Lepton Flavor Physics: Theoretical Aspects

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【IHEP Beijing】

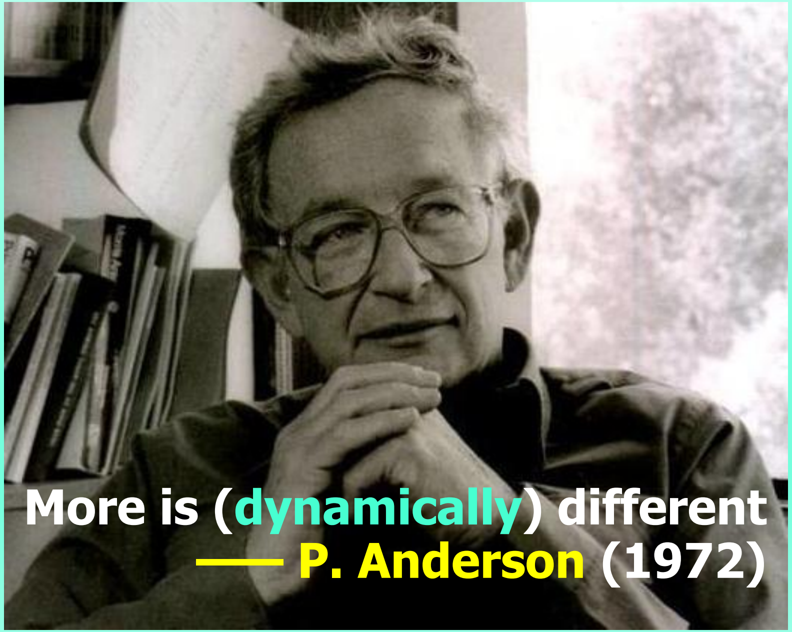
- *Historical roles of lepton flavors*
- *Origin of small neutrino masses*
- *Possible lepton favor symmetry*
- *Charged lepton flavors can help*

A personal + incomplete overview

Neutrino Oscillation Workshop, Otranto, Lecce, 2—8.9.2024

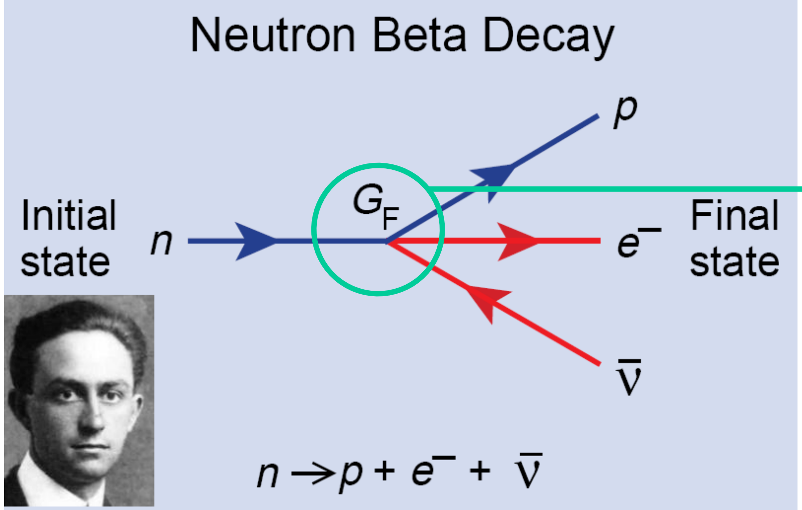
Part A

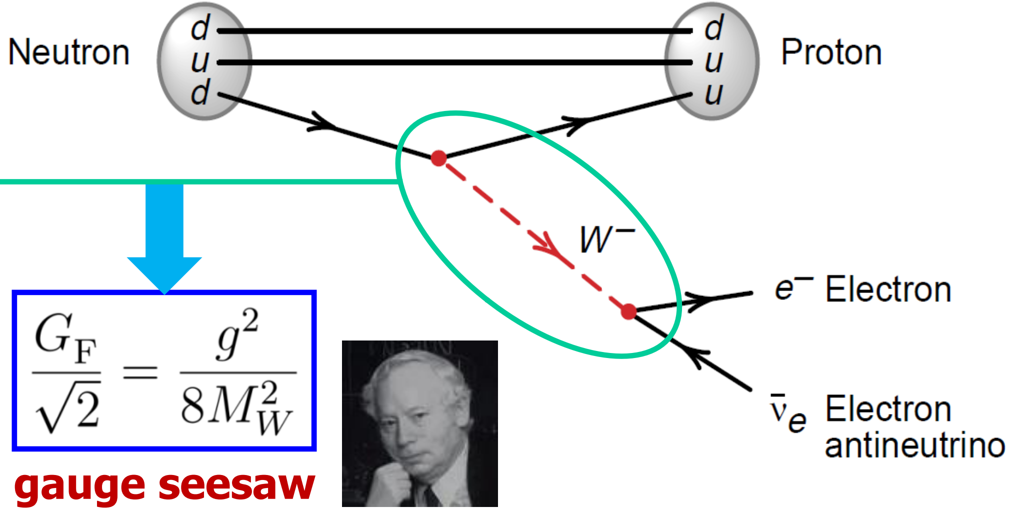
Historical roles of lepton flavors



A role of the "1 G" leptons

- **Fermi's EFT for beta decays with "1 G" leptons and quarks (1933/1934):**





$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

gauge seesaw

Fermi coupling constant

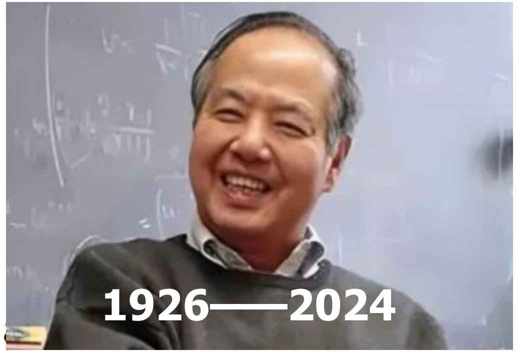
$$G_F \simeq 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Weak interaction coupling constant

$$g \simeq 0.65 \quad \textbf{vs} \quad M_W \simeq 80.4\ \mathrm{GeV}$$

A good lesson: a small effective quantity at low energies is very likely to originate from some new and heavy degrees of freedom in a more fundamental theory at much higher energy scales. History repeats itself, as we will see again and again.

Fermi's intuition is a mystery



In 2001 Fermi's PhD student T.D. Lee made the remarks on Fermi's EFT for beta decays 〖Int. J. Mod. Phys. A 16 (2001) 3633——3658, Review article〗:

$$G_{\rm F}(\overline{\psi}_p\,\gamma_\mu\,\psi_n)(\overline{\psi}_e\,\gamma^\mu\gamma_5\,\psi_\nu) \leftarrow$$ **Fermi (1933/1934)**

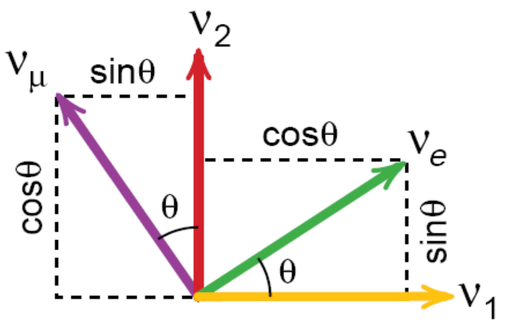
V **A**

Fermi told me that his interaction was modelled after the electromagnetic forces between charged particles, and his coupling G was inspired by Newton's constant. His paper was, however, rejected by *Nature* for being unrealistic. It was published later in Italy, and then in *Zeitschrift für Physik*.[13] Fermi wrote his γ matrices explicitly in terms of their matrix elements. His lepton current differs from his hadron current by a γ_5 factor; of course the presence of this γ_5 factor has no physical significance. Nevertheless, it is curious why Fermi should choose this particular expression, which resembles the V–A interaction, but with parity conservation. Unfortunately, by 1956, when I noticed this, it was too late to ask Fermi.

