# Role of QED Contributions to Inclusive and Semi-Inclusive Deep Inelastic Scatterings in Lepton-Hadron Collisions

- High-energy lepton-hadron collision induces both QED and QCD radiation
- Treat QED and QCD radiation equally in terms of factorization approach
- Full NLO QED contribution to inclusive DIS
- Three complementary processes from lepton-hadron scatterings
- Summary and Outlook

In collaboration with J. Cammarota, T.-B. Liu, W. Melnitchouk, N. Sato, K. Watanabe, J.-Y. Zhang, …

Jianwei Qiu
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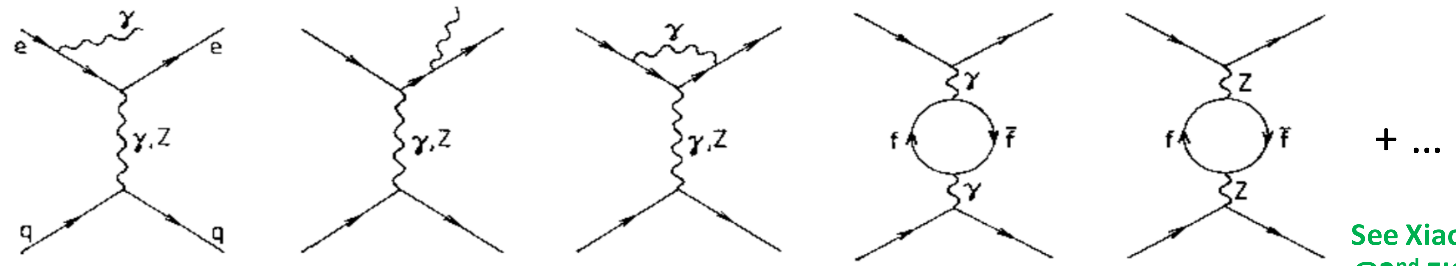
# Collision-Induced QED Radiation in Lepton-Hadron Scattering

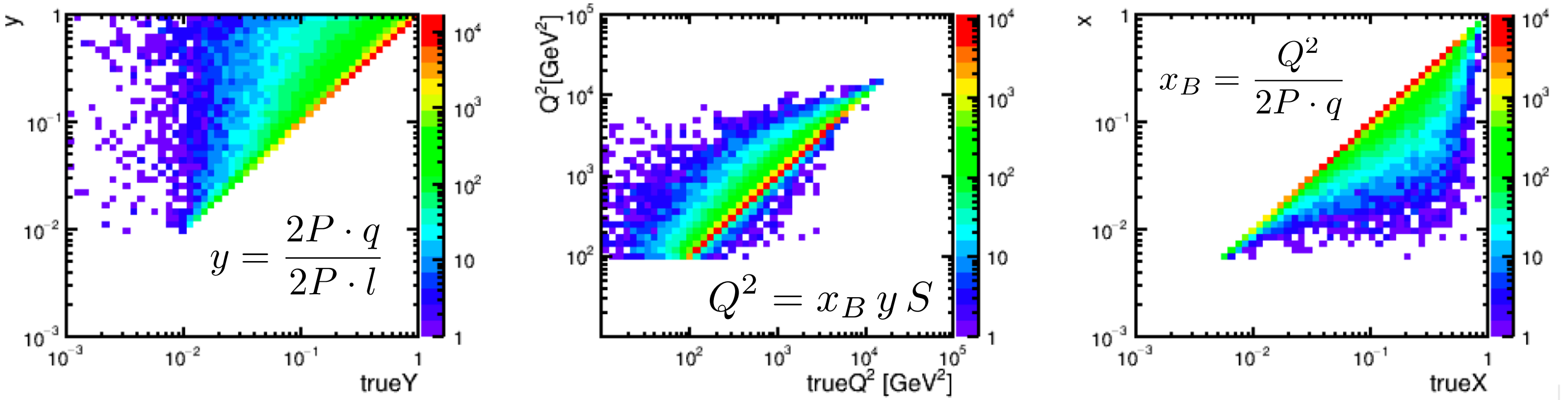
## ❑ "Probe" for the hadron @ EIC is smeared by the induced QED radiation:

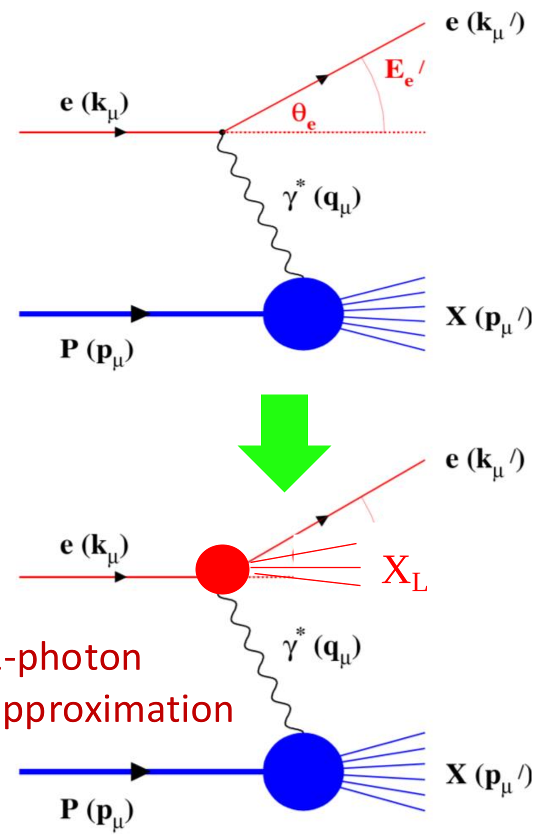
**Data sample : Int L = 10 $fb^{-1}$, Kinematics settings: 0.01<y<0.95, $10^2$ $GeV^2$ <$Q^2$<$10^5$ $GeV^2$**




**initial** **final** **vacuum** **loops**

See Xiaoxuan Chu @$2^{nd}$ EIC YR workshop




$$q_\mu \rightarrow \hat{q}_\mu$$

$$Q^2 = -q^2 \rightarrow \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \rightarrow \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

**Instead of a straight line – linear correlation,**

**the kinematic variables, y, $Q^2$ , $x_B$, from the leptons are smeared so much that the proton is not probed by the hard scale that we thought of !!!**

***Ill-defined "photon-hadron" frame?! Fitting parameter(s) needed for Radiation Corrections?***



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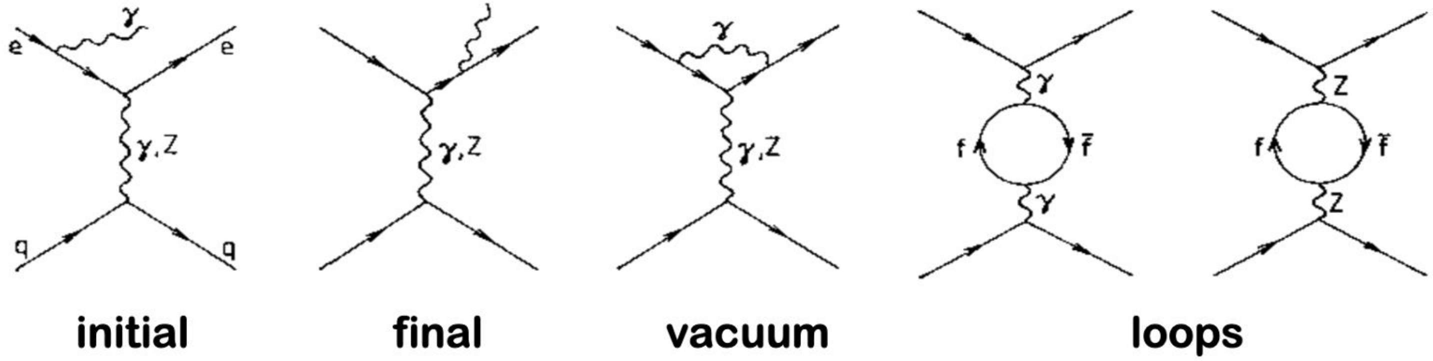
# Traditional Radiative Correction (RC) Approach

- "We know how to calculate QED radiation perturbatively!"



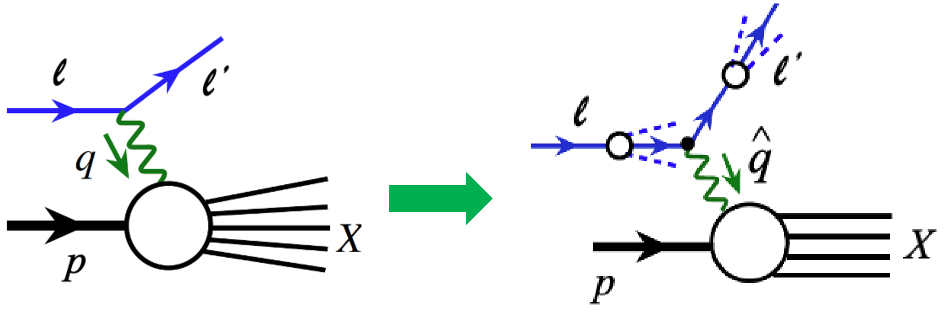

+ two-photon channel + …

→ MC program(s) for the RC with "cutoff(s)"

Always keep the γ* virtual!

