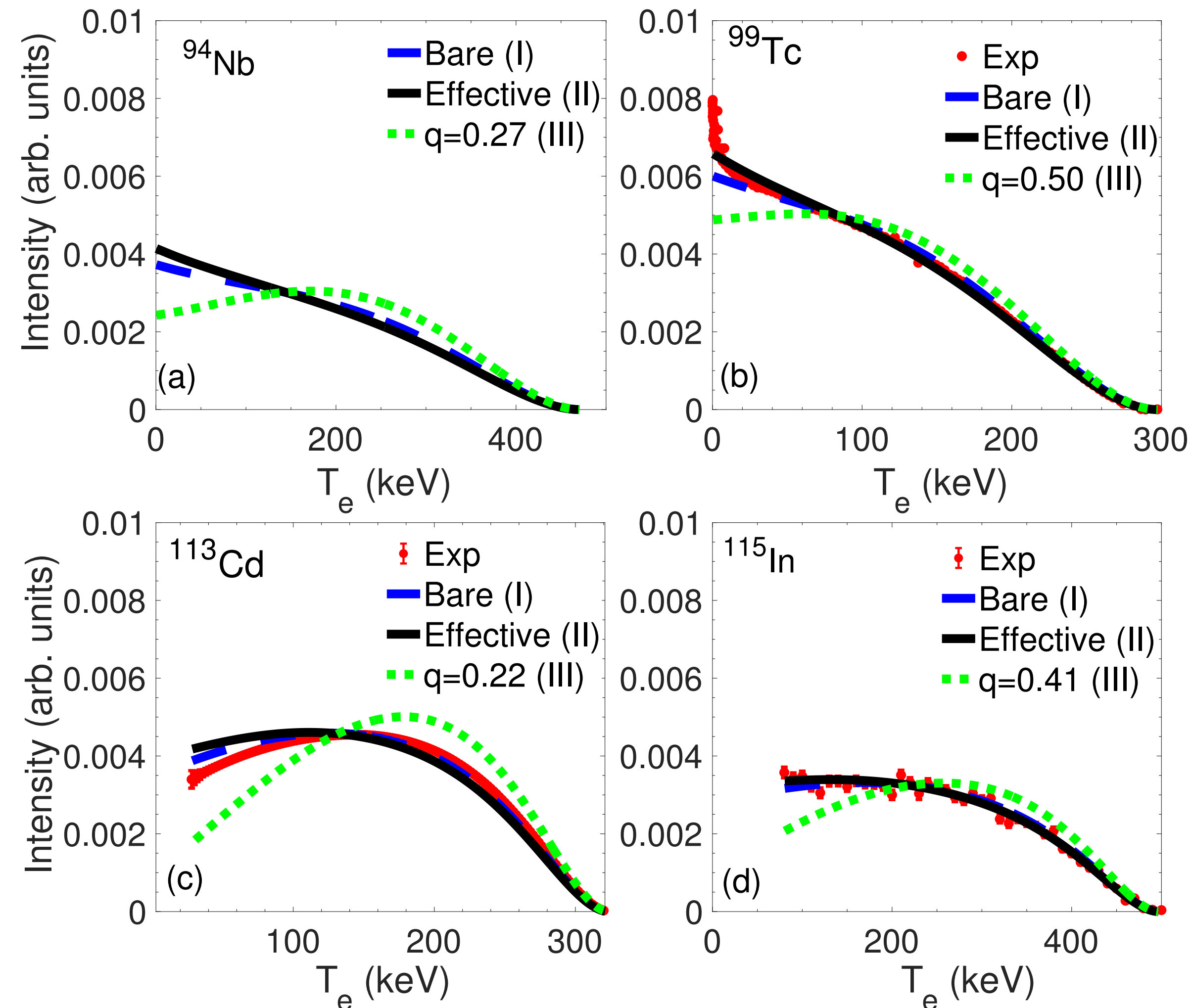
In a recent paper, the data presented were used as a term of comparison with theoretical calculations within the **Realistic Shell Model**.

