

Gravitational structure of the proton

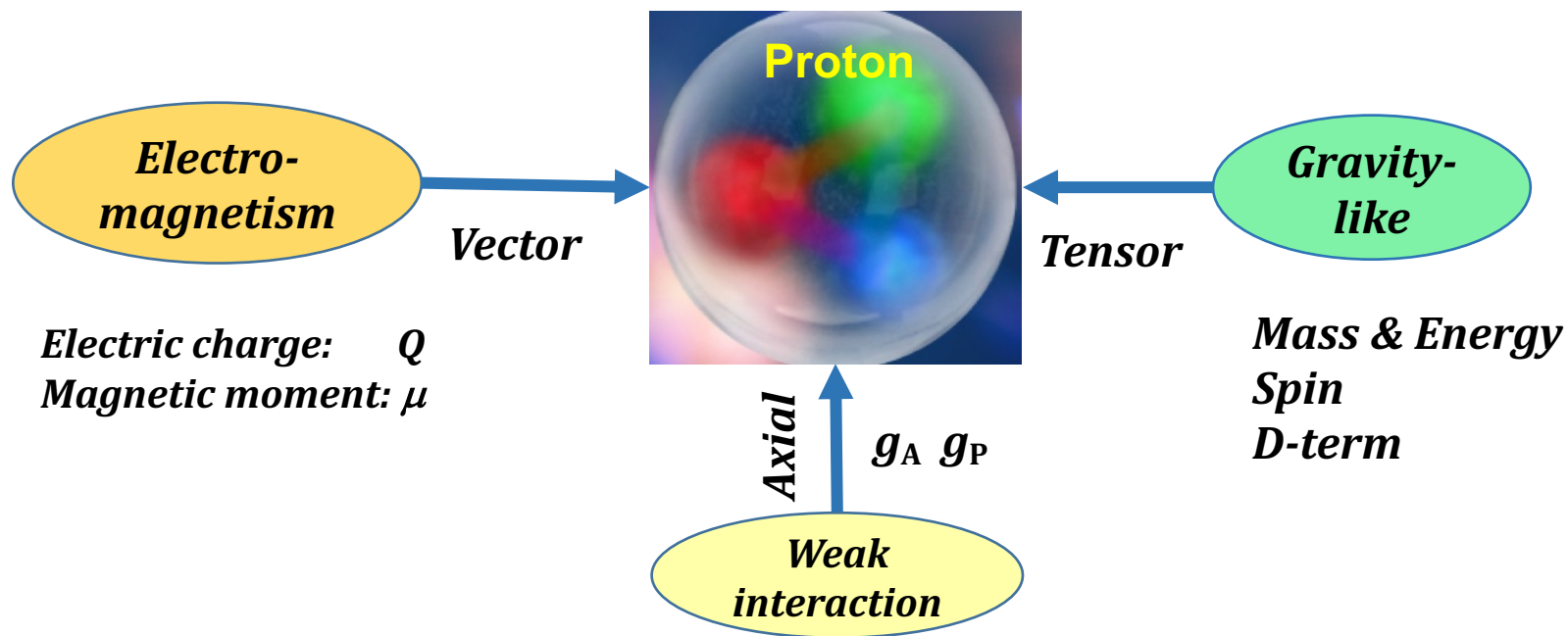
Volker D. Burkert
Jefferson Laboratory

I.Yu. Kobzarev, L.B. Okun, , Zh. Eksp. Teor. Fiz. 43 (1962)
H. Pagels, Phys.Rev. 144 (1966) 1250-1260

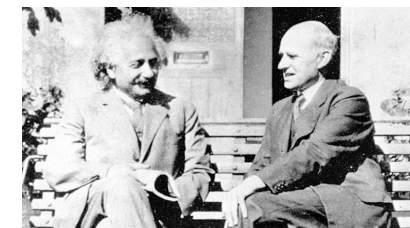
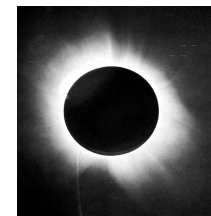
Science at the luminosity frontier: Jefferson Lab at 22 GeV
Frascati, Italy, December 9 - 13, 2024

Probing gravitational properties of the strong interaction

- The proton's internal structure has been studied through *electromagnetic and weak* interaction for over 70 years.
- What about the tensor coupling that *gravity* provides? What could we learn?



March 29, 1919



“The *D*-term is the last **unknown fundamental** global property of the proton”

M. Polyakov, P. Schweitzer

GPDs – GFFs Relations

Proton QCD matrix element of the Energy-Momentum Tensor contains three gravitational form factors (GFF) and can be written as:

$$\langle p_2 | \hat{T}_{\mu\nu}^q | p_1 \rangle = \bar{U}(p_2) \left[M_2^q(t) \frac{P_\mu P_\nu}{M} + J^q(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{2M} + d_1^q(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{5M} \right] U(p_1)$$

$M_2(t)/A(t)$: Mass & energy

$J(t)$: Angular momentum

$d_1(t)$: Forces, pressure

3 quark + 3 gluon GFF

X. Ji, Phys. Rev. D55, 7114 (1997)

$$\int dx x [\underline{H}(x, \xi, t) + \underline{E}(x, \xi, t)] = \underline{2J(t)}$$

$$\int dx x \underline{H}(x, \xi, t) = \underline{M_2(t)} + \frac{4}{5} \xi^2 \underline{d_1(t)},$$



Maxim Polyakov
1966 - 2021

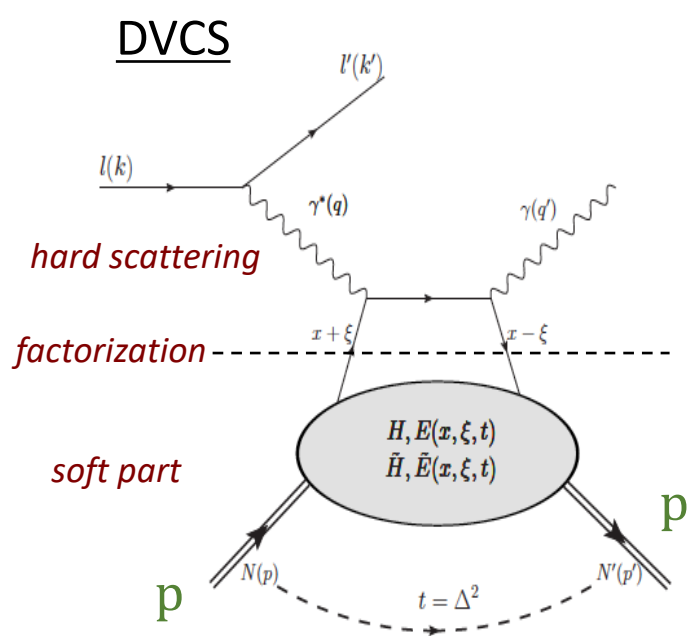
GPDs not directly accessible → integrate over the x-dependence → Compton FFs (CFFs)

Compton FFs & DVCS

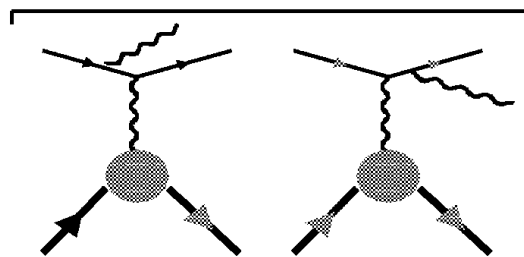
X. Ji, PRD 55 (1997) 7114 A. Radyushkin, PRD 56 (1997) 5524

$$\mathcal{H}(\xi, t) = \int_{-1}^{+1} dx H(x, \xi, t) \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

Complex valued Compton Form Factors \mathcal{H} and \mathcal{E} appear in the DVCS observables.



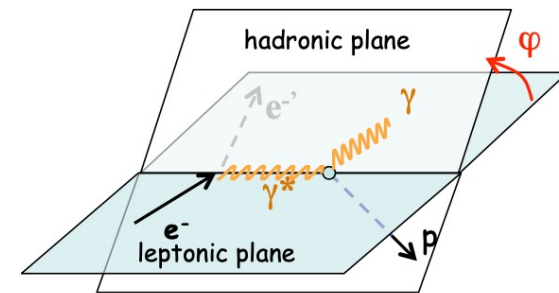
Bethe-Heitler



Elastic Form Factors

$$\xi \approx \frac{x_B}{2 - x_B}$$

DVCS-BH interference term contains $\Im m \mathcal{H}$ in the beam polarization asymmetry A_{LU} .



$$A_{LU} = \frac{d^4\sigma^{\rightarrow} - d^4\sigma^{\leftarrow}}{d^4\sigma^{\rightarrow} + d^4\sigma^{\leftarrow}} \stackrel{\text{twist-2}}{\approx} \frac{\alpha \sin \phi}{1 + \beta \cos \phi}$$