- Do we really know how to calculate QED radiation perturbatively?

  - Keep one-photon approximation (only CO radiation from leptons):

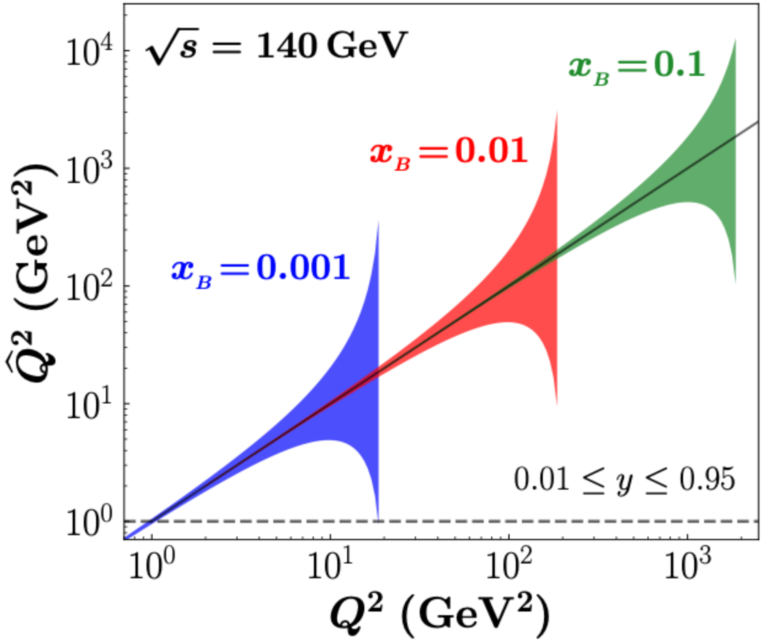



$(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$

$x_B \to \hat{x}_B \in [x_B, 1]$

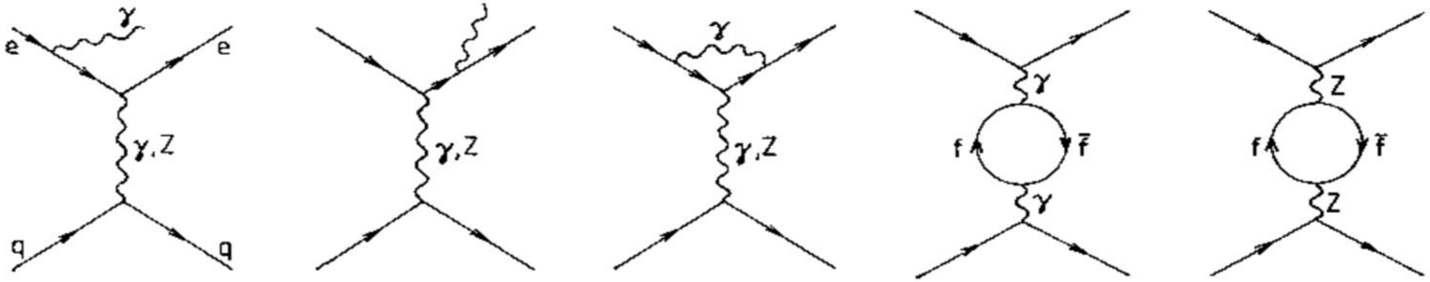
$\widehat{Q}^2_{\rm min} = Q^2 \frac{(1-y)}{(1-x_B\, y)}$

$\widehat{Q}^2_{\rm max} = Q^2 \frac{1}{(1-y+x_B\, y)}$



# Traditional Radiative Correction (RC) Approach

- "We know how to calculate QED radiation perturbatively"




initial　　final　　vacuum　　loops

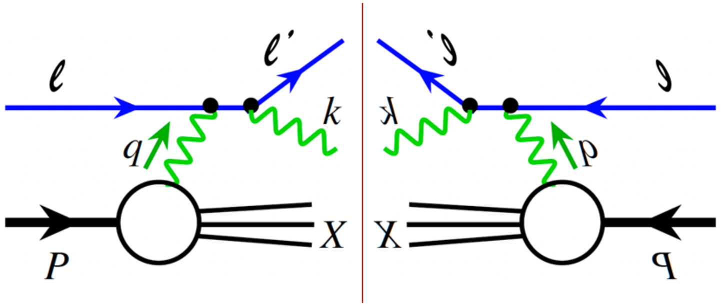
+ two-photon channel + …

→ MC program(s) for the RC with "cutoff(s)"

Always keep the $\gamma*$ virtual!

- Do we really know how to calculate QED radiation perturbatively?

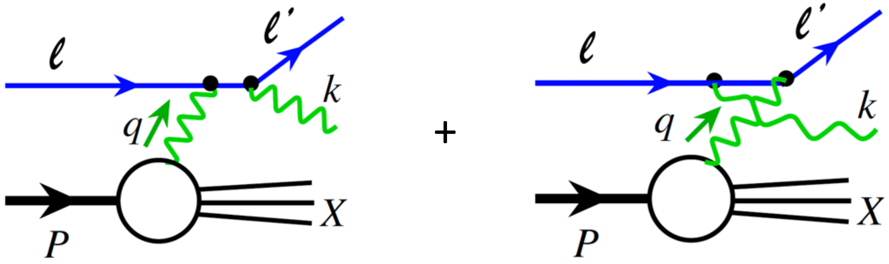
  - **Beyond CO radiation from leptons at NLO:**




$$\propto \int dq^2 \left[ L^{\mu\nu}(\ell, \ell', q^2) \frac{1}{q^2 + i\epsilon} \frac{1}{q^2 - i\epsilon} W_{\mu\nu}(q^2, P) \right]$$

$\propto \infty$　　**Perturbatively due to the pinch singularity!**

"Bethe-Heitler"-type for inclusive DIS

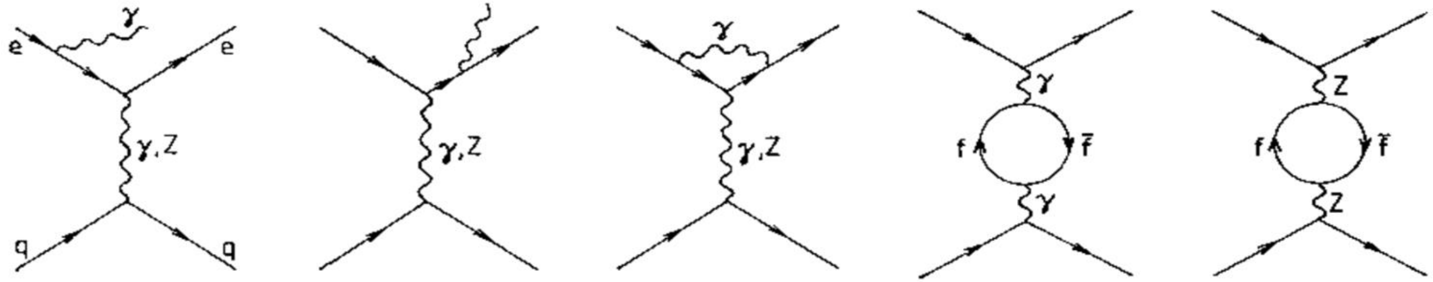



$$\propto \int dx_q \, \phi_{\gamma/h}(x_q, \mu^2) \, \hat{\sigma}_{\ell\gamma \to \ell' X}(\ell, \ell', x_q)$$

**Quark carries EM charge, it radiates photon too!**

# Traditional Radiative Correction (RC) Approach

- **"We know how to calculate QED radiation perturbatively"**




initial    final    vacuum    loops

+ two-photon channel + …

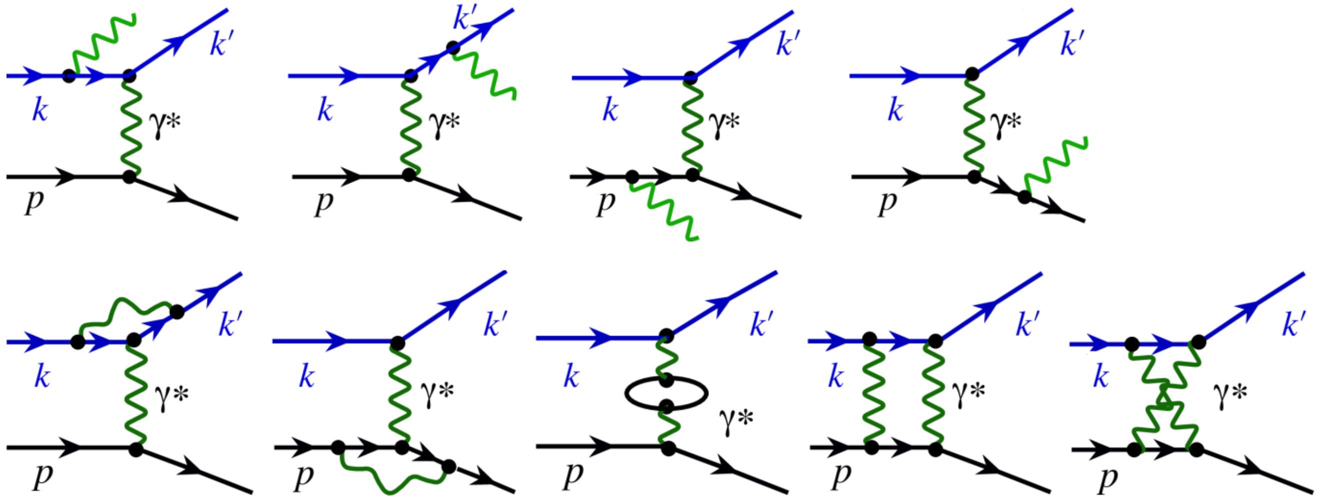
➡ MC program(s) for the RC with "cutoff(s)"

Always keep the $\gamma*$ virtual!

