

NOW 2024
Neutrino Oscillation Workshop
Otranto, Sept 2-8 2024

Effective Field Theory for single and double beta decay

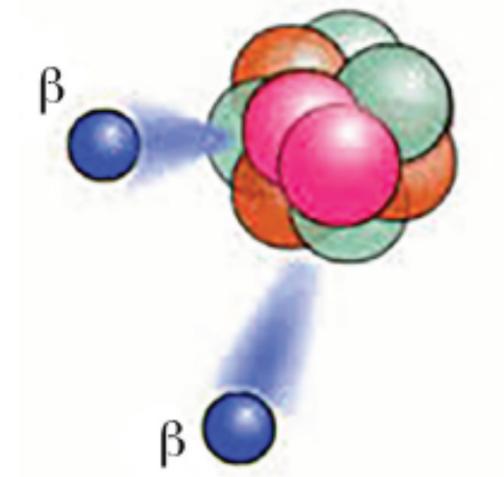
Vincenzo Cirigliano
University of Washington



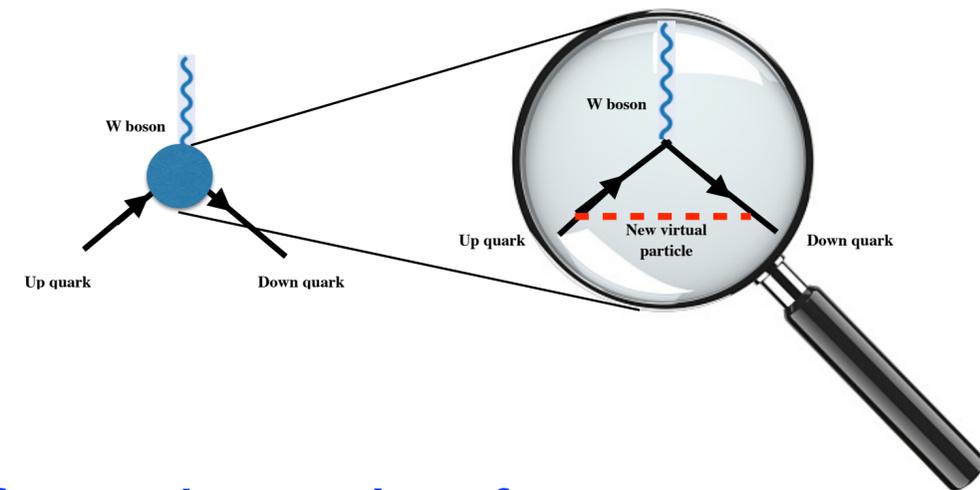
Outline

- Physics motivation and Effective Field Theory (EFT) analysis of:

- Neutrinoless double beta decay



- (Single) beta decays of neutron and nuclei



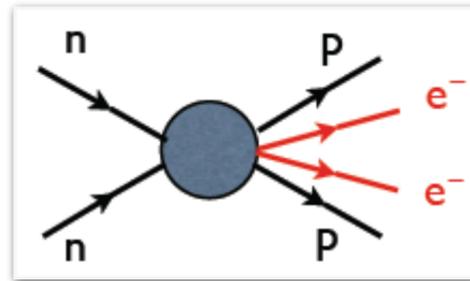
Unifying theme: end-to-end EFT approach, from the scale of new physics down to hadronic / nuclear energies

Lepton Number Violation
&
Neutrinoless double beta decay

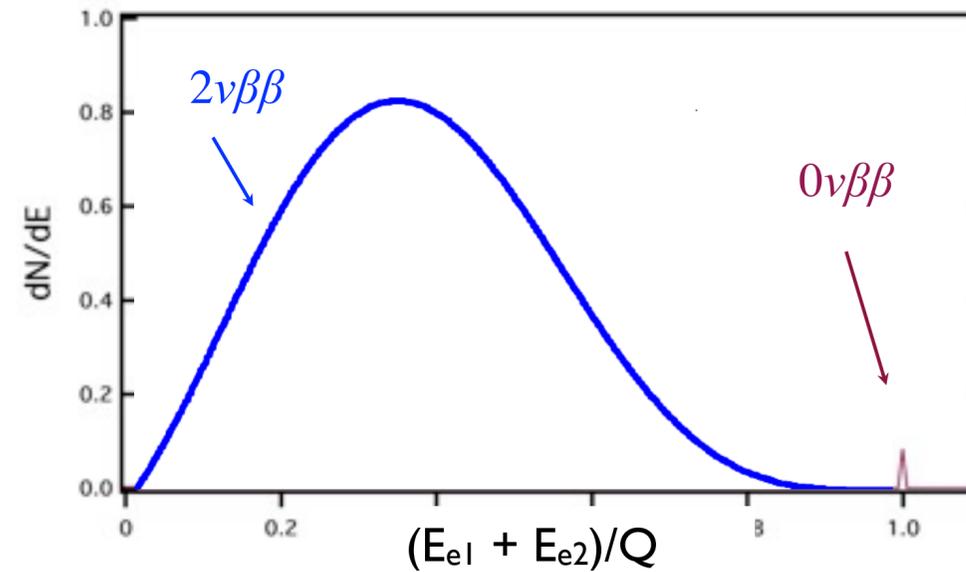
$0\nu\beta\beta$ decay

$$(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$$

See morning talks by
F. Bellini & J. Holt



$\Delta L=2$



Potentially observable only in certain even-even nuclei (⁷⁶Ge, ¹⁰⁰Mo, ¹³⁶Xe, ...) for which single beta decay is energetically forbidden

Observation \Rightarrow BSM physics, with far reaching implications

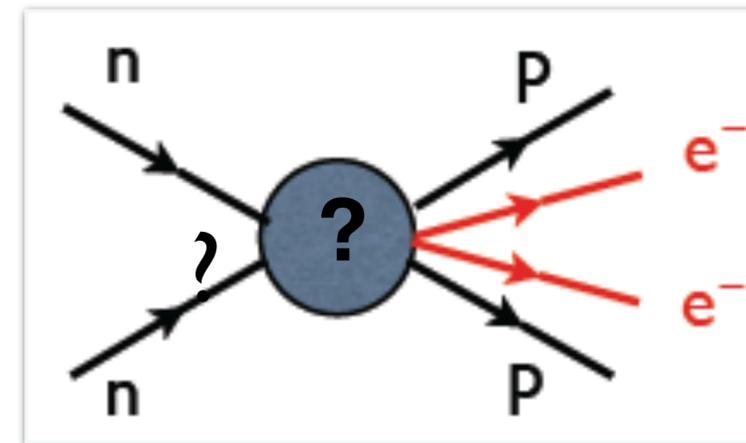
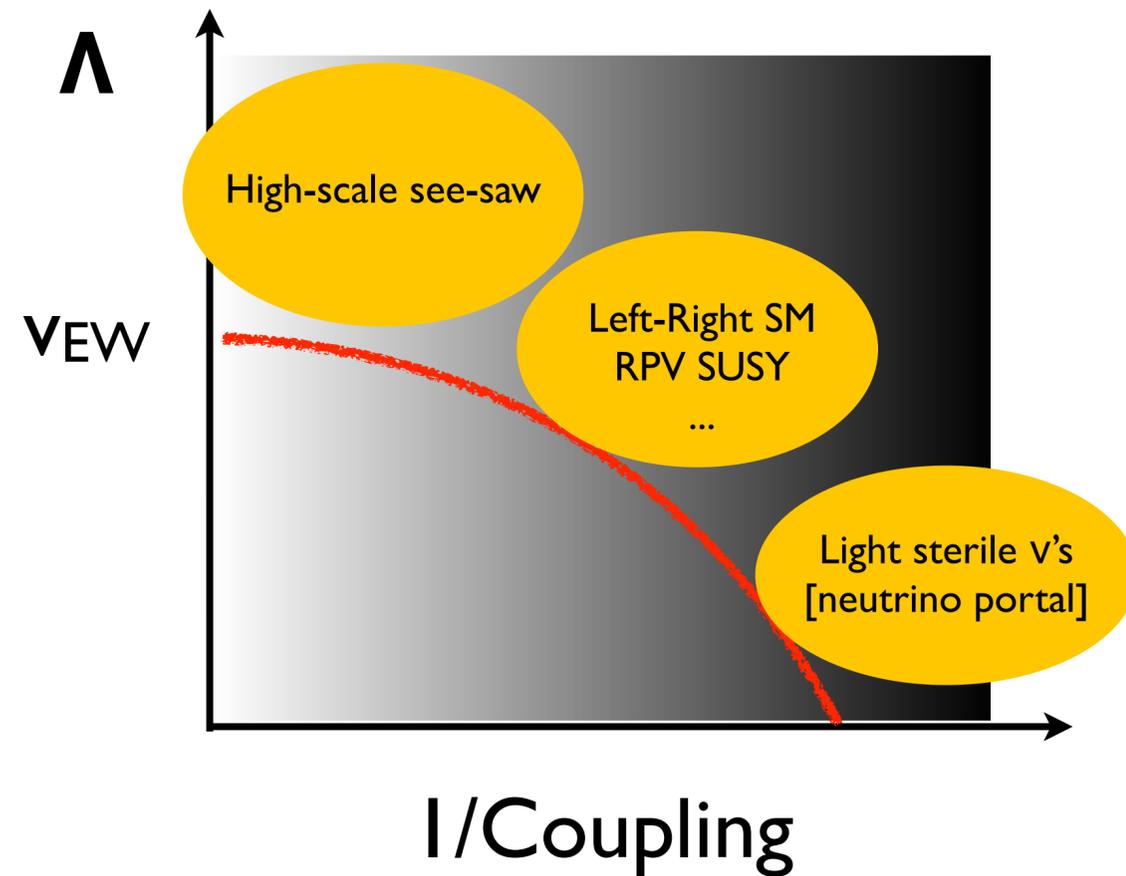
(B-L conserved in the the SM)

Demonstrate Majorana nature of neutrinos (neutrino=antineutrino)

Establish LNV, key ingredient to generate baryon asymmetry via leptogenesis

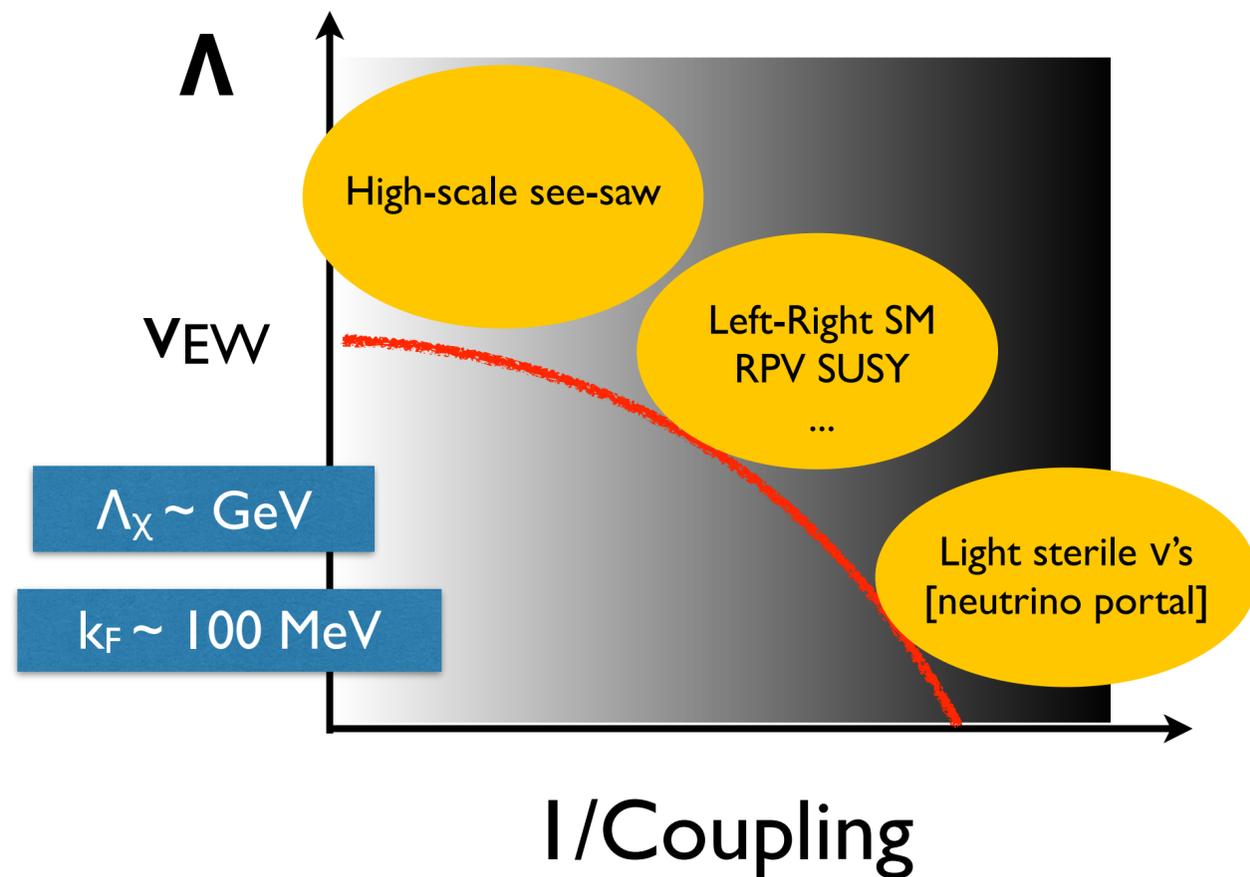
$0\nu\beta\beta$ physics reach

- $0\nu\beta\beta$ searches @ $T_{1/2} > 10^{27-28}$ yr will have broad sensitivity to mechanisms of Lepton Number Violation (LNV)



