Local-equilibrium transport of oscillating neutrinos

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Forward scattering on other particles → Matter effects (*e.g.*, MSW resonances)

Forward scattering on other neutrinos → Collective effects (*e.g.*, flavor instabilities)

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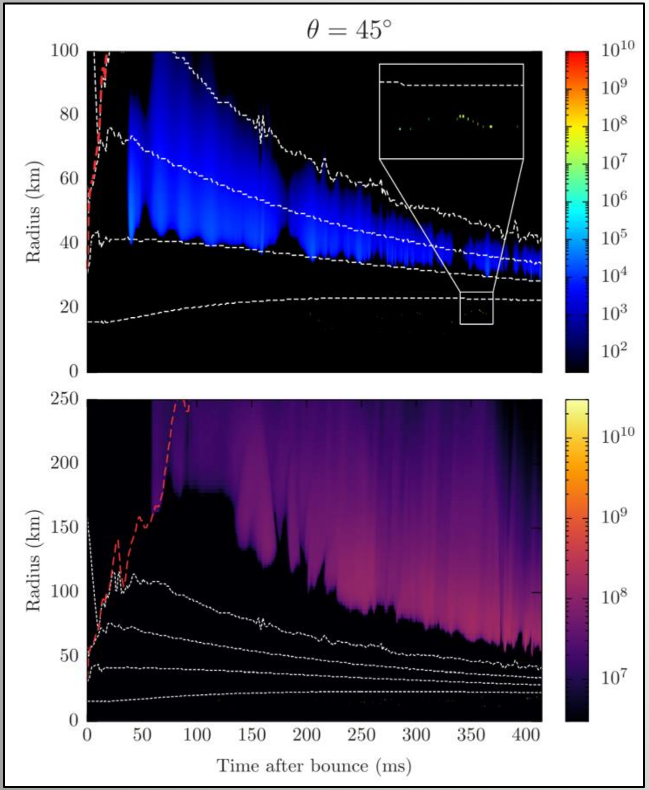
2D SN simulation



CFI growth rate (s^{-1})

FFI growth rate (s^{-1})

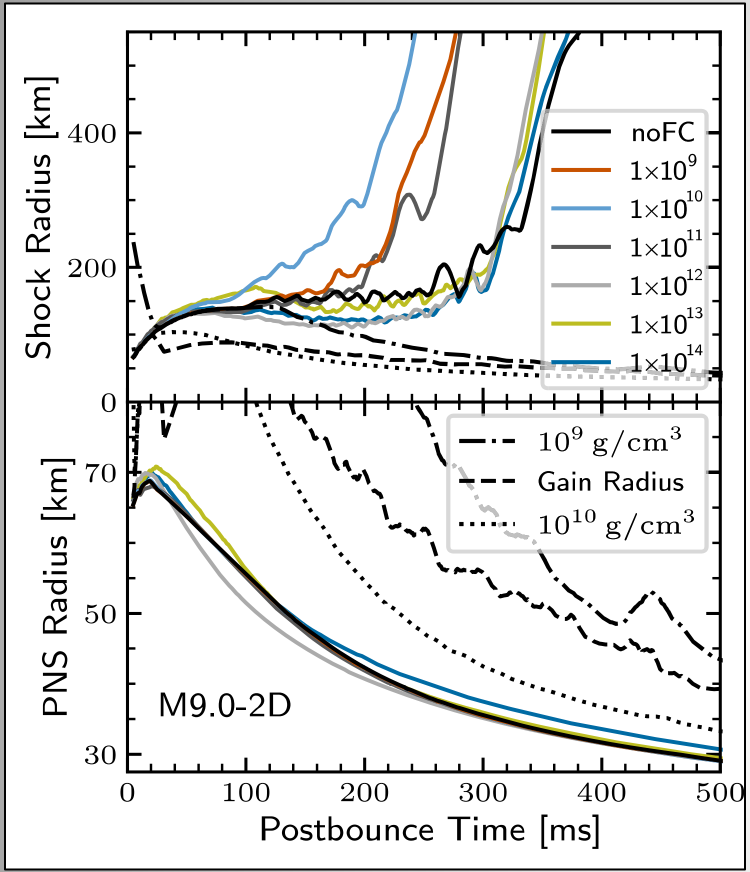
Akaho, Liu, Nagakura, Zaizen, & Yamada, PRD (2024)

Fast & collisional instabilities appear to be widespread in core-collapse supernovae & neutron star mergers.