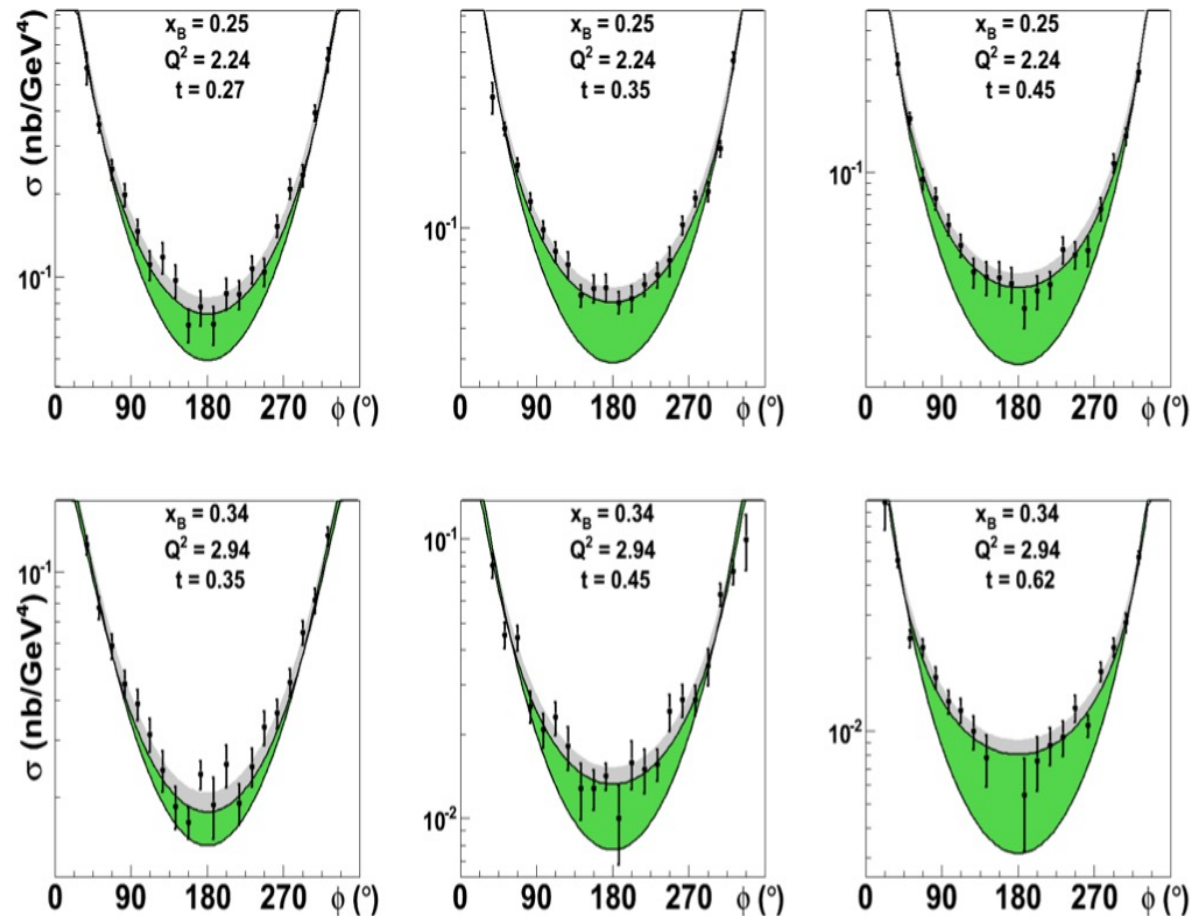
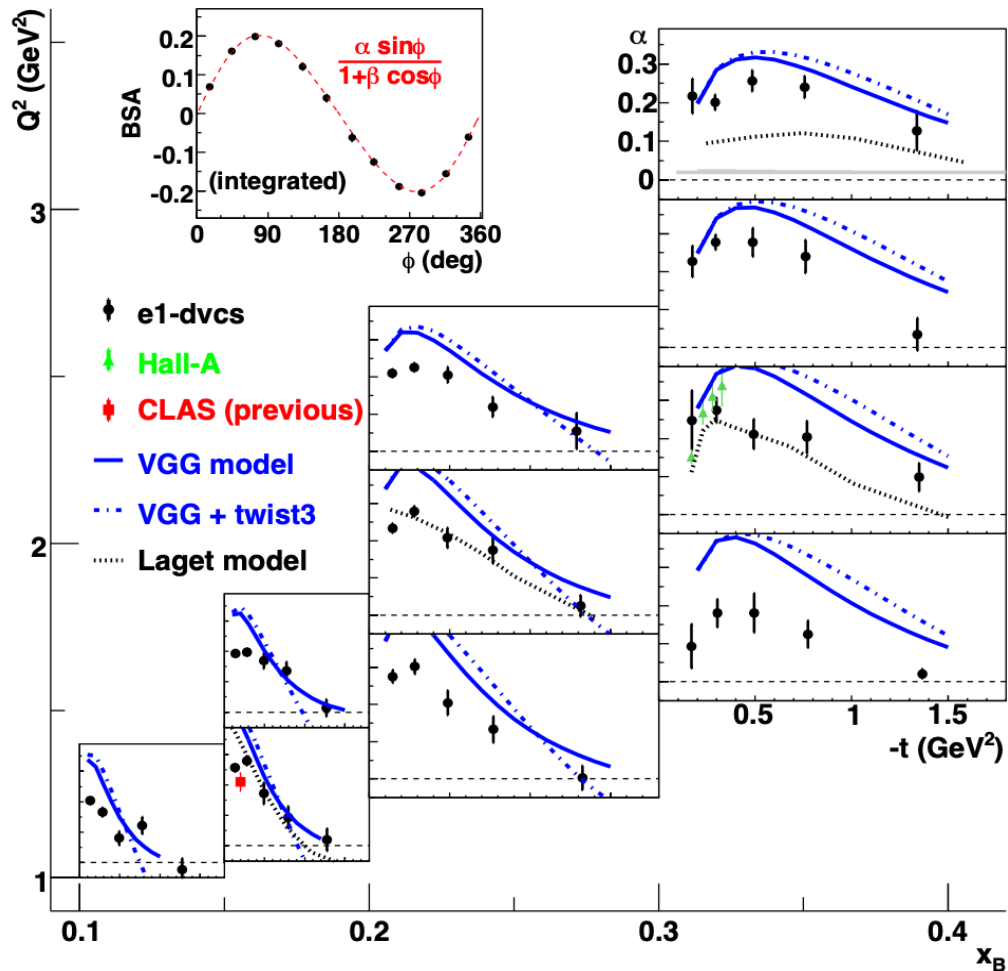
$$\alpha \propto \Im m \left(F_1 \mathcal{H} + \xi G_M \tilde{\mathcal{H}} - \frac{t}{4M^2} F_2 \mathcal{E} \right)$$

$\rightarrow \Re e \mathcal{H}$ appears in the differential cross section.

Sample DVCS BSA and cross sections

F.X. Girod et al. (CLAS), Phys.Rev.Lett. 100 (2008) 162002

H.S. Jo et al. (CLAS), Phys.Rev.Lett. 115 (2015) 21



→ Determine $\text{Im}\mathcal{H}(\xi, t)$ and $\text{Re}\mathcal{H}(\xi, t)$ from fits to BSA and the diff. cross section

First extraction of $C_{\mathcal{H}}(t)$ and the proton D-term

VB, L. Elouadrhiri, F.X. Girod, *Nature* **557**, 396–399 (2018)
 B. Pasquini, M. Polyakov, M. Vanderhaeghen, *PLB* **739** (2014) 133
 K. Goeke et al., *Phys.Rev.D* **75** (2007) 094021

Fixed-t subtracted Dispersion Relation:

$$\text{Re}\mathcal{H}(\xi, t) = C_{\mathcal{H}}(t) + \frac{1}{\pi} \mathcal{P} \int_0^1 dx \left[\frac{1}{\xi - x} - \frac{1}{\xi + x} \right] \text{Im}\mathcal{H}(x, t),$$

$$C_{\mathcal{H}}(0) = -2.27 \pm 0.16 \pm 0.33$$

$$M^2 = 1.02 \pm 0.13 \text{ GeV}^2$$

$$\alpha = 2.76 \pm 0.23$$

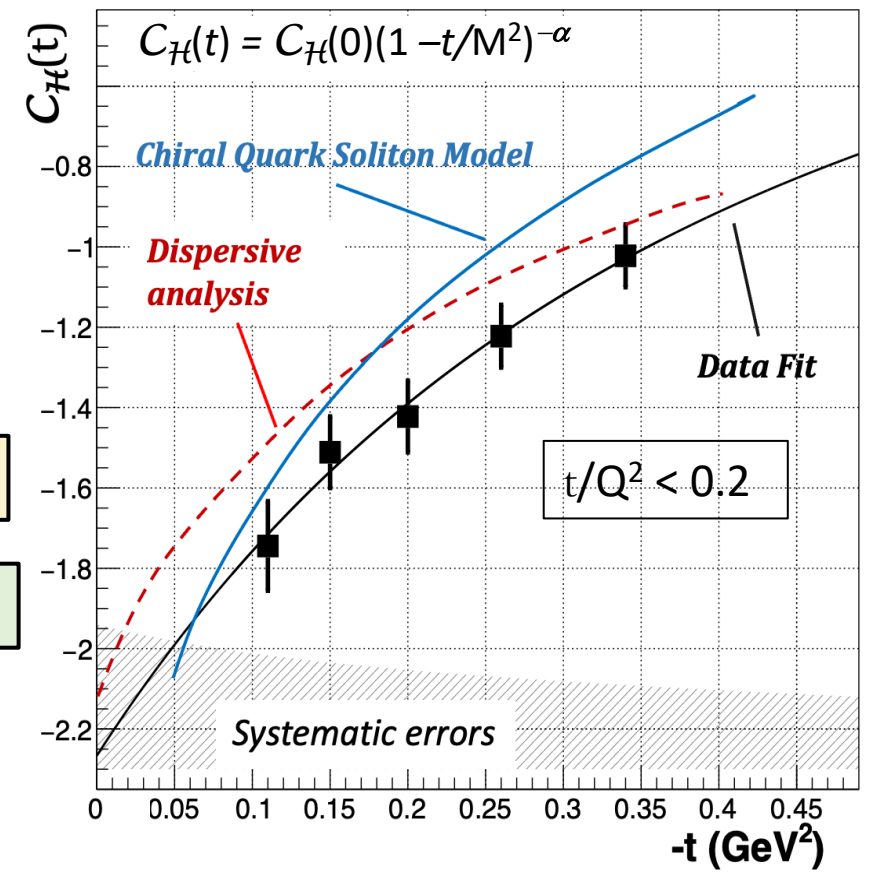
$$z = x/\xi \quad C_{\mathcal{H}}(t) = 2 \int_{-1}^1 dz \frac{D(z, t)}{1 - z^2}$$

$$D(z, t) = (1 - z^2) \left[e_u^2 + e_d^2 \frac{d_1^Q(t)}{2} \right] 3z$$

$$d_1^Q(0) = -2.04 \pm 0.14 \pm 0.30$$

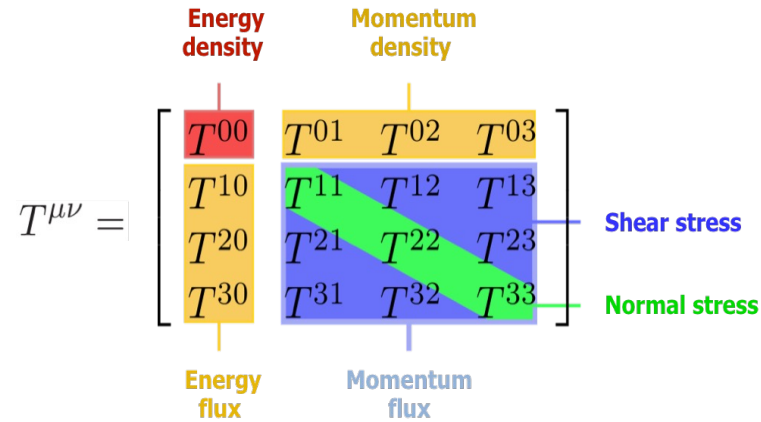
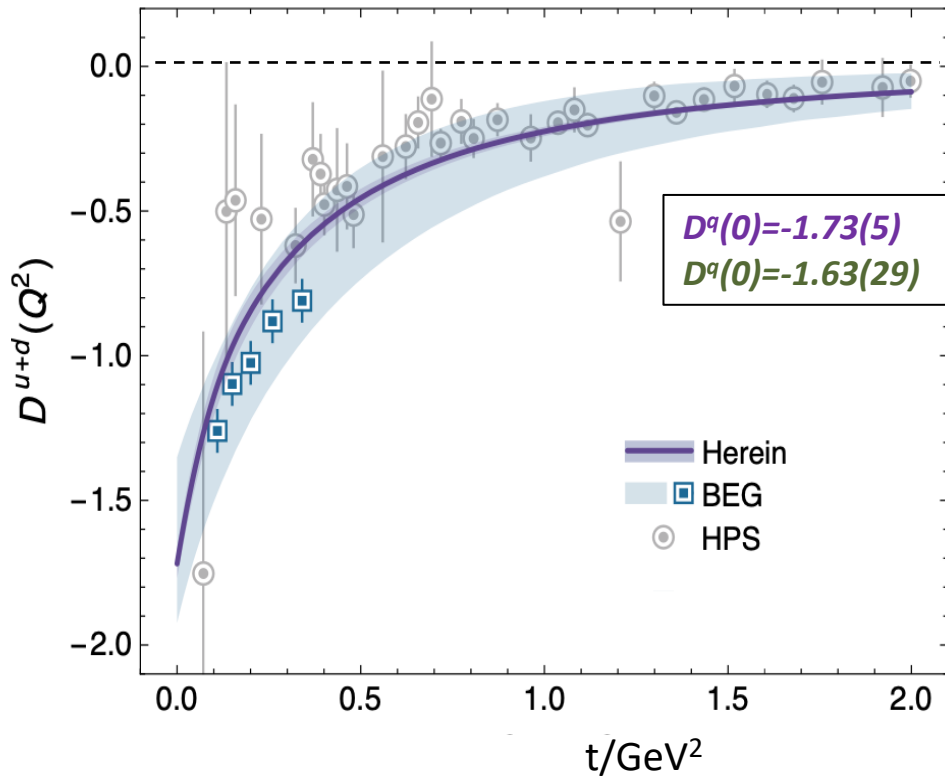
$$D^Q(0) = -1.63 \pm 0.11 \pm 0.24$$

