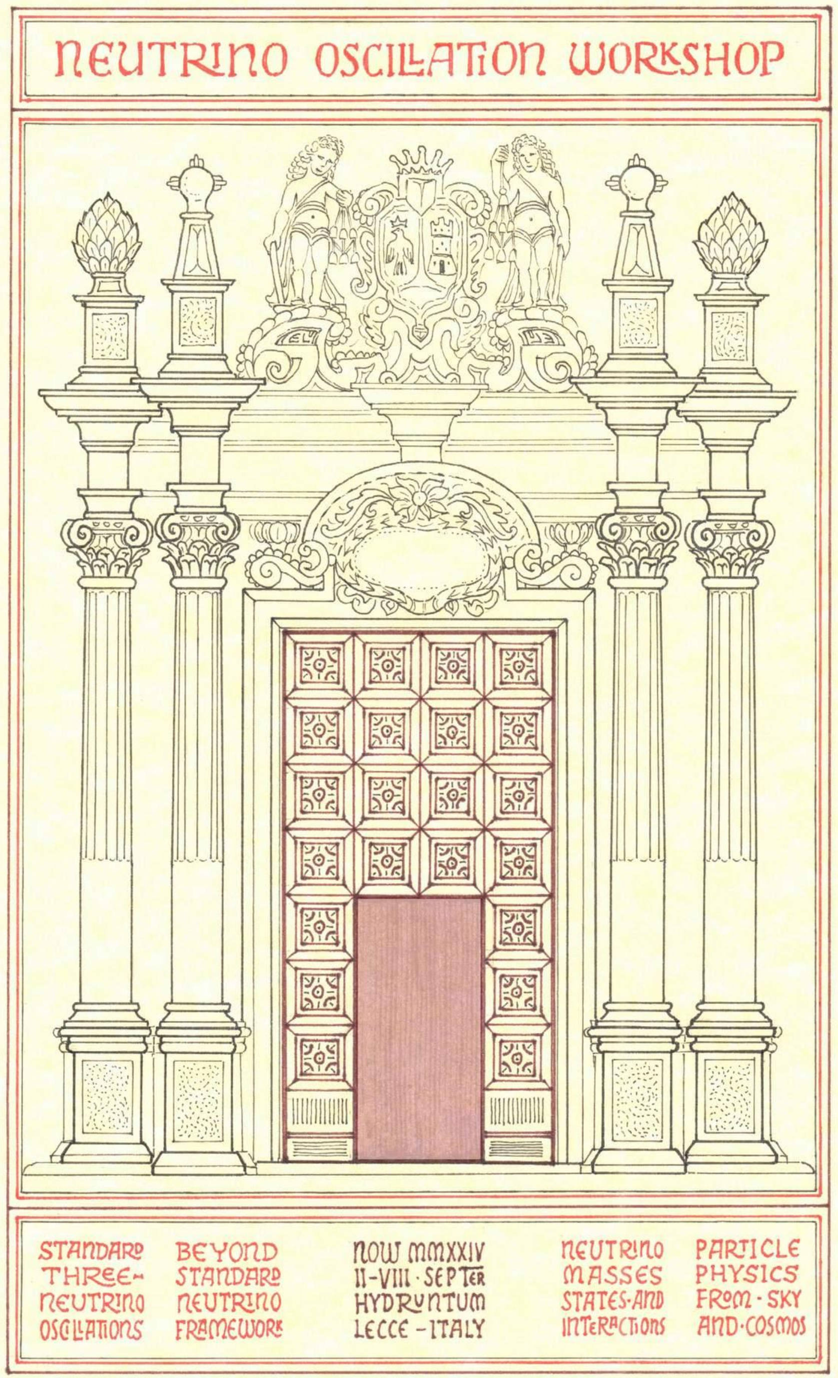

NOW 2024, Otranto
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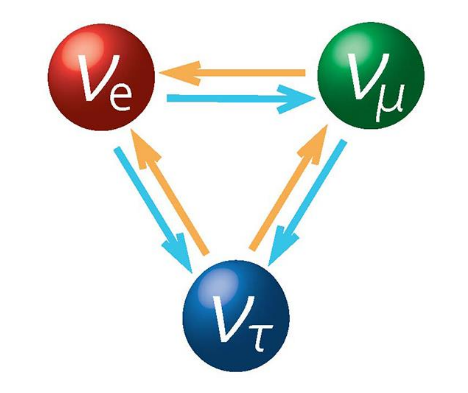
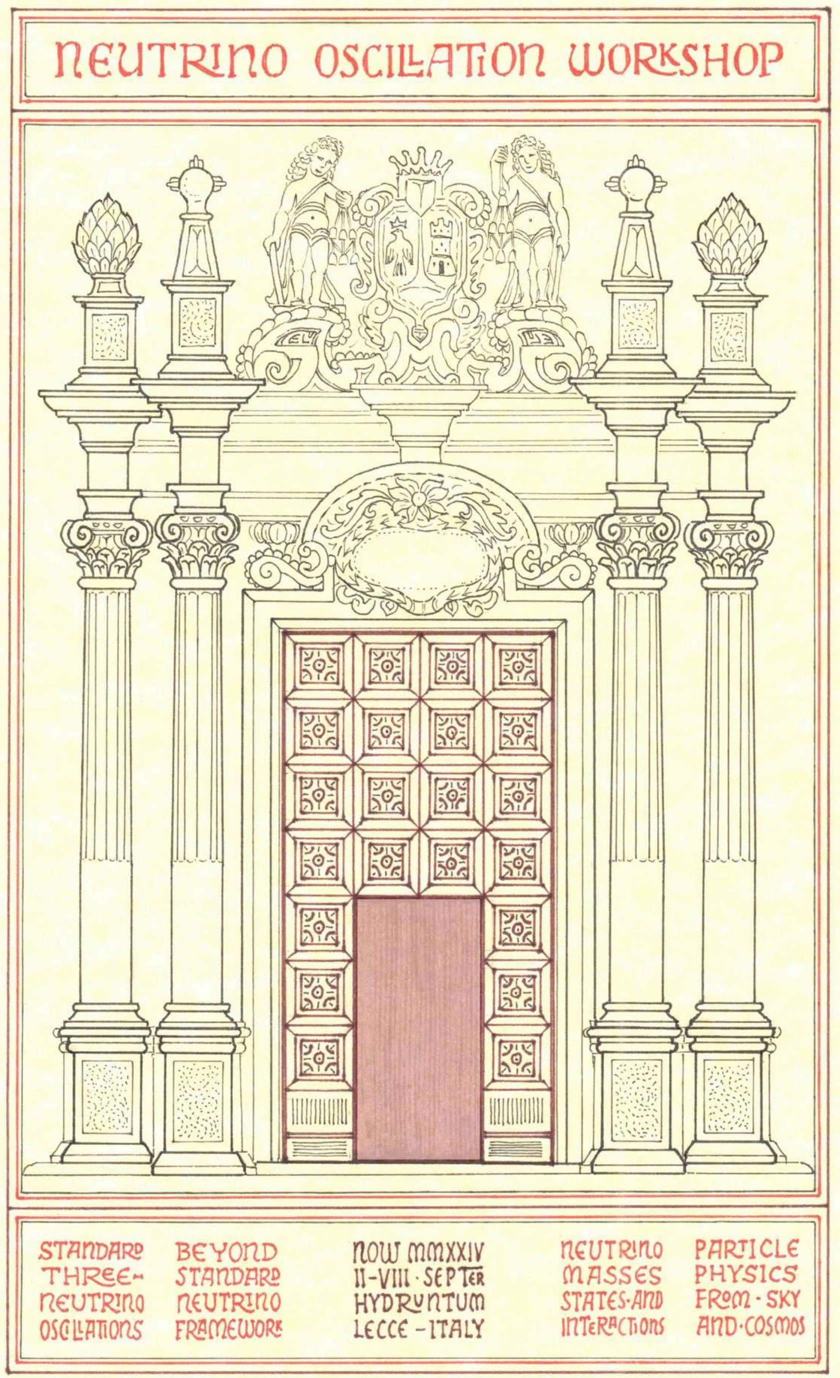
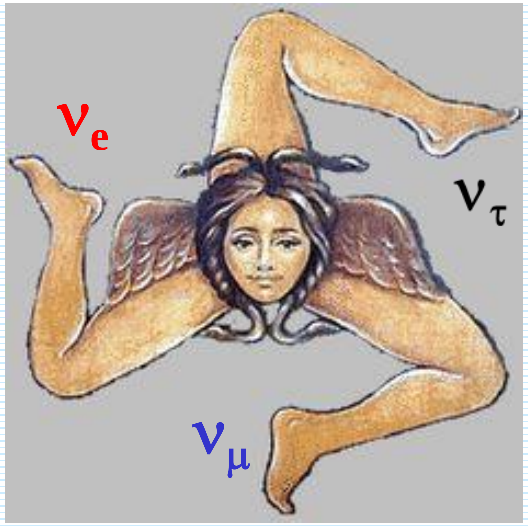


Neutrino oscillations and entanglement

Fedor Šimkovic



OUTLINE



I. Introduction

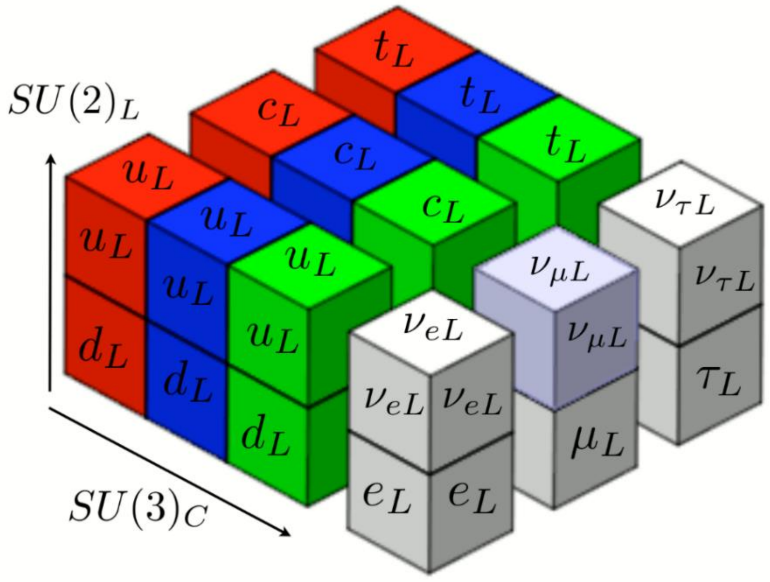
II. Neutrino oscillations as a single Feynman diagram
(QFT formalism with plane waves)