- **Do we really know how to calculate QED radiation perturbatively?**
  - **Full EM radiation at NLO:**

**Real:**

**Virtual:**




One-photon exchange & two-photon exchange are natural & gauge-invariant contributions to the DIS cross section at NLO in QED!

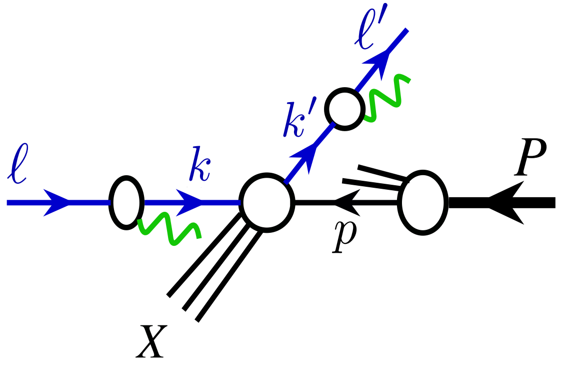
All IR divergences are completely canceled, but, CO divergences are not perturbative beyond this order and need factorization!

# Joint QCD and QED Factorization for Deep Inelastic Scattering (DIS)

## ❑ Without the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157
Cammarota, Qiu, Watanabe,
Zhang [2408.08377]

**~ Inclusive single lepton production at high transverse momentum**




$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) + \cdots$$

**LFFs** → $D_{e/j}$ **LDFs** → $f_{i/e}$ **PDFs** → $f_{a/N}$

**No structure functions, but have PDFs, LDFs, LFFs, …**

$$x_{\min} = \frac{\xi\, x_B\, y}{\xi\zeta + y - 1},$$
$$\xi_{\min} = \frac{1-y}{\zeta - x_B y},$$
$$\zeta_{\min} = 1-(1-x_B)y,$$

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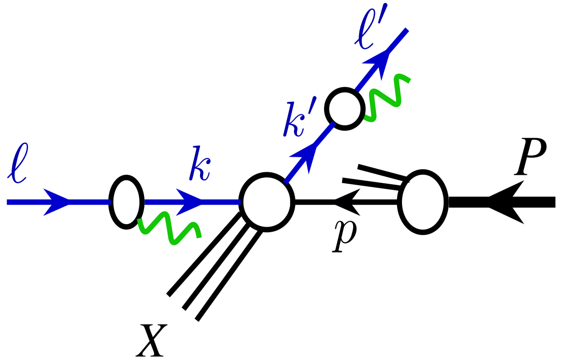


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LFFs → $D_{e/j}$; LDFs → $f_{i/e}$; PDFs → $f_{a/N}$

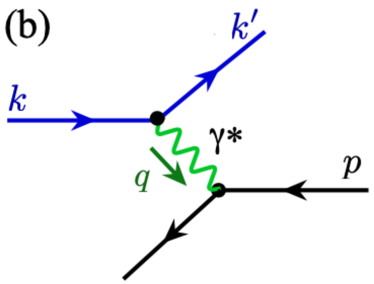
**No structure functions, but have PDFs, LDFs, LFFs, …**

## ❑ Hard parts in power of $\alpha^m\alpha_s^n$ – $\widehat{H}^{(m,n)}_{ia\to jX}$ :

*Hard parts are not sensitive to long-distance physics or the colliding states*

$$x_{\min} = \frac{\xi\,x_B\,y}{\xi\zeta + y - 1},\quad \xi_{\min} = \frac{1-y}{\zeta - x_B y},\quad \zeta_{\min} = 1-(1-x_B)y,$$

- **LO – Let h(P) to be q(P):**




$$\widehat{H}^{(2,0)}_{eq\to eX} = E_q\frac{d\sigma^{(\mathrm{LO})}_{eq\to eq}}{d^3k'} = e_q^2\,(4\alpha_{em}^2)\,\frac{x^2\zeta\left[(\zeta\xi s)^2+u^2\right]}{(\xi t)^2\,(\zeta\xi s+u)}\,\delta(x-x_{\min})$$

$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}d\xi\,D_{e/e}(\zeta,\mu^2)\,f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$
$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

$$s=(\ell+P)^2,\quad u=(\ell'-P)^2=(y-1)s,\quad t=(\ell-\ell')^2=-Q^2$$

**When:** $D_{e/e}(\zeta)=\delta(1-\zeta)\,\delta^{ee}$
$f_{e/e}(\xi)=\delta(1-\xi)\,\delta^{ee}$

➡ Recover the Born expression

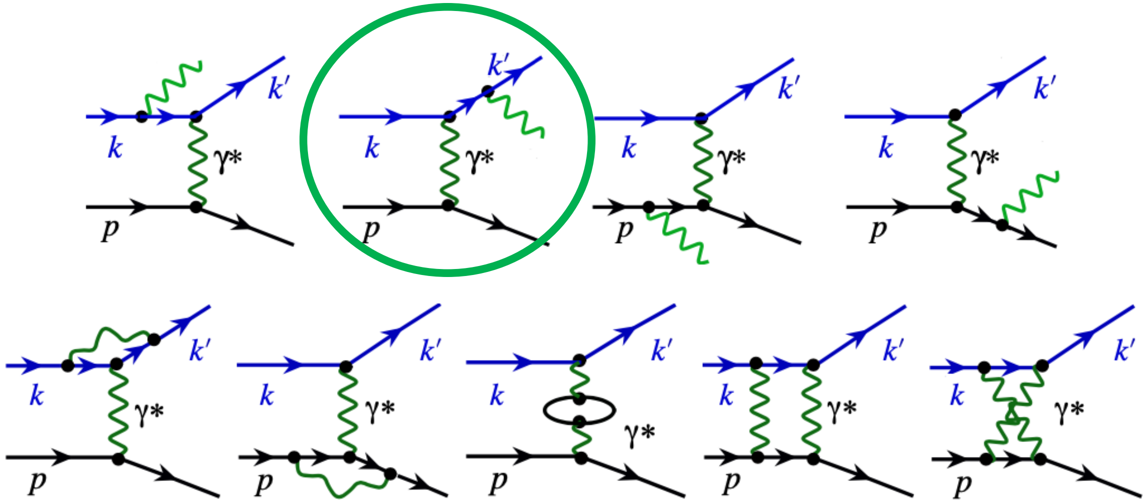
# NLO QED contribution – beyond 1-vector boson exchange

## ❑ Project the external particles to leptons or partons:

Cammarota, Qiu, Watanabe, Zhang [2408.08377]
In preparation

**NLO:** $e(\ell) \to e(k)\,, \quad e(\ell') \to e(k')\,, \quad h(P) \to q(p)\,.$

$$
\begin{aligned}
\sigma^{(1)}_{e(k)+q(p)\to e(k')+X} = {} & D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(3,0)}_{e+q\to e+X} \\
& + D^{(1)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(1)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(1)}_{q/q} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} \\
& + D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(1)}_{\gamma/q} \otimes \hat{H}^{(2,0)}_{e+\gamma\to e+X}
\end{aligned}
$$




$$
\Longrightarrow \quad \hat{H}^{(3,0)}_{e+q\to e+X} = \sigma^{(1)}_{e+q\to e+X} - D^{(1)}_{e/e} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} - f^{(1)}_{e/e} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} - f^{(1)}_{q/q} \otimes \hat{H}^{(2,0)}_{e+q\to e+X} - f^{(1)}_{\gamma/q} \otimes \hat{H}^{(2,0)}_{e+\gamma\to e+X}
$$

$$
\sigma^{(1)}_{eq\to eX} \equiv 2s\, E' \frac{d\sigma^{(3,0)}_{eq\to eX}}{d^3\ell'}
$$

$$
P_{e/e}(z) = \frac{1}{e_q^2} P_{q/q}(z) = \frac{\alpha}{2\pi} \left[ \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]
$$

$$
f^{(1)}_{i/j}(z)_{\overline{\rm MS}} = \left(-\frac{1}{\epsilon}\right)_{\rm CO} (4\pi)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} P_{i/j}(z)
$$

$$
P_{\gamma/q}(z) = \frac{\alpha}{2\pi} e_q^2 \left[ \frac{1+(1-z)^2}{z} \right]
$$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# NLO QED contribution – beyond 1-vector boson exchange