Parity violation (1956/1957) → V—A theory (1958) → Electroweak theory (1967)

A role of the "2 G" leptons

♦ In **1962** all the four "2 G" lepton members went home, making it possible to consider **lepton flavor mixing** (**Z. Maki**, **M. Nakagawa**, **S. Sakata**):



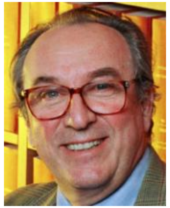
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

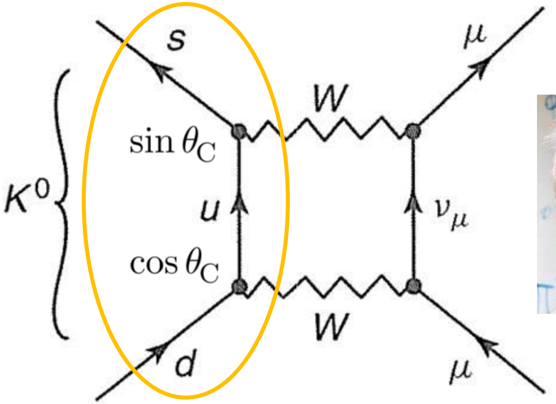
In 1967, B. Pontecorvo formulated the neutrino oscillation probability

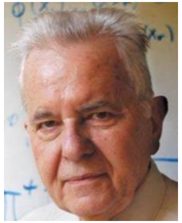
♦ In **1964** the **lepton-quark symmetry** motivated **J. Bjorken** and **S. Glashow** to propose a new quark "**charm**" with respect to ν_μ .

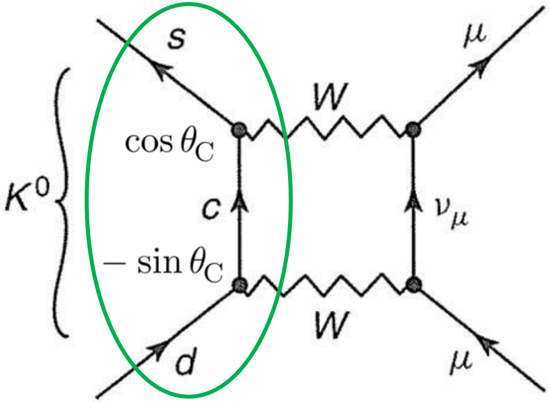
More is dynamically different

♦ In 1970, S. Glashow, J. Iliopolous and L. Maiani found that the SU(4) quark model could successfully suppress the FCNC effects of the SU(3) quark model, improved by incorporating the Cabibbo flavor mixing —— the GIM mechanism.





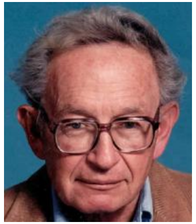




Hidden new and heavy degrees of freedom:

$$m_c \sim 2\ \mathrm{GeV}$$

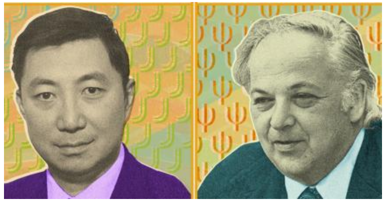
More = New dynamics?



P. Anderson

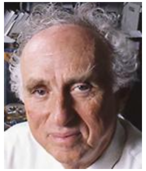
More is different (1972)

♦ In November 1974, the charm quantum number was independently discovered by S. Ting and B. Richter. A brand new "GeV" era began, calling for much higher energy machines to produce new heavy particles.



A role of the "3 G" leptons

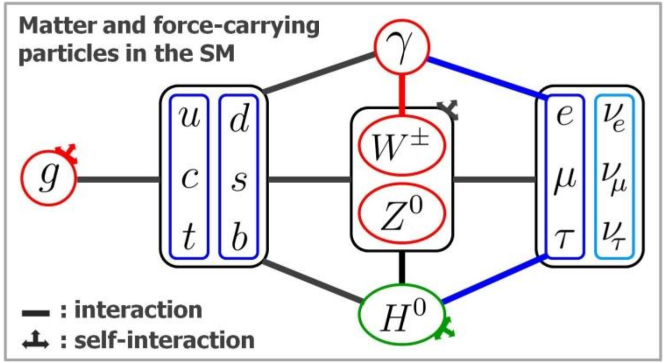
♦ In **1975** the **third** and **heaviest** charged lepton —— τ lepton was discovered by **M. Perl**, opening the "3 G" era of leptons and quarks.



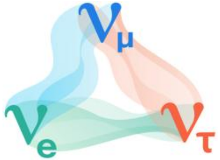
♦ **Fermilab:** let's do the rest on behalf of **Fermi**.

- ♦ **1977**: the bottom quark
- ♦ **1995**: the top quark
- ♦ **2001**: the tau neutrino

Then the "3 G" picture of fermions is complete.



♦ **The probabilities** of the standard **3-flavor** neutrino oscillations with **CP or T violation** and matter effects were first formulated by **V. Barger**, **K. Whisnant**, and **R. Phillips** in **1980**.



♦ **A global analysis** of various neutrino oscillation data in the standard **3-flavor** scheme was first made by **G. Fogli**, **E. Lisi** and **D. Montanino** in **1994** —— *proof of concept* to show its potential (predictive) power!

Weinberg's 3rd law

♦ **Going beyond the SM in the flavor sector may naturally mean going beyond the "3 G" paradigm of fundamental fermions, especially the "3 G" neutrinos, as motivated by the understanding of *neutrino mass generation* or by explaining some *puzzling anomalies*.**

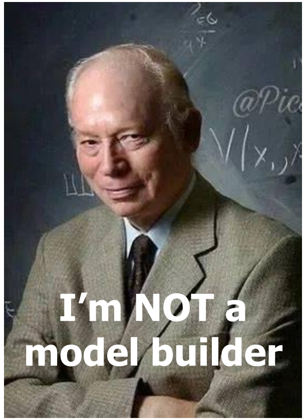
sterile species

- ♦ **3 + 1: light (eV, keV), LSND, warm DM….**
- ♦ **3 + 2: heavy (the minimal seesaw)**
- ♦ **3 + 3: heavy (the canonical seesaw)**
- ♦ **3 + 6: the double or inverse seesaw**
- ♦ **3 + n: arbitrary number and mass scales**

♦ **S. Weinberg's third Law of Progress in Theoretical Physics (1983):**

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry.