III. Neutrino-antineutrino oscillations and $0\nu\beta\beta$-decay
(QCSS scenario of ν mass generation)

IV. Towards fixing parameters of ν mass matrix
(Processes described with Frobenius covariants instead of elements of neutrino mixing matrix)

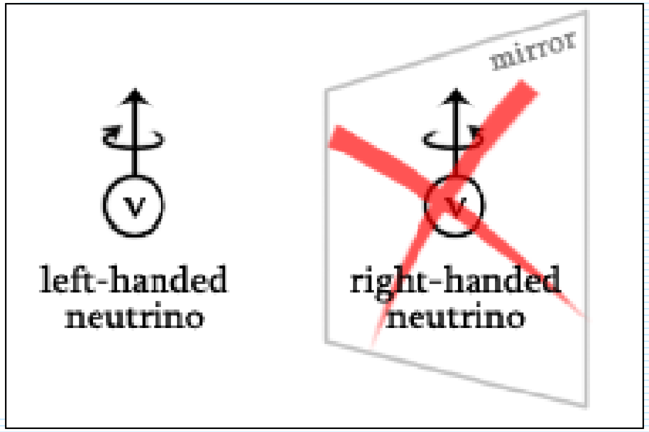
Acknowledgments: A. Khatun, S. Kovalenko, M. Krivoruchenko, and other colleagues and friends.

Standard Model
(an astonishing successful theory,
based on few principles)



ν is a special particle in SM:

- **It is the only fermion that does not carry electric charge (like γ, g, H^0)**
- **There are only left-handed ν's (ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$)**
- **ν-mass can not be generated with any renormalizable coupling with the Higgs fields through SSB**



ν's oscillations experiments
⇒ tiny neutrino masses (!)
⇒ Beyond SM physics (!)

, etc

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

3 neutrino masses, 2 mass squared differences

$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

$$U = R_{23}\tilde{R}_{13}R_{12}$$

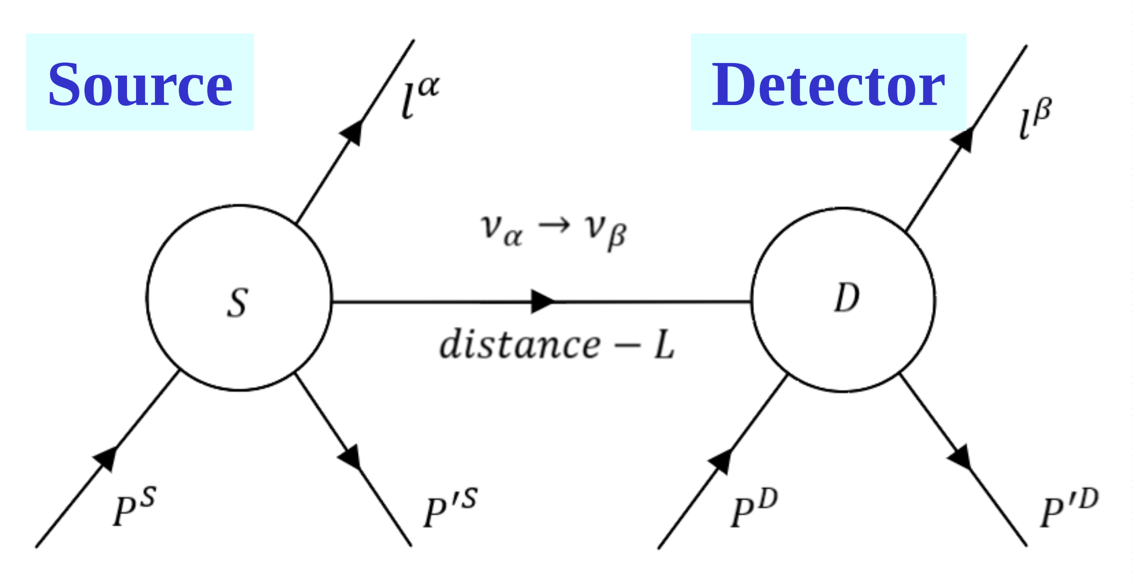
3 mixing angles
CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j}|\nu_j\rangle \qquad (\alpha = e,\ \mu, \tau)$$

	best − fit	1σ	2σ	3σ
		Normal hierarchy (NH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2/10^{-3}$ eV^2	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2\theta_{12}/10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2\theta_{13}/10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2\theta_{23}/10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ/π	1.28	1.10-1.66	0.95-1.90	0-0.07 $\oplus$ 0.81-2.00
		Inverted hierarchy (IH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.91
-$\Delta m^2/10^{-3}$ eV^2	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2\theta_{12}/10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2\theta_{13}/10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2\theta_{23}/10^{-1}$	5.51	5.17-5.67	4.60-5.82	4.39-5.96
		$\oplus$ 5.31-6.10		
δ/π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

Global neutrino oscillations analysis
(PRD 101, 116013 (2020))

Neutrino oscillations
(Quantum Mechanics Approach)



Massive neutrinos and neutrino oscillations

S. M. Bilenky

Joint Institute of Nuclear Research, Dubna, Union of Soviet Socialist Republics

S. T. Petcov

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, People's Republic of Bulgaria

The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \to e\gamma$ and $\mu \to 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double-β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double-β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.

Rev. Mod. Phys. 59, 671 (1987)

$$S \to S' + \ell^+_\alpha + \nu_\alpha$$

$$\nu_\alpha \to \nu_\beta$$

$$\nu_\beta + D \to D' + \ell^-_\beta$$

$$\Gamma_{osc} = \int \frac{d\Phi_\nu(E_\nu)}{dE_\nu} \frac{\mathcal{P}_{\alpha\beta}(E_\nu, L)}{4\pi L^2} \sigma(E_\nu)\, dE_\nu$$

Process is governed by the oscillation probability

$$S + D \to \ell^+_\alpha + \ell^-_\beta + S' + D'$$

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i\, m_j^2\, L/(2E_\nu)} \right|^2$$

Fedor Simkovic

$$\langle f|S^{(2)}|i\rangle = -i\int d^4x_1 J^{\mu}_{S}(P'_S, P_S)e^{i(P_\alpha+P'_S-P_S)\cdot x_1}\times$$
$$\int d^4x_2 J^{\mu}_{D}(P'_D, P_D)e^{i(P_\beta+P'_D-P_D)\cdot x_2}\sum_{k=1}^{3} U^*_{\alpha k}U_{\beta k}\times$$
$$\overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5)\; D(x_2-x_1,m_k)\;(1-\gamma_5)\gamma_\nu u(P_\beta;\lambda_\beta)$$

Neutrino oscillations as a single Feynman diagram

(within QFT, Walter Grimus approach revisited)]

J. Phys. G 51, 035202 (2024)

- **Choose a reference frame.**
- **The physical representation is an outgoing spherical wave of ν from the source.**

The neutrino emission and detection are identified with the charged-current vertices of a single second-order Feynman diagram for the underlying process, enclosing neutrino propagation between these two points.

$$D(x;m) = \theta(x_0)D^-(x;m) + \theta(-x_0)D^+(x;m),$$

$$D^{\pm}(x;m) = \int \frac{d\mathbf{q}}{(2\pi)^3}\frac{\mp(-\mathbf{q}\cdot\vec{\gamma}+\omega\gamma^0)+m}{2\omega}e^{\pm i(-\mathbf{q}\cdot\mathbf{x}+\omega x_0)}$$

Integration over time variables results in energy conservation and energy denominator

$$2\pi i\,\frac{\delta(E_\beta+E'_D-E_D+E_\alpha+E'_S-E_S)}{\omega+E_\alpha+E'_S-E_S+i\varepsilon}.$$

Neutrino propagation

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\not{p} + m_k}{2\omega(\omega + E_\alpha + E'_S - E_S + i\varepsilon)} e^{i\mathbf{q}\cdot(\mathbf{x}_2 - \mathbf{x}_1)}$$
$$\simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2-\mathbf{x}_1|}}{|\mathbf{x}_2 - \mathbf{x}_1|} \left(\not{Q}_k + m_k\right) \simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D}\; e^{-i\mathbf{p}_k\cdot\mathbf{x}_S}\; \frac{e^{ip_k L}}{L} \left(\not{Q}_k + m_k\right)$$

$$Q_k \equiv (E_\nu, \mathbf{p}_k), \quad \mathbf{p}_k = p_k\,(\mathbf{x}_2 - \mathbf{x}_1)\,/|\mathbf{x}_2 - \mathbf{x}_1|, \quad p_k = \sqrt{E_\nu^2 - m_k^2}$$
$$E_\nu = E_S - E'_S - E_\alpha \ (source) = E_\beta + E'_D - E_D \ (detector)$$

Energy conservation

Amplitude (there is no factorization of source and detector!)

Energy conservation

Momentum conservation at source

Momentum conservation at detector

$$\langle f|S^{(2)}|i\rangle = (2\pi)^7 \delta(E_f - E_i) \sum_k U_{\alpha k} U^*_{\beta k} \frac{e^{i p_k L}}{4\pi L} \times$$

$$T_k^{\alpha\beta}\ \delta^3_{V_S}(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\ \delta^3_{V_D}(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

with

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D)\ \overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5)\ \not{Q}_k\ \gamma_\nu u(P_\beta;\lambda_\beta)$$

Lepton current

Master formula

J. Phys. G 51, 035202 (2024)

$$d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km}$$
$$\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m)$$
$$\frac{(2\pi)^7}{4E_S E_D} \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)\times$$
$$\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha\ (2\pi)^3} \frac{d\mathbf{p}_\beta}{2E_\beta\ (2\pi)^3} \frac{d\mathbf{p}'_S}{2E'_S\ (2\pi)^3} \frac{d\mathbf{p}'_D}{2E'_D\ (2\pi)^3}$$

with

$$\mathcal{F}^{\alpha\beta}_{km} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k\right)^* \right)$$

$$\langle \Phi^{S,D}(\mathbf{P}_i) | \Phi^{S,D}(\mathbf{P}_k) \rangle = (2\pi)^3\ 2E_k\ \delta^3_{V_{S,D}}\ (\mathbf{P}_i - \mathbf{P}_k)$$

Two normalization volumes:

i) source;

ii) Detector.

