Probe QCD Confinement via π^0, η and η'

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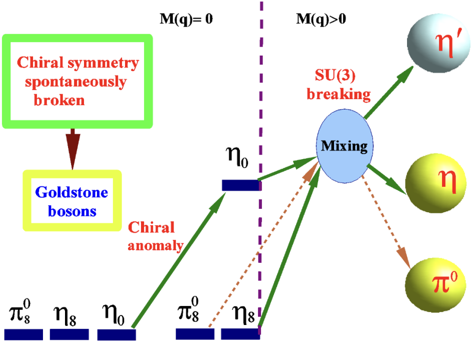
Science at the Luminosity Frontier: Jefferson Lab at 22 GeV

9-13 Dec 2024

Outline:

- QCD at low energies
- Theoretical methods
- Neutral sector
- $\pi^0 \to \gamma\gamma$
- Summary

QCD at low energies



From the 22 GeV white paper: 2306.09360

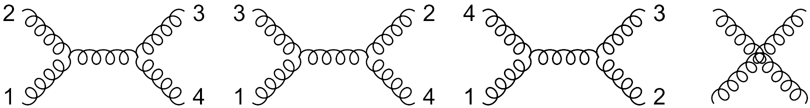
on problem of masses see previous talk by Khépani Raya

Amplitudes

- important in particle physics: Lagrangian $\to$ Feynman rules $\to$ amplitudes $\to$ cross-section
- new initiative to study these objects more deeply
- annual conferences: . . . , Prague 22, CERN 23, IAS 24, Seoul 25
- amplitudes are the key object of theoretical studies
- example $\to$ next page

QCD: gluon amplitudes

- important in high-energy collider experiments (LHC)
- using conventional methods: complicated already at the tree-level



- intermediate steps are complicated, but the final result “nice”
- standard methods hard/impossible for higher multiplicities
- surprisingly some results super simple and closed for all multiplicities

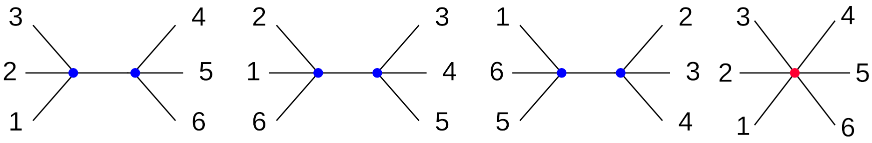
$$A_n(--+\ldots+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

(so called MHV, [Parke, Taylor '86])

pion amplitudes

[KK, Novotny, Trnka '13]

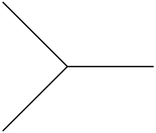
- We want to study low-energy QCD
- focus on dynamics of pions, kaons, etas
- very complicated already at the tree-level for large n
- simplify the problem: massless, large N_c (one trace $\rightarrow$ cyclic ordering)
- 4pt: $A = s_{13}$
- 6pt:



pion amplitudes: new surprising way to calculate

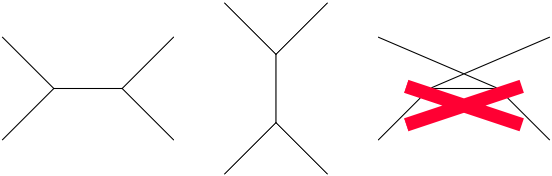
[Arkani-Hamed et al '23-'24]

The simplest model: $\mathrm{Tr}(\phi^3)$
only one vertex:



$= 1$

e.g. the 4pt amplitude:



$$A = \frac{1}{s_{12}} + \frac{1}{s_{14}}$$

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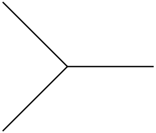

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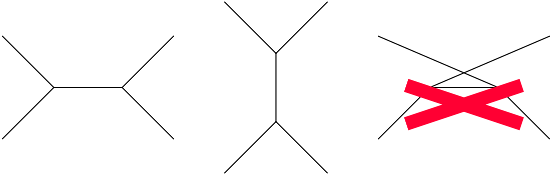
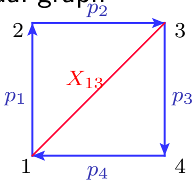


dual graph



$X_{ij} = (p_i + \ldots + p_{j-1})^2$

$$A = \frac{1}{s_{12}} + \frac{1}{s_{14}} = \frac{1}{X_{13}} + \frac{1}{X_{24}}$$

pion amplitudes: new surprising way to calculate

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The magic:

$$A = \frac{1}{X_{13}} + \frac{1}{X_{24}}$$

odd/even shifts:

$$X_{ee} \to X_{ee} + \delta, \qquad X_{oo} \to X_{oo} - \delta$$

$$X_{eo} \to X_{eo}$$

Do it in $Tr(\phi^3)$ amplitude and expand in small momenta for large δ:

$$A \to \frac{1}{X_{13} - \delta} + \frac{1}{X_{24} + \delta} \sim -X_{13} - X_{24} = s_{13}$$

which is the 4pt NLSM!

