Hadron Mass Corrections in SIDIS at 22 GeV

Alberto Accardi
Christopher Newport U. & Jefferson Lab

Science at the Luminosity Frontier: Jefferson Lab at 22 GeV

Collaborators: M. Cerutti, J. Guerrero

This work is partly supported by the DOE Office of Science, DE-SC0008724

Outline

- **Why correcting for hadron masses**
  - Quick overview of available studies
  - Mostly collinear (pT-integrated)
  - Impact on multiplicities at JLab 6, HERMES, COMPASS

- **Size of HMCs**
  - Phase-space heat maps for cross sections
  - (A bit of) theoretical systematics

- **Key messages:**
  - For the whole community:
    - ⟶ HMCs at 22 GeV are not negligible (pi) / large (K)
    - ⟶ Serious pheno / theory studies must to start now!
  - For 22 GeV:
    - ⟶ we need help, experimental expertise
      to factor in detector issues and impact studies

Why hadron mass corrections?

Why Hadron Mass Corrections?

Identified

hadron's mass

Target's

mass

$$d\sigma_h \propto d\sigma_h^{\mathrm{LT}} + O\Big(\frac{M^2}{Q^2}\Big) + O\Big(\frac{m_h^2}{Q^2}\Big) + O\Big(\frac{\delta\mu^2}{Q^2}\Big) + O\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\Big) + O\Big(\frac{\#^2}{Q^2}\Big)$$

$Q^2 \to \infty$

HMC systematics

$\longrightarrow$ Large enough, calculable

$\longrightarrow$ Match partonic & external kinematics

Why Hadron Mass Corrections?

Target's mass

Identified hadron's mass

"Residual" HTs can be fitted

$$d\sigma_h \propto d\sigma_h^{\mathrm{LT}} + O\Big(\frac{M^2}{Q^2}\Big) + O\Big(\frac{m_h^2}{Q^2}\Big) + O\Big(\frac{\delta\mu^2}{Q^2}\Big) + O\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\Big) + O\Big(\frac{\#^2}{Q^2}\Big)$$

$Q^2 \to \infty$

HMC systematics

Parton-parton correlations

Anything else

→ Large enough, calculable

→ Match partonic & external kinematics

→ Relieve fits of residual HTs

Quick overview of literature (let me know what I missed)

Inclusive DIS lots and lots of studies

- Nachtmann (1974) – elegant math
- Georgi, Politzer + de Rujula – OPE^{-1}
- Ellis, Furm., Petronzio 1986 – col.pQCD
- ....
- Kuagin, Petti (xxxx)
- Accardi, Qiu (2008)
- Guerrero, Accardi, Phys.Rev.D 106 (2022)
- ** CJ fits (2010-) ; ** AKP fits (2005-)
- ...many many more...
  - REV: Schienbein et al. (2007)
  - REV: Accardi, Brady et al. (2012)

pT integrated SIDIS

- **Albino, Kniehl, Kramer, Nucl. Phys. B (2008)
- *Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)*
- *Guerrero et al., JHEP 1509 (2015)*
- *Guerrero, Accardi, PRD 97 (2018)*

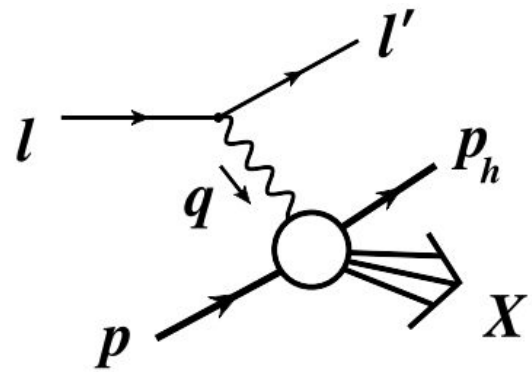
TMD SIDIS (unpolarized)

- Boglione et al., JHEP 10 (2019)
- **Scimemi, Vladimirov, JHEP 06 (2020)
- Scimemi, Moos, Vladimirov, JHEP 01 (2022)

** global QCD fits

Collinear SIDIS

Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)



- **Invariant momentum fractions**

$$x_B = \frac{-q^2}{2q\cdot p}$$

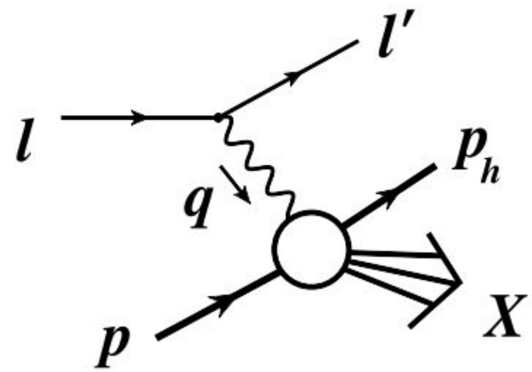
$$\begin{cases} z_h = \dfrac{p_h\cdot q}{p\cdot q} \\[2ex] z_e = \dfrac{2p_h\cdot q}{-q2} \end{cases}$$

Mixes initial and final state (← z_h)

like in e+e- (← z_e)
+ decouples the final state

Collinear SIDIS

Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)



- **Invariant momentum fractions**

$$x_B = \frac{-q^2}{2q \cdot p}$$

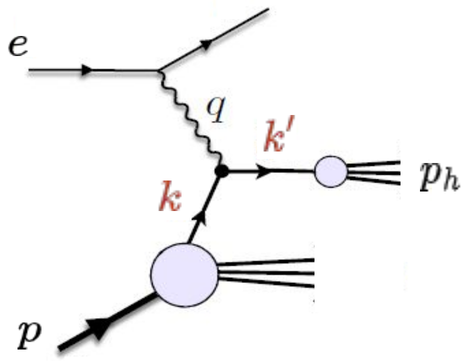
$$\begin{cases} z_h = \dfrac{p_h \cdot q}{p \cdot q} \\ z_e = \dfrac{2p_h \cdot q}{-q2} \end{cases}$$

- **Lightcone momentum fractions**
    - Suitable for QCD factorization

$$\xi = -\frac{q^+}{p^+} \qquad \zeta = \frac{p_h^-}{q^-}$$

Collinear SIDIS

Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)



- **Partons live on the light cone**

$$x = \frac{k^+}{p^+} \overset{LO}{=} \xi$$

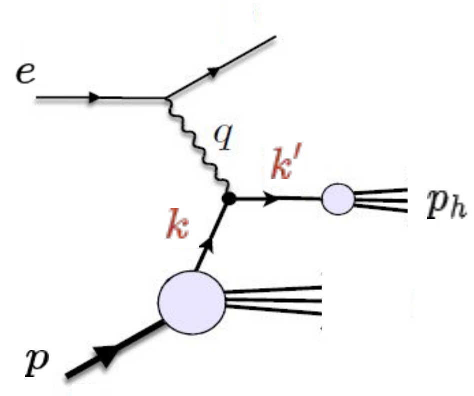
$$z = \frac{p_h^-}{k'^-} \overset{LO}{\gtrsim} \zeta\left(1 + \frac{m_h^2 + k_T'^2}{Q^2}\right)$$

