Neutrino quantum kinetics in core-collapse supernova and compact object mergers

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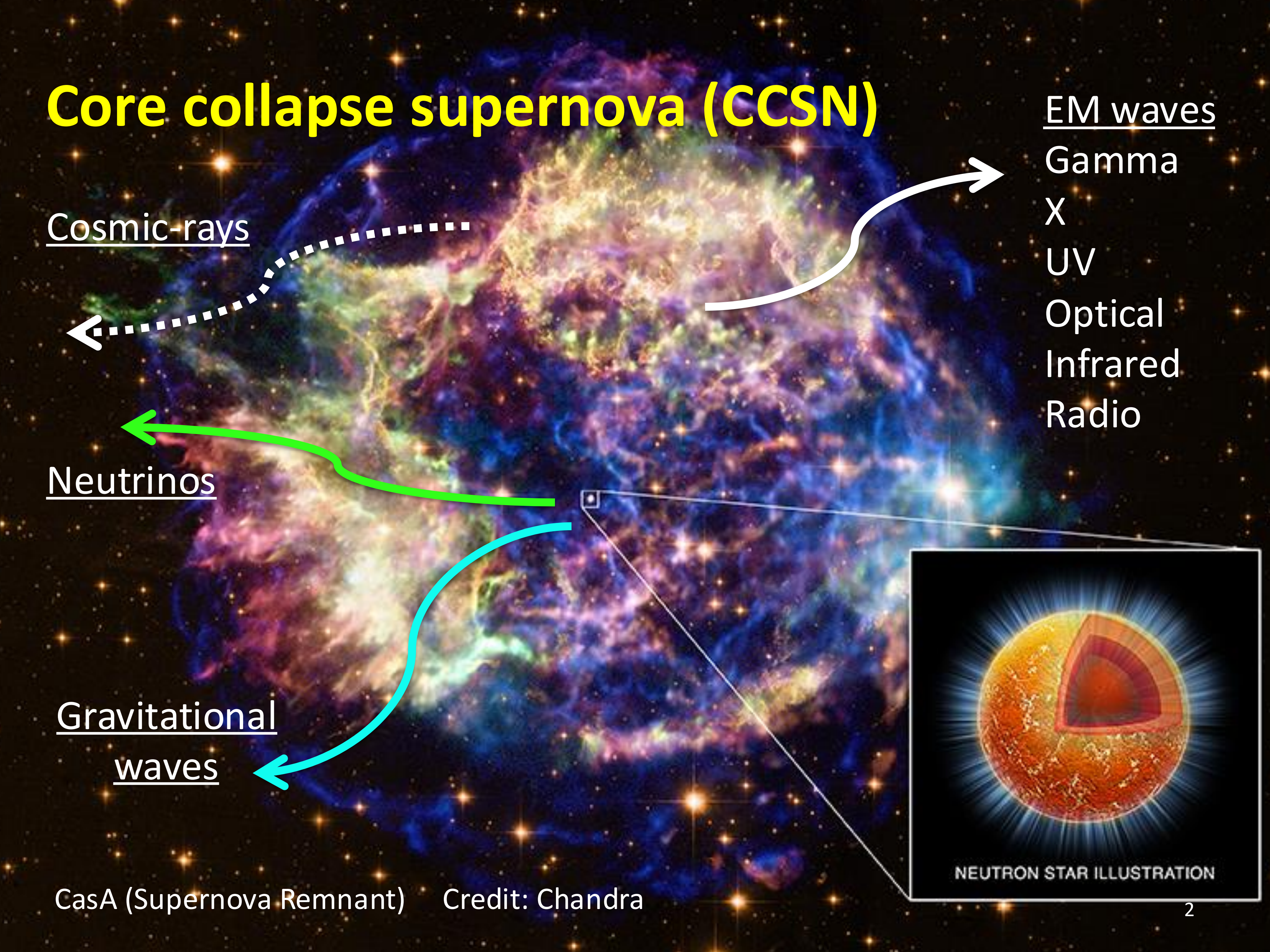
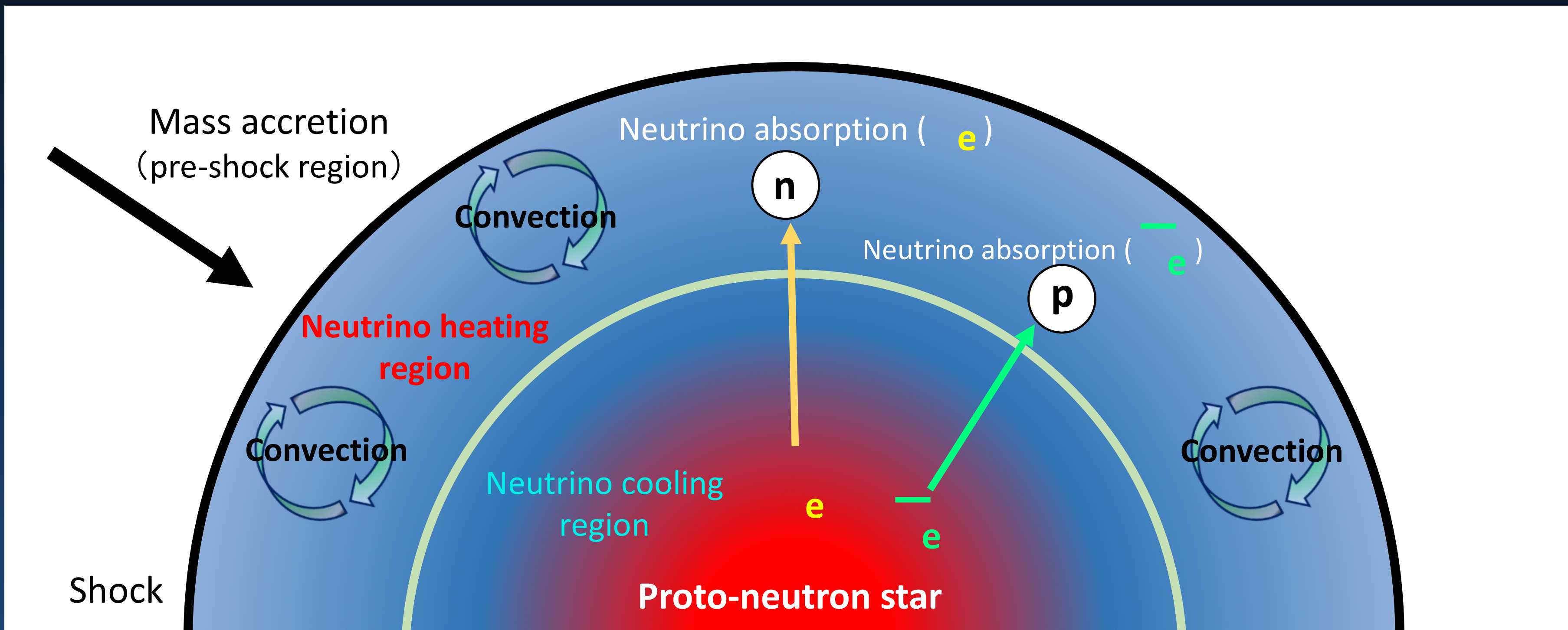
Core collapse supernova (CCSN)



CasA (Supernova Remnant) Credit: Chandra

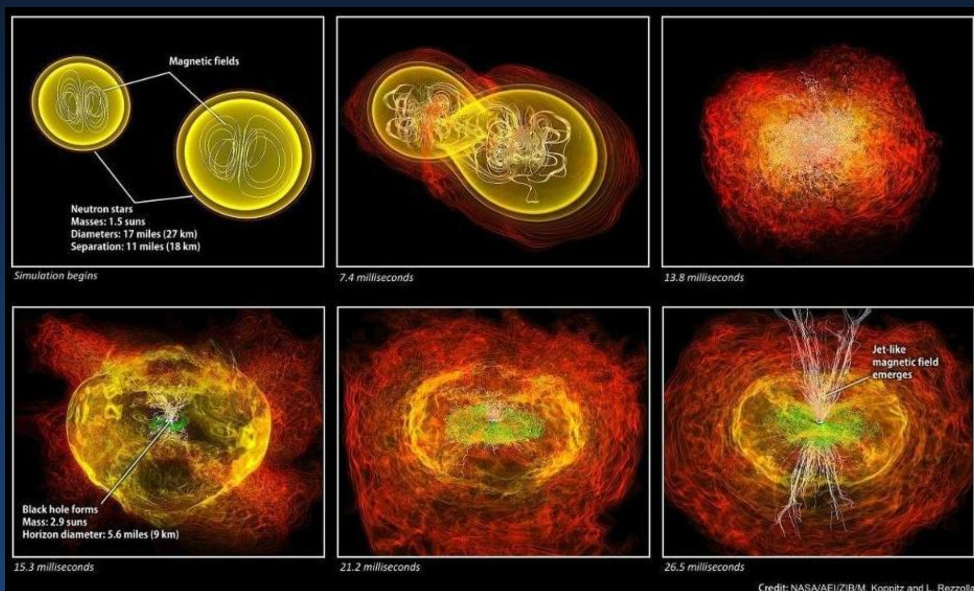
Neutrinos play a pivotal role on CCSN explosion



Multi-physics + Multi-dimensional (3D) problems

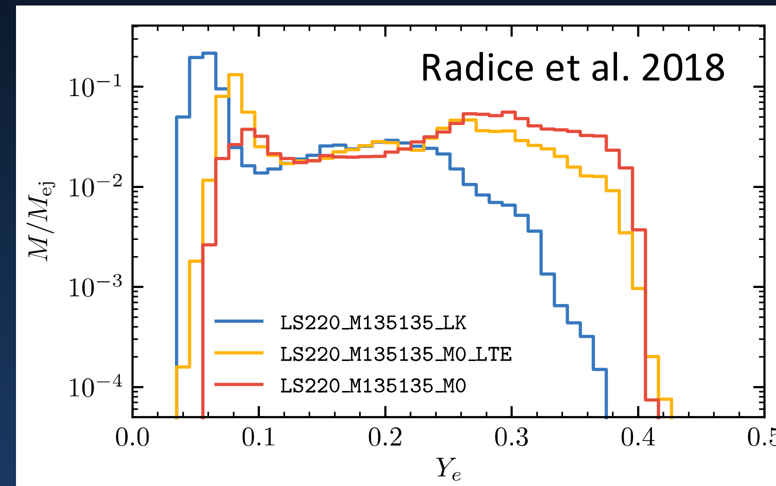
Numerical simulation is necessary to study the complex non-linear dynamics

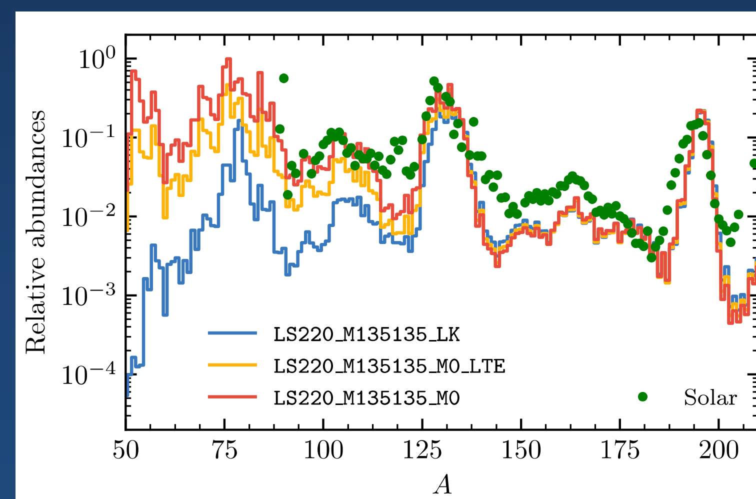
Binary neutron star merger (BNSM)



Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

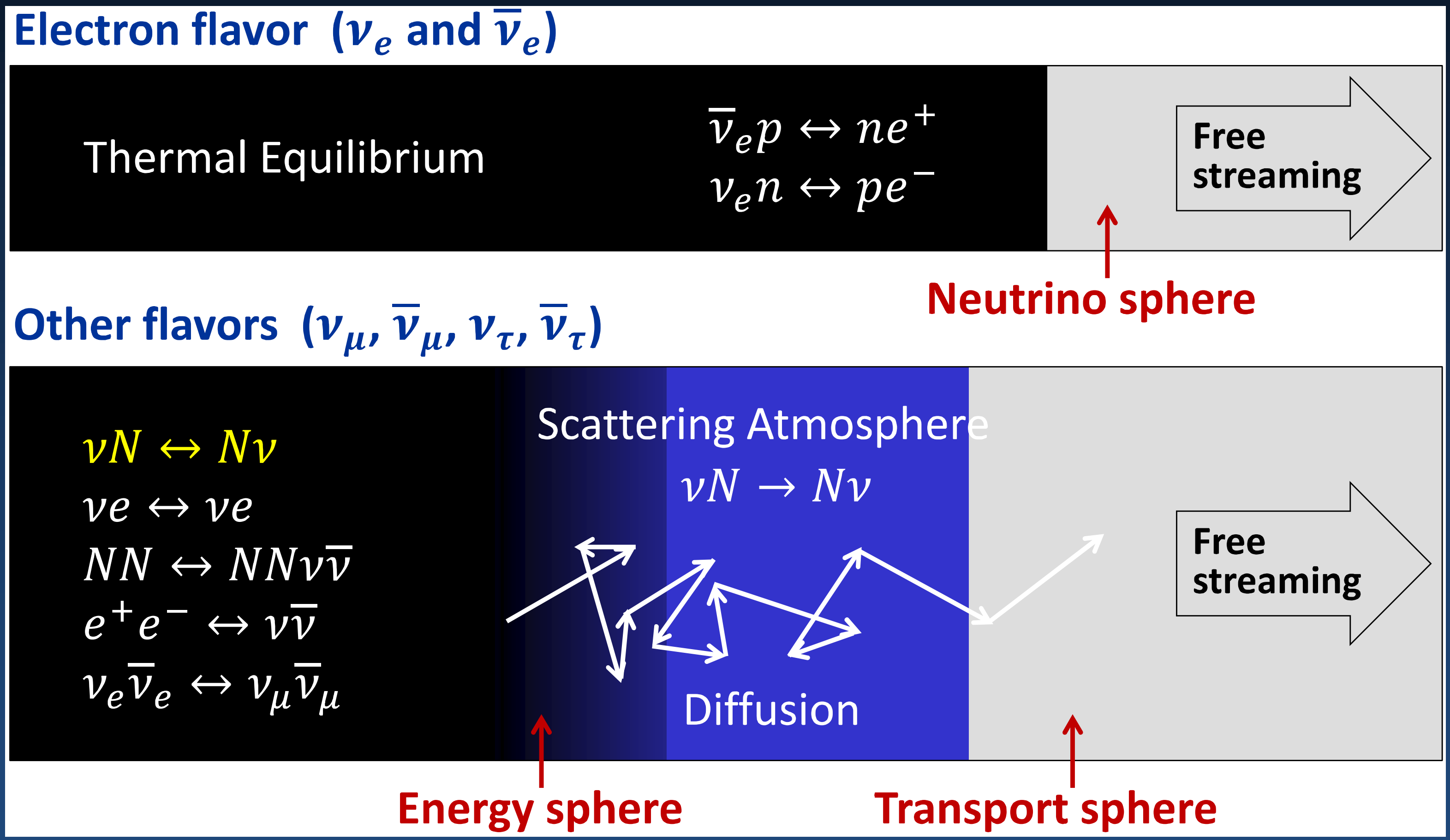
Lepton number transport by neutrinos is a key player to determine r-process nucleosynthesis.





