Measurements of α_s with JLab@22 GeV

A. Deur, Jefferson Lab

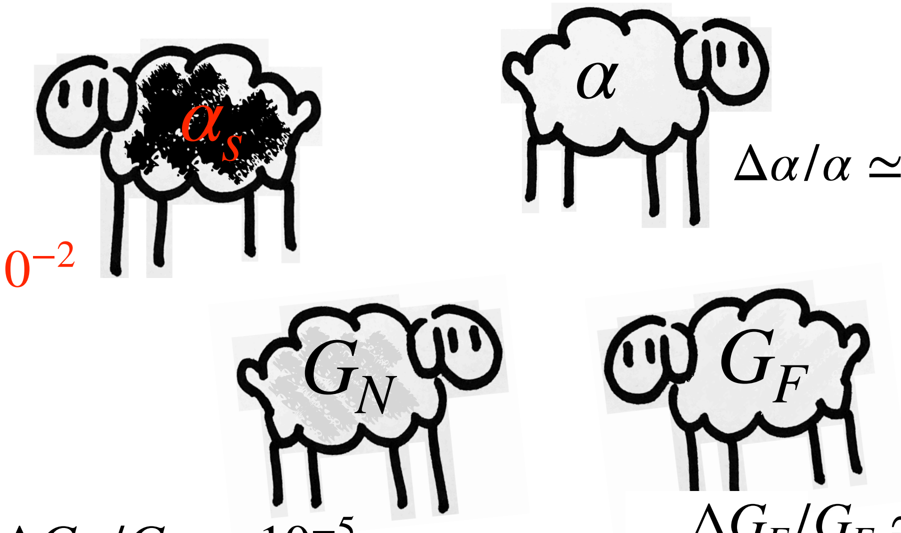
12/09/2024

Science at the Luminosity Frontier: Jefferson Lab at 22 GeV. INFN Frascati.

- Measurement of $\alpha_s(M_z^2)$
- Mapping of $\alpha_s(Q^2)$ for $1 < Q^2 < 22$ GeV^2

Importance of measuring $\alpha_s(M_z)$

• α_s: most important quantity of QCD, key parameter of the Standard Model, but (by far) the least known fundamental coupling:



$\Delta\alpha_s/\alpha_s \simeq 10^{-2}$

$\Delta\alpha/\alpha \simeq 10^{-10}$

$\Delta G_N/G_N \simeq 10^{-5}$

$\Delta G_F/G_F \simeq 10^{-6}$

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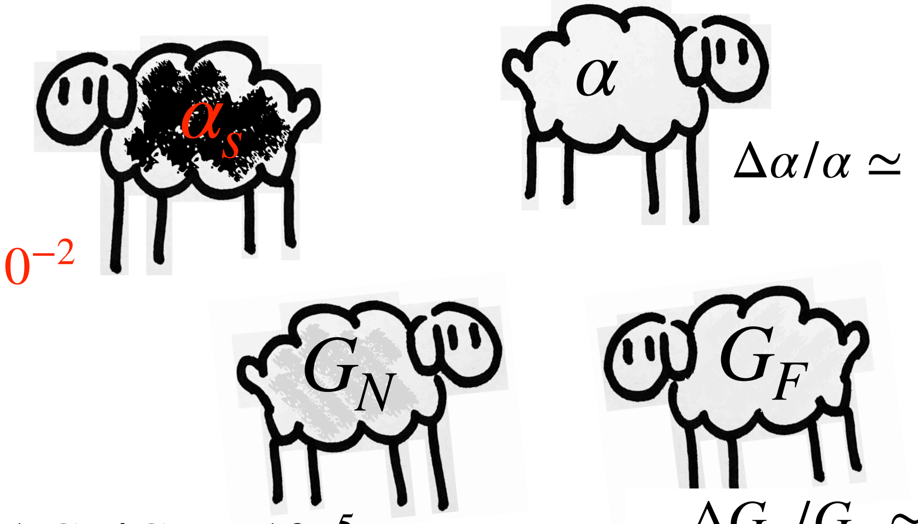

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•Large efforts ongoing to reduce $\Delta\alpha_s/\alpha_s$ (Snowmass 2022, J.Phys.G 51 (2024) 9, 090501 arXiv:2203.08271)

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$\Rightarrow$ Strategy: combine many independent measurements with larger uncertainties.

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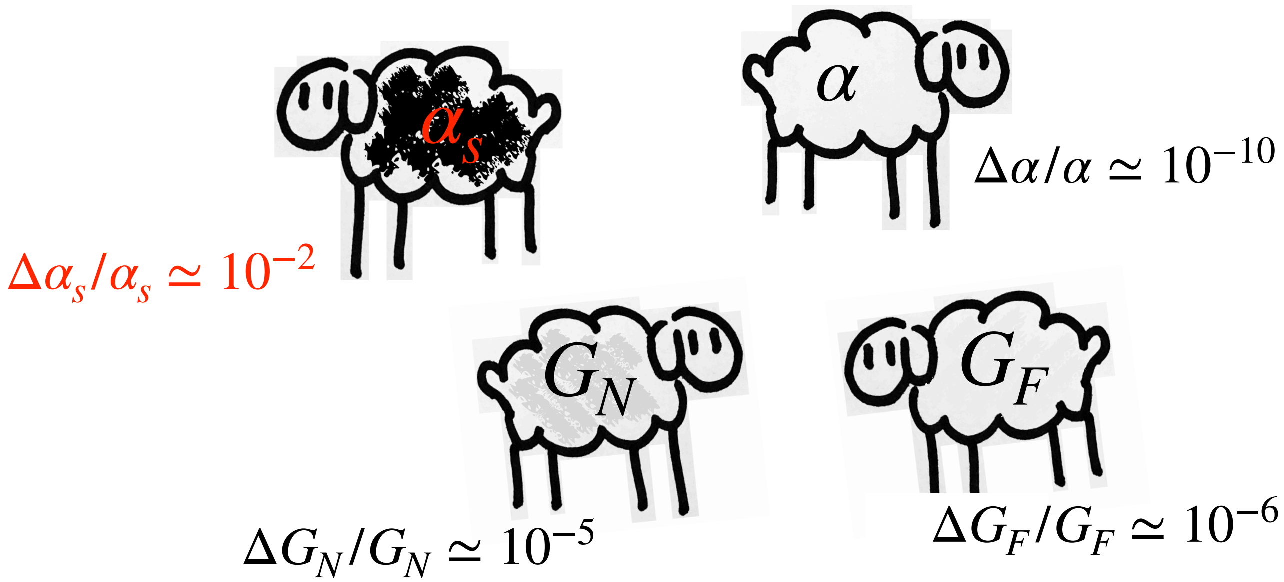

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- Good prospects of measuring precisely $\alpha_s(M_z)$ at JLab@22 GeV with Bjorken sum rule:

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- With polarized NH_3 and ^3He targets: 5% systematics (experimental, i.e., not counting low-x uncert. Mitigated for Q^2-dep. meas.)

Main issue with sum rules:

Unmeasured low-x part: $\int_0^1$ integrant dx.

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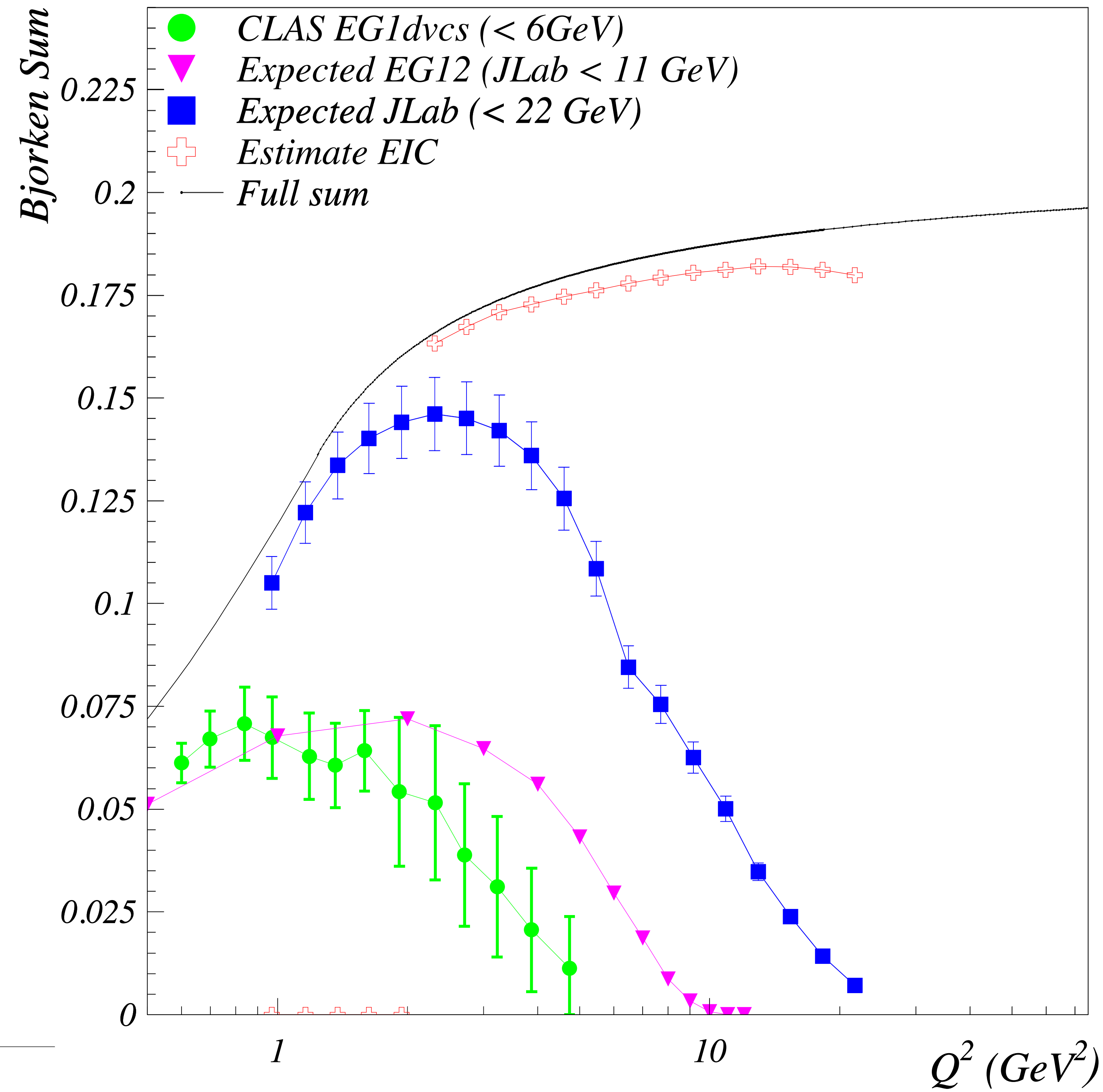

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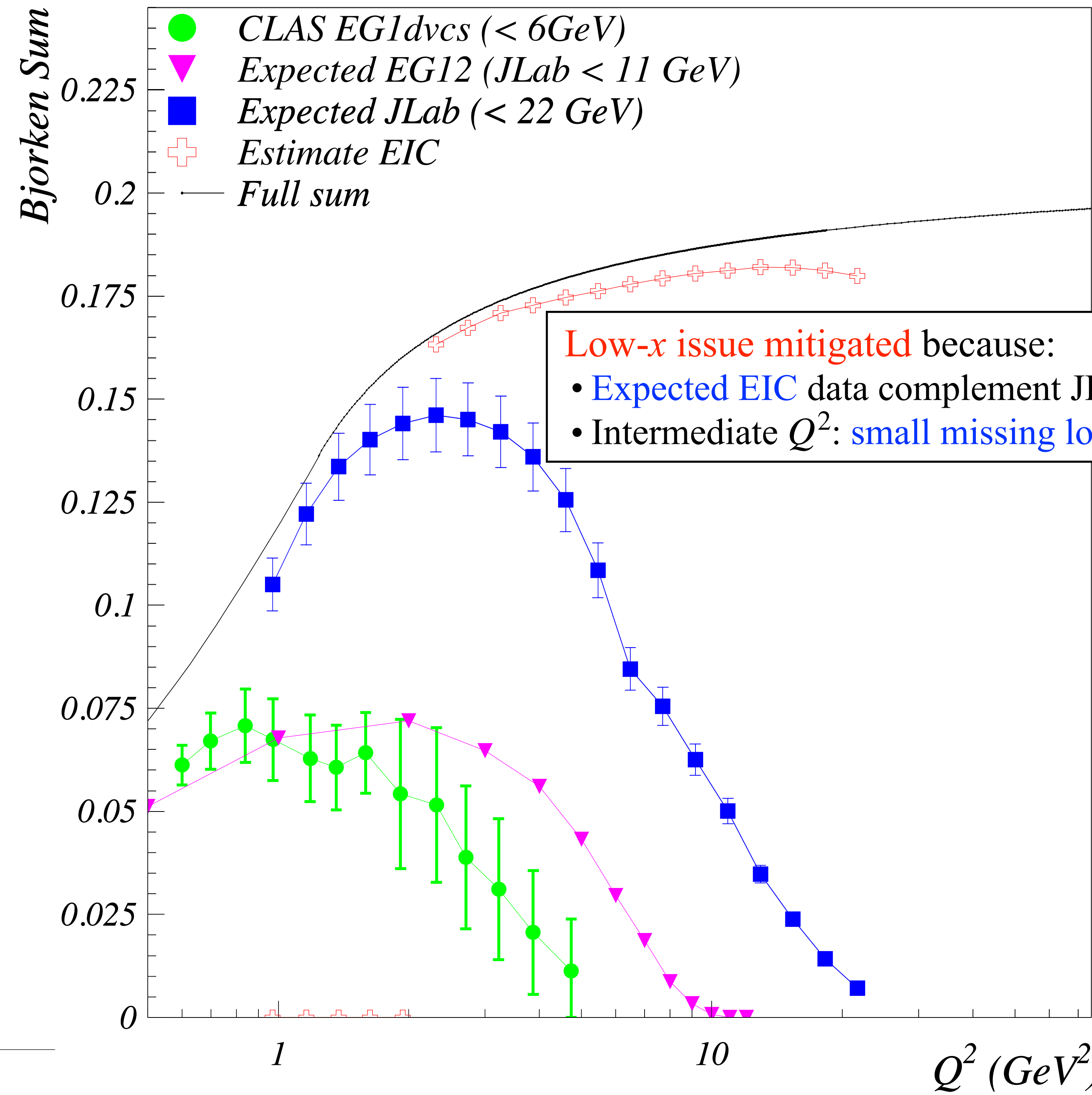

Low-x issue mitigated because:

- Expected EIC data complement JLab data;
- Intermediate Q^2: small missing low-x.

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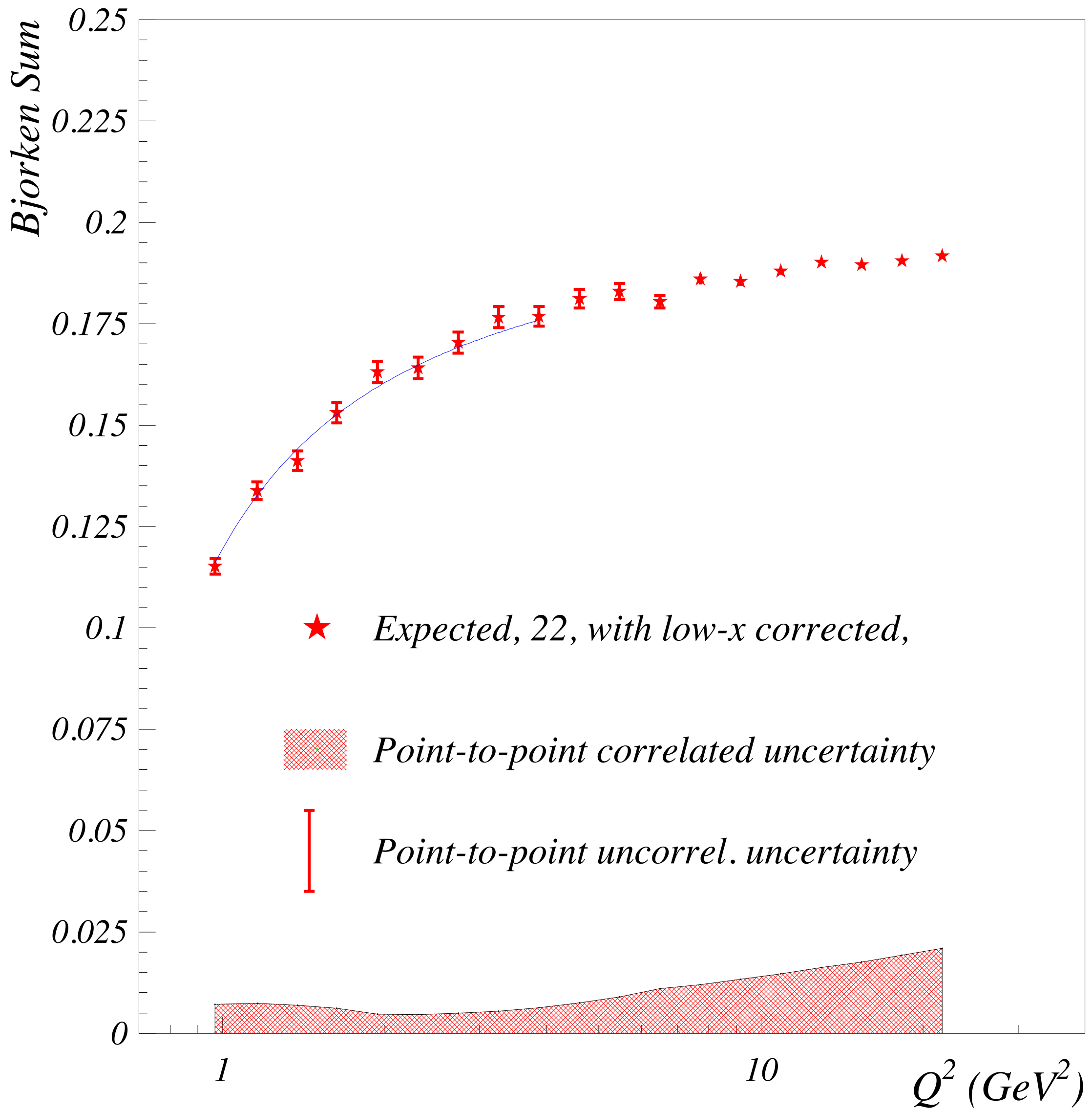

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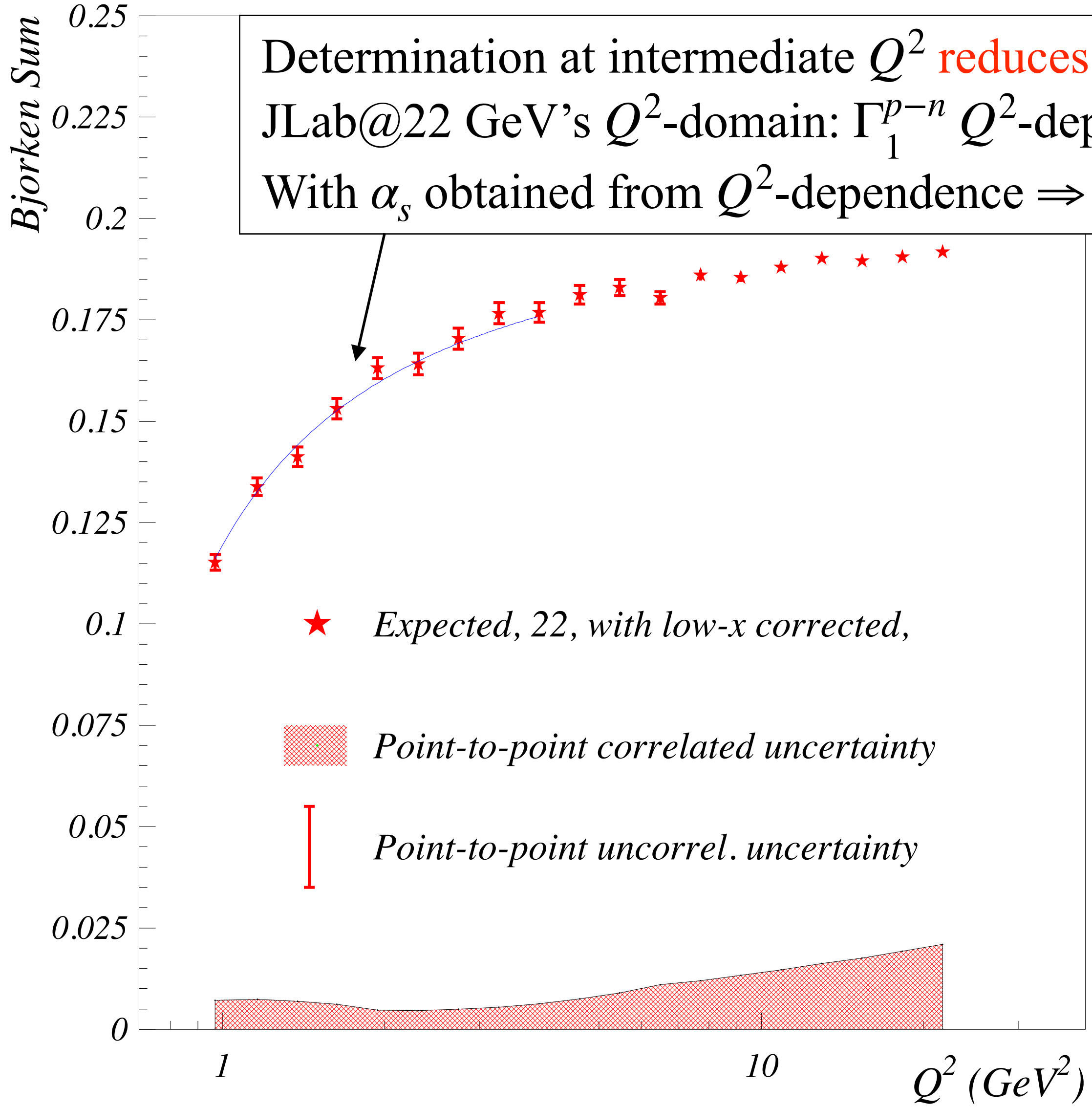

Determination at intermediate Q^2 reduces uncertainty by a factor of ~5 compared to determinations near M_Z^2
JLab@22 GeV's Q^2-domain: Γ_1^{p-n} Q^2-dependence is ~50 times steeper than for EIC.
With α_s obtained from Q^2-dependence ⇒ strongest α_s sensitivity.