Very good match of the spectral shape  $\rightarrow$  **without  $g_A$  quenching**

**Half-life systematically underestimated (factor of 2 - 8)**



	$\log(ft)$	Bare	$\log(ft)$	Effective	$\log(ft)$	Exp.
$^{94}\text{Nb}$		11.30		11.58		11.95 (7)
$^{99}\text{Tc}$		11.580		11.876		12.325 (12)
$^{113}\text{Cd}$		21.902		22.493		23.127 (14)
$^{115}\text{In}$		21.22		21.64		22.53 (3)



# Summary

- **Forbidden beta decays**, and in particular their spectral shape, **challenge the nuclear models**
- **Nuclear models** and **computation techniques** improved a lot
- Renewed experimental efforts ongoing to provide **novel high-quality data**



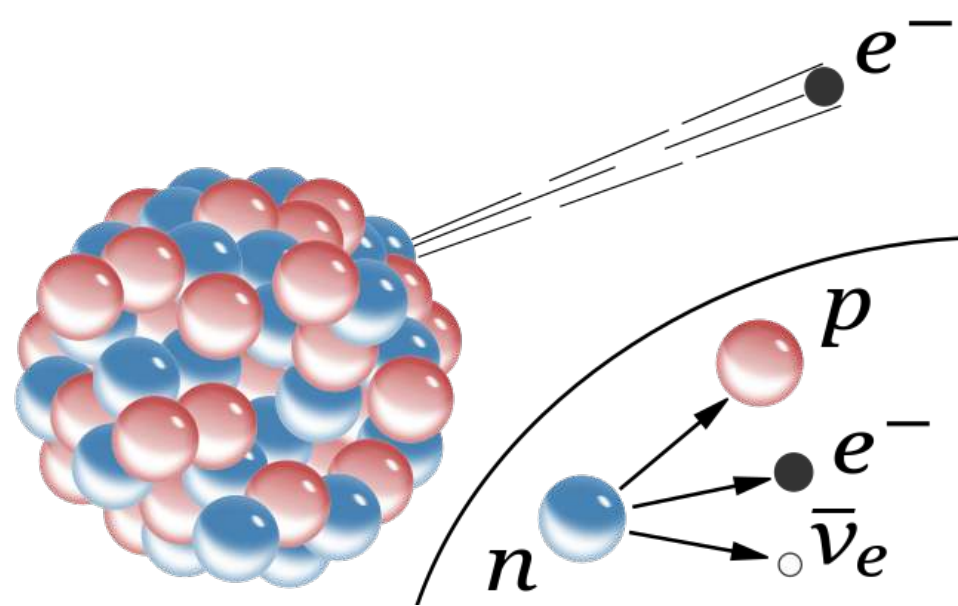
**Mapping the spectral shape of several forbidden  $\beta$ -decays in terms of effective nuclear parameters**



**We could shed light on the nuclear physics behind the phenomenological  $g_A$  quenching and try to avoid it**

**Backup slides**

# Forbidden $\beta$ -decays



$$\log_{10} ft = \log_{10}(f(Z, E_0) \cdot T_{1/2})$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$\Delta\pi = (-1)^{\Delta J}$$

Forbidden non-unique

$$\Delta\pi = (-1)^{\Delta J - 1}$$

Forbidden unique\*

Transition	$\Delta L$	$\Delta J$	$\Delta\pi$	$\log_{10} ft$
<b>Fermi</b> "super-allowed"	0	0	0	3-4
<b>Gamow- Teller</b> "allowed"	0	0,1	0	3 - 10
<b>Forbidden</b>	n	n, n+1	0/1	5 -10 (1 <sup>st</sup> ) 22-24 (4 <sup>th</sup> )

Forbiddenness index

K	1	2	3	4	5
$\Delta J$	0,1,2	2,3	3,4	4,5	5,6
$\Delta\pi$	-1	1	-1	1	-1

As a rule of thumb, each degree of forbiddenness gives 5-6 orders of magnitude in  $\sim T_{1/2}$

See B. Singh et al., Nucl. Data Sheets 84 (1998) 487.

# Spectral shape description - 1

$$S(E) = C(E) \cdot S_{all}(E)$$

$$S_{all}(E) \propto F(Z_d, E) \cdot (Q_\beta - E)^2$$

$C(E)$  → empirical correction function

numerically calculated

Being  $E$  the total energy of the electron, while  $P$  and  $Q$  the momenta of the electron and the anti-neutrino, respectively.