- **Lightcone momentum fractions**
  - Suitable for QCD factorization

$$\xi = -\frac{q^+}{p^+} \qquad \zeta = \frac{p_h^-}{q^-}$$

Collinear SIDIS

Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)



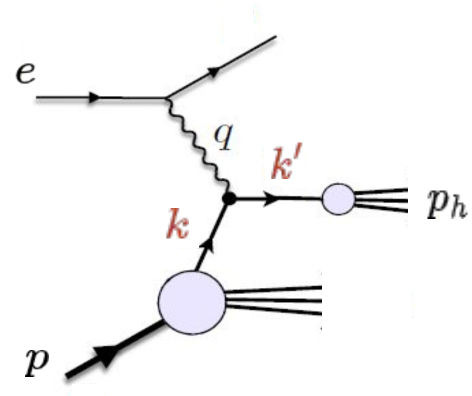
- **Partons live on the light cone**

$$x = \frac{k^+}{p^+} \overset{LO}{=} \xi \quad \longrightarrow \quad \xi = -\frac{q^+}{p^+} = \frac{2x_B}{1+\sqrt{1+4x_B^2\frac{M^2}{Q^2}}}\xi$$

$$z = \frac{p_h^-}{k'^-} \overset{LO}{\gtrsim} \zeta\left(1+\frac{m_h^2+k_T'{}^2}{Q^2}\right)$$

Collinear SIDIS

Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)



- **Partons live on the light cone**

$$x = \frac{k^+}{p^+} \overset{LO}{=} \xi$$

$$z = \frac{p_h^-}{k'^-} \overset{LO}{\gtrsim} \zeta \left(1 + \frac{m_h^2 + k'_T 2}{Q^2}\right)$$

z_h:

$$\zeta = \frac{p_h^-}{q^-} = \frac{z_h}{2} \frac{\xi}{x_B} \left(1 + \sqrt{1 - 4 \frac{x_B^2}{z_h^2} \frac{M^2}{Q^2} \frac{m_h^2}{Q^4}}\right)$$

z_e:

$$\zeta = \frac{p_h^-}{q^-} = \frac{z_e}{2} \left(1 + \sqrt{1 + \frac{4}{z_e^2} \frac{m_{hT}^2}{Q^2}}\right)$$

Impact - Hadron Multiplicities

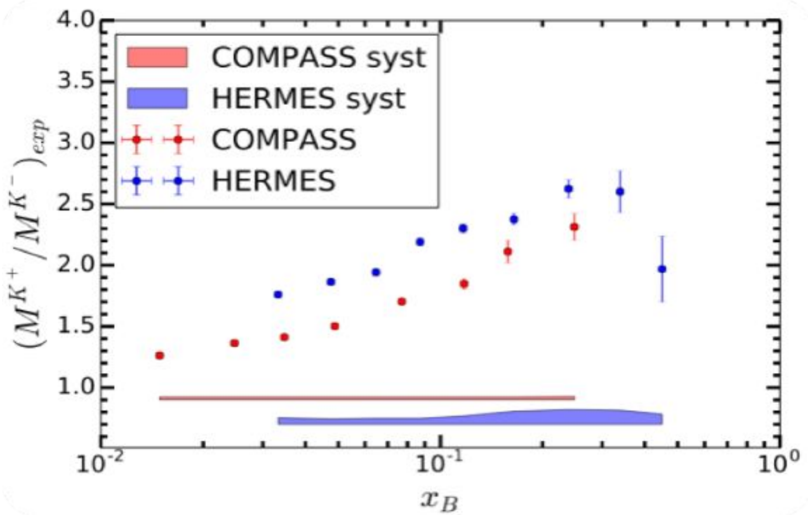
Guerrero, Accardi, PRD 97 (2018)
Guerrero et al., JHEP 1509 (2015)
Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009)

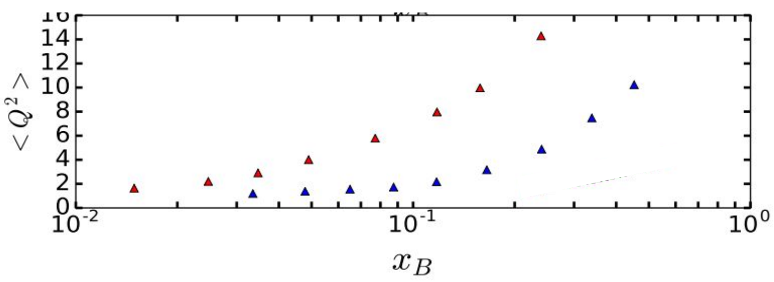
- **Kinematic shift** $x_B \to \xi \qquad z_{h(e)} \to \zeta_h$
  - Calc with HMCs: $$M^h = J(\xi, \zeta_h) \frac{\sum_q e_q^2\, q(\xi) D_h(\zeta_h)}{\sum_q e_q^2\, q(\xi)}$$
  - Without: $$M^{h(0)} = \frac{\sum_q e_q^2\, q(x_B, z_h)}{\sum_q e_q^2\, q(x_B)}$$
- **Mass correction ratio**
  - To “remove” HMCs from data
  - Visually compare different experiments

$$M_{exp}^{h(0)} = \frac{M^{h(0)}}{M^h} M_{exp}^h$$

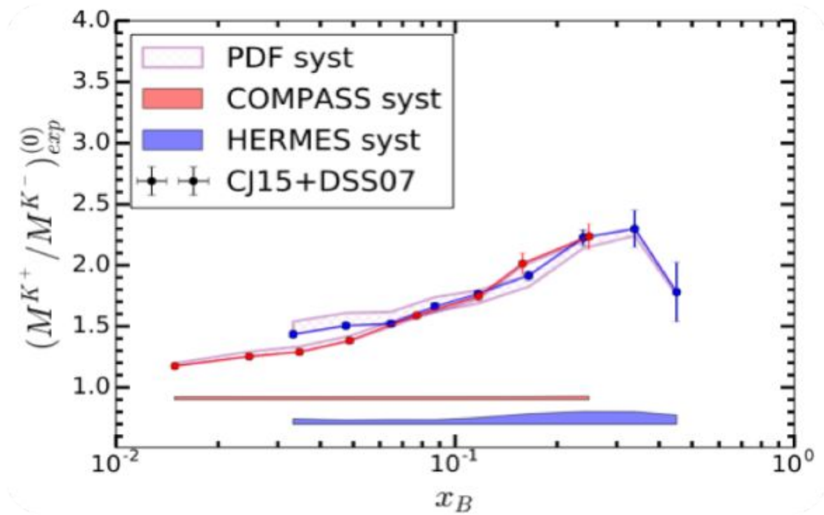
Kaons (integrated over z, Q^2)