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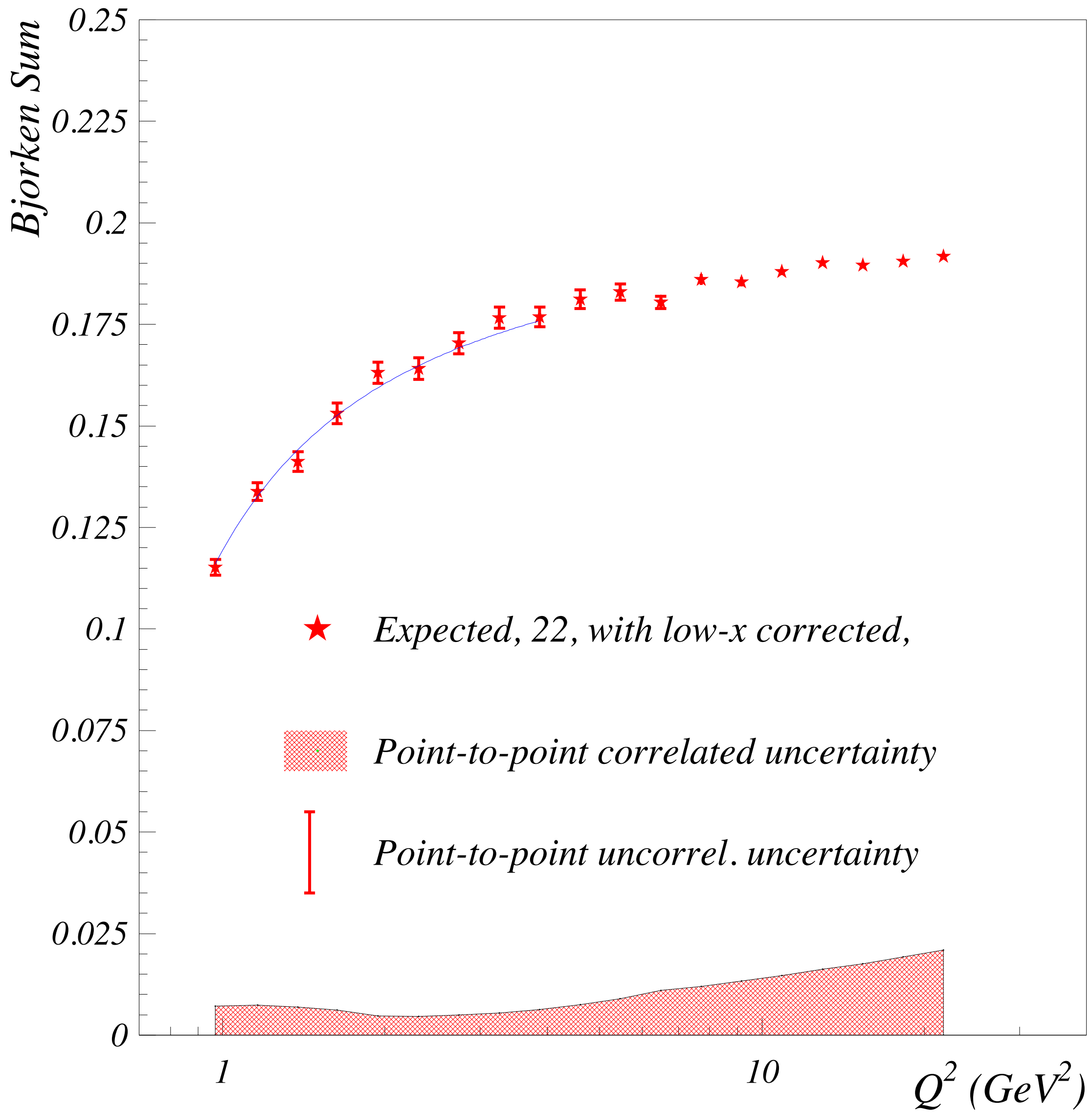
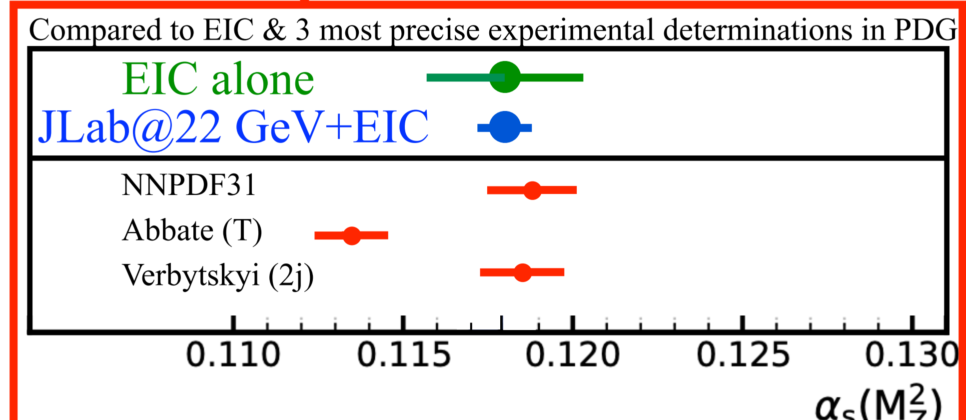


Fitting simulated Bjorken sum data yields:

$\Delta\alpha_s/\alpha_s \simeq 6.1 \times 10^{-3}$
$\pm 4.2(\text{uncor.}) \pm 3.6(\text{cor.}) \pm 2.6(\text{theo.})] \times 10^{-3}$

Details given in talk at JLab@22 GeV Workshop, Jan. 2024. See also back-up slides



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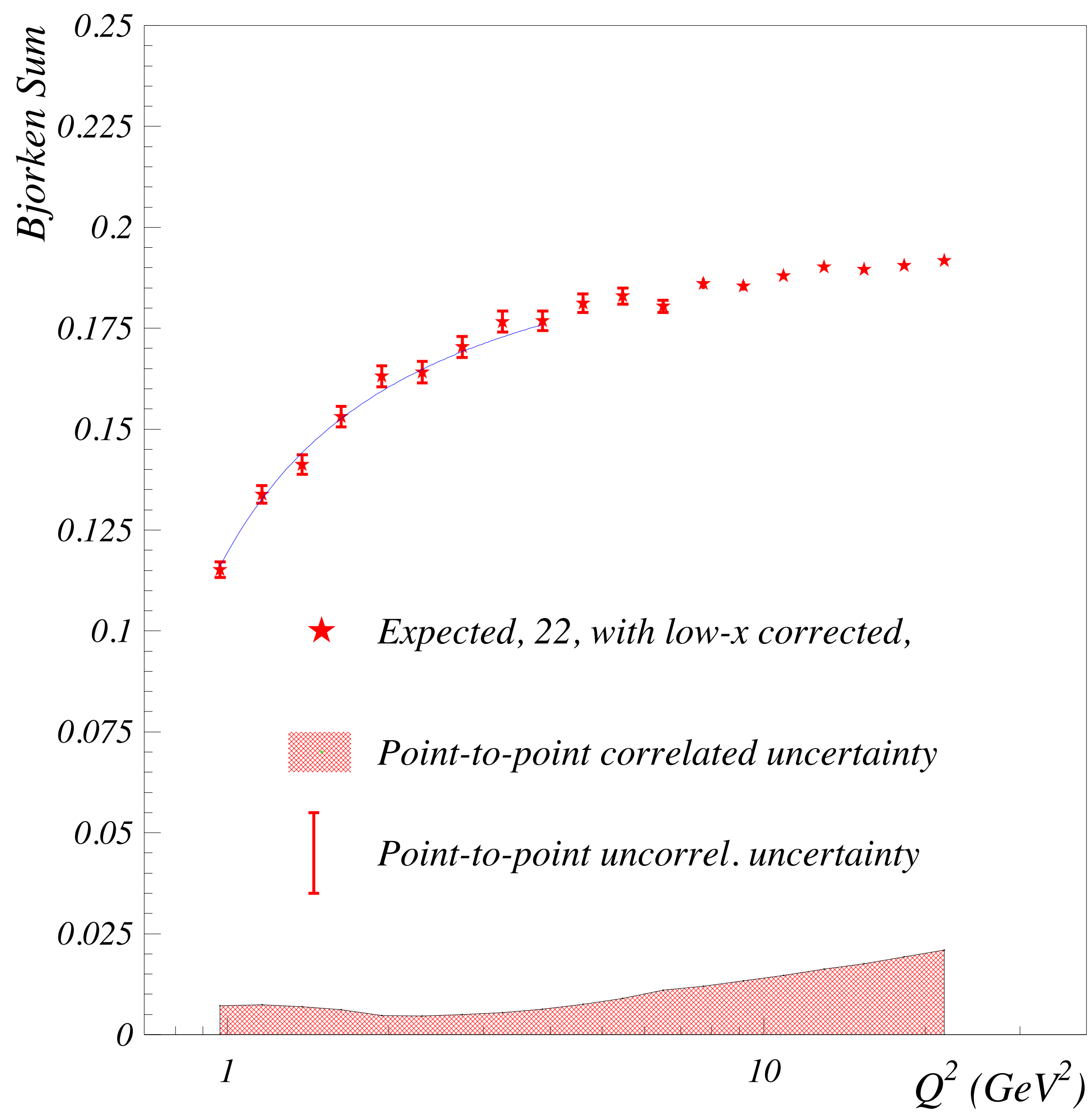

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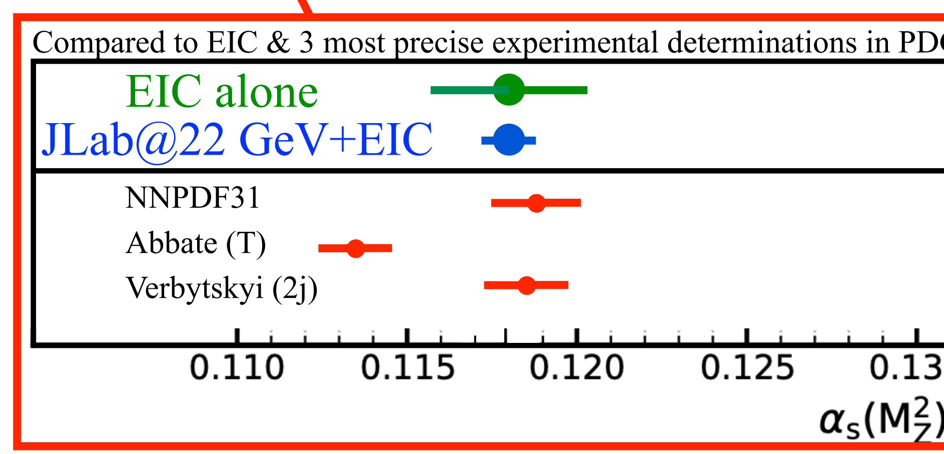
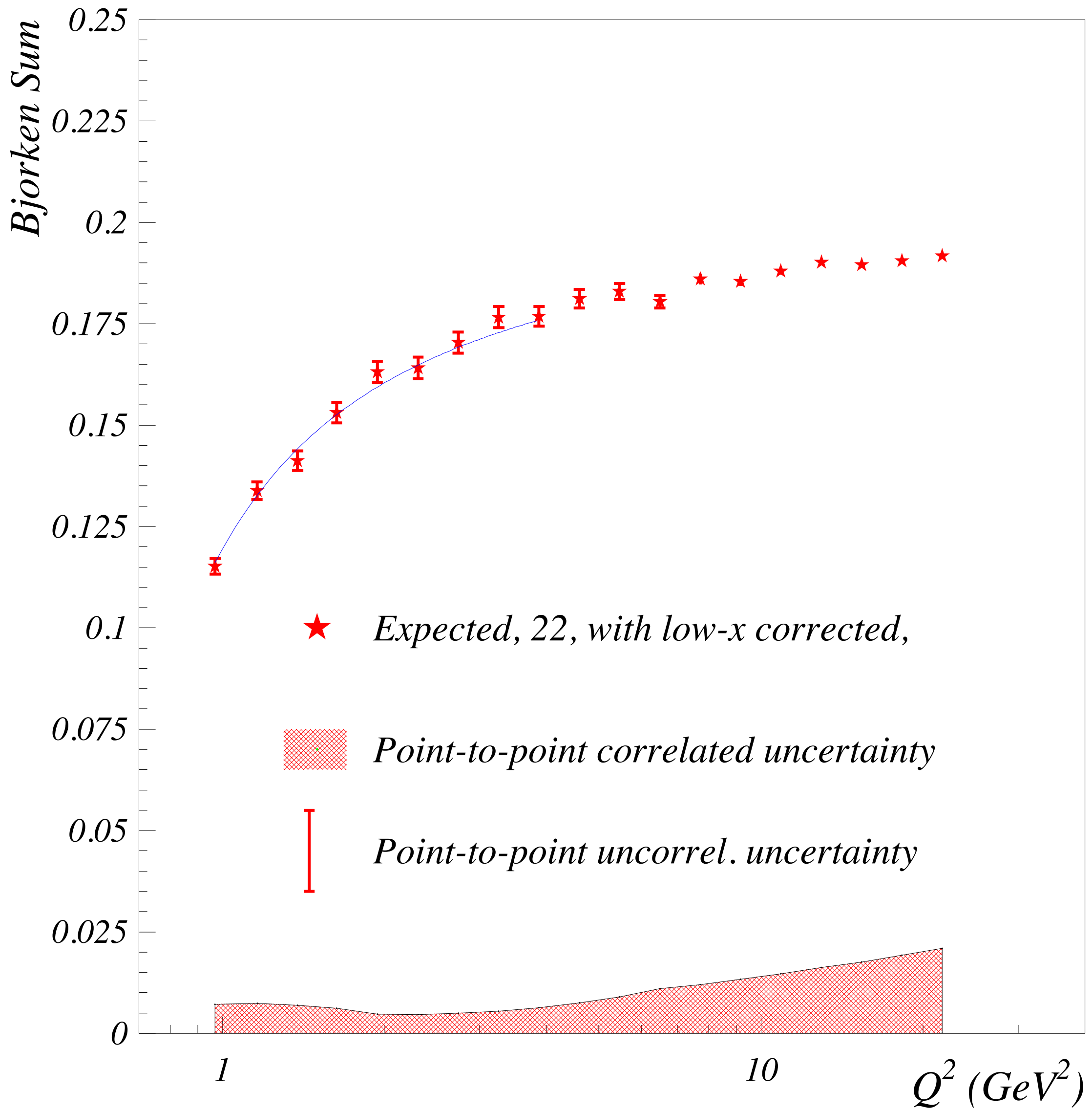


- One extraction from JLab@22 can yield α_s with greater accuracy than world data combined.
- Same exercise with EIC yields $\Delta\alpha_s/\alpha_s \gtrsim 1.3\ \%$. PRD 110, 074004 (2024) [arXiv:2406.05591]
- Yet, EIC data required to minimize the low-x uncertainty of JLab's determination.

Measuring α_s

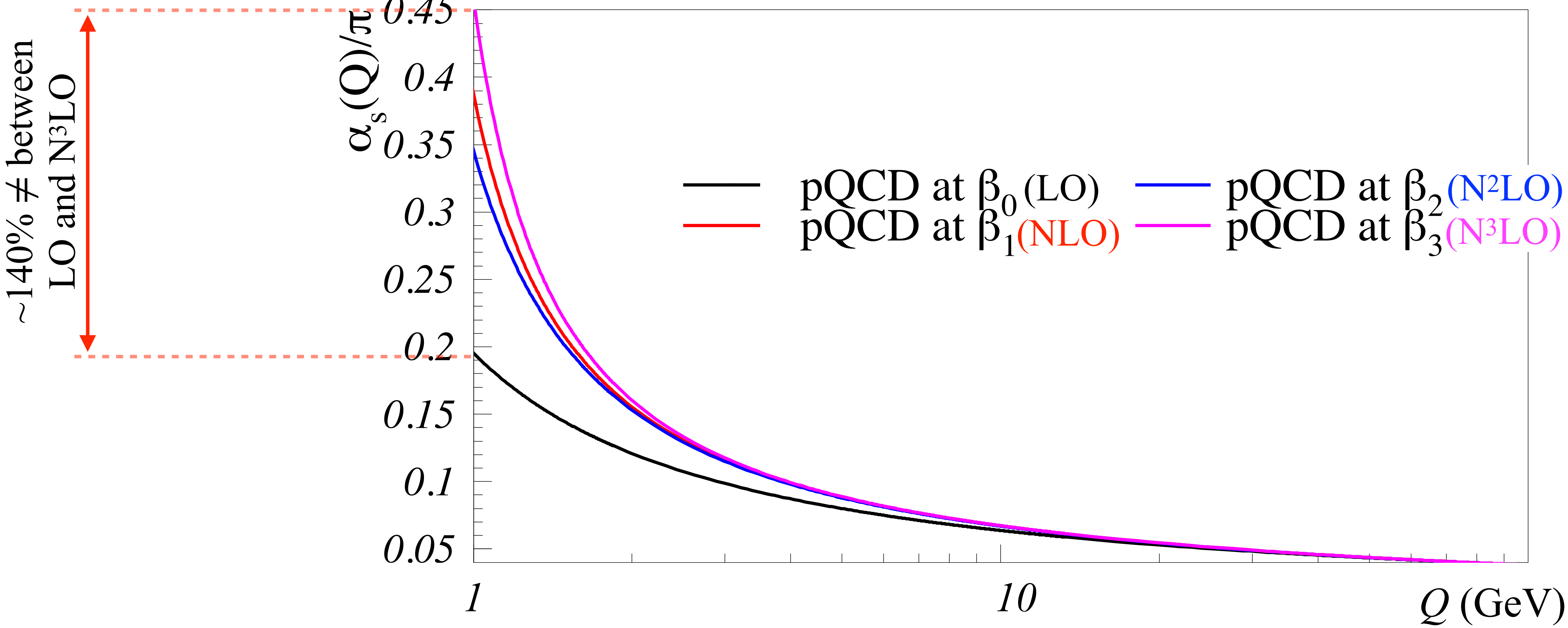
Another possibility: Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve the Bj SR for $\alpha_s(Q^2)$:

- One α_s per Γ_1^{p-n} experimental data point.
- Lower systematic accuracy makes this not competitive for $\alpha_s(M_z)$.
- Small uncorrelated uncertainty (Q^2**-dependence**) provides good relative $\alpha_s(Q^2)$ mapping.



⇒Sensitivity to high-order QCD loops that have not yet been directly measured.