Modeling of neutrino radiation field: necessitating a kinetic treatment

Figure by Janka 2017

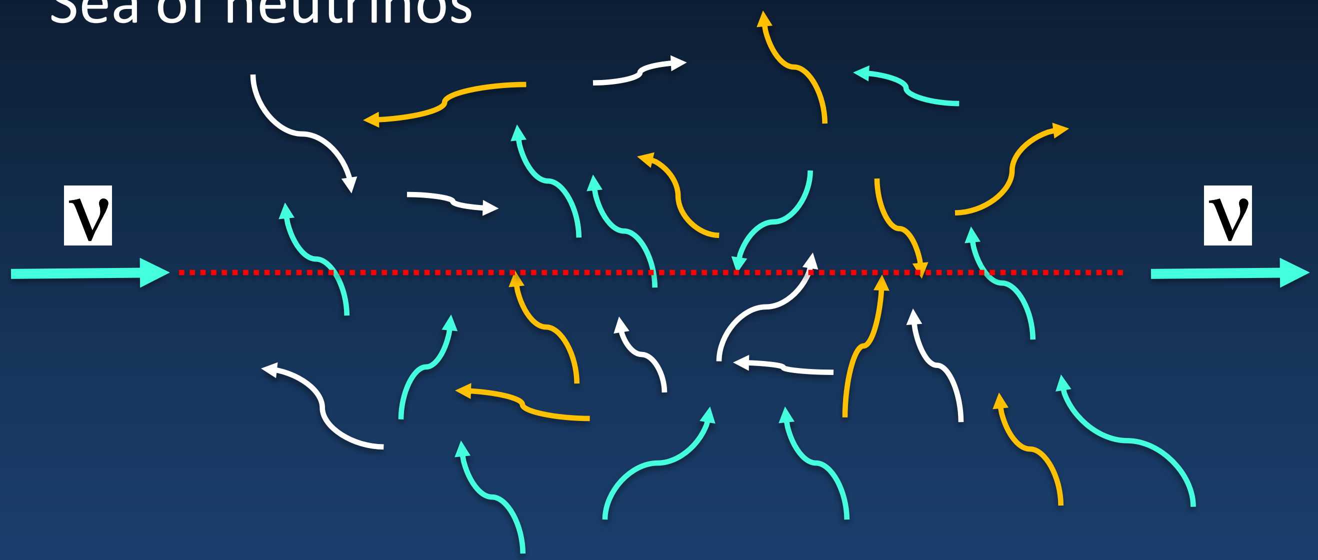


Optically thick ↔ Optically thin

Neutrino oscillation induced by self-interactions

Pantalone 1992

Sea of neutrinos



1. Refractions by self-interactions induce neutrino flavor conversions, which is analogy to matter effects (e.g., MSW resonance).

2. The oscillation timescale is much shorter than the global scale of CCSN/BNSM.

3. Collective neutrino oscillation induced by neutrino-self interactions commonly occurs in CCSNe and BNSM environments.

Quantum Kinetics neutrino transport:

(See also Christina Volpe's talk tomorrow)

Vlasenko et al. 2014, Volpe 2015, Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\mathrm{col}} + i p^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}],$$

Advection terms
(Same as Boltz eq.)

Collision term

Oscillation term

f is not a "distribution function"

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

$$\overset{(-)}{H} = \overset{(-)}{H}_{\mathrm{vac}} + \overset{(-)}{H}_{\mathrm{mat}} + \overset{(-)}{H}_{\nu\nu},$$

$$H_{\mathrm{vac}} = \frac{1}{2\nu} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger,$$

$$H_{\mathrm{mat}} = D \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} \left(1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}\right) \left(f(q') - \bar{f}^*(q')\right),$$

Rich flavor-conversion phenomena driven by neutrino-neutrino self-interactions

Vacuum:	$\omega = \frac{\Delta m^2}{2E_\nu},$
Matter:	$\lambda = \sqrt{2} G_F n_e,$
Self-int:	$\mu = \sqrt{2} G_F n_\nu,$

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional instability (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\,\Omega_{\pm} = \pm \frac{\Gamma - \bar{\Gamma}}{2} \frac{\mu S}{\sqrt{(\mu D)^2 + 4\omega\mu S}} - \frac{\Gamma + \bar{\Gamma}}{2}$$

Γ: Matter-interaction rate

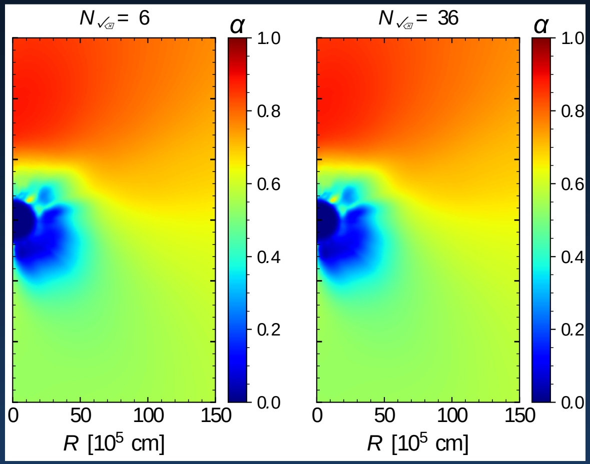
- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

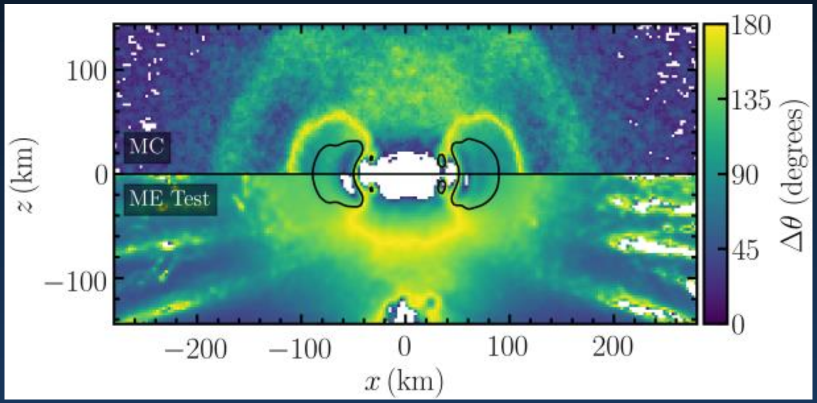
FFC occurs in both CCSN and BNSM

Core-collapse supernova

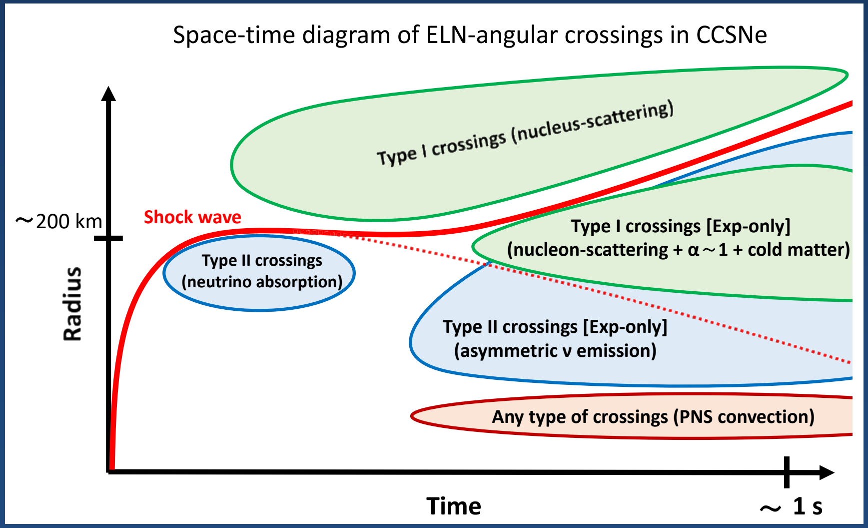


Abbar et al. 2018

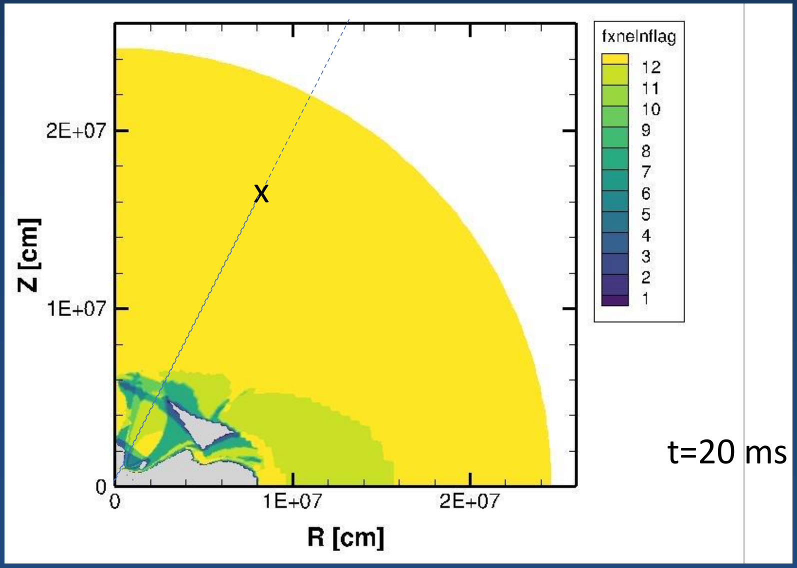
Binary neutron star merger



Richers 2022



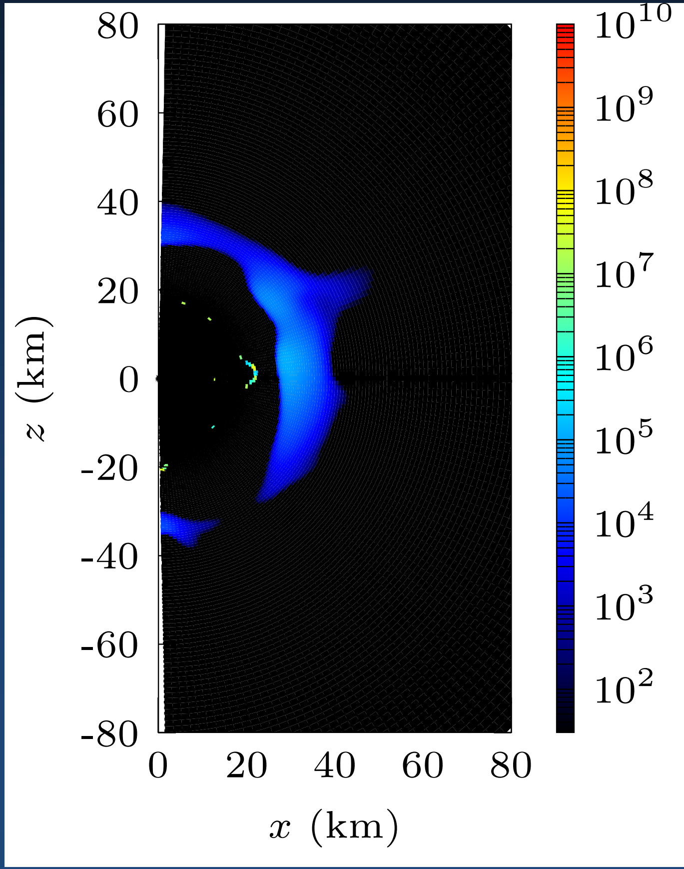
Nagakura et al. 2021



Sumiyoshi et al. in prep

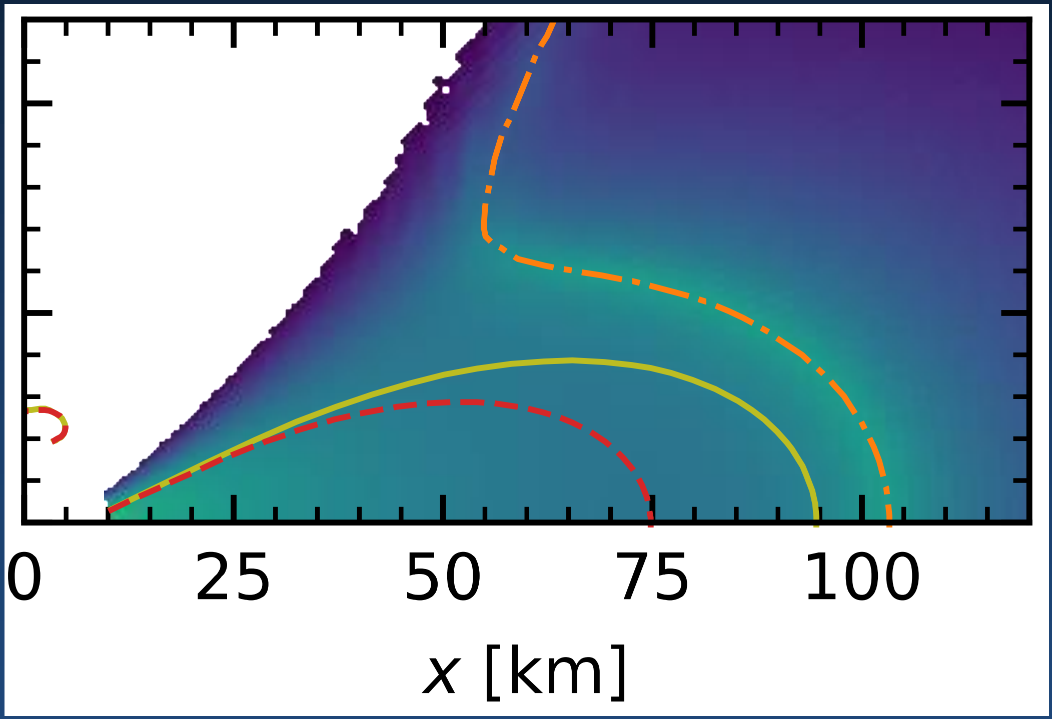
Collisional instability also occurs in both CCSN and BNSM