→ The D-term is negative

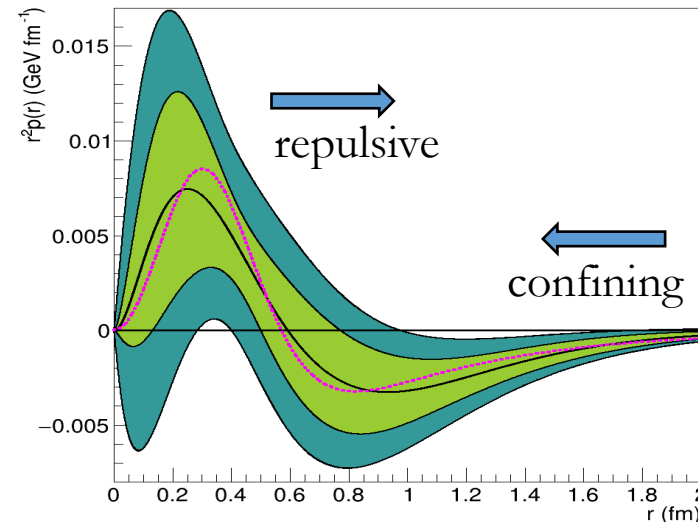


Summary of Proton $D^q(t)$ results

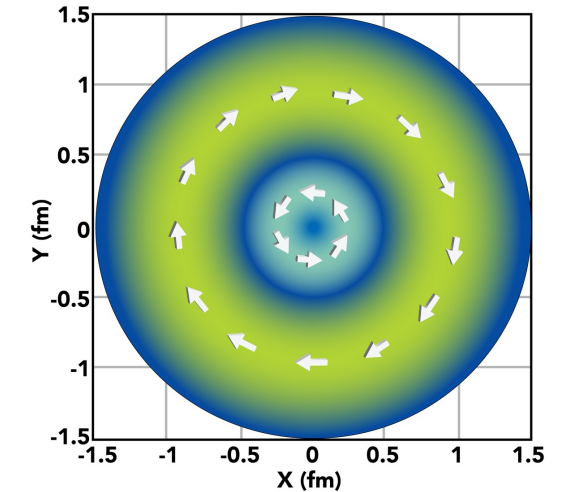
Burkert-Elouadrhiri-Girod, *Nature* 557, 396 (2018) (BEG)
 Hackett, Pefkou, Shanahan, *Phys.Rev.Lett.*132, 251904 (2024)
[Z.Q. Yao et al., arXiv:2409.15547](https://arxiv.org/abs/2409.15547)



$$T^{ij}(\vec{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$



Tangential stress at $r=0.6\text{fm}$
 $\sim 40 \times 10^3 \text{ N}$



$$\int r^2 p(r) dr = 0;$$

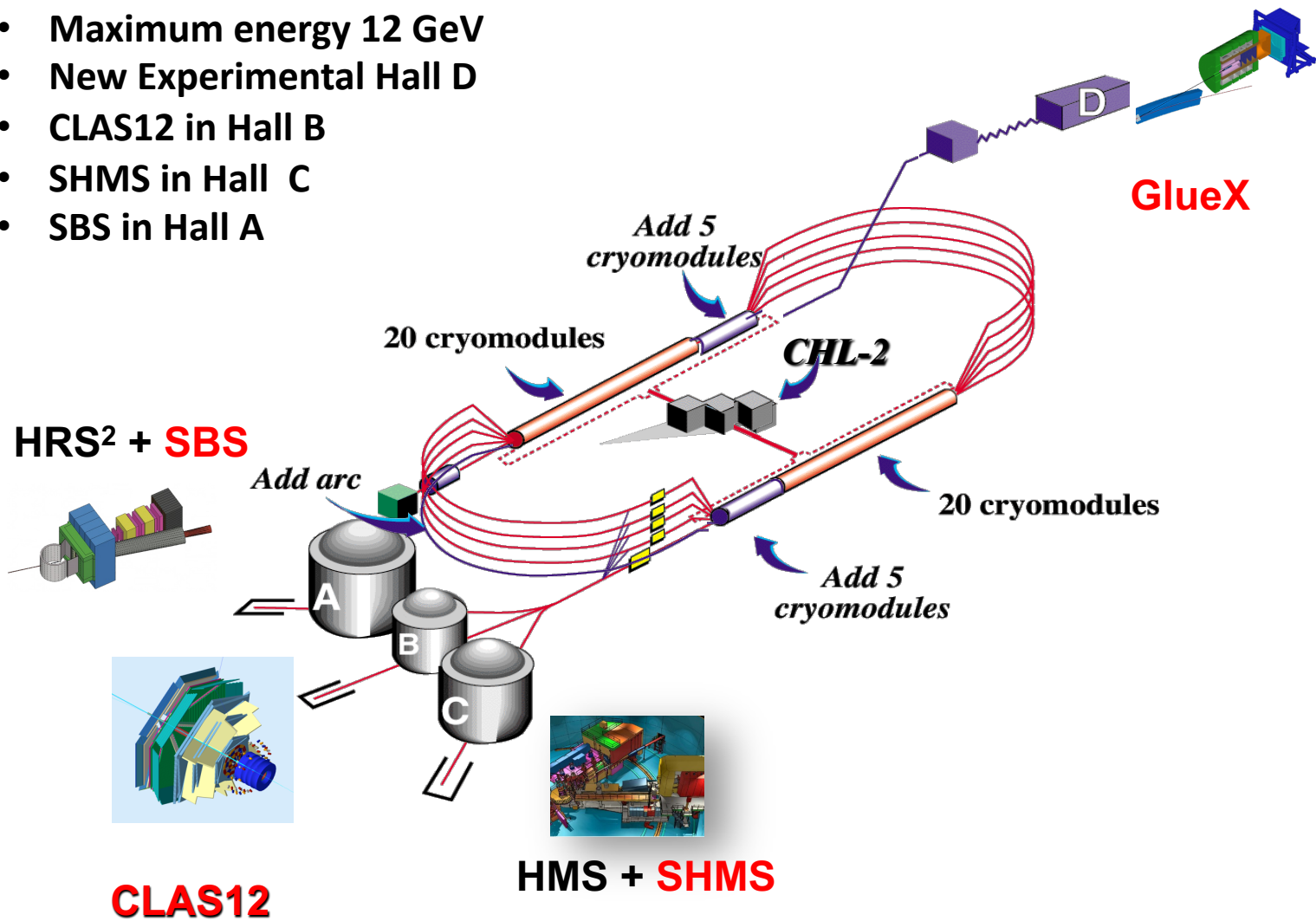
v. Laue, Annalen Phys. (1911)

Peak pressure ($r=0$) = $1.2 \times 10^{35} \text{ Pa}$
 Atmosphere = 10^5 Pa

The Jefferson Lab Energy Upgrade

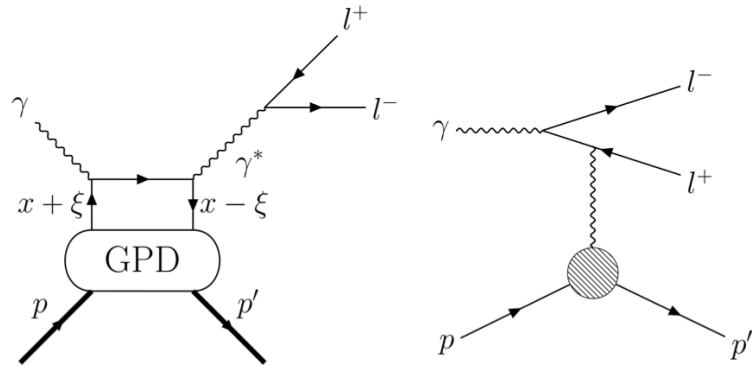


- Maximum energy 12 GeV
- New Experimental Hall D
- CLAS12 in Hall B
- SHMS in Hall C
- SBS in Hall A

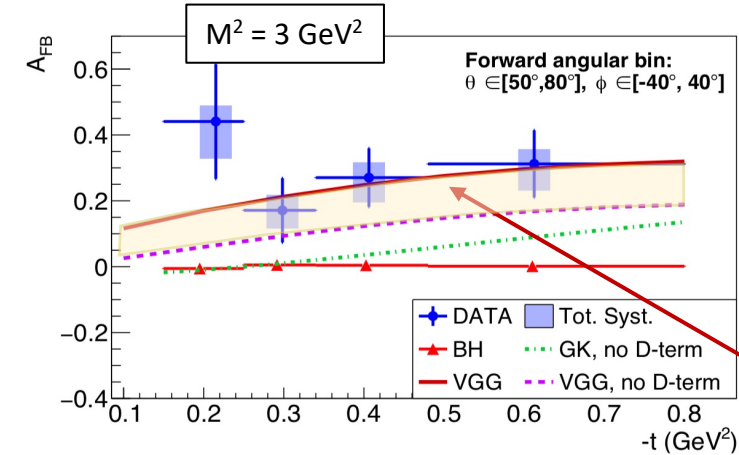


Time-like Compton Scattering $\gamma p \rightarrow e^+e^-p$

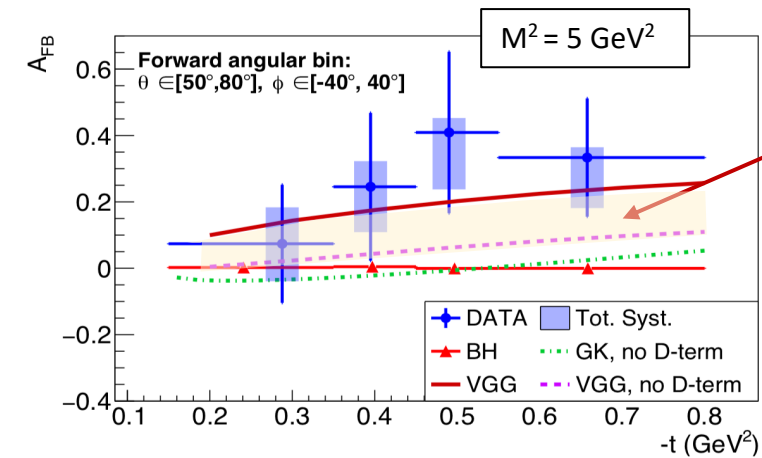
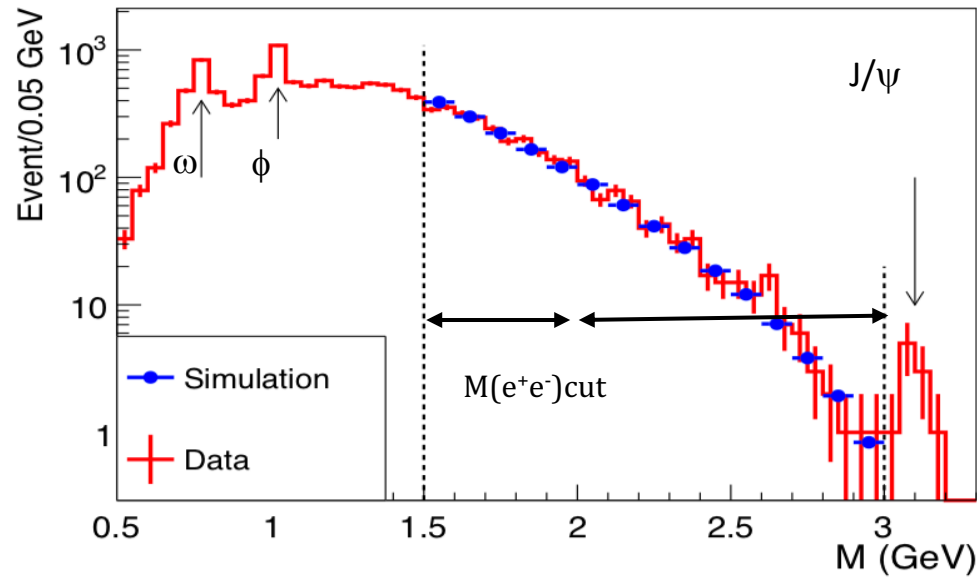
P. Chatagnon et al. (CLAS), PRL 127 (2021) 26, 262501