**Forbidden Non-unique**

$$C_1(E) = 1 + \frac{a_1}{E} + a_2 E + a_3 E^2 + a_4 E^3$$

**Forbidden Unique**

$$C(E) = C_1 \cdot C_2$$

$$1^{\text{st}} \quad C_1 = P^2 + c_1 Q^2$$

$$2^{\text{nd}} \quad C_2 = P^4 + c_1 Q^2 P^2 + c_2 Q^4$$

# Spectral shape description - 2

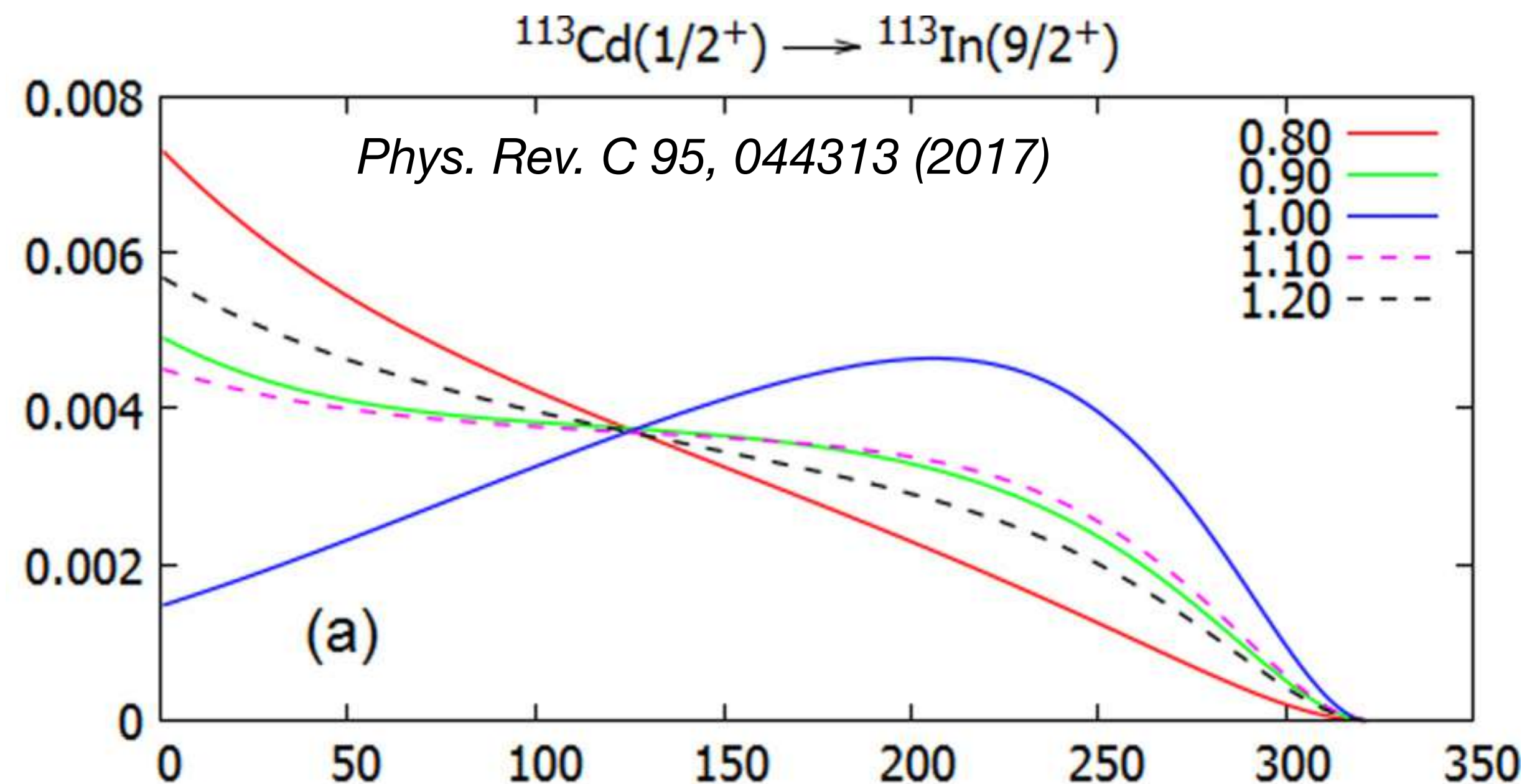
$$S(E) = C(E) \cdot S_{all}(E)$$

$$S_{all}(E) \propto F(Z_d, E) \cdot (Q_\beta - E)^2$$

$C(E)$  → empirical correction function

numerically calculated

The “shape factor” encodes the nuclear-structure information and can be decomposed into vector, axial-vector, and vector-axial-vector parts.