Experimental data





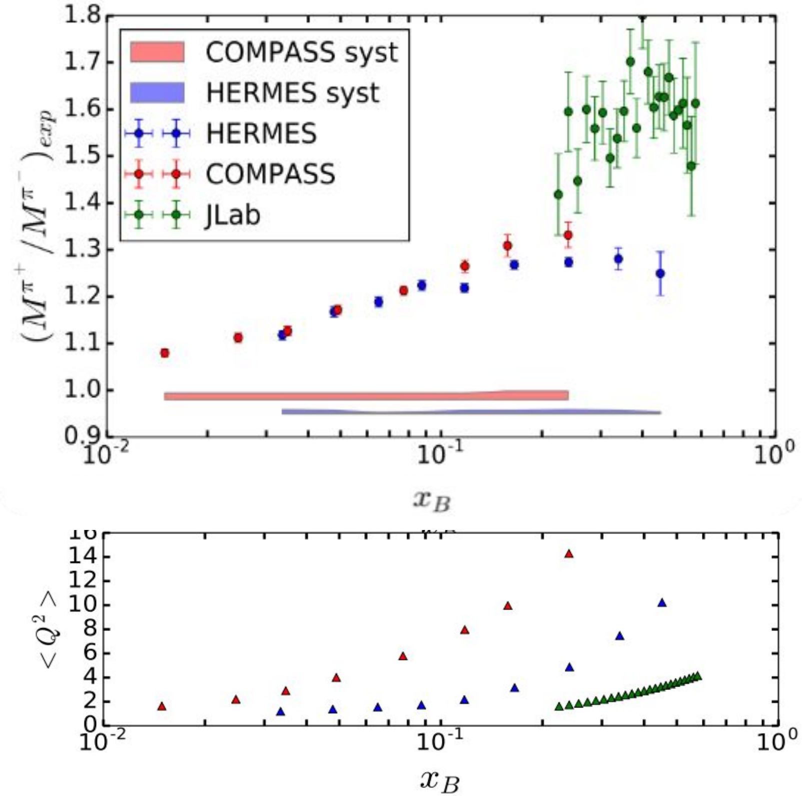
HMCs removed



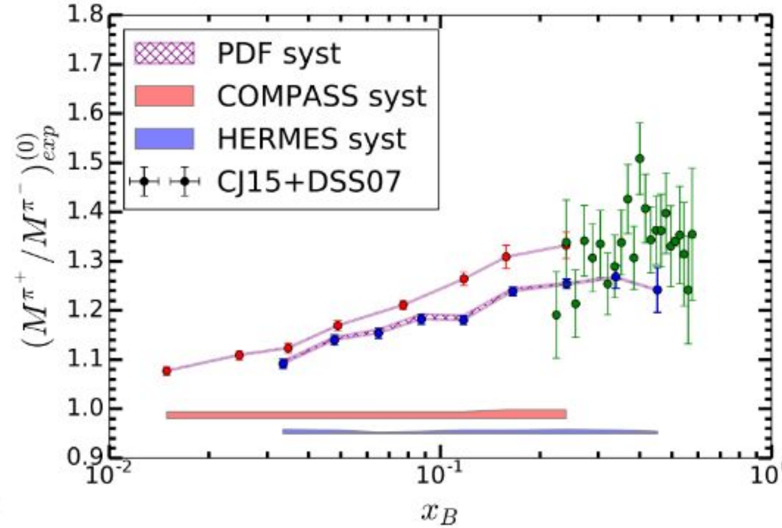
Guerrero, Accardi, PRD 97 (2018)

Pions (integrated over z, Q^2)

Experimental data



HMCs removed



Guerrero, Accardi, PRD 97 (2018)

HMC size: heat maps

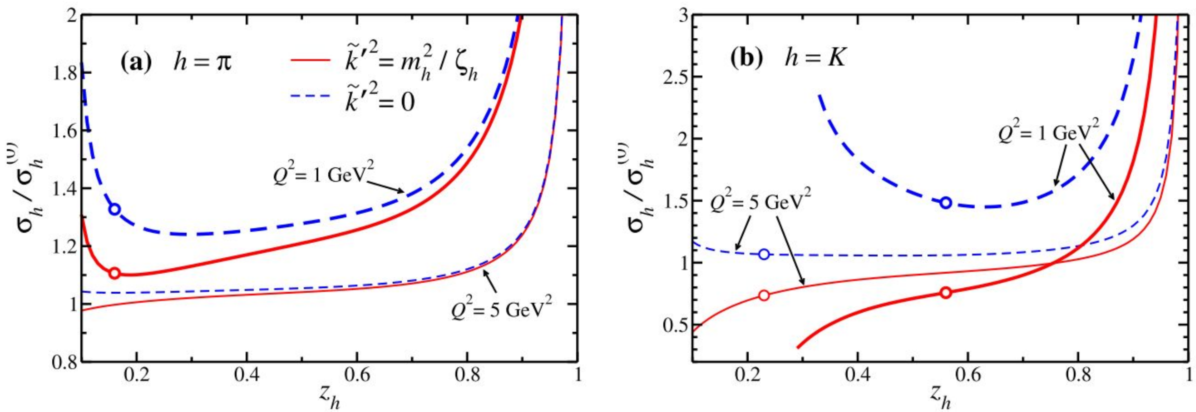
HMC heat maps

- **HMC relative effect for cross sections**

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

  - That is, what mistake would we make if we analyzed the data with massless calculation?

- **Example**



(Circles are rough estimate of target/current region boundary)

HMC heat maps

- **HMC relative effect for cross sections**

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

  - That is, what mistake would we make if we analyzed the data with massless calculation?

- **Heat maps**
  - HMCs depend on 3 variables: $x_B,\ z_h,\ Q^2$
    $\longrightarrow$ (z_e would really be better, but not enough time for this workshop…)
  - 2 variables at a time, fix the 3rd
  - Will show 22 GeV kinematics
    $\longrightarrow$ Pions, then kaons