Forward-Backward asymmetry



$t/M^2 < 0.2$



$d_1(t)$

A_{FB} confirms the large contribution of the D-term to $Re\mathcal{H}$

Status and prospects of DVCS at high energy

RG-A

Liquid H₂ target
 Beam spin asymmetry
 Cross sections in preparations →

RG-B

Liquid D₂ target
 Beam spin asymmetry

RG-C

Long. polarized proton NH₃ and ND₃
 Sensitivity to CFF $Im\tilde{\mathcal{H}}_p$

RG-H

Transverse polarized proton in NH₃
 New target and beamline in design stage.
 Sensitivity to CFF $\mathcal{E}_p \rightarrow$ essential to extract $J^q(t)$

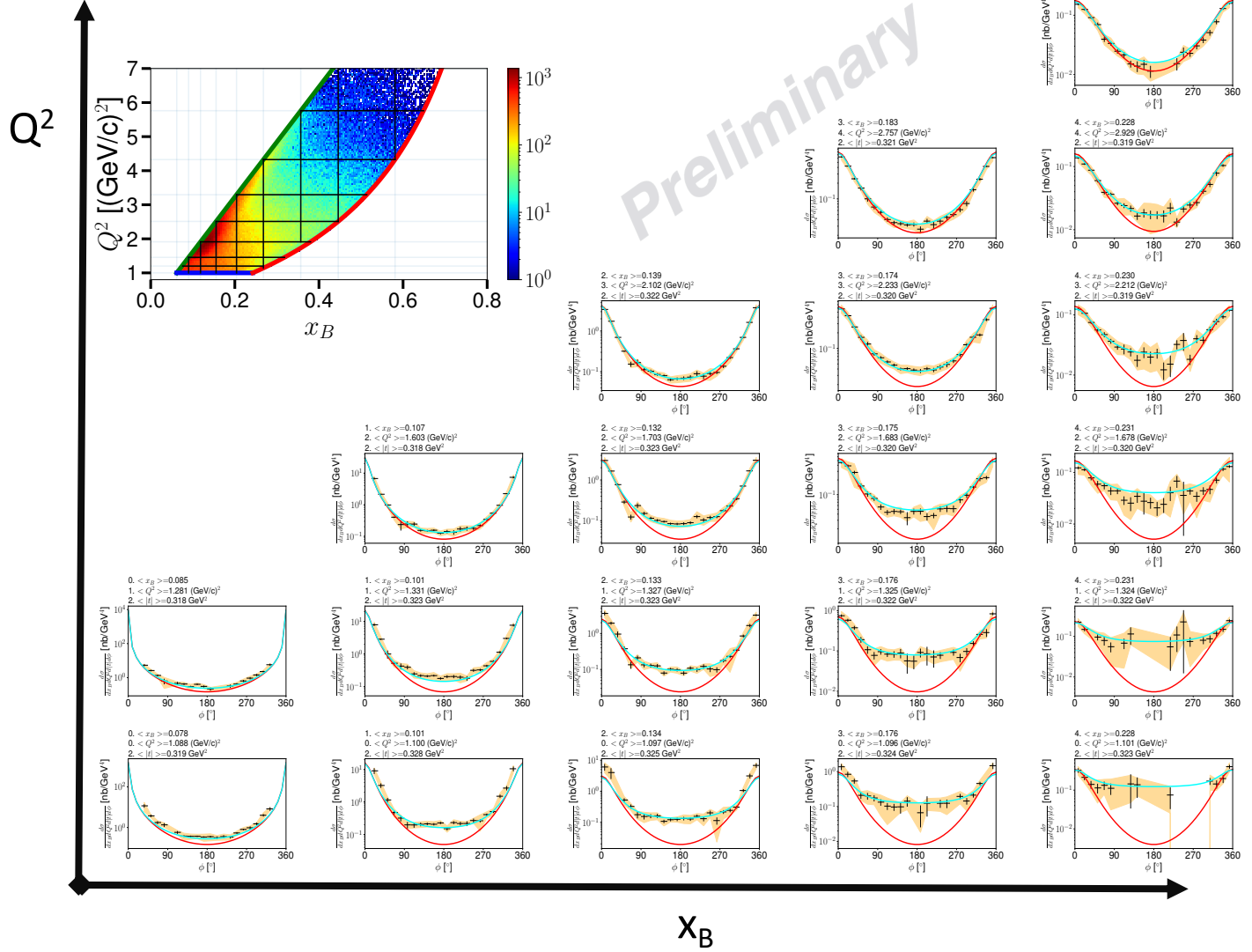
ALERT

DVCS on ⁴He target
 Run in early 2025, sensitivity to CFF $Im\mathcal{H}_{He}$

Hall A

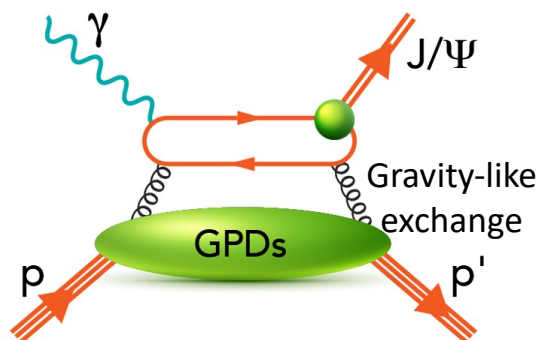
Liquid H₂ target
 high Q^2 & x_B , published CFF

Sample diff. cross sections



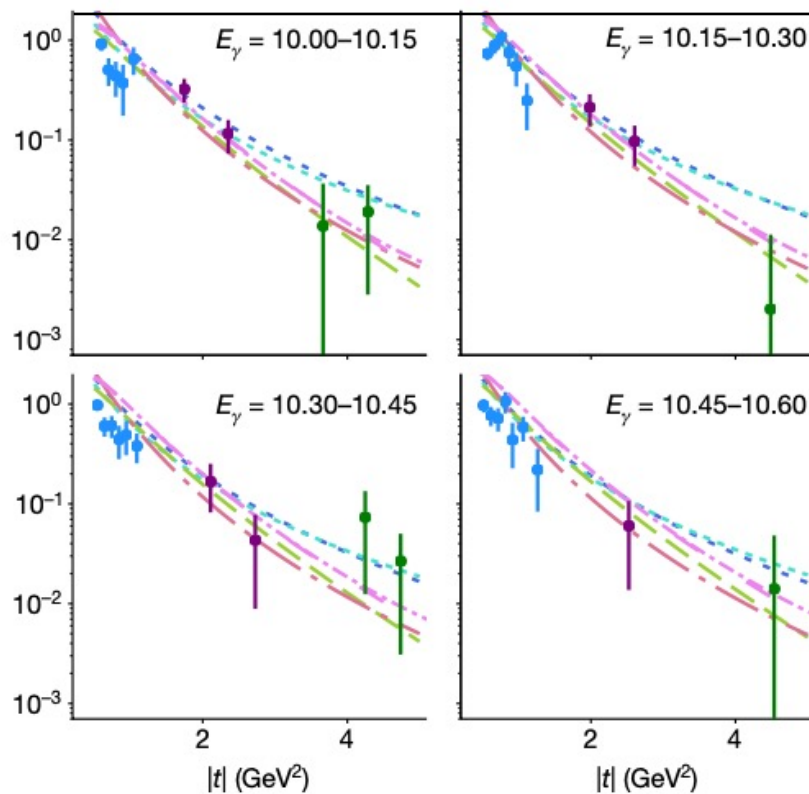
Gluon GFF $A_g(t)$ and $D_g(t)$

<https://journals.aps.org/prd/pdf/10.1103/PhysRevD.104.054015>



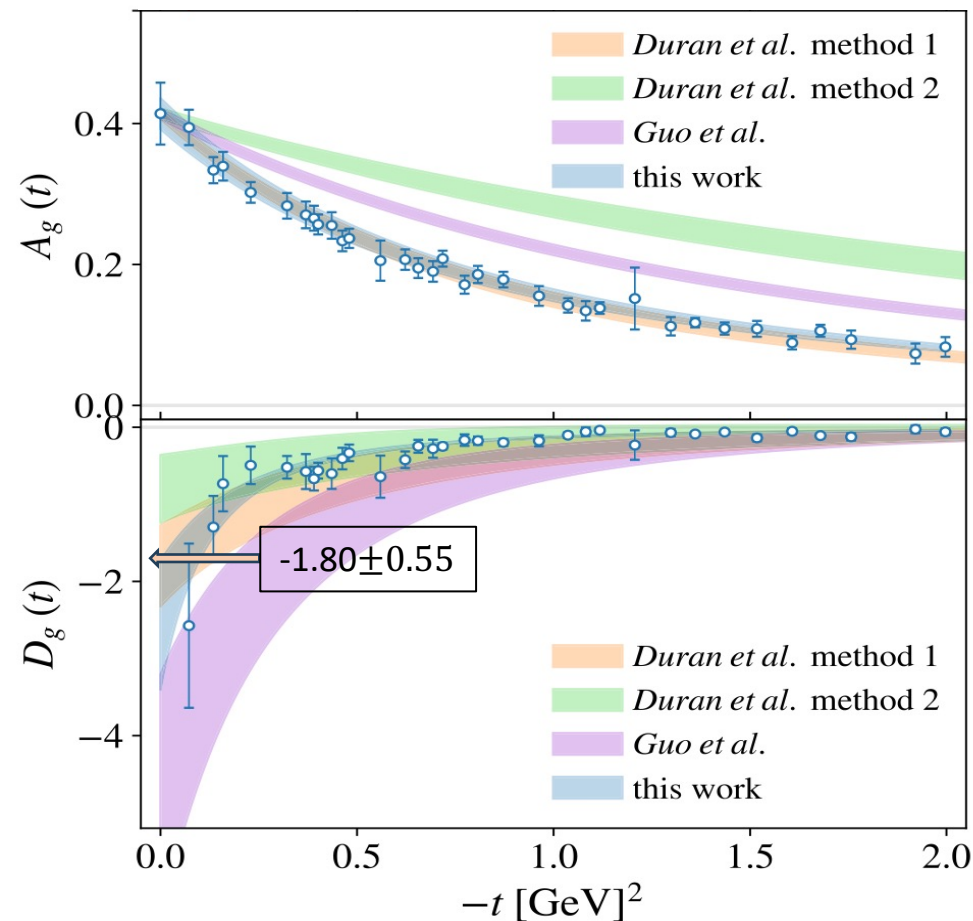
Exclusive J/ψ photoproduction at threshold sensitive to gluon GFFs.

B. Duran et al., Nature 615 (2023) 7954, 813

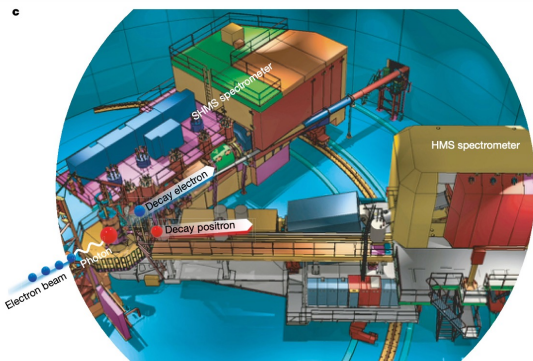


Yields of J/ψ photoproduction near threshold for different energies vs $|t|$, fitted with model approaches to determine GFF $A_g(t)$ and D_g -term.

D.C. Hackett et al., PRL 132 (2024) 25, 251904



Gluonic GFF A_g and D_g using holographic QCD, Lattice QCD and the GPD approach.