**Template spectra** can be produced within Nuclear Models as a function of **nuclear parameters ( $g_A$ )** or with **different approximations**, and then **compared with experimental data**.

# Shape Function: $g_A$ and sNME

$$T_{1/2} = \frac{6289}{\tilde{C}}$$

Decay half-life

Nuclear Matrix Elements enter the Shape Functions!

$$\tilde{C} = \int_0^{Q_\beta} S(E) \cdot dE$$

Integrated  
shape function

$$\boxed{V \mathcal{M}_{KK-11}^{(0)}} = \left( \frac{(-M_n c^2 + M_p c^2 + W_0) \times R}{\hbar c} + \frac{6}{5} \alpha Z \right) \times \boxed{V \mathcal{M}_{KK0}^{(0)}}$$

$M_{n/p}$  = neutron/proton mass       $K$  = order of forbiddenness

$Z$  = atomic number of the daughter nucleus

$R$  = atomic radius       $W_0$  = Q-value

$$S(E) = C(E) \cdot p \cdot E \cdot (Q_\beta - E)^2 \cdot F(Z_d, E)$$

Shape Function

**Formula valid only in the ideal case:**

- infinite valence spaces
- perfect nuclear many-body theory

$$C(E) = g_V^2 C_V(E) + g_A^2 C_A(E) + g_V g_A C_{VA}(E)$$

Shape Function Decomposition

**I-NME fixed by the nuclear calculations**

**sNME is treated as free parameter**

# NLDBD vs Heavy Ion DCER

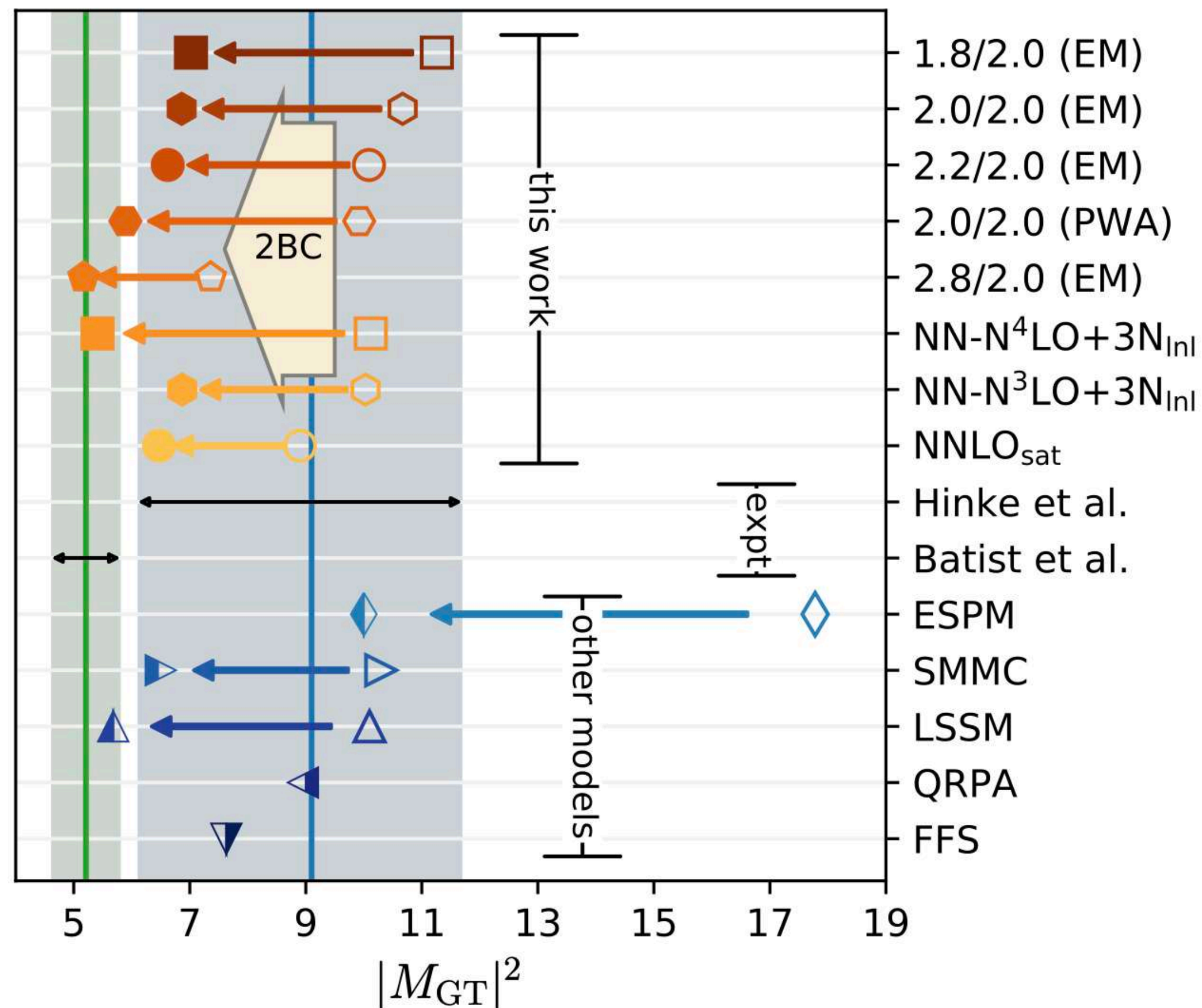
Credits to  
Horst Lenske

1. **Initial and final states:** Parent/daughter states of the  $0\nu\beta\beta$  are the same as those of the target/residual nuclei in the DCE;
2. **Spin-Isospin mathematical structure** of the transition operator: Fermi, Gamow-Teller and rank-2 tensor together with higher L components are present in both cases;
3. **Large momentum transfer:** A linear momentum transfer as high as 100 MeV/c or so is characteristic of both processes;
4. **Non-locality:** both processes are characterized by two vertices localized in two valence nucleons. In the ground to ground state transitions in particular a pair of protons/neutrons is converted in a pair of neutrons/protons so the non-locality is affected by basic pairing correlation length;
5. **In-medium** processes: both processes happen in the same nuclear medium, thus quenching phenomena are expected to be similar;
6. Relevant **off-shell propagation** in the intermediate channel: both processes proceed via the same intermediate nuclei off-energy-shell even up to 100 MeV.



# Ab initio calculations

(<sup>100</sup>Sn) P. Gysbers et al, *Nature Phys.* 15 (2019) 5, 428-431



Ab initio calculations including **2 body currents** improve the match with the half-life for **Gammow-Teller transitions**.