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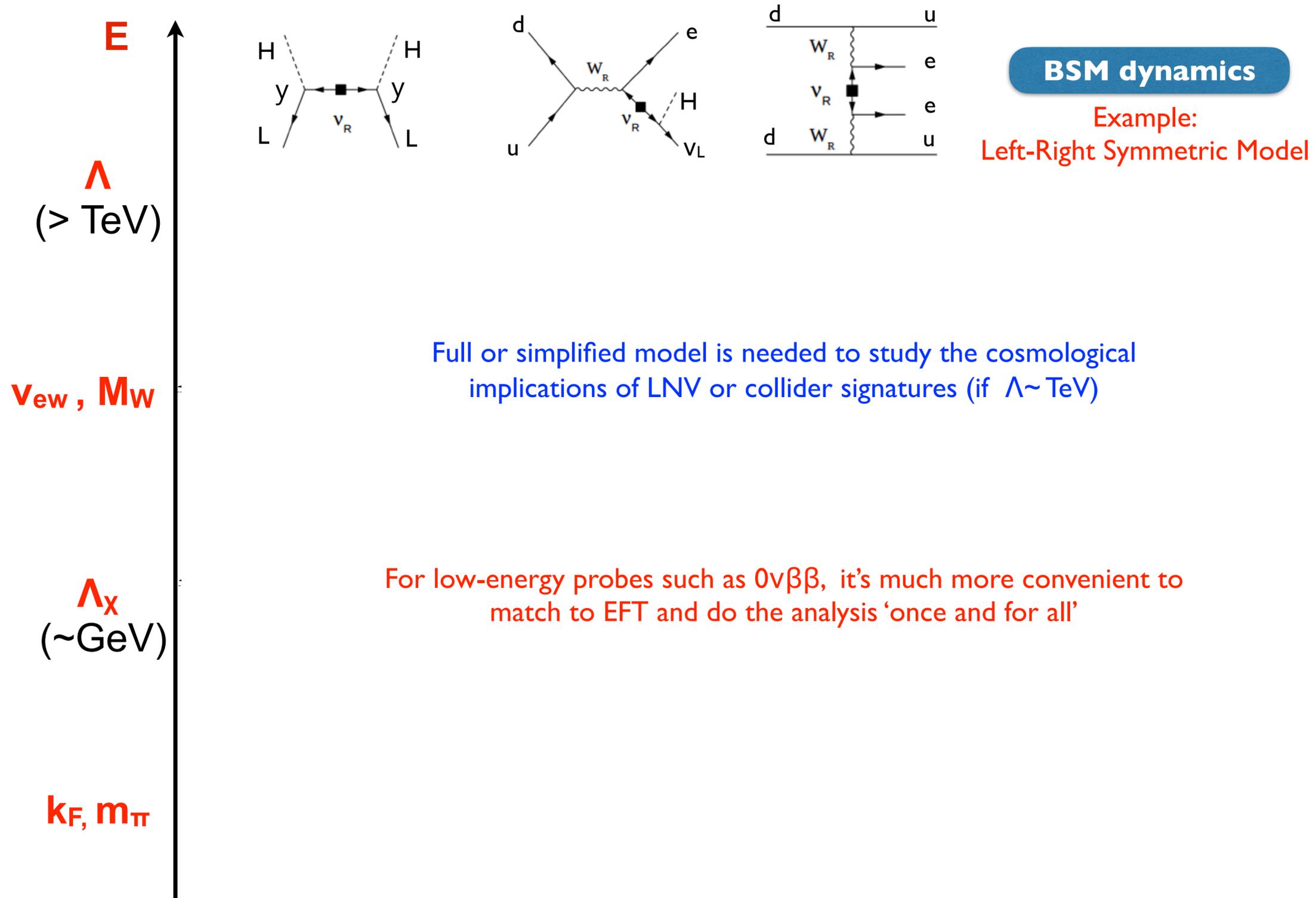
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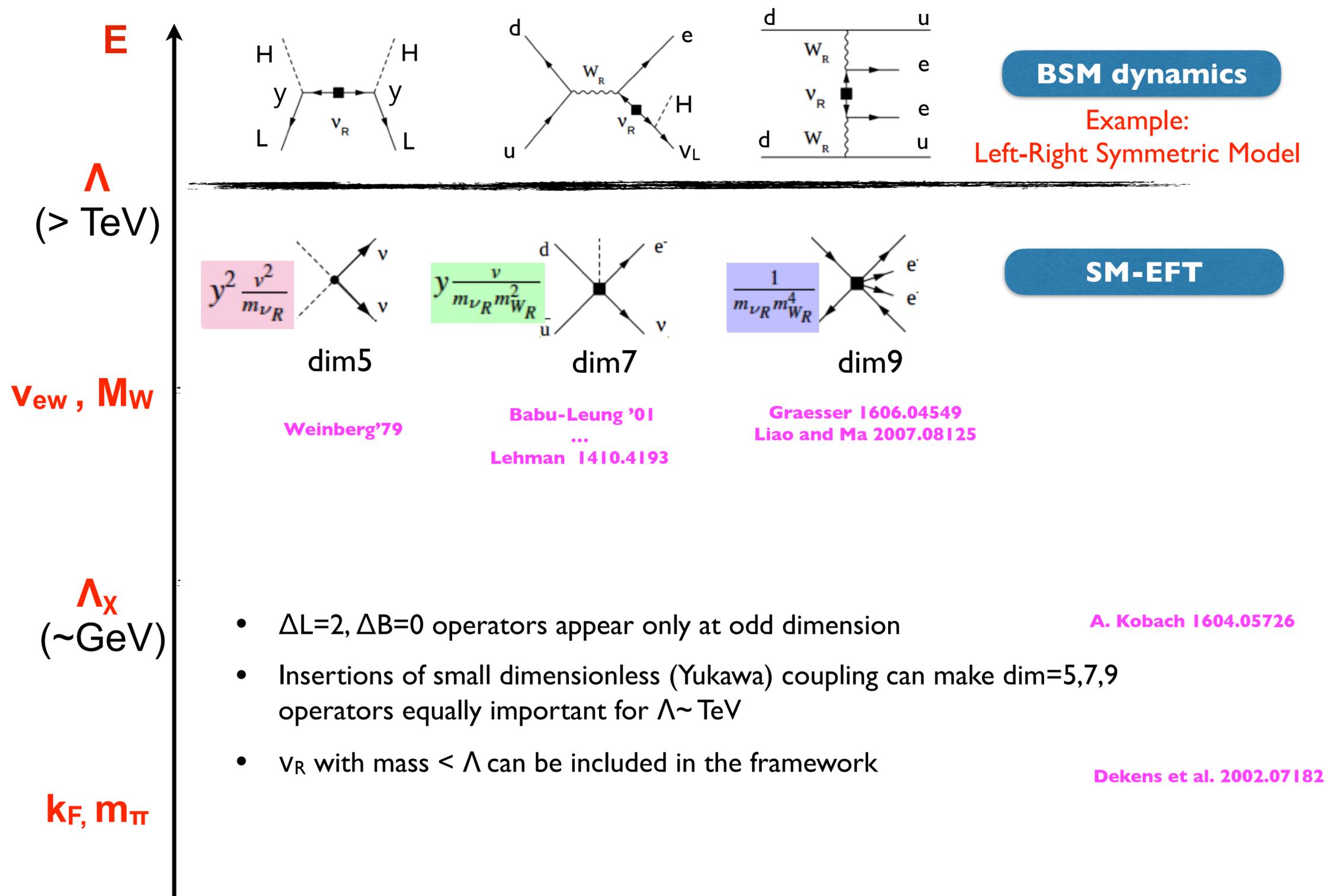
- Given the widely-separated scales, the impact of $0\nu\beta\beta$ searches and the relation to other probes of LNV is best analyzed through a **tower of EFTs** that connect the LNV scale Λ to nuclear scales, with controllable uncertainties

See Snowmass white paper 2203.21169 and refs therein

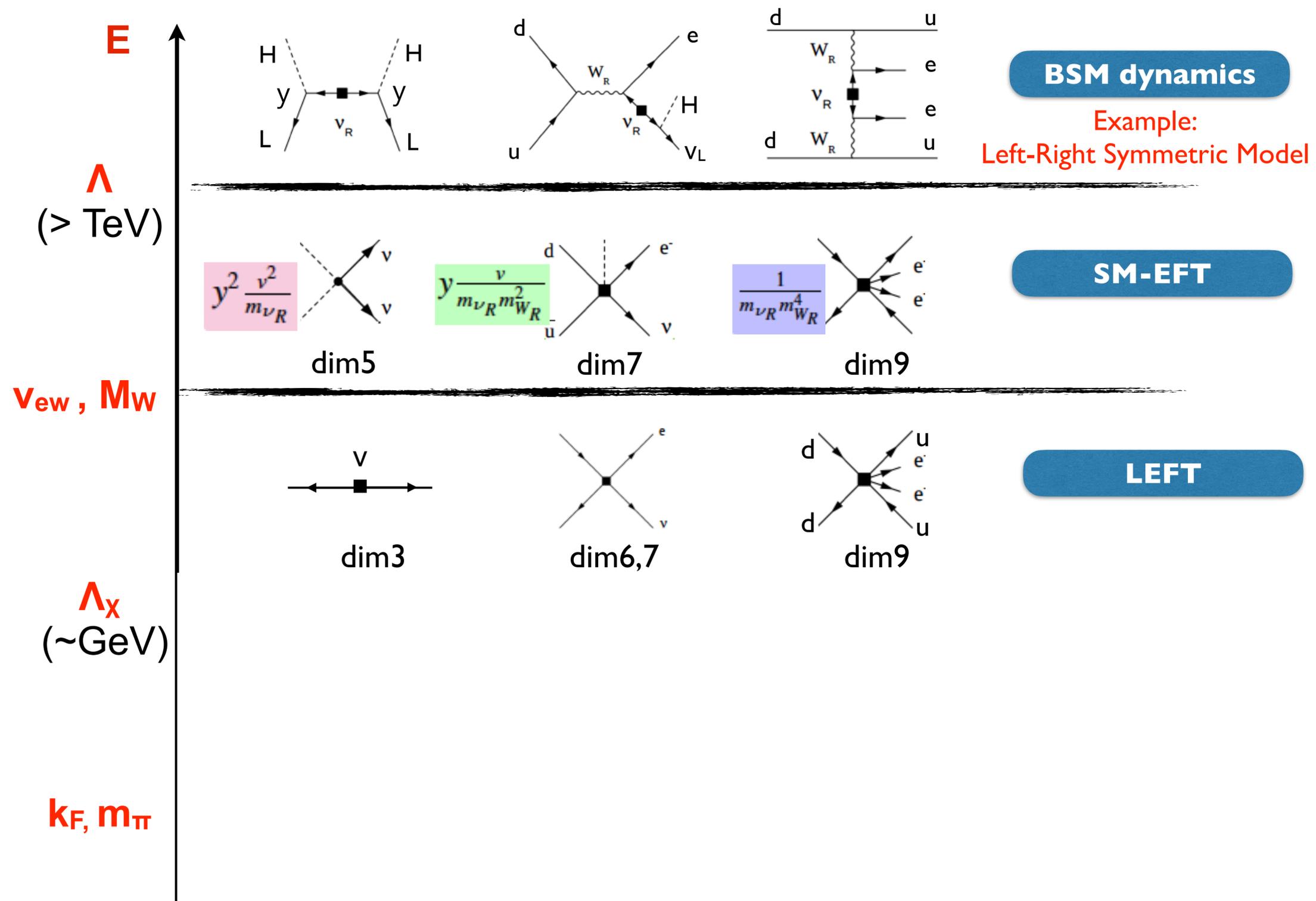
'End-to-end' EFT framework



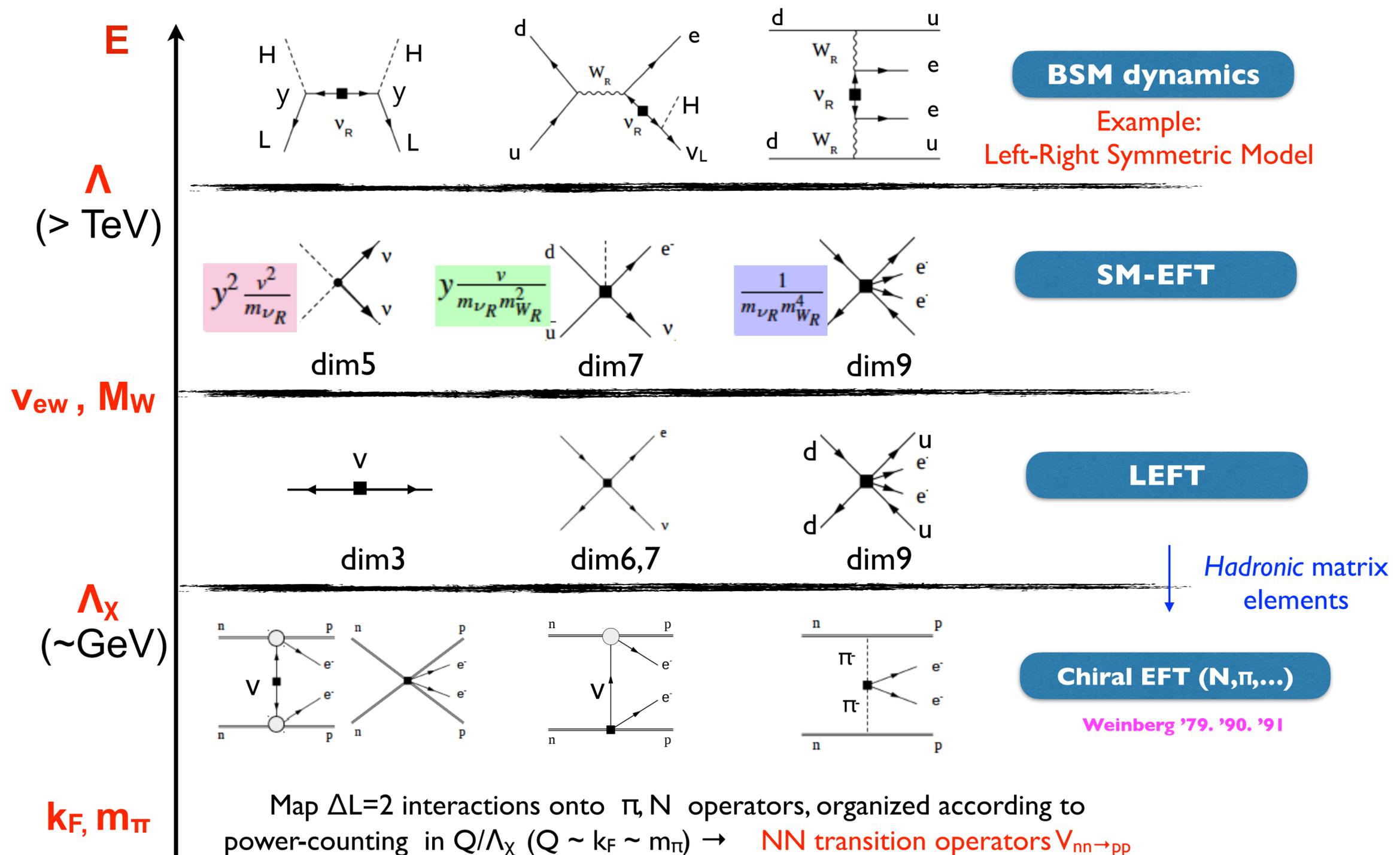
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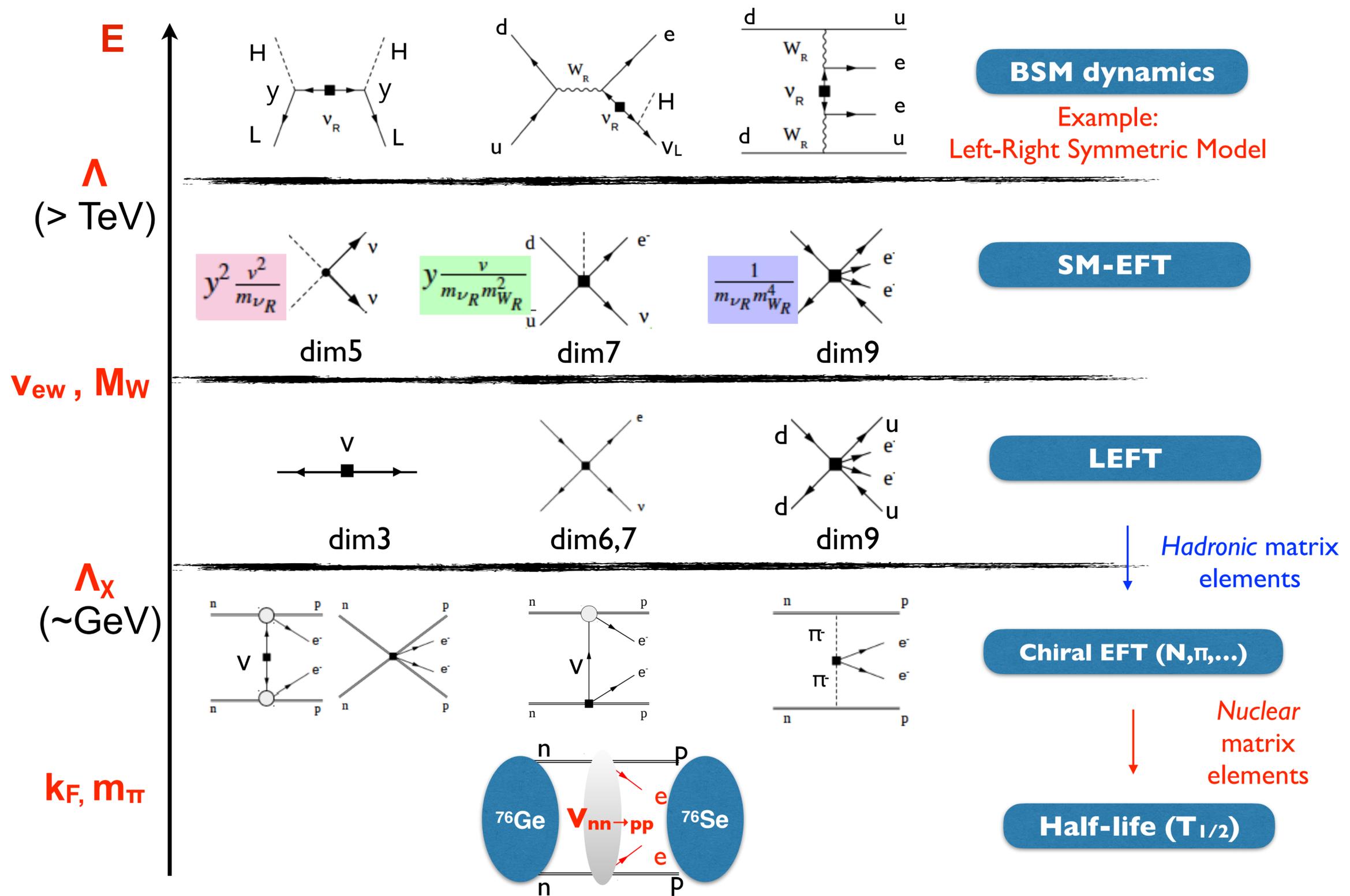
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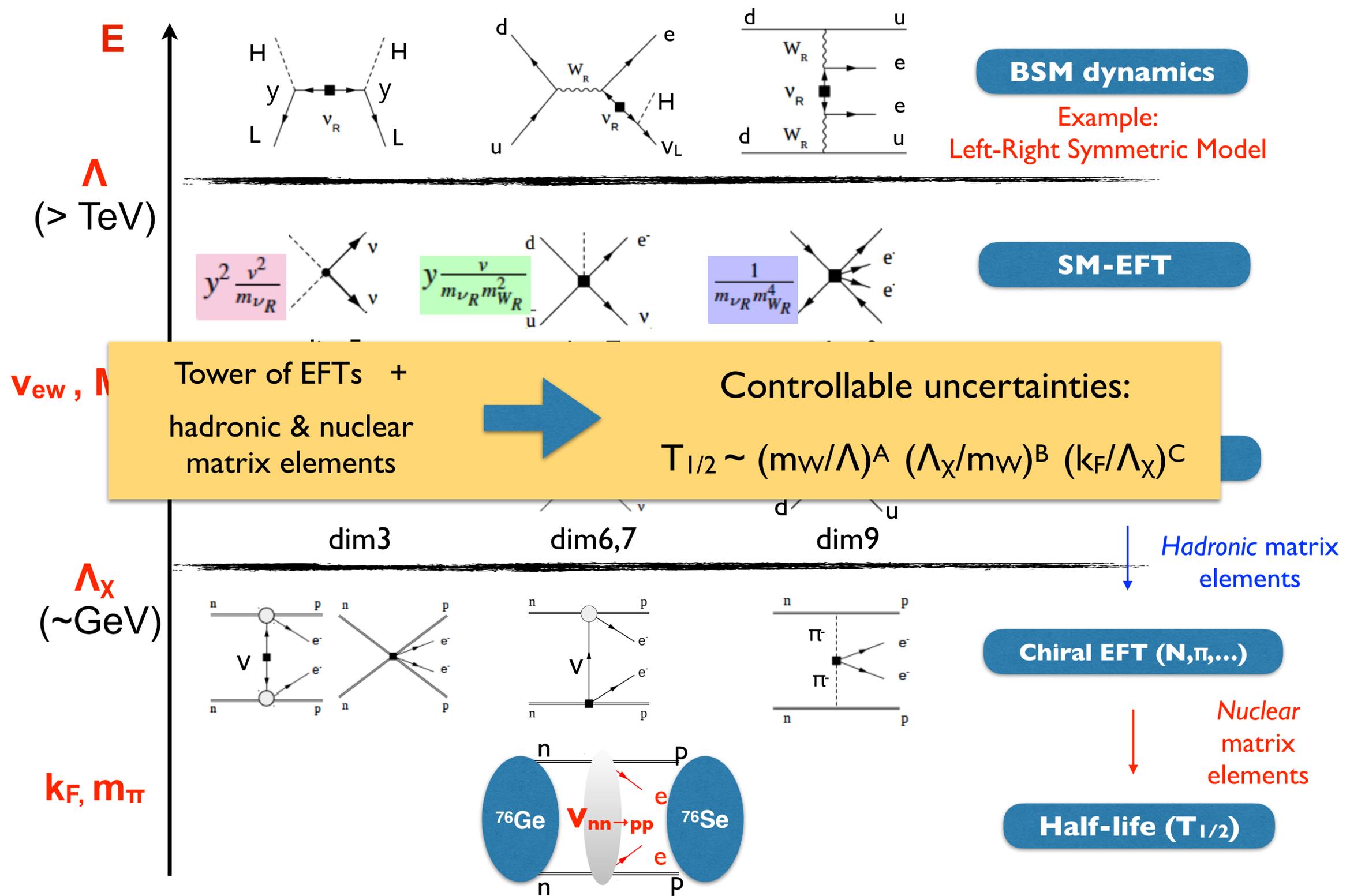
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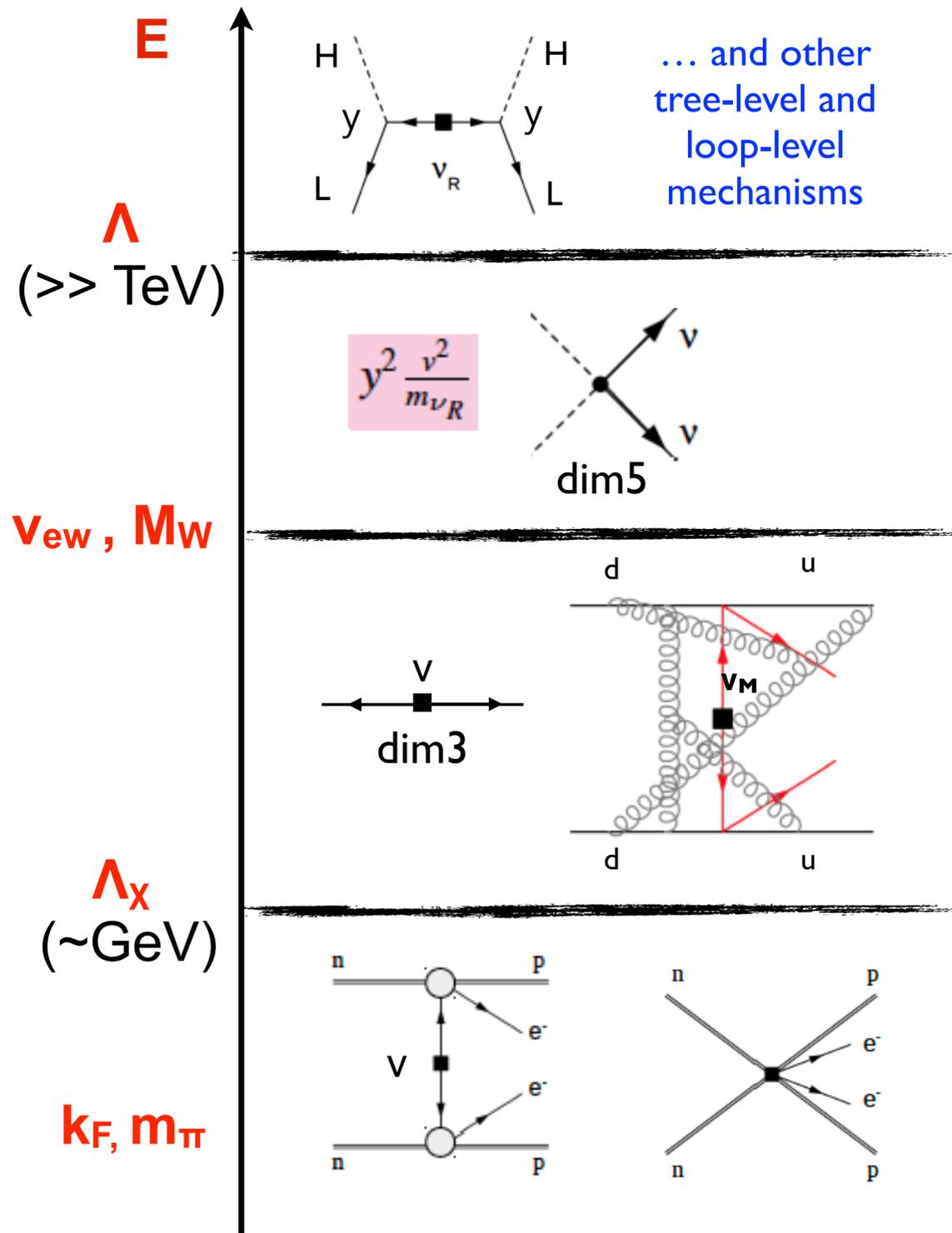
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High scale LNV