## ❑ NLO QED contribution:

$$\hat{H}^{(3,0)}_{eq\to eX} \propto \alpha^3\, e_q^2 \left\{ e_l^2\, \frac{2(1+\hat{v}^2)}{9\hat{v}} \left[3\ln\frac{(1-\hat{v})s}{\mu^2} - 5\right] \delta(1-\hat{w}) \right.$$
$$+ e_q \left[a_1 \delta(1-\hat{w}) + \frac{a_7}{(1-\hat{w})_+} + a_6\right]$$
$$+ e_q^2 \left[b_1 \delta(1-\hat{w}) + b_2 \left(\frac{1}{1-\hat{w}}\right)_+ + b_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + b_4\right]$$
$$\left. + c_1 \delta(1-\hat{w}) + c_2 \left(\frac{1}{1-\hat{w}}\right)_+ + c_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + c_4 \right\}$$

$$\hat{v} = 1 - \frac{x_B}{x}\frac{y}{\zeta}$$

$$\hat{w} = \frac{1-y}{\xi\,(\zeta - (x_B/x)\,y)}$$

$a_1, a_6, a_7$
$b_1, b_2, b_3, b_4$
$c_1, c_2, c_3, c_4$
are analytic functions of $\hat{v}$ and $\hat{w}$.

$$e_l^2 = \sum_f N_c^f\, e_f^2$$ Sum over the flavors appeared in the photon vacuum polarization

Completely IR and CO safe! Only depends on factorization scale μ, same in all partonic scattering channels
No need for any "cut-off" parameter(s) in the traditional "Radiative Correction"

➡ **In joint QCD & QED factorization: Lepton-distributions are not pure QED !**
**Hadron's parton distributions are not pure QCD !**

# Impact of Factorized QED Contribution to Lepton-Hadron Scattering

## ❑ Perturbative lepton distributions:

$$f_{e/e}^{(\mathrm{NLO})}(\xi,\mu^2) = \delta(1-\xi) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D_{e/e}^{(\mathrm{NLO})}(\zeta,\mu^2) = \delta(1-\zeta) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

Electron mass to regularize the CO divergence
– Only defined under the integration.

## ❑ Model distributions:

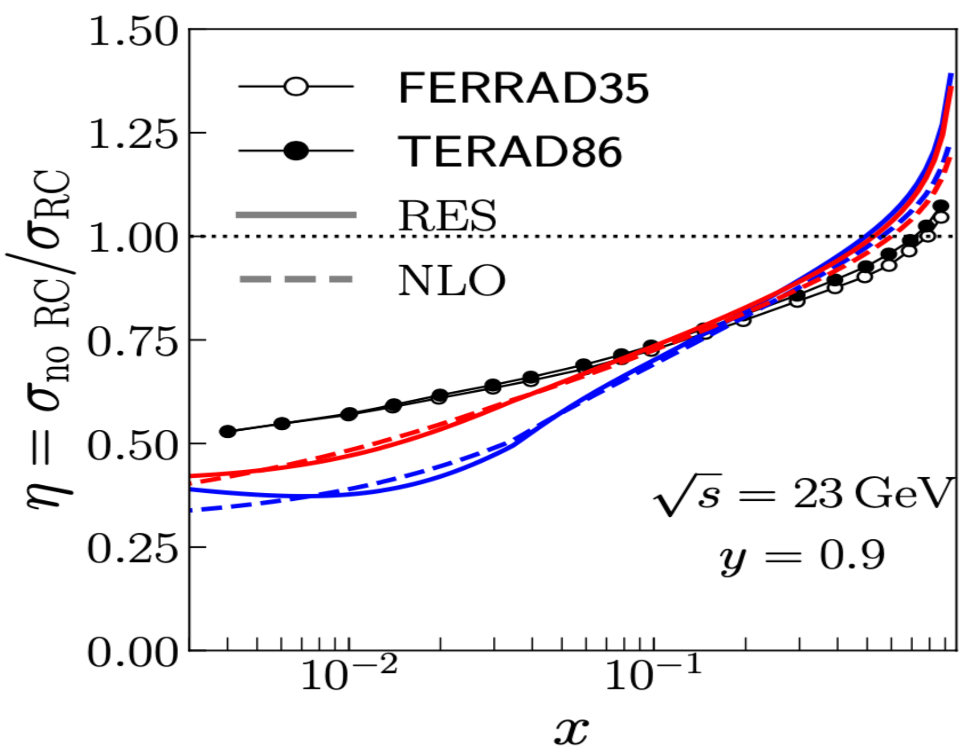
**LDFs:** Very different from PDFs, peaked
**LFFs:** at larger momentum fraction

$$f_{e/e}(x) \approx D_{e/e}(x) = N_e \frac{x^\alpha (1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

with $N_e = 1$

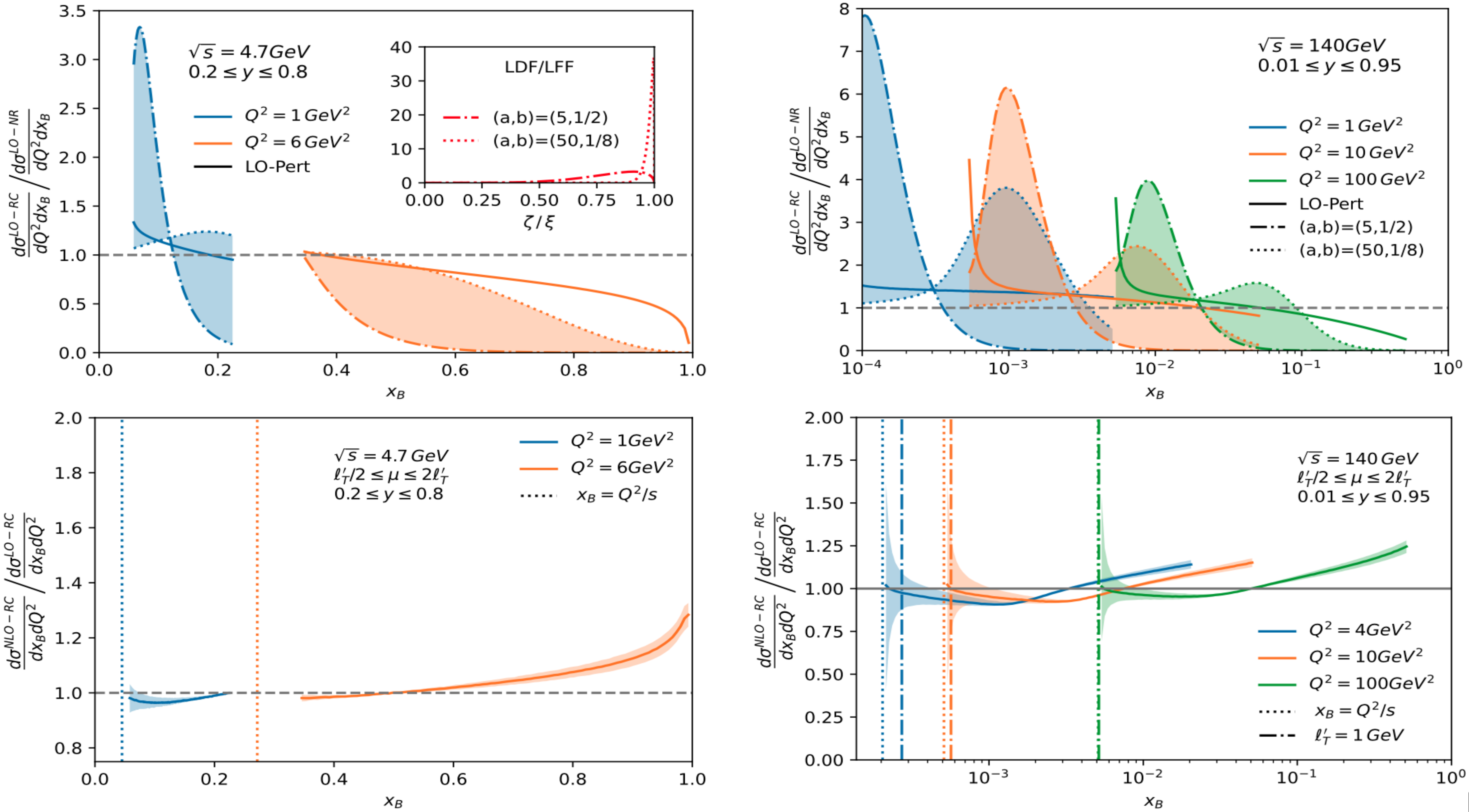
$$(\alpha,\beta) = (5, 1/2),\ (50, 1/8)$$