Quark & gluon GFF and radii

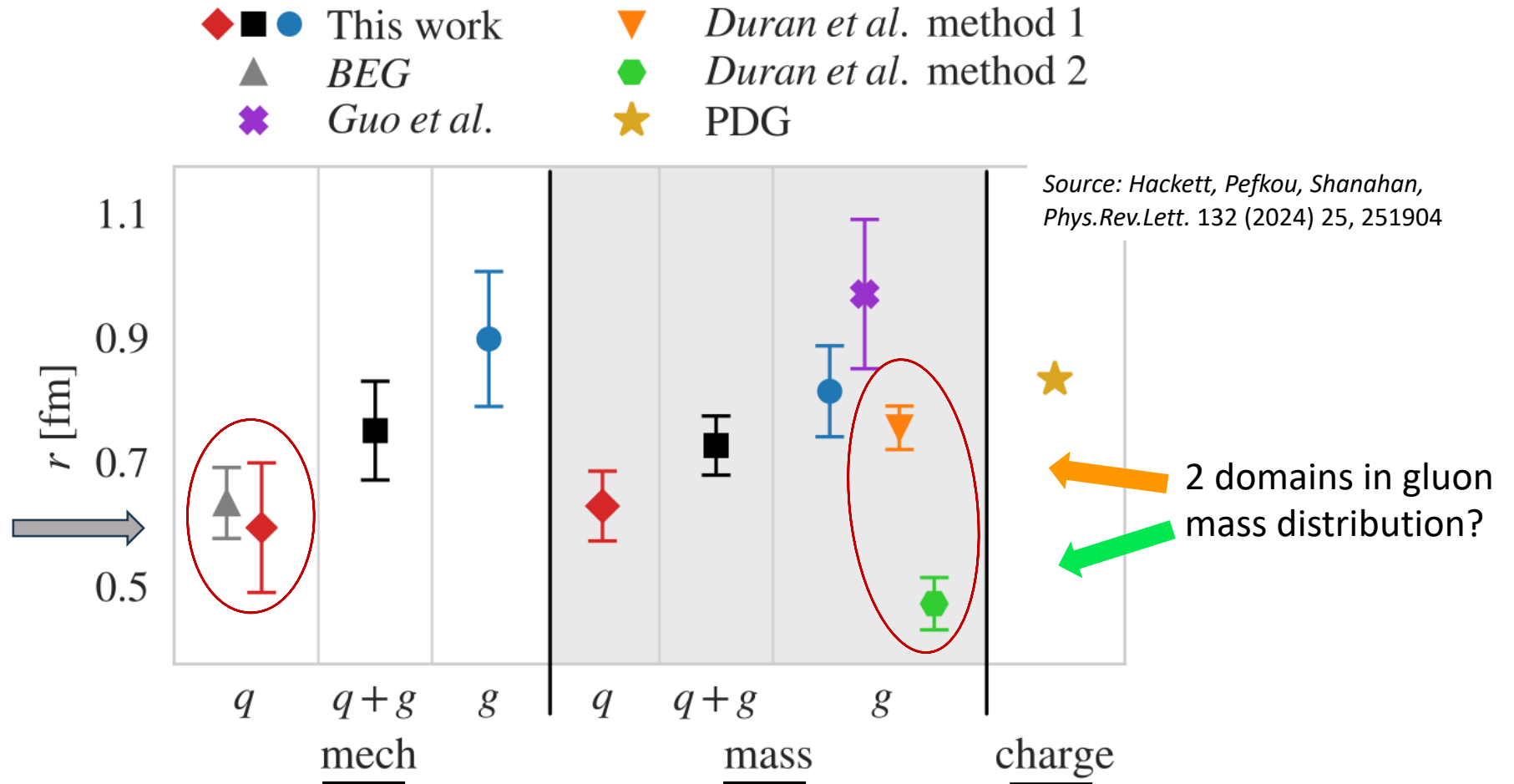
Mechanical radius:

$$\langle r^2 \rangle_{\text{mech}} = 6 \frac{D}{\int_{-\infty}^0 dt D(t)}$$

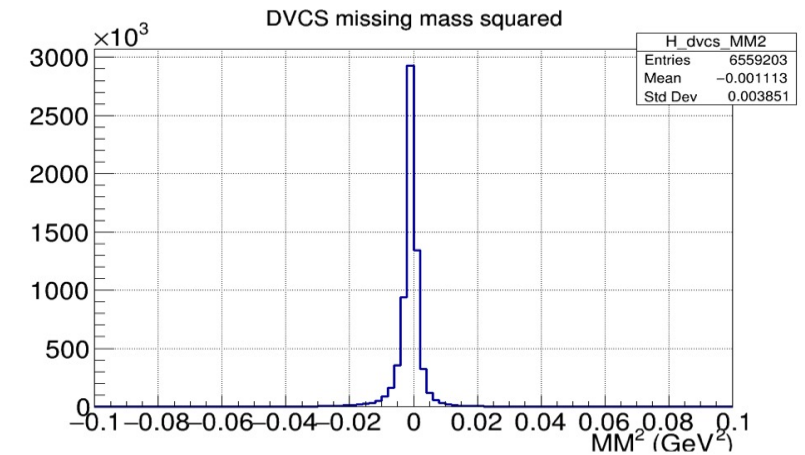
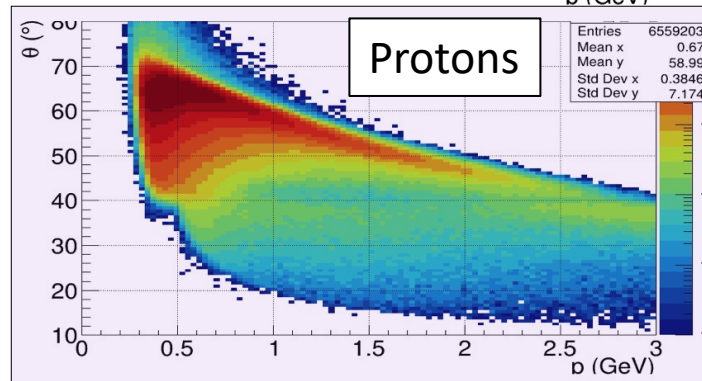
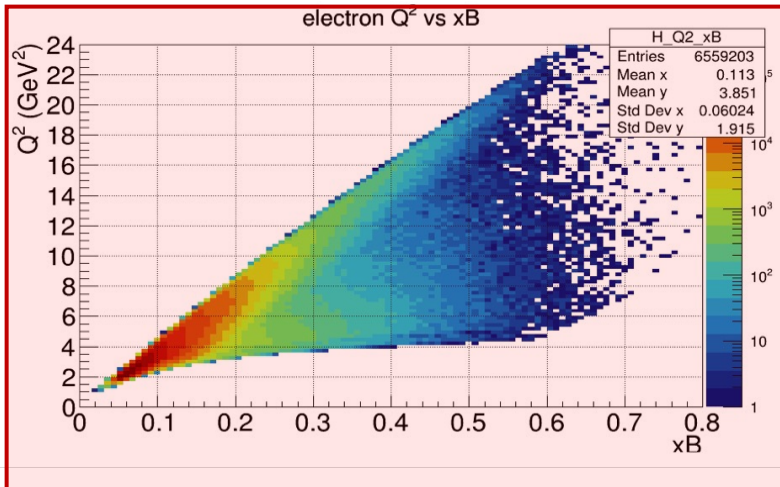
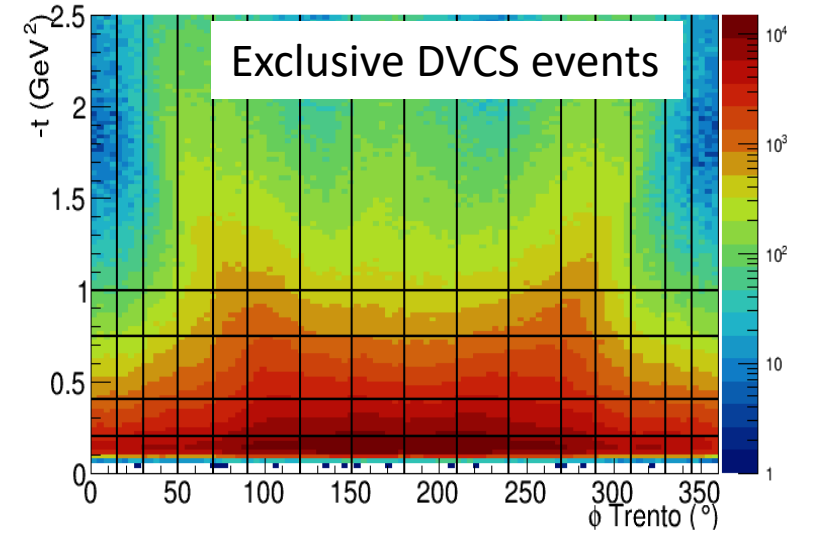
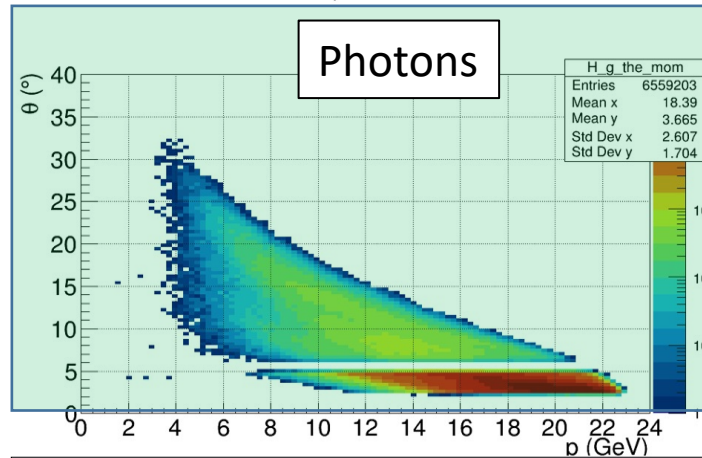
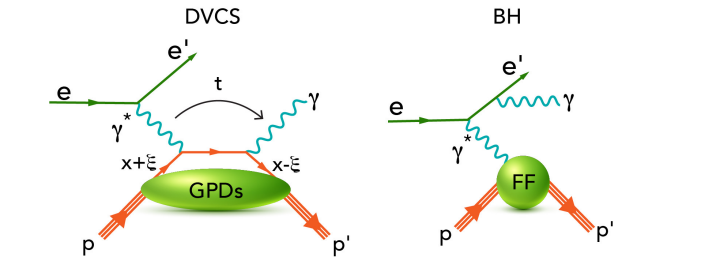
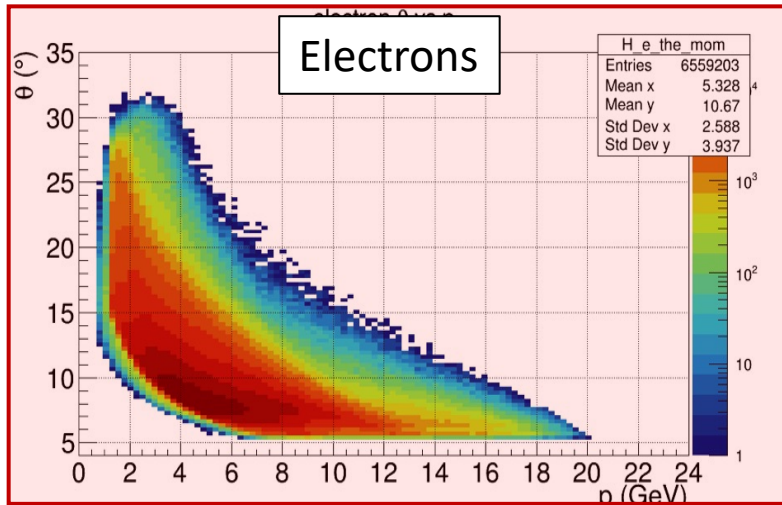
$$\langle r_p^2 \rangle_{\text{mech}} = 0.402 \pm 0.072 \text{ fm}^2$$

$$\sqrt{\langle r_p^2 \rangle_{\text{mech}}} = 0.634 \pm 0.057 \text{ fm}$$

Proton mechanical quark radius is $\approx 30\%$ smaller than its charge radius. Agreement with LQCD.