"more" maybe stupid



♦ **A good lesson: the history of particle physics tells us that a *real* new degree of freedom must be able to help solve at least one fundamental problem and make the theory more natural, exact and powerful.**

Part B

Origin of small neutrino masses

Going beyond SM: ν's = Majorana

♦ **Fundamentals** of the electroweak SM structure → **reasons** for zero ν-masses:

- ♦ Quantum mechanics + Lorentz invariance
- ♦ Local $SU(2)_L \times U(1)_Y$ gauge symmetries
- ♦ The Higgs mechanism
- ♦ Renormalizability (no d ≥ 5 operators)

Plus *economical* particle content:

- No right-handed neutrino fields
- Only one Higgs doublet

$$m_1 = m_2 = m_3 = 0$$

Go beyond the d=4 operators

SMEFT

The "unique" d=5 operator

$$\mathcal{O}_{\rm W} = \frac{\overline{\ell_{\rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\rm L}}{\Lambda}$$

S. Weinberg 1979

E. Majorana 1937

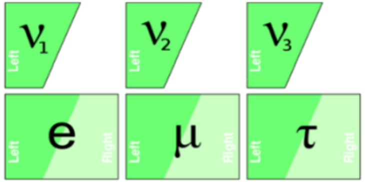
SSB → ν-masses; LNV → ν's = the Majorana fermions

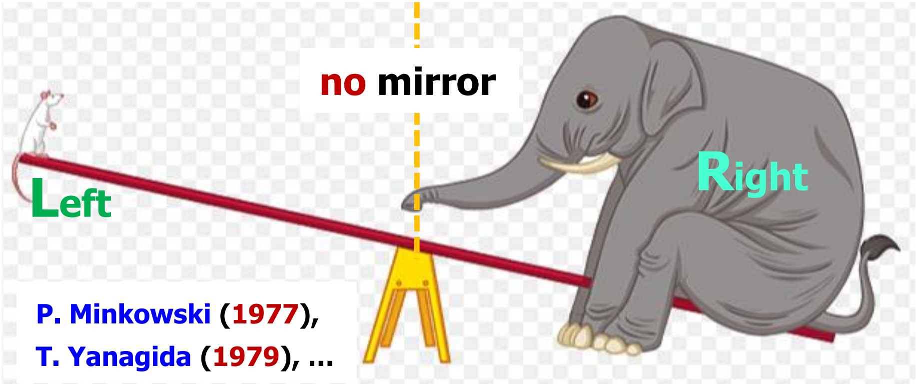
Supported by ν-oscillations ← ν-masses

ν's = the Majorana fermions

The simplest UV realization of $\mathcal{O}_{\rm W}$

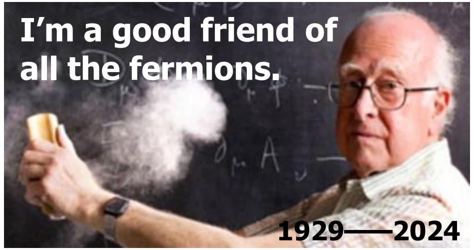
- Right-handed neutrino fields are added, not mirror counterparts of left-handed ones.





N_1 N_2 N_3

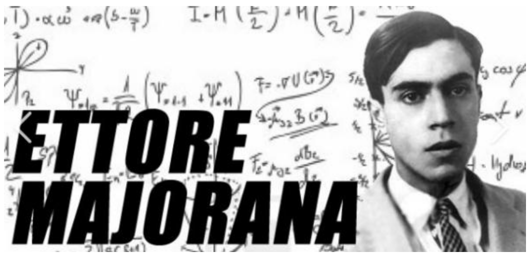
- Yukawa interactions —— the Higgs fields play a crucial role, as they do in generating masses for the charged fermions in the SM



- The Majorana nature of massive neutrinos:
  N and *N*^{c} may have *self-interactions*, respecting all the fundamental symmetries of the SM.

Gell-Mann's totalitarian principle (1956)
Everything not forbidden is compulsory!

$$\frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R}$$



- More is different: the Majorana fermions are new physics and a new form of matter!

Seesaw works before and after SSB

- The **seesaw** mechanism formally works above the **Fermi** scale **before SSB** (**ZZX 2023**)

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\left[\nu_{\rm L} \quad (N_{\rm R})^c\right]} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

- The **basis transformation** for the origin of three active **Majorana** neutrino masses:

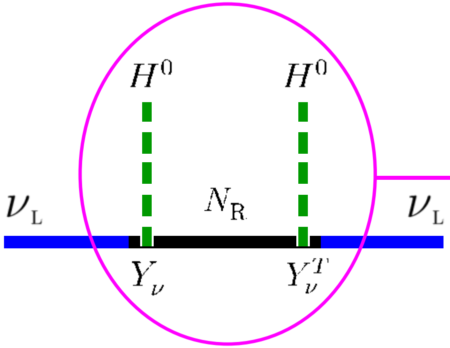
$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R} \end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

working masses:
$D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$ **light**
$D_N \equiv {\rm Diag}\{M_4, M_5, M_6\}$ **heavy**

SSB

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix}$$

6×6 mass matrix



Integrating out the heavy degrees of freedom:

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.} \qquad M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

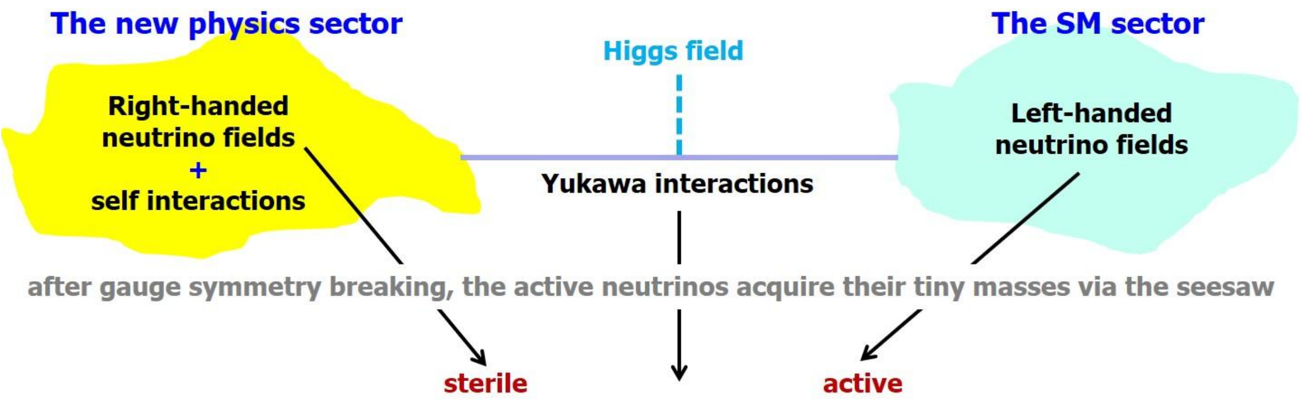
Consistent with the dim-5 Weinberg operator!