**PDFs:** CTEQ CT18
not much difference from using other set of PDFs




*Liu, Melnitchouk, Qiu, Sato, JHEP 11 (2021) 157*

# Impact of Factorized QED Contribution to Lepton-Hadron Scattering



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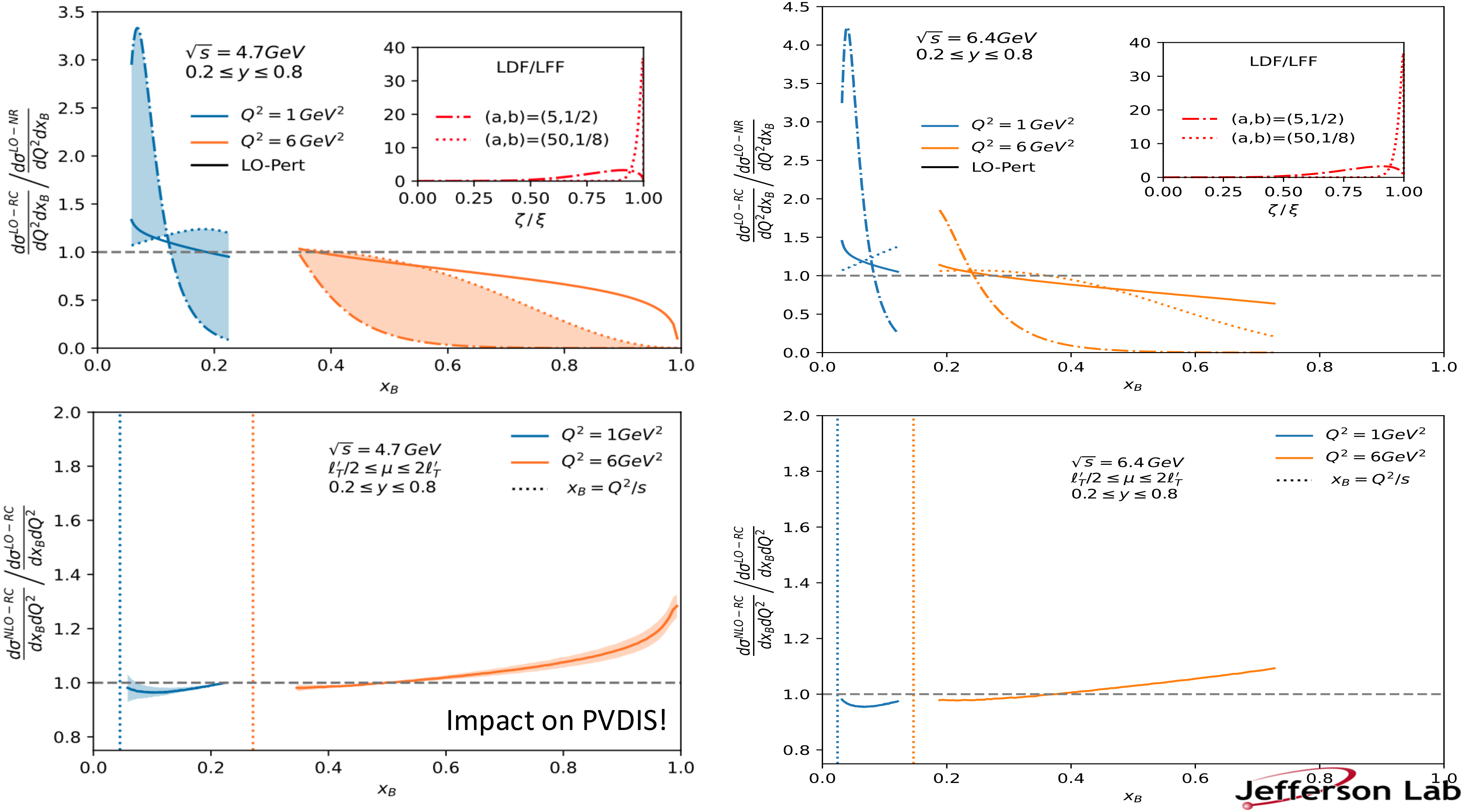
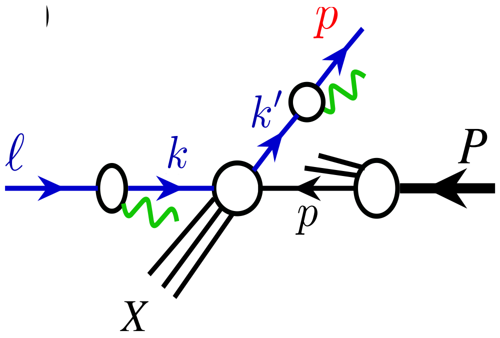
# Impact of Factorized QED Contribution to Lepton-Hadron Scattering



# Impact of Factorized QED Contribution to Lepton-Hadron Scattering

- **$Q^2$ is NOT an ideal hard scale at small $x_B$ and/or beyond LO in QED:**

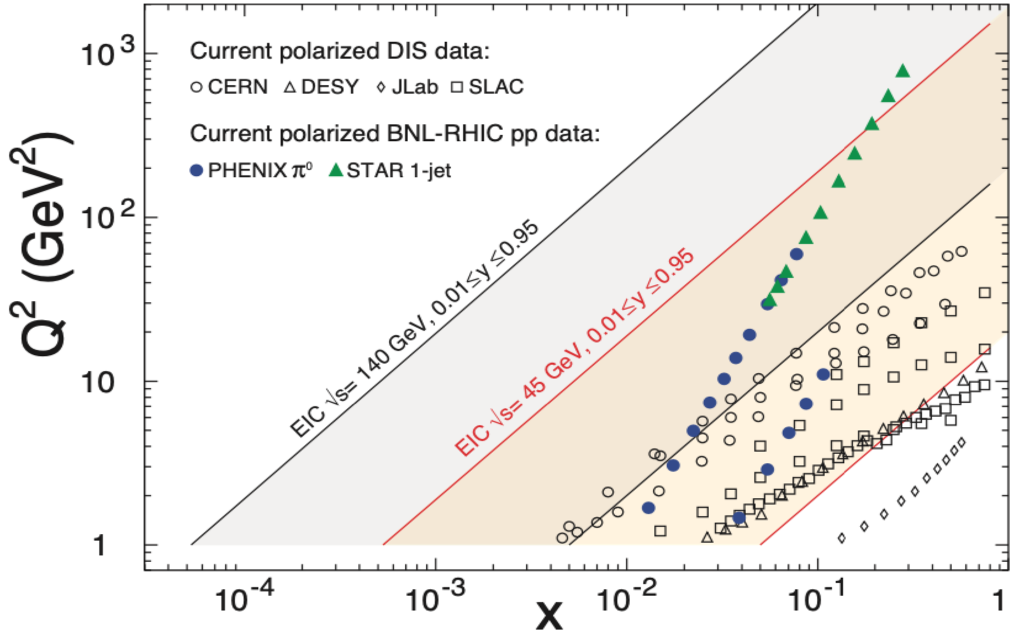
$$p_T^2 = Q^2(1-y) = Q^2\left(1 - \frac{Q^2}{x_B s}\right)$$