Sawyer, PRL (2016) [FFI]
Johns, PRL (2023) [CFI]

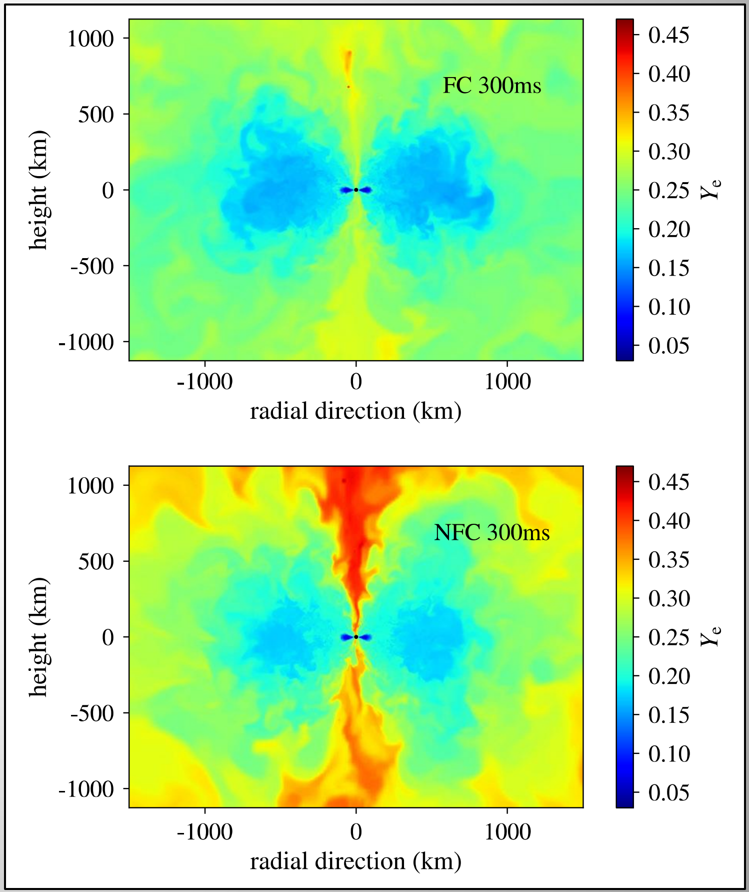
Flavor mixing can significantly change **SN explosion dynamics** (*left*) and **post-merger chemical evolution** (*below*).

Radii of the SN shock & protoneutron star



Ehring, Abbar, Janka, Raffelt, & Tamborra, PRL (2023)

Electron fraction in a post-merger accretion disk



Li & Siegel, PRL (2021)

Problem:
The quantum kinetic equation (QKE) is computationally intractable.

$$\underbrace{i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right)\rho}_{\textbf{Particle advection}} = \underbrace{[H, \rho]}_{\textbf{Flavor mixing}} + \underbrace{iC}_{\textbf{Collisions}}$$

Problem:
The quantum kinetic equation (QKE) is computationally intractable.

Proposed solution:
A coarse-grained transport theory based on local mixing equilibrium.
Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)
Johns, 2401.15247 (*Subgrid modeling of neutrino oscillations in astrophysics*)