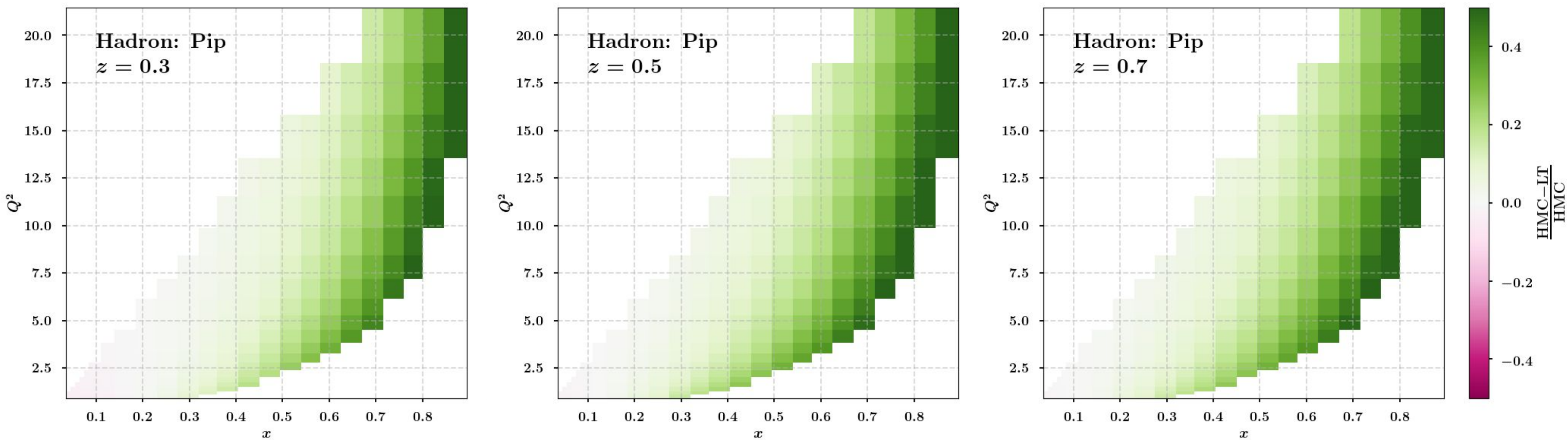
Heat maps: pions

(apologies for the semi-random color maps, we'll do better asap)

Pions: x vs. Q^2

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

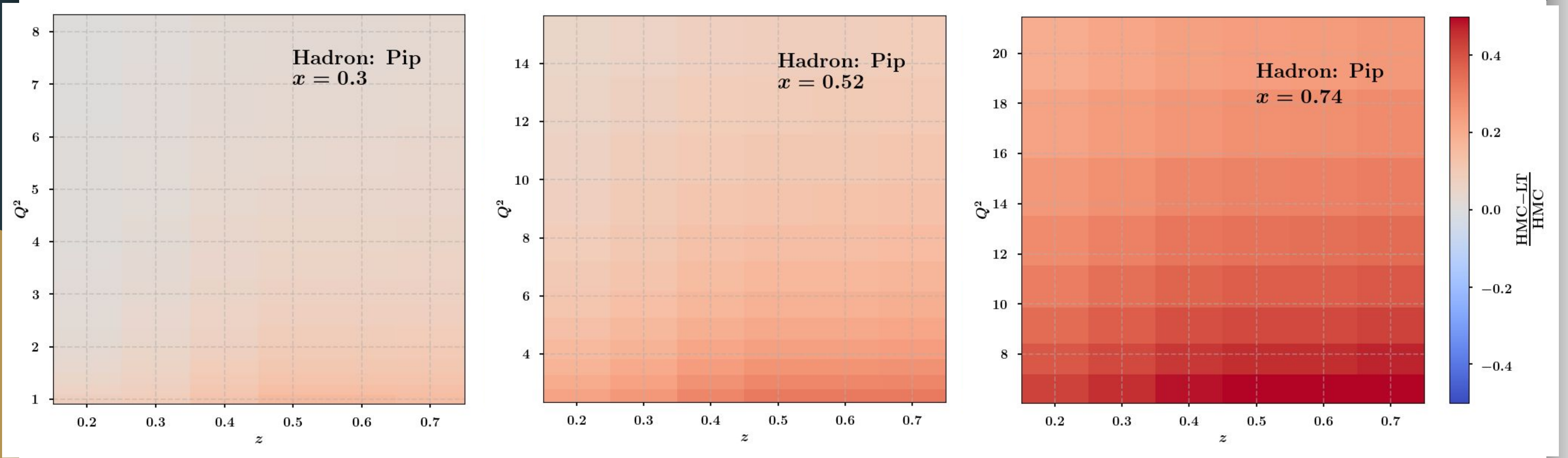
- **Non-negligible effect**
  - Especially towards large x, near the $W^2 > 4$ GeV^2 cut



Pions: z_h vs. Q^2

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

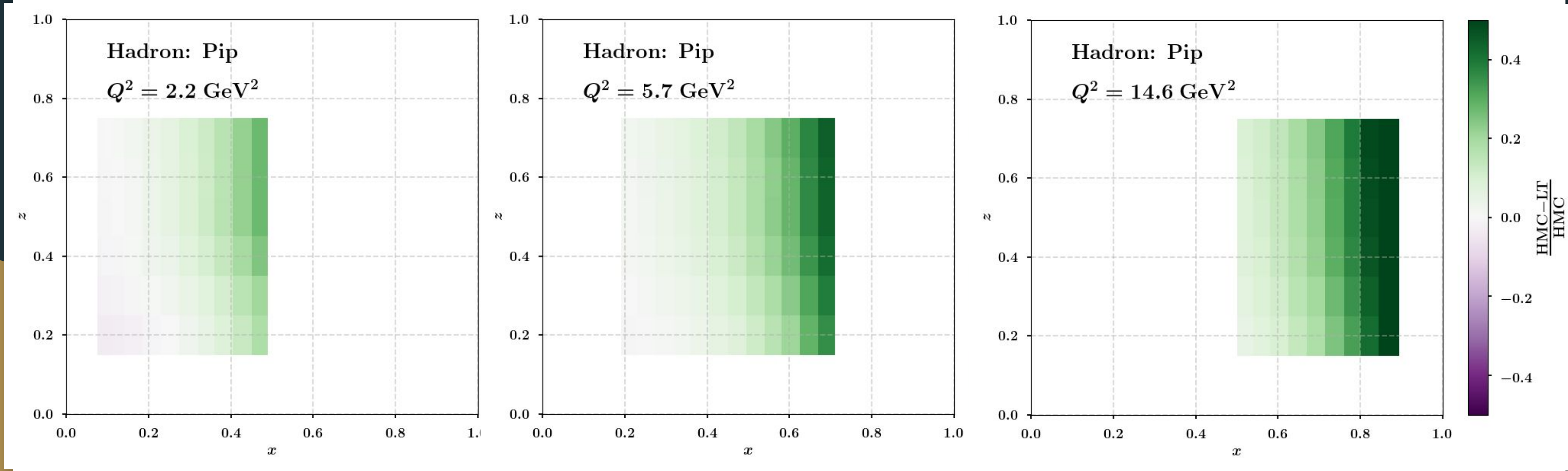
- **Increases with z and 1/Q²**



Pions: x vs. z_h

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

- **X – zh correlation**
  - Look well here, it will become more obvious with the kaons