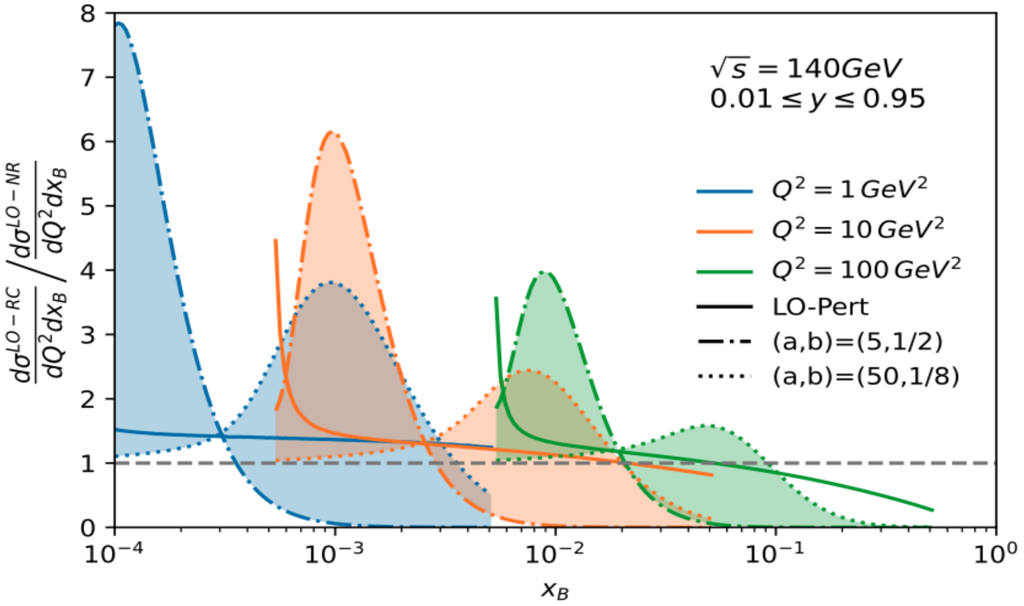


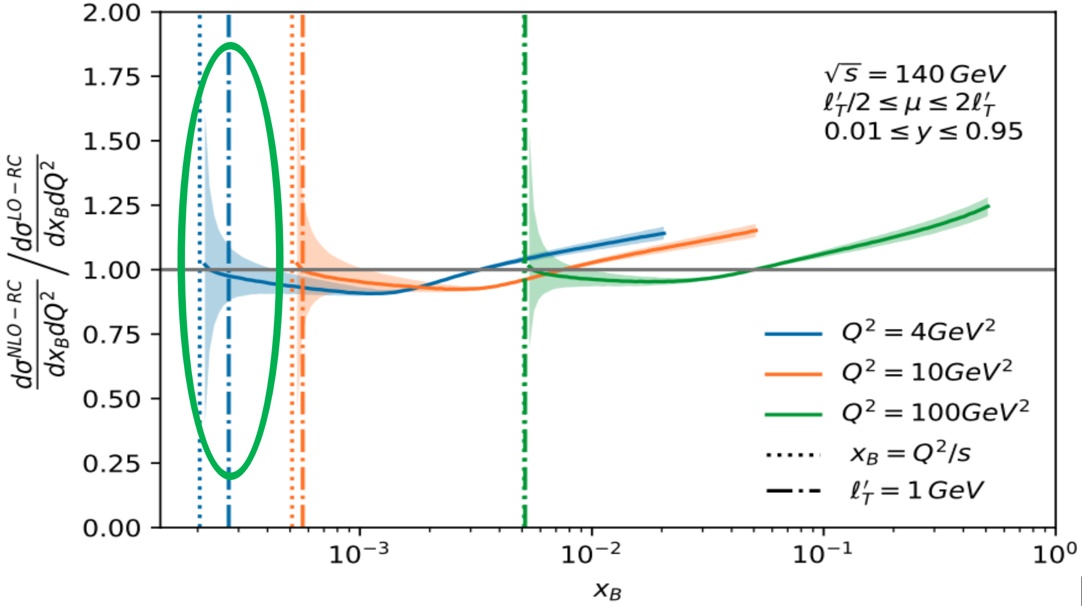

For EIC: $y \leq 0.95$

$$p_{T_{\min}}^2 = Q^2(1-y_{\max}) = \frac{Q^2}{20}!!!$$

Factorization does not work if $p_T^2$ is too small!

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# Joint QCD + QED Evolution of Universal Lepton Distribution Functions

Qiu, Watanabe
In preparation

## ❑ Modified DGLAP equation for LDFs:

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

### Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,,\quad N_F\,,\quad N_l$

- **Factorization scale:**

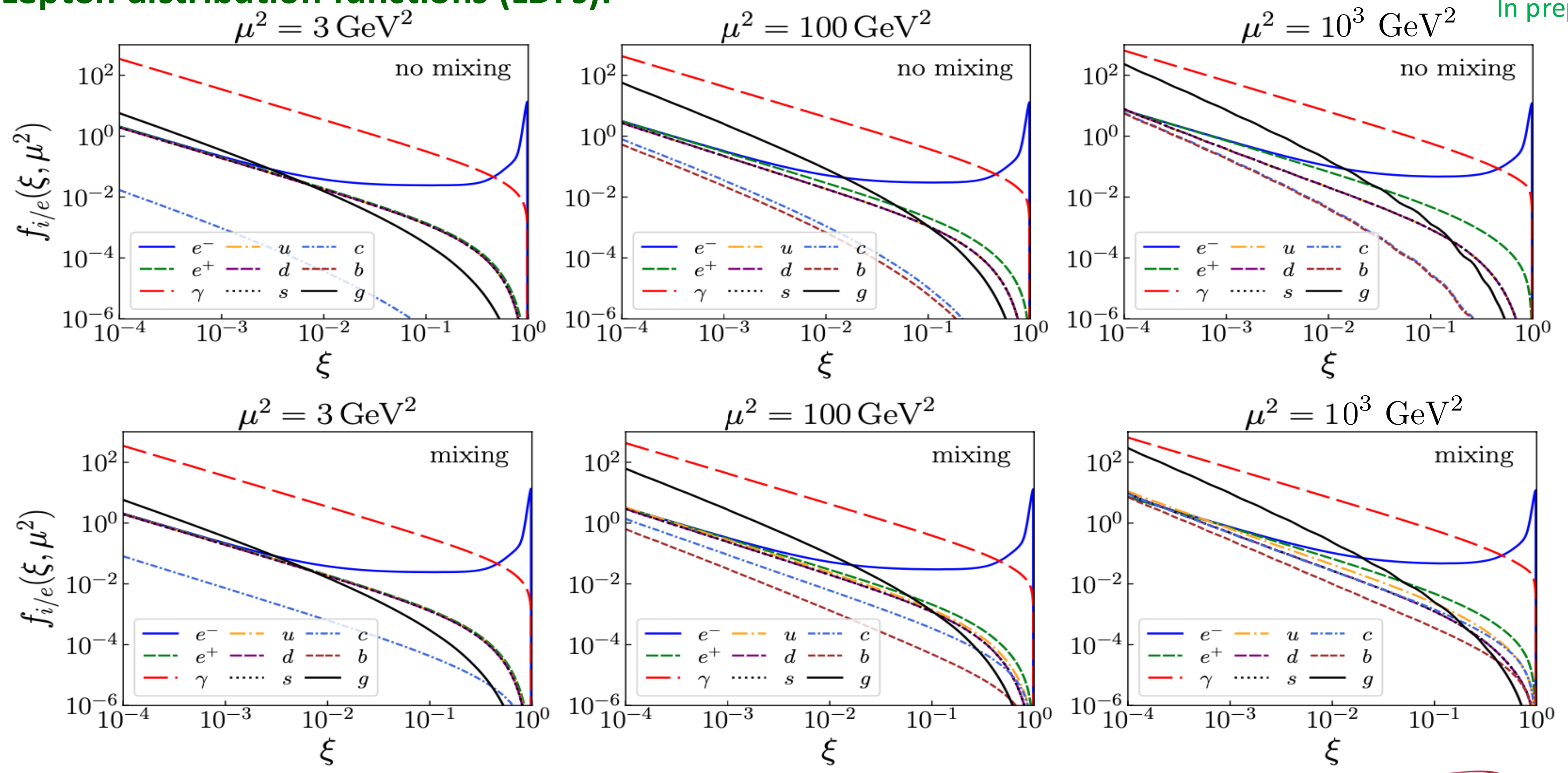
  $\mu^2 \sim m_c^2$

- **Input LDFs at $\mu^2$:**
  - Perturbatively generated by solving QED evolution from lepton mass threshold
  - With perturbatively calculated fixed-order MSbar LDFs
  - Test the size of non-perturbative hadronic contribution
  - …

# Evolution of Lepton Distribution Functions (LDFs)

## ❑ Lepton distribution functions (LDFs):

Qiu, Watanabe
In preparation




**With LDFs, we calculated single hadron production, including J/ψ production at the EIC**



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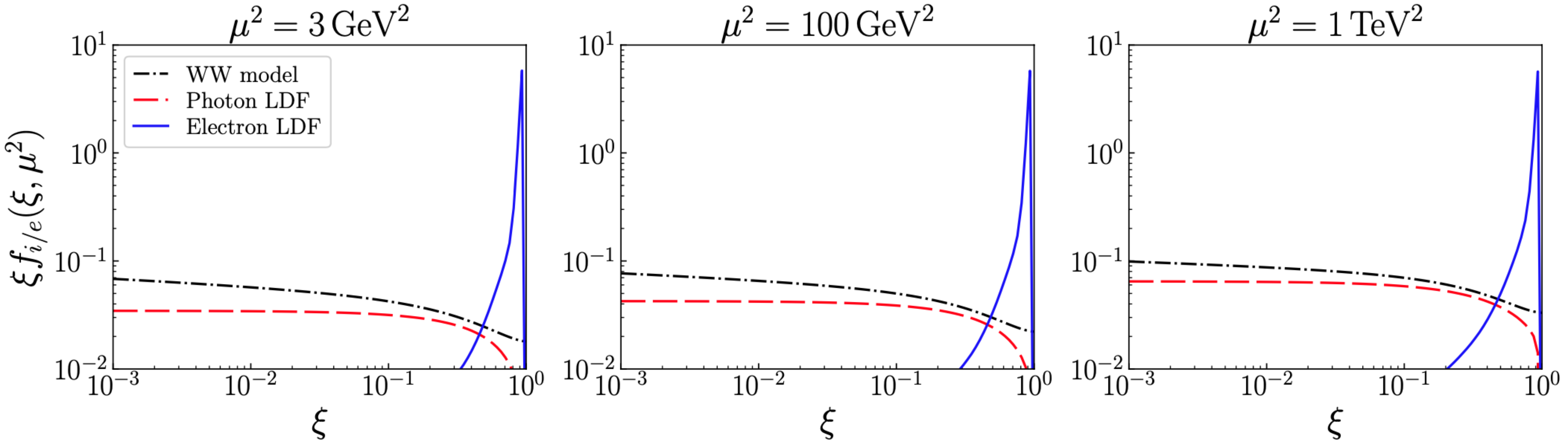
# Evolution of Lepton Distribution Functions (LDFs)