- **If you can untie Weinberg's knot, you will find *new* and *heavier* degrees of freedom**

A full parameterization of seesaw

A ***block parametrization*** **of active-sterile flavor mixing in the seesaw framework:**

- ♦ **reflects salient features of the seesaw dynamics**
- ♦ **offers *generic* + *explicit* expressions of observables using the Euler-like angles and phases (ZZX, 2012)**



Block parametrization

$$\mathbb{U} = \begin{pmatrix} I & 0 \\ 0 & U_0' \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix}$$

interplay

3 angles + 3 phases → $O_{56}O_{46}O_{45}$

$O_{23}O_{13}O_{12}$ ← **3 angles + 3 phases**

$$O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}$$

9 mixing angles + 9 CP-violating phases

The weak charged-current interactions of leptons:

U **=** ***AU***$_0$**: the PMNS matrix;**
R **: an analogue for heavy.**

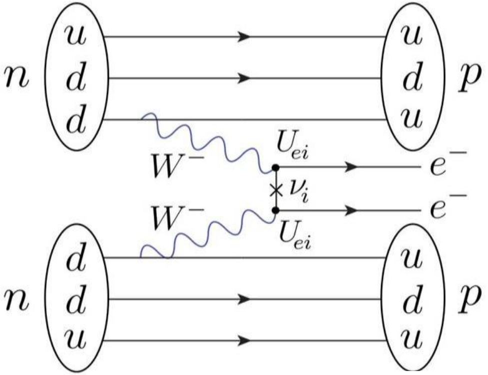
$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\; \mu \;\; \tau)_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

oscillations ⟵ light　　heavy ⟶ leptogenesis

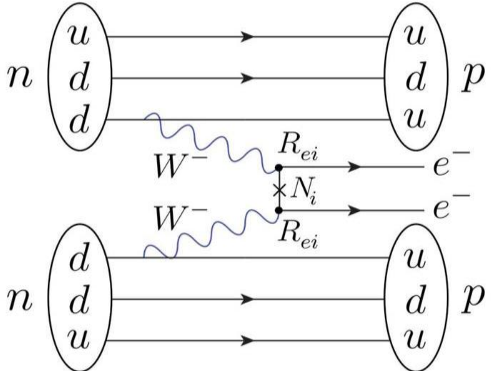
seesaw + unitarity:
$$\begin{cases} U D_\nu U^T + R D_N R^T = \mathbf{0} \\ U U^\dagger + R R^\dagger = I \end{cases}$$

To see Prof. Majorana in 0ν2β decays

- The seesaw-induced Majorana nature of massive neutrinos assure the 0ν2β decays to occur, a unique LNV place to meet Prof. Majorana.



light neutrinos



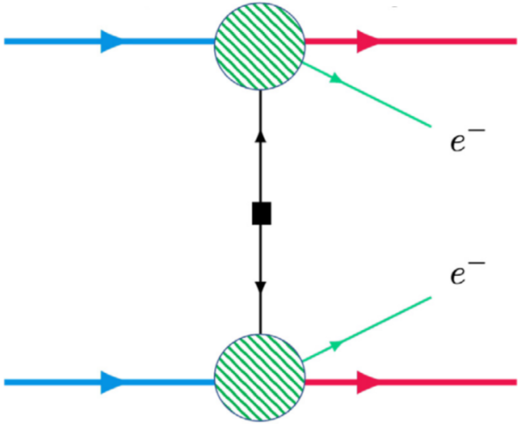
heavy neutrinos

Seesaw + Unitarity:

$$\sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{j=1}^{3} M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{j=1}^{3} |R_{ej}|^2 = 1$$

Interplay between propagators + NMEs



J.M. Yao et al, PPNP 2022

- **Stupid question**: which channel is more fundamental?
- **Correct answer**: they are equally fundamental, thanks to the Yukawa interactions (i.e., $R = 0 \implies m_i = 0$).
- In most cases, the contribution from heavy Majorana neutrinos to the 0ν2β decays are negligibly small in the seesaw mechanism (ZZX, 2009; W. Rodejohann, 2010).

Pros and cons of the seesaw

♦ **Pros A**: neutrinos have the right to be right (*handed*) to keep a left-right symmetry —— the most natural and economical extension of the SM: high gain + low costs.

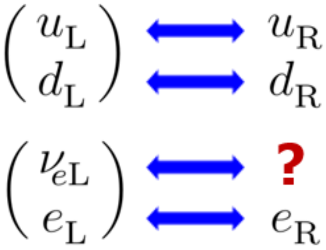
♦ **Pros B**: The Majorana mass term as new dof is highly nontrivial and has a profound effect on the SM, making the seesaw framework consistent with Weinberg's EFT.

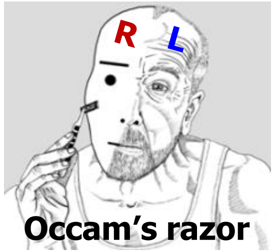
♦ **Pros C**: A big bonus is baryogenesis via leptogenesis, making it possible to kill two birds with one stone.

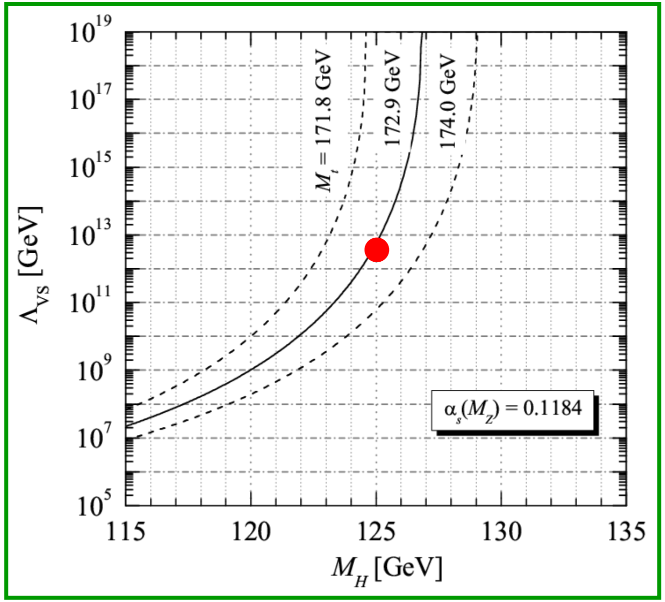
♦ **Cons A**: Naturalness of the seesaw demands its scale far above the Fermi scale, making its testability dim.

♦ **Cons B**: Seesaw-induced fine-tuning issue associated with the Higgs mass (F. Vissani 1998, Casas et al 2004, Abada et al 2007).

