

# The EMC effect of light-nuclei within the light-front Hamiltonian dynamics

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Sezione di Perugia



# Outline

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- 🌐 The EMC effect
- 🌐 The Light-Front Poincaré covariant approach
- 🌐 Nuclear Structure Functions (SFs) with Relativistic Hamiltonian Dynamics
- 🌐 Numerical results for the EMC effect
- 🌐 Numerical results for the  ${}^3\text{He}$  Spin dependent SFs
- 🌐 Conclusions

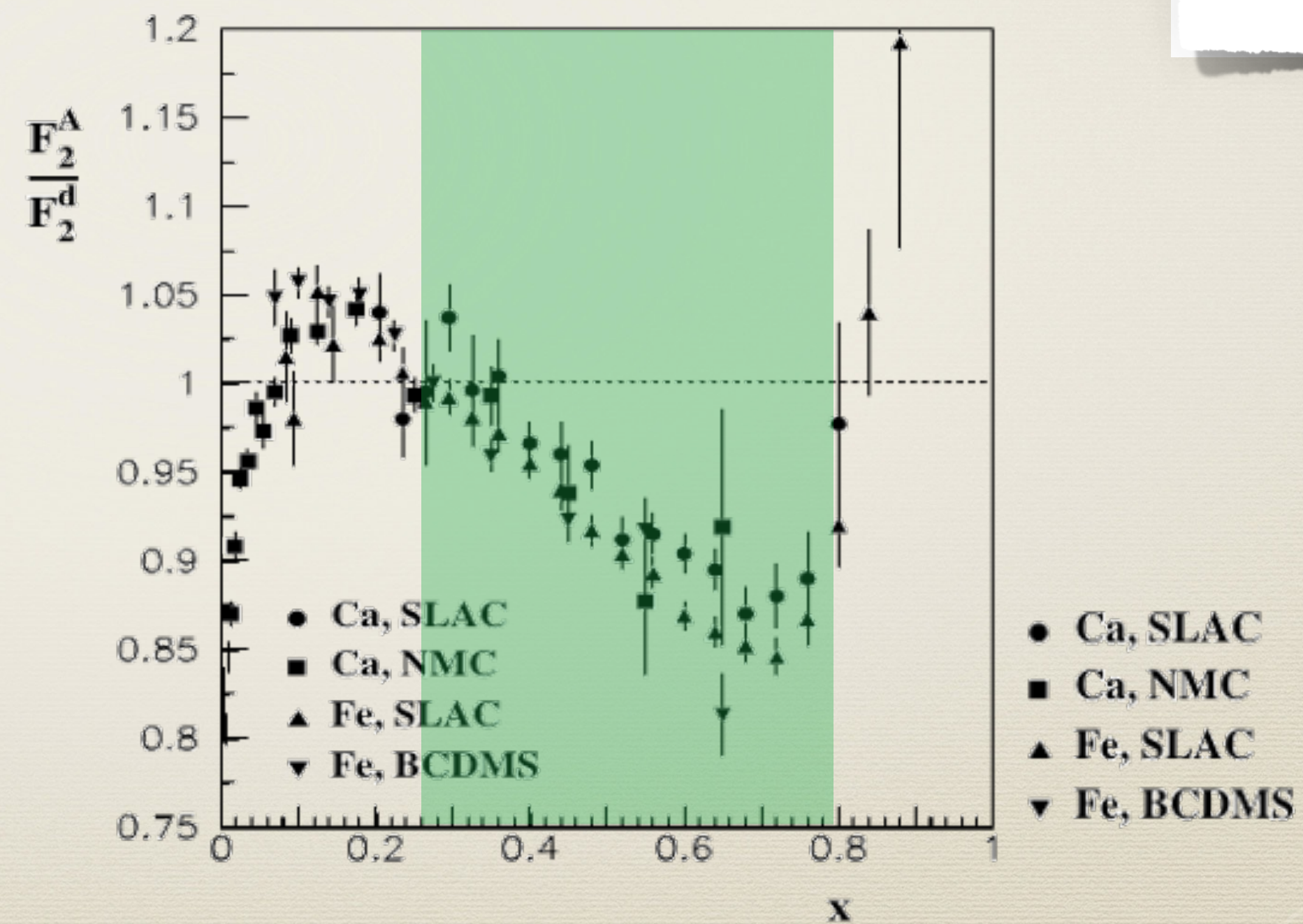
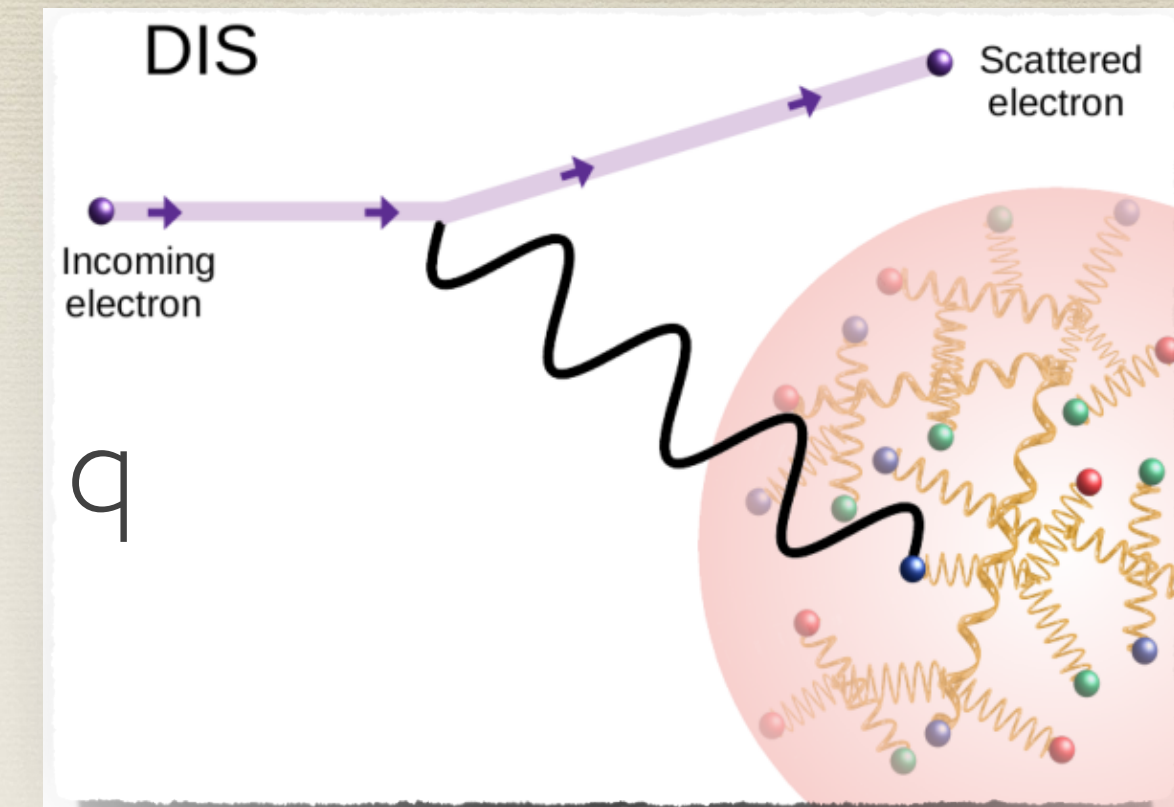
# The EMC effect