An example:
J. Phys. G 51, 035202 (2024)

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

$$\Gamma^{\pi^+ n}_{osc} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu \nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu)\, dE'_\nu$$

$$= \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \nu_e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2)\ p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Standard QM approach

New QFT approach (no decoherence, no factorization of two processes)

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{\mathcal{P}^{QFT}_{\mu e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2)\ p_e E_e$$

with

$$\mathcal{P}^{QFT}_{\alpha\beta}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U^*_{\alpha k}\ U^*_{\beta k} U_{\alpha k}\ e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2}\right)$$

Is there an entanglement of ν production and detection? (7 momenta in a game)

$$\mu^- + n \to e^- + \overline{\nu}_e + e^- + p$$

$$\mu^- \to \nu_\mu + e^- + \overline{\nu}_e, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

$P_\mu,\ P_{e_s},\ P_{\nu_e},$

$\mathbf{P}_k$

$P_n,\ P_p, P_{e_d}$

source $$\int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e} - m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s}+m_e\right)\times$$

ν **– prop.** $$\mathrm{Tr}\ \gamma^\sigma \not{P}_k \gamma^\rho (1-\gamma_5)\left(\not{P}_\mu + m_\mu\right)\gamma^\lambda \not{P}_k \gamma^\delta (1-\gamma_5)\left(\not{P}_{e_d}+m_e\right)\times$$

detector $$\frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V - g_A\gamma_5)\left(\not{P}_n + m_n\right)\gamma_\delta(g_V - g_A\gamma_5)\left(\not{P}_p + m_p\right) \times \frac{2\ 2}{256}$$

$$= 16\ 4\ (P_\mu\cdot P_{\overline{\nu}_e})((P_\mu - P_{e_s} - P_k)\cdot P_k)\ + (P_{e_d}\cdot P_p)(P_n\cdot P_k)$$

Additional terms

$$-8\ 4\ \mathbf{P}_k^2\ (P_\mu\cdot(P_\mu - P_{e_s} - P_k))\ (P_{e_d}\cdot P_p)(P_{e_s}\cdot P_n)$$
$$-2\epsilon_{P_{e_d}P_{e_s}P_\mu P_k}\ \left((P_{e_s}\cdot P_p)\ (P_n\cdot P_k) - (P_\mu\cdot P_p)\ (P_n\cdot P_k)\right.$$
$$\left. +(P_p\cdot P_k)\left((P_{e_s}\cdot P_n) - (P_n\cdot P_k)\right)\right)$$
$$+2\epsilon_{P_{e_s}P_{e_\mu}P_n P_p}\ (P_{e_d}\cdot P_k)\ \left((P_\mu\cdot P_k) + P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_p P_k}\ \left(2(P_{e_s}\cdot P_\mu)\ (P_n\cdot P_k) - P_\mu^2\ (P_n\cdot P_k) - (P_\mu\cdot P_n)\ P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_n P_k}\ \left(2(P_{e_s}\cdot P_\mu)\ (P_p\cdot P_k) - P_\mu^2\ (P_p\cdot P_k)\right.$$
$$\left. -2(P_\mu\cdot P_p)\ (P_p\cdot P_k) + (P_\mu\cdot P_p)\ P_k^2\right)$$
$$+\epsilon_{P_{e_d}P_{e_s}P_\mu P_p}\ (\cdots) + \epsilon_{P_{e_d}P_{e_s}P_\mu P_n}\ (\cdots) + \epsilon_{P_{e_d}P_\mu P_p P_k}\ (\cdots) + \epsilon_{P_{e_d}P_\mu P_n P_k}\ (\cdots)$$
$$+2\epsilon_{P_\mu P_n P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_n P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_\mu P_p P_k}\ (\cdots) + \epsilon_{P_{e_s}P_\mu P_n P_k}\ (\cdots)$$

Simplification:
$g_V = g_A = 1$
$P_k = P_l$

Product of traces under the phase-space integration

$$\text{source} \quad \int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e}-m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s}+m_e\right)\times$$

$$\nu-\text{prop.} \quad \mathrm{Tr}\ \gamma^\sigma\not{P}_k\gamma^\rho(1-\gamma_5)\left(\not{P}_\mu+m_\mu\right)\gamma^\lambda\not{P}_k\gamma^\delta(1-\gamma_5)\left(\not{P}_{e_d}+m_e\right)\times$$

$$\text{detector} \quad \frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V-g_A\gamma_5)\left(\not{P}_n+m_n\right)\gamma_\delta(g_V-g_A\gamma_5)\left(\not{P}_p+m_p\right)\times$$

$$\delta\left(P_{e_s}+P_{\nu_e}-P_Q\right)\frac{d\mathbf{p}_{e_s}}{E_{e_s}}\frac{d\mathbf{p}_{\nu_e}}{E_{\nu_e}}=\frac{1}{4}256\times16\times$$

$$\{\ C_g\ (P_\mu\cdot P_k)\ +\ C_{PP}\ \left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \left((P_\mu\cdot P_k)-P_k^2\right)\ )\times$$

$$\left[\left(g_A^2-g_V^2\right)\ m_n m_p\ (P_{e_d}\cdot P_k)+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot P_k)\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot P_k)\right]$$

$$-\frac{1}{4}256\times8\times\mathbf{P}_k^2\ \left(\ C_g\ -\ C_{PP}\ \left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \right\}\times$$

$$\left[\left(g_A^2-g_V^2\right)\ m_n m_p\ (P_{e_d}\cdot(P_k-P_\mu))+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot(P_k-P_\mu))\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot(P_k-P_\mu))\right]$$

$$C_g=\frac{1}{6}\frac{P_Q^2-m_e^2}{P_Q^2}\ \pi\ \frac{\left(P_Q^2-m_e^2\right)^2}{P_Q^2}$$

$$C_{PP}=\frac{1}{3}\frac{P_Q^2+2m_e^2}{(P_Q^2)^2}\ \pi\ \frac{\left(P_Q^2-m_e^2\right)^2}{P_Q^2}$$

$$P_Q^2=(P_\mu-P_\nu)^2=m_\mu^2-2E_\mu E_\nu+p_\mu E_\nu\cos\left(\theta_{\mu\nu}\right)$$

Entanglement of production and detection processes:
additional term ~ ν–mass squared:
Processes with intermediate-charged lepton processes need to be investigated!

Separately for production and detection processes

$$\text{source} \quad \int\int \frac{1}{2}\mathrm{Tr}\ \gamma_\rho(1-\gamma_5)\left(\not{P}_{\overline{\nu}_e}-m_\nu\right)\gamma_\lambda(1-\gamma_5)\left(\not{P}_{e_s}+m_e\right)\times$$

$$\delta\left(P_{e_s}+P_{\nu_e}-P_Q\right)\frac{d\mathbf{p}_{e_s}}{E_{e_s}}\frac{d\mathbf{p}_{\nu_e}}{E_{\nu_e}}$$

$$=\frac{1}{2}256\times\left(\ C_g\ (P_\mu\cdot P_k)\ +\ C_{PP}\ \left(P_\mu^2-(P_\mu\cdot P_k)\right)\ \left((P_\mu\cdot P_k)-P_k^2\right)\ \right)$$

$$\text{detector} \quad \frac{1}{2}\mathrm{Tr}\ \gamma_\sigma(g_V-g_A\gamma_5)\left(\not{P}_n+m_n\right)\gamma_\delta(g_V-g_A\gamma_5)\left(\not{P}_p+m_p\right)$$

$$=\frac{1}{2}64\times\left[\left(g_A^2-g_V^2\right)\ m_n m_p\ (P_{e_d}\cdot P_k)+(g_A-g_V)^2\,(P_{e_d}\cdot P_n)\ (P_p\cdot P_k)\right.$$

$$\left.+(g_V+g_A)^2\,(P_{e_d}\cdot P_p)(P_n\cdot P_k)\right]$$

Neutrino Mass Term

Dirac

$$L^D_{\text{mass}} = -\sum_{\alpha\beta} \overline{\nu}_{\alpha R}\ M^D_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{3} m_k \overline{\nu}_k \nu_k$$

$$\alpha, \beta = e, \mu, \tau, \qquad V^\dagger\ M^D\ U = M^D_{\text{diag}}$$

Majorana

$$L^M_{\text{mass}} = \frac{1}{2} \sum_{\alpha\beta} \nu^T_{\alpha L} C^\dagger\ M^L_{\alpha\beta}\ \nu_{\beta_L} + H.c.$$

$$= \frac{1}{2} \sum_{k=1}^{3} m_k \nu^T_k C^\dagger \nu_k$$

$$\alpha, \beta = e, \mu, \tau$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \qquad U^T\ M^M\ U = M^M_{\text{diag}}$$

Dirac-Majorana

$$L^{D+M}_{\text{mass}} = -\sum_{\alpha\beta} \overline{N}_{\alpha R}\ M^{D+M}_{\alpha\beta}\ N_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{6} m_k \overline{\nu}_k \nu_k$$

$$N = \begin{pmatrix} \nu_L \\ \nu^C_R \end{pmatrix}, \qquad \alpha, \beta = e, \mu, \tau, s_1, s_2, s_3$$

$$U^T\ M^{D+M}\ U = M^{D+M}_{\text{diag}}$$

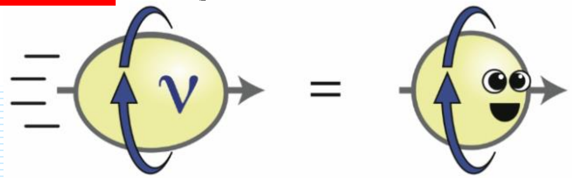
Majorana fermions

Ettore Majorana
Teoria simmetrica dell'elettrone e del positrone (A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]

Bruno Pontecorvo
Inverse beta processes and nonconservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)