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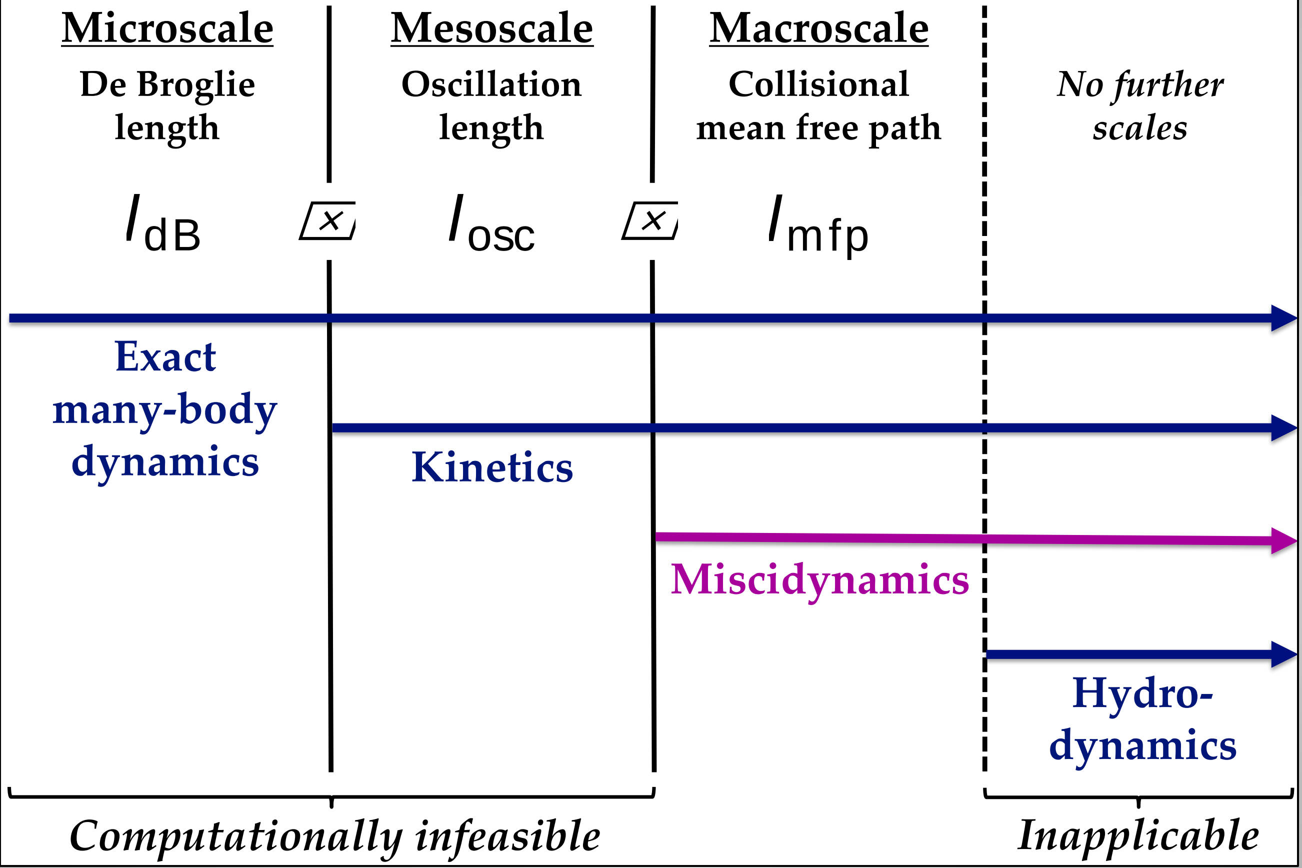
Vanishes in *mixing* eq

Vanishes in *collisional* eq

Length scales, coarse-grainings, & transport theories



Microscale

De Broglie length

Mesoscale

Oscillation length

Macroscale

Collisional mean free path

No further scales

$l_{dB} \boxed{\times} l_{osc} \boxed{\times} l_{mfp}$

Exact many-body dynamics

Kinetics

Miscidynamics

Hydro-dynamics

Computationally infeasible

Inapplicable

The miscidynamic equation

$$i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right) \rho^{\mathrm{eq}} = i C^{\mathrm{eq}}_{\mathrm{non}}$$

Evolution is driven by collisions & astrophysical gradients.

with $$C = \underbrace{C_{\mathrm{uni}}}_{\text{unitary}} + \underbrace{C_{\mathrm{non}}}_{\text{non-unitary}}$$

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What changes need to be made to current simulations?

(1) Distribution functions ⟶ Density matrices.

(2) Add off-diagonals to collision terms.

(3) Re-equilibrate ρ after each step.