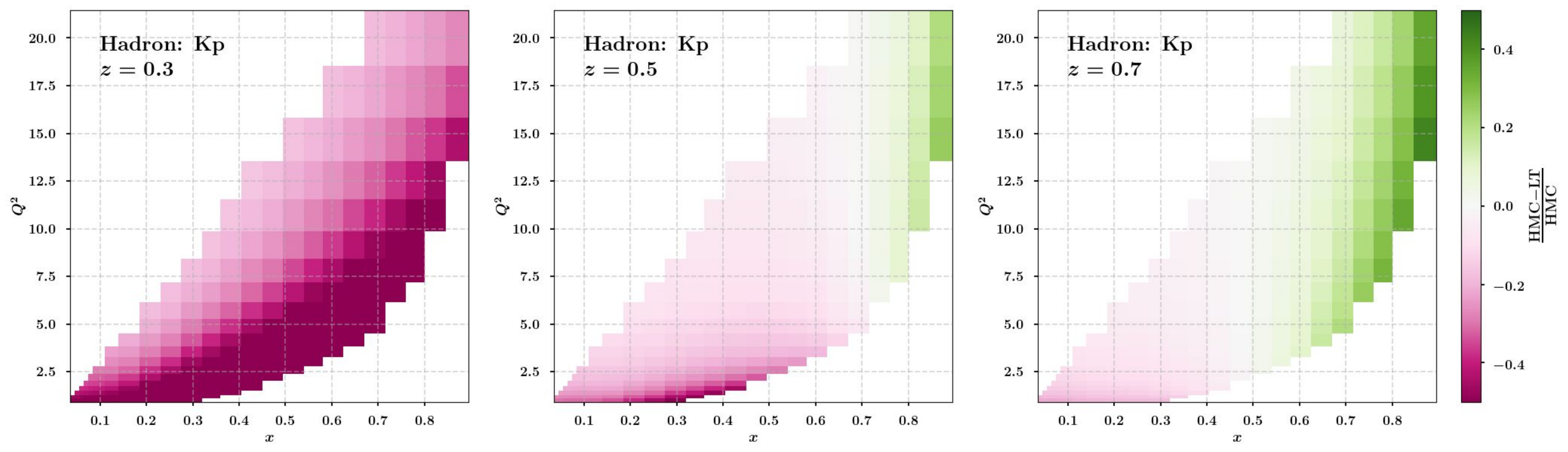
Heat maps:
kaons

(apologies for the semi-random color maps, we'll do better asap)

Kaons: x vs. Q^2

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

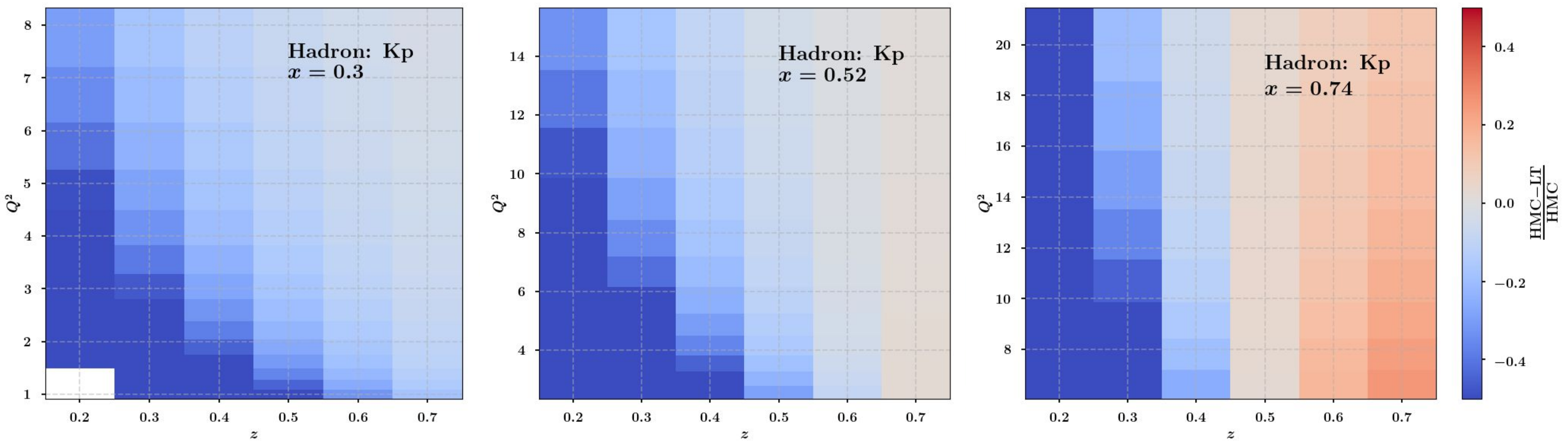
- **Larger effects!** Positive and (mostly) negative



Kaons: z_h vs. Q^2

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

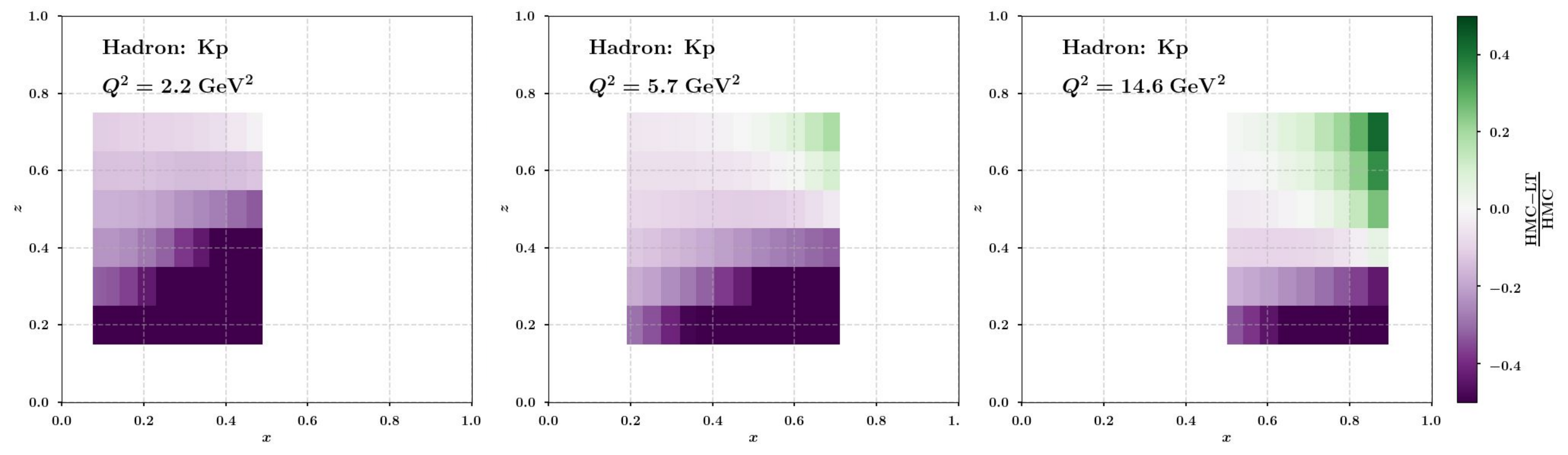
- **Larger effects!** Positive and (mostly!) negative