ν ↔ anti-ν oscillation

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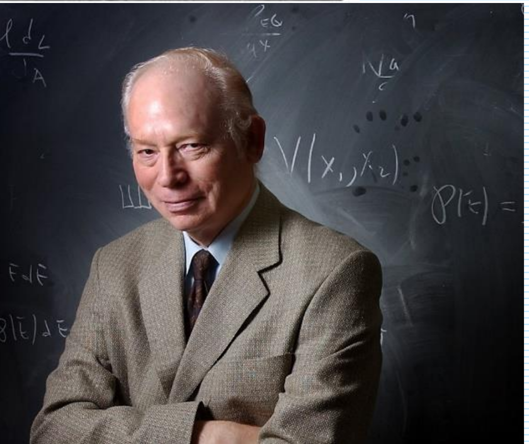
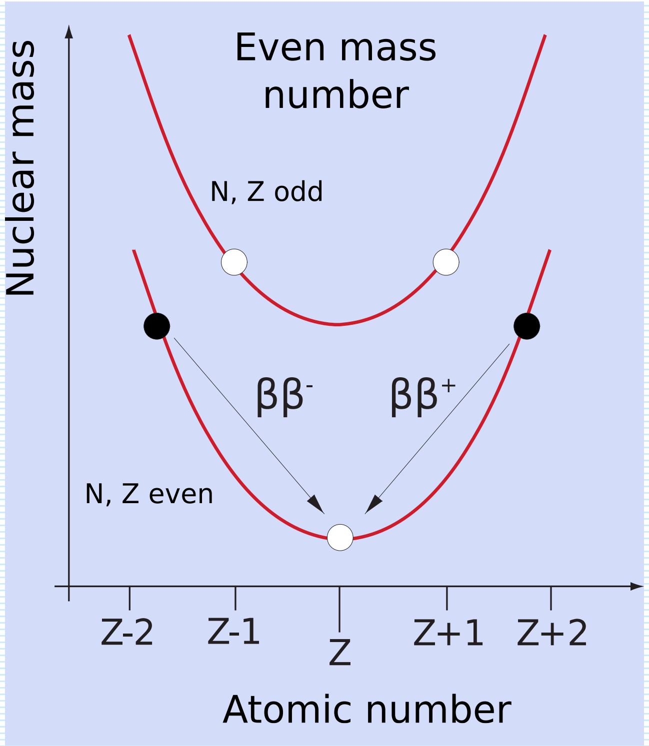
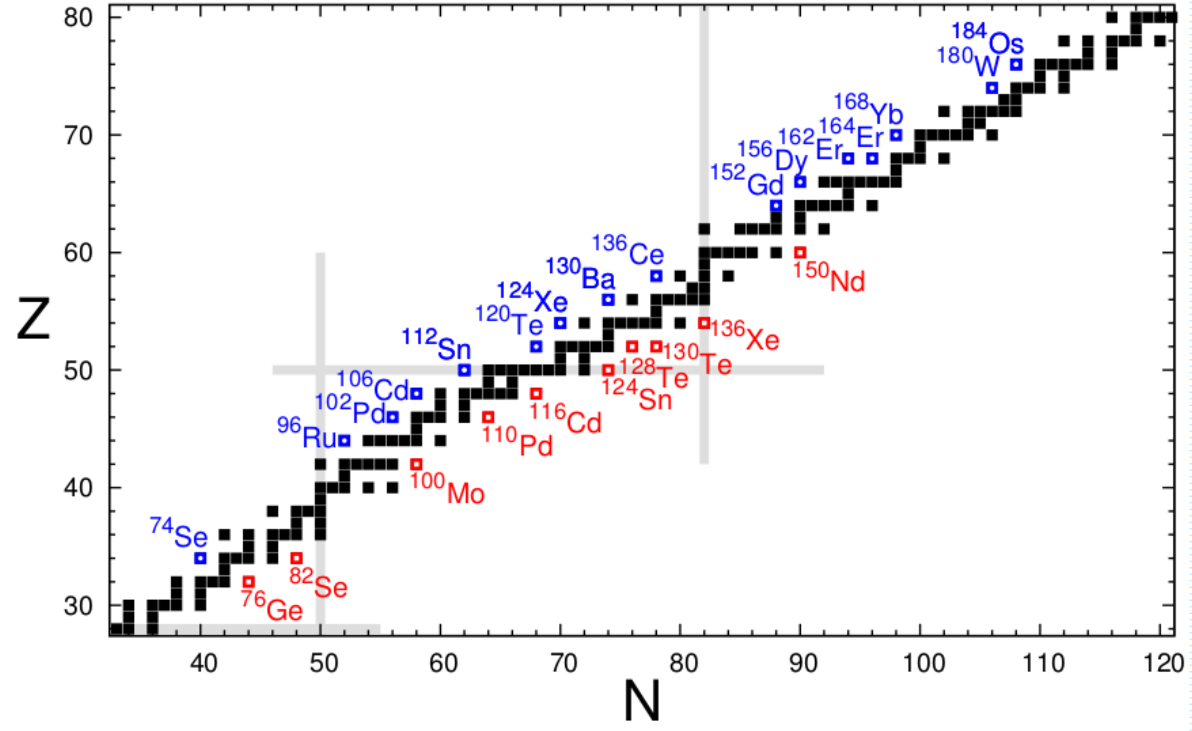
thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."



Steve Weinberg
ν-mass generation via d=5 eff. oper. related to unknown high energy scale (GUT?)

Nuclear double-β decay
(even-even nuclei, pairing int.)





Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$$

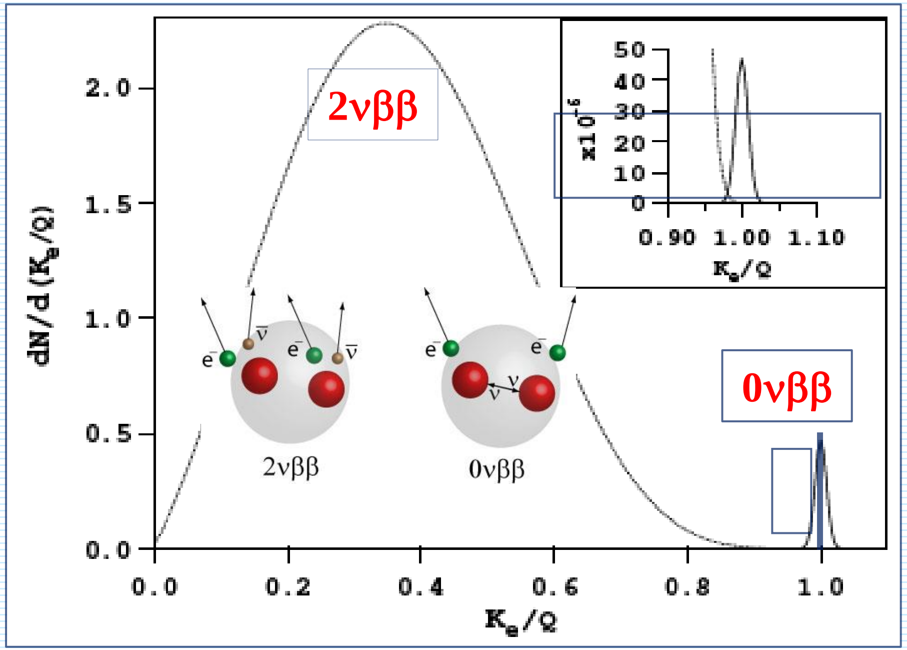
Goepert-Mayer – 1935. 1st observation in 1987

Nuovo Cim. 14, 322 (1937) | Phys. Rev. 56, 1184 (1939)

Neutrinoless double-β decay – LN violated

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- \quad \text{(Furry 1937)}$$

Not observed yet. Requires massive Majorana ν's



Nuovo Cim. 14, 322 (1937)

neutrino ↔ antineutrino oscillations

Second order process with real intermediate neutrinos

$$S + D \to \ell_\alpha^+ + \ell_\beta^+ + S' + D'$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha, \ \ \nu_\alpha \to \overline{\nu}_\beta, \ \ \overline{\nu}_\beta + D \to D' + \ell_\beta^+$$

Amplitude proportional to ν–mass

$$T_k^{\alpha\beta} = J_S^\mu(P'_S, P_S) J_D^\nu(P'_D, P_D) \times \overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5) m_k \gamma_\nu u(P_\beta;\lambda_\beta)$$

Oscillation probability

$$\mathcal{P}_{\alpha\bar{\beta}}^{\mathrm{QFT}}(E_\nu, L) \equiv |\langle \nu_\beta | \overline{\nu}_\alpha \rangle|^2 = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j}^* \frac{m_j}{E_\nu} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Replacement:

$$U_{\alpha k} \to U_{\alpha k}^*$$
$$U_{\beta m}^* \to U_{\beta m}$$

Particular process:

$$\pi^+ + p \to \mu^+ + e^+ + n$$

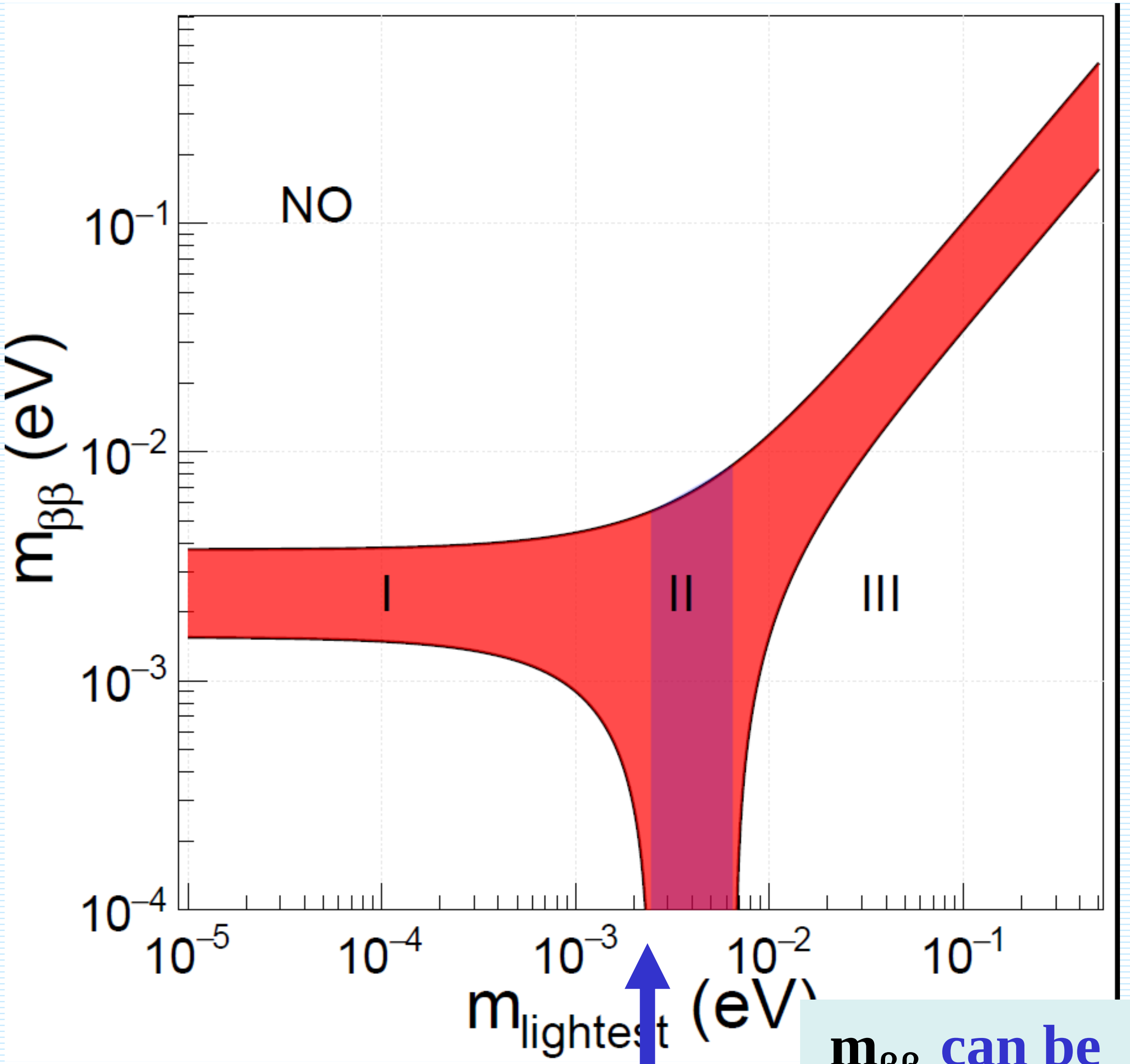
Production rate

$$\Gamma_{QFT}^{\pi^+ p} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \overline{\nu}_e}^{\mathrm{QFT}}(E_\nu, L)}{4\pi L^2} (g_V^2 + 3 g_A^2) \ p_e E_e$$

A connection of neutrino-antineutrino oscillation with 0νββ-decay

$$m^{L=0}_{ee} = m_{\beta\beta}$$

$$\mathcal{P}^{\mathrm{QFT}}_{e\bar{e}}(E_\nu, L=0) \equiv |\langle\nu_e|\overline{\nu}_e\rangle|^2 = \left|\sum_{k=1}^{3} U^*_{ek}U^*_{ek}\frac{m_k}{E_\nu}\right|^2 = \frac{(m^{L=0}_{ee})^2}{E^2_\nu} = \frac{(m_{\beta\beta})^2}{E^2_\nu}$$



$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c^2_{12}c^2_{13}m_1,\ \rho_2 = s^2_{12}c^2_{13}m_2,\ \rho_3 = s^2_{13}m_3$$

$$\min_{\phi_1,\phi_2} m_{\beta\beta} = \begin{cases} |\rho_2-\rho_3|-\rho_1, & \text{if } \rho_1 < |\rho_2-\rho_3| \quad : \text{region I}, \\ 0, & \text{if } |\rho_2-\rho_3| \le \rho_1 \le \rho_2+\rho_3 \quad : \text{region II}, \\ \rho_1-(\rho_2+\rho_3), & \text{if } \rho_2+\rho_3 < \rho_1 \quad : \text{region III}. \end{cases}$$

$m_{\beta\beta}$ can be strongly \suppressed (?!)

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021)

This operator contributes to the Majorana-neutrino mass matrix due to chiral symmetry breaking via the light-quark condensate.

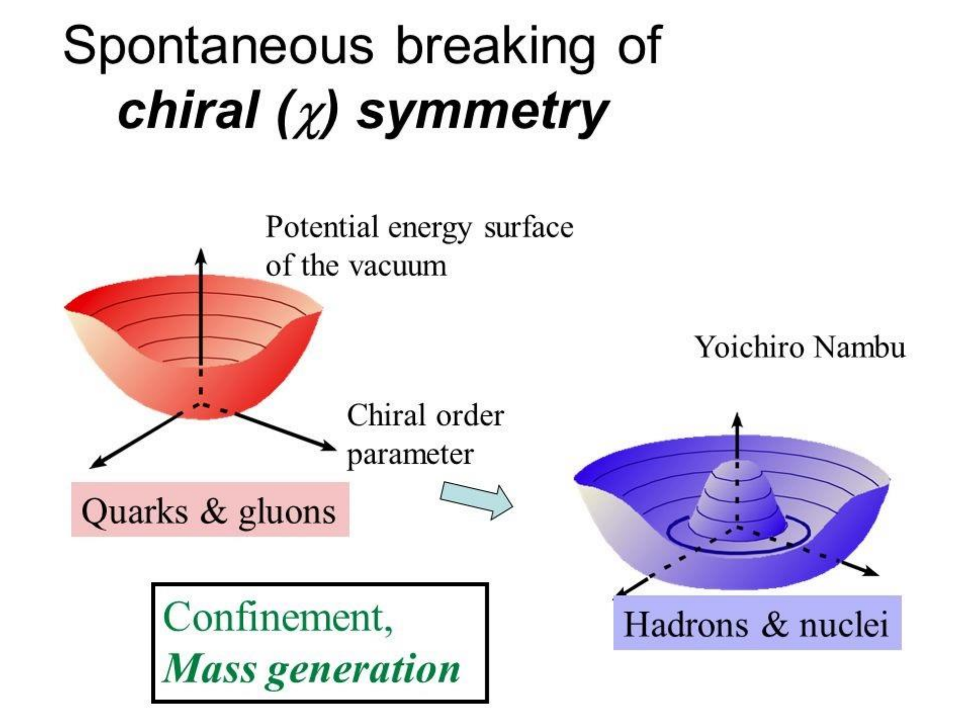
The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3}\, \overline{L_\alpha^C}\, L_\beta\, H \left\{ (\overline{Q}\, u_R),\ (\overline{d_R}\, Q) \right\}$$

After the EWSB and ChSB one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} m_{\alpha\beta}{}^{\nu} &= g_{\alpha\beta}\, v\, \frac{\langle \overline{q}q \rangle}{\Lambda^3} \\ &= g_{\alpha\beta}\, v \left(\frac{\omega}{\Lambda} \right)^3 \end{aligned}$$

$$\begin{aligned} g_{\alpha\beta} &= g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle \\ \omega &= -\langle \overline{q}q \rangle^{1/3}, \quad \langle \overline{q}q \rangle^{1/3} \approx -283\ \mathrm{MeV} \end{aligned}$$



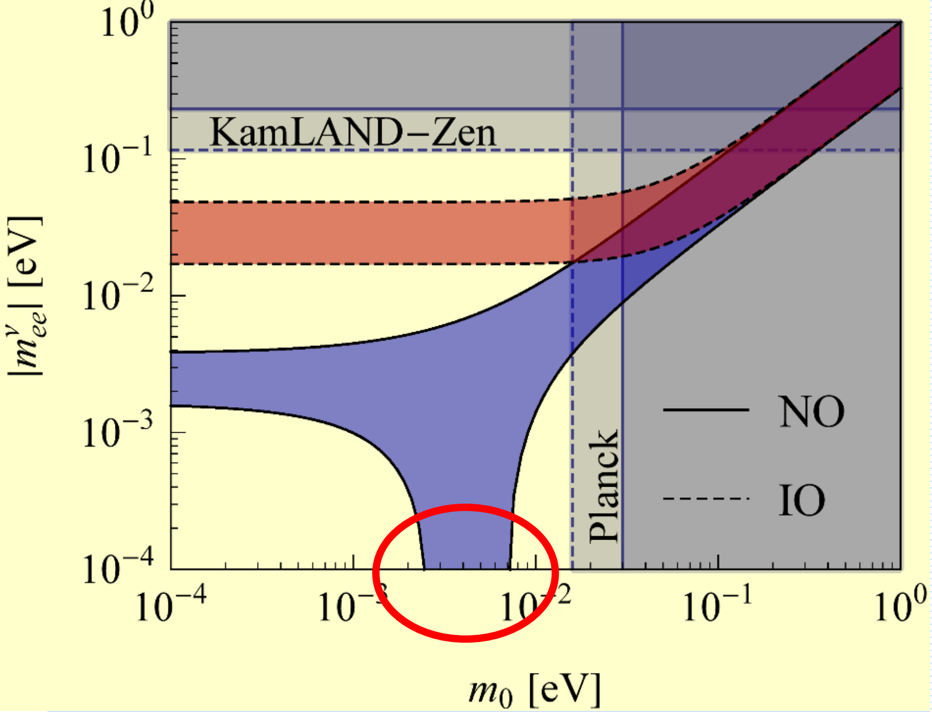
Λ ~ a few TeV
we get the neutrino mass in the sub-eV ballpark

The genuine QCSS scenario

(predicts NH and ν-mass spectrum)