Measuring $\alpha_s(Q)$



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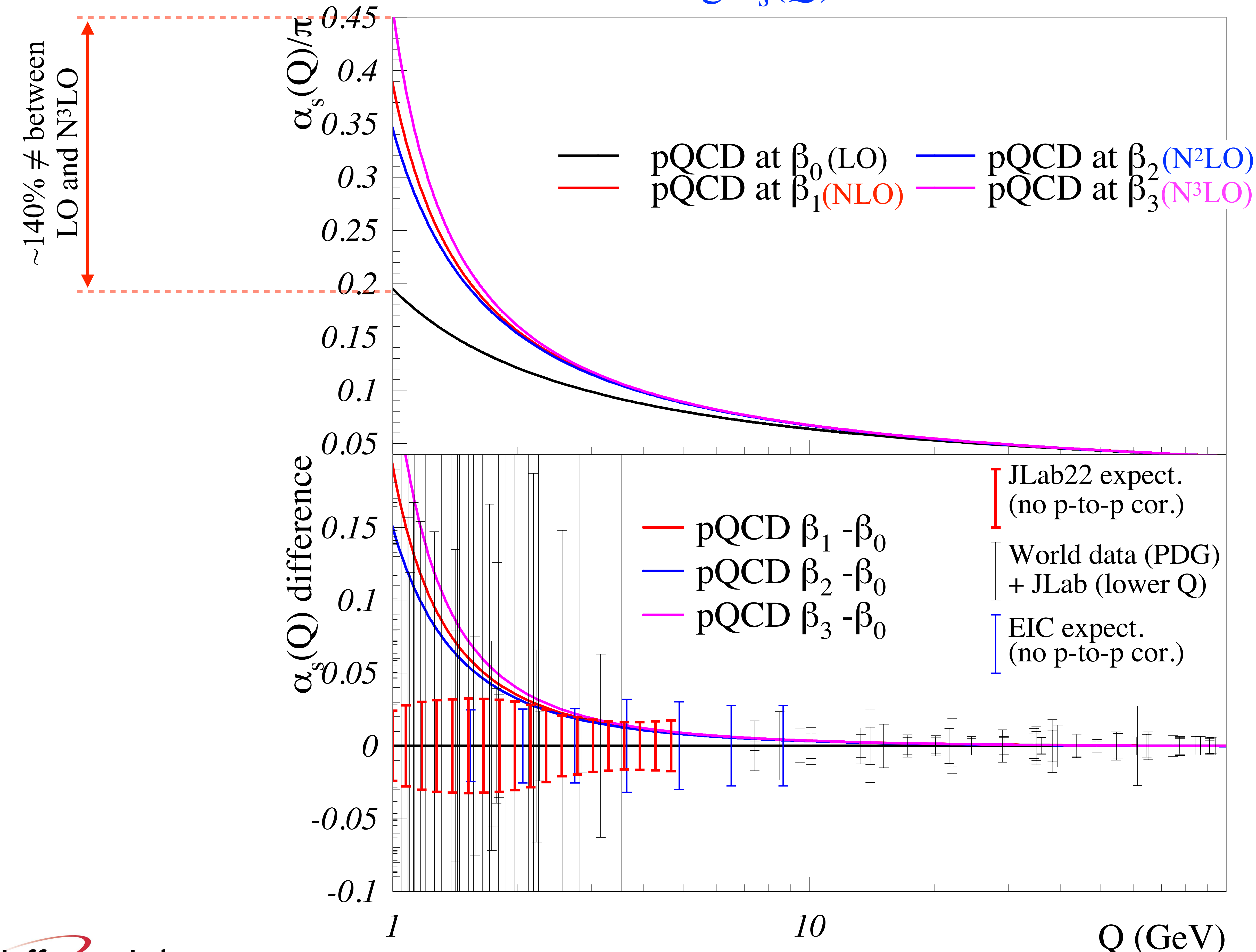

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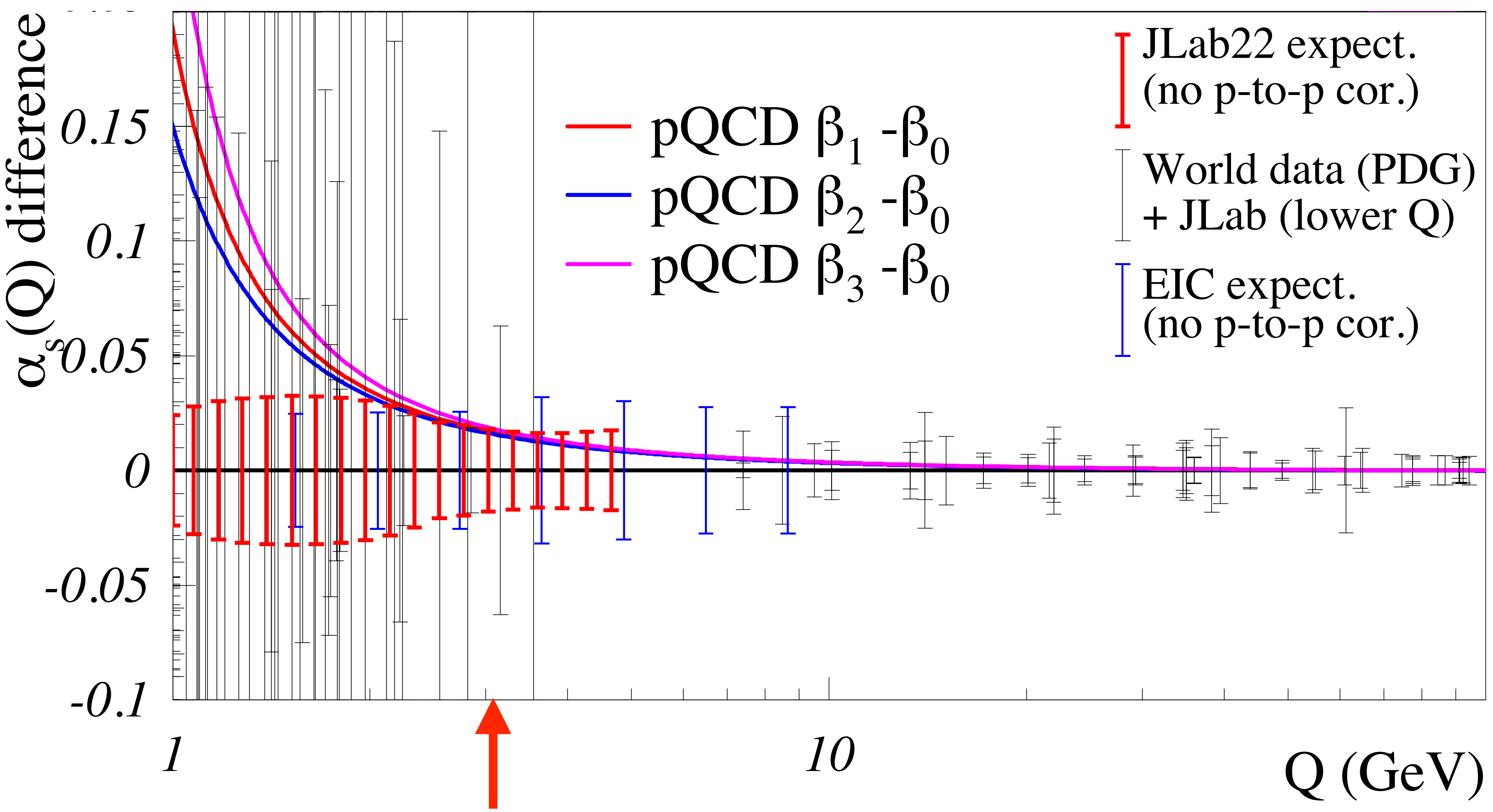

$Q^2 < 5.4$ GeV^2: start being sensitive to $\beta_{1,2}$...

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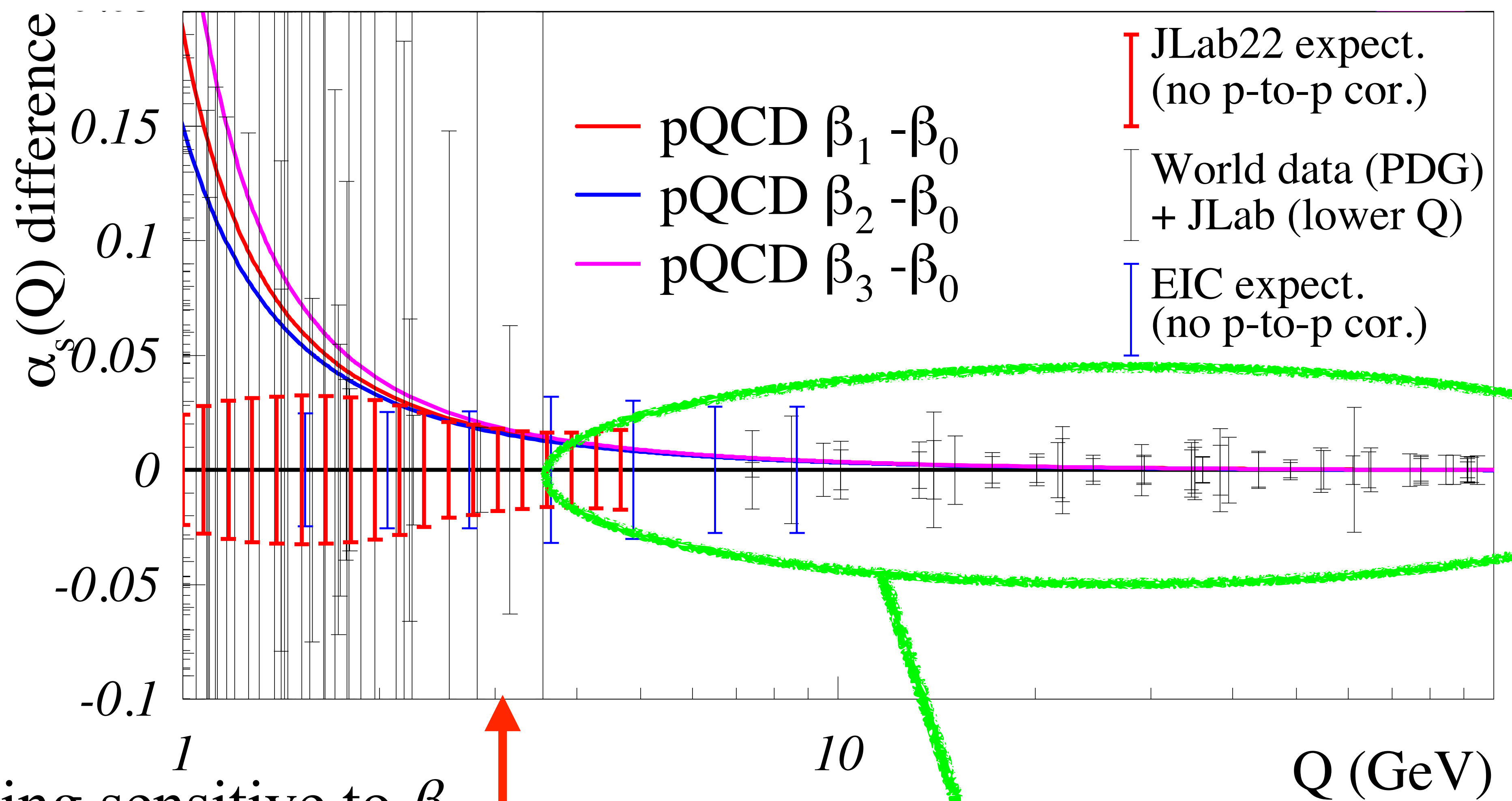
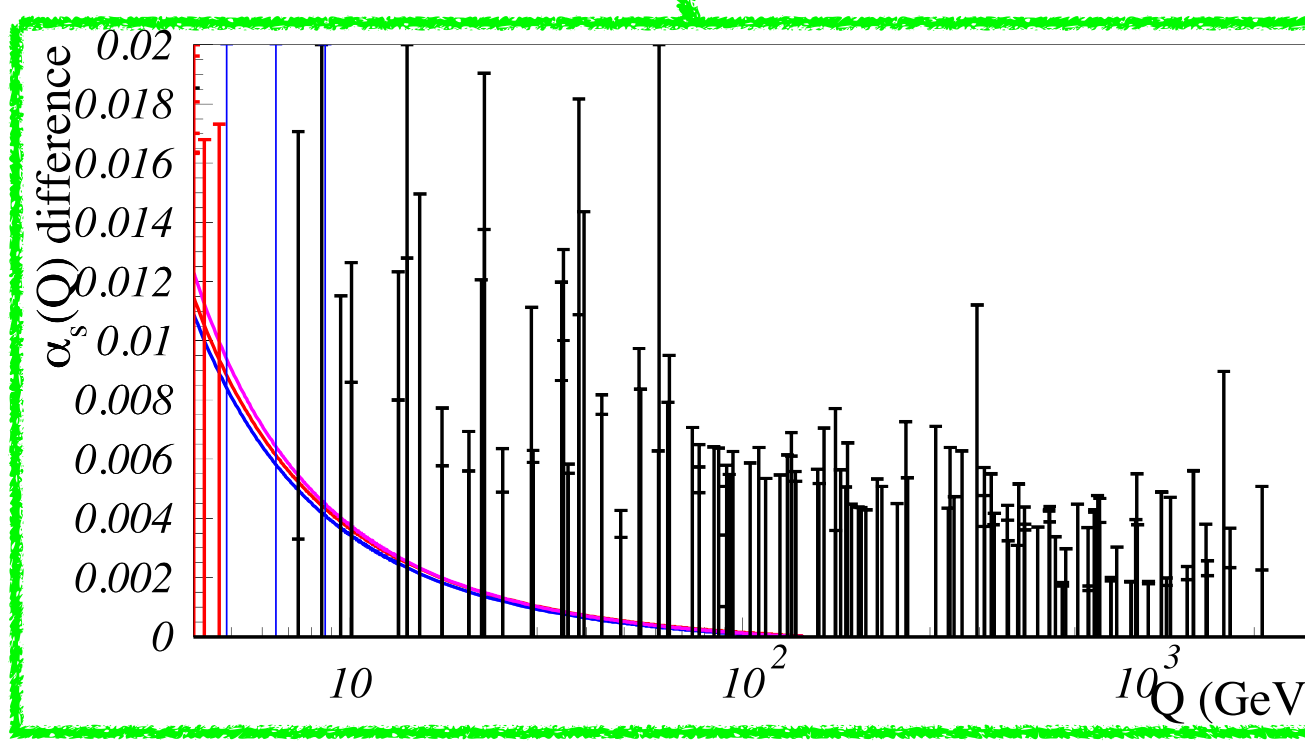


$Q^2 < 5.4\ \text{GeV}^2$: start being sensitive to $\beta_{1,2}$...

Unique direct sensitivity to 2+ loops.

Despite higher accuracy, large Q^2 world data never sensitive to it. (Also, often: single point measurement.)



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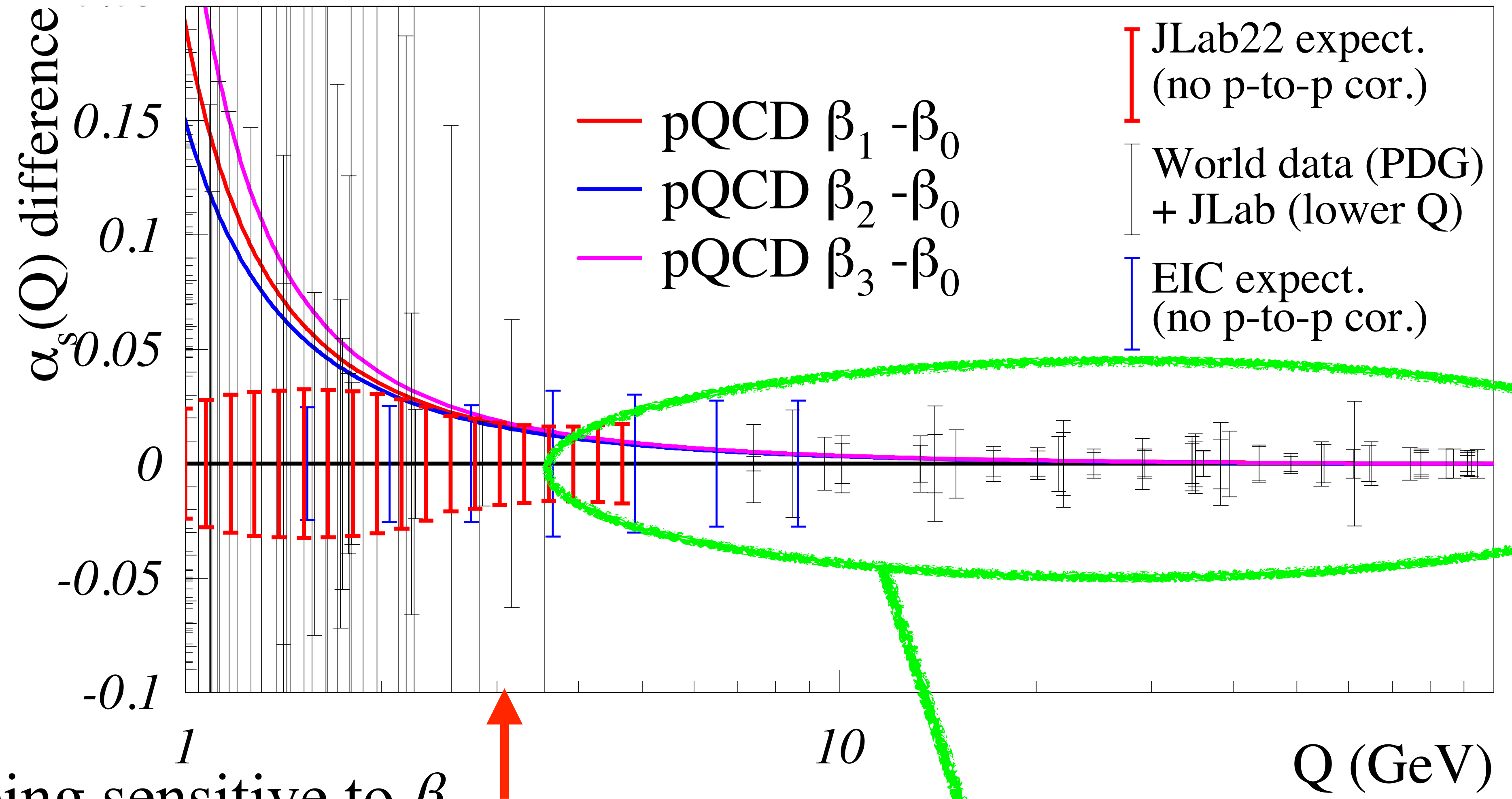

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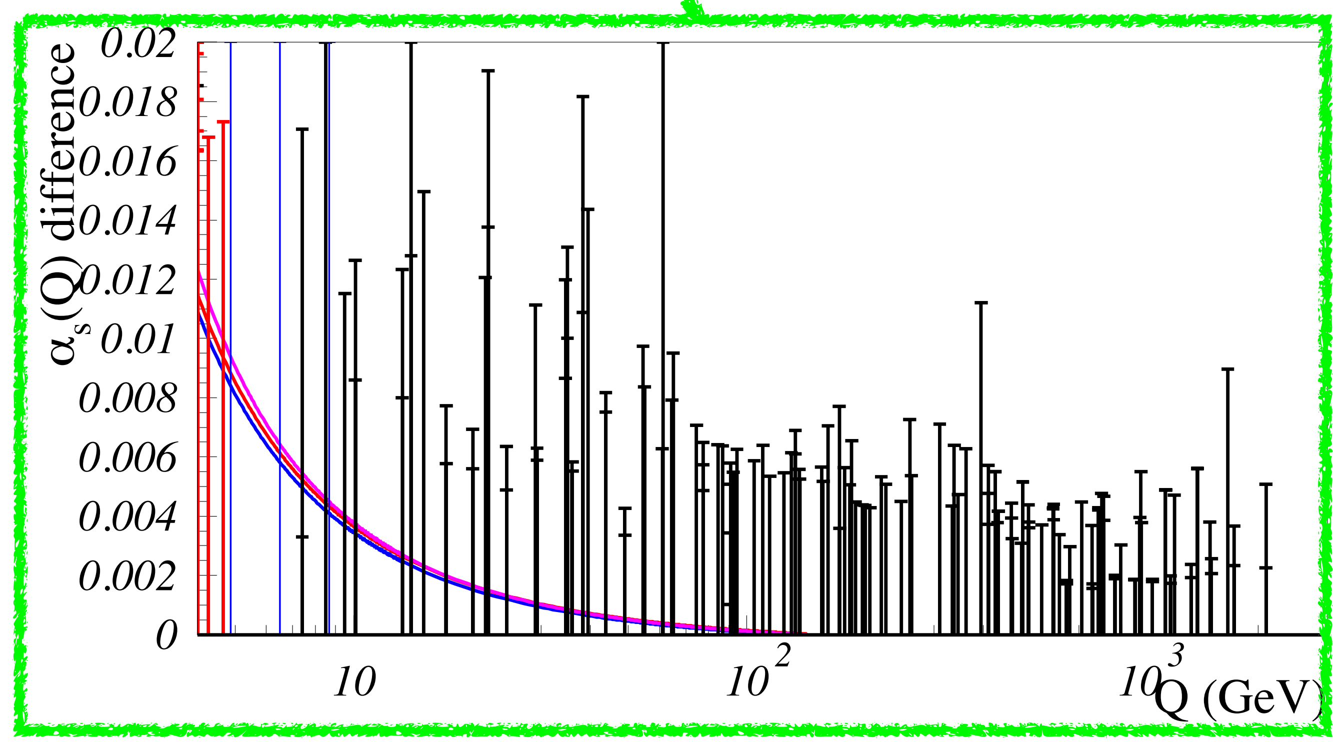