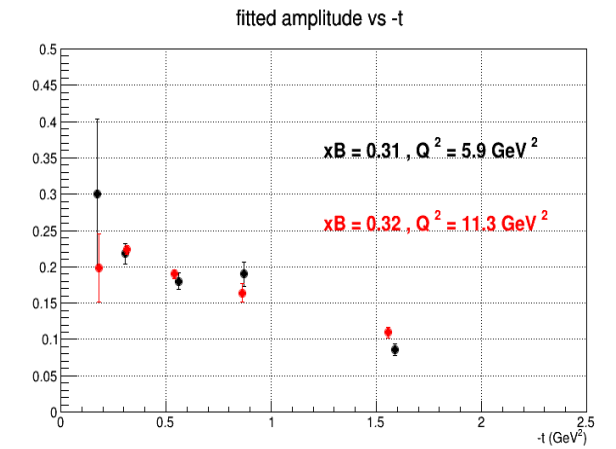
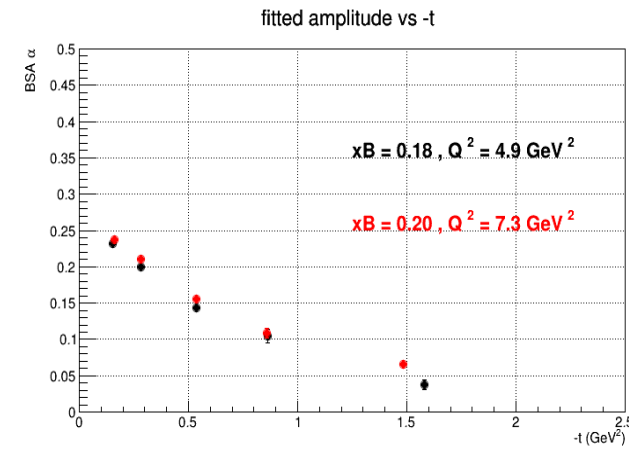
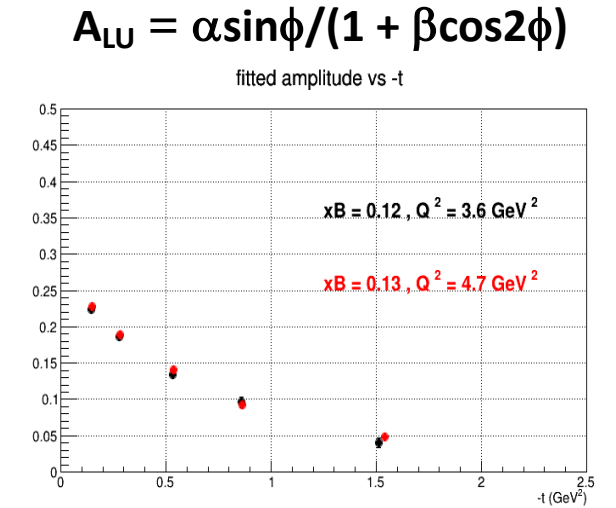
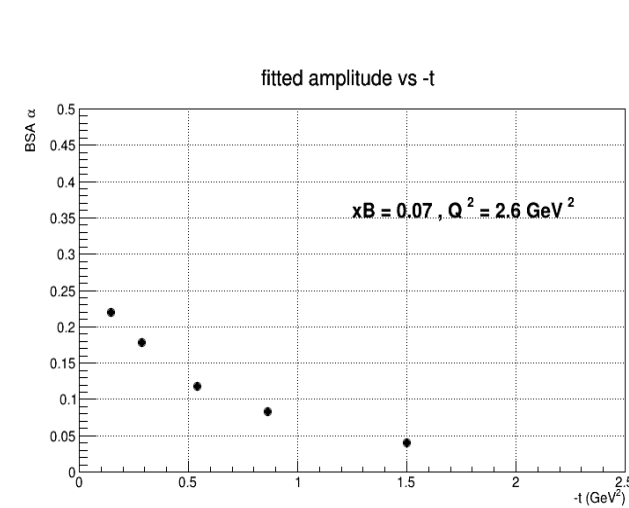
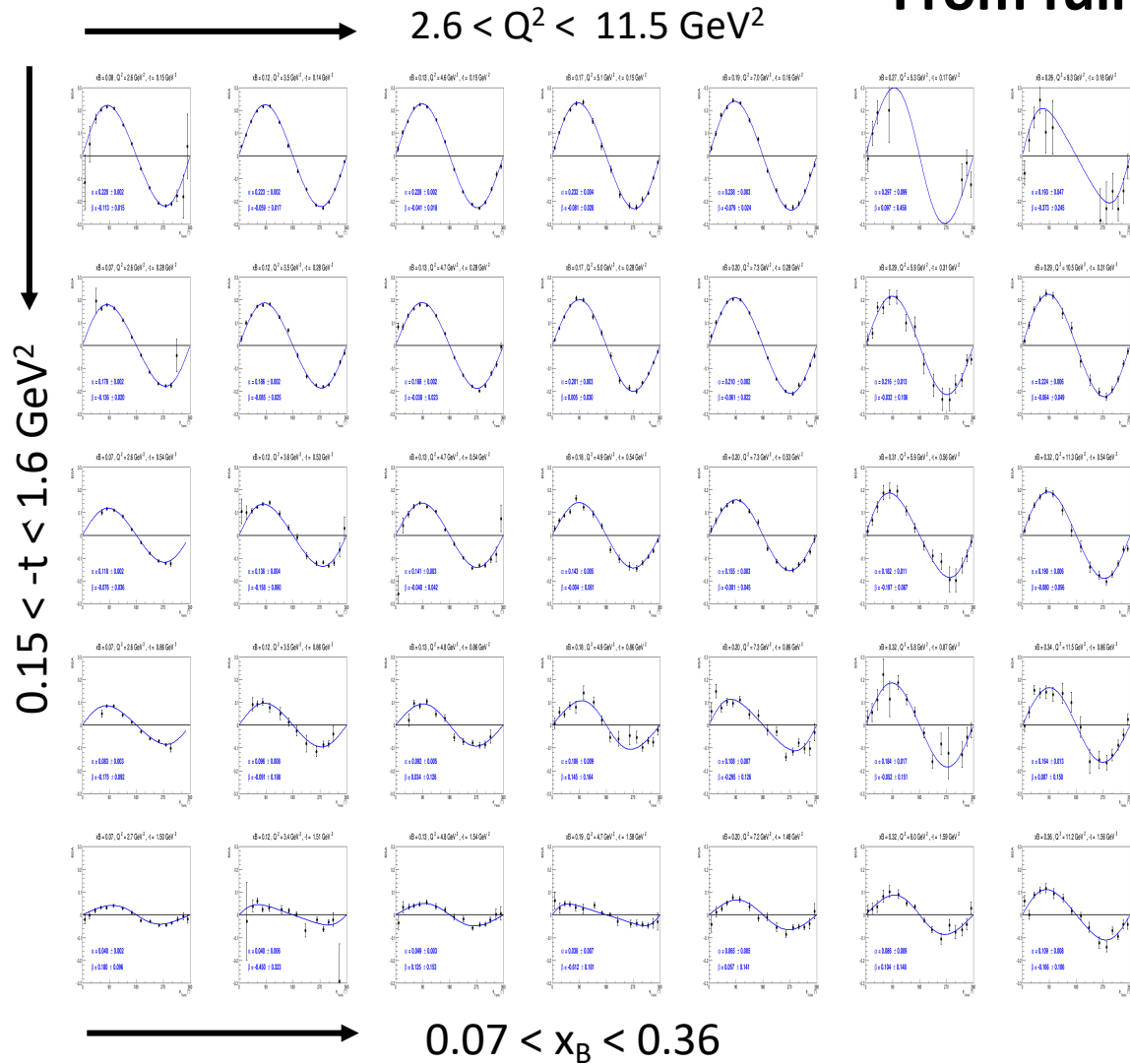
Exclusive DVCS @ 22/24 GeV with CLAS12



F.X. Girod

Beam spin asymmetry A_{LU} ($E=22/24\text{GeV}$)

From full simulations & reconstructions

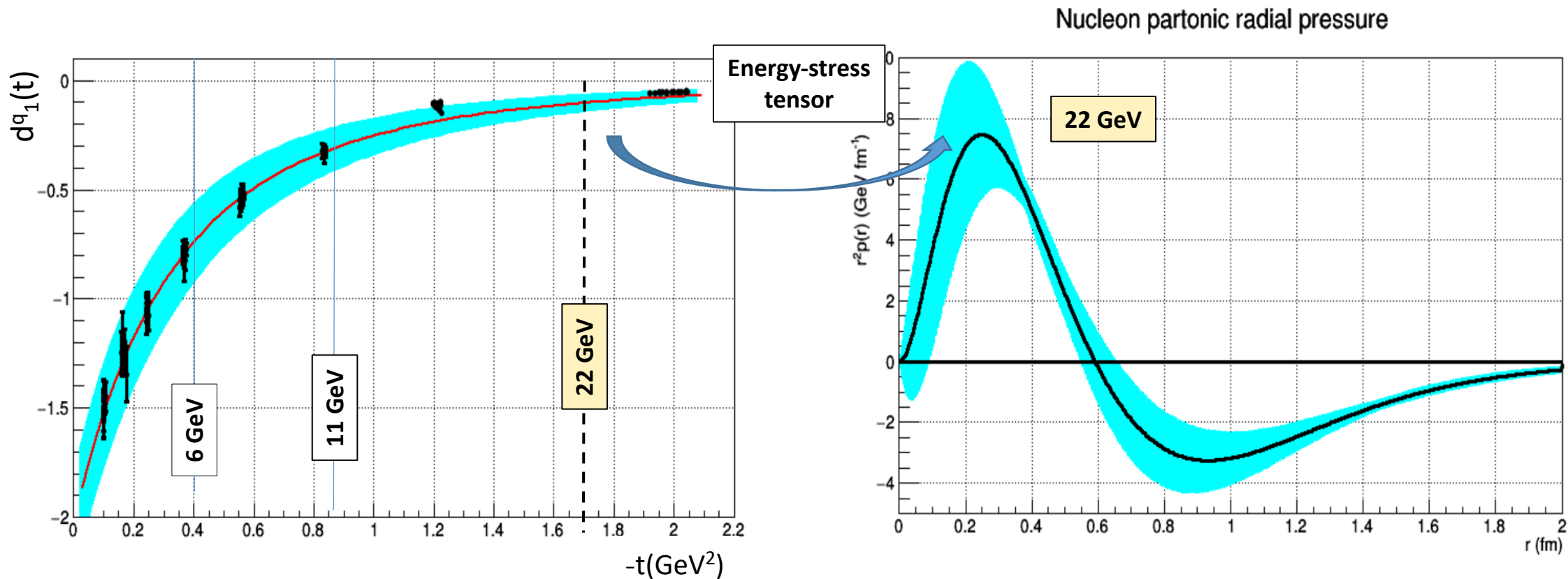


F.X. Girod

The proton's $D^q(t)$ and pressure at 22 GeV

Fitting the dispersion relation to $\text{Im}\mathcal{H}$, $\text{Re}\mathcal{H}$

$$-t/Q^2 < 0.2$$



22 GeV will cover a large range in $-t$ and may discover the existence of pressure domains.

Summary

- The exploration of the proton's **gravitational** structure presents a very broad experimental scope.
 - Gravity couples to **mass, spin, forces/pressure** for quarks and gluons, resulting in **6 GFF**.
 - DVCS@6 GeV and at 10.6 GeV provide first access to the **quark GFF $D^q(t)$** of the proton.
 - The **GFF $J^q(t)$** will be accessible with transverse polarized target measurements of CFF $\mathcal{E}(\xi, t)$
 - J/ψ photoproduction near threshold provides model-based data interpretation of the gluon mass GFF **$A^g(t)$** and the gluon D-GFF **$D^g(t)$** .
- **DVCS @22 GeV** expands the kinematic allowing application of the DR and providing a large t -range even with $t/Q^2 < 0.2$.
- The study of gravitational structure of particles, including light nuclei, could become a comprehensive experimental program at JLab. **The 22 GeV energy reach is a perfect match to reveal the underlying novel science of the proton's GFFs.**

Sidebar 3.2 The Pressure Inside the Proton

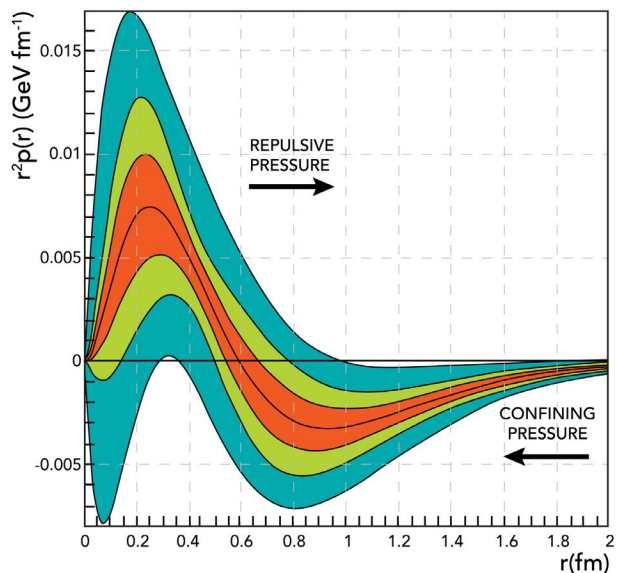


Figure 1: Pressure distribution in the proton weighted as $r^2 p(r)$. The peak pressure at $r = 0$ corresponds to 1035 Pascal. The green shaded bands represent uncertainties if only initial world data (dark) or more recent high statistics data (light) are included. The red band represent projections of future experiments [S10].