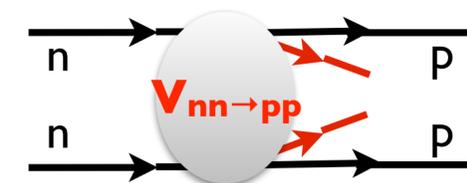
- LNV originates at very high scale ($\Lambda \gg v$) \rightarrow dominant low-energy remnant is Weinberg's dim-5 operator:

$$\mathcal{L}_5 = \frac{w_{\alpha\alpha'}}{\Lambda} L_\alpha^T C \epsilon H H^T \epsilon L_{\alpha'}$$

- Below the weak scale this is just the neutrino Majorana mass ($m_{\beta\beta} \sim w_{ee} v^2/\Lambda$), but let's not forget QCD!

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} - \frac{4G_F}{\sqrt{2}} V_{ud} \bar{u}_L \gamma^\mu d_L \bar{e}_L \gamma_\mu \nu_{eL} - \frac{m_{\beta\beta}}{2} \nu_{eL}^T C \nu_{eL} + \text{H.c.}$$

- $0\nu\beta\beta$ mediated by active V_M with potential $V_{nn \rightarrow pp}$ with long- and short-range components proportional to $m_{\beta\beta}$



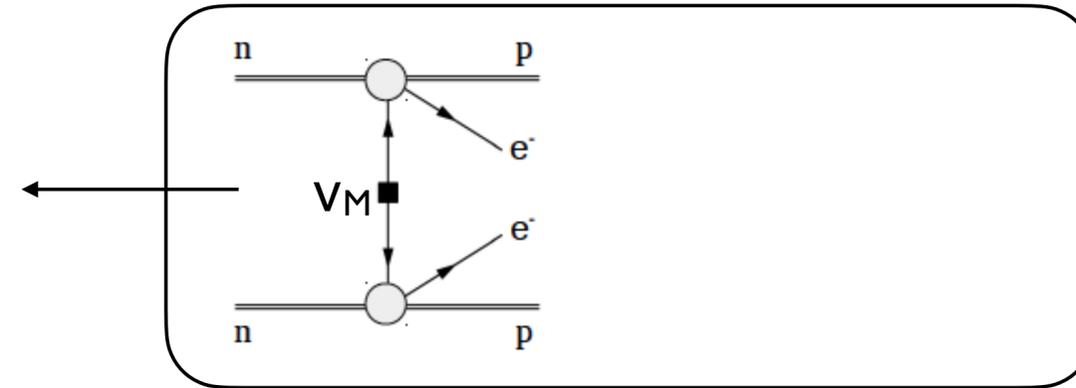
Insights from EFT

VC, W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, U. van Kolck 1802.10097

- Transition operator to leading order (LO) in Q/Λ_χ ($Q \sim k_F \sim m_\pi$, $\Lambda_\chi \sim \text{GeV}$)

'Usual' V_M exchange $\sim 1/k_F^2 \sim 1/Q^2$
Coulomb-like potential



$$V_\nu^{(a,b)} = \tau^{+,a} \tau^{+,b} \frac{1}{q^2} \left(J_V^{(a)}(\mathbf{q}) J_V^{(b)}(-\mathbf{q}) + J_A^{(a)}(\mathbf{q}) J_A^{(b)}(-\mathbf{q}) \right)$$

$$J_V \sim 1$$

$$J_A \sim g_A \sigma$$

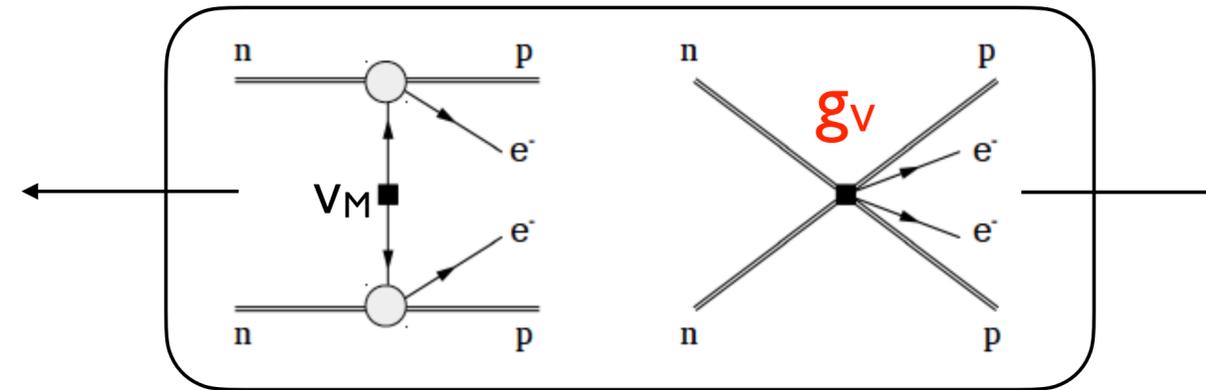
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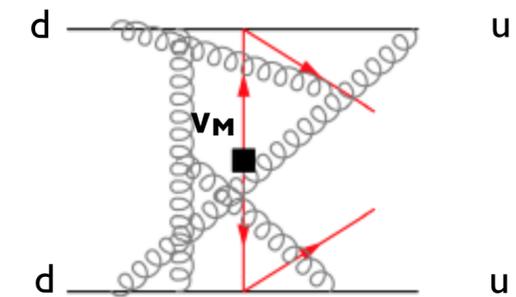
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'New': short-range coupling $g_V \sim 1/Q^2$



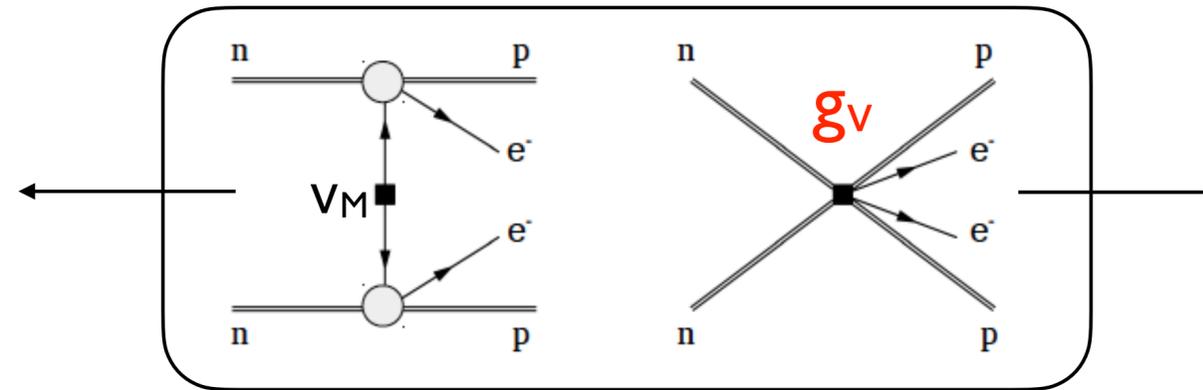
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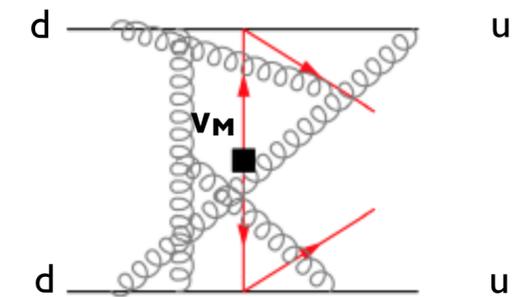
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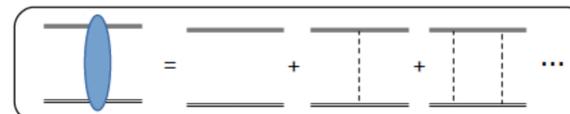
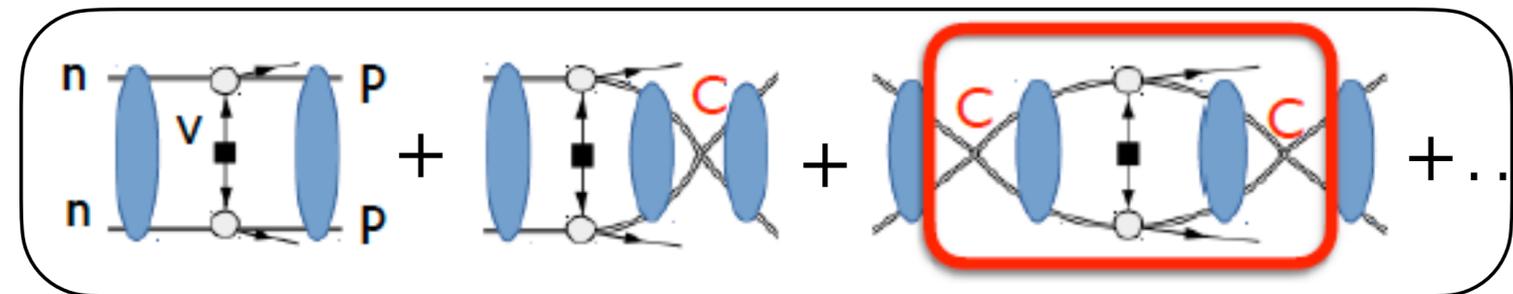
- LO scaling is required by renormalization of $nn \rightarrow pp$ amplitude in presence of strong interactions

LO strong potential



$$C \sim 4\pi/(m_N Q)$$

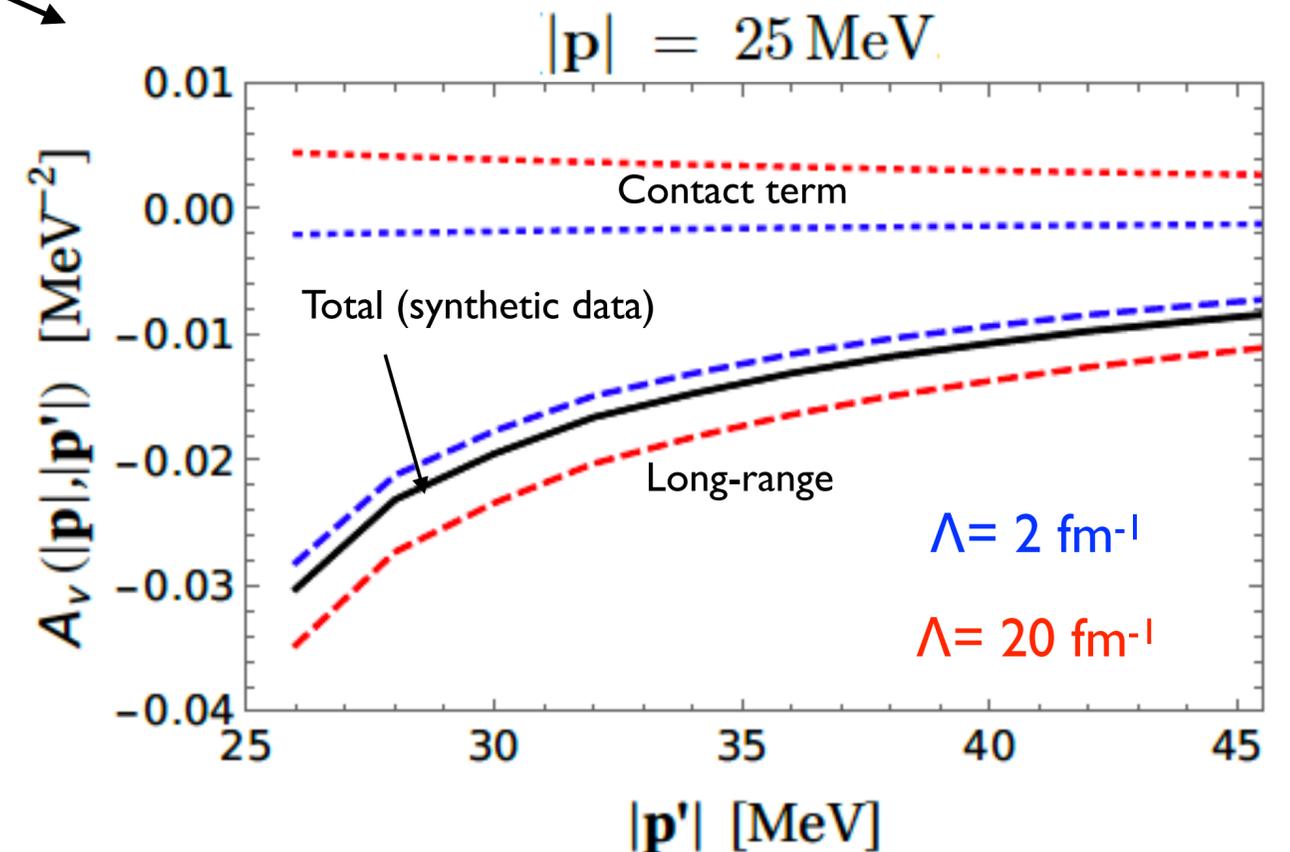
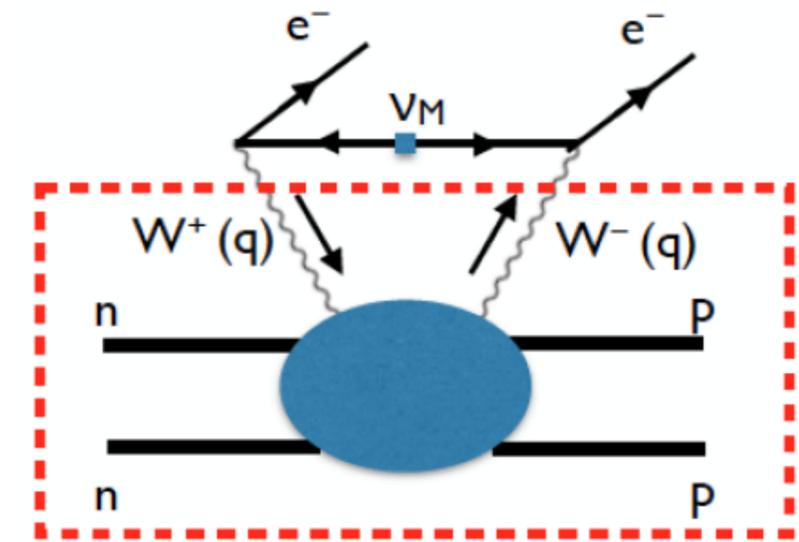
UV divergence $\propto (m_N C/4\pi)^2 \sim 1/Q^2$