Qiu, Watanabe
In preparation

## Photon distribution of the electron:

- **Weizsa ̈cker-William photon distribution:**

$$f_{\gamma/e}^{\rm WW}(\xi,\mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi) \left[ \ln\left(\frac{\mu^2}{\xi^2 m_e^2}\right) - 1 \right]$$




- **LDFs are not purely perturbative in QED or perturbative – need global analysis!**
  - **Precision measurements for BSM physics at the EIC needs reliable lepton distributions**
  - **Joint global analysis of lepton and hadron distribution functions should be carried out.**
  - **Impact on searching BSM at ILC or CEPC, FCC, …**



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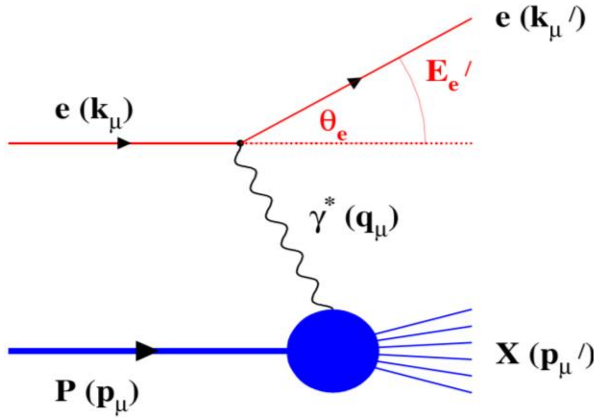
# Inclusive Single Prompt Hadron Production

- Recall: Photoproduction in ep collision is important & sensitive to how the "photon" is defined




- Real or quasi-photon is defined by

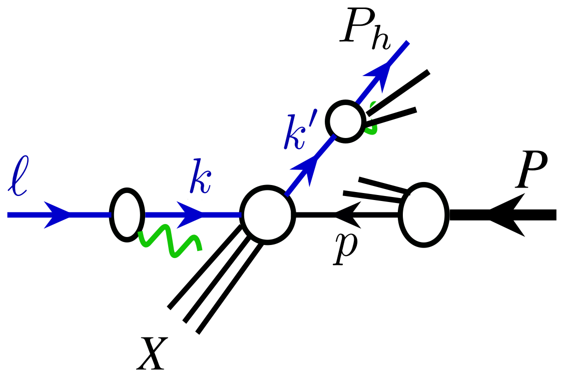
  $k'_T \leq k_{T_{\text{cut}}}$ or $\theta_e \leq \theta_{\text{cut}}$

- Photon flux is derived by

  Evaluating the photon shower with above "cut"
  Weizsaecker-Williiams photon distribution, …

- Inclusive single hadron (jet) production in ep collision:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation




***Without measuring the scattered electron!***

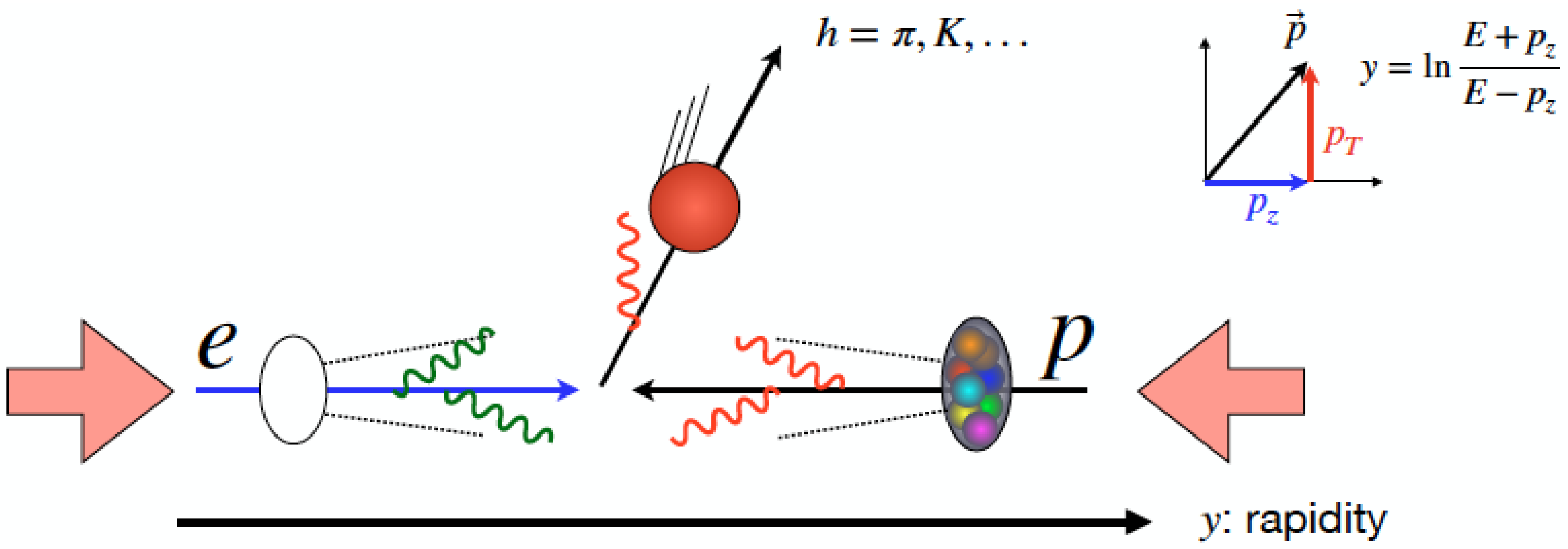
Single hard scale, collinear factorization

$$E_h \frac{d\sigma_{\ell P \to P_h x}}{d^3 P_h} = \frac{1}{2s} \sum_{i,a,b} \int_{z_{\min}}^{1} \frac{dz}{z^2} \int_{\xi_{\min}}^{1} \frac{d\xi}{\xi} D_{h/b}(z,\mu^2) f_{i/e}(\xi,\mu^2)$$
$$\times \int_{x_{\min}}^{1} \frac{dx}{x} f_{a/N}(x,\mu^2)\, \widehat{H}_{ia\to bX}(\xi\ell, xP, P_h/z, \mu^2) + \ldots$$

- Universal lepton distribution functions (LDFs)
- No artificial cut to define the "photon"
- Single factorization scale: μ

# Inclusive Single Prompt Hadron Production

- **Inclusive hadron production in lepton-hadron (ep) scatterings in the ep center of mass frame:**

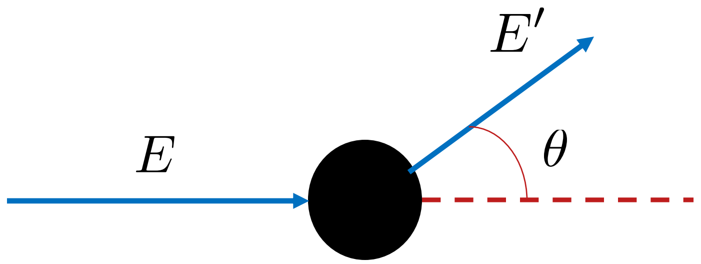



Clean test of the fragmentation production
A prerequisite for using the factorized formula for SIDIS!