The scale of the SM vacuum stability seems consistent with the seesaw + leptogenesis scale —— *suggestive?* (J. Elias-Miro et al 2012, ZZX, H. Zhang, S. Zhou 2012)







The SM vacuum stability for a light Higgs

Recent TH progress for seesaw

♦ Complete one-loop matching of the seesaw onto the SMEFT (D. Zhang, S. Zhou 2021; Y. Du, X.X. Li, J.H. Yu 2022)

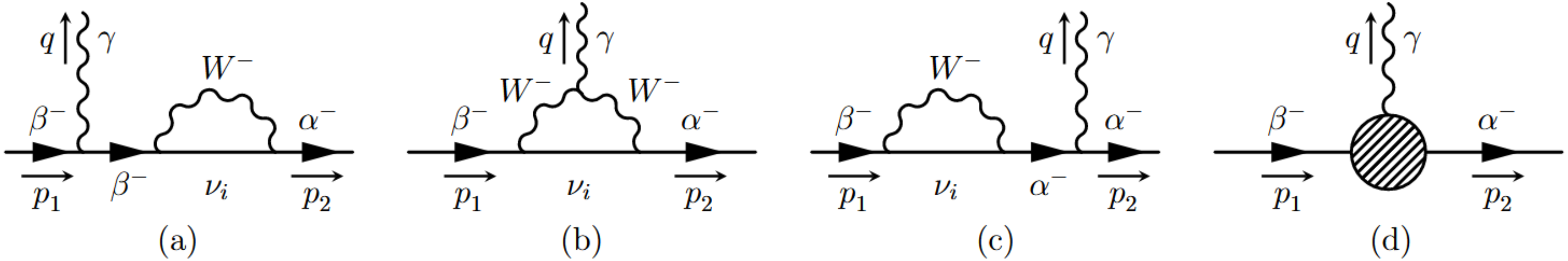


Diagram (d) is generated by the *dim-6* operator at the *one-loop* level and is crucial for the seesaw EFT to correctly calculate the *cLFV decays*, consistent with the full seesaw.

♦ Complete one-loop RGEs in the seesaw EFT framework including the effects of PMNS non-unitarity (Y. Wang, D. Zhang, S. Zhou 2023)

♦ The latest constraints on the PMNS non-unitarity (M. Blennow et al 2023, and talk to be given by E. Fernandez-Martinez)

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \;\; \mu \;\; \tau)_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

seesaw + unitarity:

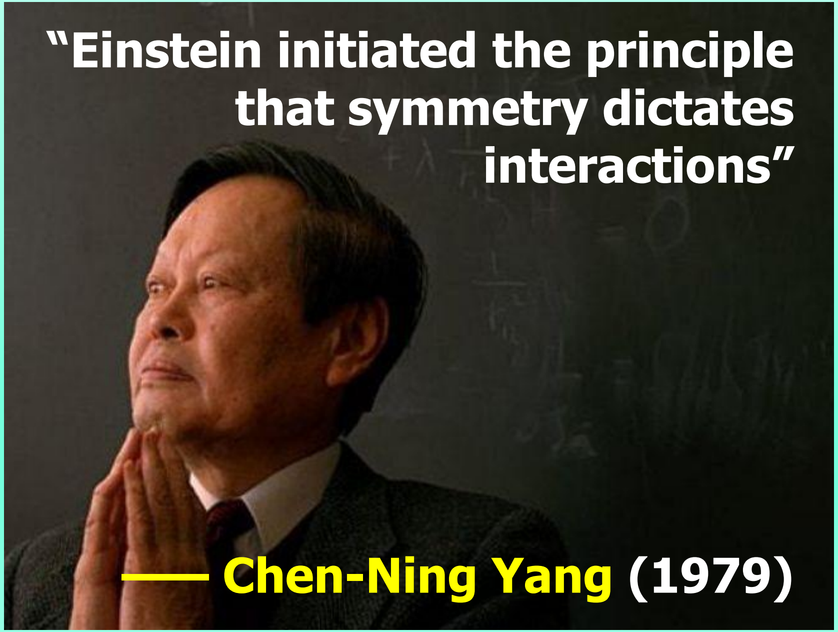
$$UD_\nu U^T + RD_N R^T = \mathbf{0}$$

$$UU^\dagger + RR^\dagger = I$$

Model-independent way to constrain the seesaw parameter space at *low* energy scales.

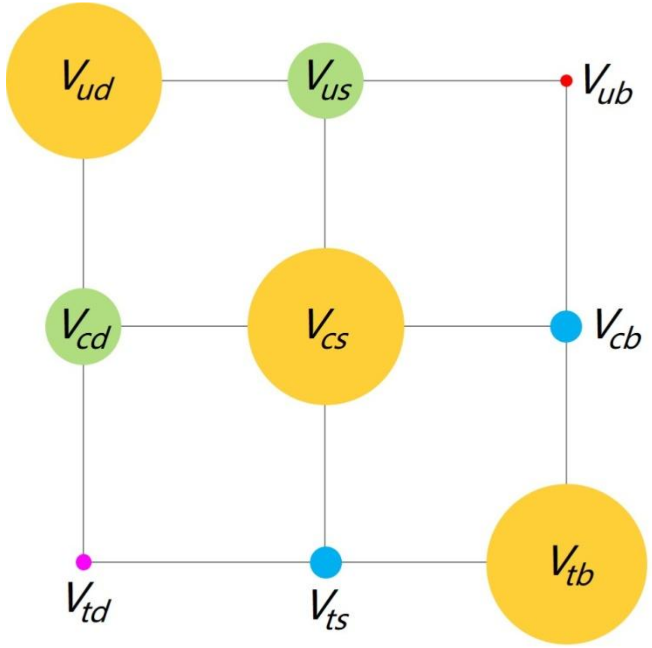
Part C

Possible lepton flavor symmetry



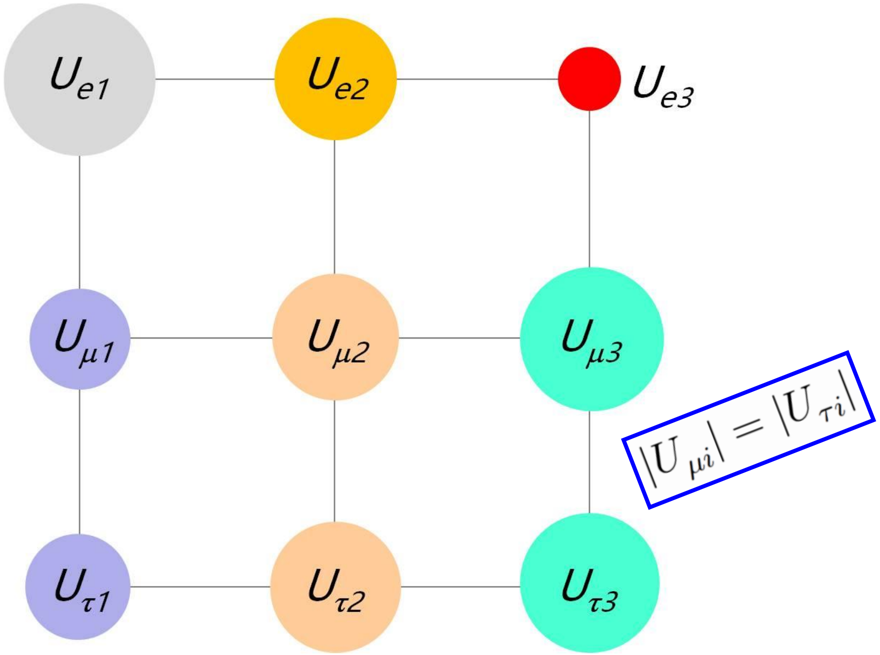
The patterns of flavor mixing

♦ **The data tell us that quarks and leptons have rather different flavor mixing patterns:**