Kaons: x vs. z_h

$$\frac{HMC - LT}{HMC} = \frac{\sigma_h - \sigma_h^{(0)}}{\sigma_h}$$

- $x - z_h$ correlations now are quite visible



Theoretical uncertainty
- 1st pass -

Transverse momentum effects

- Fragmentation scaling variable and kinematic shifts depend on
  - Final state hadron's transverse momentum
    $\longrightarrow$ would need TMD formalism
  - And mass of undetected hadrons
    $\longrightarrow$ this we cannot control
  - But it is a $1/Q^2$ effect

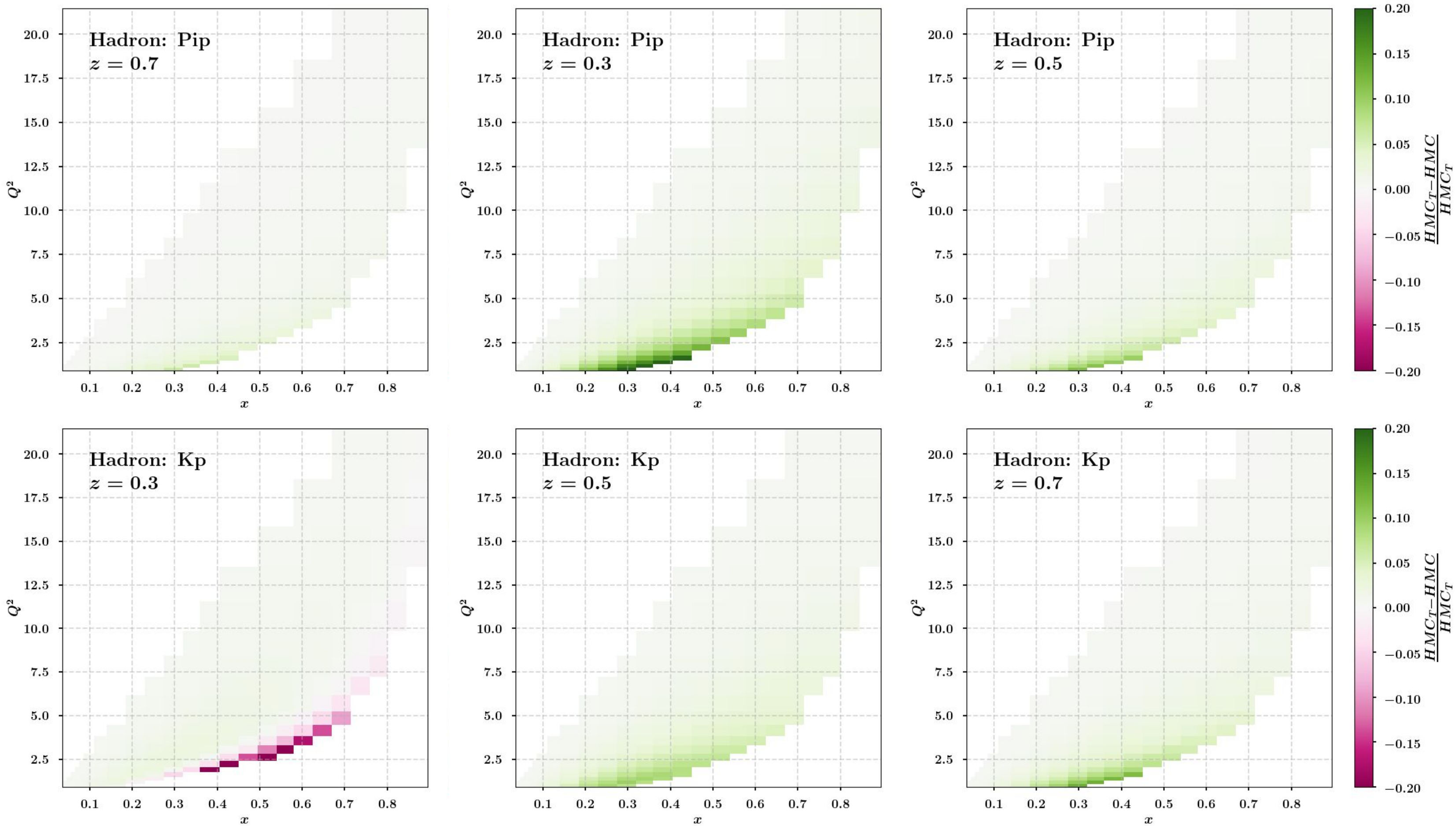
$$z = \frac{p_h^-}{k'^-} \overset{LO}{\gtrsim} \zeta \left(1 + \frac{m_h^2 + k_T'^2}{Q^2}\right)$$

- **Estimate the effect:**
  - In previous plots, $m_{hT}^2 \approx m_h^2$
  - Now compare with $m_{hT}^2 \approx m_h^2 + \langle k_T'^2 \rangle_{\text{TMD fits}}$

$\longrightarrow$ Plot heat maps of $\dfrac{HMC_T - HMC}{HMC_T} = O\left(\dfrac{1}{Q^2}\right)$

With / without transverse momentum

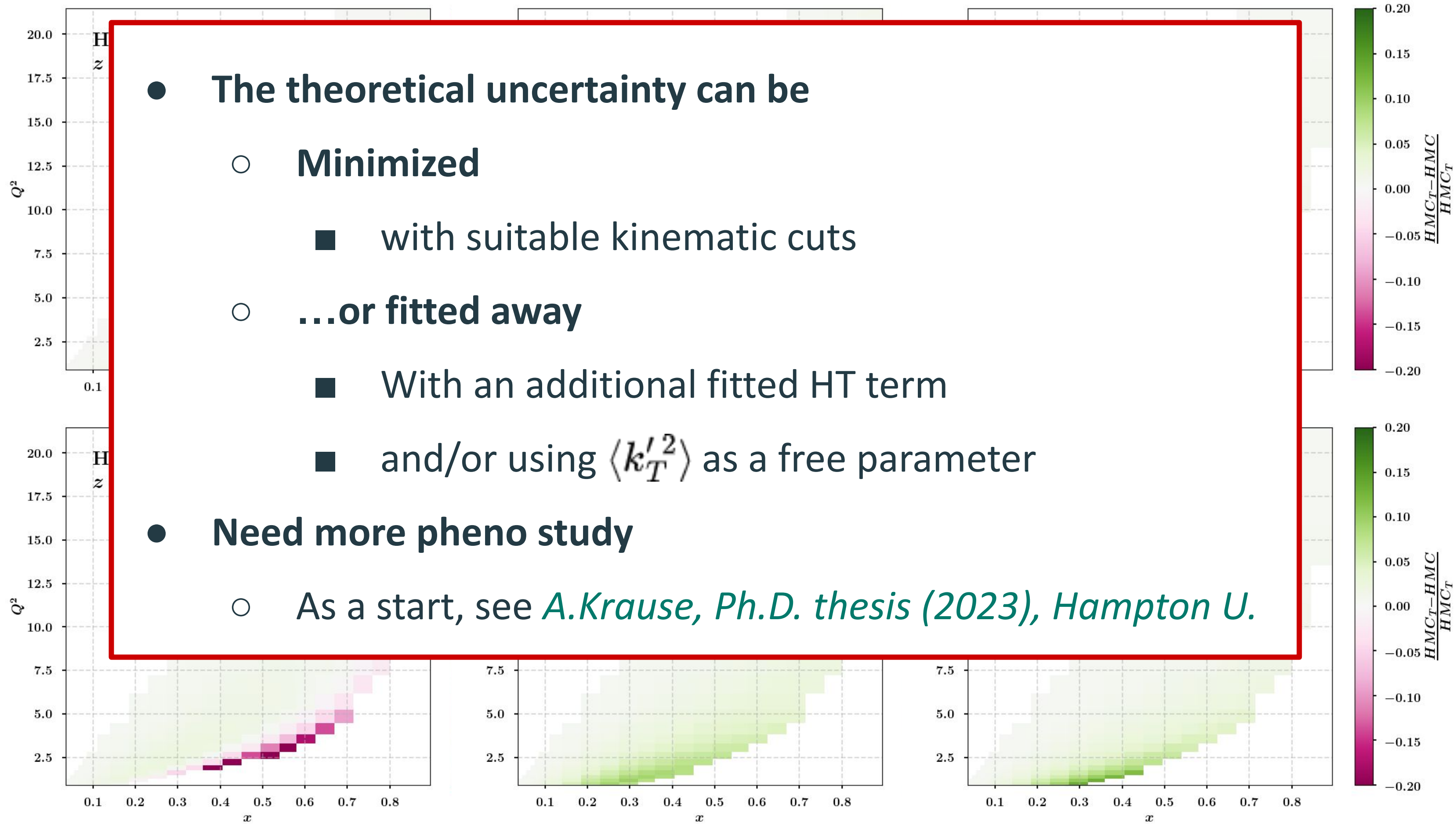
$$\frac{HMC_T - HMC}{HMC_T} = O\left(\frac{1}{Q^2}\right)$$