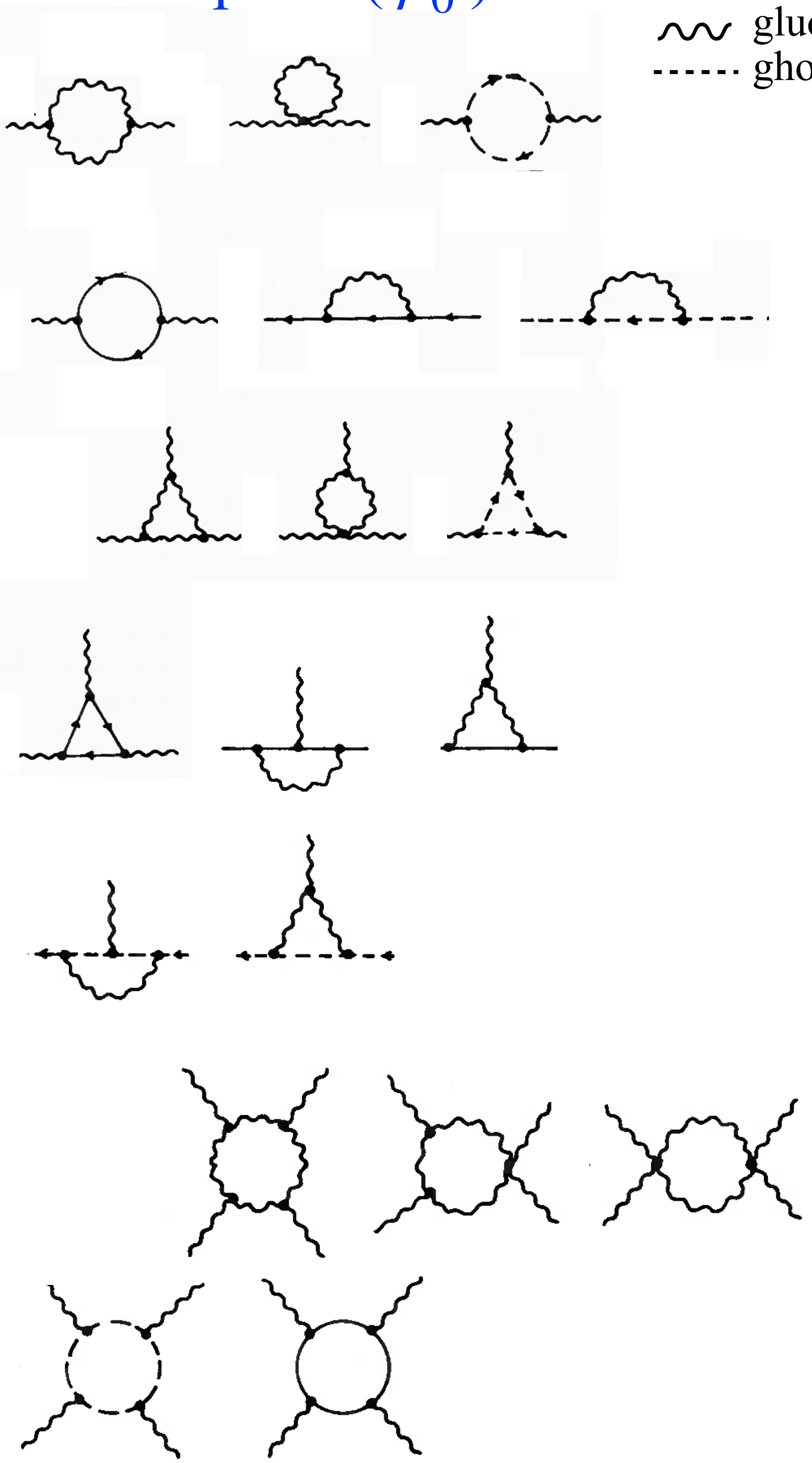
pQCD Q^2-evolution: so far, tested beyond LO for α_s-expansion (*e.g.*, DGLAP: higher order gluon bremsstrahlung).

This test isolates loop effects. Tests $\hbar$-expansion (β-series, with higher loops).

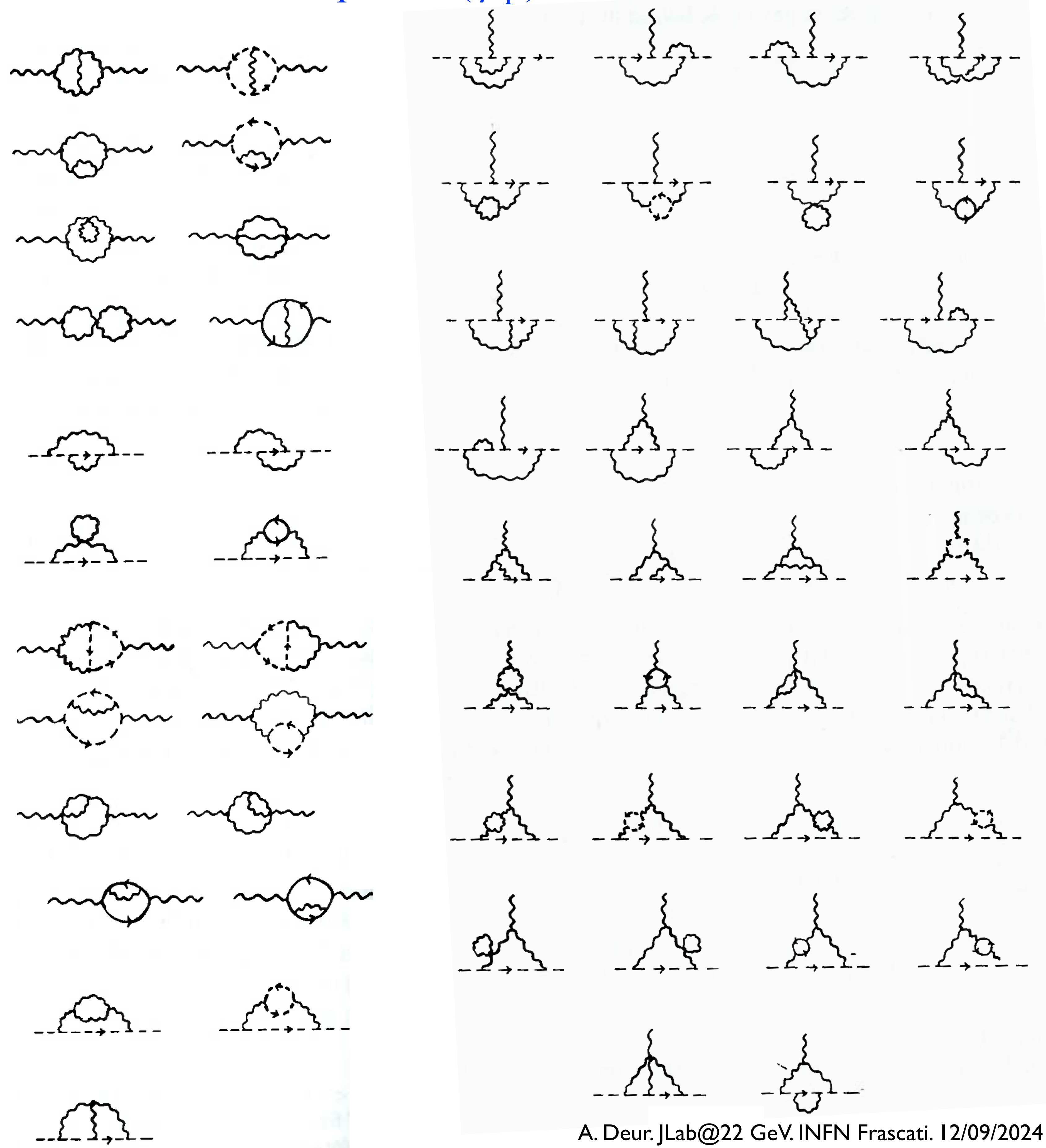
What do we learn from measuring 2-loop corrections ?

1-loop LO (β_0)

quark
gluon
ghost



2-loop NLO (β_1)



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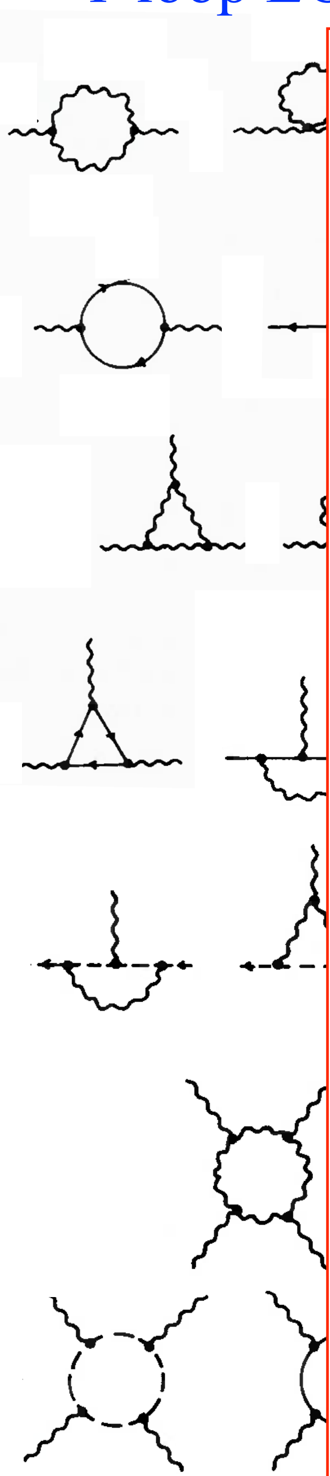
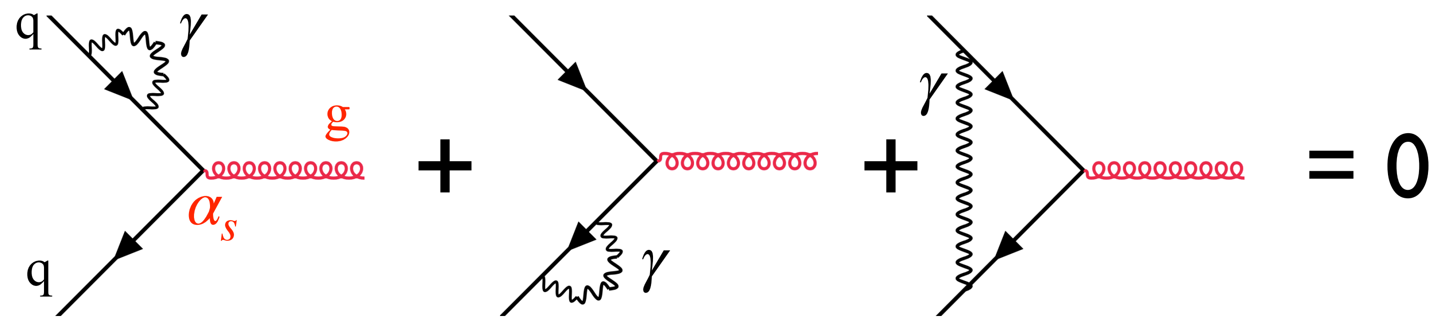
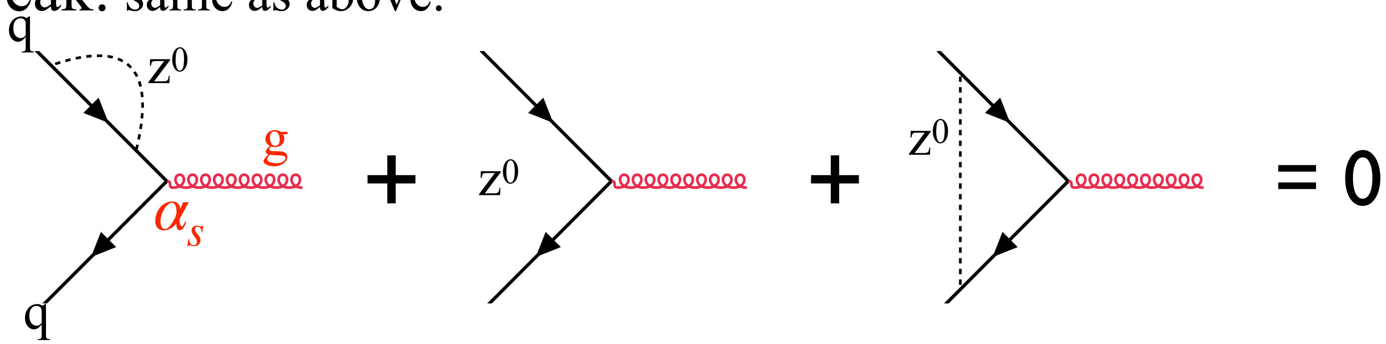


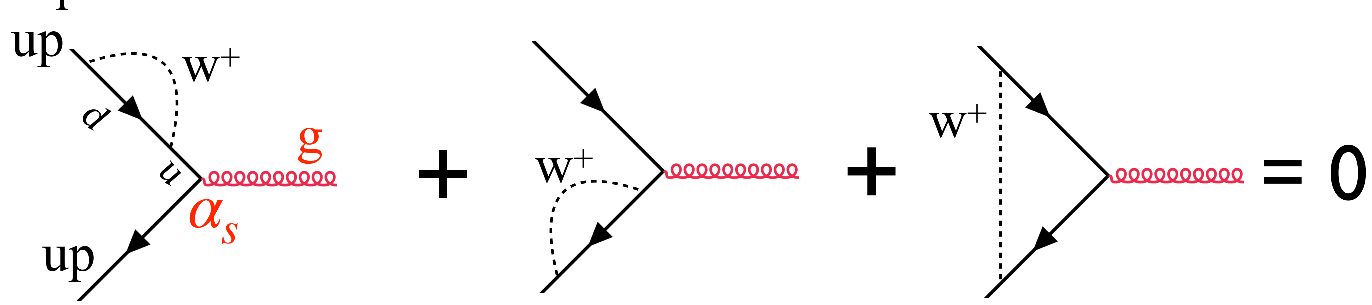
β_0: complete set of 1-loop graphs (ignoring gravity):

- QED: add quark self-energy + vertex correction, but sum cancels (Ward-Takahashi identities)

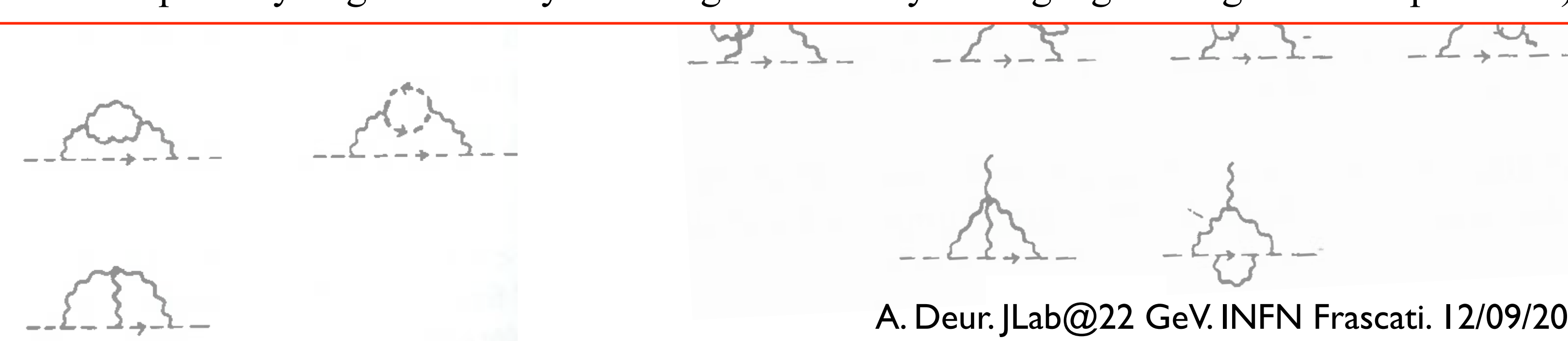


- Weak: same as above.





- Cannot add any other graphs because gluons (no electric charge nor weak isospin) couple only to quarks. (Same for ghosts: couple only to gluons. They can be ignored in any case: gauge-fixing fictitious particles.)