Core-collapse supernova



Akaho et al. 2023

Binary neutron star merger

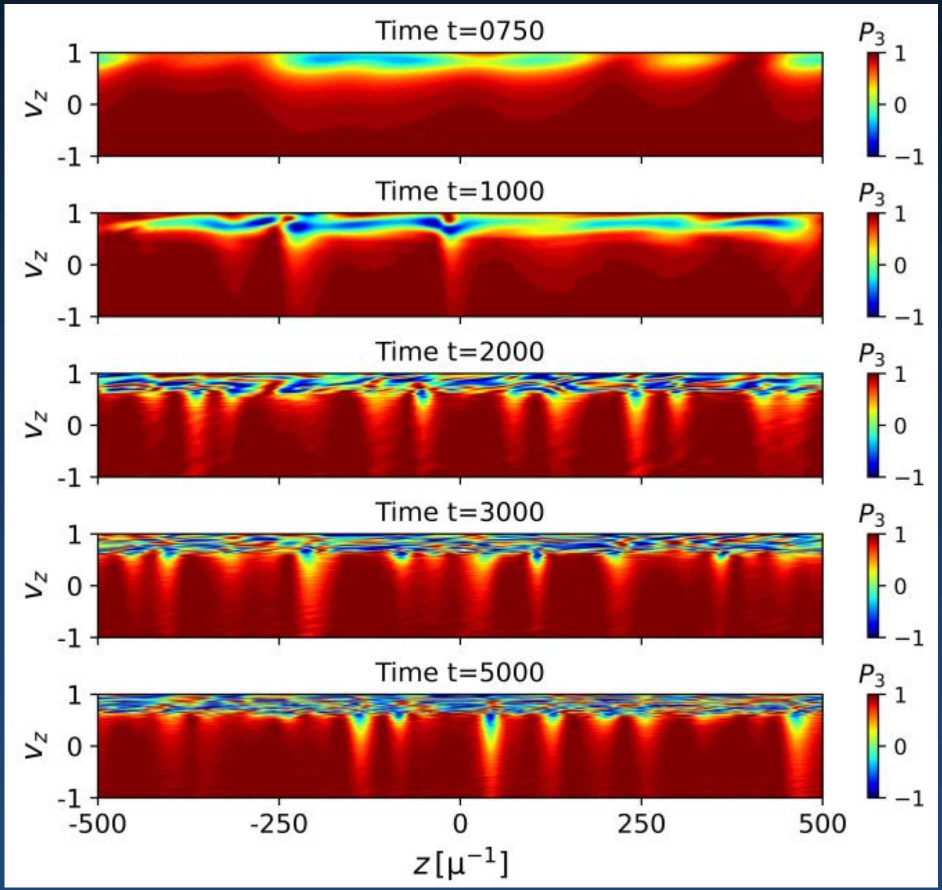


Xiong et al. 2022

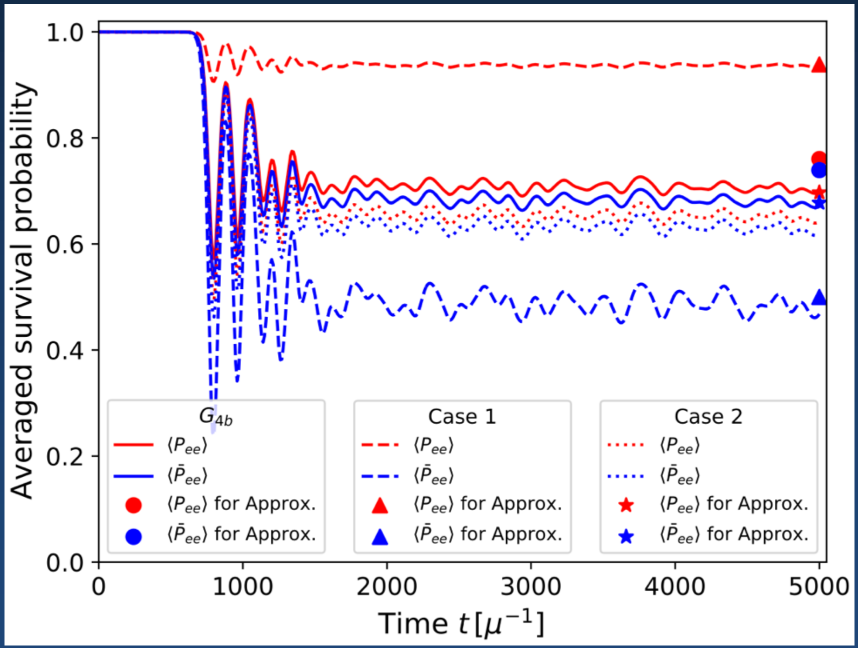
- Local simulations:

Asymptotic state FFC can be estimated ”analytically”

Conservation law of neutrinos
+
Stability condition
(disappearance of ELN-XLN crossings)



Zaizen and Nagakura 2022



But see Zaizen and Nagakura 2023, 204 for dependence of boundary conditions

- Global simulations:

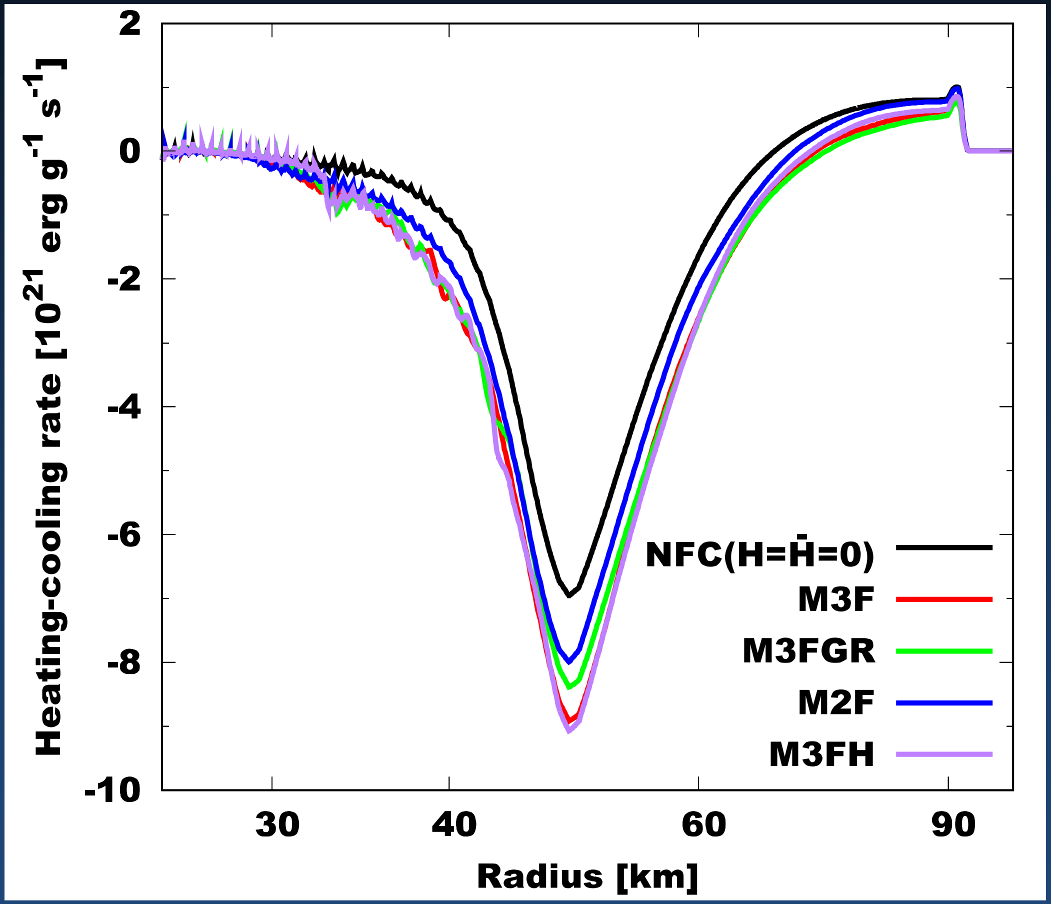
General-relativistic quantum-kinetic neutrino transport (GRQKNT)

Nagakura 2022

$$p^{\mu}\frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^{i}} = -p^{\mu}u_{\mu}\overset{(-)}{S}_{\rm col} + ip^{\mu}n_{\mu}[\overset{(-)}{H}, \overset{(-)}{f}],$$

- Fully general relativistic (3+1 formalism) neutrino transport
- Multi-Dimension (6-dimensional phase space)
- Neutrino matter interactions (emission, absorption, and scatterings)
- Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- 3 flavors + their anti-neutrinos
- Solving the equation with Sn method (explicit evolution: WENO-5th order)
- Hybrid OpenMP/MPI parallelization

- Global Simulations of FFC (in CCSN) Nagakura PRL 2023



Numerical setup:

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

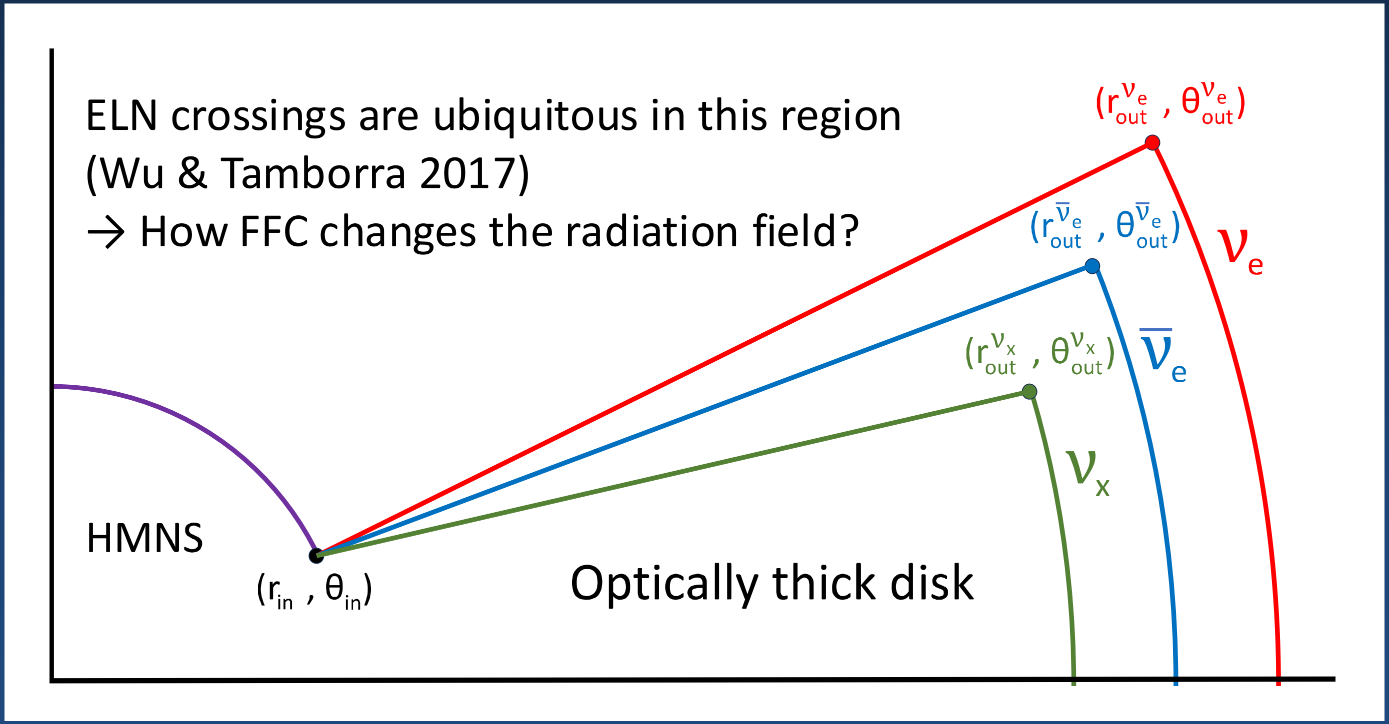
Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

→ Impacts on the explodability of CCSN

- Global simulations of FFC (in BNSM) Nagakura 2023

- Setup:

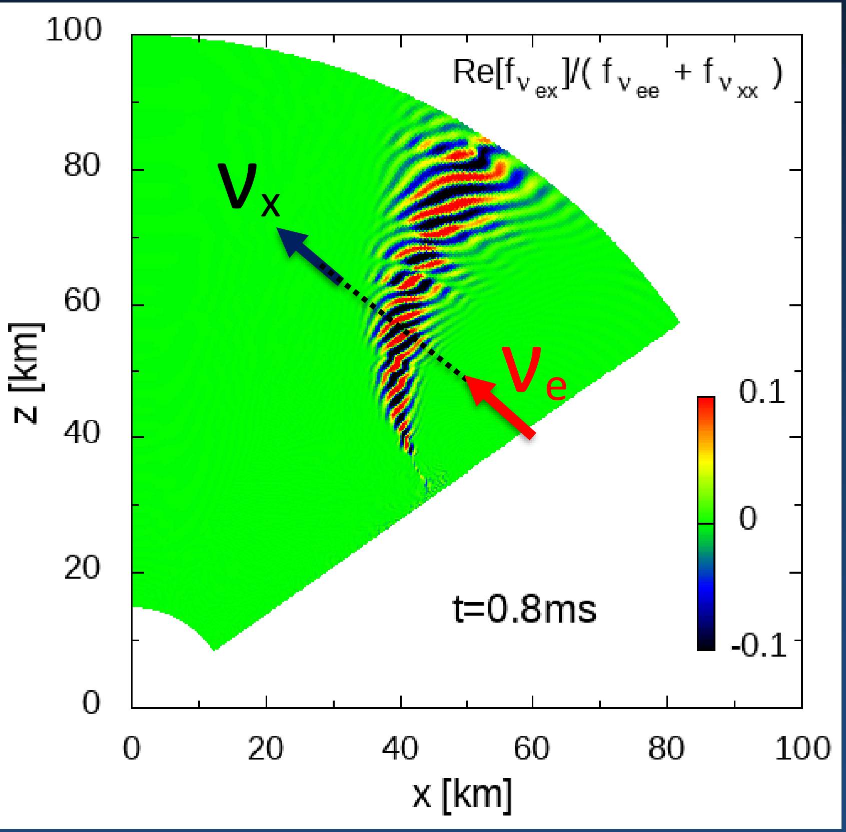
  - Hypermassive neutron star (HMNS) + disk geometry
  - Thermal emission on the neutrino sphere
  - QKE (FFC) simulations in axisymmetry
  - Resolutions: 1152 (r) × 384 (θ) × 98 ($θ_ν$) × 48 ($ϕ_ν$)



- Global simulations of FFC (in BNSM)

- Appearance of <u>flavor swap and EXZS</u>:

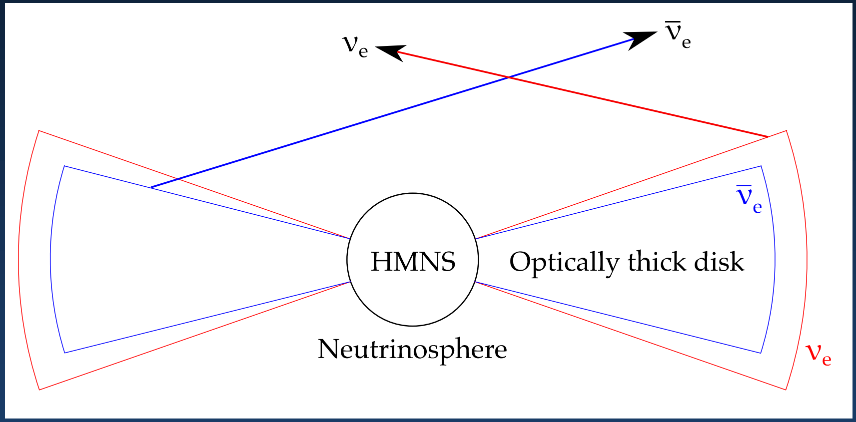
Flavor coherency



Nagakura 2023

Colliding-beam model

Zaizen and Nagakura 2024



$$\partial_{t'}^2 P_3 \sim -4\mu^2 \left(1 - (P_3)^2\right)$$

Neutrino flavor swaps are inevitable from the perspective of stability .

- BGK Subgrid model Nagakura et al. 2024.

See also John's talk:
thermodynamics of neutrino oscillations

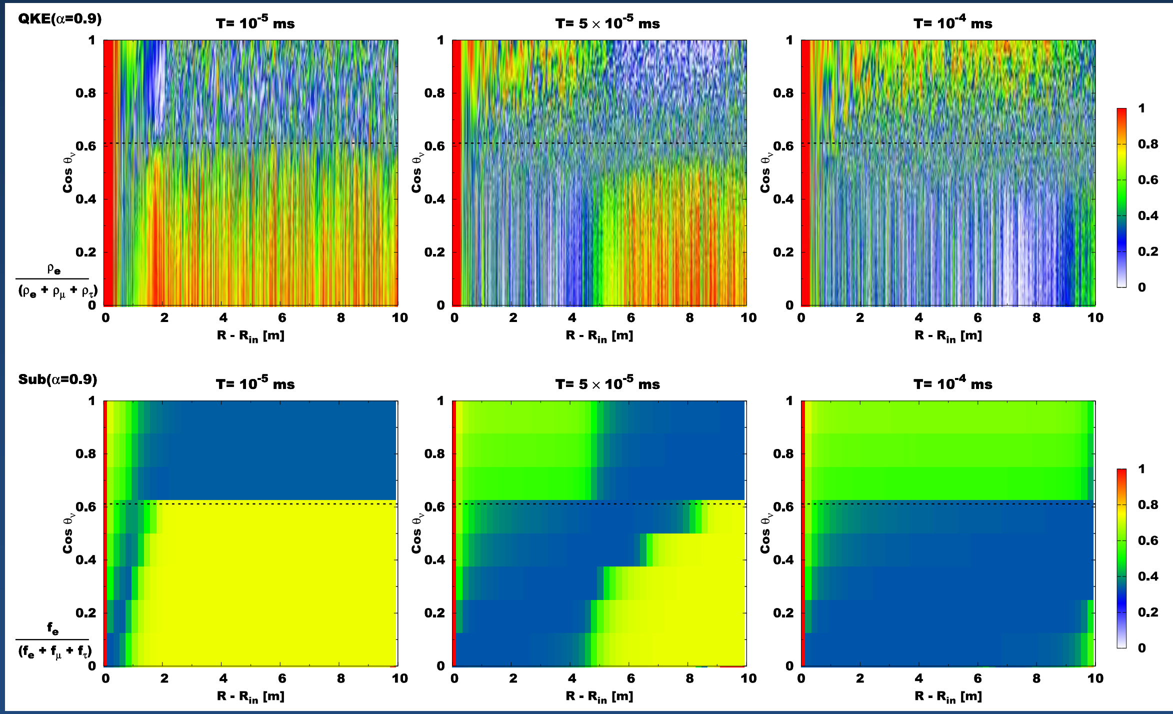
$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i p^\mu n_\mu [H, f]$$

: Full QKE

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$

: Relaxation-time approximation

Radial-angular distributions for survival probability of electron-type neutrinos



Summary

- Radiation-hydrodynamic simulations under classical treatments of neutrino kinetics have been matured in CCSN and BNSM community.
- Collective neutrino oscillations, one of the quantum kinetics features of neutrinos, ubiquitously occur in CCSN and BNSM environments.
- Fast neutrino-flavor conversion (FFC) and collisional flavor instabilities potentially gives a radical impact on fluid-dynamics, nucleosynthesis, and neutrino signal.
- We developed a new GRQKNT code for time-dependent global simulations of neutrino quantum kinetics (QKE).
- Global simulations are currently available, which shows qualitatively different features of flavor conversions from those found in local simulations.

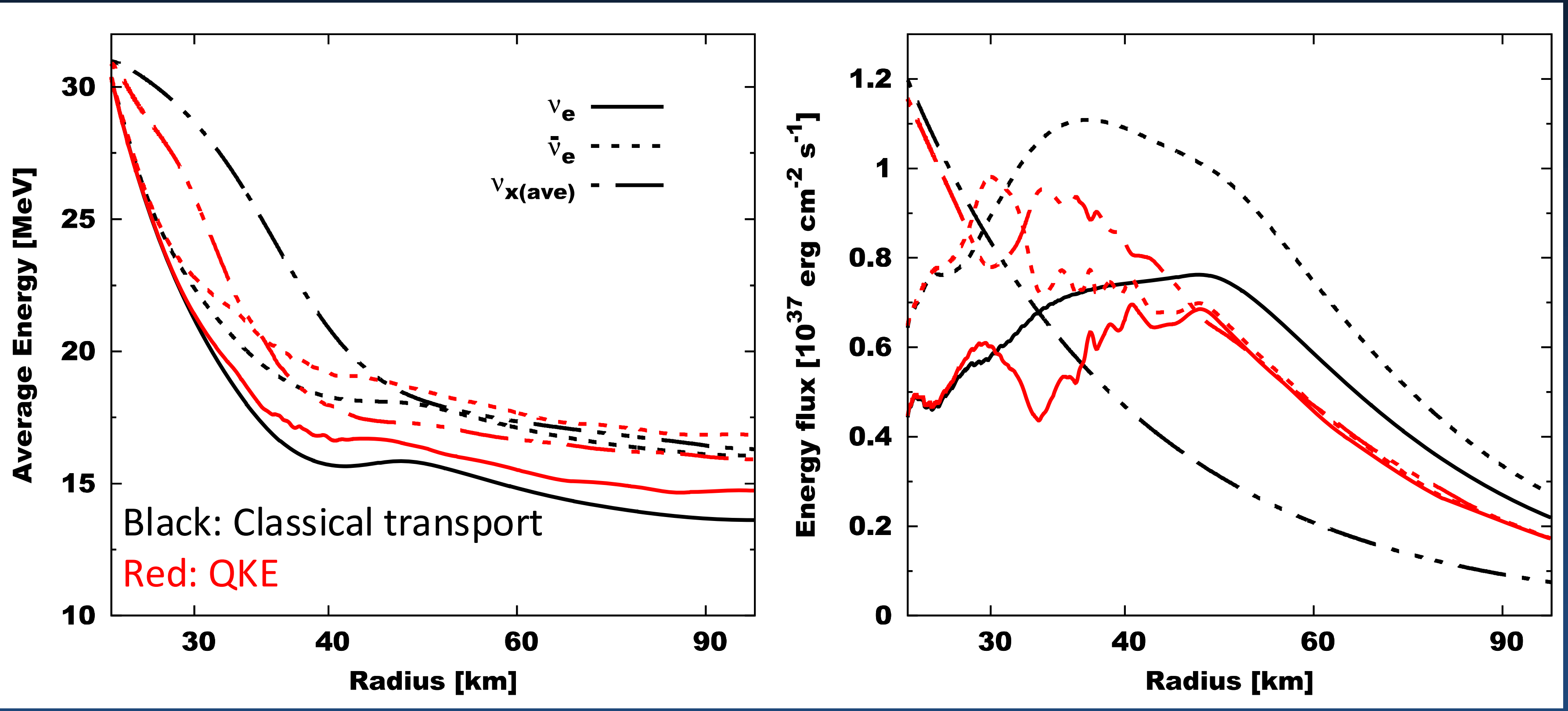
Backup

- Global Simulations of FFC (in CCSN)

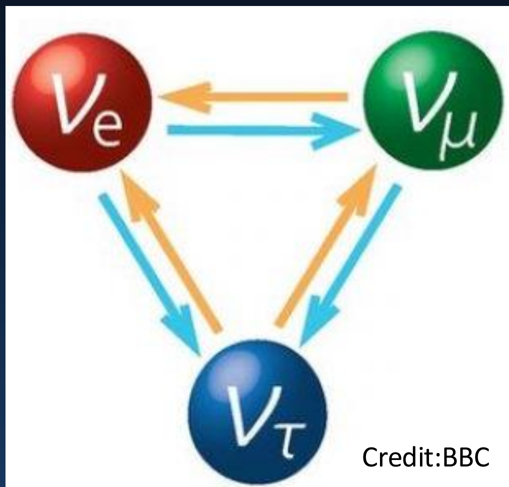
Nagakura PRL 2023

Average energy

Energy flux



Neutrino oscillations



- There are many experimental evidences that neutrinos can go through flavor conversion.
- Neutrinos have at least three different masses.
- Flavor eigenstates are different from mass eigenstates.

Mass state $|\nu_i\rangle = \sum_\alpha U^*_{\alpha i}\,|\nu_\alpha\rangle\,,$

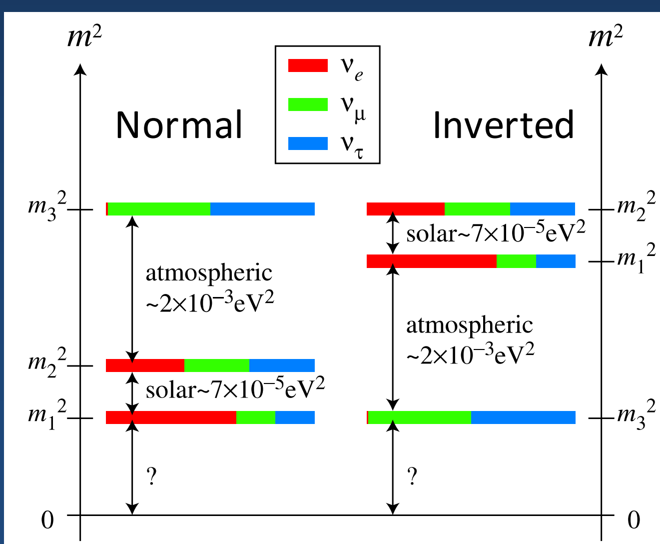
Flavor state $|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,,$

U: Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix)