With / without transverse momentum

$$\frac{HMC_T - HMC}{HMC_T} = O\left(\frac{1}{Q^2}\right)$$

- **The theoretical uncertainty can be**
  - **Minimized**
    - with suitable kinematic cuts
  - **…or fitted away**
    - With an additional fitted HT term
    - and/or using $\langle k_T'^2 \rangle$ as a free parameter
- **Need more pheno study**
  - As a start, see *A.Krause, Ph.D. thesis (2023), Hampton U.*



Takeaways

Takeaways

- **Key messages:**
  - For the whole community:
    → HMCs at 22 GeV are not negligible (pi) / large (K)
    → Serious pheno / theory studies must to start now!
  - For 22 GeV:
    → we need help, experimental expertise
    to factor in detector issues and impact studies
- **Theoretical uncertainties in HMCs**
  - Can be controlled / fitted away
- **We all need to also look at HMC in TMD observables!**

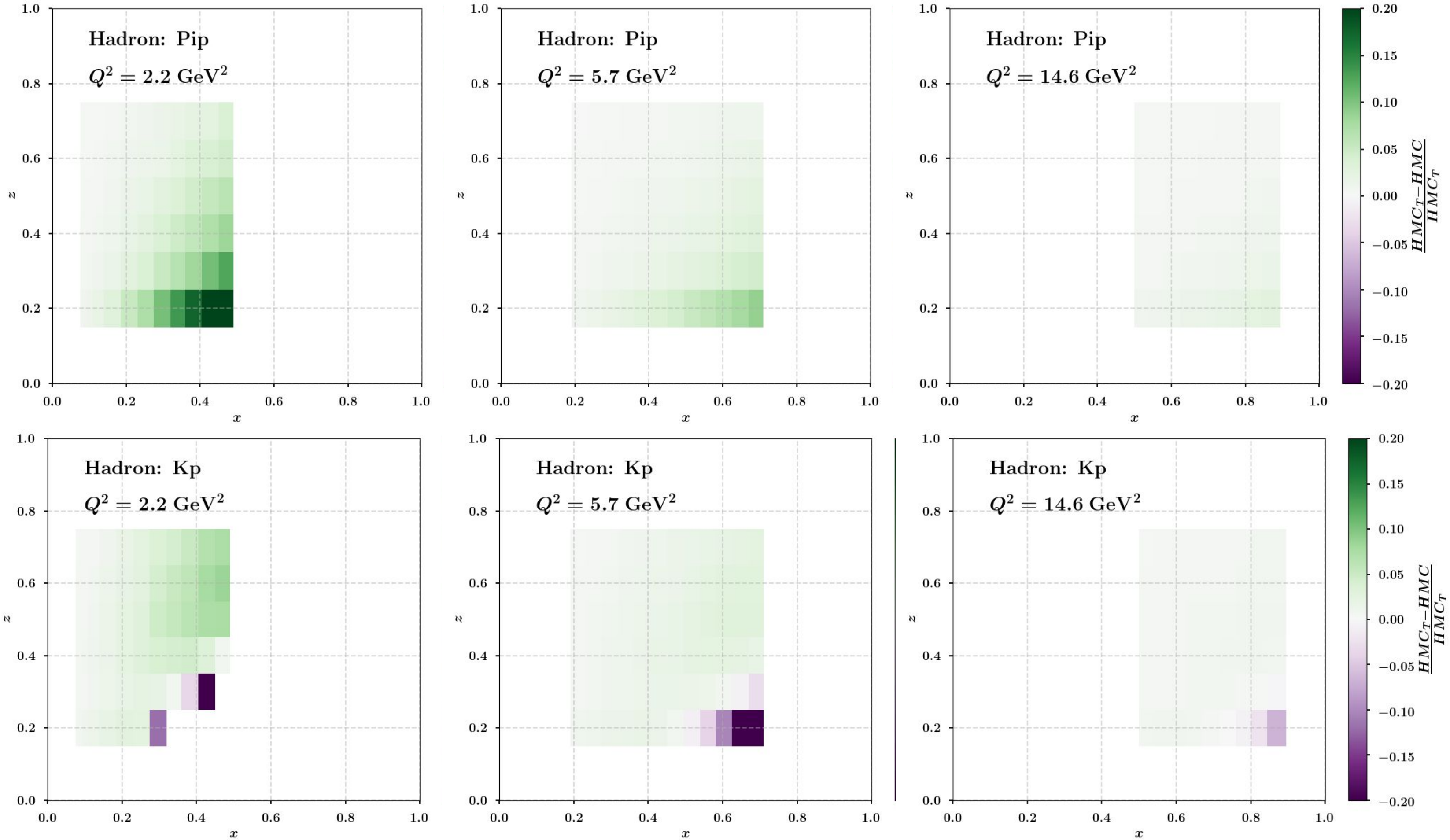
Thank you!