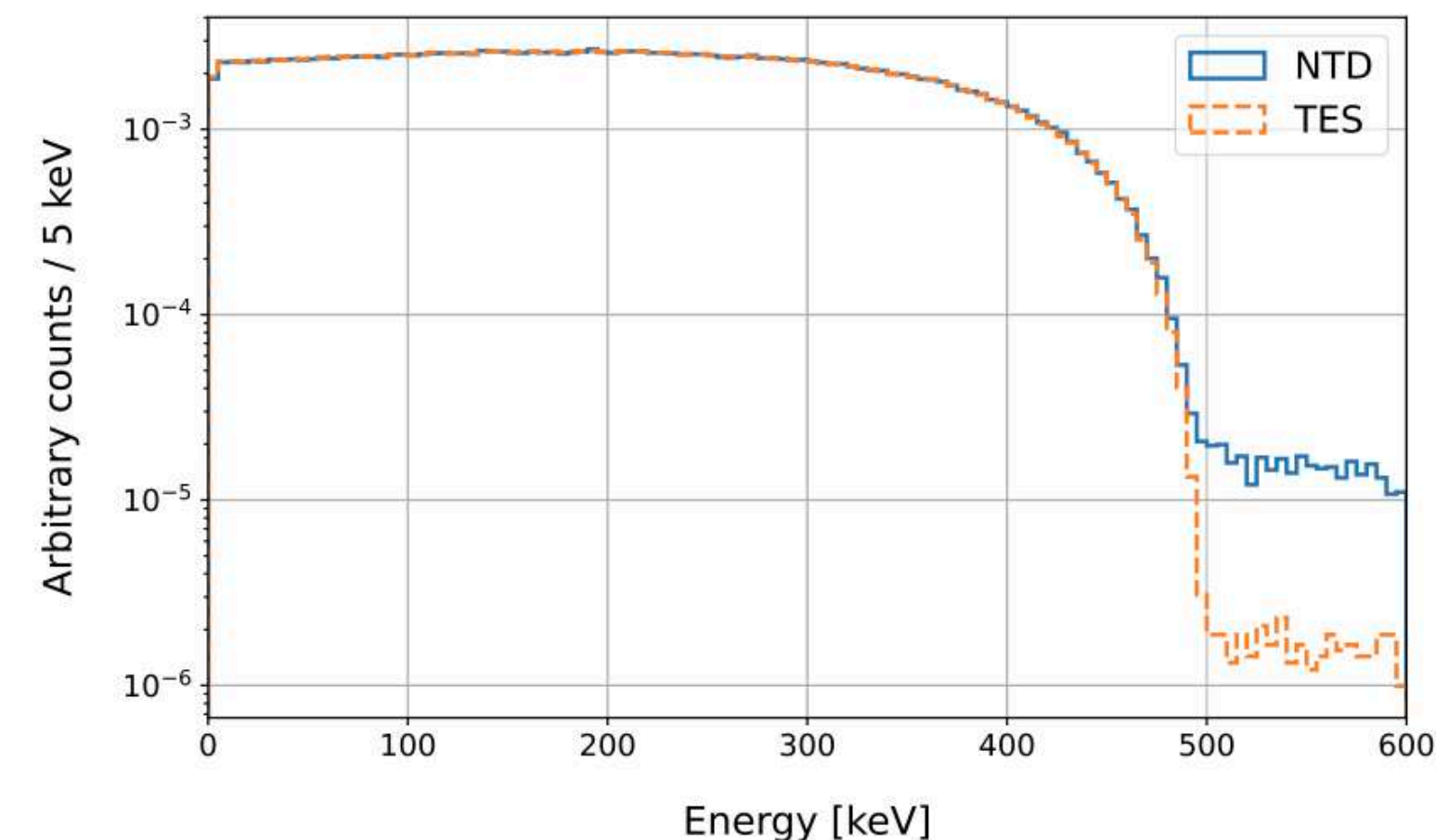
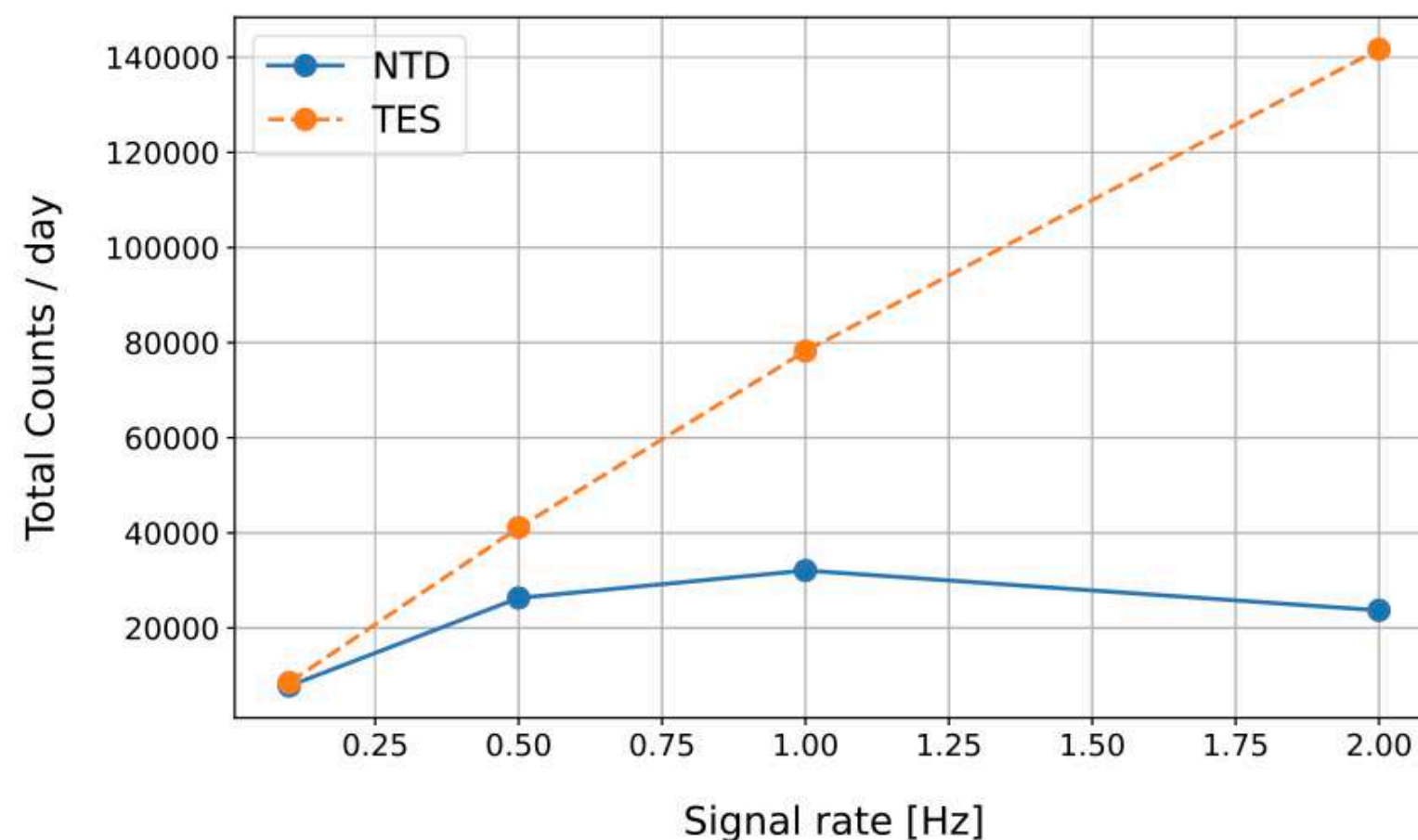
it is unclear what's happen for spectral shape and for forbidden decay.