In DIS off a nuclear target with A nucleons:

$$0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$$

$0.2 \leq x \leq 0.8$  "EMC (binding) region":  
mainly valence quarks involved

$$\frac{d\sigma}{d\Omega dE'} \propto F_2^A(x)$$



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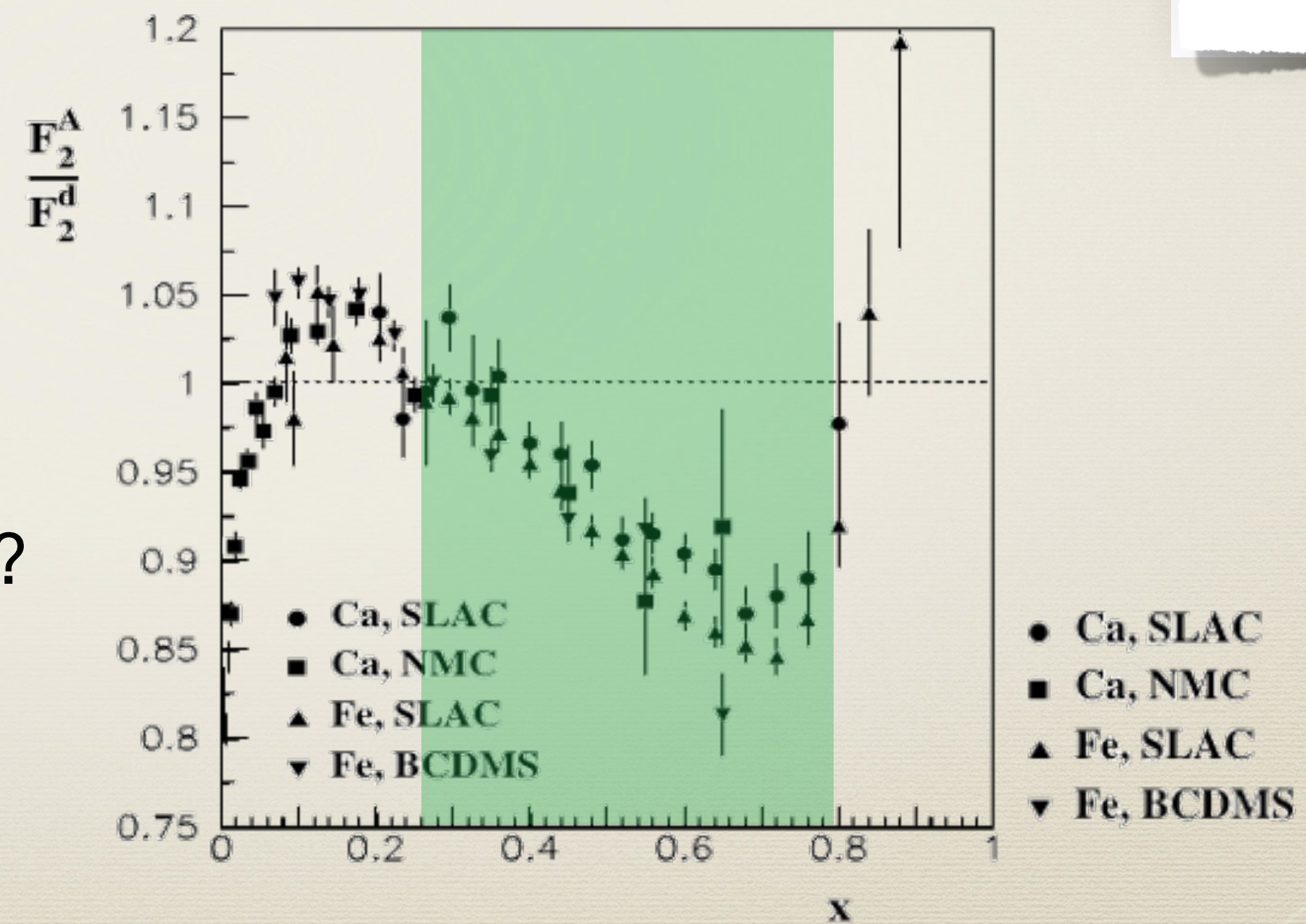
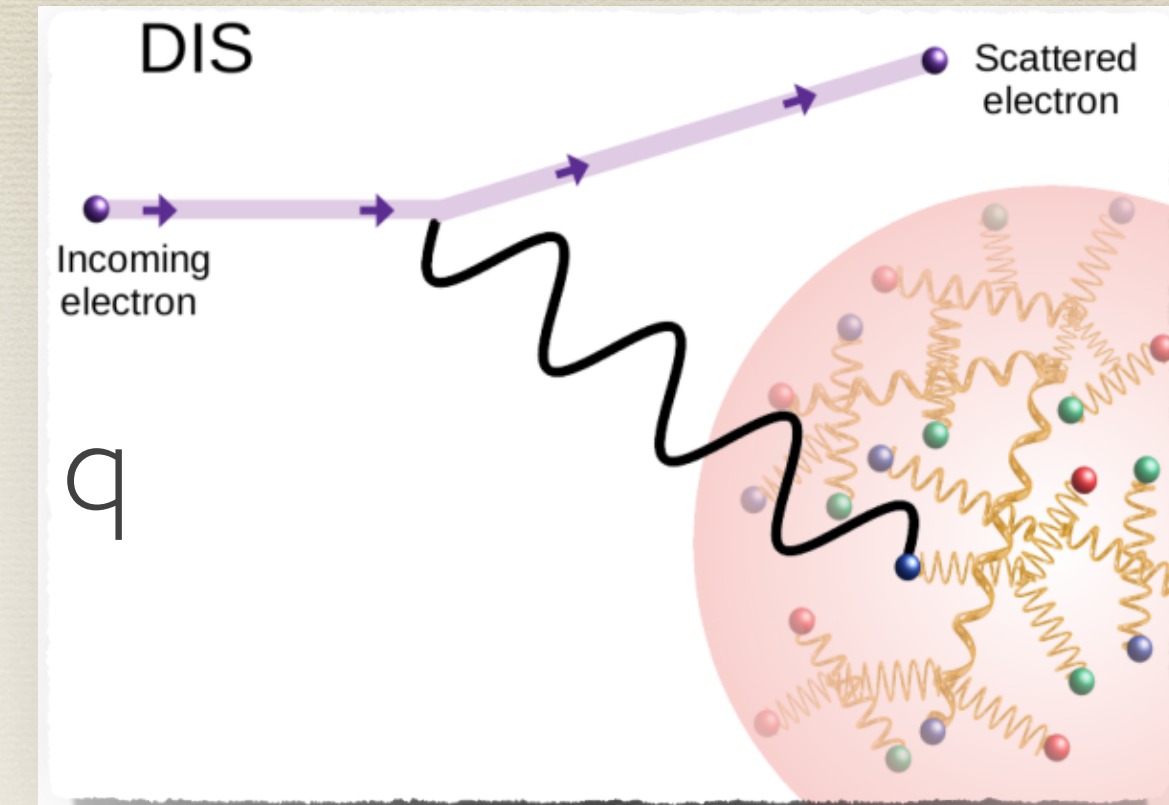
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Naive parton model interpretation:

"Valence quarks, in the bound nucleon, are in average slower than in the free nucleon"

Is the bound proton bigger than the free one??

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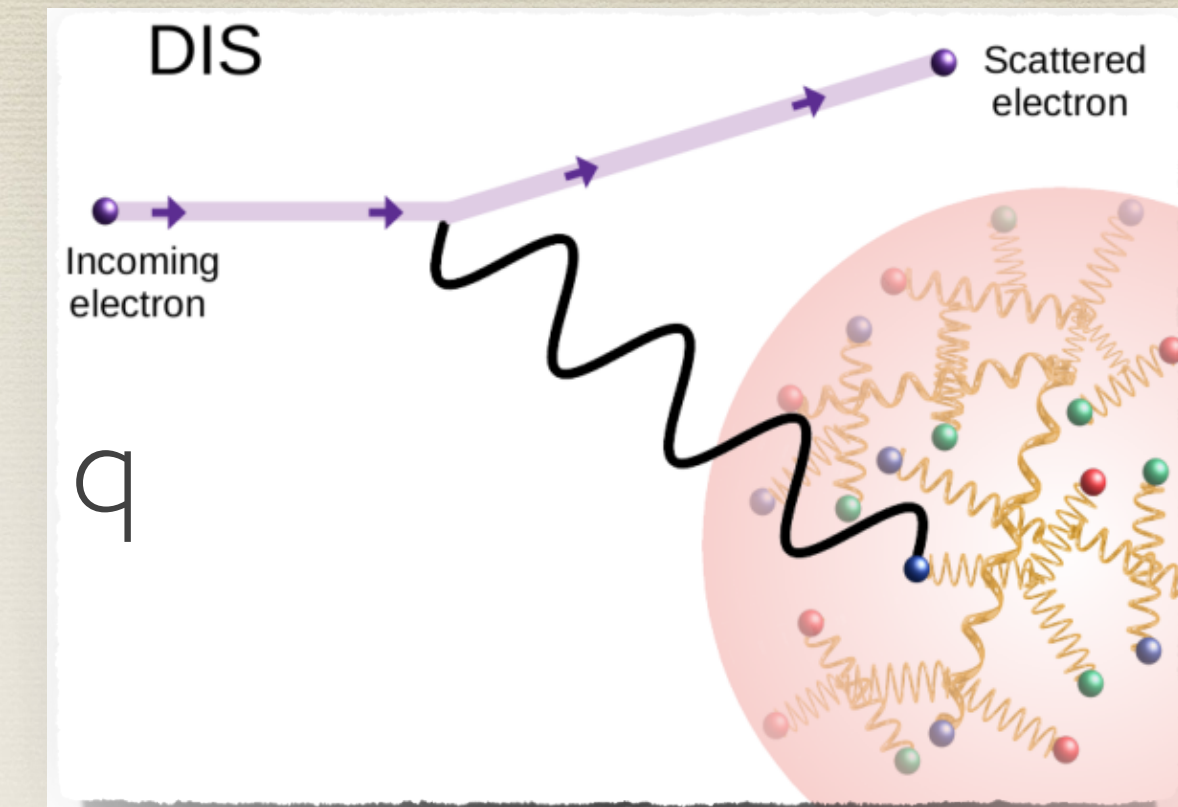
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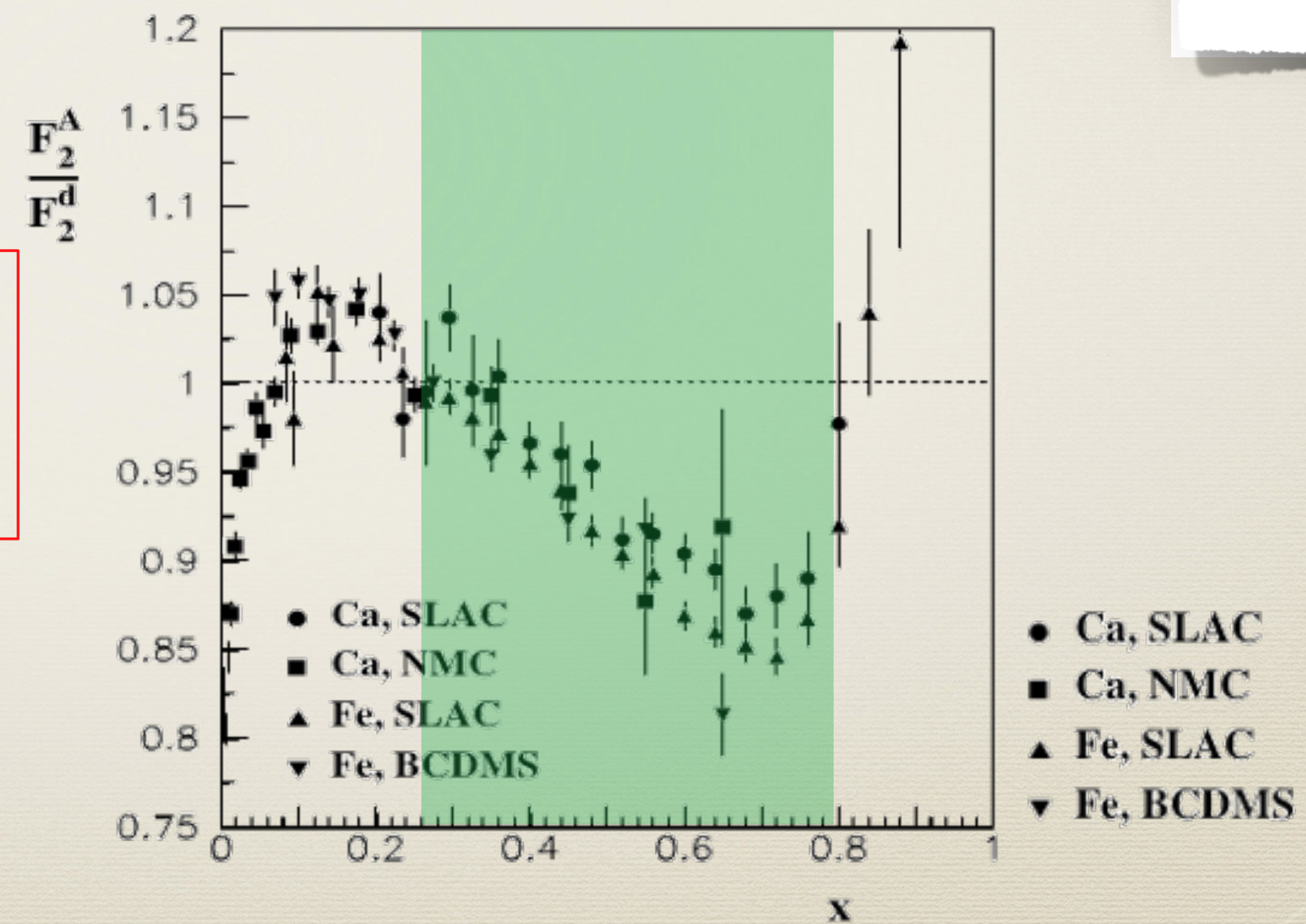
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**main features:** universal behavior independent on  $Q^2$ ;  
weakly dependent on A; Scales with the density  $\rho \rightarrow$   
global property?  
Or due to SRC  $\rightarrow$  local property?