Appendix:

More theory uncertainty plots
&
Phase space limitations

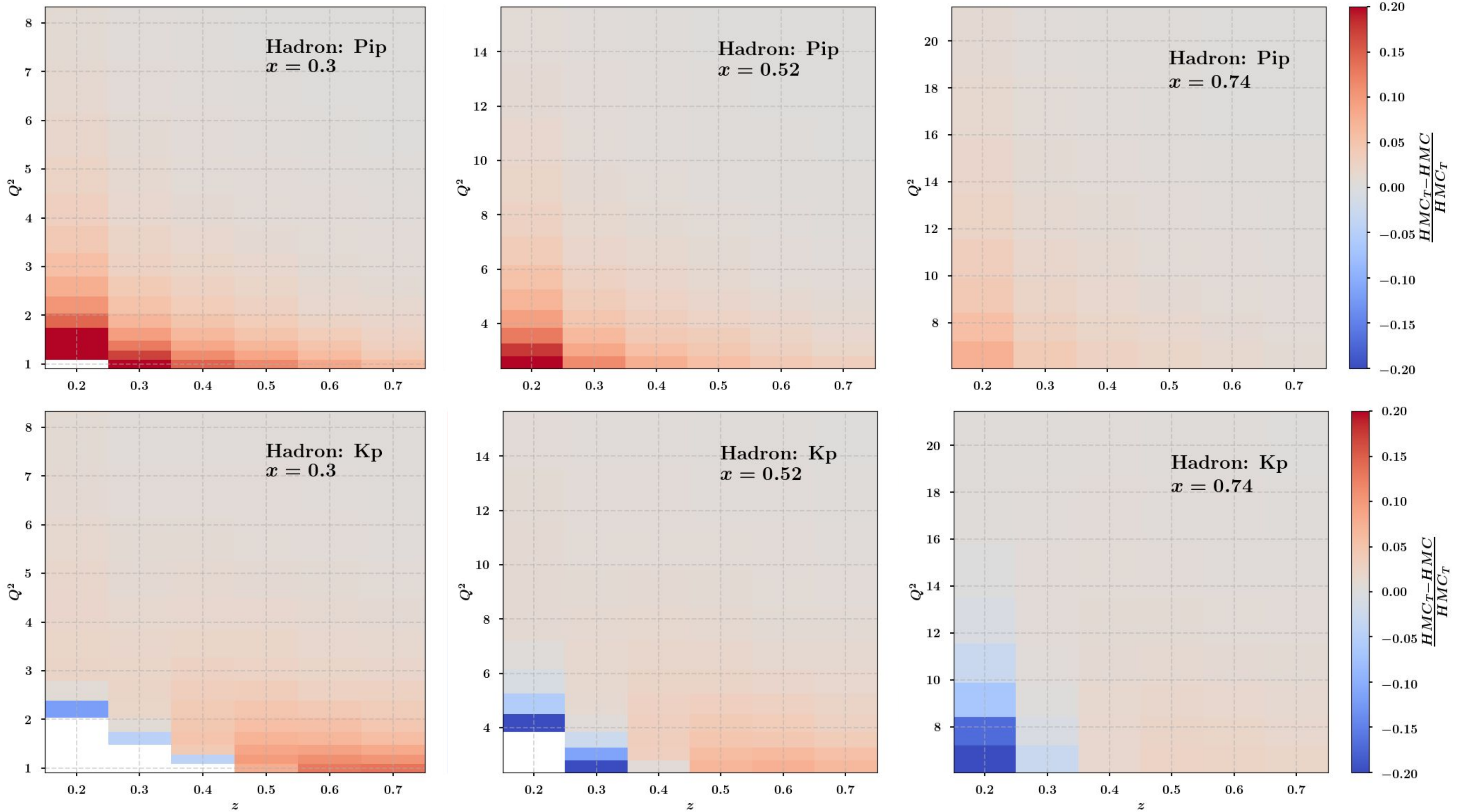
$$\frac{HMC_T - HMC}{HMC_T} = O\left(\frac{1}{Q^2}\right)$$

With/out transverse mom: x vs. z



$$\frac{HMC_T - HMC}{HMC_T} = O\left(\frac{1}{Q^2}\right)$$

With/out transverse mom: z vs. Q^2



Phase space limitations

Guerrero et al., JHEP 09 (2015) 169

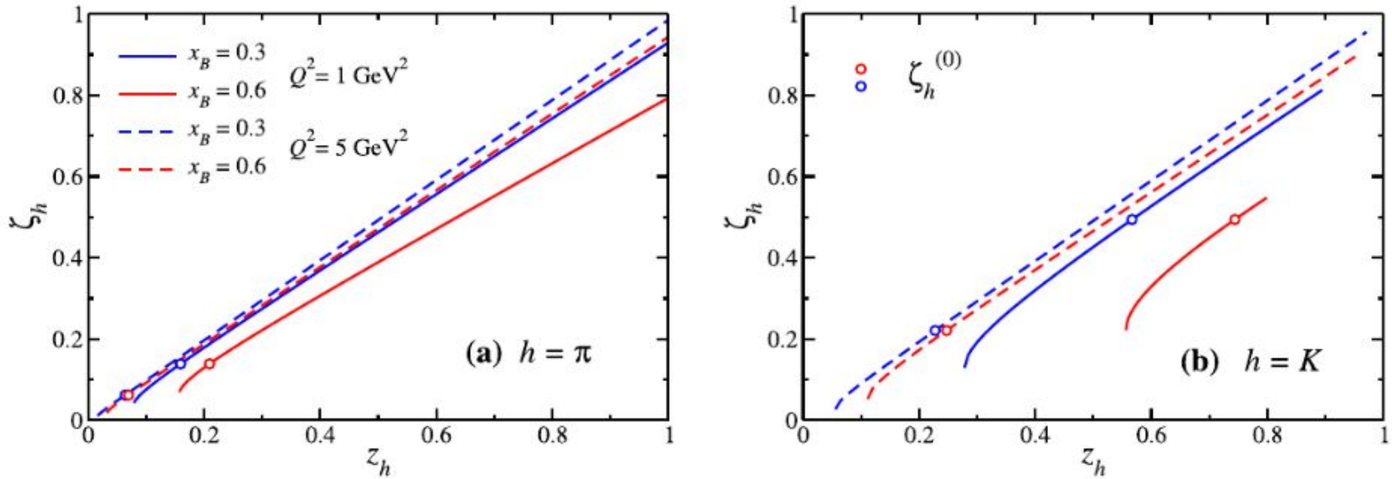


Figure 2. Finite-Q^2 fragmentation variable ζ_h versus z_h for the semi-inclusive production of (a) pions, $h = \pi$ and (b) kaons, $h = K$, at fixed values of $x_B = 0.3$ (blue curves) and 0.6 (red curves) for $Q^2 = 1$ (solid curves) and $5\,\mathrm{GeV}^2$ (dashed curves). The curves are shown only in the kinematically allowed z_h regions, and the boundaries between the current ($\zeta_h > \zeta_h^{(0)}$) and target ($\zeta_h < \zeta_h^{(0)}$) fragmentation regions are indicated by the open circles.

$$\zeta_h^{(0)} = \zeta_h(z_e = 0)$$