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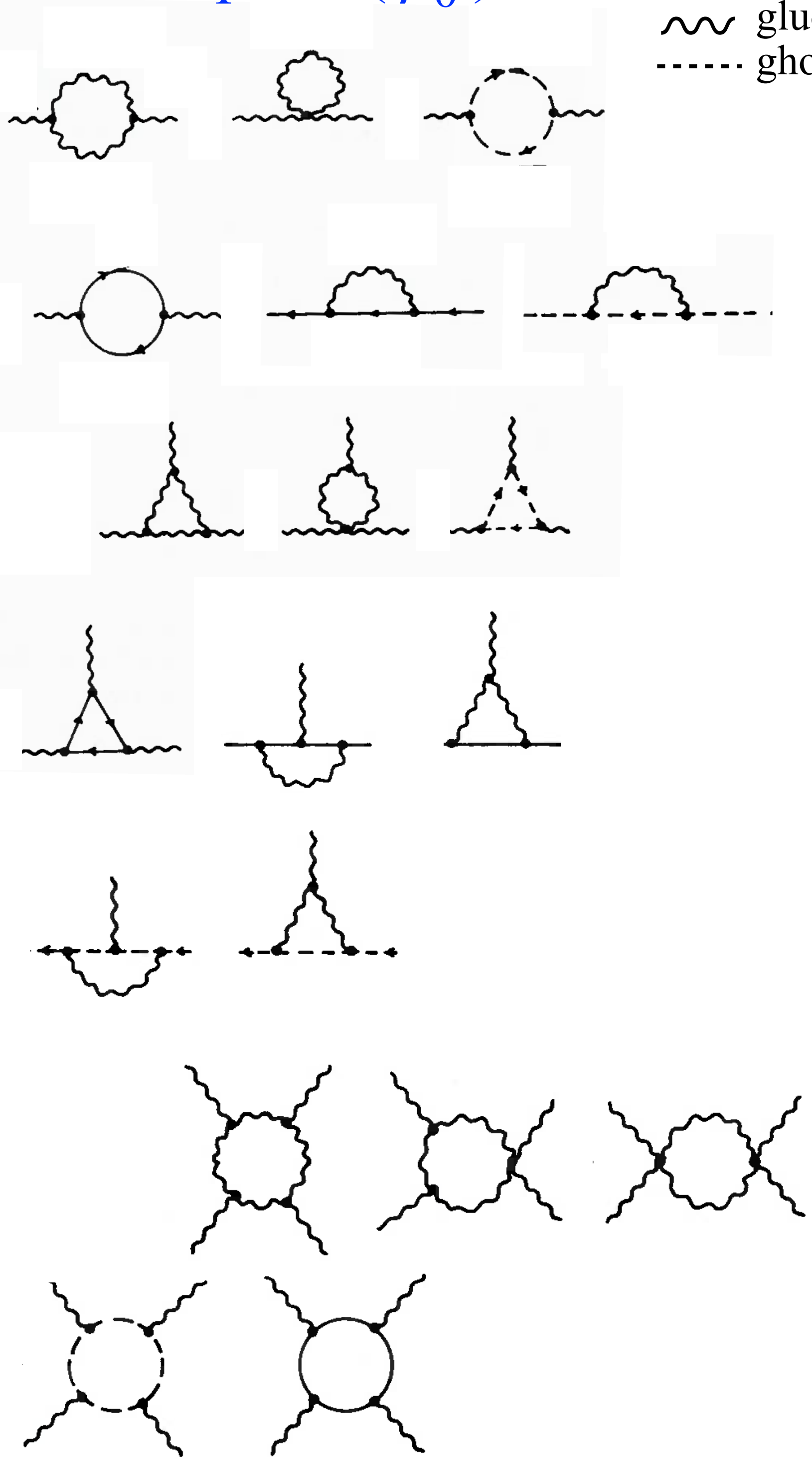
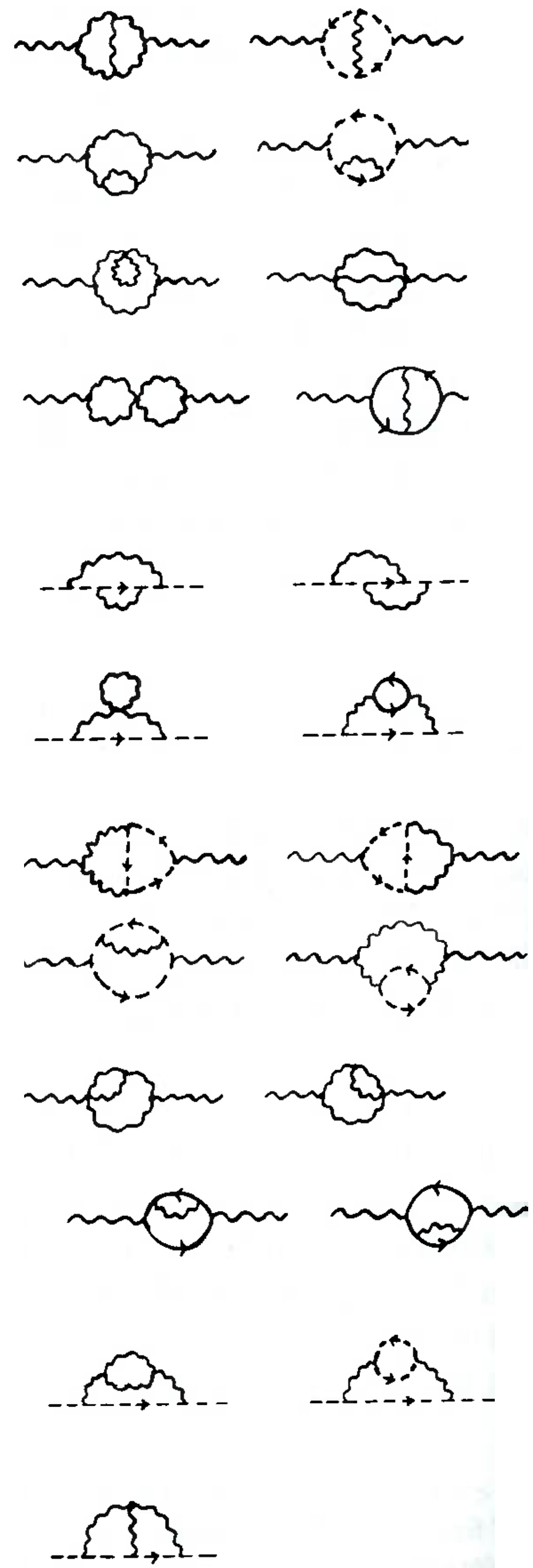
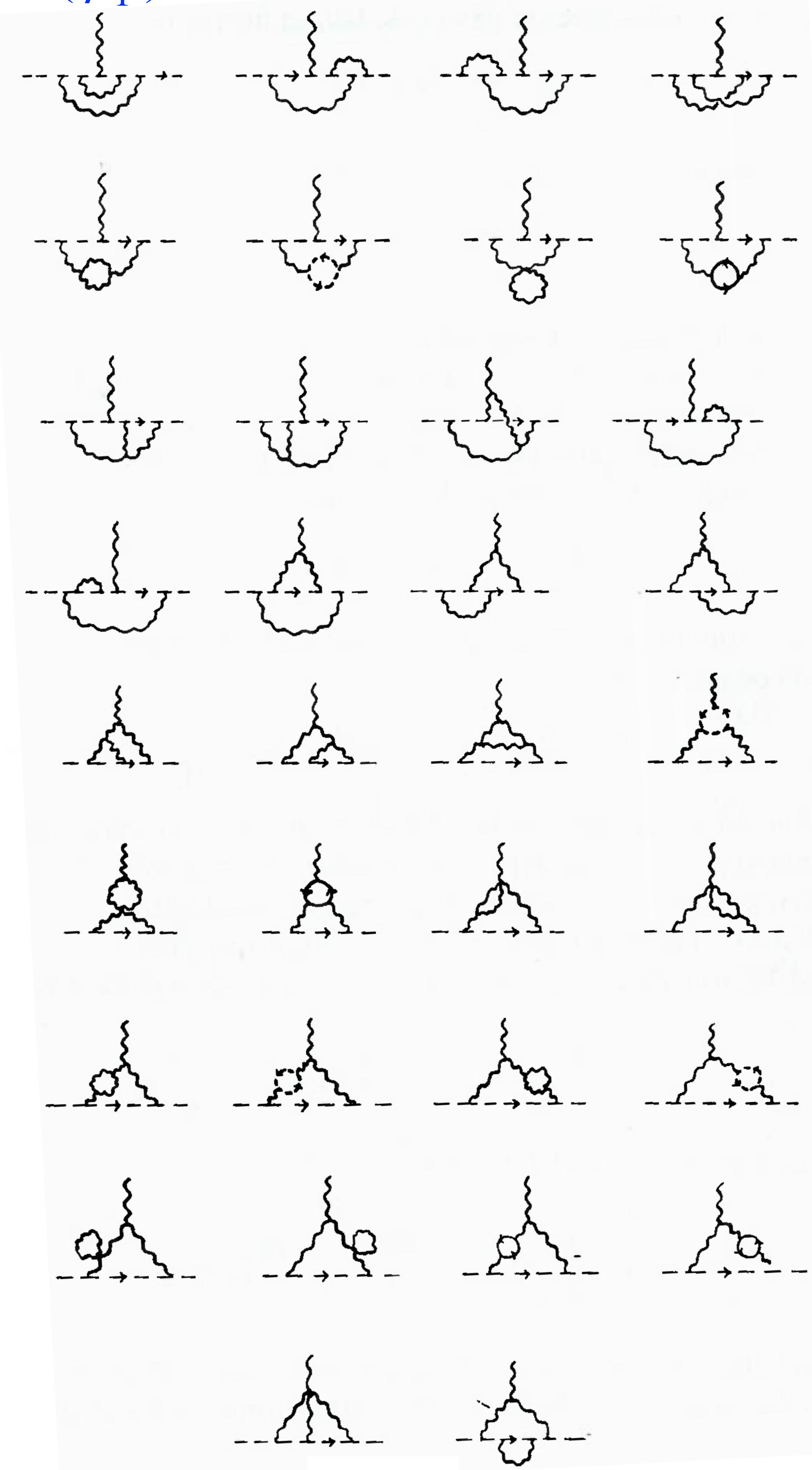


2-loop NLO (β_1)





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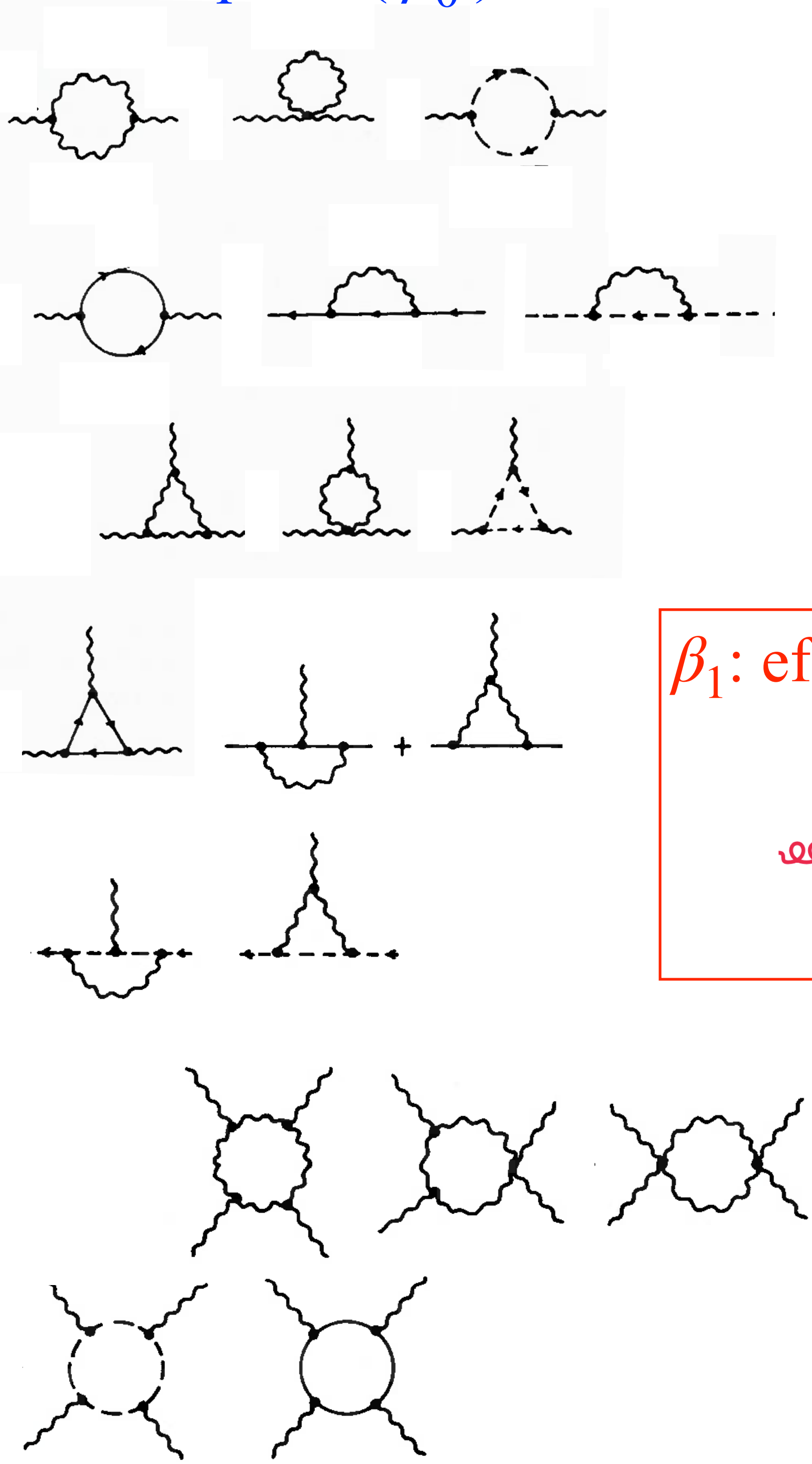
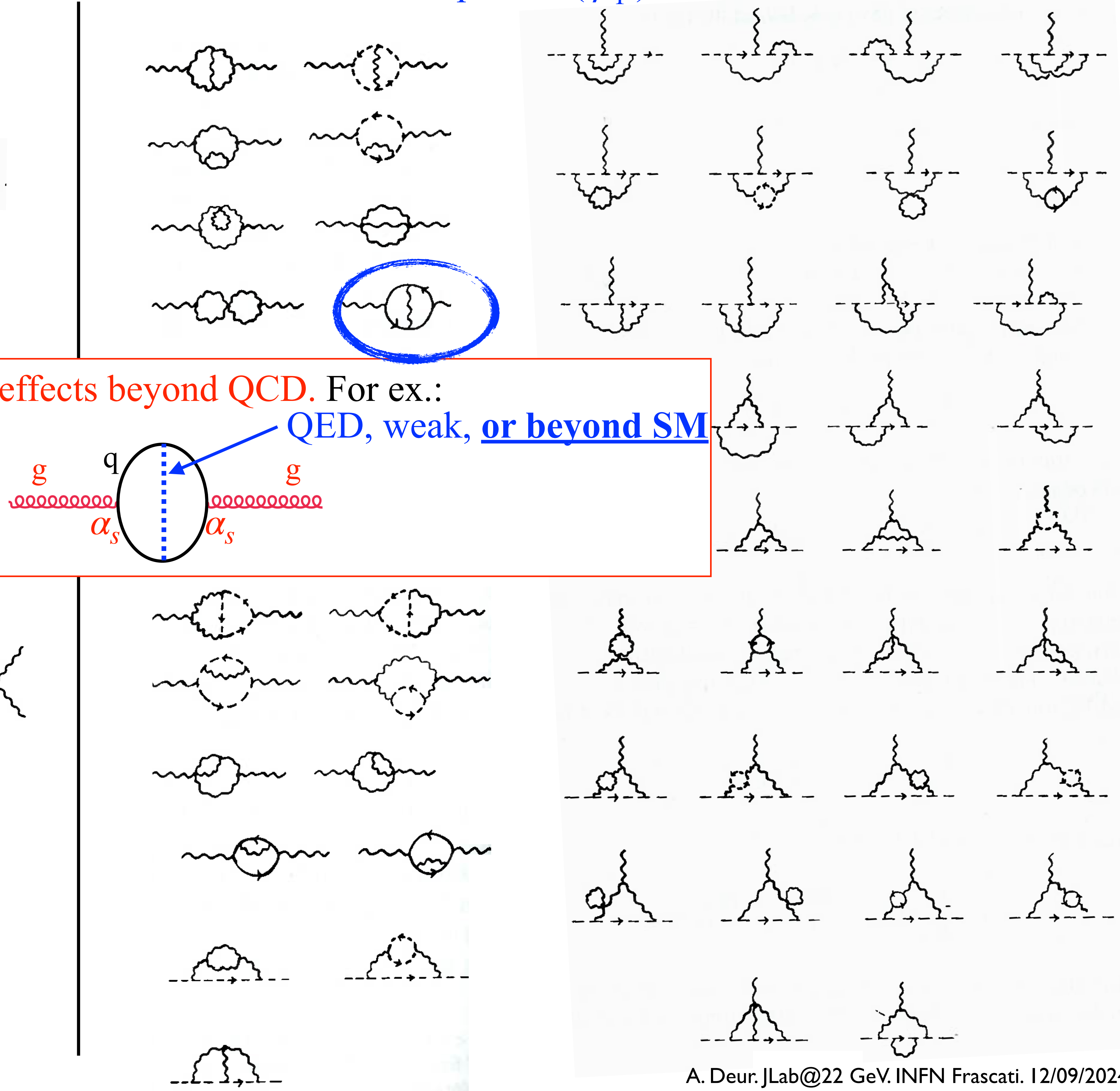


2-loop NLO (β_1)

β_1: effects beyond QCD. For ex.:

QED, weak, **or beyond SM**



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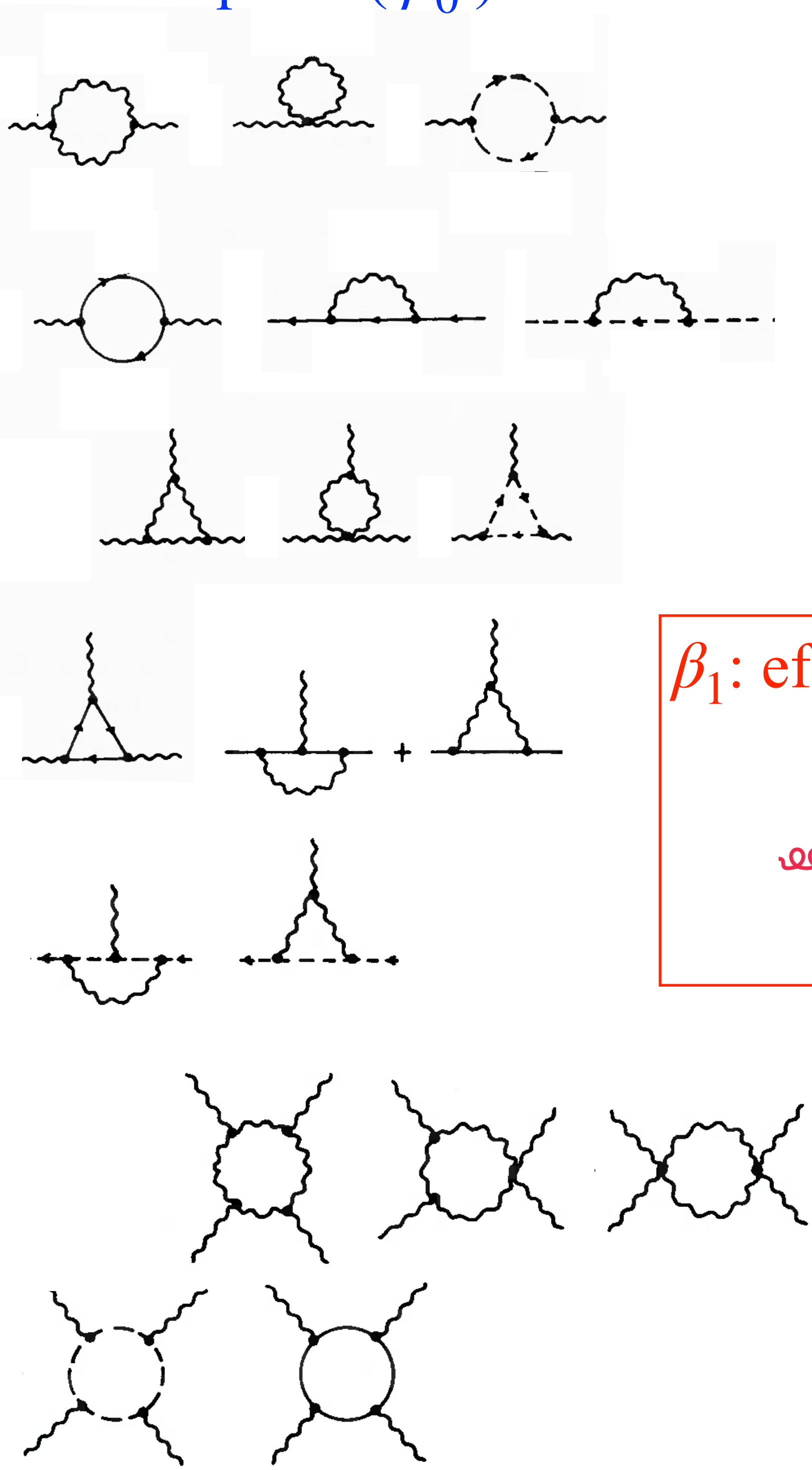

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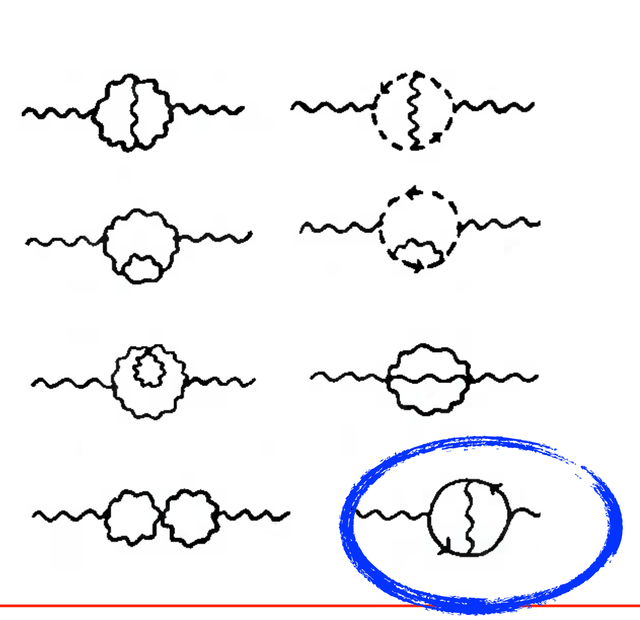