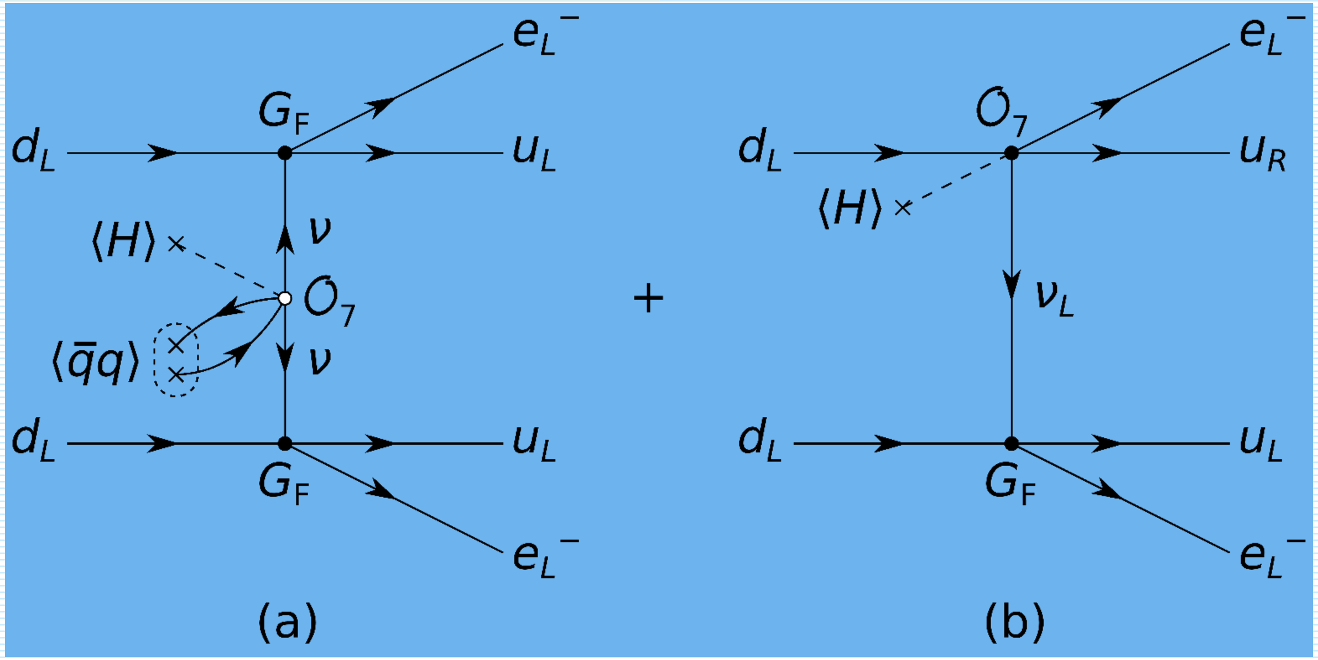
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \overline{\nu^C_{\alpha L}} \nu_{\beta L} \left(g^u_{\alpha\beta} \overline{u_L} u_R + g^d_{\alpha\beta} \overline{d_R} d_L \right) + \text{H.c.}$$

$$m^\nu_{\alpha\beta} = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q} q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$



(a) PRL 112, 142503 (2014).

(b) PLB 453, 194 (1999).



Neutrino spectrum (NH) !!!

(PRD 103, 015007 (2021))

2 meV < m_1 < 7 meV

9 meV < m_2 < 11 meV

50 meV < m_2 < 51 meV

Prediction for m_β

9 meV < m_β < 12 meV

Prediction for cosmology (Σ)

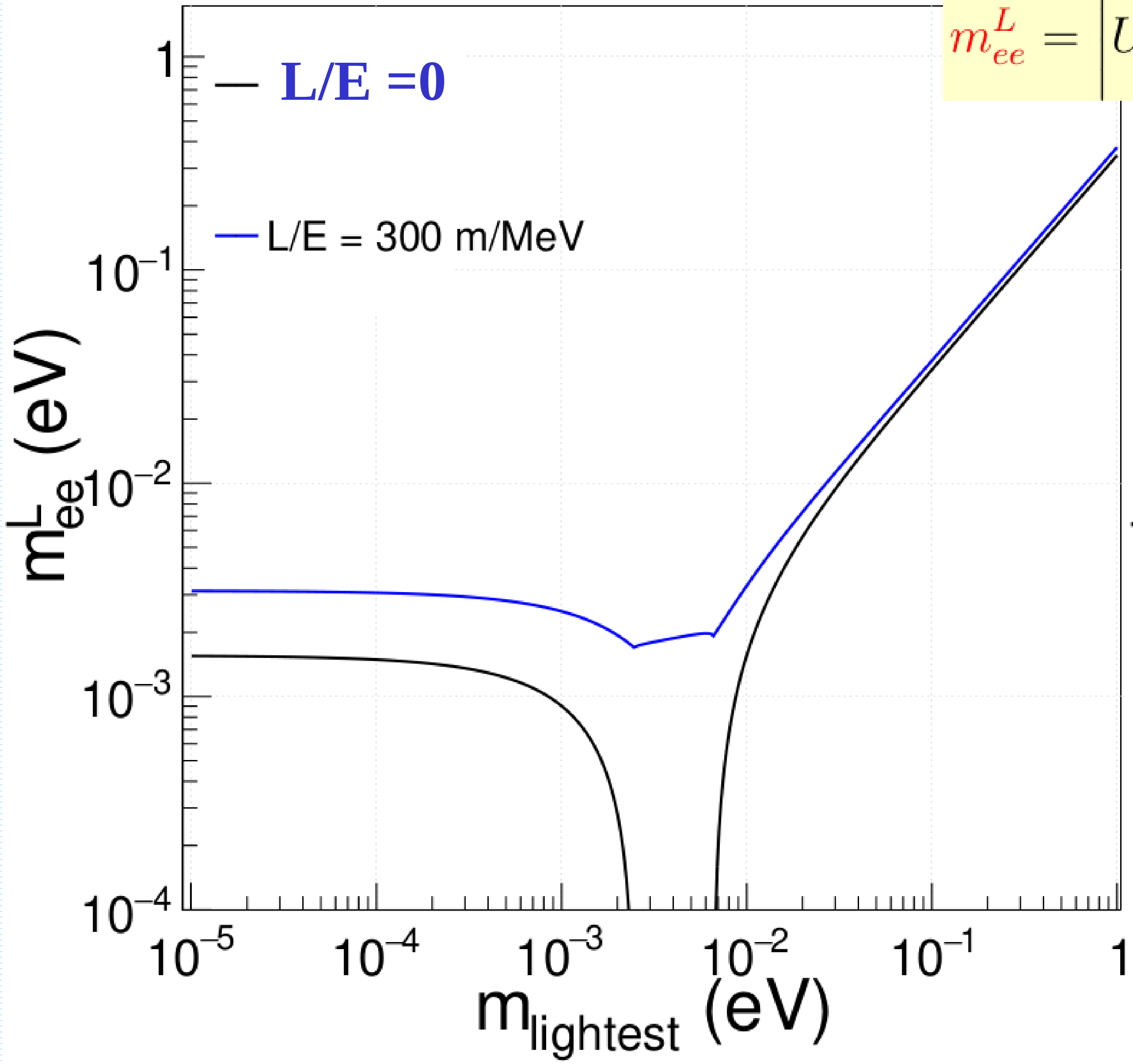
62 meV < $m_1 + m_2 + m_3$ < 69 meV

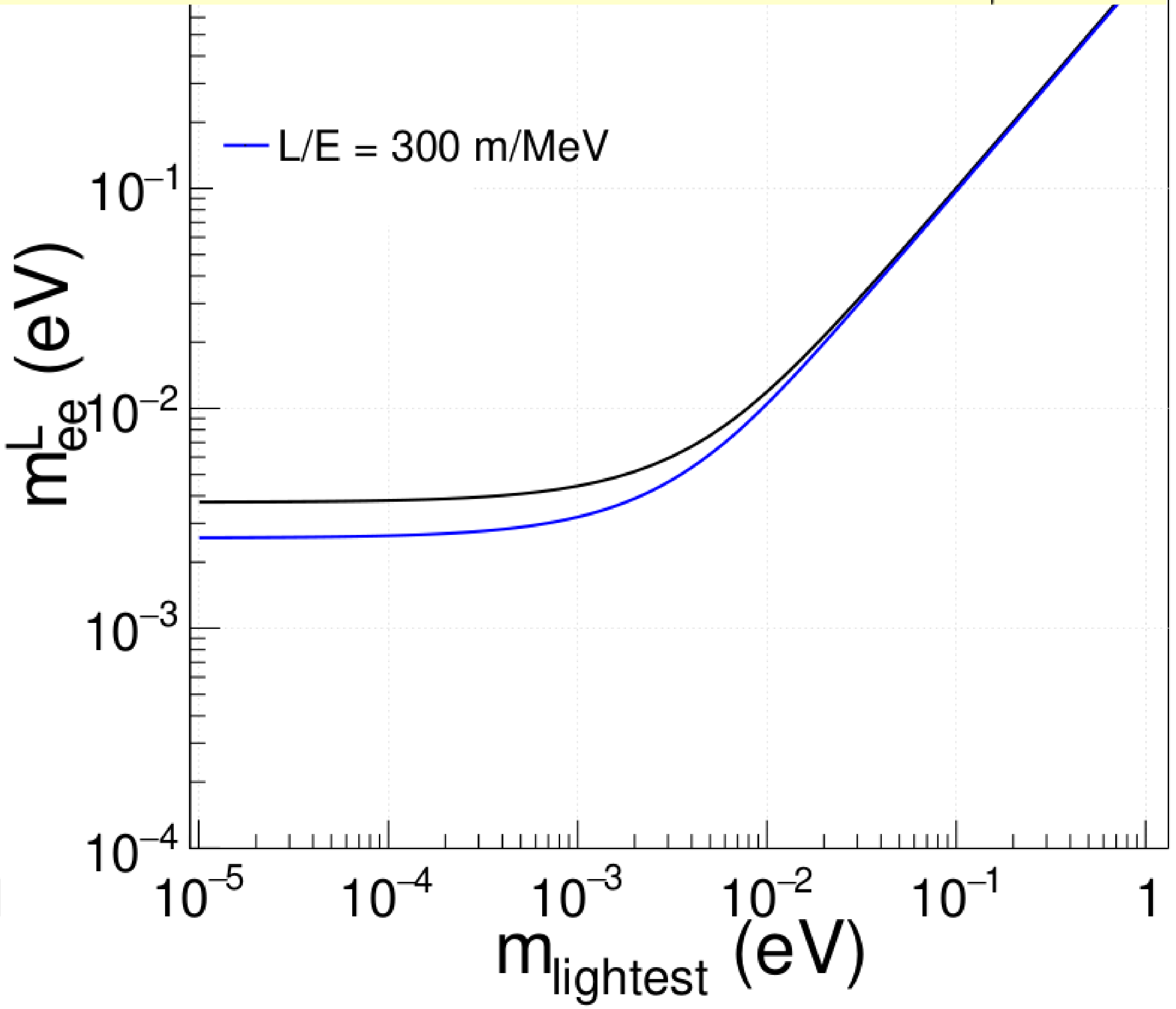
Dependence of m^{L}_{ee} on $m_{lightest}$ and L/E