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

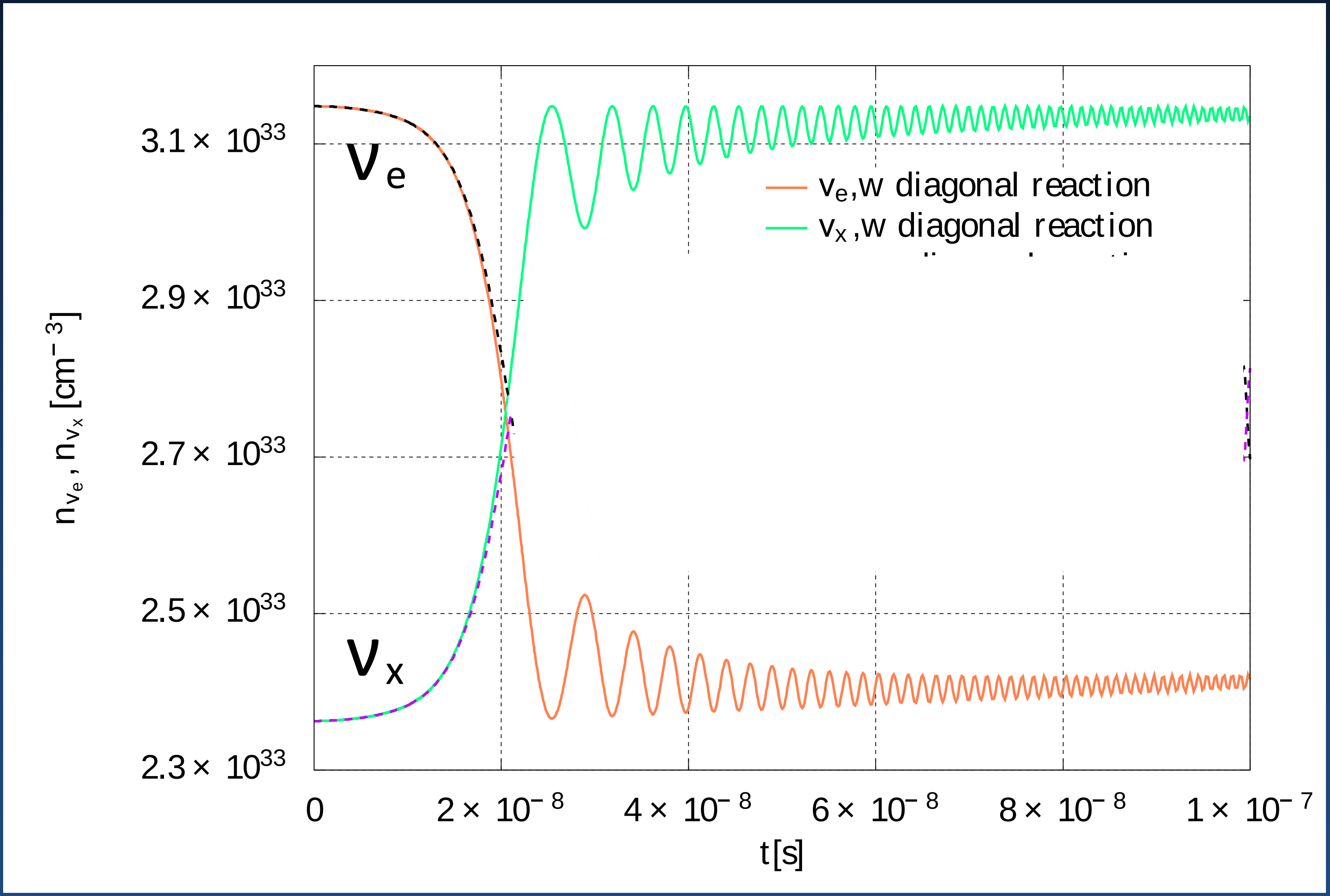
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$



Feruglio et al. 2003

Collisional flavor swap
(associated with resonance-like collisional instability)

Kato, Nagakura, and Johns 2024.



Time-dependent global simulations of FFC

Nagakura and Zaizen PRL 2022, PRD 2023

- Issue:

$$\ell_{n_\nu} \equiv c T_{n_\nu} = 0.235 \text{ cm} \left(\frac{L_\nu}{4 \times 10^{52} \text{erg/s}} \right)^{-1} \left(\frac{E_{\text{ave}}}{12\text{MeV}} \right) \left(\frac{R}{50\text{km}} \right)^2 \left(\frac{\kappa}{1/3} \right)$$

Oscillation wavelength is an order of sub-centimeter.

Too short !!!!

How can we make FFC simulations tractable???

- Strategy:

$$\frac{\partial \overset{(-)}{f}}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cos\theta_\nu \overset{(-)}{f}) - \frac{1}{r \sin\theta_\nu} \frac{\partial}{\partial \theta_\nu} (\sin^2\theta_\nu \overset{(-)}{f}) = -i\,\xi\, [\overset{(-)}{H}, \overset{(-)}{f}],$$

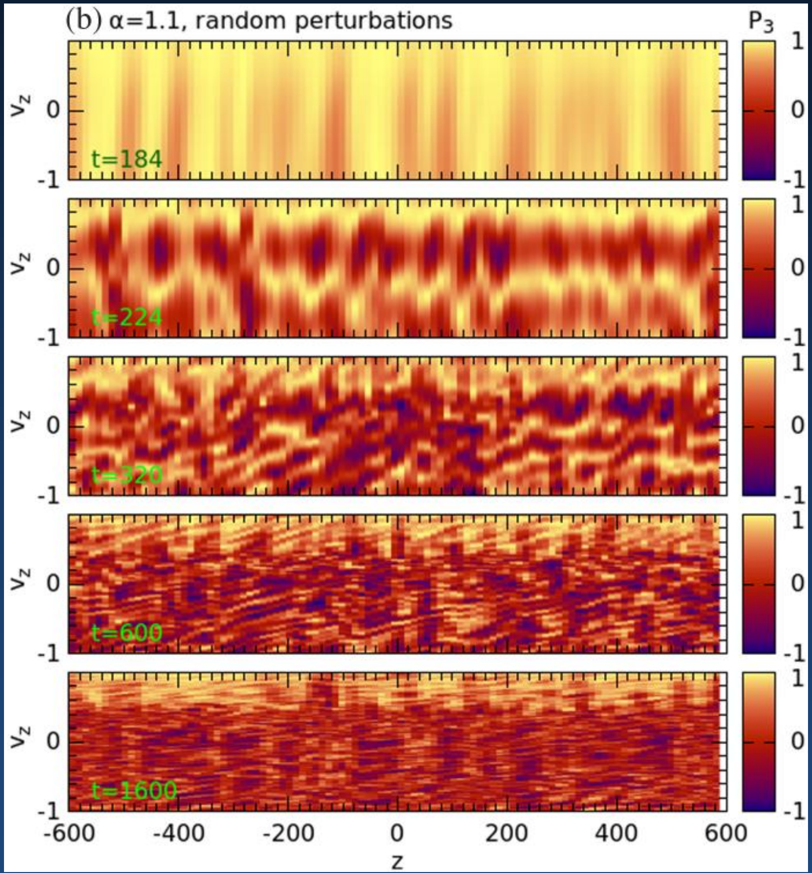
Attenuation parameter ($0 \leqq \xi \leqq 1$)

- Attenuating Hamiltonian makes global QKE simulations tractable.
- Realistic features can be extracted by a convergence study of ξ ($\rightarrow$ 1).

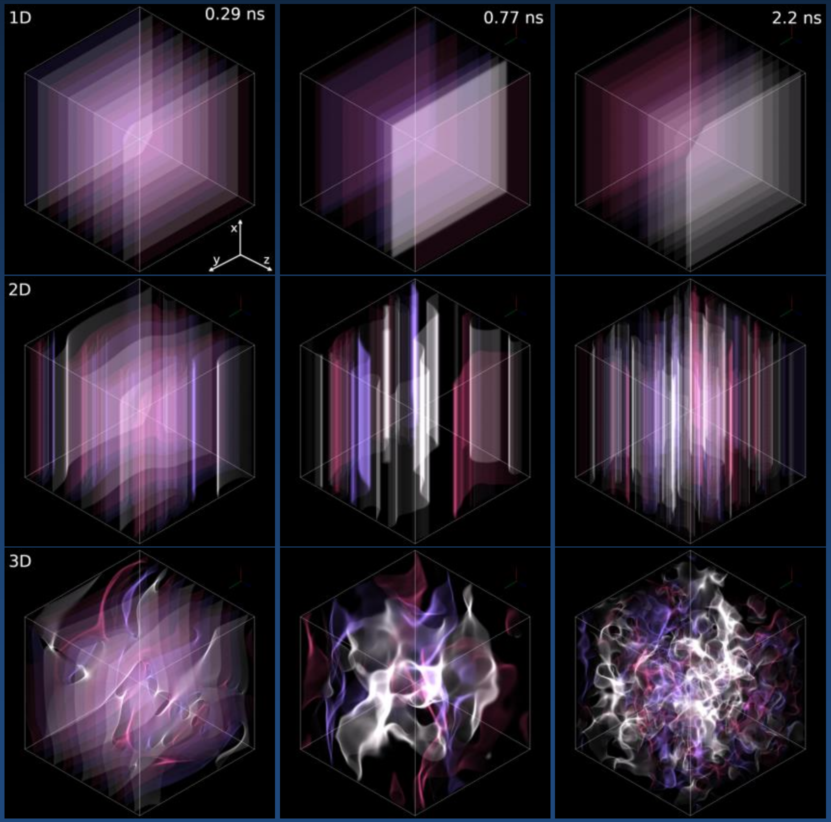
- Local simulations:

$$\frac{\partial f_{ab}}{\partial t} + c\mathbf{\Omega} \cdot \nabla f_{ab} = \mathcal{C}_{ab} - \frac{i}{\hbar}\left[\mathcal{H}, f\right]_{ab}$$

Advection terms (flat + cartesian-coordinate)