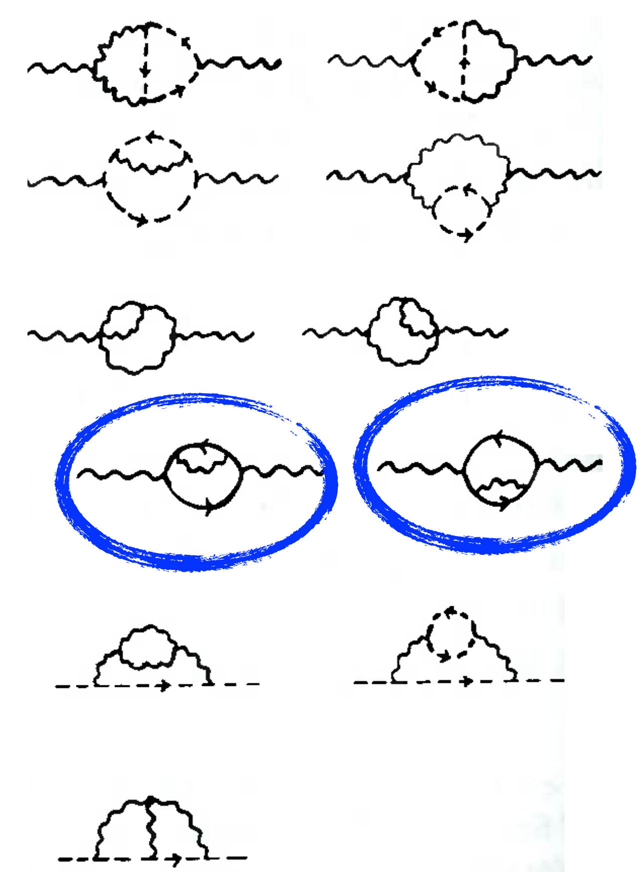

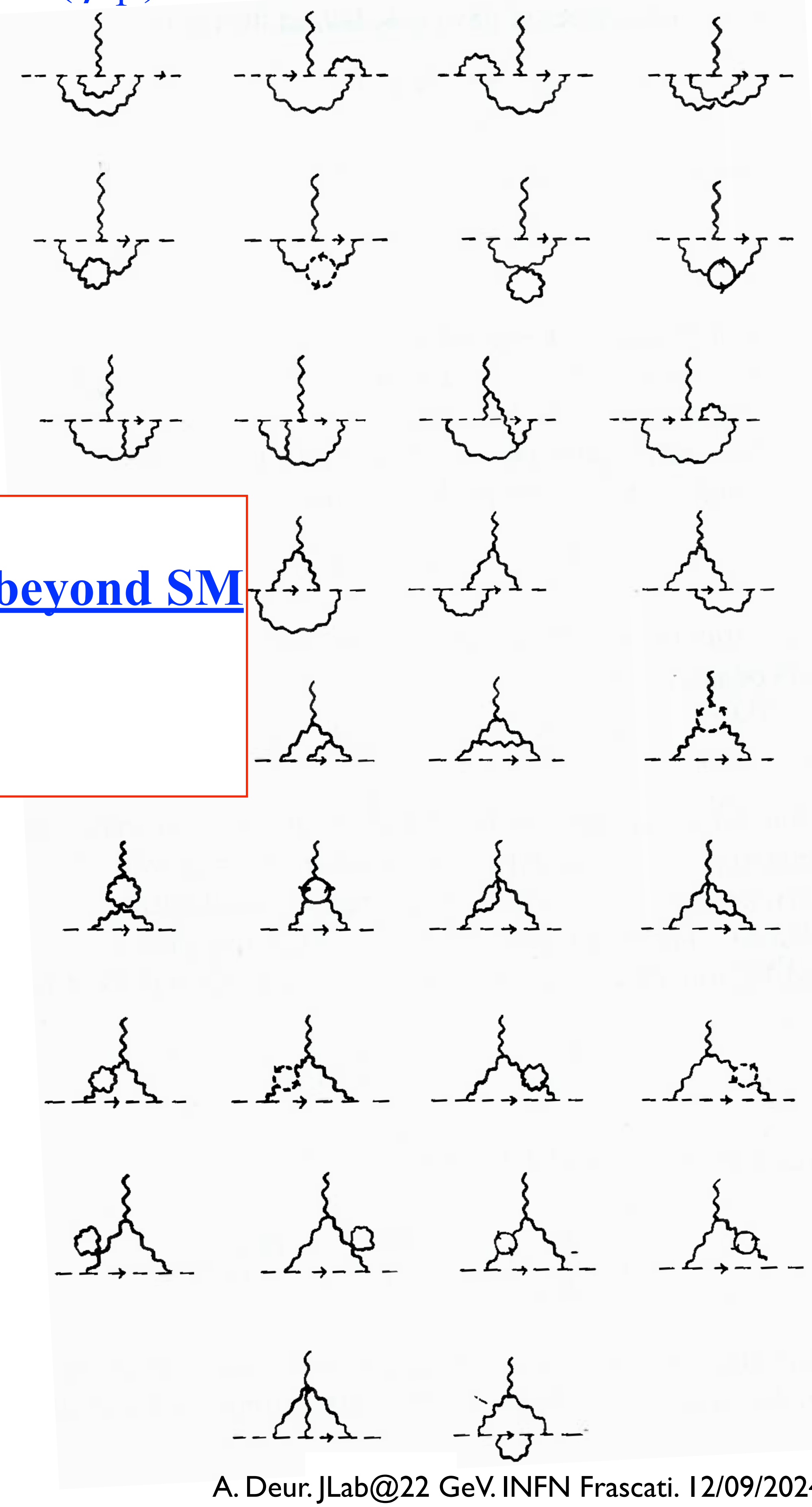

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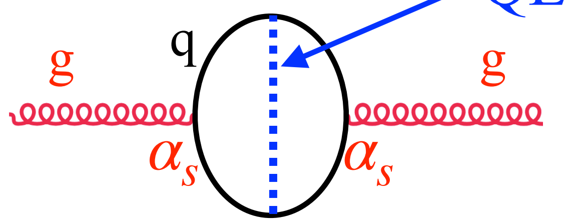

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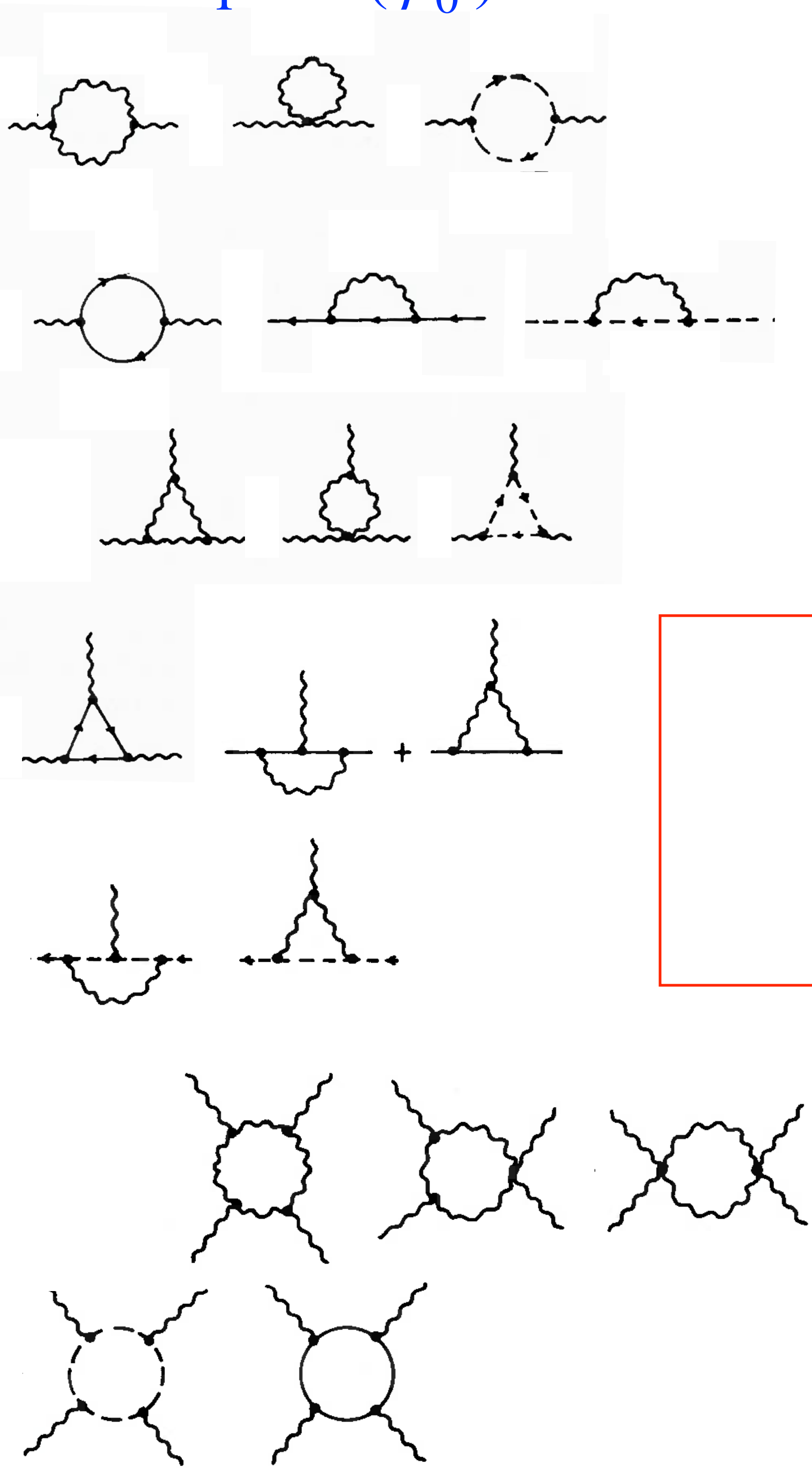

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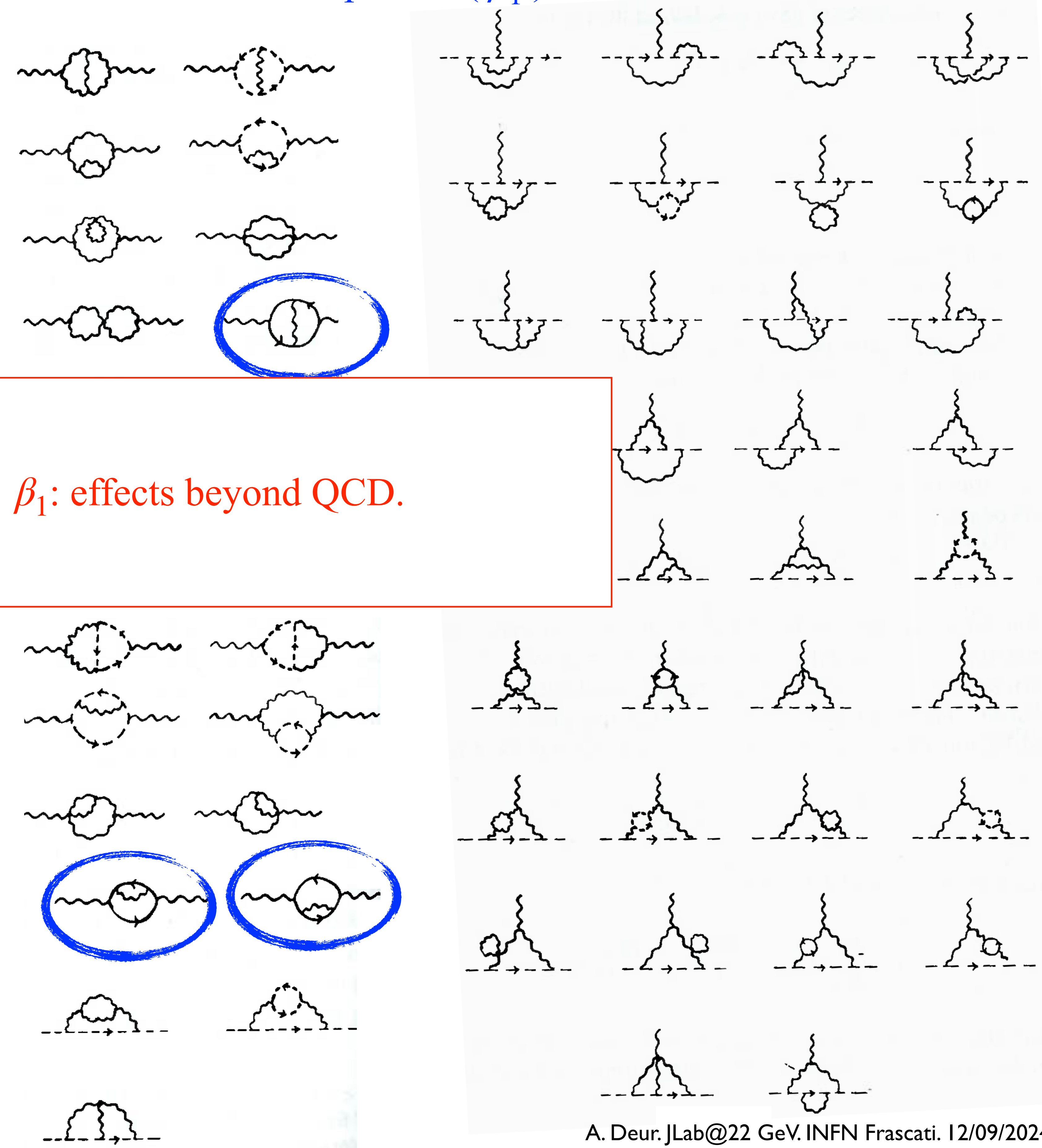


β_1: effects beyond QCD.

Conclusions

- Of the 4 fundamental couplings, α_s has by far the lowest accuracy.
- Accurate experimental determinations of $\alpha_s(Q^2)$ are crucial for QCD, SM and beyond SM studies.
  - The Bjorken sum $\Gamma_1^{p-n}(Q^2)=\int g_1^{p-n}(x,Q^2)dx$ offers a simple and competitive method to determine α_s.
  - Study indicates that JLab@22 GeV can provide a determination of $\alpha_s(M_Z^2)$ at the ~0.6% level.
  - Polarized data at low-x from EIC are essential. A EIC-only determination of $\alpha_s(M_Z)$ with the Bjorken sum would reach a ~1.3% accuracy.
  - This is but one of several ways to determine $\alpha_s(M_Z^2)$ with JLab@22. Others, e.g., global fits of (un)polarized PDFs should also provide competitive measurements. Put together, they have the potential to be provide a leading contribution toward a better determination of α_s.
- One may also map the Q^2-dependence of $\alpha_s(Q^2)$ in the 1-22 GeV^2 domain.
  - $Q^2 < 5.3$ GeV^2: JLab@22 mapping sensitive to 2-loop (β_1) effect. First time this would be the case.
  - Effects beyond QCD start at β_1. (None at β_0)
  - Mapping tests QCD and opens a new window for BSM physics.
  - Sensitivity to BSM needs to be calculated.

Thank you

Back-up slides

Importance of measuring $\alpha_s(M_z)$

- α_s: most important quantity of QCD, key parameter of the Standard Model, but (by far) the least known fundamental coupling: $\Delta\alpha_s/\alpha_s \simeq 10^{-2}$ ($\Delta\alpha/\alpha \simeq 10^{-10}$, $\Delta G_F/G_F \simeq 10^{-6}$, $\Delta G_N/G_N \simeq 10^{-5}$)
- Large efforts ongoing to reduce $\Delta\alpha_s/\alpha_s$ (Snowmass 2022, J.Phys.G 51 (2024) 9, 090501 arXiv:2203.08271)
- No "silver bullet" experiment can exquisitely determine α_s.

  $\Rightarrow$ Strategy: combine many independent measurements with larger uncertainties.

  Currently, best individual experimental determinations are ~1%-2% level.

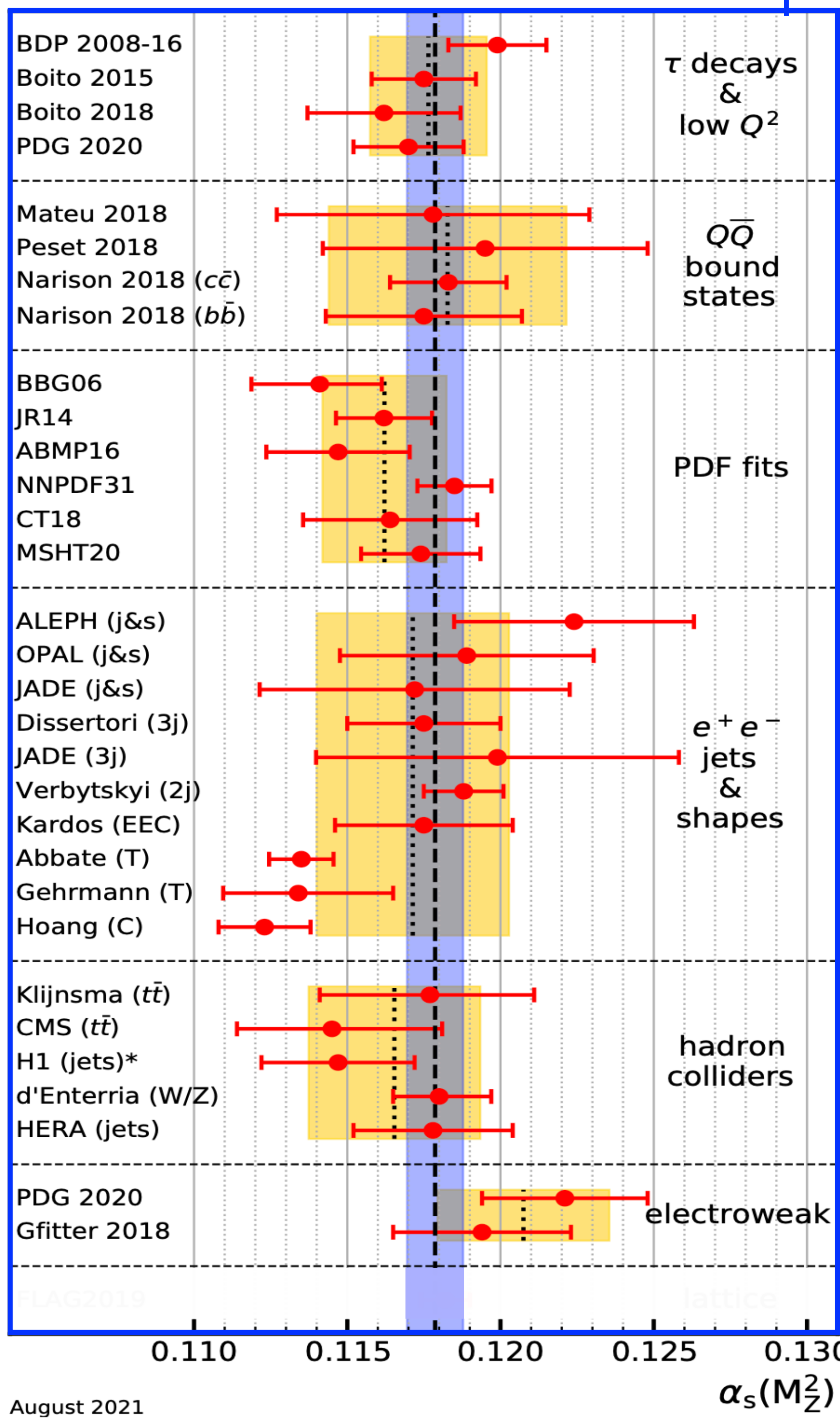


Fig. from Part. Data Group.

Bjorken sum rule

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893\left(\frac{\alpha_s}{\pi}\right)^5\right]$$

$$+ \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon's First spin structure function

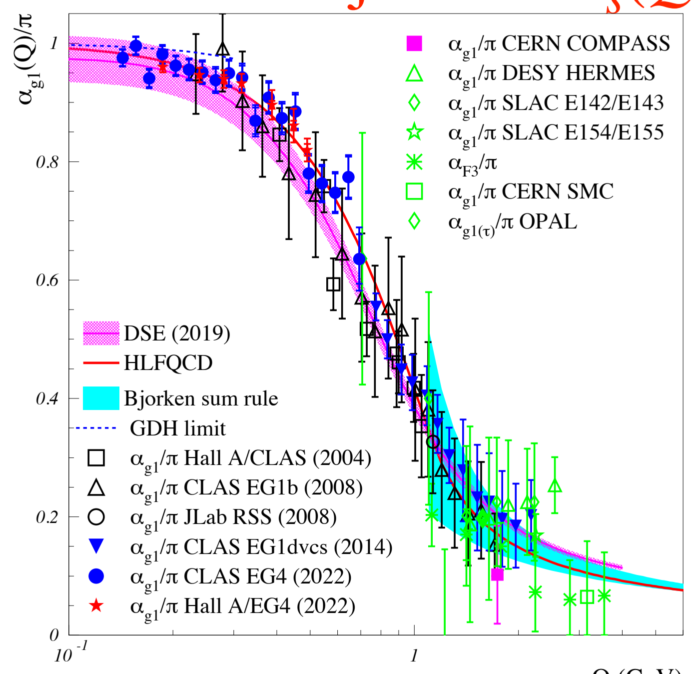
Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative 1/Q^{2n} power corrections. (+rad. corr.)

$\Rightarrow$ Two possibilities to extract $\alpha_s(M_Z)$:

- Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve the Bj SR for $\alpha_s(Q^2)$.
  - One α_s per Γ_1^{p-n} experimental data point.



The Bj SR allows to extract $\alpha_s(Q^2)$ at all scale!

- Poor systematic accuracy, typically $\Delta\alpha_s/\alpha_s$ ~10% at high energy $\Rightarrow$ Not competitive.

Bjorken sum rule

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  - Poor systematic accuracy, typically $\Delta\alpha_s/\alpha_s$ ~10% at high energy $\Rightarrow$ Not competitive.