$m^{L=0}_{ee} = m_{\beta\beta}$

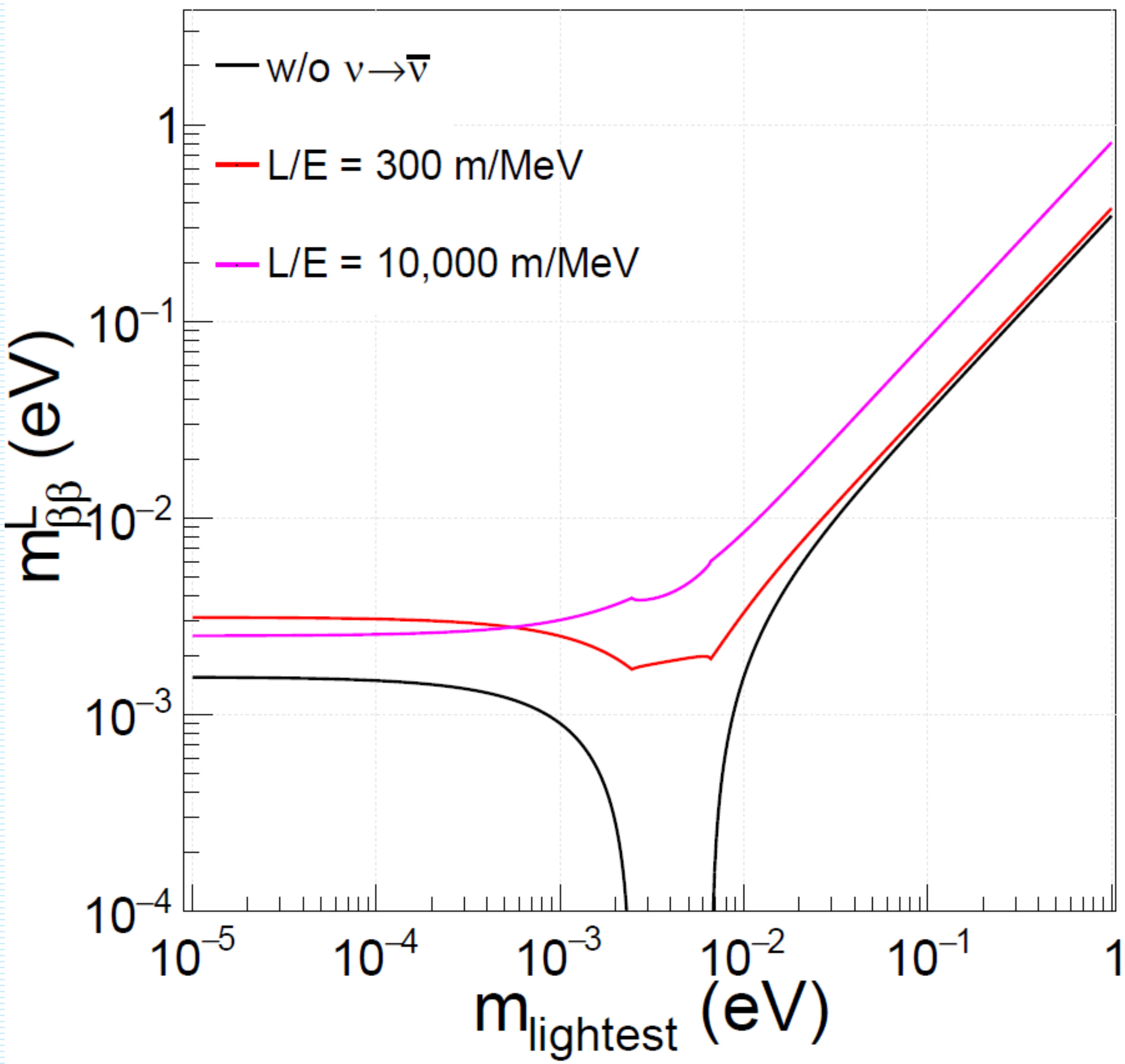
$$m_{\beta\beta} = \left|\sum_{j}^{3} U_{ej}^2\, m_j\right| = |c_{12}^2 c_{13}^2 m_1 e^{2i\phi_1} + s_{12}^2 c_{13}^2 m_2 e^{2i\phi_2} + s_{13}^2 m_3|$$

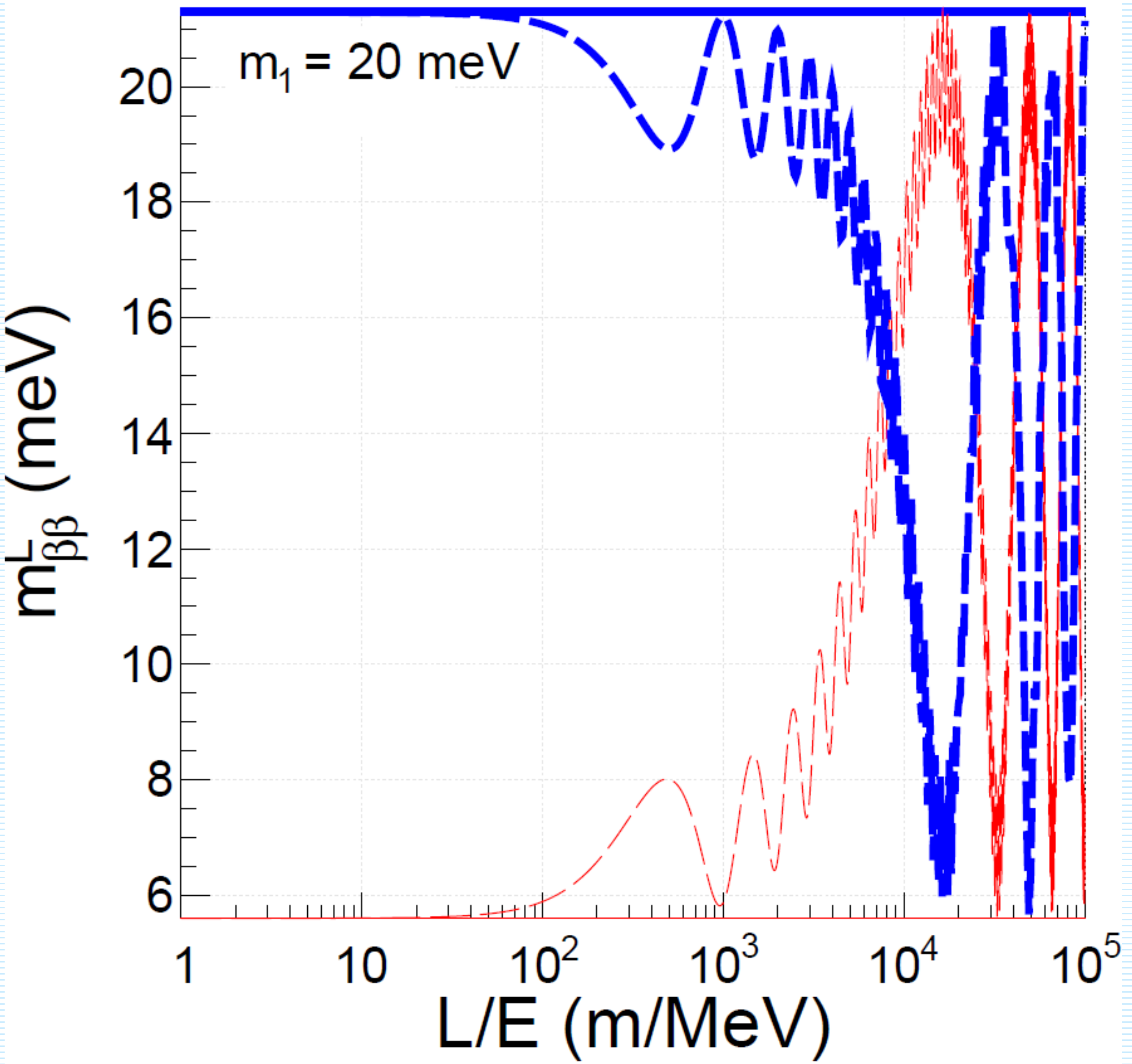
$$m_{ee}^{L} = \left|U_{e1}^2\, m_1 \; + U_{e2}^2\, m_2\; e^{-i\frac{\Delta m_{21}^2}{2E}L} + U_{e3}^2\, m_3\; e^{-i\frac{\Delta m_{31}^2}{2E}L}\right|$$





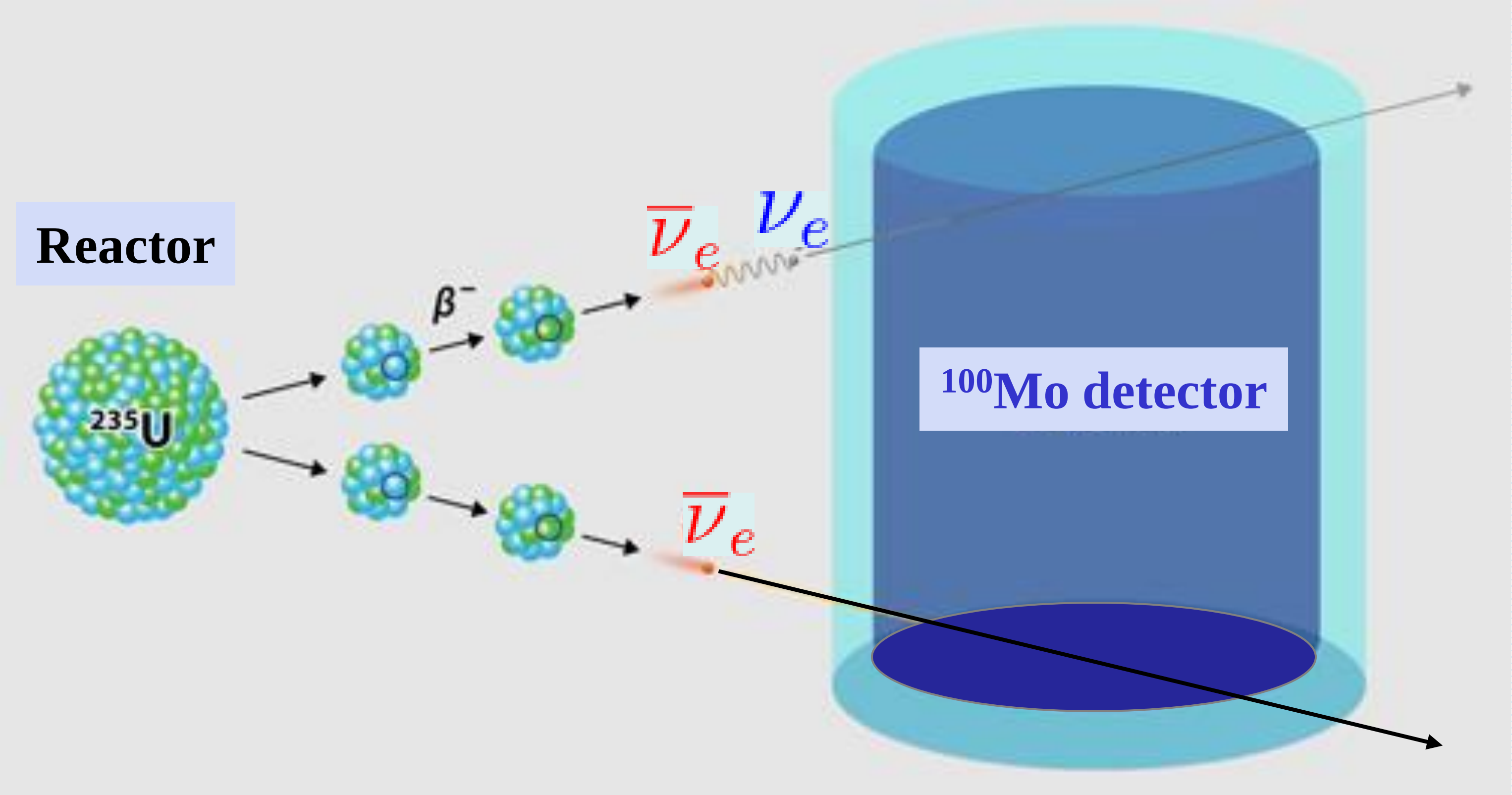
Dependence of m^L_{ee} on $m_{lightest}$ and L/E