The CKM quark flavor mixing

$$V = I + \mathcal{O}(\lesssim \theta_{\rm C})$$



The PMNS lepton flavor mixing

$$U = U_0 + \mathcal{O}(\lesssim \theta_{13})?$$

♦ **Quarks: approximate up-down parallelism**

♦ **Leptons: approximate μ-τ interchange symmetry**

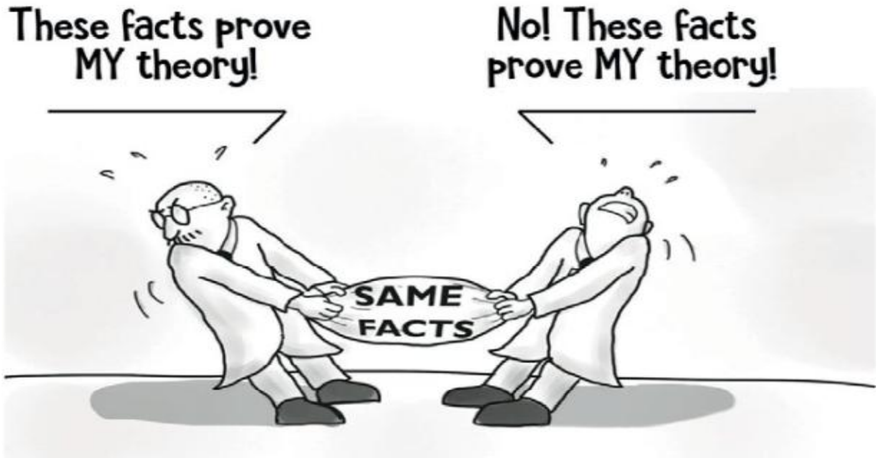
Which symmetry is closer to the truth?

♦ So far a lot of flavor symmetries have been taken into account for model building [recent reviews: ZZX 2020 (Phys. Rept.); F. Feruglio, A. Romanino 2020 (Rev. Mod. Phys.); G.J. Ding, S. King 2024 (Rept. Prog. Phys.)]

S_3 , S_4 , A_4 , A_5 , D_4 , D_7 , T_7 , T′, Δ(27), Δ(48), …
$U(1)_F$, $SU(2)_F$, modular, translational, ….

♦ What is the guiding principle? The bottom line is that the models should be compatible with data

If data is King, where is Queen?



www.nu-fit.org

(T. Schwetz et al)

♦ Almost all the flavor symmetries cannot explain tiny ν-masses. Many of them invoke the seesaw.

Modular invariance is the best seller

♦ The modular invariant model building (G. Altarelli, F. Feruglio 2006; F. Feruglio 2017)

♦ Orbifold compactification: 10D string theory → 4D SM + 3 copies of 2D torus.

♦ A single complex modulus τ is enough to parameterize the shape of torus. The modular invariant super-potential gives rise to the modular form of the Yukawa coupling matrices which depend on τ.

♦ The "*seesaw mechanism*" is invoked.

Comment A: physical meaning of the complex modular parameter τ is unclear?

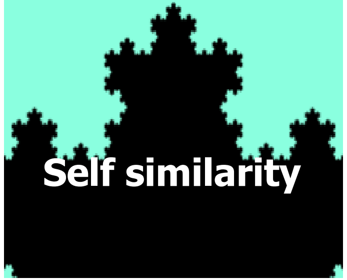
Comment B: flavor textures are not transparent due to a *nonlinear* realization of modular symmetry, and hence a careful numerical fitting has to be done?

Comment C: no good reason for a strong mass hierarchy of charged fermions?

♦ In contrast, the conventional (*discrete*) flavor symmetries can linearly predict flavor mixing with CG coefficients, and thus more transparent in physics. *None is simple!*

Symmetry breaking is more subtle

- Symmetry or form invariance of a theory means that *behind it* there is something unobservable. But symmetry breaking is highly nontrivial as it usually makes things observable.



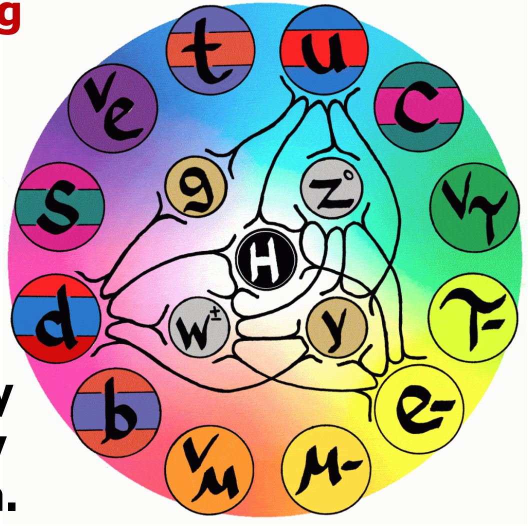
- A natural source of symmetry breaking is from *quantum corrections* from a super-high energy scale down to the Fermi scale.

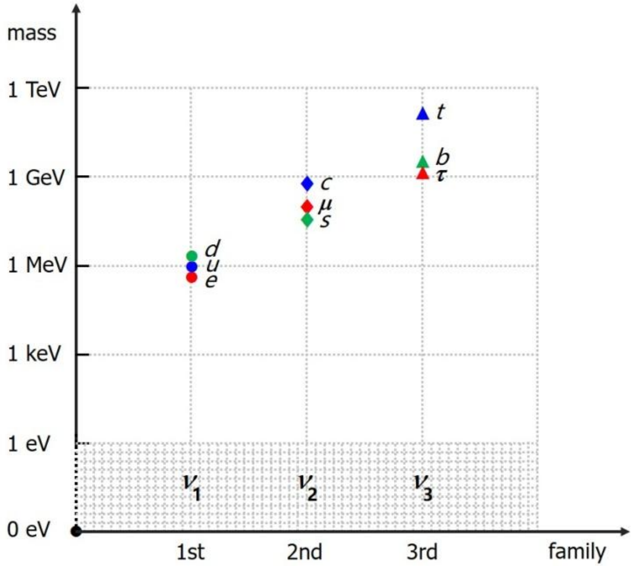
- Other ways of symmetry breaking is often of high costs and low gain.

- Examples of symmetry breaking in the SM framework:

- Parity: weak V–A structure
- Local gauge $SU(2)_L \times U(1)_Y$: the BEH mechanism
- CP violation: the KM phase

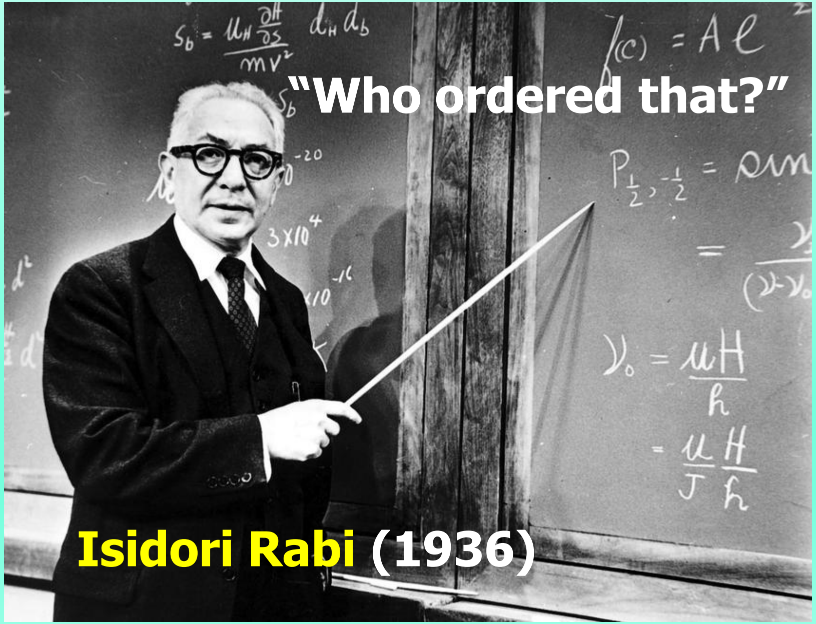
- The flavor sector involves many free parameters, and we can only qualitatively understand the data.