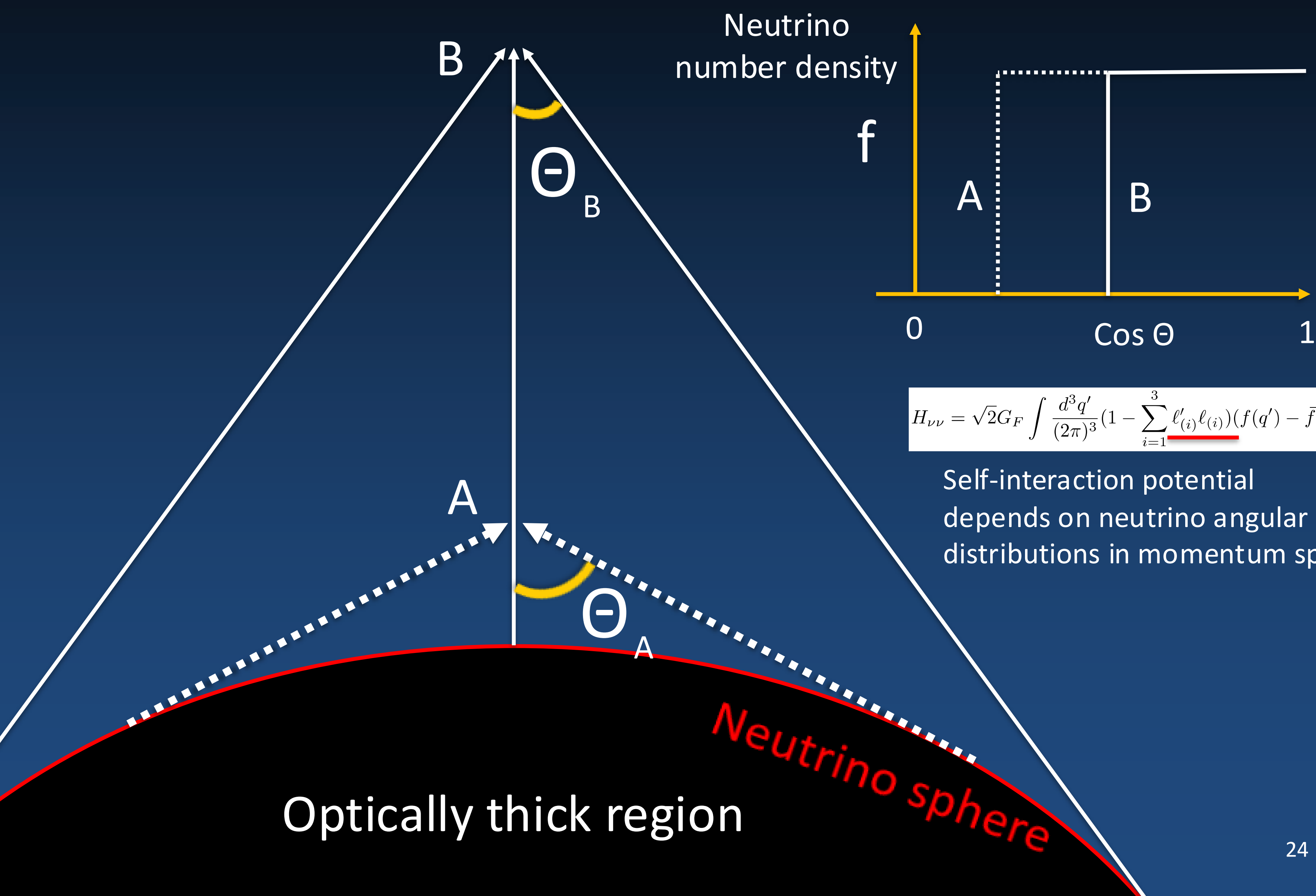


Wu et al. 2021



Richers et al. 2021

- Need of global simulations in the study of flavor conversions in CCSN/BNSM

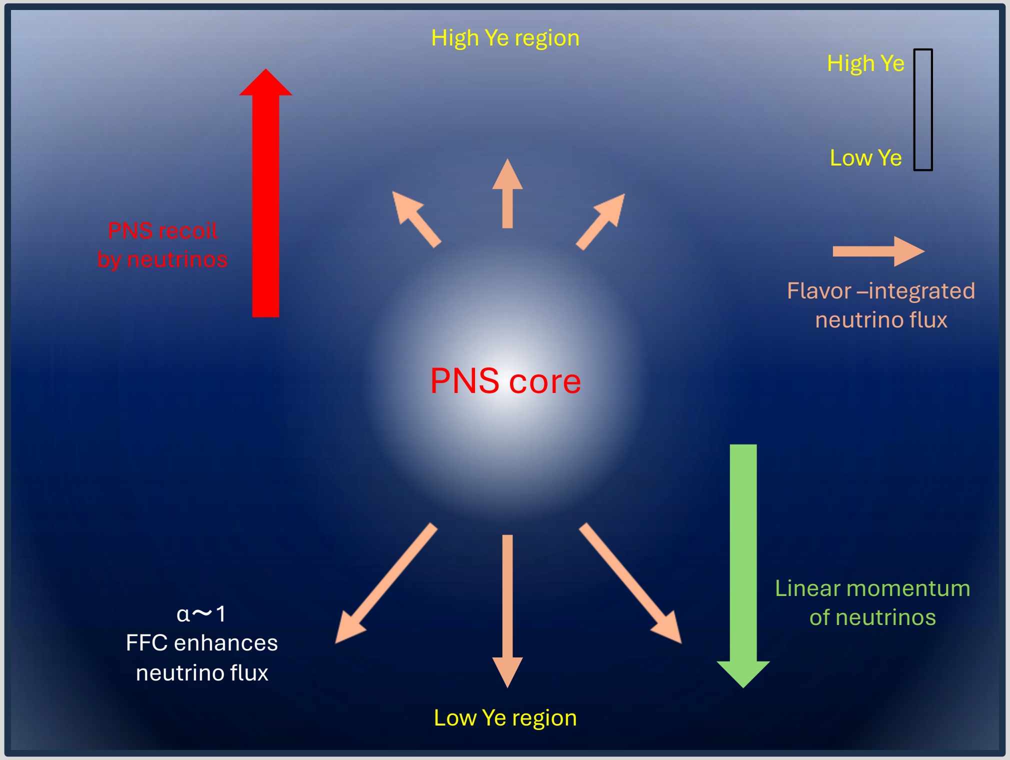


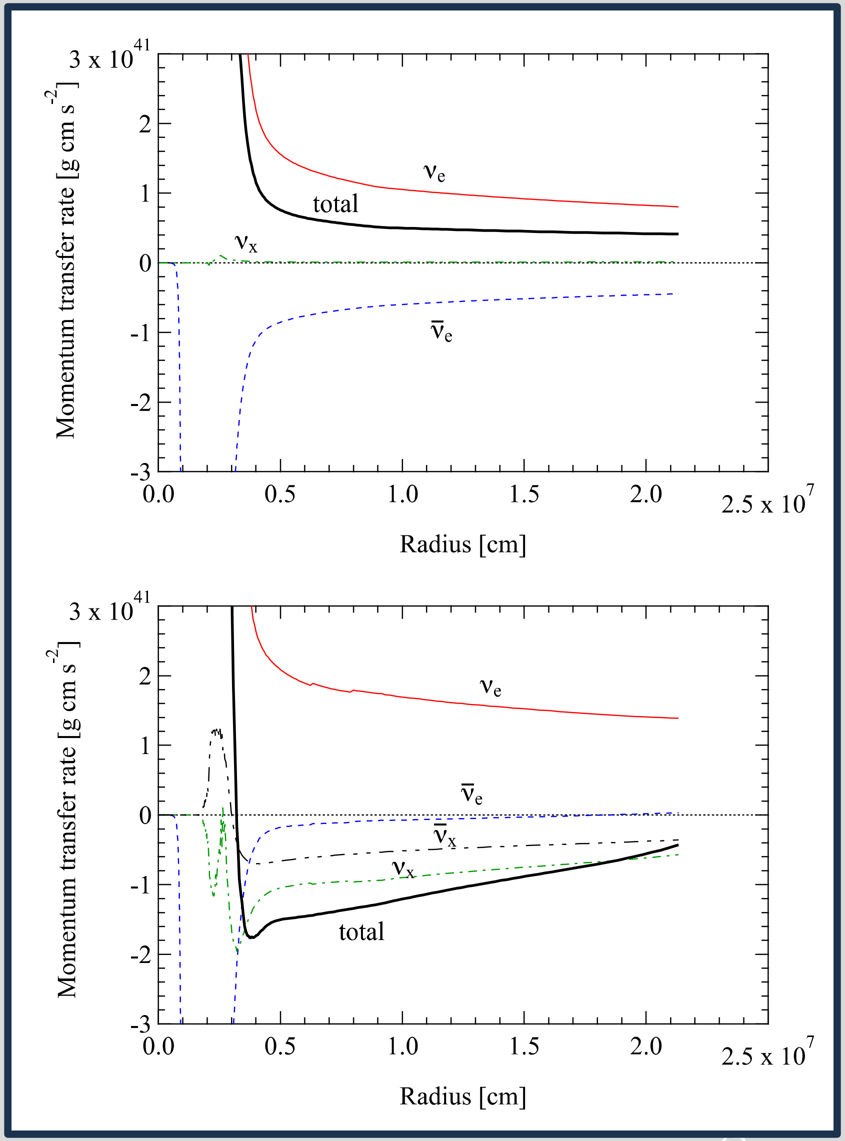
$$H_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3q'}{(2\pi)^3}\left(1-\sum_{i=1}^{3} \ell'_{(i)}\ell_{(i)}\right)\left(f(q')-\bar{f}^*(q')\right),$$

Self-interaction potential depends on neutrino angular distributions in momentum space

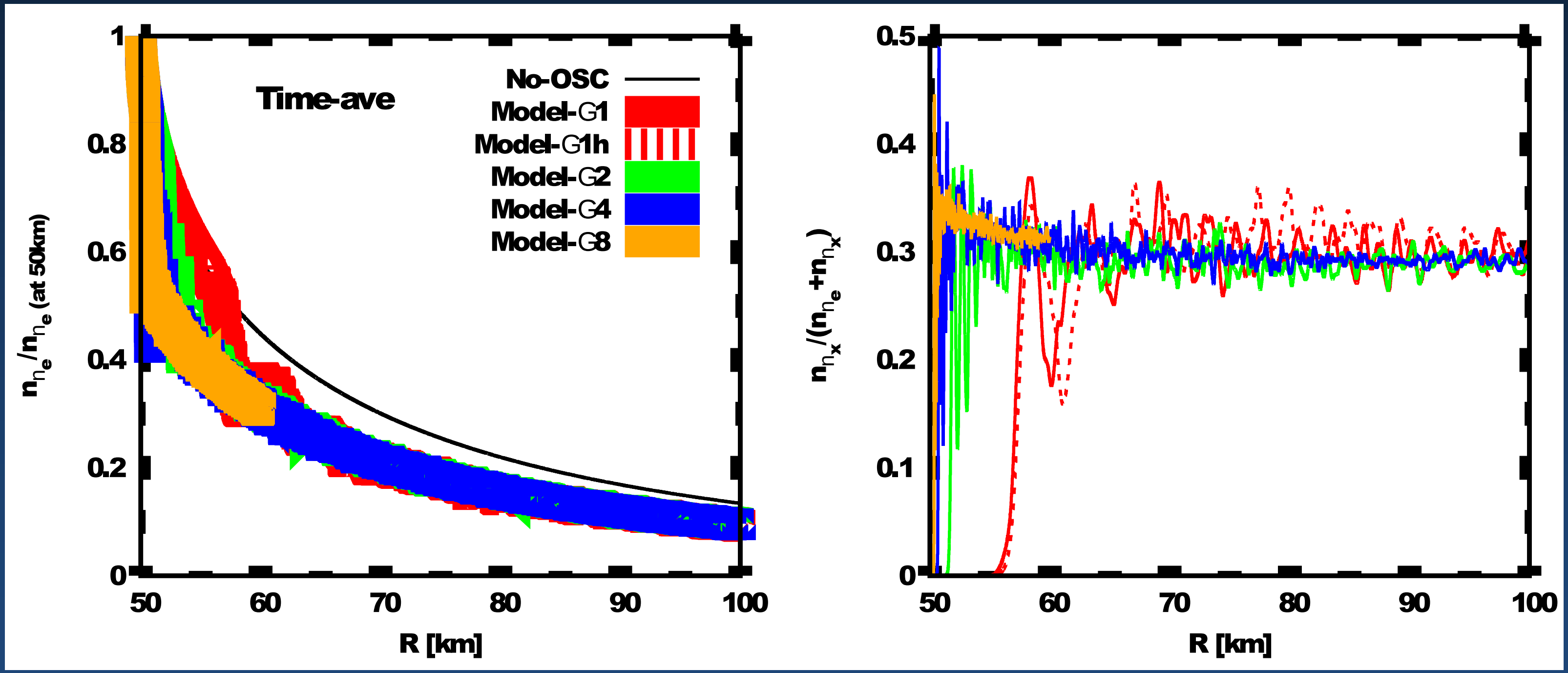
- Neutron star kick powered by neutrino flavor conversions

Nagakura and Sumiyoshi: arXiv:2401.15180



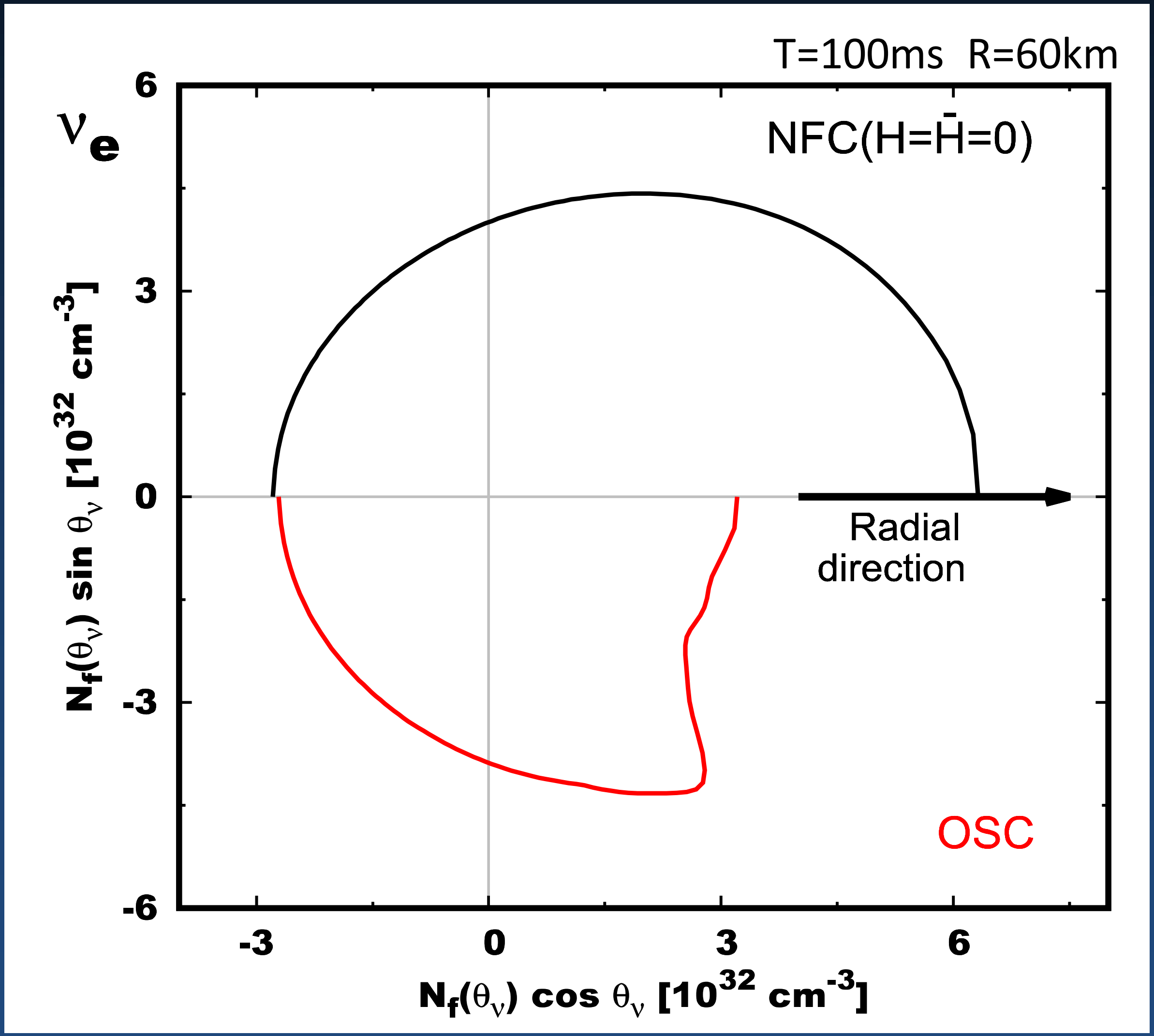


Attenuating Hamiltonian potential does not change the degree of flavor conversion in asymptotic states.