Impact of leading-order contact term

- g_V estimated through dispersive analysis, with $\sim 30\%$ uncertainty (validated with $\Delta I=2$ NN electromagnetic coupling)
- Provided 'synthetic data' for the $nn \rightarrow pp$ amplitude to be used to fit g_V with regulators used in nuclear calculations

[1] VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371



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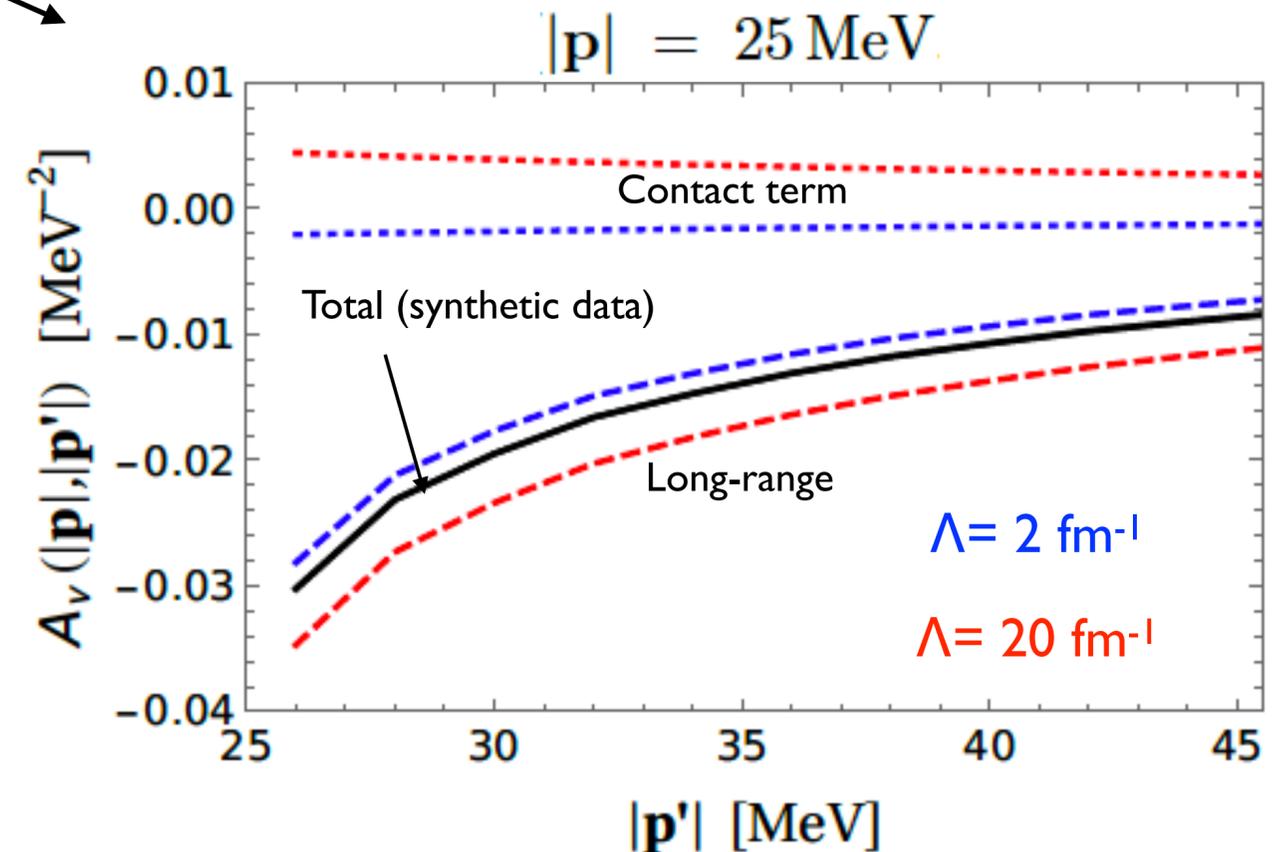
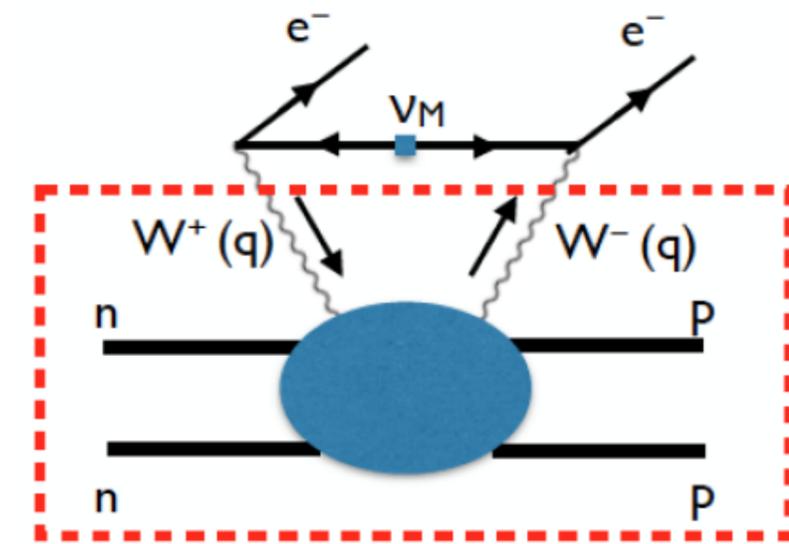
- Contact term fitted to synthetic data [2] and used in ab-initio calculations in ^{48}Ca [2], ^{130}Te [3], ^{136}Xe , [3], ^{76}Ge [4]: enhances matrix elements by $\sim 40\%$ [Ge] and $>50\%$ [Te, Xe]

[2] Wirth, Yao, Hergert, 2105.05415

[3] Belley et al, 2307.15156

[4] Belley et al, 2308.15634

See talk by Jason Holt for details and implications on $m_{\beta\beta}$



Impact of leading-order contact term

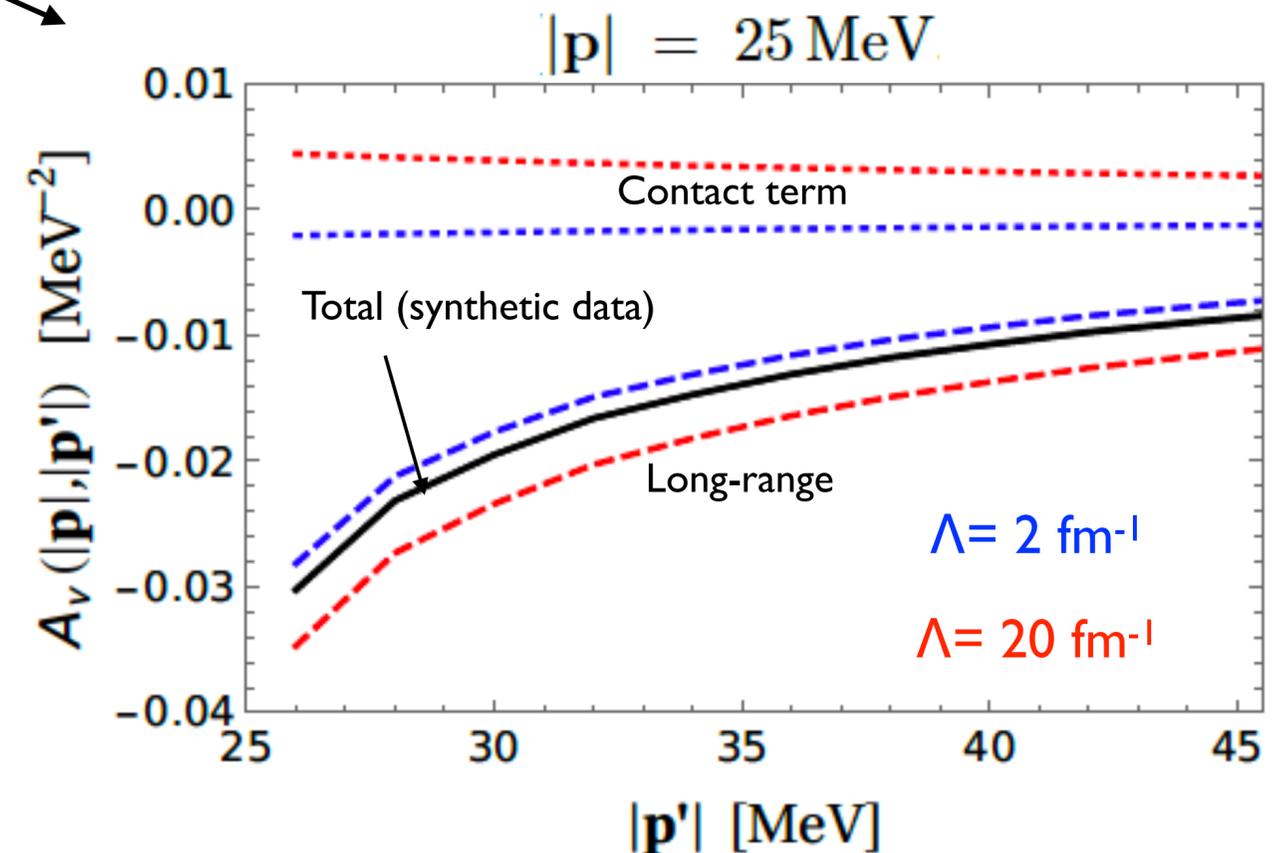
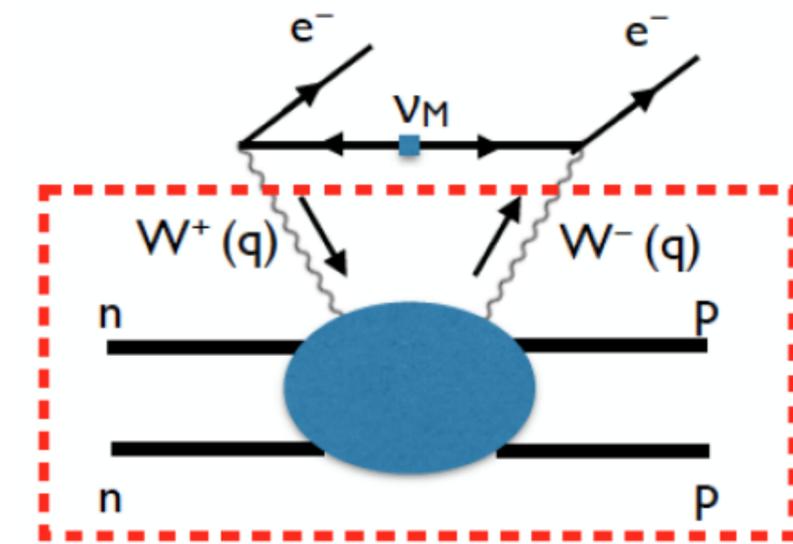
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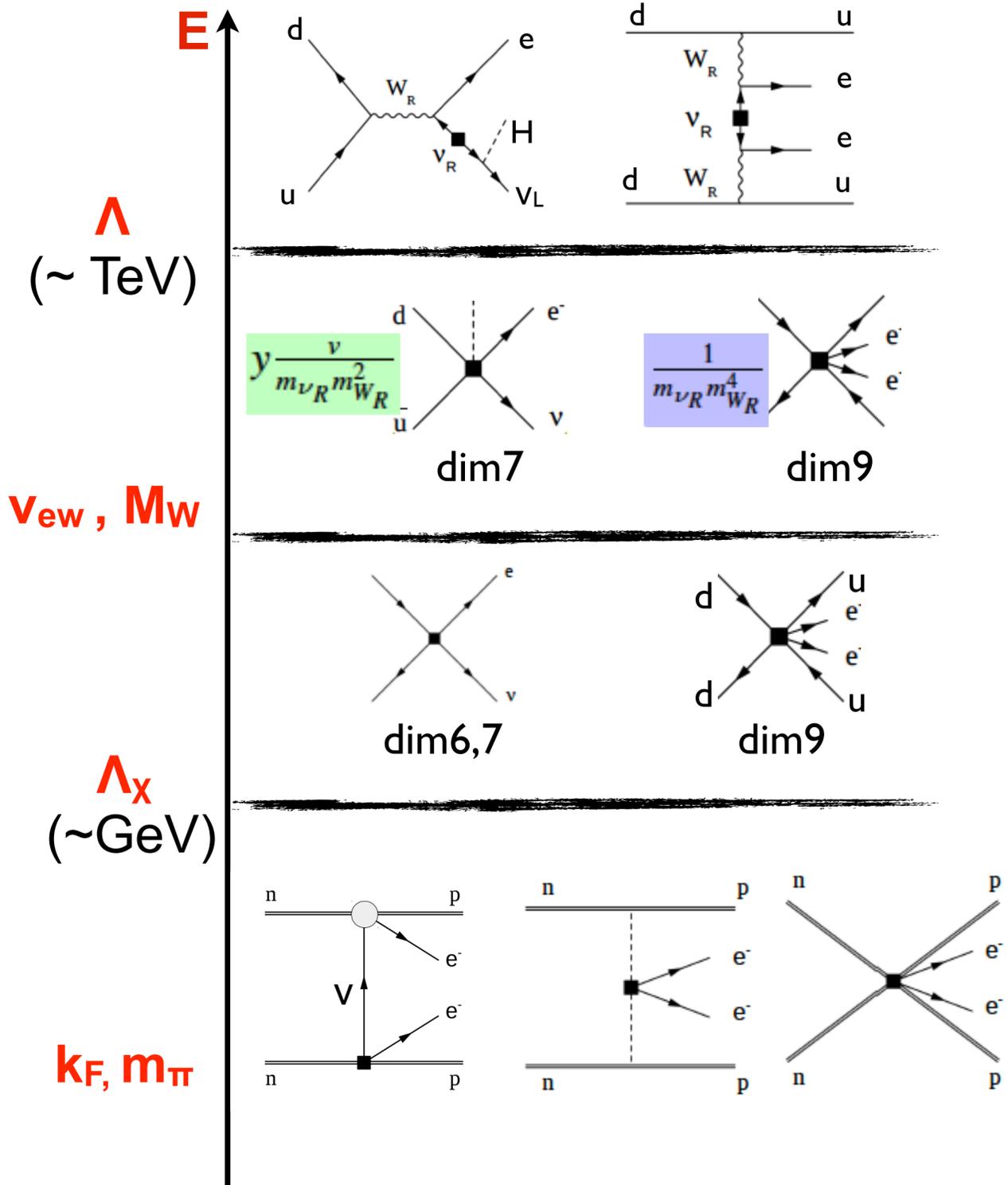
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Overall uncertainties still sizable but improvable!
Progress requires theoretical activity at the interface of EFT, lattice QCD, and nuclear structure



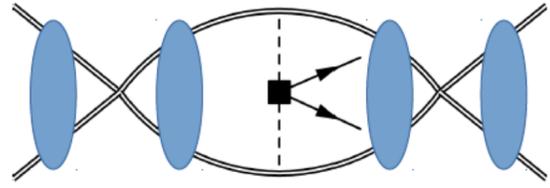
~TeV-scale LNV



- Higher dim operators arise in well motivated models. Can compete with Dim=5 operator if $\Lambda \sim O(1-10 \text{ TeV})$