- Measurement of Q^2**-dependence** of $\Gamma_1^{p-n}(Q^2)$.
  - Need Γ_1^{p-n} at several Q^2 points. Only one (or a few) value of α_s.
  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_Z)$=0.120±0.009

Altarelli, Ball, Forte, Ridolfi, Nucl.Phys. B496 337 (1997)

Bjorken sum rule at JLab@22 GeV

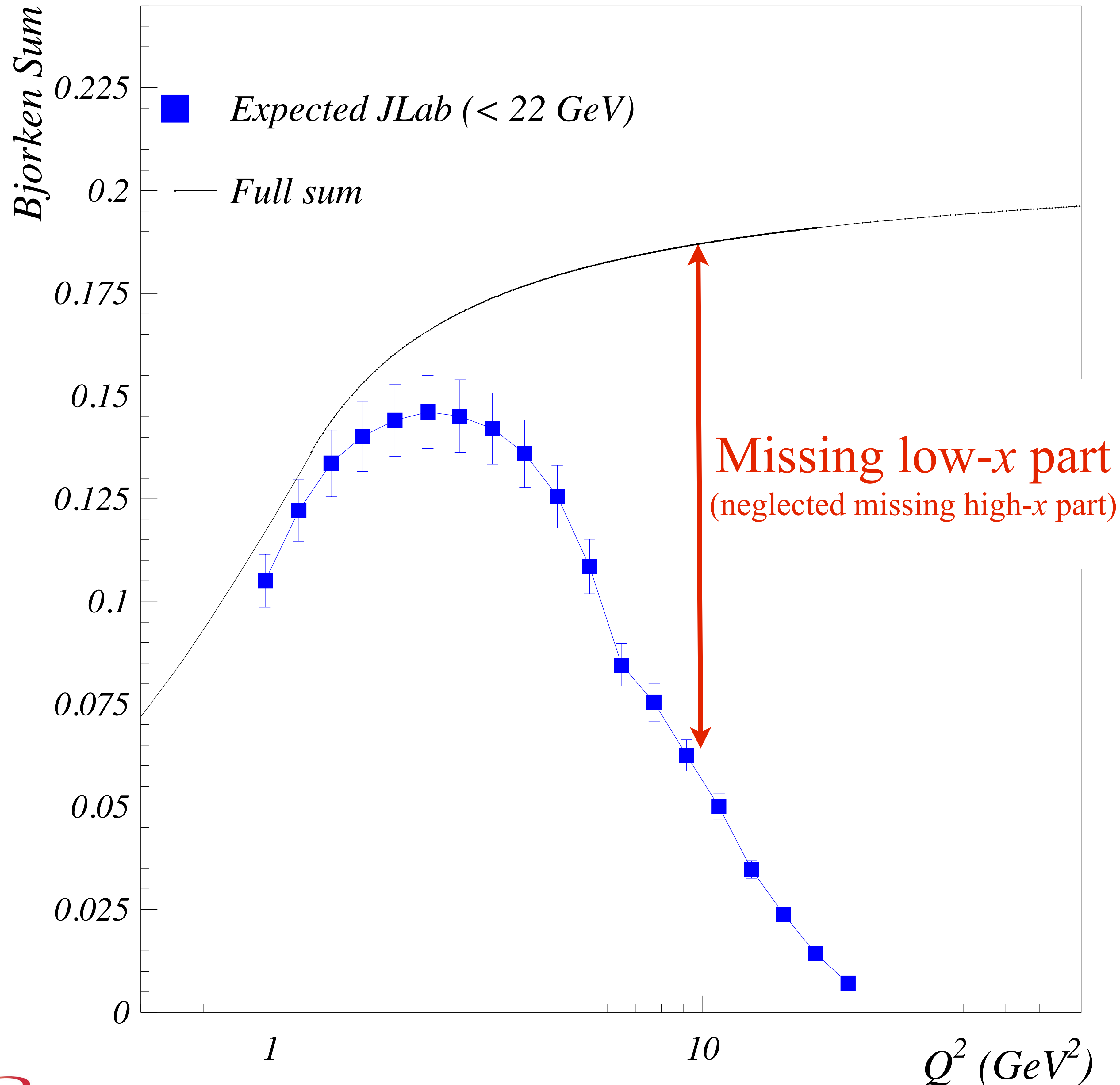
- Statistical uncertainties are expected to be negligible:
  - JLab is a high-luminosity facility;
  - A JLab@22 GeV program would include polarized DVCS and TMD experiments. Those imply long running times compared to those needed for inclusive data gathering;
  - High precision data already available from 6 GeV and 12 GeV for the lower Q^2 bins and moderate x.

- Looking at the 6 GeV CLAS EG1dvcs data, required statistics for DVCS and TMD experiments imply statistical uncertainties < 0.1% on the Bjorken sum. For the present exercise we will use 0.1% on all Q^2-points with Q^2-bin sizes increasing exponentially with Q^2.

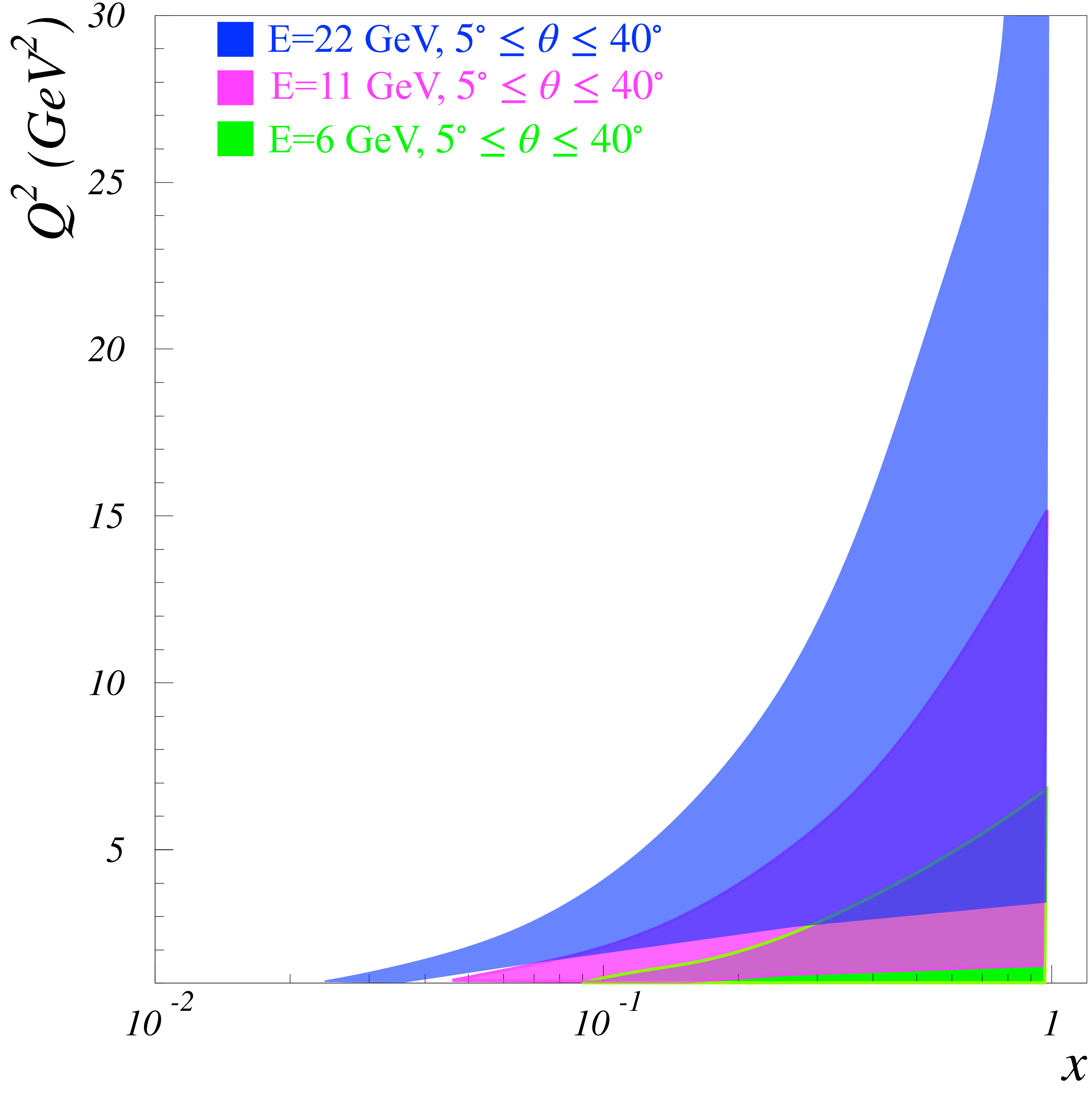
- Use 5% for experimental systematics (i.e. not including the uncertainty on unmeasured low-x).
  - Nuclear corrections:
    - D: negligible assuming we can tag the ~spectator proton
    - ^{3}He: 2% (5% on n, which contribute to 1/3 to the Bjorken sum: 5%/3≃2%)
  - Polarimetries: Assume ΔP_e-$\Delta P_N = 3\%$.
  - Radiative corrections: 1%
  - F_1 to form g_1 from A_1: 2%
  - g_2 contribution to longitudinal asym: Negligible, assuming it will be measured.
  - Dilution/purity:
    - Bjorken sum from P & D: 4%
    - Bjorken sum from P & ^{3}He: 3%
  - Contamination from particle miss-identification: Assumed negligible.
  - Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

Adding in quadrature: ~5%

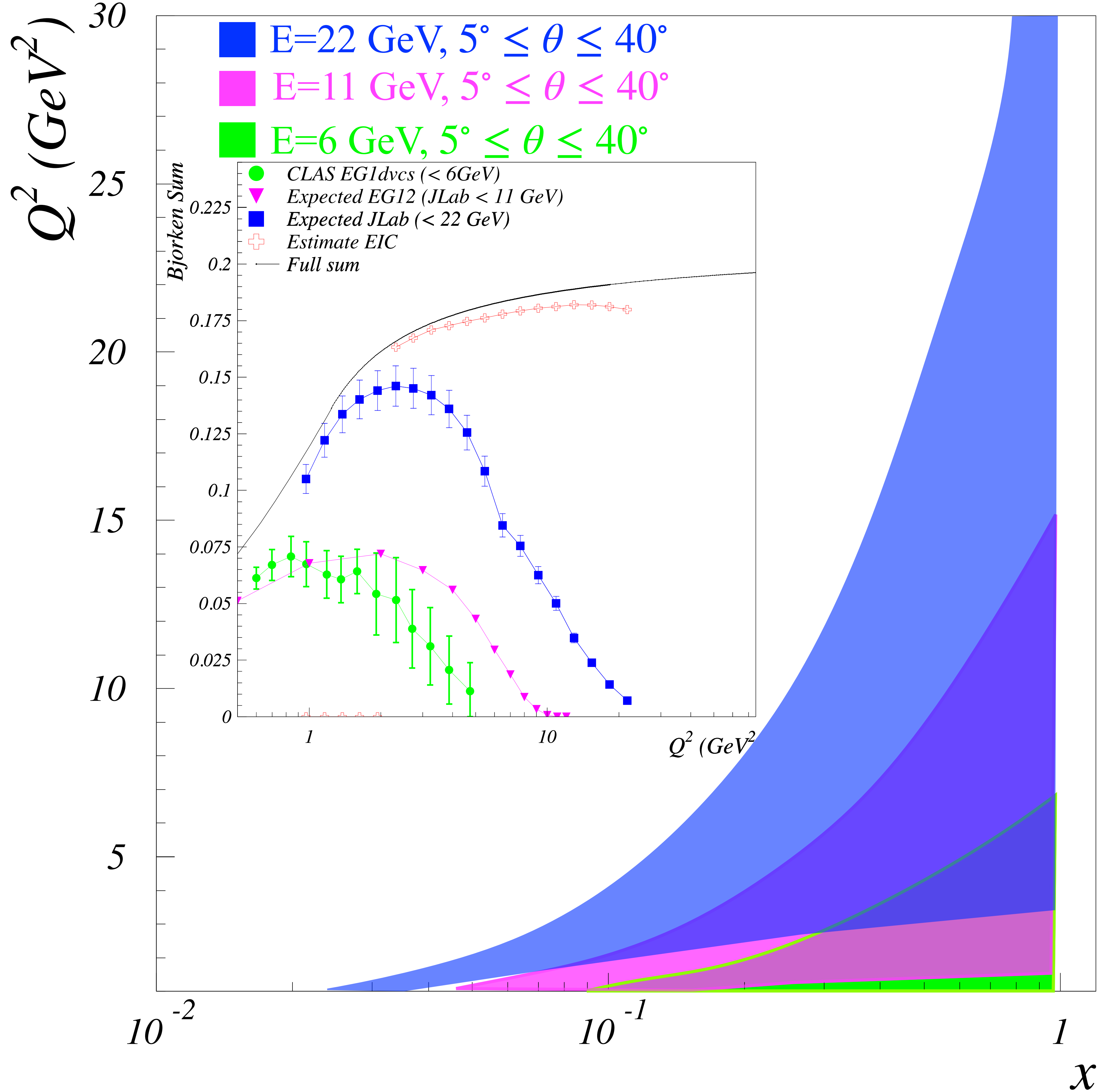
Under these assumptions:



Comparison with JLab at 6 and 11 GeV



Comparison with JLab at 6 and 11 GeV



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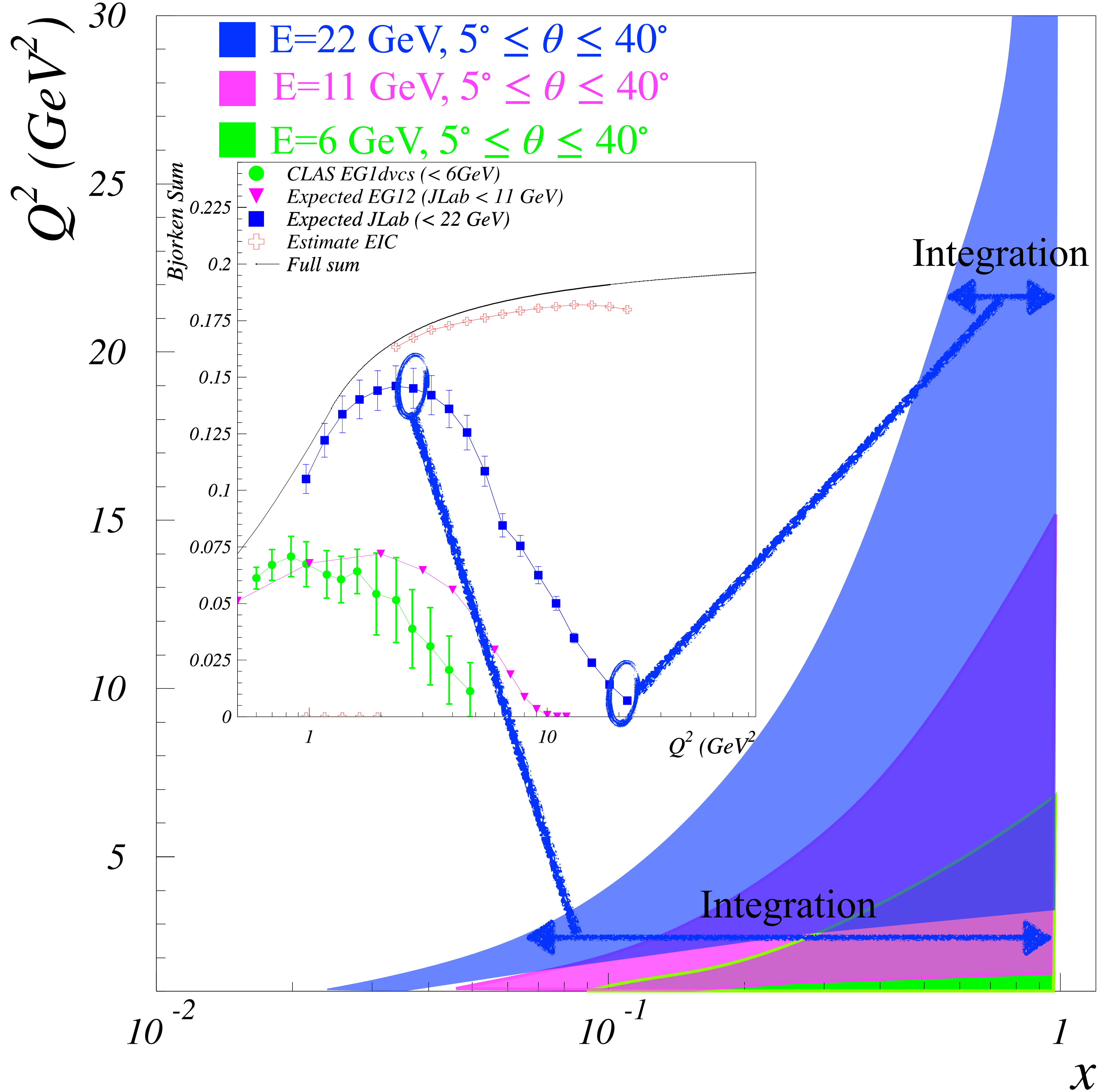
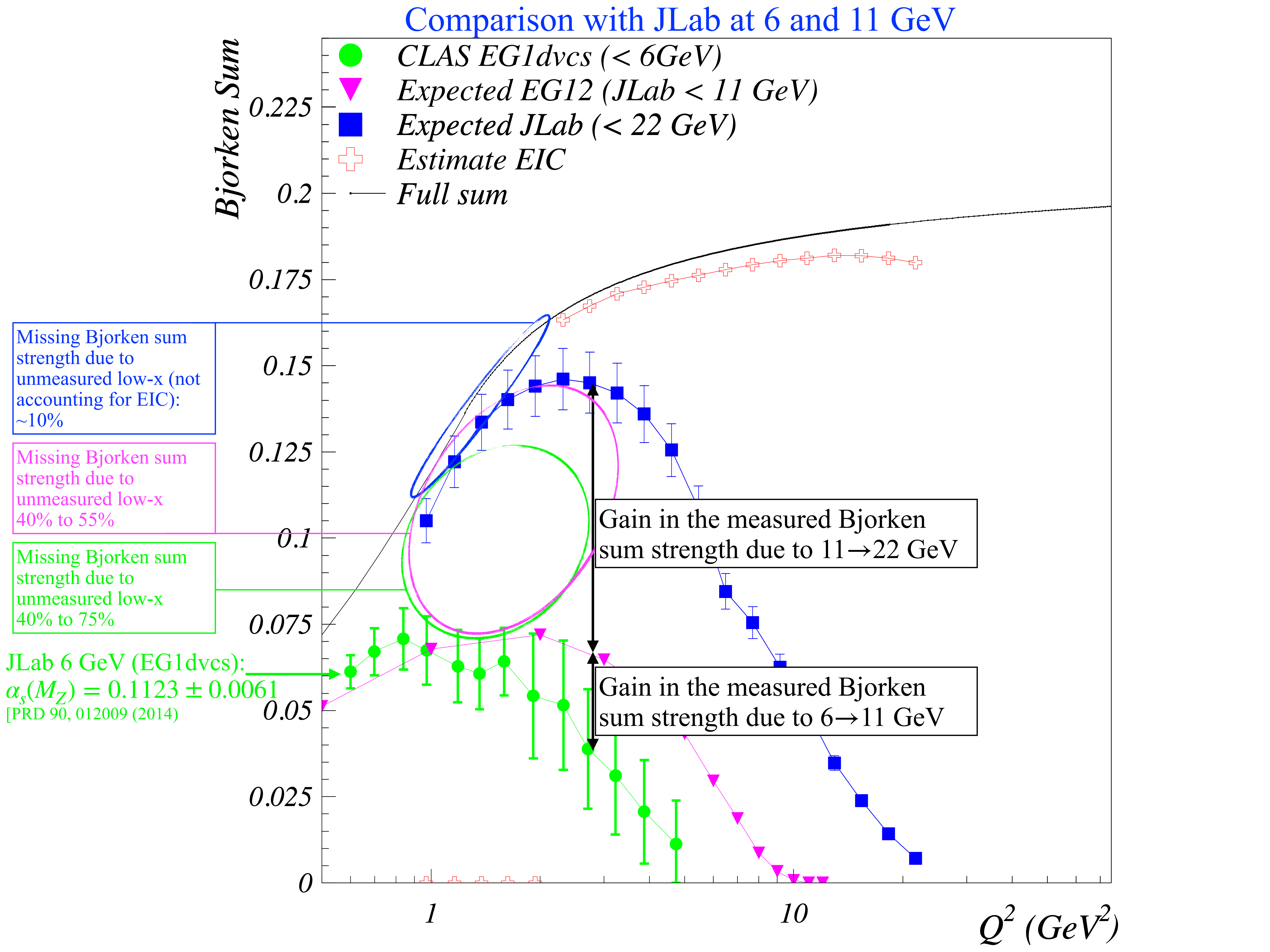
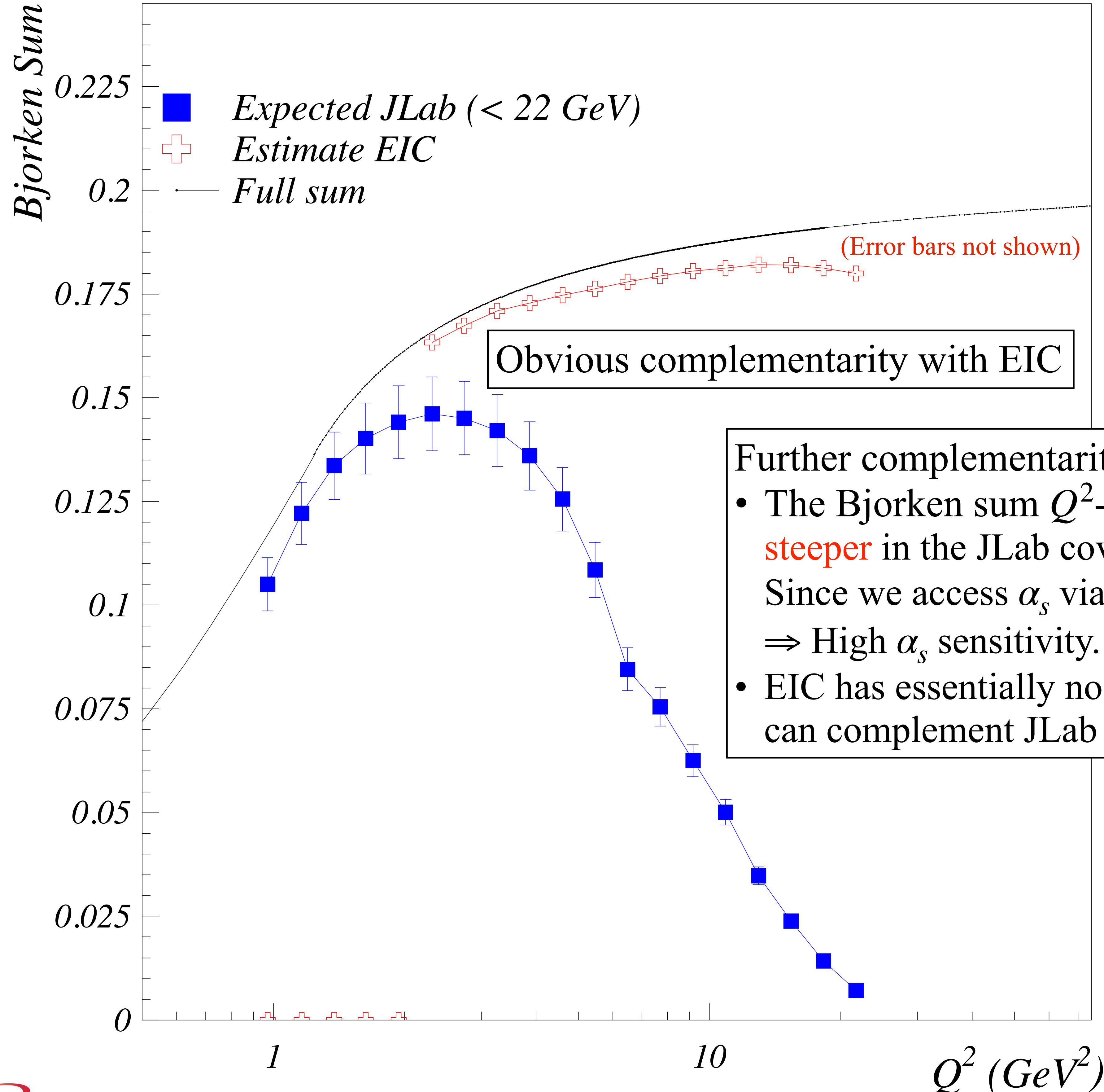