Neutrino quantum thermodynamics might explain *why* flavor evolves near local mixing equilibrium.

$$S = -V \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \, \mathrm{Tr}\left[\rho_{\boldsymbol{p}} \log \rho_{\boldsymbol{p}} + (1-\rho_{\boldsymbol{p}}) \log(1-\rho_{\boldsymbol{p}})\right]$$

Fix total energy &
neutrino number
at each $\boldsymbol{p}$

$$\rho_{\boldsymbol{p}}^{\mathrm{eq}} = \frac{1}{e^{\beta(H_{\boldsymbol{p}}^{\mathrm{eq}} - \mu_{\boldsymbol{p}})} + 1}$$

First law of thermodynamics

$$\Delta U = W + Q$$

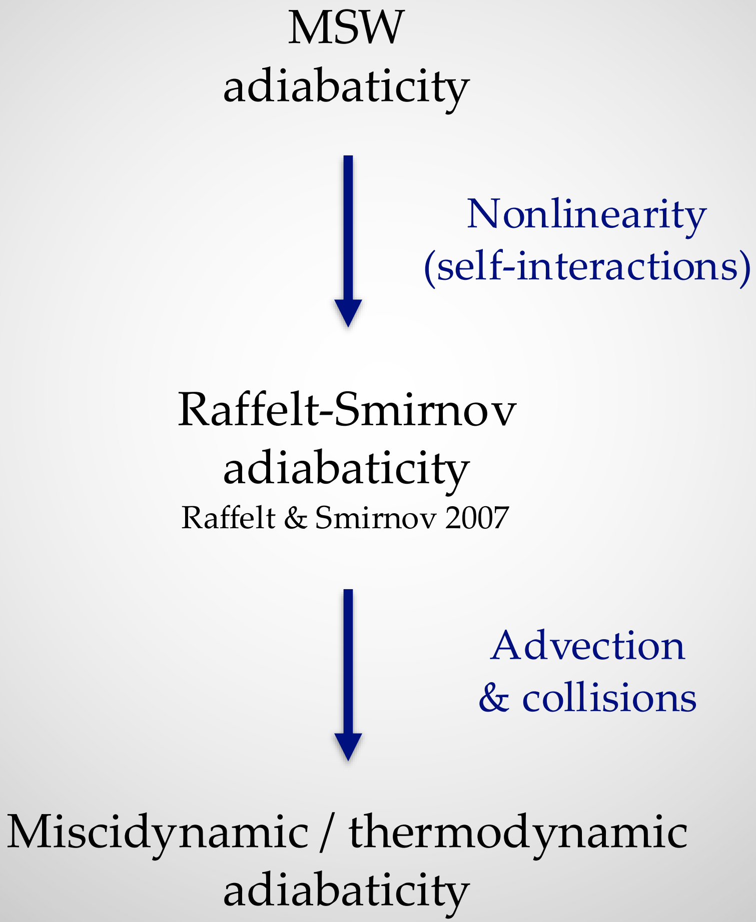
with W and Q appropriately defined.

$$\Delta U = \overbrace{\frac{1}{N_f} H_0 \Delta P_0 + \frac{1}{2}\vec{H}\cdot\Delta|\vec{P}|\hat{P}}^{\equiv Q^{\mathrm{env}}} + \overbrace{\frac{1}{2}|\vec{H}||\vec{P}|\Delta\left(\hat{H}\cdot\hat{P}\right)}^{\equiv Q^{\mathrm{kin}}}$$
$$+ \underbrace{\frac{1}{N_f}\Delta H_0 P_0 + \frac{1}{2}\Delta|\vec{H}||\vec{P}|\hat{H}\cdot\hat{P}}_{\equiv W}$$

$$\rho = \frac{1}{2}\left(P_0 + \vec{P}\cdot\vec{\sigma}\right)$$

From here it's easy to show that *quantum* adiabatic effects (*e.g.*, MSW, spectral swaps) are adiabatic processes in the *thermodynamic* sense as well.

Miscidynamics generalizes adiabatic quantum evolution.