# In-115 by ACCESS: Full Numerical Results

Model	<i>Positive solution</i>				<i>Negative solution</i>			
	$g_A$	sNME	$T_{1/2}$ [ $\times 10^{14}$ yr]	$\chi_{red}^2$	$g_A$	sNME	$T_{1/2}$ [ $\times 10^{14}$ yr]	$\chi_{red}^2$
<i>Best fit</i>								
ISM	$0.964^{+0.010}_{-0.006}$	$1.75^{+0.13}_{-0.08}$	$5.26 \pm 0.06$	1.55	$0.774^{+0.046}_{-0.042}$	$-5.43^{+0.40}_{-0.22}$ (*)	$5.40 \pm 0.07$	2.27
MQPM	$1.104^{+0.019}_{-0.017}$	$2.88^{+0.49}_{-0.71}$	$5.26 \pm 0.07$	1.65	$0.978^{+0.022}_{-0.021}$	$-5.40^{+0.38}_{-0.53}$ (*)	$5.46 \pm 0.07$	2.26
IBFM-2	$1.172^{+0.022}_{-0.017}$	$0.81^{+0.52}_{-0.24}$	$5.25 \pm 0.07$	1.66	$0.739^{+0.069}_{-0.058}$	$-5.20^{+0.63}_{-0.41}$ (*)	$5.40 \pm 0.06$	1.97
<i>Matched half-life</i>								
ISM	$0.965^{+0.013}_{-0.010}$	$1.10 \pm 0.03$	$5.20 \pm 0.07$	1.78	$0.869^{+0.004}_{-0.004}$	$-1.15 \pm 0.03$	$5.50 \pm 0.06$	2.94
MQPM	$1.093^{+0.009}_{-0.007}$	$0.90 \pm 0.03$	$5.05 \pm 0.06$	2.32	$0.992^{+0.004}_{-0.004}$	$-1.00 \pm 0.03$	$5.64 \pm 0.07$	3.22
IBFM-2	$1.163^{+0.036}_{-0.010}$	$1.10 \pm 0.03$	$5.28 \pm 0.06$	1.67	$0.958^{+0.012}_{-0.015}$	$-1.15 \pm 0.03$	$5.46 \pm 0.07$	2.28

## Next steps with increasing challenges:

- $^{113}\text{Cd}$  with natural  $\text{CdWO}_4$
- $^{99}\text{Tc}$  with  $\text{Li}_2\text{MoO}_4$ ,  $\text{Na}_2\text{Mo}_2\text{O}_7$  irradiated at nuclear reactor / doped
- move from semiconductor sensors (NTD) to superconductive sensors (TES)
  - better threshold, energy resolution and detection efficiency (pile-up)



## Beyond ACCESS:

- $^{37}\text{Cl}$  with irradiated  $\text{NaCl}$  irradiated at nuclear reactor
- $^{97}\text{Zr}$  with irradiated  $\text{ZrO}_2$  irradiated at nuclear reactor