- Global Simulations of FFC (in CCSN)

Neutrino angular distributions

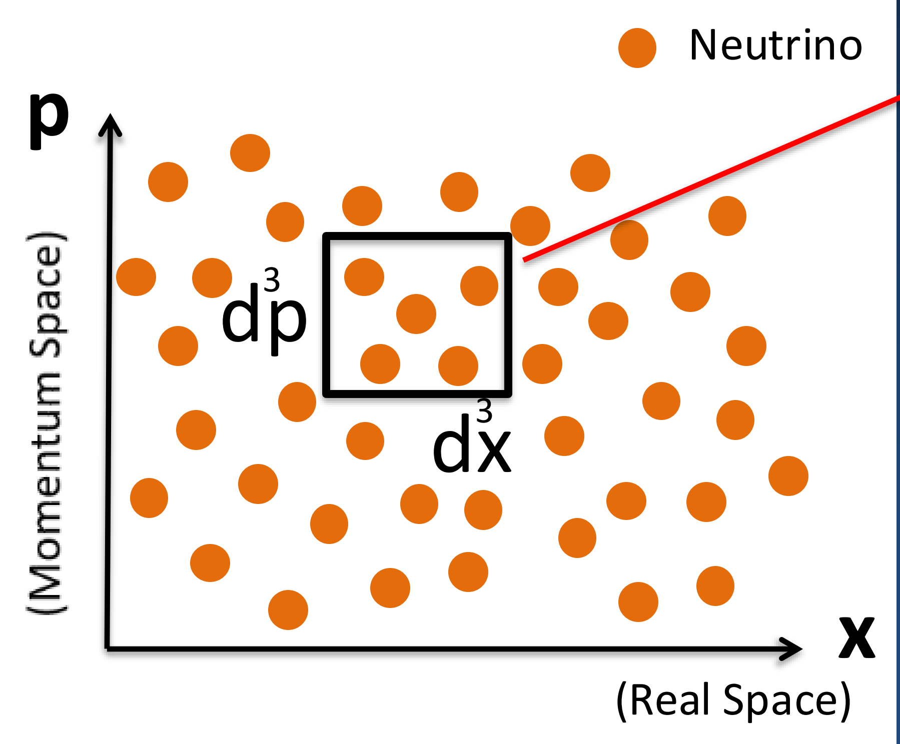


Boltzmann neutrino transport

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial f}{\partial p^{i}} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}},$$

(Time evolution + Advection Term) (Collision Term)

6 dimensional phase space



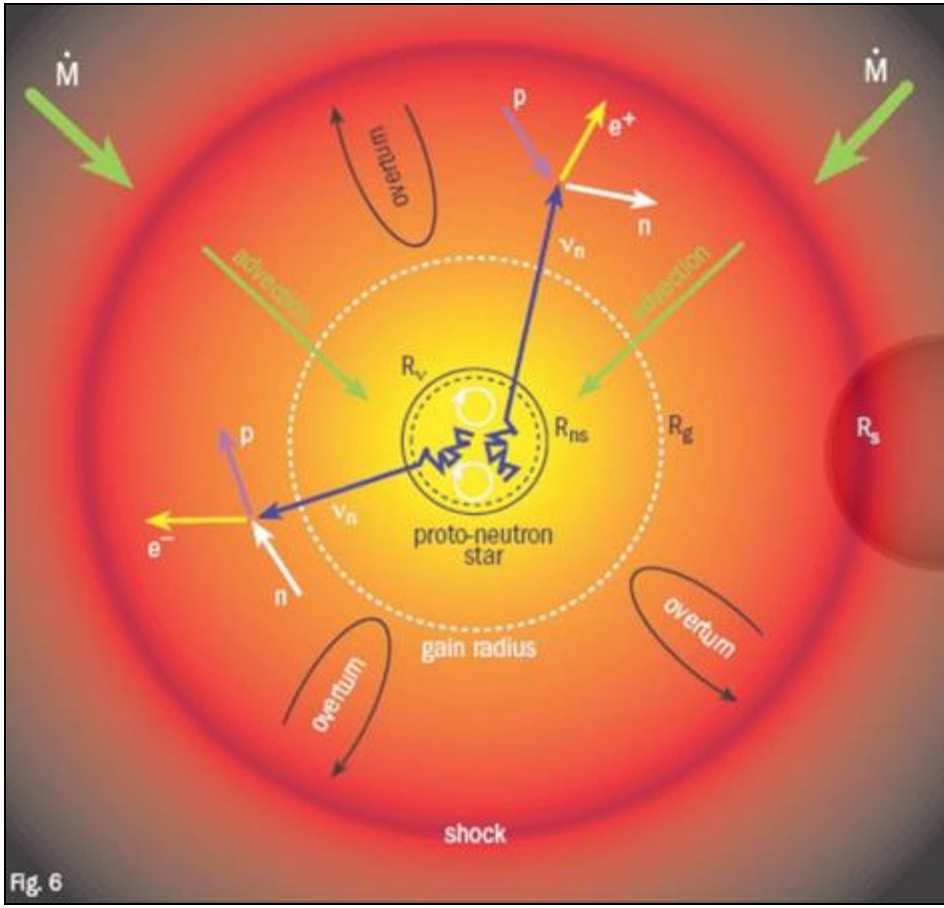
$$dN = f(t, \boldsymbol{p}, \boldsymbol{x}) d^3p d^3x$$

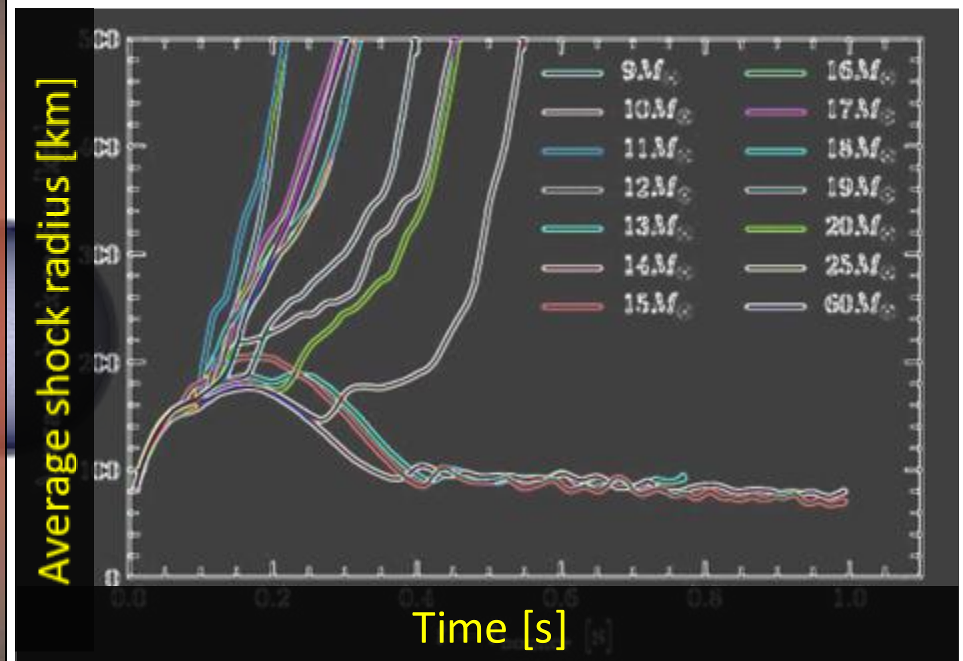
Conservative form of GR Boltzmann eq.

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\Big|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial\nu}(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)})$$
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta}f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+\frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\mathrm{rad}},$$

Shibata, Nagakura, Sekiguchi, and Yamada (2014) t = 0.010 s

Nagakura et al. 2019





- Core-collapse supernova (CCSN)

- Massive stars (larger than ～8 times solar masses) end up their life as core collapse supernovae (CCSNe)
- High energy transient/astrophysical phenomena
- Disseminate heavy elements into ISM
- Birth places for neutron stars and black holes
- Astrophysical laboratories for high energy physics
- Amusement parks for physicists

Linear stability analysis of flavor instabilities

Dispersion relation approach

Example: FFC (Izaguirre et al. 2017)

1. $$\rho_\nu = \frac{f_{\nu_e} + f_{\nu_x}}{2} I + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s_v & S_v \\ S_v^* & -s_v \end{pmatrix}.$$

2. $$i(\partial_t + \boldsymbol{v} \cdot \nabla_r) S_v = -v^\mu (\Lambda_\mu + \Phi_\mu) S_v + \int d\Gamma' v^\mu v'_\mu G_{v'} S_{v'},$$

Momentum-space integration

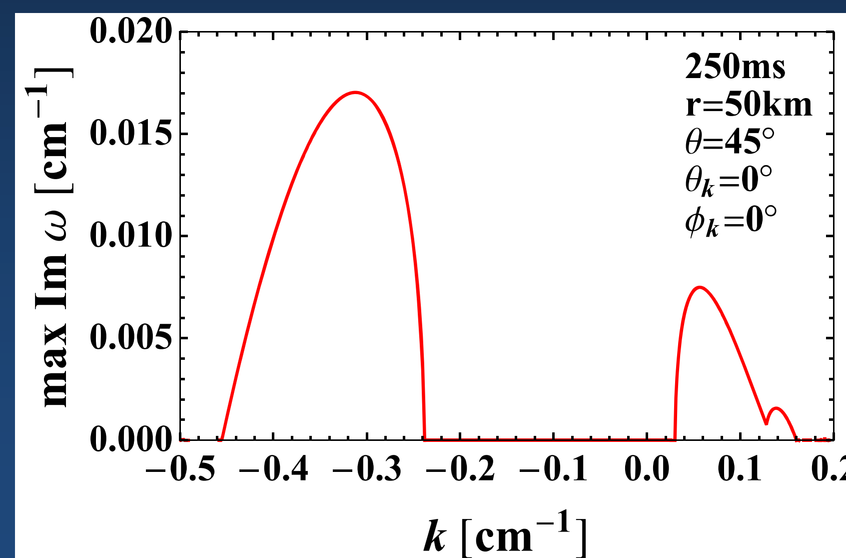
3. $$S_v = Q_v \exp[-i(\Omega t - \boldsymbol{k} \cdot \boldsymbol{r})].$$

4. $$\Pi^{\mu\nu} \equiv \eta^{\mu\nu} + \int d\Gamma G_v \frac{v^\mu v^\nu}{v^\gamma k_\gamma} = \eta^{\mu\nu} - \int d\Gamma G_v \frac{v^\mu v^\nu}{\omega - \boldsymbol{v} \cdot \boldsymbol{k}}.$$

$$(G_v \equiv \sqrt{2} G_F f_{\nu_e}(\boldsymbol{v}),)$$

$$\det \Pi = 0,$$

1. Decomposing traceless part
2. Linearizing QKE equation
3. Plane-wave ansatz
4. Computing Dispersion relation

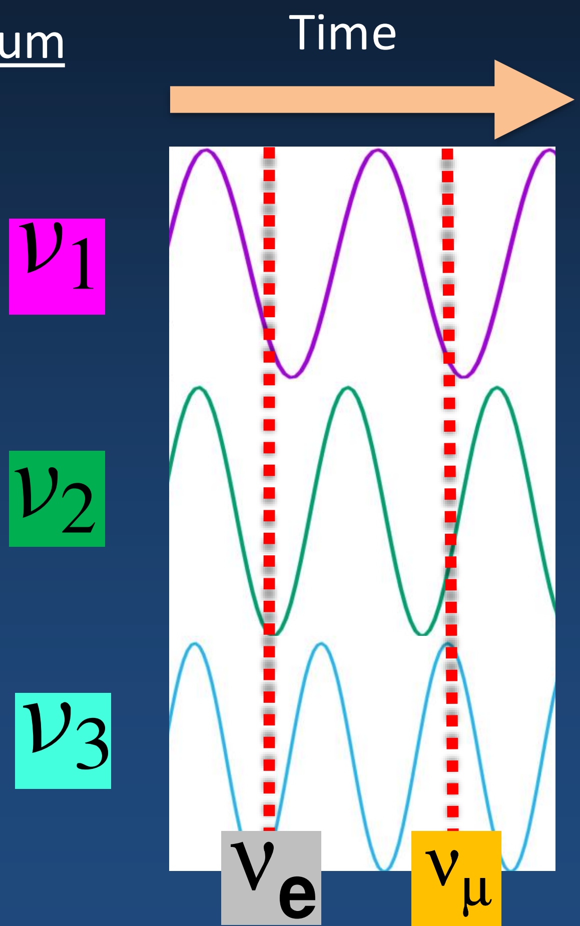


Neutrino oscillation with a plane-wave picture

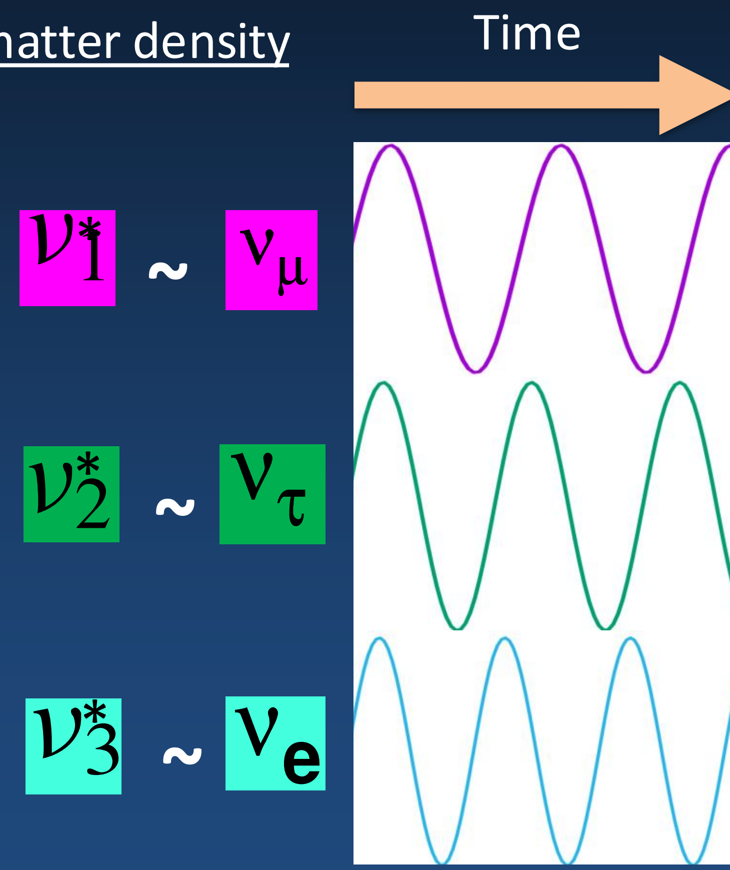
$$\psi_\ell(0,0) \equiv |\nu_\ell\rangle = \sum_{i=1}^{3} U^*_{\ell i}\,|\nu_i\rangle$$

$$\psi_\ell(t,x) = \sum_{i=1}^{3} U^*_{\ell i}\,|\nu_i\rangle\, e^{-i\phi_i}, \qquad \text{phase } \phi_i = E_i\,t - p_i\,x$$