Symmetry 14, 1383 (2022)

Oscillations of reactor antineutrinos to neutrinos



$$\nu_e + ^{100} Mo \rightarrow ^{100} Tc^* + e^-$$
$$^{100}Tc^* \rightarrow ^{100} Tc + \gamma' s$$
$$^{100}Tc \rightarrow ^{100} Ru + e^- + \overline{\nu}_e$$

M_T = 1 ton
W = 1 Gwatt
L = 10 m
$F(Z,E_e)$ B_{GT} = 5 x 0.33
$\mu \approx 100$ g mol^{-1}

$$R_L^{\overline{\nu}\nu} = 5.33 \times 10^{-13} \left(\frac{M_{\mathrm{T}}}{1\mathrm{kg}}\right) \left(\frac{1\mathrm{g}\cdot\mathrm{mol}^{-1}}{\mu}\right) \times \left(\frac{m_{\beta\beta}}{1\mathrm{eV}}\right)^2 \left(\frac{\mathrm{W}}{1\mathrm{GWatt}}\right) \left(\frac{1\mathrm{m}}{\mathrm{L}}\right)^2 F(Z,E_e)B_{GT} \left(\frac{\mathrm{events}}{\mathrm{s}}\right)$$

$$R_L^{\overline{\nu}\nu} \approx 10^{-6} \left(\frac{m_{\beta\beta}}{1\mathrm{eV}}\right)^2 \left(\frac{\mathrm{events}}{\mathrm{year}}\right)$$

Towards fixing parameters of ν mass matrix

Phys. Atom. Nucl. 86 (2023) 5, 709-724
Phys. Part. Nucl. Lett. 21 (2024) 1, 1-4

Fitting 9 parameters: 6 of neutrino mixing matric plus 3 masses, assumption NH or IH

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Fitting 9 parmeters of 3x3 Majorana neutrino mass matrix

$$\mathbb{M}_+ = \begin{vmatrix} M^{\mathrm{M}}_{ee} & M^{\mathrm{M}}_{e\mu}\, e^{i\phi_{e\mu}} & M^{\mathrm{M}}_{e\tau}\, e^{i\phi_{e\tau}} \\ M^{\mathrm{M}}_{e\mu}\, e^{i\phi_{e\mu}} & M^{\mathrm{M}}_{\mu\mu} & M^{\mathrm{M}}_{\mu\tau}\, e^{i\phi_{\mu\tau}} \\ M^{\mathrm{M}}_{e\tau}\, e^{i\phi_{e\tau}} & M^{\mathrm{M}}_{\mu\tau}\, e^{i\phi_{\mu\tau}} & M^{\mathrm{M}}_{\tau\tau} \end{vmatrix}$$

Neutrino flavor states are projected onto mass states with Frobenius covariants

$$\mathbb{M}_+^\dagger = \mathbb{M}_+^* = \mathbb{M}_-$$

$$\mathbb{F}_{r\pm} \equiv |r\pm\rangle\langle r\pm| = \prod_{s\neq r} \frac{\mathbb{M}_\pm \mathbb{M}_\mp - \lambda_s}{\lambda_r - \lambda_s}$$

All the processes can be rewritten with Frobenius covariants (instead of mixing matrices)

$$p(\lambda) = \det ||\lambda - \mathbb{M}_\pm \mathbb{M}_\mp|| = 0$$

Tri-bimaximal mixing model of Majorana neutrinos

Phys. Atom. Nucl. 86 (2023) 5, 709-724
Phys. Part. Nucl. Lett. 21 (2024) 1, 1-4

ν mass matrix

$$\mathbb{M}_L = \begin{bmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{bmatrix}$$

$$x = \frac{2}{3}m_1 + \frac{1}{3}m_2$$
$$y = -\frac{1}{3}m_1 + \frac{1}{3}m_2$$
$$v = -\frac{1}{2}m_1 + \frac{1}{2}m_3$$

Frobenius covariants

$$\mathbb{F}_{1\pm} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 1/6 & 1/6 \\ -1/3 & 1/6 & 1/6 \end{bmatrix}$$

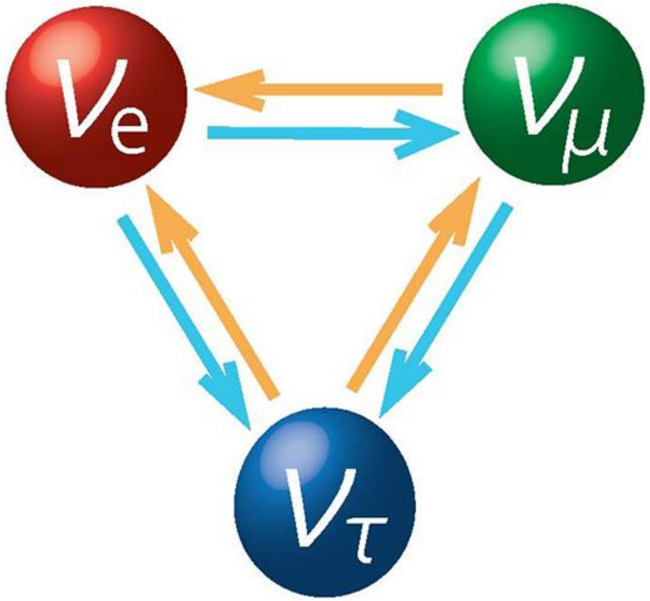
$$\mathbb{F}_{2\pm} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\mathbb{F}_{3\pm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$$

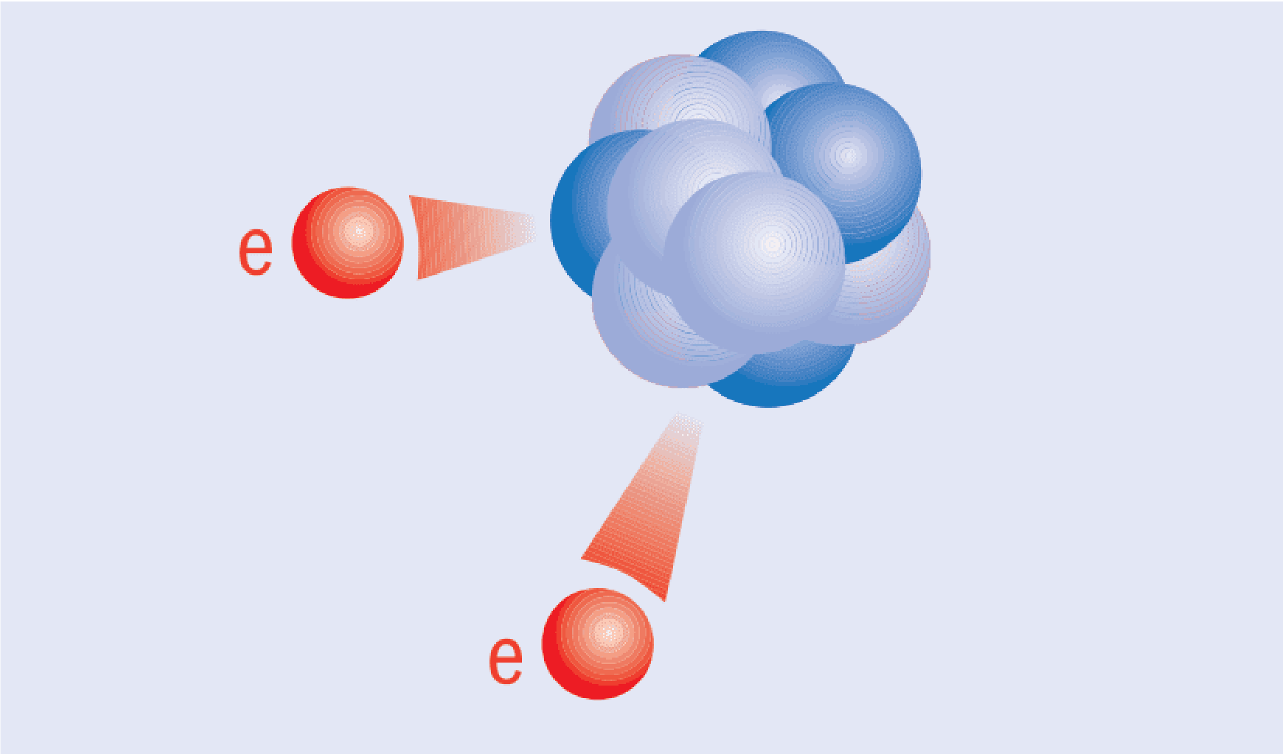
tri-bimaximal mixing matrix

$$\mathbb{U} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix}$$


Time flies when
you are having fun.
Albert Einstein
quotefancy


ν_e
ν_μ
ν_τ


THANK YOU!


e
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