In the history of the universe, protons were formed microseconds after the Big Bang, when the universe expanded and cooled sufficiently for the binding forces to become strong enough to freeze quarks and gluons together into protons and neutrons, the building blocks of the atom's nucleus.

The internal structure of the proton has been studied in great detail using the electromagnetic interaction as a probe. The elastic form factors, its internal distribution of charge and magnetism, have been studied for the past 70 years. Its quark structure has been studied for over 55 years, and its helicity, or spin structure, for over 40 years. In contrast, we know very little about the proton's mechanical properties: its internal mass distribution, angular momentum, pressure and shear stress. These properties are encoded in gravitational form factors, which can be probed directly only in the proton's interaction with gravity- a practical impossibility due to the extreme weakness of the gravitational force. Thus, the mechanical properties were completely unknown until recently.

A theoretical breakthrough enabled the first experimental extraction of one of the gravitational form factors, $D(t)$, and the determination of the pressure distribution inside the proton shown in Fig. 1 obtained in 2018 by scientists from

Jefferson Lab. These results were based on the analysis of deeply virtual Compton scattering (DVCS) data, measured with the CEBAF Large Acceptance Spectrometer CLAS, and combined with information provided by generalized parton distributions (GPDs), a theoretical framework for mapping out the internal structure of protons.

The comparison of peak pressure measured in various regions and objects on Earth, in the solar system, and in the universe are displayed in Fig. 2. The tiny proton with a peak pressure of 1035 Pascal beats them all, including the most densely packed known macroscopic objects in the universe -the cores of neutron stars.

With current and planned state-of-the-art experimental facilities, further development and breakthroughs in theory and in lattice QCD, we will be able to reveal more of the mystery of the strong force, the most powerful force in nature, that binds quarks together to form the fundamental building blocks of the atomic nuclei.

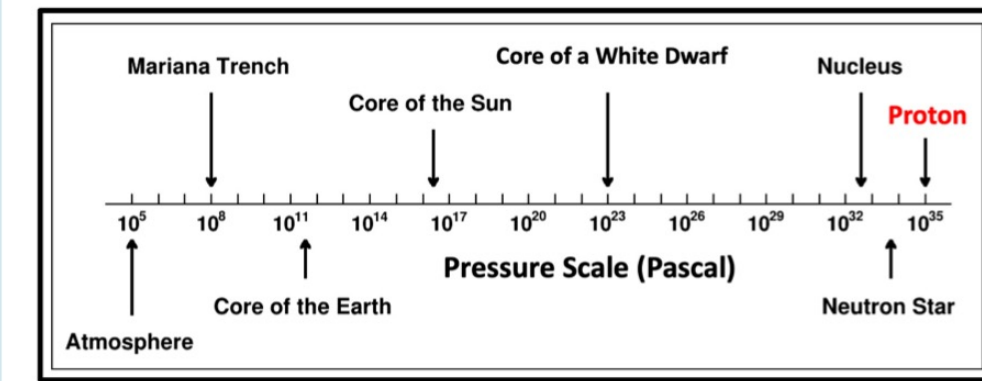


Figure 2: Peak pressure of objects in the universe, including the peak pressure inside the proton [S11].

“the measurements of this (mass) distribution, combined with measurements of other “mechanical” properties of the proton, such as the pressure distribution, **will definitely advance our understanding of the quantum origin of mass”**

D. Karzeev

Additional slides

The proton's D-term(s)

The proton gravitational formfactors $D^q(0)$ and $D^g(0)$ have been **determined experimentally** as follows:

$$D^q(0) = -1.63 \pm 0.29 \text{ (BEG)}$$

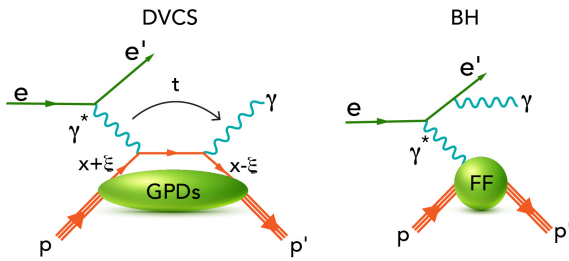
$$D^g(0) = -1.80 \pm 0.55 \text{ (Duran)}$$

Proton D-term

$$D(0) = -3.43 \pm 0.58 \text{ (BEG + Duran)}$$

Isolating Compton Form Factors in DVCS

Probing gravitational structure through DVCS



Polarized beam, **unpolarized** target:

$$\Delta\sigma_{LU} \sim \sin\phi \operatorname{Im}\{F_1 H + \xi(F_1 + F_2)\tilde{H} + kF_2 E\}d\phi \quad \Rightarrow \quad \mathcal{H}(\xi, t)$$

Unpolarized beam, **longitudinal** target:

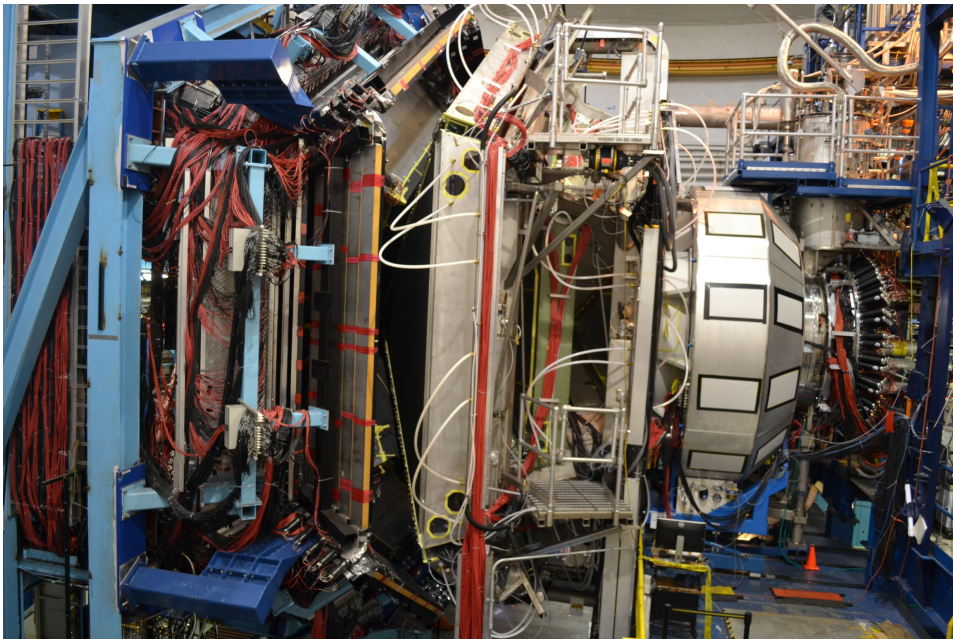
$$\Delta\sigma_{UL} \sim \sin\phi \operatorname{Im}\{F_1 \tilde{H} + \xi(F_1 + F_2)(H + \xi/(1 + \xi)E)\}d\phi \quad \Rightarrow \quad \tilde{\mathcal{H}}(\xi, t)$$

Unpolarized beam, **transverse** target:

$$\Delta\sigma_{UT} \sim \cos\phi \sin(\phi_s - \phi) \operatorname{Im}\{k(F_1 E - F_2 H)\}d\phi \quad \Rightarrow \quad \mathcal{E}(\xi, t)$$

Unpolarized cross section:

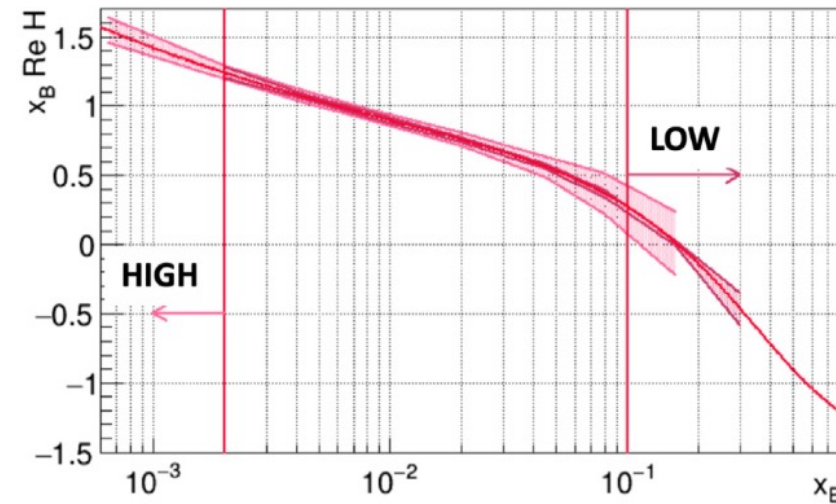
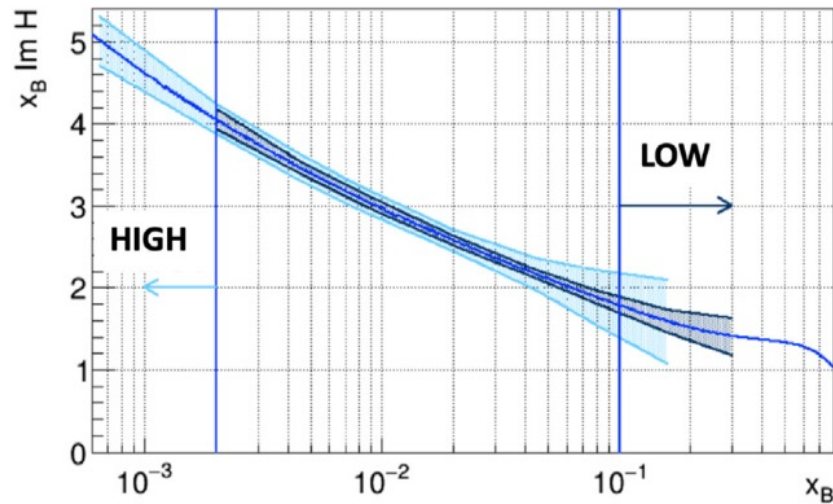
$$\operatorname{Re}(\text{CFFs}), \text{ separate h.t. contributions to DVCS} \quad \Rightarrow \quad \operatorname{Re}(T^{\text{DVCS}})$$



US Electron-Ion-Collider

Simulated DVCS CFF on protons at the EIC at CM 31.6 GeV and at 100 GeV.