Vacuum

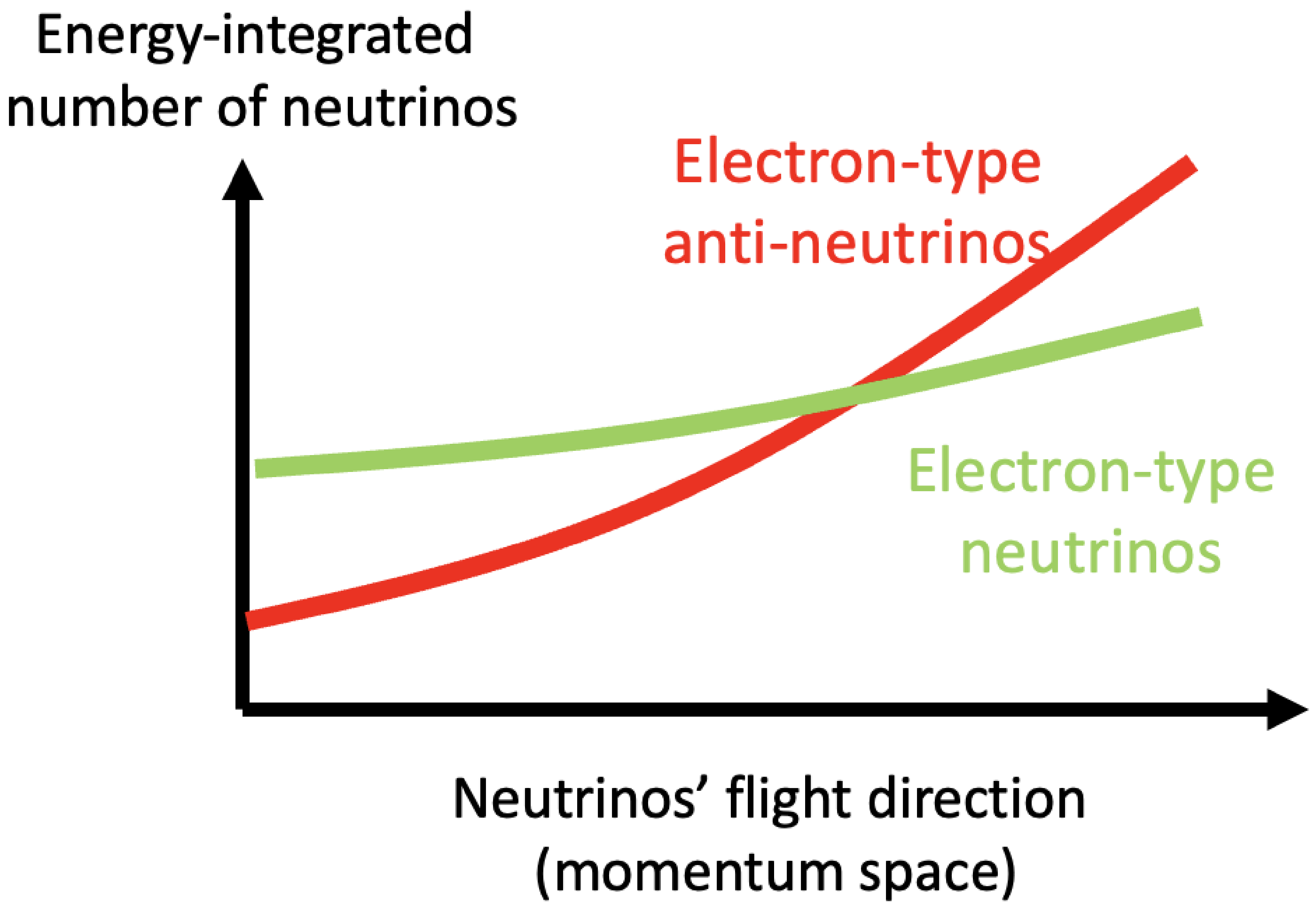


High matter density



Boltzmann transport becomes a reasonable approximation.

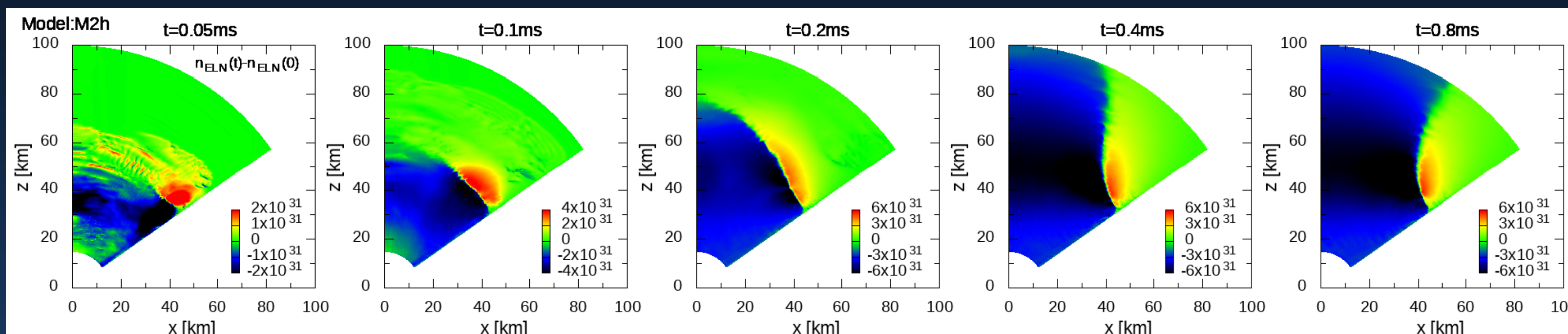
Instability criterion of FFC (ELN angular crossing)



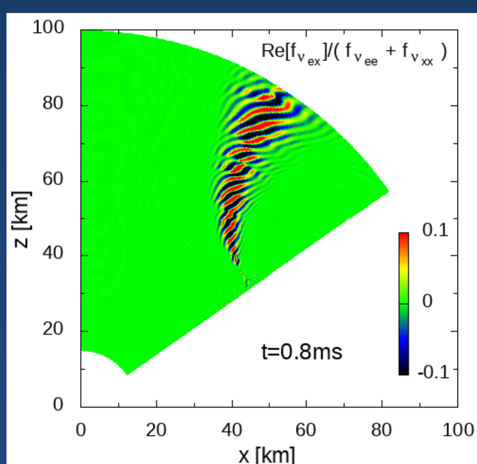
Global Simulations of FFC in a BNSM environment

Nagakura 2023

- Temporal evolution of FFCs in global scale:



- Off-diagonal component of the density matrix (coherency of flavor states):



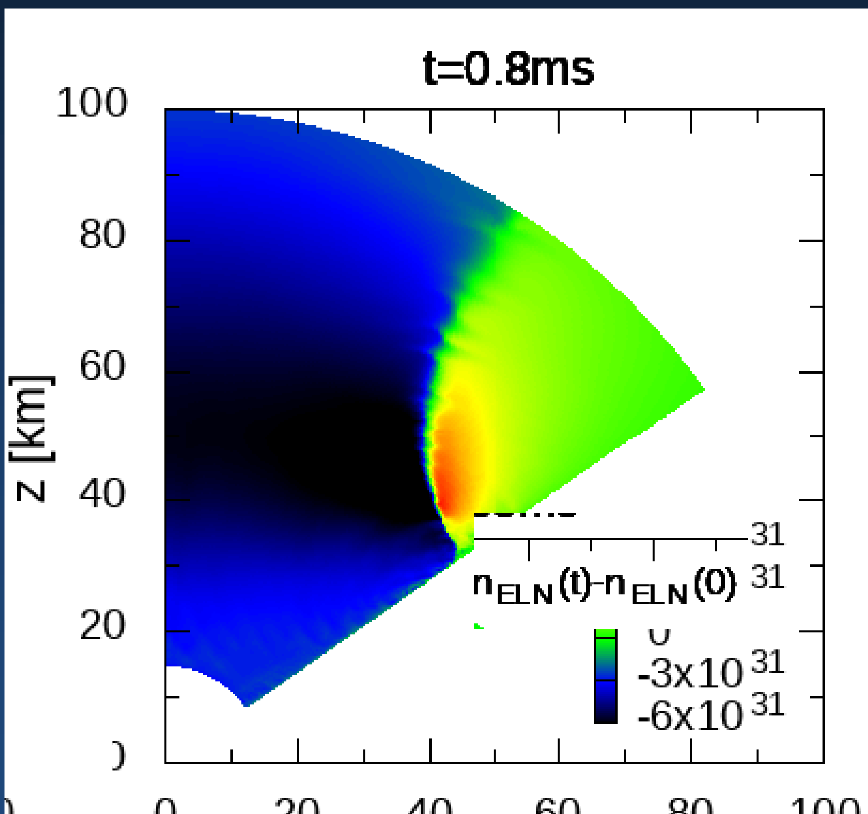
Take-home messages:

- FFC occurs vividly in a narrow region.
- The converted neutrinos spread in space by advections, leading to a radical change of neutrino radiation field.

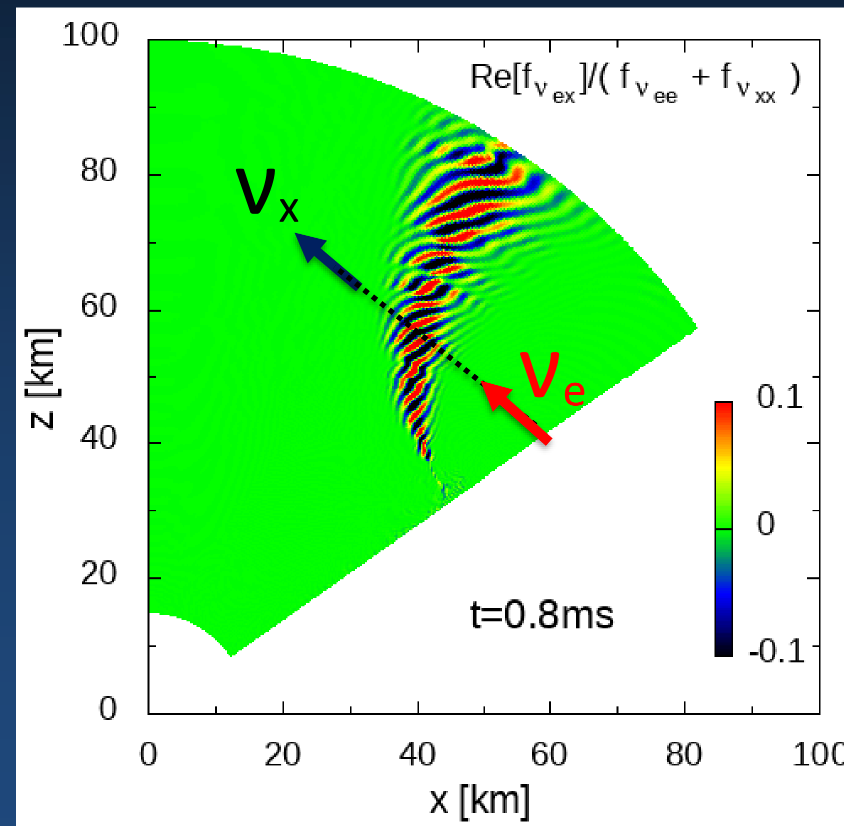
- Global simulations of FFC (in BNSM) Nagakura 2023

- Appearance of flavor swap and EXZS (ELN-XLN Zero Surface):

ELN - XLN



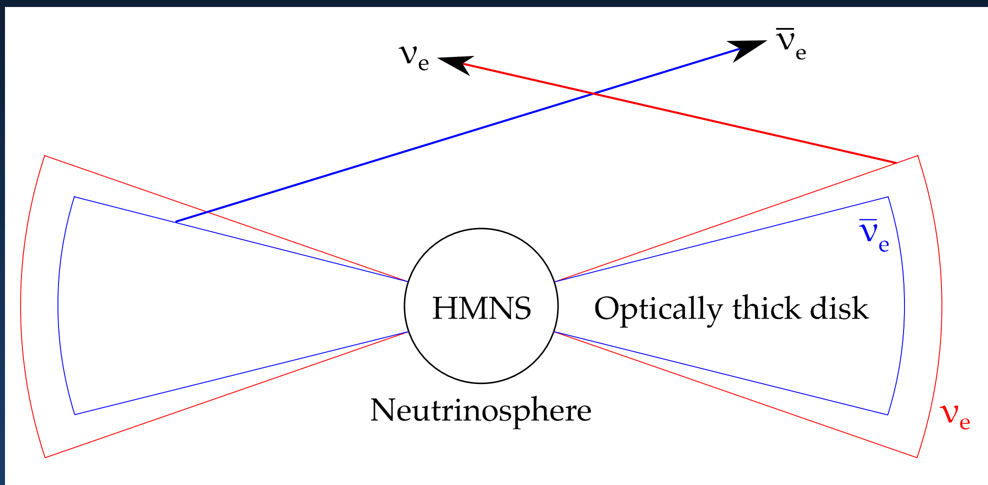
Flavor coherency

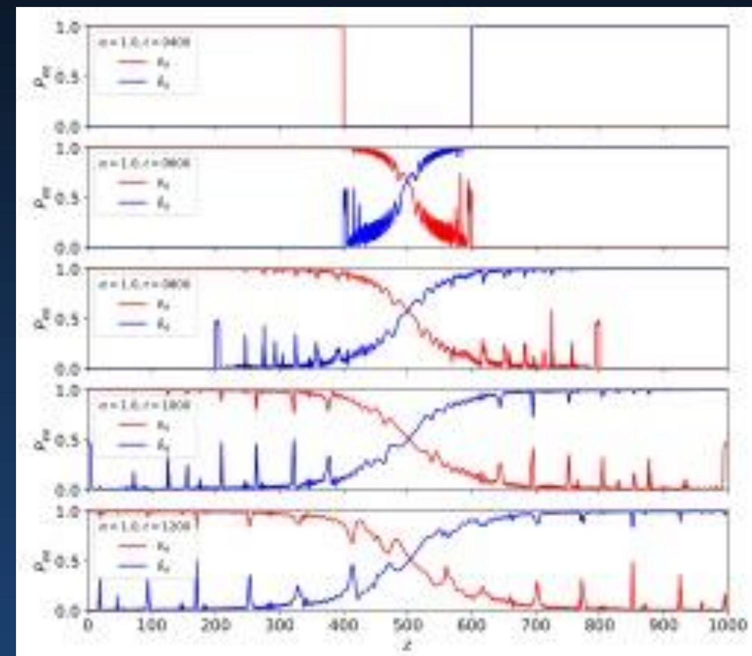


Fast flavor swap would be ubiquitous in BNSM

Zaizen and Nagakura 2024

Colliding-beam model





$$\partial_{t'}^2 P_3 \sim -4\mu^2\left(1-(P_3)^2\right)$$

P_3 = 1 : electron-type → Unstable

P_3 = 0 : equipartition → Non-steady

P_3 = -1 : heavy-lepton type → Stable

Neutrino flavor swaps are inevitable from the perspective of stability .