MSW
adiabaticity

Nonlinearity
(self-interactions)

Raffelt-Smirnov
adiabaticity
Raffelt & Smirnov 2007

Advection
& collisions

Miscidynamic / thermodynamic
adiabaticity

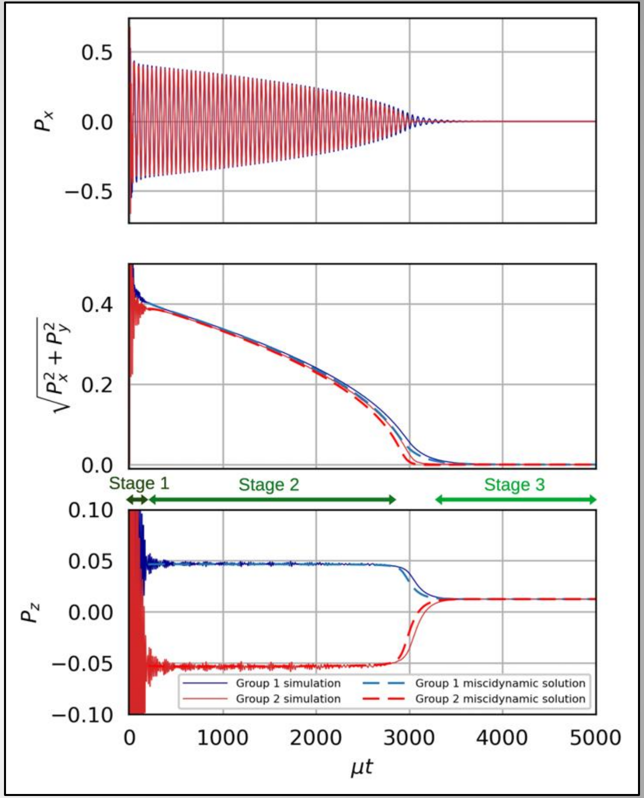
Successful applications of miscidynamics

- MSW conversion and spectral swaps

- Collisional flavor instabilities
  Johns & Rodriguez, 2312.10340
  (*Collisional flavor pendula and neutrino quantum thermodynamics*)

- Other forms of collisional relaxation (*right*)
  Kost, **Johns**, & Duan, 2402.05022
  (*Once-in-a-lifetime encounter models for neutrino media: I. From coherent oscillations to flavor equilibration*)
  Kost, **Johns**, & Duan, in preparation
  (*Once-in-a-lifetime encounter models for neutrino media: II. Self-driven adiabatic flavor relaxation*)

Neutral-current flavor relaxation



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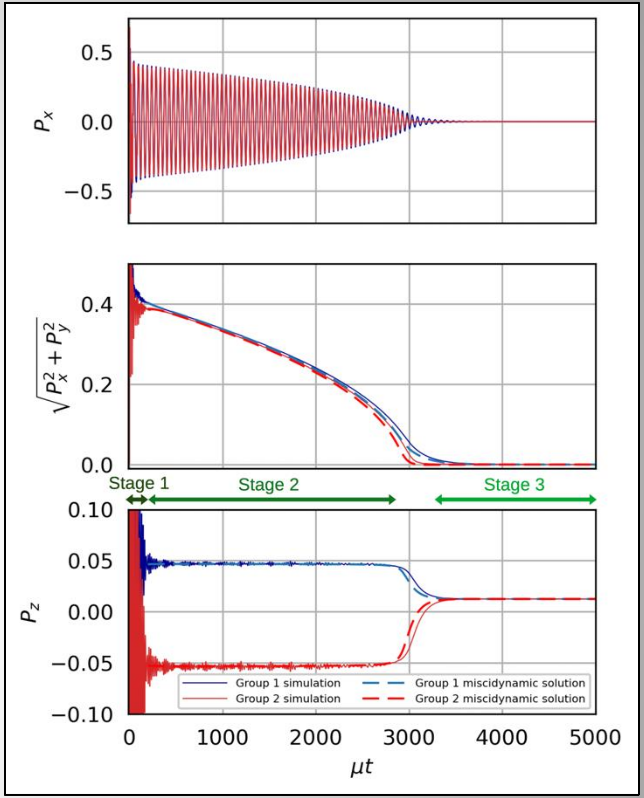
Neutral-current flavor relaxation



Application to **fast flavor conversion** will
require further development of the theory.

Current status of miscidynamics

- The **adiabatic theory** (this talk) is largely complete & simulation-ready.
- The **nonadiabatic theory** is still being developed.
- The nonadiabatic theory is definitely needed for some models, but it's unclear whether it's needed for astrophysics.

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