Progress in Particle and Nuclear Physics 131 (2023) 104032



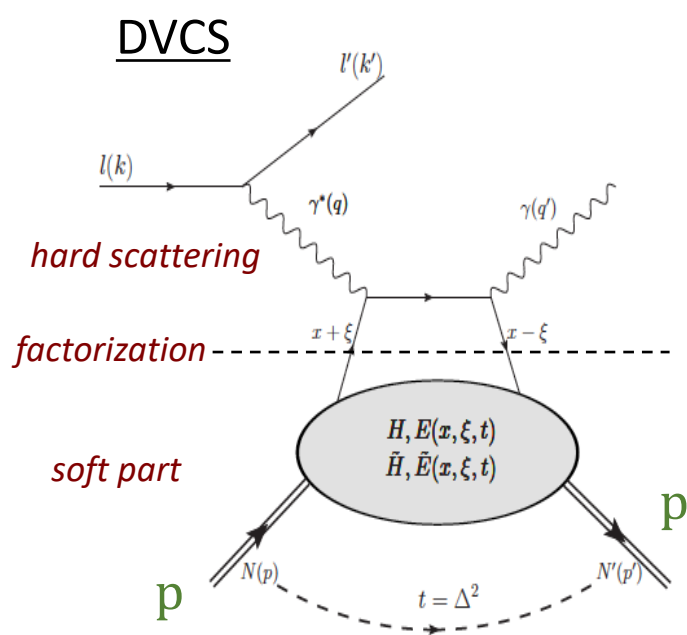
The CFF $\text{Im}\mathcal{H}$ (left) and $\text{Re}\mathcal{H}$ (right) simulated for $\sqrt{s} = 31.6$ GeV and $\sqrt{s} = 100$ GeV for same luminosity. Note that the CFF model does not include the D-term contribution given thus unrealistic $\text{Re}(H)$ values. The x_B areas marked as LOW can only be accessed with reasonable statistics with the 31.6 GeV. The areas marked as HIGH can only be accessed with the high energy operation.

Compton FFs & DVCS

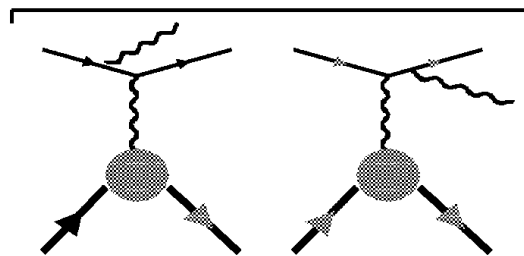
X. Ji, PRD 55 (1997) 7114 A. Radyushkin, PRD 56 (1997) 5524

$$\mathcal{H}(\xi, t) = \int_{-1}^{+1} dx H(x, \xi, t) \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)$$

Complex valued Compton Form Factors \mathcal{H} and \mathcal{E} appear in the DVCS observables.



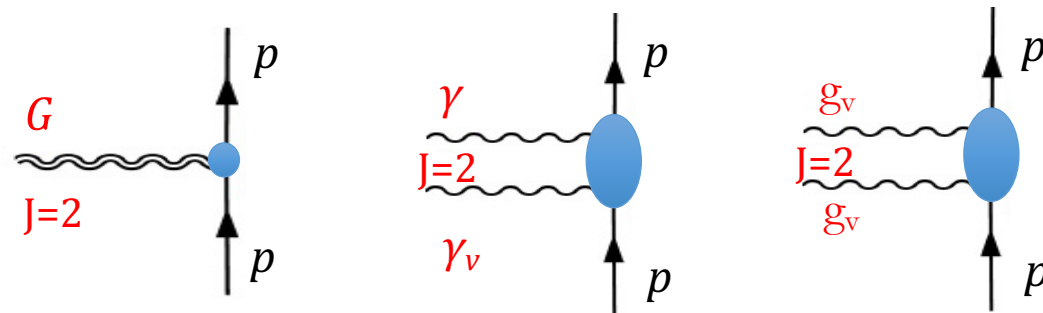
Bethe-Heitler



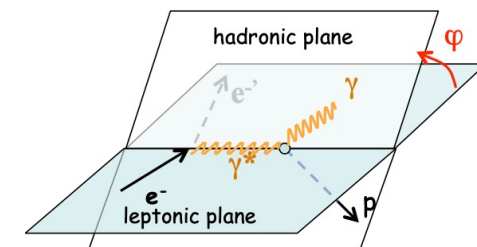
Elastic Form Factors

$$\xi \approx \frac{x_B}{2 - x_B}$$

Gravity & gravity-like couplings



DVCS-BH interference term contains $\Im m \mathcal{H}$ in the beam polarization asymmetry A_{LU} .



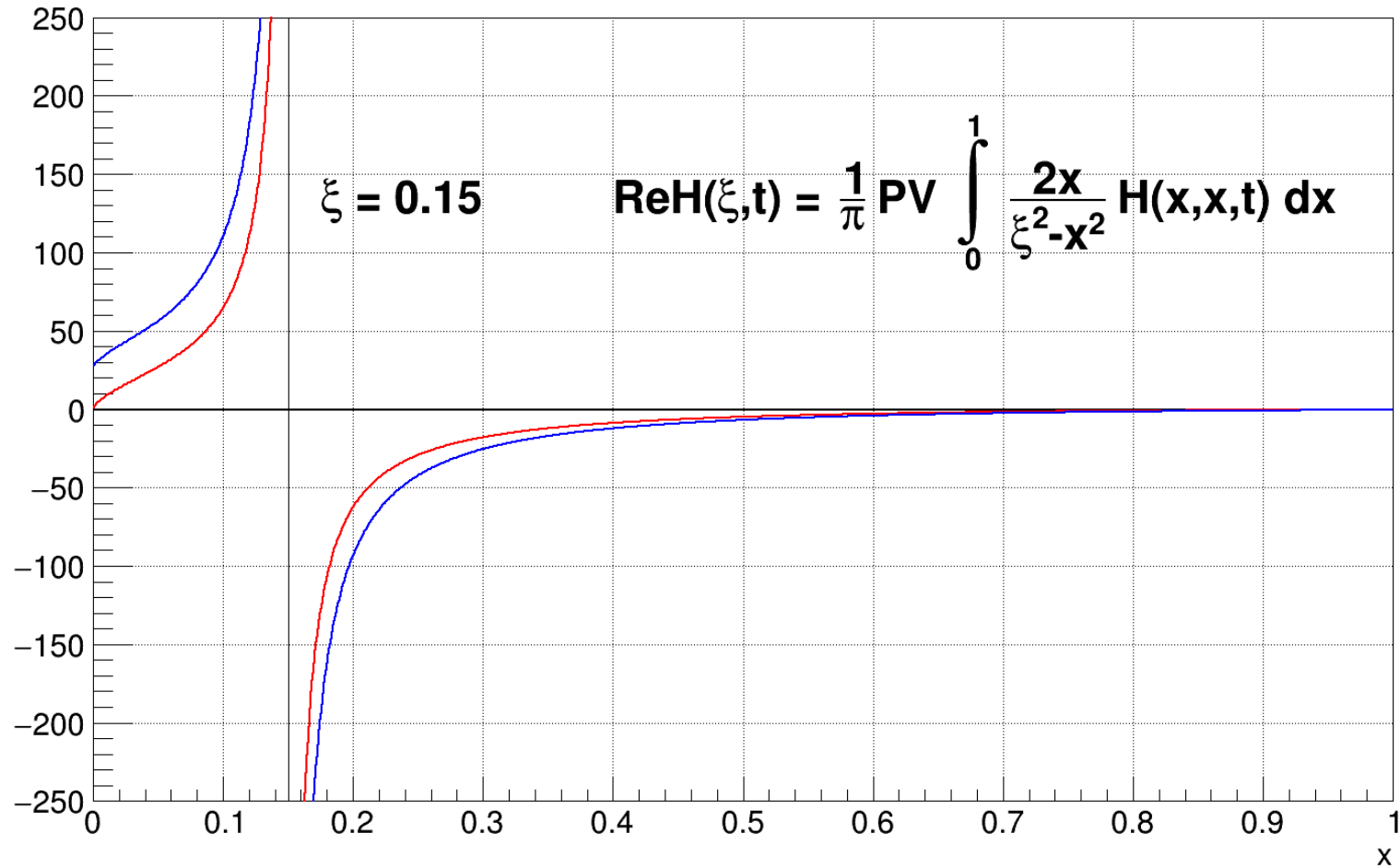
$$A_{LU} = \frac{d^4\sigma^{\rightarrow} - d^4\sigma^{\leftarrow}}{d^4\sigma^{\rightarrow} + d^4\sigma^{\leftarrow}} \stackrel{\text{twist-2}}{\approx} \frac{\alpha \sin \phi}{1 + \beta \cos \phi}$$

$$\alpha \propto \text{Im} \left(F_1 \mathcal{H} + \xi G_M \tilde{\mathcal{H}} - \frac{t}{4M^2} F_2 \mathcal{E} \right)$$

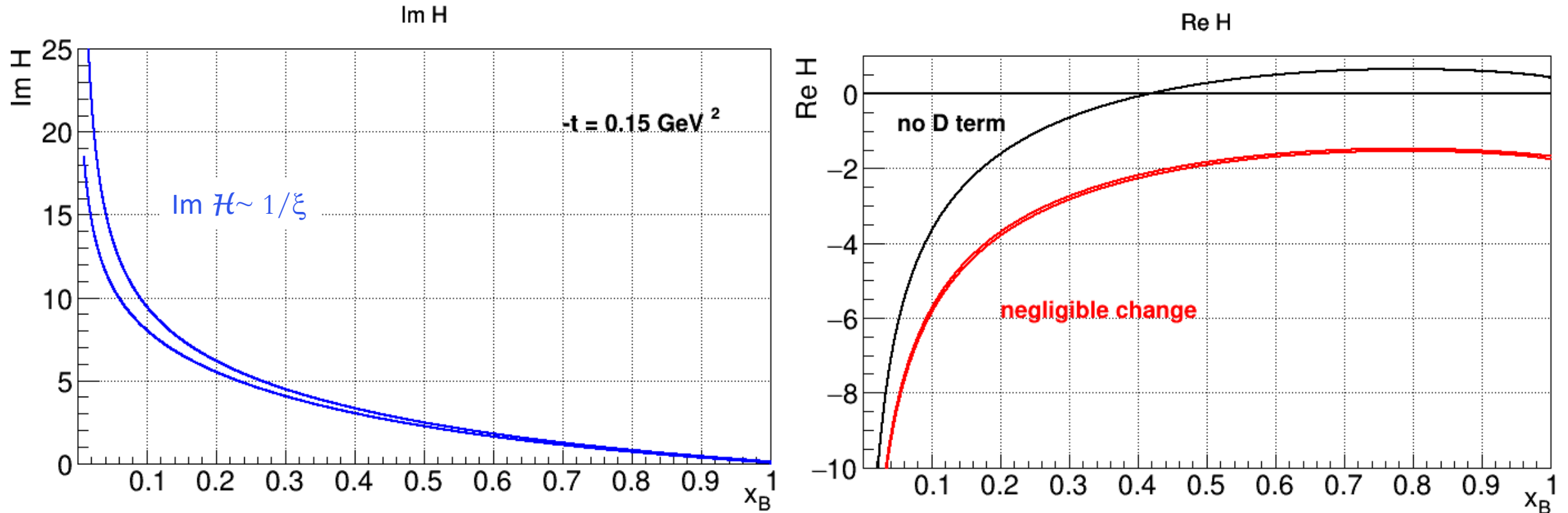
$\rightarrow \Re e \mathcal{H}$ appears in the differential cross section.

Dispersion Integral

dispersion kernel comparison at $\xi=0.15$ and $-t = 0.15 \text{ GeV}^2$



Effect of $\text{Im}\mathcal{H}$ parameterization on $\text{Re}\mathcal{H}$



- Changing the parameterization of $\text{Im}\mathcal{H}$ at small ξ so that $\text{Im}\mathcal{H} \sim 1/\xi$ has negligible effect on $\text{Re}\mathcal{H}$.
- Eliminating the D-term has large effects and is incompatible with the data.