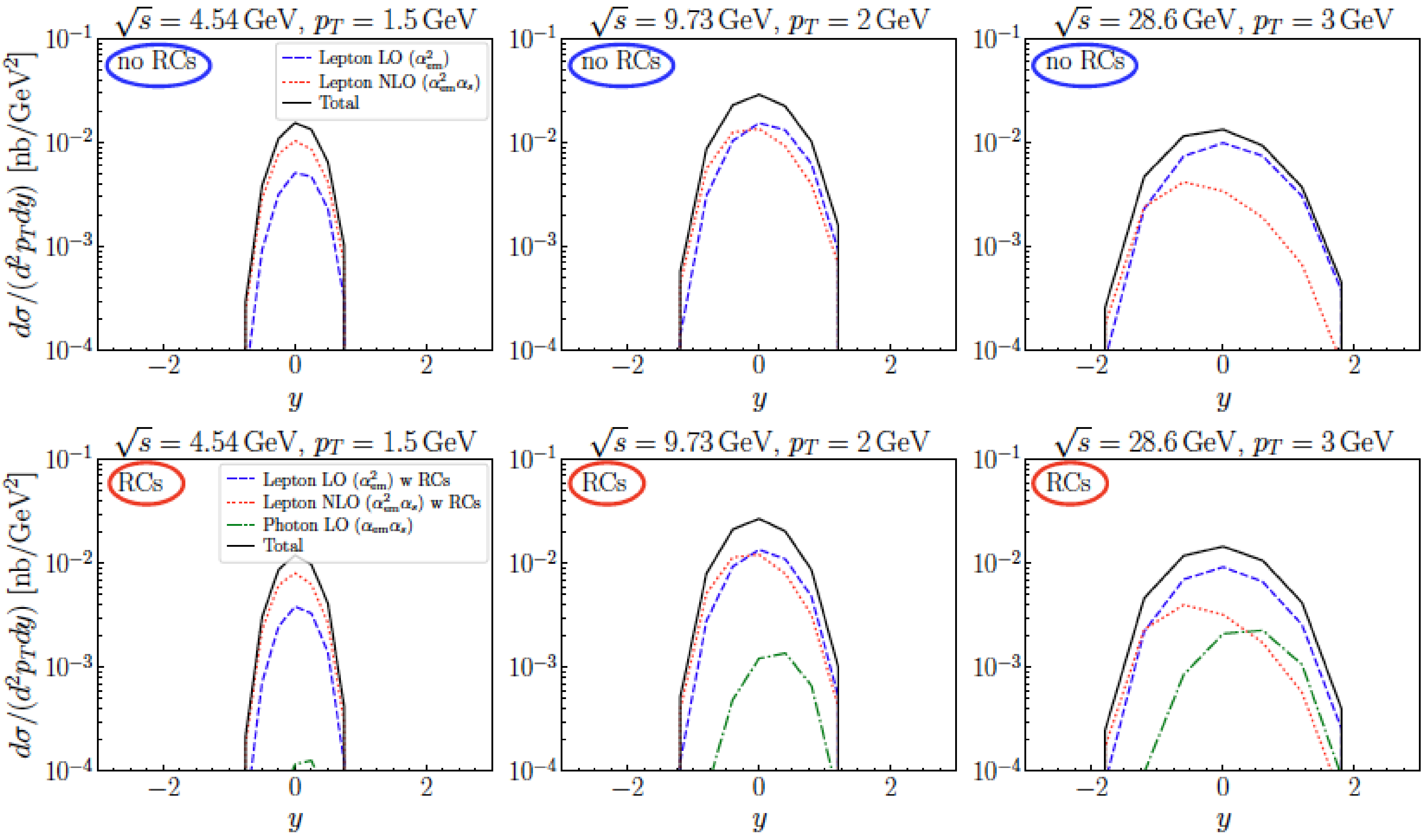
Taking positive (negative) rapidity for the direction of an incident electron (target proton) in the center of mass frame

- **Transverse momentum is boost invariant:**

In target rest frame: $p_T = E' \sin(\theta)$
$\gtrsim 1 \text{ GeV?}$



# $\pi^+$ cross-section: rapidity dependence at higher $p_T$



# Treat QED and QCD Radiation Equally for All Lepton-Hadron Scattering

- **Three complementary processes for high-energy lepton-hadron scattering:**
  - **Inclusive single high-pT lepton:** $e(\ell) + H(P) \to e(\ell') + X$ Inclusive DIS
  - **Inclusive single high-pT hadron (or jet):** $e(\ell) + H(P) \to h(p)(\text{or jet(p)}) + X$ "Photoproduction"
  - **Inclusive high-pT lepton + hadron:** $e(\ell) + H(P) \to e(\ell') + h(p) + X$ Semi-Inclusive DIS

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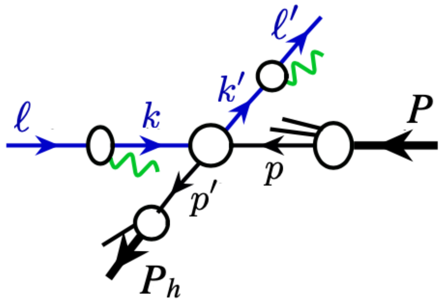
## Factorization for SIDIS (beyond one-photon exchange approximation):




TMD regime: lepton and hadron almost back-to-back

Hybrid Factorization: Collinear factorization for QED + TMD factorization for QCD

$$E_{\ell'} E_{P_h} \frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\,\mathrm{d}^3 P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 \mathrm{d}\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \times \left[E_{k'} E_{P_h} \frac{\mathrm{d}^6\hat\sigma_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3 k'\,\mathrm{d}^3 P_h}\right]_{k=\xi\ell,\,k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- **One-photon approximation:** $i = j = e$

$$\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}x_B\mathrm{d}y\,\mathrm{d}\psi\,\mathrm{d}z_h\,\mathrm{d}\phi_h\mathrm{d}P_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{\mathrm{d}\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta) \times \frac{\hat x_B}{x_B\xi\zeta}\left[\frac{\alpha^2}{\hat x_B\,\hat y\,\widehat{Q}^2}\frac{\hat y^2}{2(1-\hat\varepsilon)}\left(1+\frac{\hat\gamma^2}{2\hat x_B}\right)\sum_n \hat w_n F_n^h(\hat x_B, \widehat{Q}^2, \hat z_h, \widehat{P}_{hT}^2)\right]$$

Evaluated in a "virtual photon-hadron" frame

$L(\xi,\zeta) : \{\hat q, P, \widehat{P}_h\} \Rightarrow \{q, P, P_h\}$

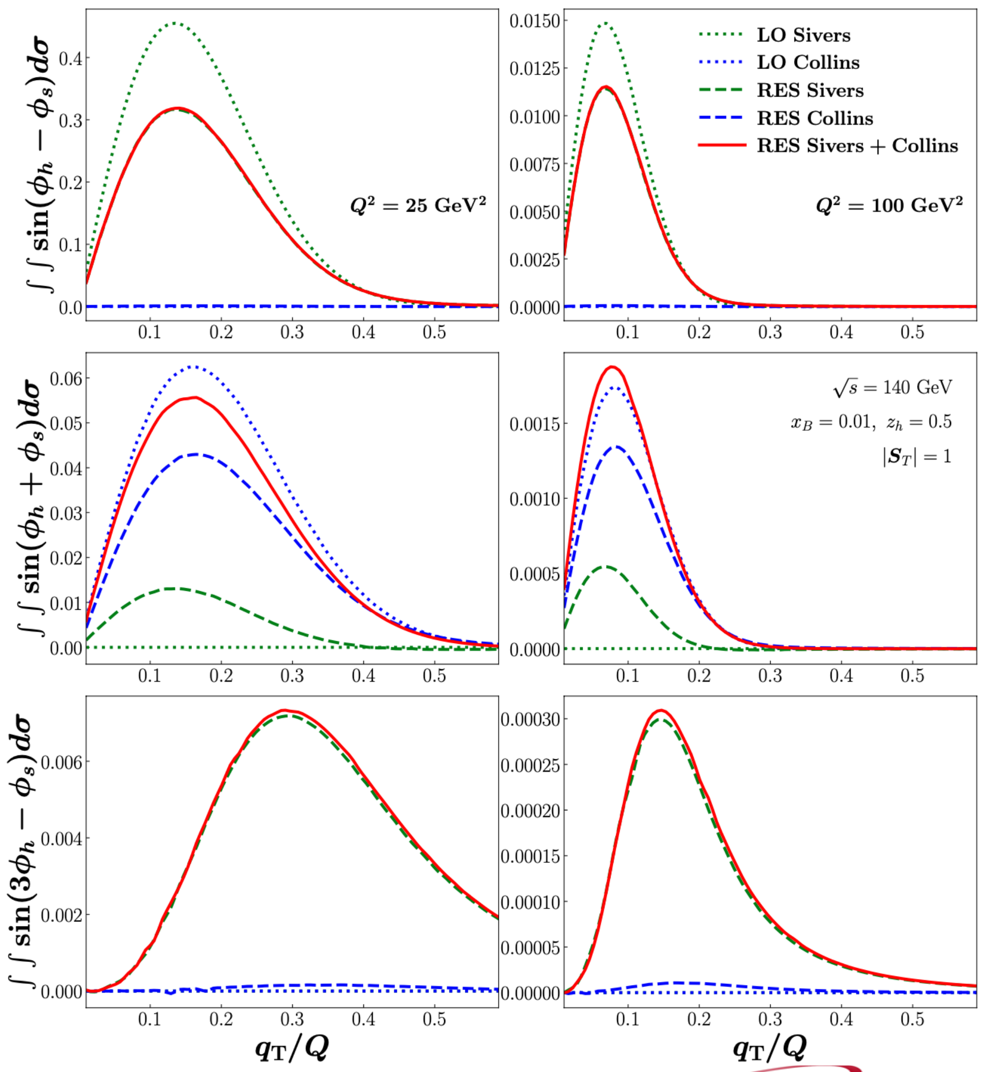
Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033; JHEP 11 (2021) 157

# Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad+S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big] \\
&\quad+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] \\
&\quad+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad+\varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$



# Summary and Outlook – Thank you!

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons (Have not be able to extend this to full EW+QCD factorization!)
  - No artificial and/or process dependent parameter(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)
- Two-photon exchange is a natural part of NLO QED contribution – better controlled calculations for precision, in particular, for PVDIS, …