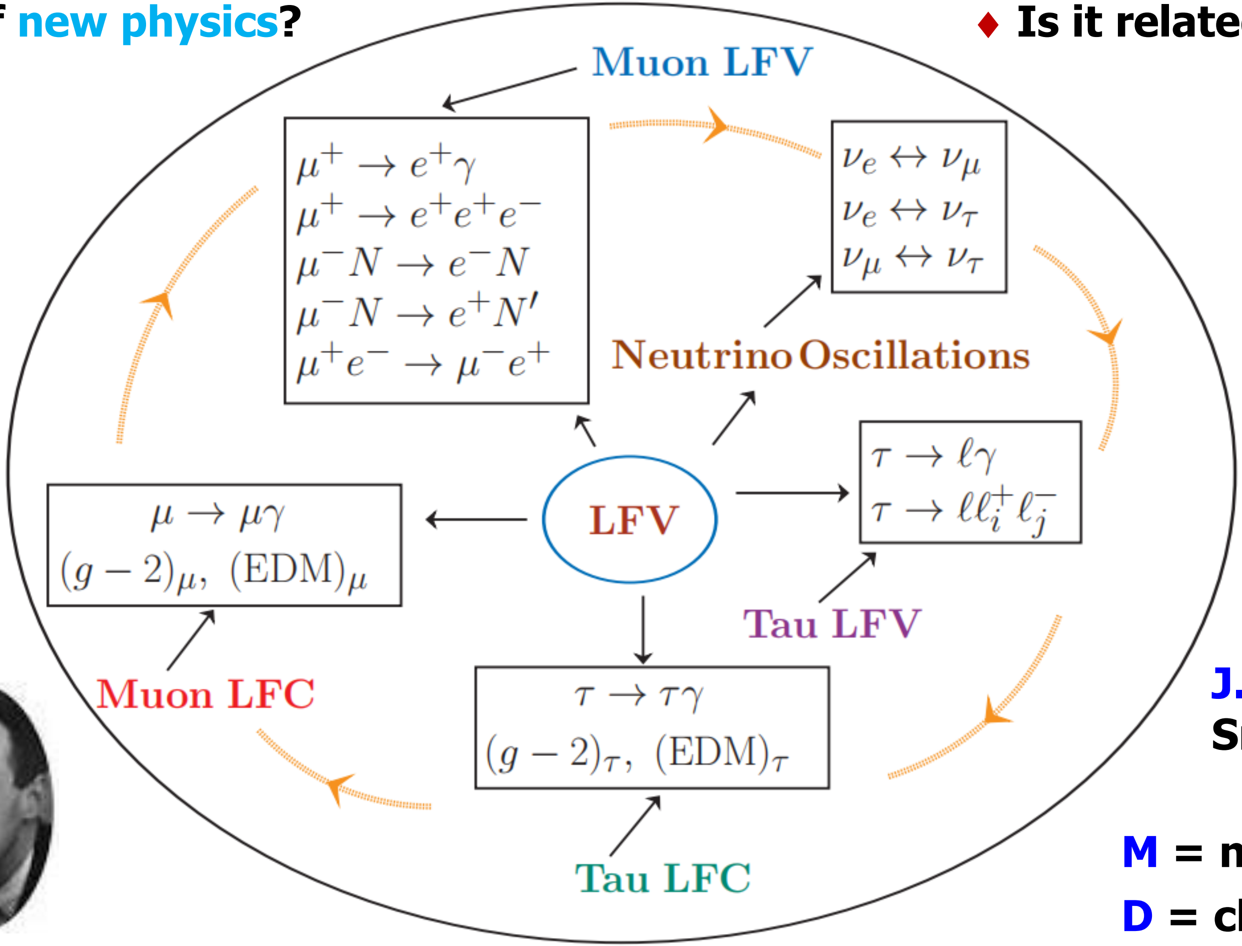
Part D

Charged lepton flavors can help



Some typical LFV vs LFC processes

- What kind of new physics?
- Is it related to ν-masses?



J. Albrecht *et al*
Snowmass 2013

M = neutrinos

D = charged leptons

The 1st **seesaw** paper was on $\mu \to e + \gamma$!

Volume 67B, number 4 PHYSICS LETTERS 25 April 1977

$\mu \to e\gamma$ AT A RATE OF ONE OUT OF 10^9 MUON DECAYS?

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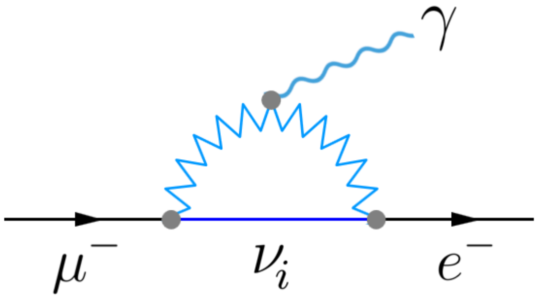
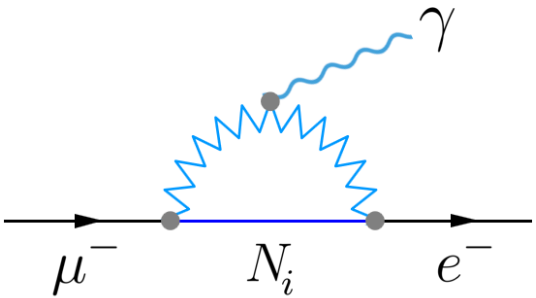
Received 28 February 1977

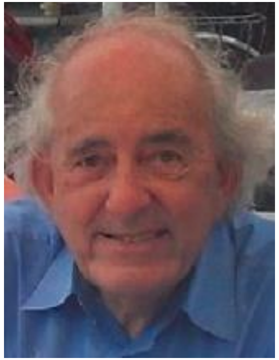
It is proposed that lepton number conservation, purely left-handed charged weak currents and vanishing neutrino masses are a limiting case of a parity symmetric $SU2_L \times SU_R \times U2^V$ gauge theory. Right-handed neutrinos acquire a lepton number violating mass, leaving an $SU2_L \times U1$ subgroup unbroken. Consequences for the decay $\mu \to e\gamma$ are studied.