Comparison with JLab at 6 and 11 GeV
Bjorken Sum
CLAS EG1dvcs (< 6GeV)
Expected EG12 (JLab < 11 GeV)
Expected JLab (< 22 GeV)
Estimate EIC
Full sum
0.225
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
1
10
Q^2 (GeV^2)
Missing Bjorken sum strength due to unmeasured low-x (not accounting for EIC): ~10%
Missing Bjorken sum strength due to unmeasured low-x 40% to 55%
Missing Bjorken sum strength due to unmeasured low-x 40% to 75%
JLab 6 GeV (EG1dvcs):
$\alpha_s(M_Z) = 0.1123 \pm 0.0061$
[PRD 90, 012009 (2014)
Gain in the measured Bjorken sum strength due to 11→22 GeV
Gain in the measured Bjorken sum strength due to 6→11 GeV

Comparison with EIC



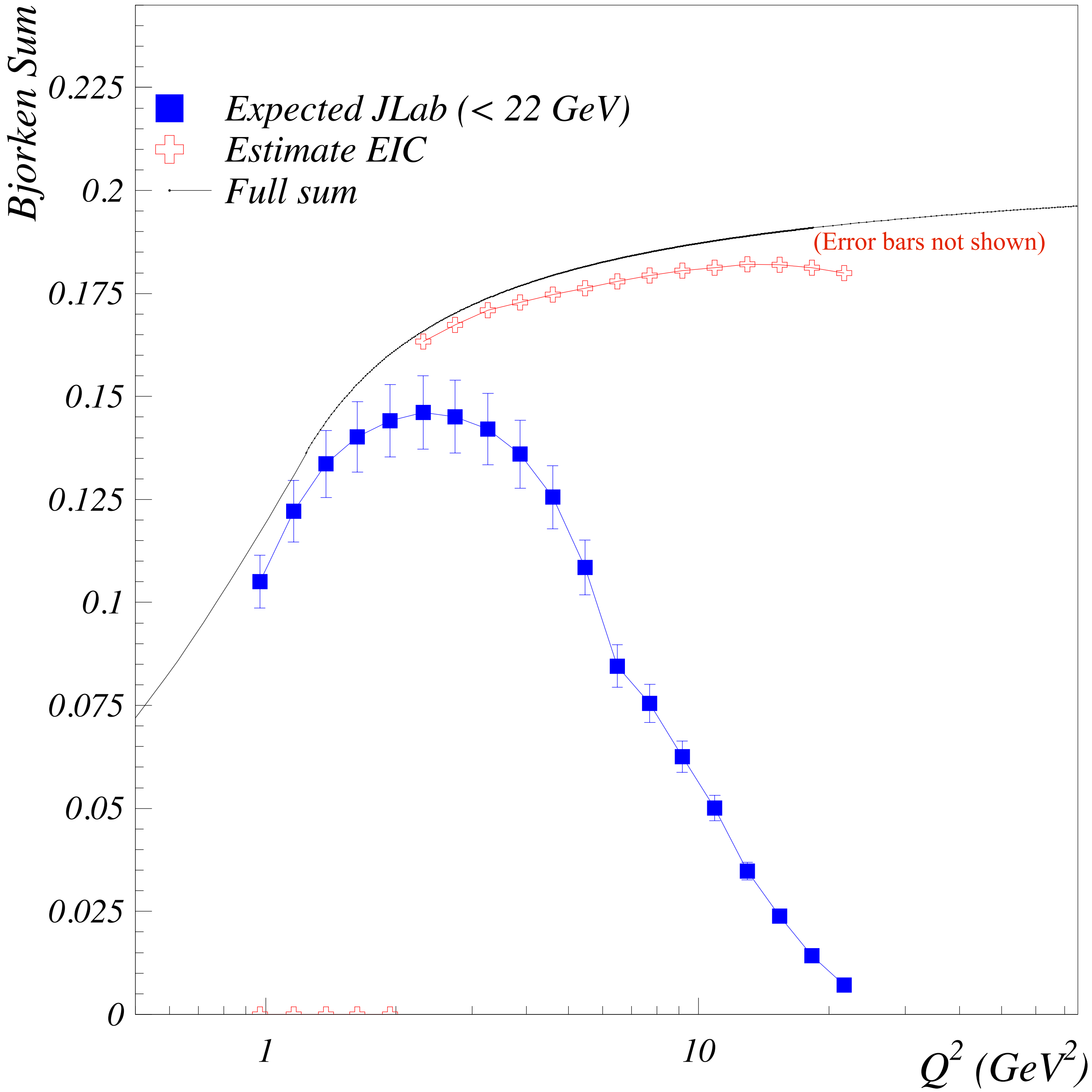
Further complementarity:

- The Bjorken sum Q^2-dependence is up to 50 times steeper in the JLab covered range than that of EIC. Since we access α_s via relative Q^2-dependence $\Rightarrow$ High α_s sensitivity.
- EIC has essentially no unmeasured low-x issue and can complement JLab data.

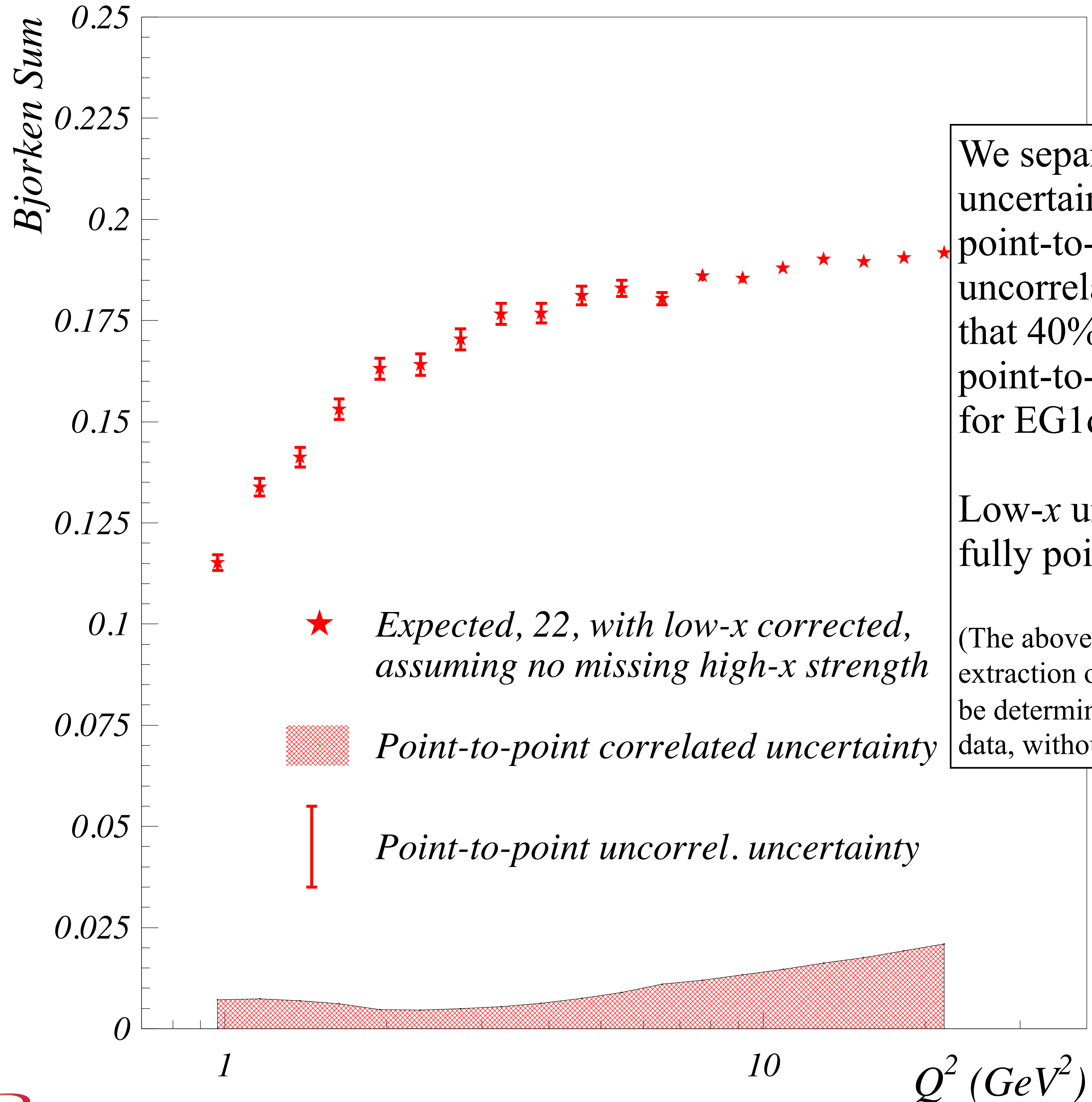
Low-x uncertainty

- For the Q^2 bins covered by EIC, global fits will be available up to the lowest x covered by EIC.
  ⇒ assume 10% uncertainty on that missing (for the JLab measurement) low-x part.
  Assume 100% for the very small-x contribution not covered by EIC.

- For the 5 lowest Q^2 bins not covered by EIC:
  - Bin #5 close to the EIC coverage ⇒ Constrained extrapolation, assume 20% uncertainty on missing low-x part.
  - Bin #4, assume 40% uncertainty, Bin #3, assume 60%, Bin #2, assume 80%, Bin #1, assume 100%.



Bjorken sum rule at JLab@22 GeV (meas.+low-x)

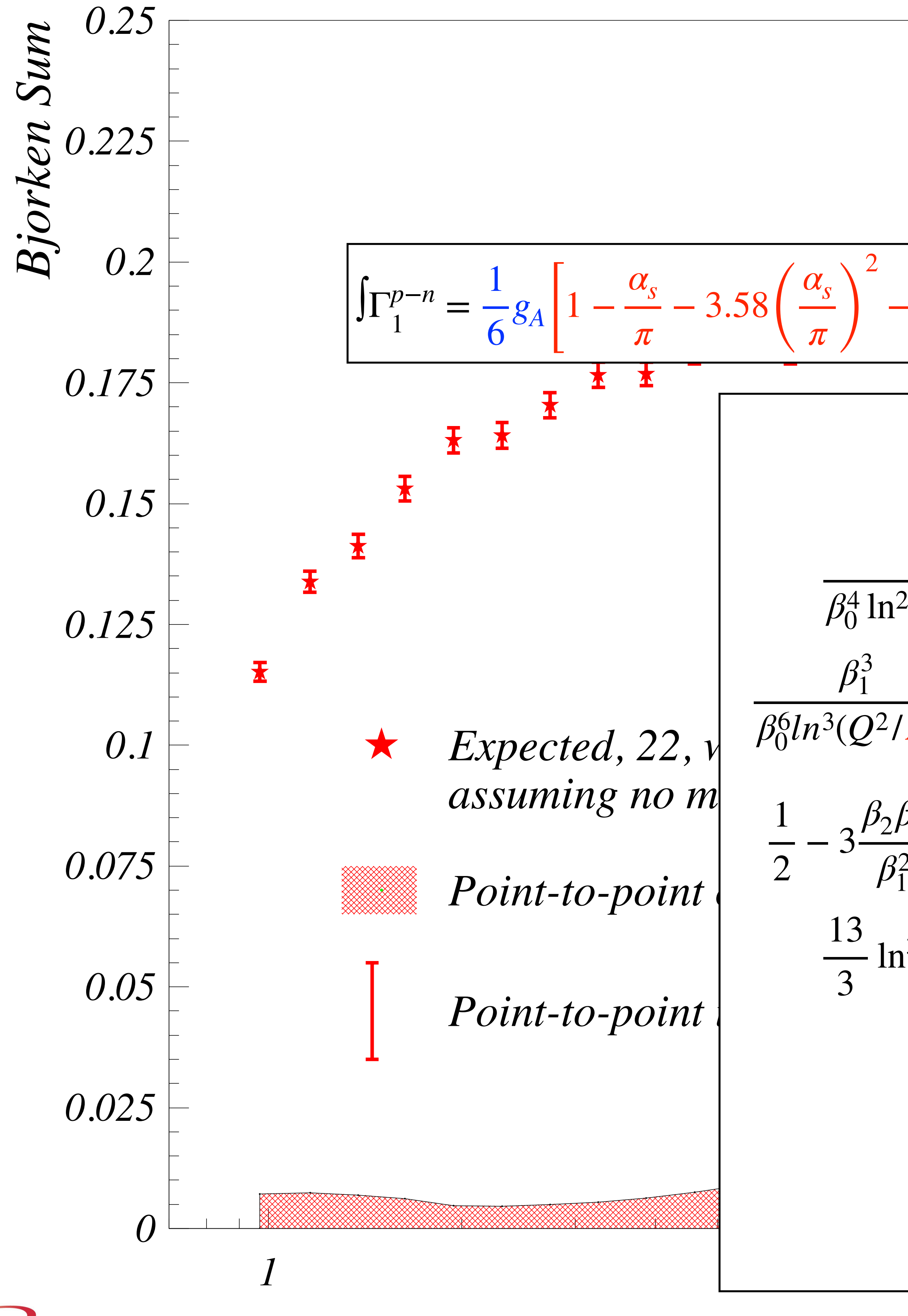


We separate the total experimental uncertainty (i.e. excluding the low-x error) in point-to-point correlated and uncorrelated contributions, assuming that 40% of the total uncertainty is point-to-point correlated (as obtained for EG1dvcs Bjorken sum analysis).

Low-x uncertainty is assumed to be fully point-to-point correlated.

(The above assumptions are not crucial for the extraction of α_s. Also, the proper separation would be determined from analysis of the actual 22 GeV data, without assumption.)

Extraction of $\alpha_s(M_Z)$



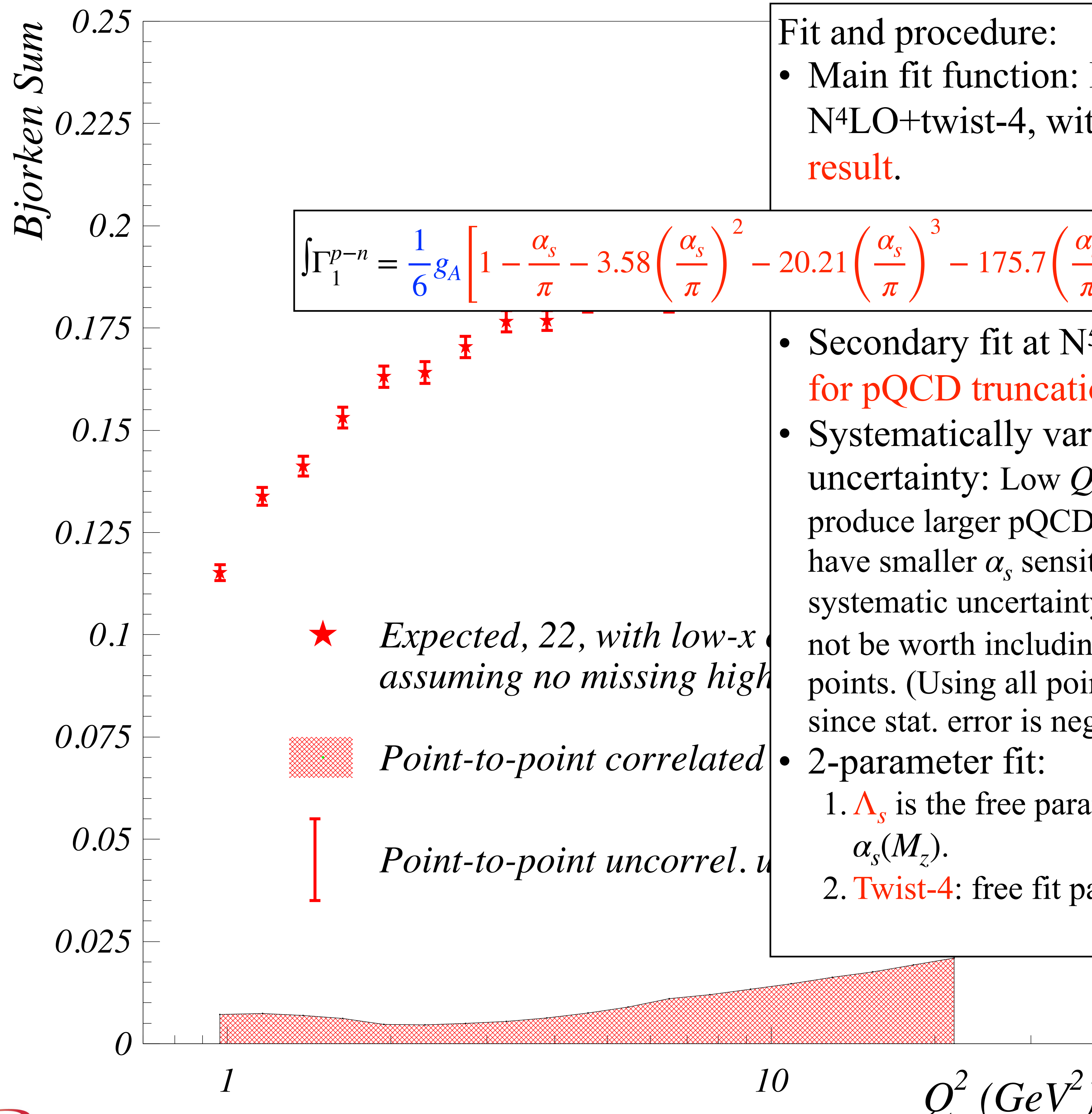
Fit and procedure:

- Main fit function: Bjorken sum approximant at N^{4}LO+twist-4, with α_s at 4-loop (i.e. β_3), for main result.

$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4\right] + \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right]$$

$$\alpha_s^{\overline{\mathrm{MS}}}(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_s^2)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)} + \frac{\beta_1^2}{\beta_0^4 \ln^2(Q^2/\Lambda_s^2)}\left(\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 + \frac{\beta_2\beta_0}{\beta_1^2}\right) + \frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)}\left(-\ln^3(\ln(Q^2/\Lambda_s^2)) + \frac{5}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 2\ln(\ln(Q^2/\Lambda_s^2)) - \frac{1}{2} - 3\frac{\beta_2\beta_0}{\beta_1^2}\ln(\ln(Q^2/\Lambda_s^2)) + \frac{\beta_3\beta_0^2}{2\beta_1^3}\right) + \frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)}\left(\ln^4(\ln(Q^2/\Lambda_s^2)) - \frac{13}{3}\ln^3(\ln(Q^2/\Lambda_s^2)) - \frac{3}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 4\ln(\ln(Q^2/\Lambda_s^2)) + \frac{7}{6} + \frac{7}{6} + \frac{3\beta_2\beta_0}{\beta_1^2}\left(2\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1\right) - \frac{\beta_3\beta_0^2}{\beta_1^3}\left(2\ln(\ln(Q^2/\Lambda_s^2)) + \frac{1}{6}\right)\right]$$

Extraction of $\alpha_s(M_Z)$



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- Secondary fit at N⁵LO+twist-4 and α_s at 4-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2 range to minimize total uncertainty: Low Q^2 points have high α_s sensitivity but produce larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity and larger experimental systematic uncertainty but smaller pQCD error. ⇒May not be worth including the lowest and/or highest Q^2 points. (Using all points for statistics sake is not worth it, since stat. error is negligible.)
- 2-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  2. Twist-4: free fit parameter.

Extraction of $\alpha_s(M_Z)$