- 31 operators up to dimension 9

- New mechanisms at the hadronic scale: need appropriate chiral EFT treatment



Renormalization requires a contact terms at the same order as pion-range

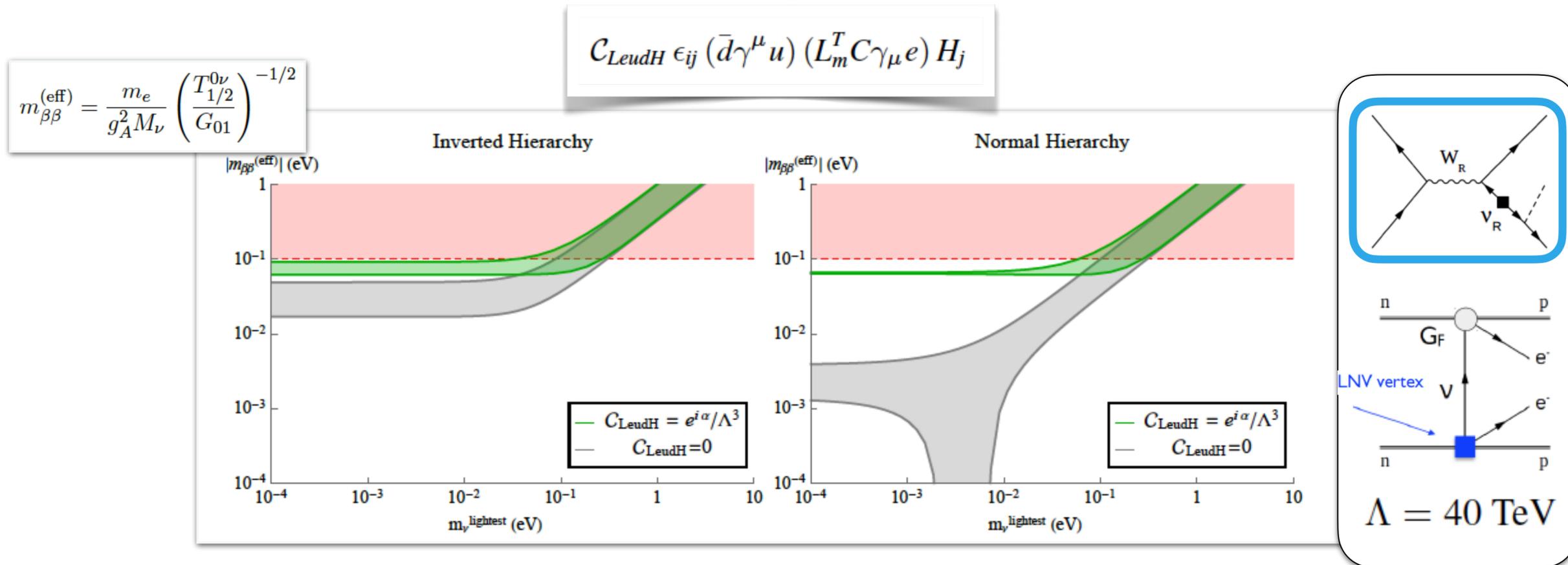
VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti [1806.02780]

- Not including pion- and short-range effects leads to factor $\sim (Q/\Lambda_\chi)^2 \sim 1/100$ reduction in sensitivity to new physics!

Phenomenological interest

- TeV-scale LNV induces contributions to $0\nu\beta\beta$ not directly related to the exchange of light neutrinos, within reach of planned experiments

New contributions can add incoherently or interfere with $m_{\beta\beta}$, significantly affecting the interpretation of experimental results

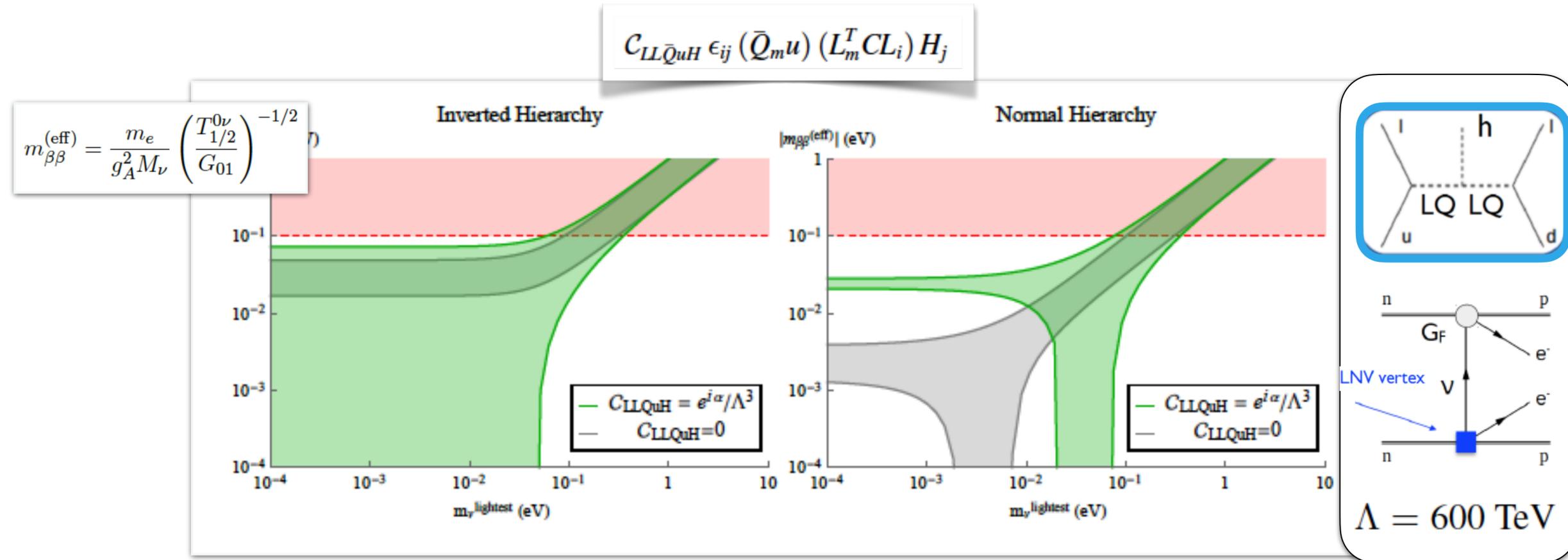


VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390

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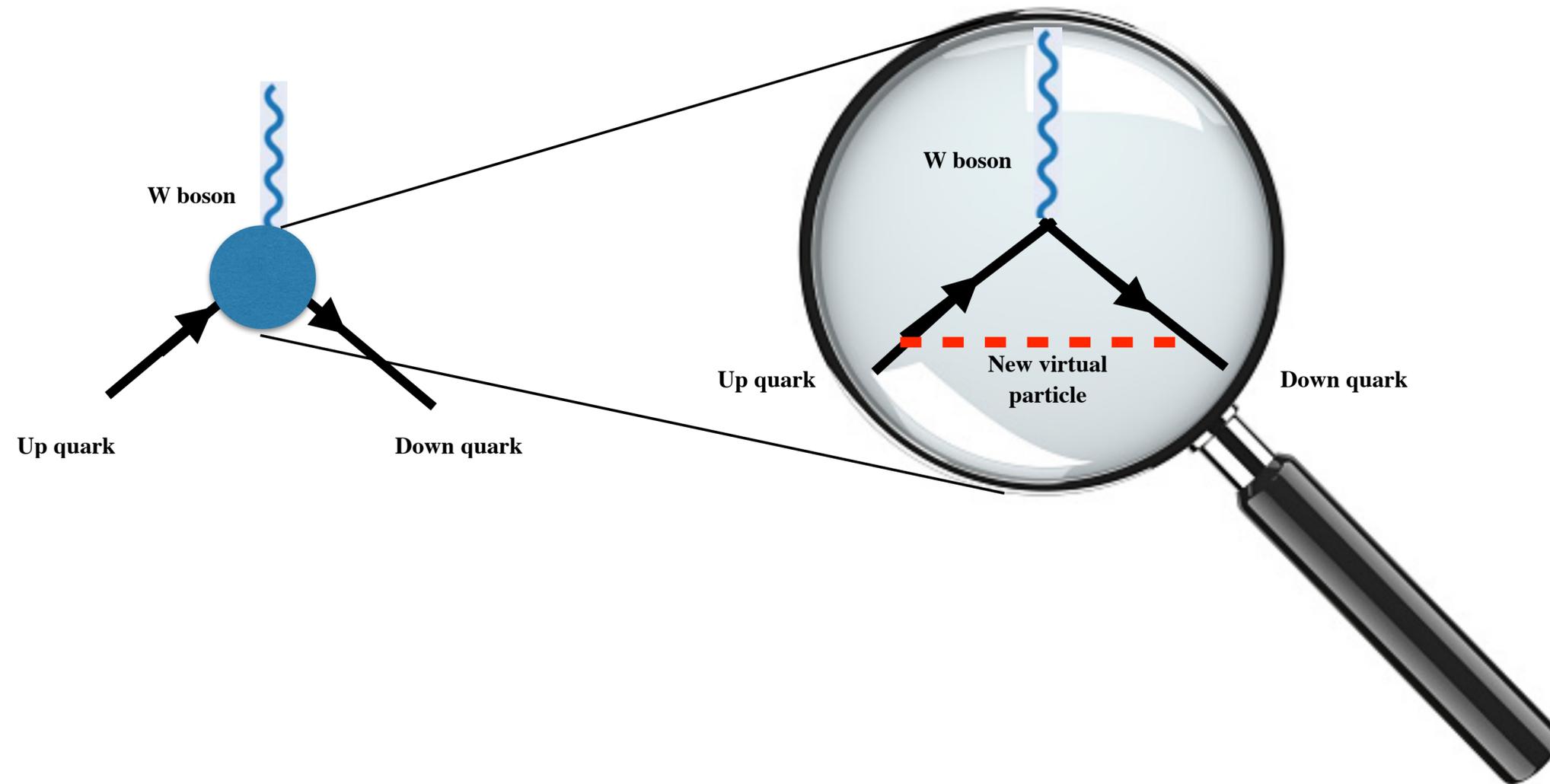
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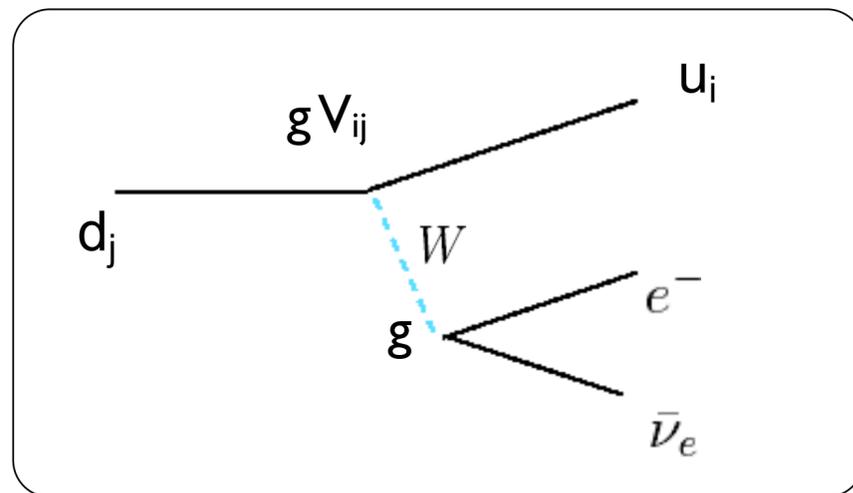
VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390

Precision beta decays



β decays in the SM and beyond

- In the SM, mediated by W exchange \Rightarrow only “V-A”; Cabibbo universality; lepton universality



$$G_F^{(\beta)} \sim G_F^{(\mu)} V_{ij} \sim 1/v^2 V_{ij}$$

Cabibbo Universality

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

$$[G_F]_e / [G_F]_\mu = 1$$

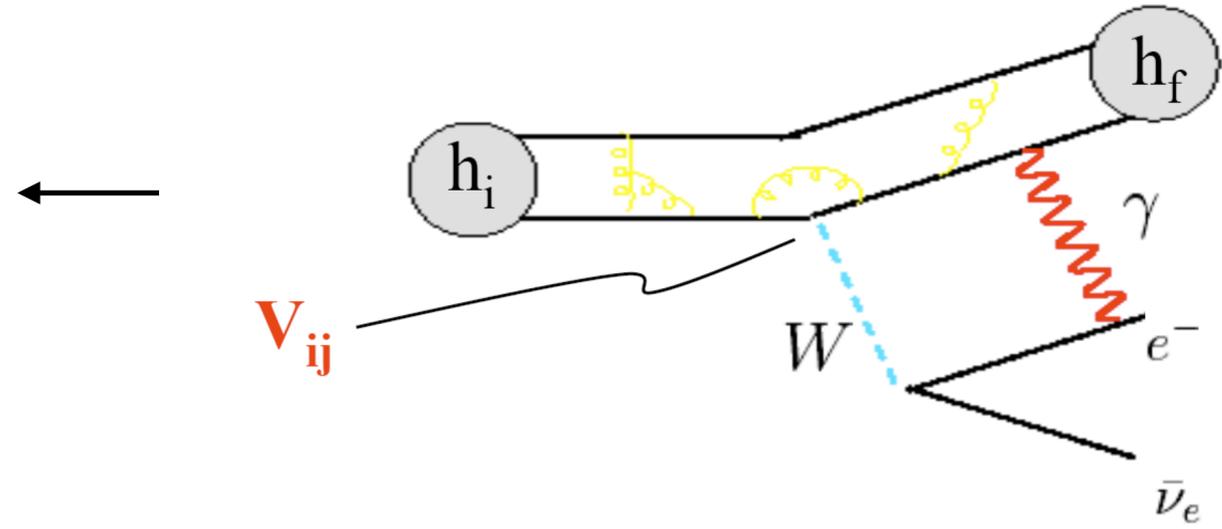
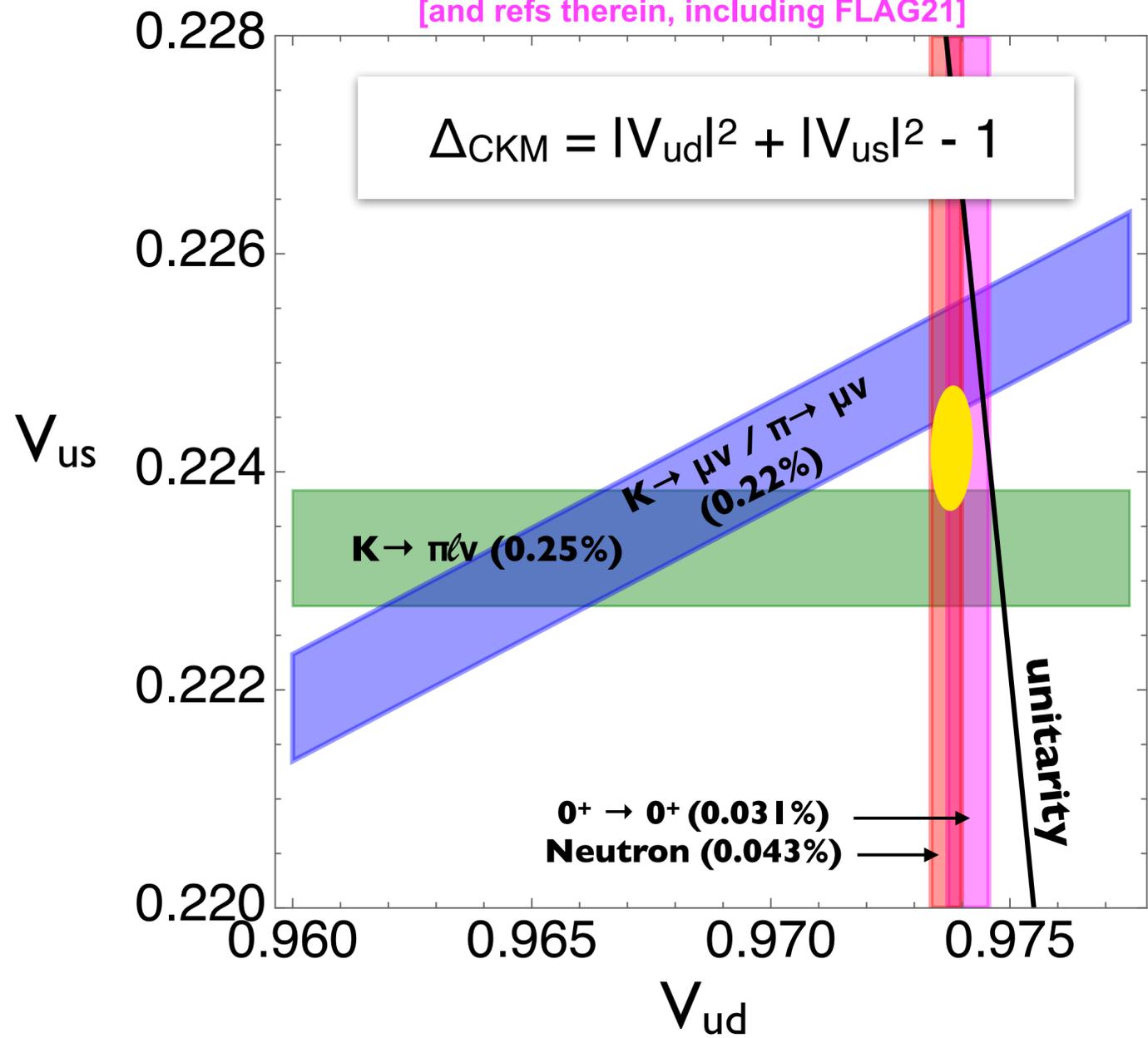
Lepton Flavor Universality (LFU)