light neutrinos

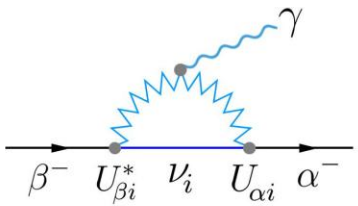


heavy neutrinos

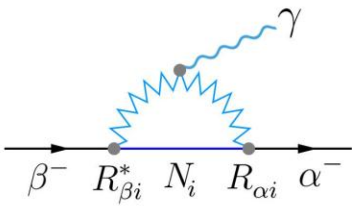


A strongest constraint on PMNS nonunitarity

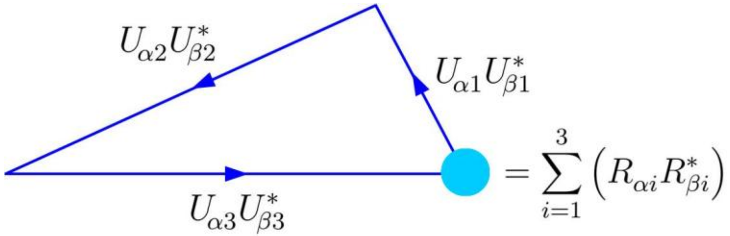
- **It can help constrain unitarity of the 3×3 *PMNS* matrix through the *cLFV* processes.**



light neutrinos



heavy neutrinos



In the full seesaw (ZZX, D. Zhang, 2009.09717) or its EFT with one-loop matching (D. Zhang, S. Zhou, 2107.12133):

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6} + \frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2 \simeq \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2$$

which allows us to constrain the unitarity hexagon using current experimental data on three radiative cLFV decays:

$$\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right| = \left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right| \simeq \sqrt{\frac{8\pi\xi_{\alpha\beta}}{3\alpha_{\rm em}}} \simeq 33.88\sqrt{\xi_{\alpha\beta}}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\mu i}\right| < 2.20\times10^{-5}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < 1.46\times10^{-2}$$

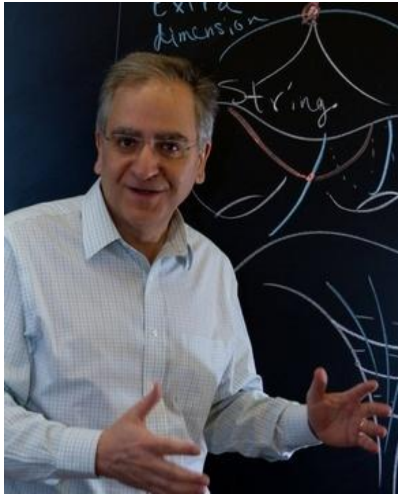
$$\left|\sum_{i=1}^{3} U_{\mu i}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{\mu i}R^*_{\tau i}\right| < 1.70\times10^{-2}$$

- **Imposing the mu-tau reflection symmetry:**

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < 2.20\times10^{-5}$$

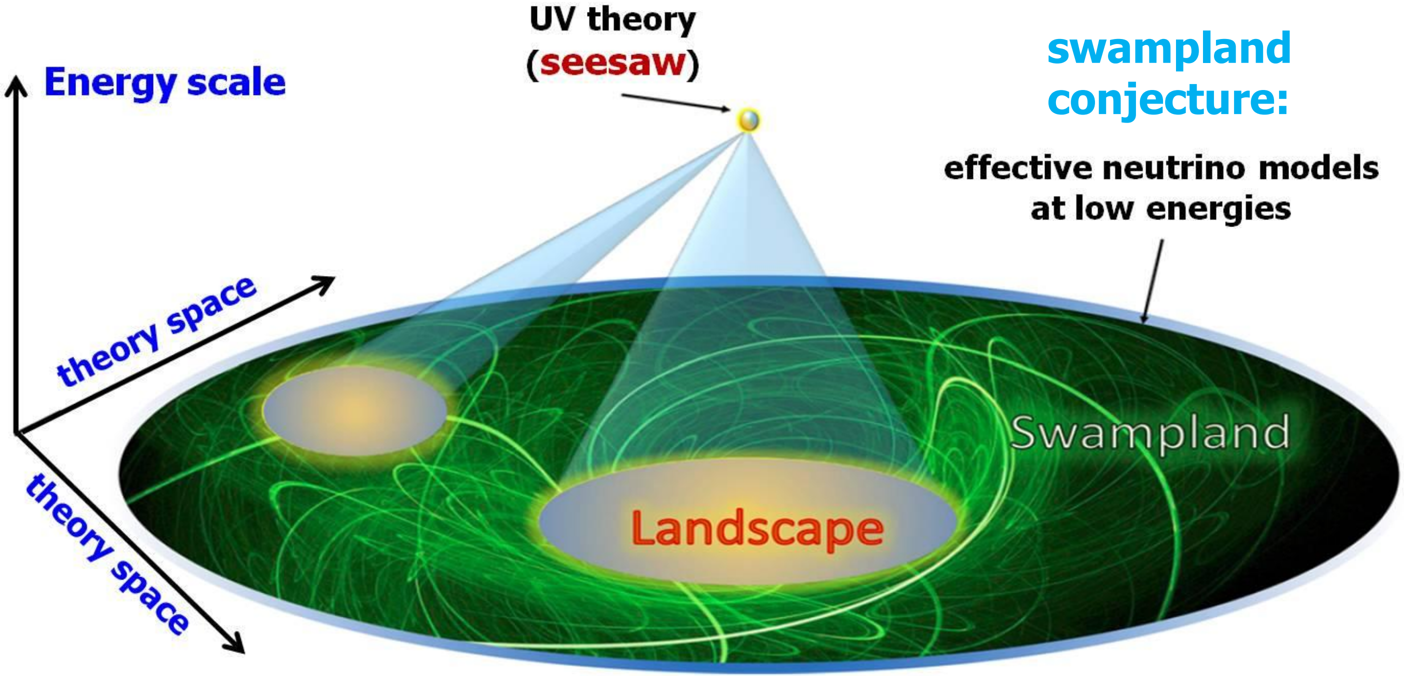
Conclusions

♦ Following the *naturalness* and *simplicity* principles to extend the SM, I foresee that the known neutrinos are Majorana fermions, and their very tiny masses originate from the seesaw mechanism. This picture is fully in agreement with the spirit of Weinberg's EFT and thus should be located in Vafa's landscape of particle physics.



Cumrun Vafa 2005

swampland conjecture:



♦ In the precision measurement era, *model-independent* TH or PH studies are needed.

An example of this kind

♦ **For the first time**, a **model-independent** expression of the **Jarlskog** invariant of **CPV** in terms of **18 original seesaw parameters** has been calculated (**ZZX**, 2406.01142, **PLB**).

Theorists
The type-I seesaw mechanism

3 heavy neutrino masses
9 active-sterile mixing angles
6 CP-violating phases

$$\mathcal{J}_\nu = \sum_{i=1}^{3} \left(C_{\alpha i} \sin \alpha_i + C_{\beta i} \sin \beta_i \right)$$

$$P(\nu_\mu \to \nu_e) = -4 \sum_{i<j} \left(\mathcal{R}_{ij} \sin^2 \frac{\Delta_{ji} L}{4E} \right) - 8 \mathcal{J}_\nu \prod_{i<j} \sin \frac{\Delta_{ji} L}{4E}$$

Experimentalists
Low-energy measurements

3 light neutrino masses
3 active mixing angles
3 CP-violating phases

♦ **To really test the seesaw, one has to calculate everything observable by use of those original seesaw parameters instead of the derivational ones or a mixture.**