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which is the 4pt NLSM!

True up to all multiplicities!!

and can be extended to the loop level

Theoretical methods

- amplitudes methods are important to uncover hidden structures
- especially true for the low-energy QCD
- we hope that the above miracles have some footprint in the low-energy data
- BSM? double copy of YM $\rightarrow$ gravity
  double copy of NLSM $\rightarrow$ special galileon

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problem: masses

- ongoing investigation (in collaboration with N. Arkani-Hamed)
- possible to extend it to $SU(3)$ limit ($m_u = m_d = m_s \neq 0$)
- very hard beyond this limit, for the physical masses

the most complicated is the neutral sector, i.e. π^0, η, η'

Neutral sector

it is necessary to study and understand all details of π^0, η and η'
important properties/channels:

- $\eta, \eta' \to 3\pi$
- their lifetimes

Decays of π^0

neutral pion: the lightest hadron

rich experimental activity: Hades, KLOE-2, A2), JLab, NA62, ...

accessible also at lattice

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Five measured decays (with corresponding Γ_i/Γ)

- $\pi^0 \to \gamma\gamma$ (98.8%) $\leftarrow$ this talk
- $\pi^0 \to e^+e^-\gamma$ (1.17%)
  - $\pi^0 \to$ positronium γ (2×10^{-9})
- $\pi^0 \to e^+e^-e^+e^-$ (3×10^{-5})
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[Bernstein, Holstein: Neutral Pion Lifetime Measurements and the QCD Chiral Anomaly]

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other possible decay modes

- $\pi^0 \to 4\gamma$ (theory vs. exp: 2×10^{-11} vs $< 2 \times 10^{-8}$)
- $\pi^0 \to \nu\bar{\nu}$
  helicity supression – limit: $< 4 \times 10^{-9}$ (NA62 '21, 60x improvement), theory: $< 5 \times 10^{-10}$ (direct) or $< 10^{-24}$ (cosmology)
- $\pi^0 \to \nu\bar{\nu}\gamma$ (Wolfram mode)
  SM: $\sim 10^{-18}$, exp.limit $< 2 \times 10^{-7}$ (NA62 '19)

EFT $\rightarrow$ ChPT

EFT

- separated degrees of freedom (simplification)
- building the most general Lagrangian
- ordering principle (powercounting)

example: ChPT

- goldstone bosons (spontaneous symmetry breakdown of chiral symmetry)
- momentum power counting
- even sector: Lagrangian up to NNNLO
- odd sector? $\Rightarrow$
  symmetry pattern of QCD must be studied more carefully

Odd sector

- first we need to add EM interaction:
  $\partial_\mu U \to D_\mu U = \partial_\mu U + i[U, v_\mu], \qquad v_\mu \sim Q A_\mu$
- and add by hands monomial to $\mathcal{L}$:

$$U F_{\mu\nu} \tilde{F}^{\mu\nu}$$

  U can be transformed out: we have to add (at least two) derivatives on U – vanishes in chiral limit (Sutherland theorem)

way out: anomaly, in fact two anomalies

$$\partial^\mu A^i_\mu \neq 0$$

(non-trivial for $i = 0, 3, 8$, or for π^0, η, η' states)

incorporated to the action by Wess, Zumino and Witten (WZW)
two-flavour case: [Kaiser'01]

$\pi^0 \to \gamma\gamma$: LO

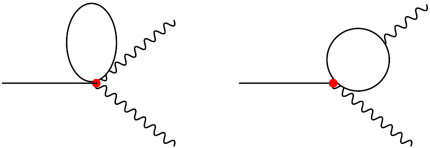
- π^0 lightest hadron

  $\Rightarrow$ primary decay mode $\pi^0 \to \gamma\gamma$
- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\, eV$$

$\pi^0 \to \gamma\gamma$: Correction to the current algebra prediction

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_\pi$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]



- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

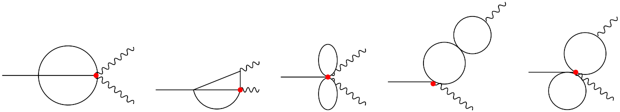
  in 2-flavour case EM corrections [Ananth., Moussallam '02]:

$$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

- study based on dispersion relations [Ioffe, Oganesian '07]:

$$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT [KK, Moussallam'09]

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1 - \vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- two-loop 
- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency relations

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$
\begin{aligned}
A_{NNLO} = & \frac{e^2}{F_\pi}\Big\{\frac{1}{4\pi^2} \\
& + \frac{16}{3}m_\pi^2(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9}B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
& - \frac{M^4}{24\pi^2F^4}\left(\frac{1}{16\pi^2}L_\pi\right)^2 + \frac{M^4}{16\pi^2F^4}L_\pi\Big[\frac{3}{256\pi^4} + \frac{32F^2}{3}\left(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\right)\Big] \\
& + \frac{32M^2B(m_d - m_u)}{48\pi^2F^4}L_\pi\left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right] \\
& + \frac{M^4}{F^4}\lambda_+ + \frac{M^2B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_+ = & \frac{1}{\pi^2}\left[-\frac{2}{3}d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right] \\
& + \frac{16}{3}F^2\left[8l_3^r(c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right] \\
\lambda_- = & \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r\,(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right] \\
\lambda_{--} = & d_{--}^{Wr}(\mu) - 128F^2 l_7(c_3^{Wr} + c_7^{Wr})\ .
\end{aligned}
$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\Bigg\{\frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right] + 32B(m_d - m_u)\left[\frac{4}{3} C_7^{Wr} + 4 C_8^{Wr}\left(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right) - \frac{1}{16\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3} L_\eta)\right)\right] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2\Bigg\}$$

$\pi \to \gamma\gamma$: Phenomenology inputs

- $F_\pi = 92.22 \pm 0.07$ MeV (using updated value of V_{ud} [Towner, Hardy'08]). using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])

- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}$GeV^{-2} (from $\eta \to 2\gamma$)

result [KK, Moussallam'09]: $\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)$ eV

$\pi^0 \to \gamma\gamma$: comments

1) F_π is a crucial ingredient

- F_π vs $\hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]
- using $\pi^0 \to \gamma\gamma$:

$$F_\pi = 93.85 \pm 1.4\,\text{MeV}$$

$$\text{cf with } \hat{F}_\pi = 92.22(7)$$

$$(1.2\sigma \text{ difference})$$

- our F_π from PDG is based on π_{l2} and SM using [Marciano, Sirlin'93]
- important input V_{ud}: new update by [Hardy,Towner '20]

$$0.97418(26) \to 0.97373(31)$$

we should change $F_\pi \to 92.3$ ...

$\pi^0 \to \gamma\gamma$: comments

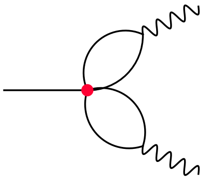
2) η/η' decays

$\eta' \to \gamma\gamma$:

- interconnected with $\pi^0 \to \gamma\gamma$ (via constant C_7)

$\eta \to \gamma\gamma$:

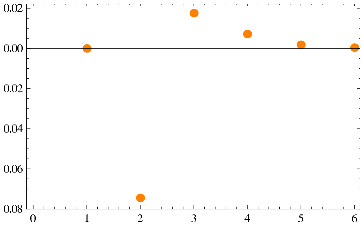
- interconnected with $\pi^0 \to \gamma\gamma$ ($\to$ constant C_8)
- ongoing project to calculate it also at two loop (with J.Bijnens)
- much complicated than $\pi^0 \to \gamma\gamma$



$\pi^0 \to \gamma\gamma$: comments

3) still interesting from purely theoretical point of view

a) Leading logarithm contribution of individual orders in percent of the leading order: [Bijnens,KK,Lanz'12]



b) surprising connections [Bijnens,KK,Lanz'12]

Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to two loops for leading logs beyond the soft-photon limit

Summary

- neutral sector of low energy QCD: the important theoretical challenge of understanding EFT
- BSM: tantalizing theoretical connection with a different BSM sector!
- In this talk focused mainly on $\pi^0 \to \gamma\gamma$ decay.
- summary: more theoretical understanding needed, and maybe more calculations needed

$$\text{theory: } (8.09 \pm 0.11) \text{ eV}$$
$$\text{PrimEx I+II: } (7.80 \pm 0.12) \text{ eV}$$

or equivalently π^0 lifetime:

$$\text{theory: } 8.04 \pm 0.11 \times 10^{-17} \text{ s}$$
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