- New physics can spoil universality relations. Precision of 0.1-0.01% probes $\Lambda > 10$ TeV
- Focus on Cabibbo universality test (1st row CKM unitarity)

The Cabibbo angle “anomaly”

$$\Gamma = G_F^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \Delta_R) \times F_{\text{kin}}$$

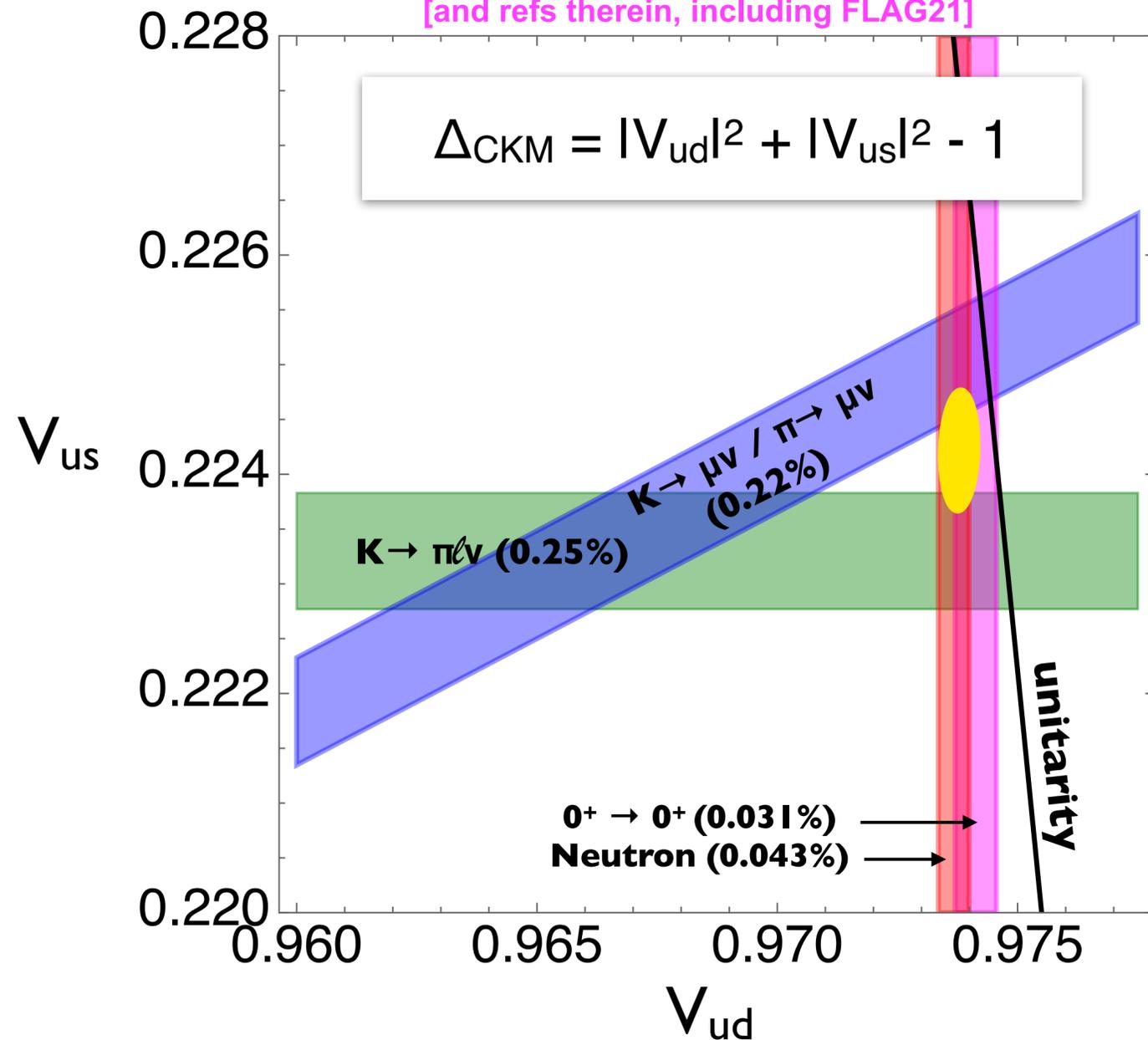
VC-Crivellini-Hoferichter-Moulson 2208.11707
[and refs therein, including FLAG21]



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VC-Crivellin-Hoferichter-Moulson 2208.11707
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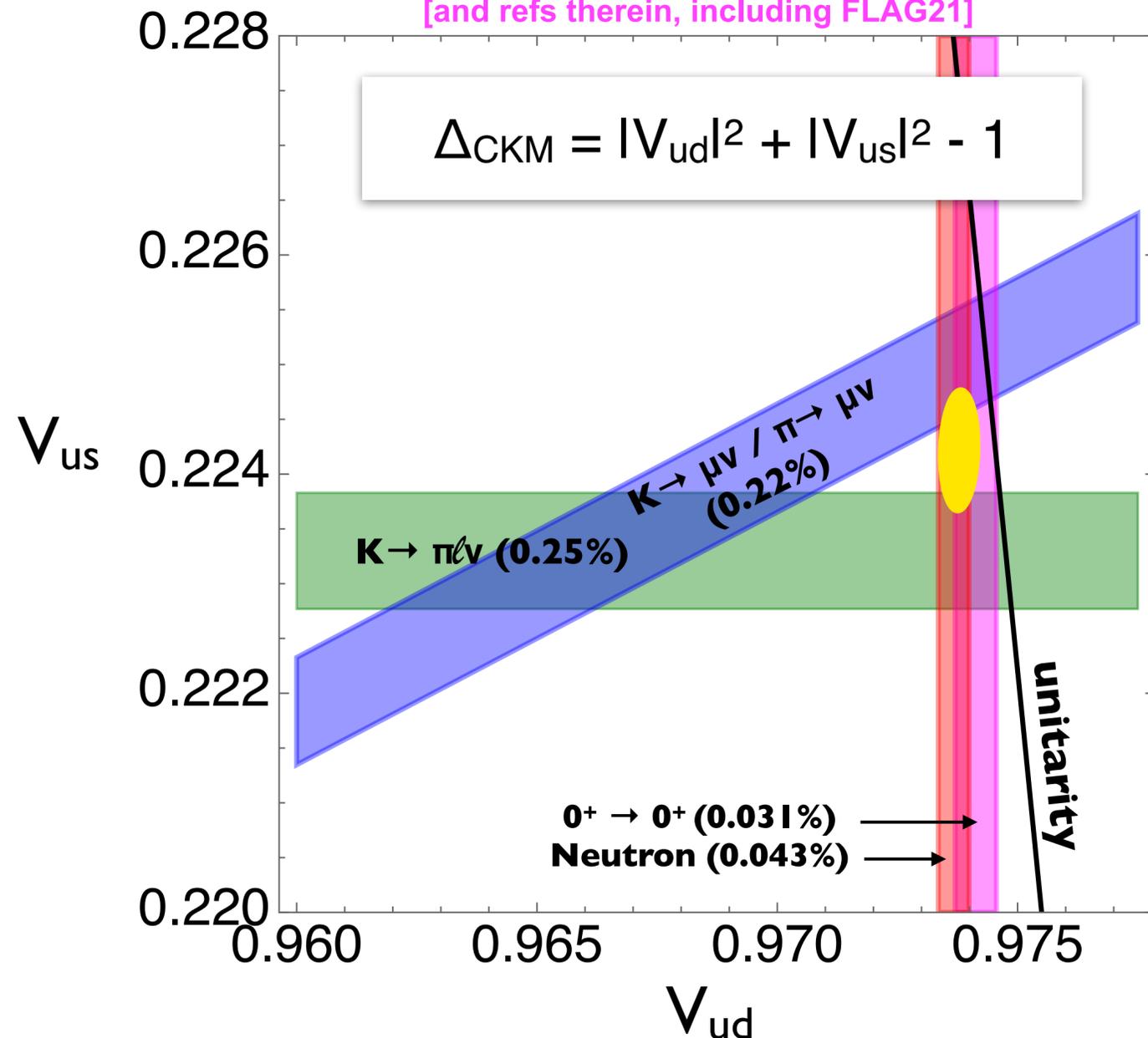
- The ‘anomalies’:
 - $\sim 3\sigma$ effect in global fit ($\Delta_{\text{CKM}} = -1.48(53) \times 10^{-3}$)
 - $\sim 3\sigma$ problem in meson sector (Kl2 vs Kl3)
- Can be explained by BSM physics (R-handed quark currents), but need to scrutinize the SM input!

Grossman-Passemar-Schacht 1911.07821
VC-Crivellin-Hoferichter-Moulson 2208.11707
Belfatto-Berezhiani 1906.02714, 2103.05549; Belfatto-Trifinopoulos 2302.14097
VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, 2311.00021

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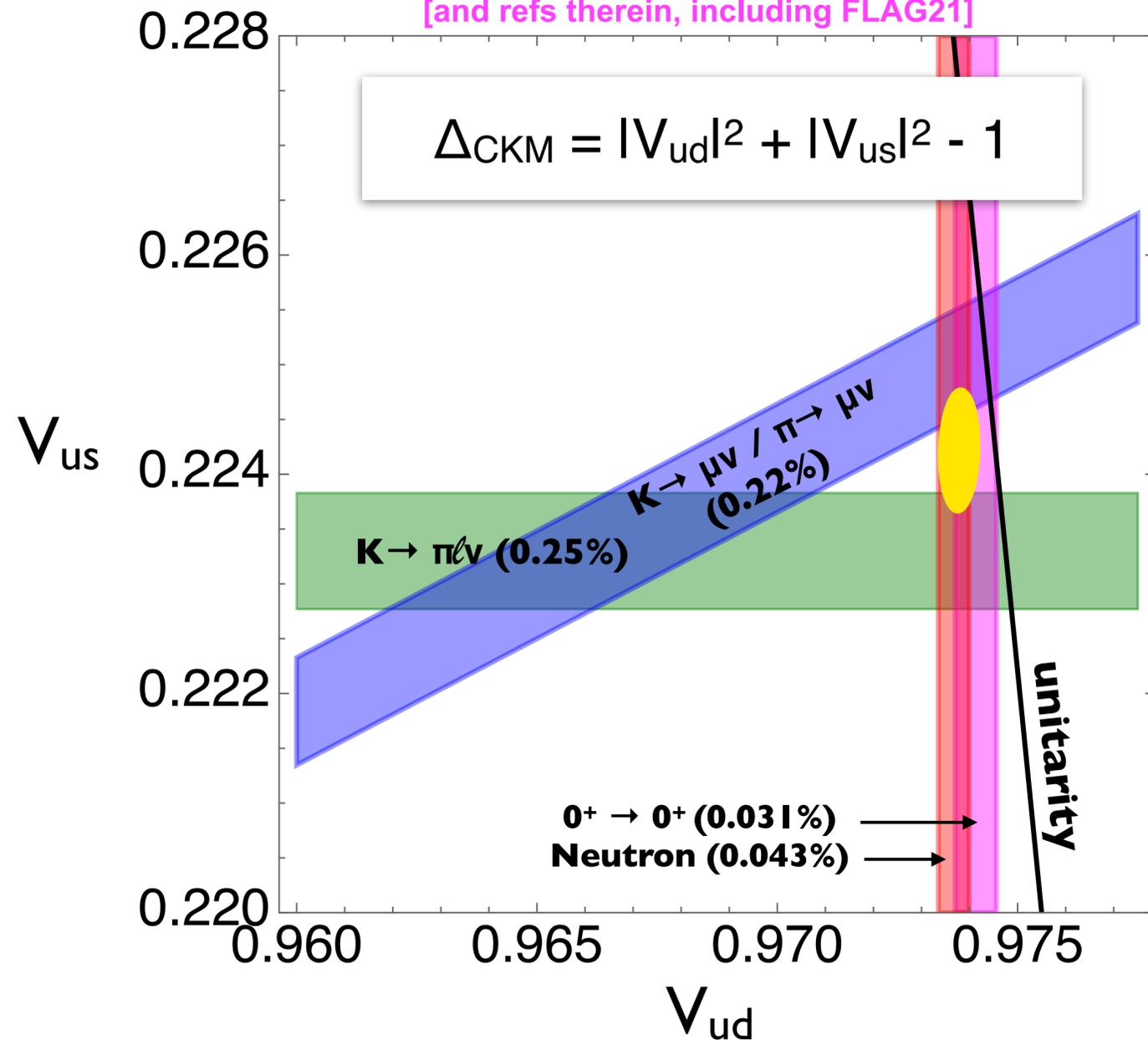


- **Expected experimental improvements:**
 - neutron decay
 - pion beta decay (PIONEER @PSI)
 - new $K_{\mu 3}/K_{\mu 2}$ BR measurement at NA62
- **Further theoretical scrutiny**
 - Lattice QCD: $K \rightarrow \pi$ vector f.f., rad. corr. for K_{l3}
 - EFT for radiative corrections neutron and nuclei, with precision goal $\sim 2\text{-}3 \times 10^{-4}$
 - ...

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VC-Crivellin-Hoferichter-Moulson 2208.11707
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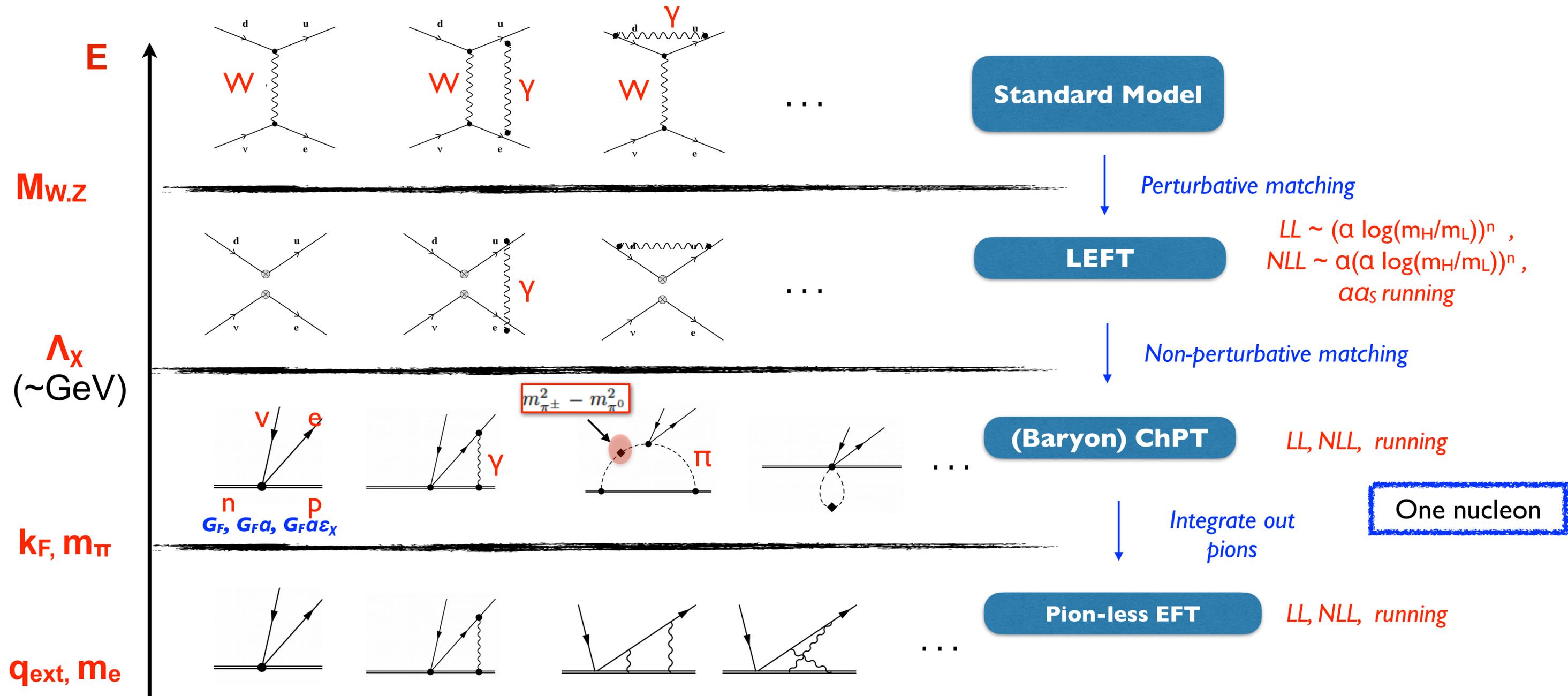
Multi-step strategy

- Matching and running in a tower of EFTs: $SM \rightarrow LEFT \rightarrow ChPT \rightarrow \not{t}EFT, \chi EFT$



Multi-step strategy

- Matching and running in a tower of EFTs: SM \rightarrow LEFT \rightarrow ChPT \rightarrow ν EFT, χ EFT



New effects in neutron decay

- Larger radiative correction to decay rate shifts V_{ud} by -0.013% [effect due to large NLL $\sim \alpha^2 \text{Log}(m_N/m_e)$]

VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138

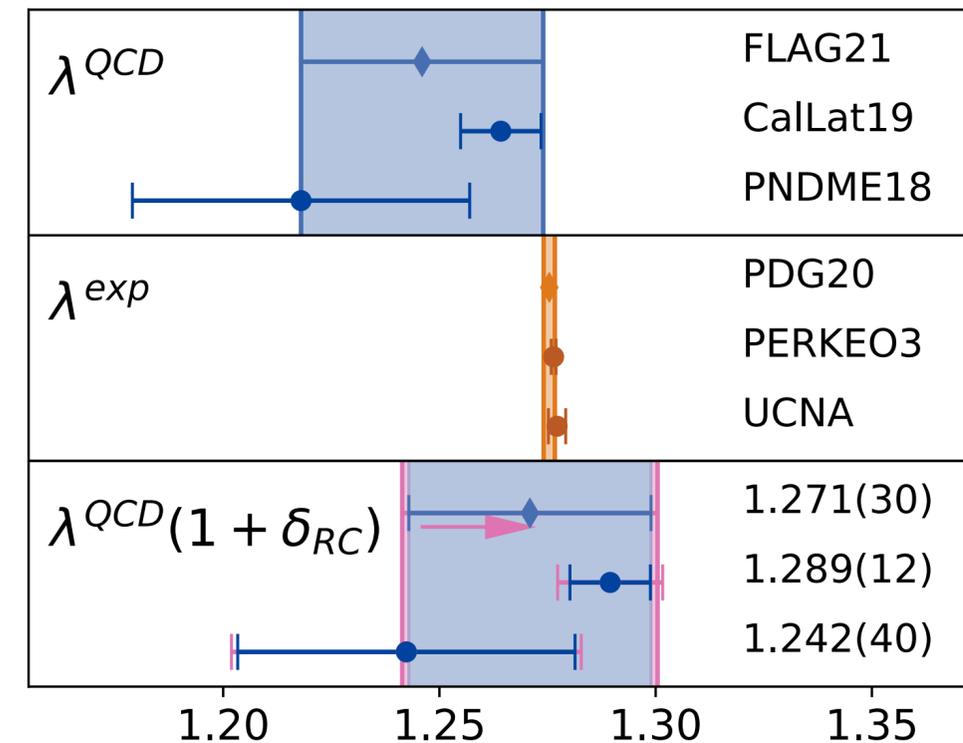
- (g_A/g_V) gets %-level corrections proportional to the pion EM mass splitting, 100x larger than previous estimate

VC, J. de Vries, L. Hayen, E. Mereghetti, A. Walker-Loud 2202.10439

$$\frac{\lambda^{\text{exp}}}{\lambda^{\text{QCD}}} = 1 + \delta_{\text{RC}}$$

$$\delta_{\text{RC}} \simeq (2.0 \pm 0.6)\%$$

Large uncertainty due to unknown LEC that could be determined by future lattice calculations



Radiative corrections generally improve agreement between data and Lattice QCD

New effects in nuclear decays

VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E. Mereghetti, 2405.18469, 2405.18464

- Hard photons induce $NN \rightarrow NNe\nu$ contact interactions \Rightarrow 'Weak potentials' of $O(G_F a \epsilon_\chi)$ involve two currently unknown LECs $\Rightarrow \delta V_{ud}$ (nuclear structure) $\sim 0.0003 |_{\text{models}} \rightarrow 0.00052_{\text{EFT}}$
- LECs can be obtained by fitting data, once NME calculations for several isotopes become available

k_F, m_π

$$\mathcal{L}_{\chi PT} = -\sqrt{2}G_F V_{ud} \bar{e}_L \gamma_\mu \nu_L \bar{N} \tau^+ N [e^2 g_{V1}^{NN} \bar{N} N + e^2 g_{V2}^{NN} \bar{N} \tau_3 N]$$

$Q \sim q_{\text{ext}}, m_e$

$\epsilon_\chi = m_\pi / \Lambda_\chi \sim 0.1$

$\epsilon_\pi = q_{\text{ext}} / m_\pi \sim 10^{-2}$

$$H_{EW} = \sqrt{2}G_F V_{ud} \bar{e}_L \gamma_\mu \nu_L \mathcal{J}_W^\mu,$$

$$\mathcal{J}_W^\mu = \sum_{n=1}^A (g_V \delta^{\mu 0} - g_A \delta^{\mu i} \sigma^{(n)i}) \tau^{(n)+} + (\mathcal{J}^{2b})^\mu + \dots$$

$$+ \delta^{\mu 0} (\mathcal{V}^0 + E_0 \mathcal{V}_E^0) + \delta^{\mu i} \mathcal{V}_i + p_e^\mu \mathcal{V}_{m_e} + \dots$$

EFT has identified new correction and (temporarily) increased the uncertainty....
 But in the long run it's the only viable approach to quantify the uncertainties.

Conclusions

- EFT is a great tool to connect electroweak and higher scales to nuclear energies, with quantifiable uncertainties
- Illustrated with two examples

EFT is crucial to assess impact of ton-scale
 $0\nu\beta\beta$ searches

- Relates $0\nu\beta\beta$ to underlying LNV dynamics (and collider & cosmology)
- Organizes contributions to hadronic and nuclear matrix elements \rightarrow control uncertainties

EFT methods can shed light on the ‘Cabibbo angle anomaly’ one of few low-energy “cracks” in the SM

- Seamlessly include next-to-leading logarithmic perturbative effects
- Uncovered new short-range nuclear structure-dependent effects needed to reach the $\sim 3 \times 10^{-4}$ precision in V_{ud}
- This framework may become of interest for neutrino-nucleus scattering at low energy

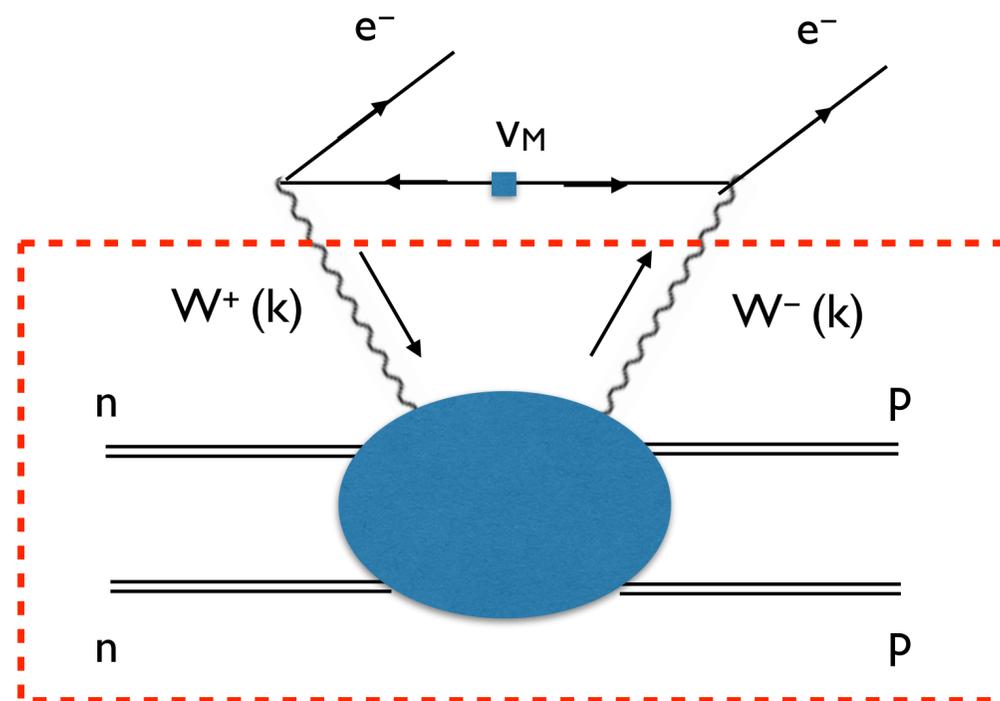
Backup: double beta decay

Estimating the contact term (I)

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

- Useful representation of the amplitude

$$\mathcal{A}_\nu \propto \int \frac{d^4 k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4 x e^{ik \cdot x} \langle pp | T \{ j_W^\alpha(x) j_W^\beta(0) \} | nn \rangle$$



Forward “Compton” amplitude

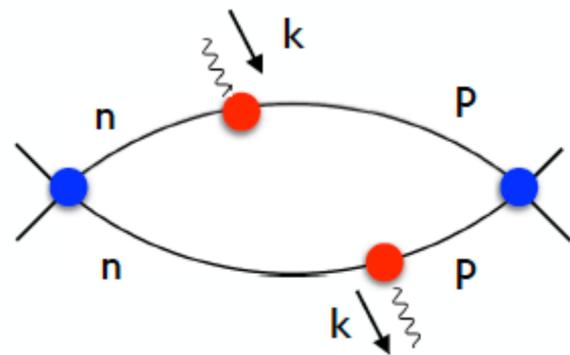
Estimating the contact term (I)

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

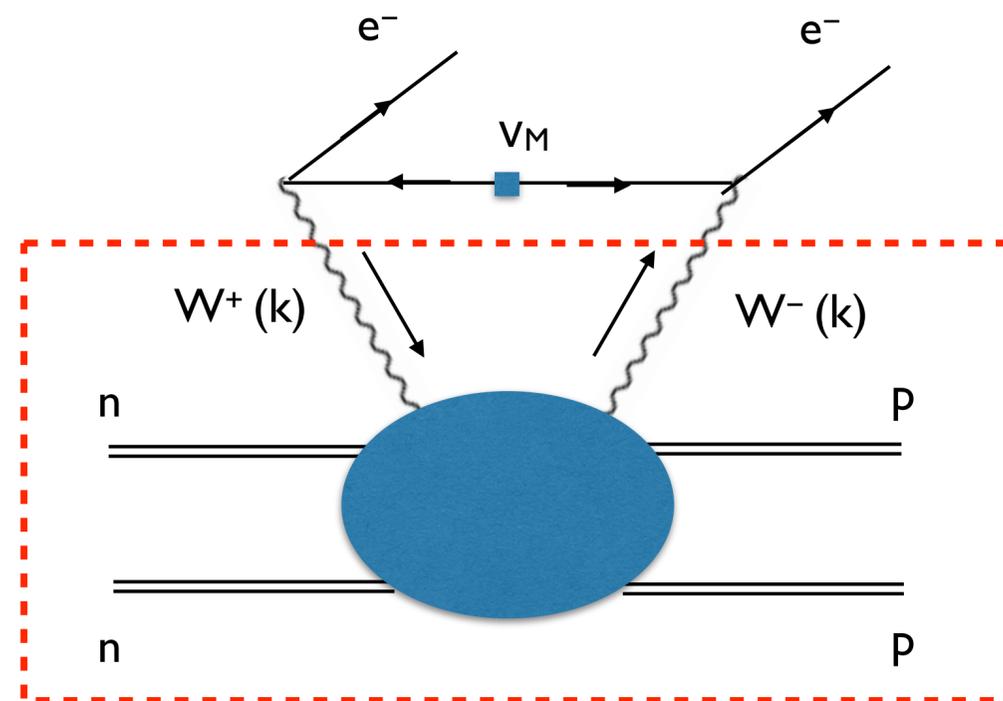
- Useful representation of the amplitude

$$\mathcal{A}_\nu \propto \int \frac{d^4 k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4 x e^{ik \cdot x} \langle pp | T \{ j_W^\alpha(x) j_W^\beta(0) \} | nn \rangle$$

Low k: chiral EFT to NLO

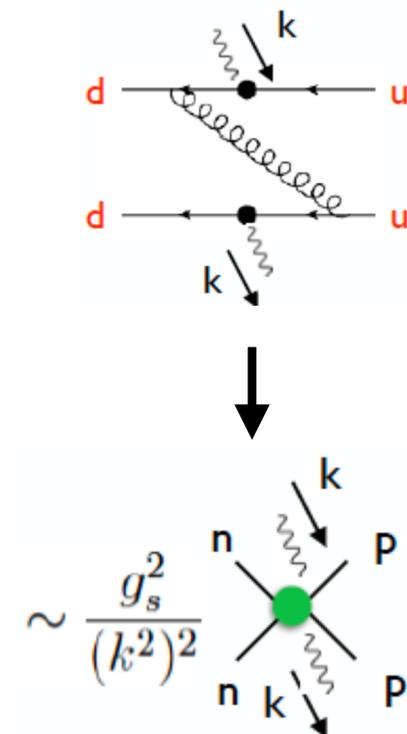


Intermediate k: resonance contributions in \blacksquare and \bullet , π NN intermediate state, ...



Forward "Compton" amplitude

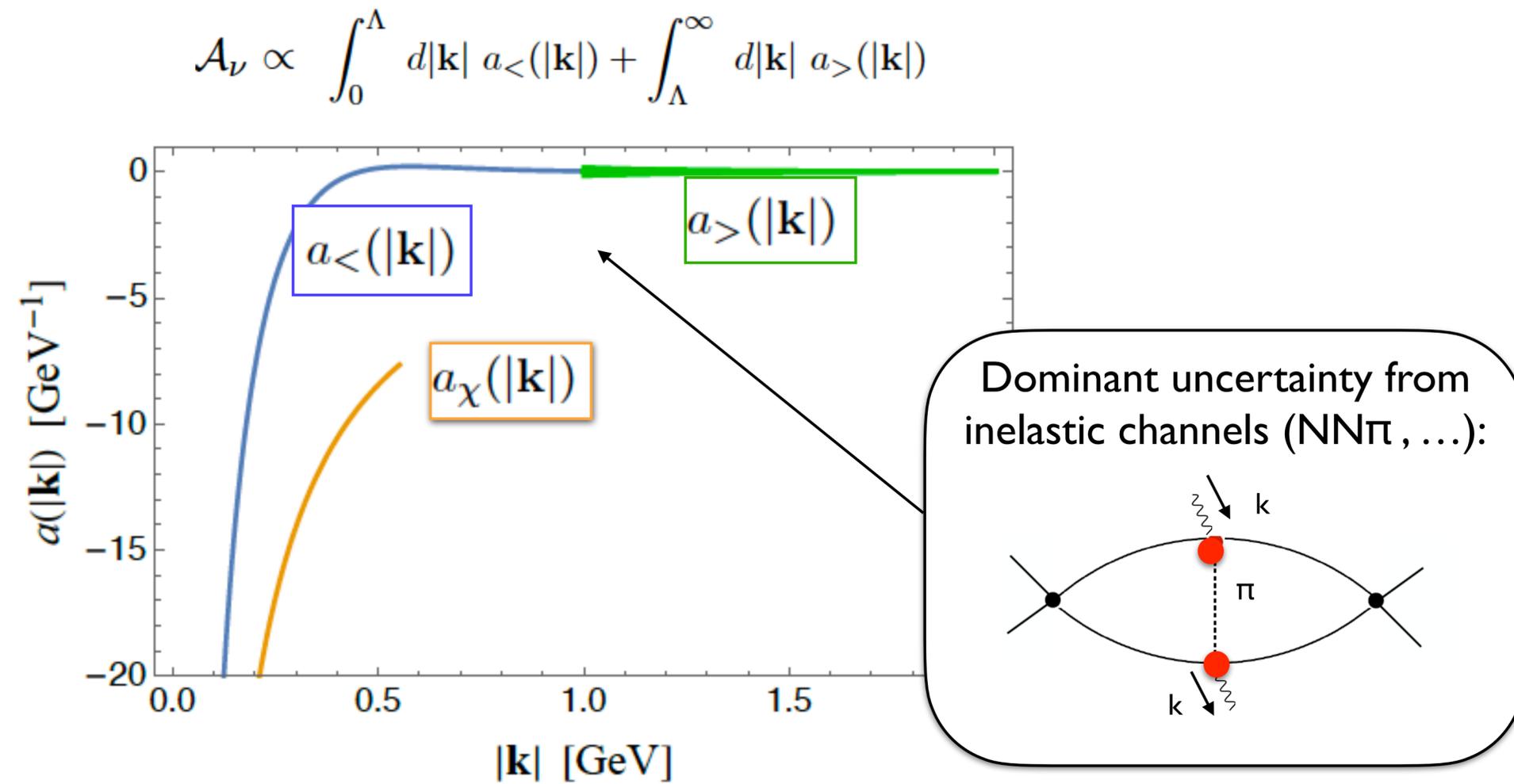
High k: QCD OPE



Estimating the contact term (2)

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

- Determine $C_{1,2}$ with $\sim 30\%$ uncertainty (dominated by intermediate k)



Estimating the contact term (2)

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

- Determine $C_{1,2}$ with $\sim 30\%$ uncertainty (dominated by intermediate k)
- Validation: $C_1 + C_2 \Rightarrow (a_{nn} + a_{pp})/2 - a_{np} = 15.5(4.5)$ fm versus $10.4(2)$ fm (exp)
- Provided 'synthetic data' for the $nn \rightarrow pp$ amplitude at threshold
- First calculation of $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ with contact fitted to synthetic data \Rightarrow contact term enhances nuclear matrix element by $(43 \pm 7)\%$

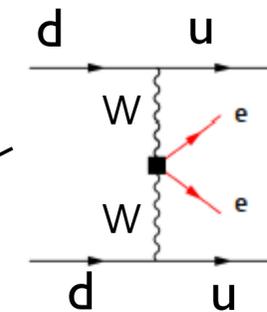
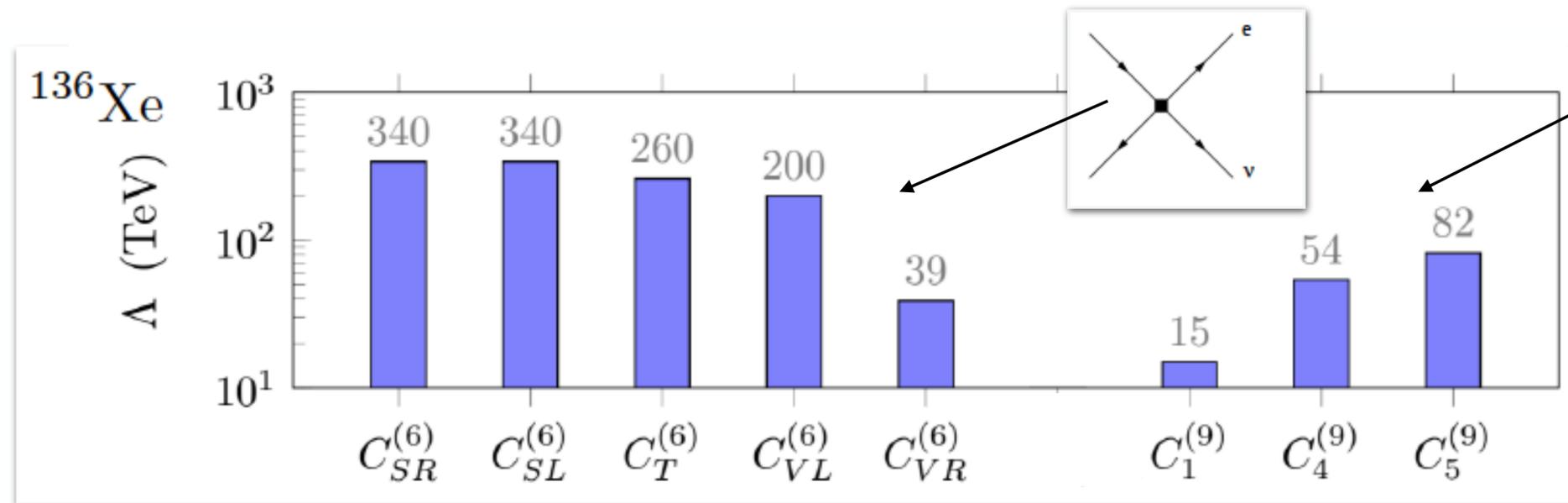
Wirth, Yao, Hergert, 2105.05415

What scales are we probing?

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1806.02780

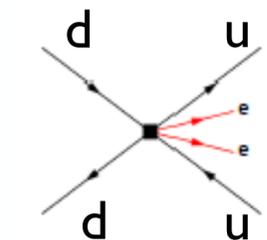
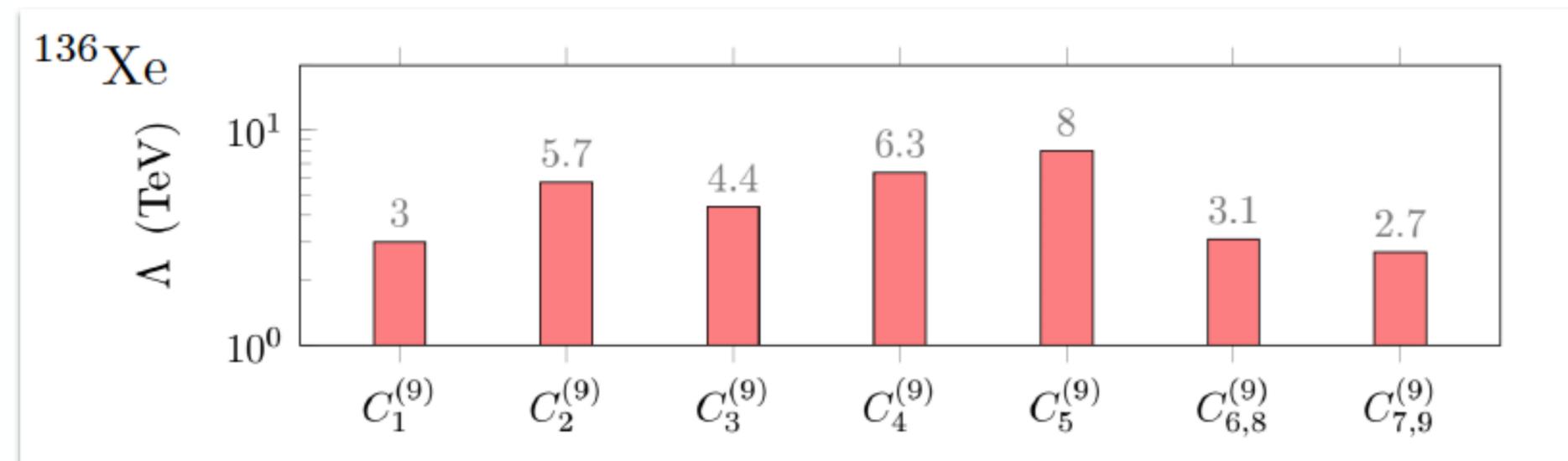
Dim 7 in SM-EFT

$(\nu/\Lambda)^3$



Dim 9 in SM-EFT

$(\nu/\Lambda)^5$



Bounds reflect dependence on Λ_x/Λ and Q/Λ_x

Phenomenological interest

- TeV-scale LNV induces contributions to $0\nu\beta\beta$ not directly related to the exchange of light neutrinos, within reach of planned experiments

New contributions can add incoherently or interfere with $m_{\beta\beta}$, significantly affecting the interpretation of experimental results

- TeV-scale LNV may lead to correlated or precursor signal at LHC: $pp \rightarrow ee jj$ (important to unravel the mechanism)

Keung-Senjanovic '83

Maiezza-Nemevesek-
Nesti- Senjanovic
1005.5160

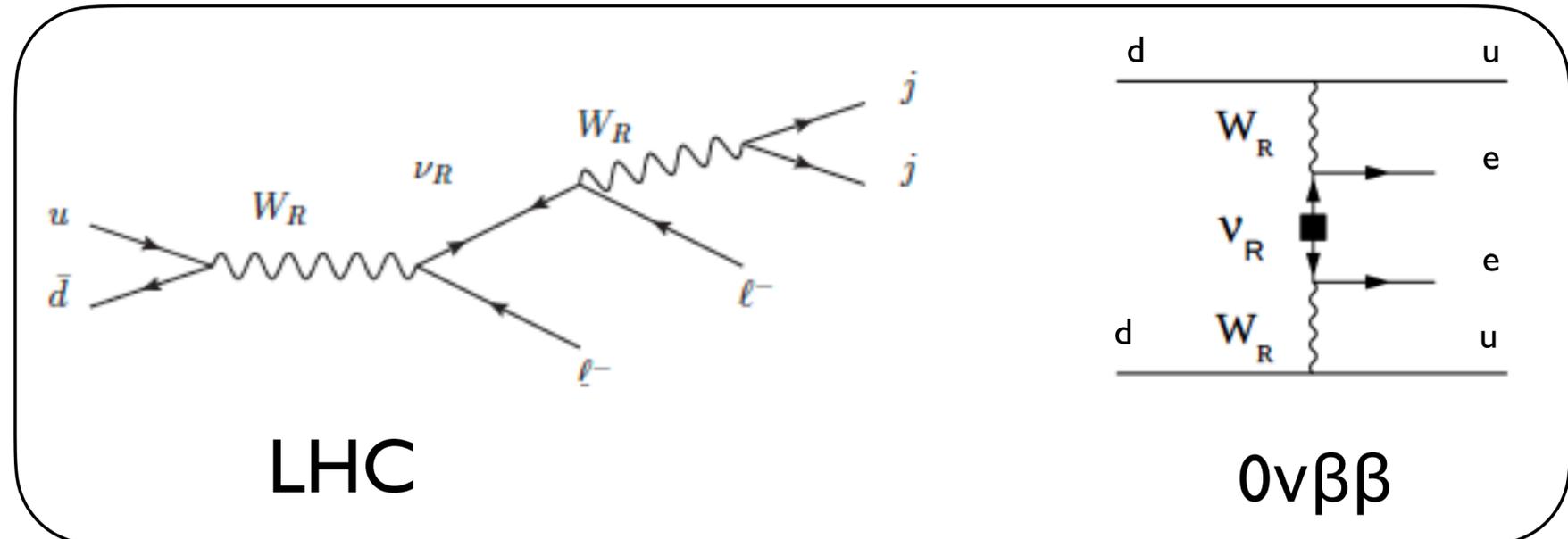
Helo-Kovalenko-Hirsch-
Pas 1303.0899, 1307.4849

Cai, Han, Li, Ruiz
1711.02180

Peng, Ramsey-Musolf,
Winslow, 1508.0444

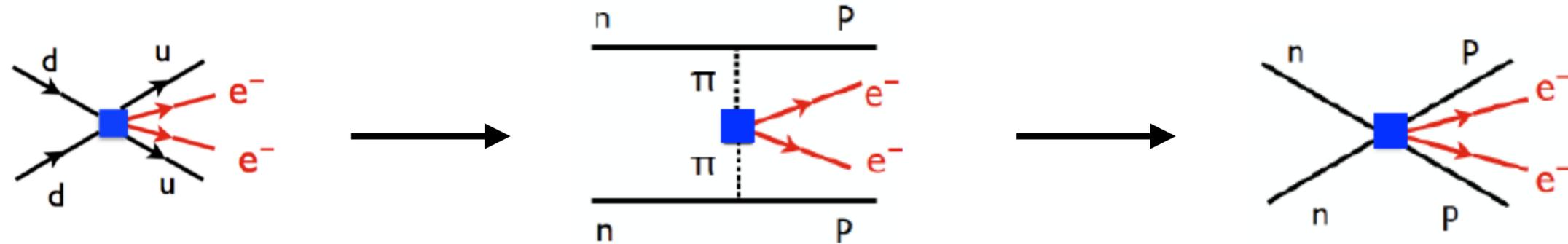
...

Classic LRSM example



Hadronic theory developments

- Leading order hadronic realization of dim-9 operators:



In Weinberg's counting, pion-exchange contribution dominates

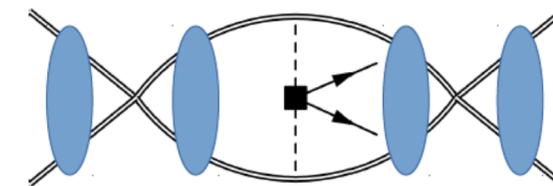
Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205

Vergados 1982, Faessler, Kovalenko, Simkovic, Schweiger 1996

$\pi\pi$ matrix element known from Lattice QCD at <10%

Nicholson et al (CalLat), 1805.02634

Renormalization requires a contact at the same order!



VC, W. Dekens, J. de Vries, M. Graesser, Mereghetti [1806.02780]

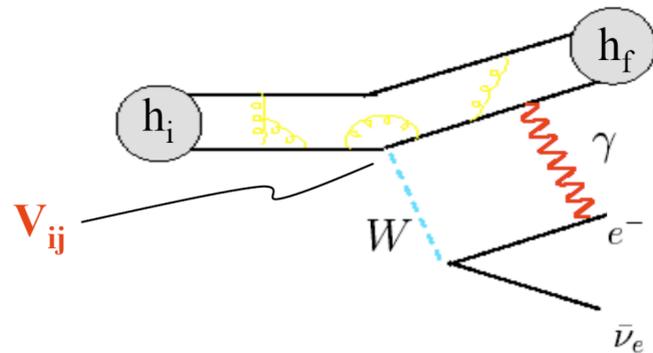
E.

- Several unknown LO NN contact couplings! Opportunity for LQCD

Backup: beta decay

EFT for radiative corrections: why?

- Widely separated mass scales play a role in neutron and nuclear beta decays



$$M_{W,Z}$$

$$\gg \Lambda_\chi \sim m_N \sim 4\pi F_\pi \sim 1 \text{ GeV}$$

$$\gg m_\pi \sim 140 \text{ MeV}$$

$$\gg q_{\text{ext}} \sim m_n - m_p \sim m_e \sim 1 \text{ MeV}$$

Weak scale

χ SB & nucleon mass scale

Pion mass / hadronic structure

Q value, nuclear excitations

- Small ratios appear as expansion parameters and arguments of logarithms

$$\epsilon_W = \Lambda_\chi / M_W \sim 10^{-2} \quad \epsilon_\chi = m_\pi / \Lambda_\chi \sim 0.1 \quad \epsilon_{\text{recoil}} = q_{\text{ext}} / \Lambda_\chi \sim 10^{-3} \sim \alpha / \pi \quad \epsilon_{\pi} = q_{\text{ext}} / m_\pi \sim 10^{-2}$$

- At the required precision ($\sim 10^{-4}$), need to keep terms of $\mathcal{O}(G_F \alpha)$, $\mathcal{O}(G_F \alpha \epsilon_\chi)$, $\mathcal{O}(G_F \epsilon_{\text{recoil}})$, along with leading logarithms (LL $\sim (\alpha \ln(\epsilon))^n$) and next-to-leading logarithms (NLL $\sim \alpha (\alpha_s \ln(\epsilon_W))^n$, $\alpha (\alpha \ln(\epsilon))^n$)

Neutron decay (I): decay rate and V_{ud}

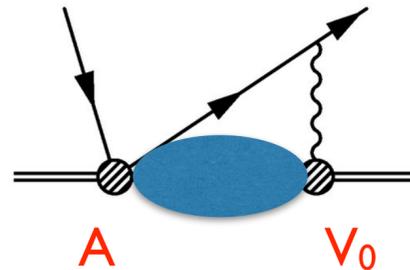
VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R), \quad \lambda = g_A/g_V$$

$$\Delta_f = 3.573(5)\%$$

$$\Delta_R = 4.044(24)_{\text{Had}}(8)_{\alpha\alpha_s^2}(7)_{\alpha\epsilon_\chi^2}(5)_{\mu_\chi} [27]_{\text{total}} \times 10^{-2}$$

Non-perturbative contribution
proportional to the γ - W 'box'
[Seng et al. 1807.10197]



Neutron decay (I): decay rate and V_{ud}

VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306.03138

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R), \quad \lambda = g_A/g_V$$

CORRECTION	COMPARISON with LITERATURE**	MAIN SOURCE of DISCREPANCY
$\Delta_f = 3.573(5)\%$	-0.035%	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; background-color: #e6f2ff; width: fit-content; margin: 10px auto;"> NR vs relativistic Fermi function $\alpha^2 \text{Log}(m_N/m_e)$ </div> <p style="color: blue; margin-top: 10px;">Both related to the treatment of NLL corrections in the hadronic EFT</p>
$\Delta_R = 4.044(27)\%$	+0.061%	
$\Delta_{\text{TOT}} = 7.761(27)\%$	+0.026%	

** As compiled in VC, A. Crivellin, M. Hoferichter, M. Moulson, 2208.11707. Non-perturbative input in Δ_R is the same

Overall shift of -0.013% in V_{ud} (neutron) compared to previous literature

$0^+ \rightarrow 0^+$ nuclear decay rate

- EFT-based decay rate formula reorganizes ‘traditional’ corrections in terms of ‘matching and running’ (e.g. $C_{\text{eff}}^{(gV)}$)

Courtesy of Wouter Dekens

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{2(G_F V_{ud})^2}{(2\pi)^5} W(E_e, \mathbf{p}_e, \mathbf{p}_\nu) \tilde{C}(E_e) \bar{F}(\beta, \mu) [1 + \tilde{\delta}'_R(E_e, \mu)] (1 - \bar{\delta}_C) [1 + \tilde{\delta}_{\text{NS}}(E_e, \mu)] [C_{\text{eff}}^{(gV)}(\mu)]^2$$

The diagram illustrates the EFT-based decay rate formula with various correction terms highlighted in colored boxes and ovals:

- Fermi function** (π^2, Z enhanced) is associated with $\tilde{C}(E_e)$ (grey oval).
- Isospin correction** is associated with $\bar{F}(\beta, \mu)$ (brown oval).
- Shape/atomic/recoil corrections** are associated with $W(E_e, \mathbf{p}_e, \mathbf{p}_\nu)$ (black box).
- Outer correction** ($\mathcal{O}(\alpha/\pi)$) is associated with $[1 + \tilde{\delta}'_R(E_e, \mu)]$ (orange oval).
- Isospin correction** is also associated with $(1 - \bar{\delta}_C)$ (green oval).
- Nuclear structure dependence** is associated with $[1 + \tilde{\delta}_{\text{NS}}(E_e, \mu)]$ (blue oval).
- RG/matching effects** ($\mu \gtrsim m_e$) are associated with $[C_{\text{eff}}^{(gV)}(\mu)]^2$ (purple oval).

- $\tilde{C}(E_e)$, $\tilde{\delta}_C$, $\tilde{\delta}_{\text{NS}}$ require nuclear structure input: good prospects of using ‘ab initio’ methods
- Significant new effect is in $\tilde{\delta}_{\text{NS}}$: short range potentials associated with currently unknown LECs

EFT for nuclear decays: impact on V_{ud}

- Exploratory study in $^{14}\text{O} \rightarrow ^{14}\text{N}$ decay (Quantum Monte Carlo calculation of relevant matrix element)

$$V_{ud} = 0.97364(12)_{g_V(10)_{\text{exp}}(22)_{\bar{f}}(13)_{\delta_{NS}^{\text{non-LEC}}(44)_{\delta_{NS}^{\text{LEC}}(12)_{\delta_c}[55]_{\text{total}}}$$

Residual scale dependence due to missing terms of $\mathcal{O}(\alpha^2 Z)$ in the Fermi function

Largest uncertainty. Assumes $g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)$
 LECs can be obtained by fitting data, once NME calculations for several isotopes become available.
 Dispersive methods and lattice QCD can also be useful

- To be compared with Towner-Hardy 2020 result (from $^{14}\text{O} \rightarrow ^{14}\text{N}$ decay alone):

$$V_{ud} = 0.97405(37)_{\text{total}}$$

(31) from δ_{NS}