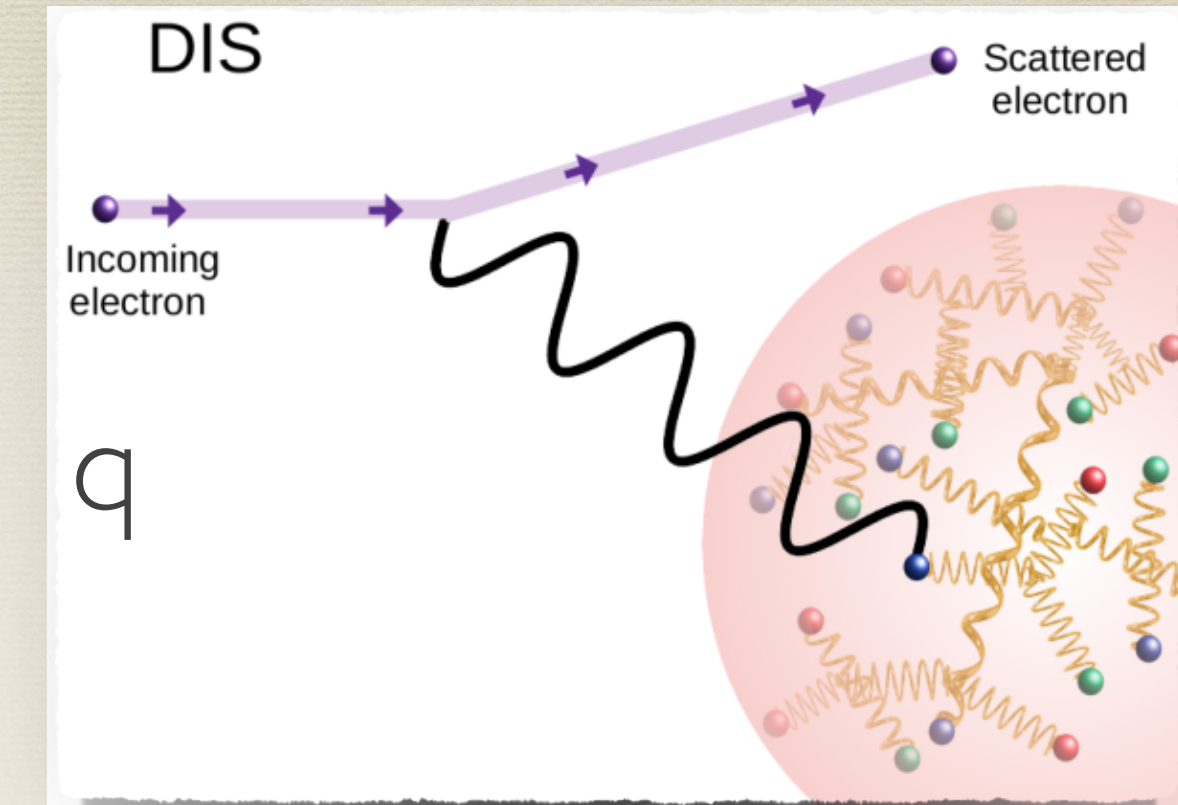
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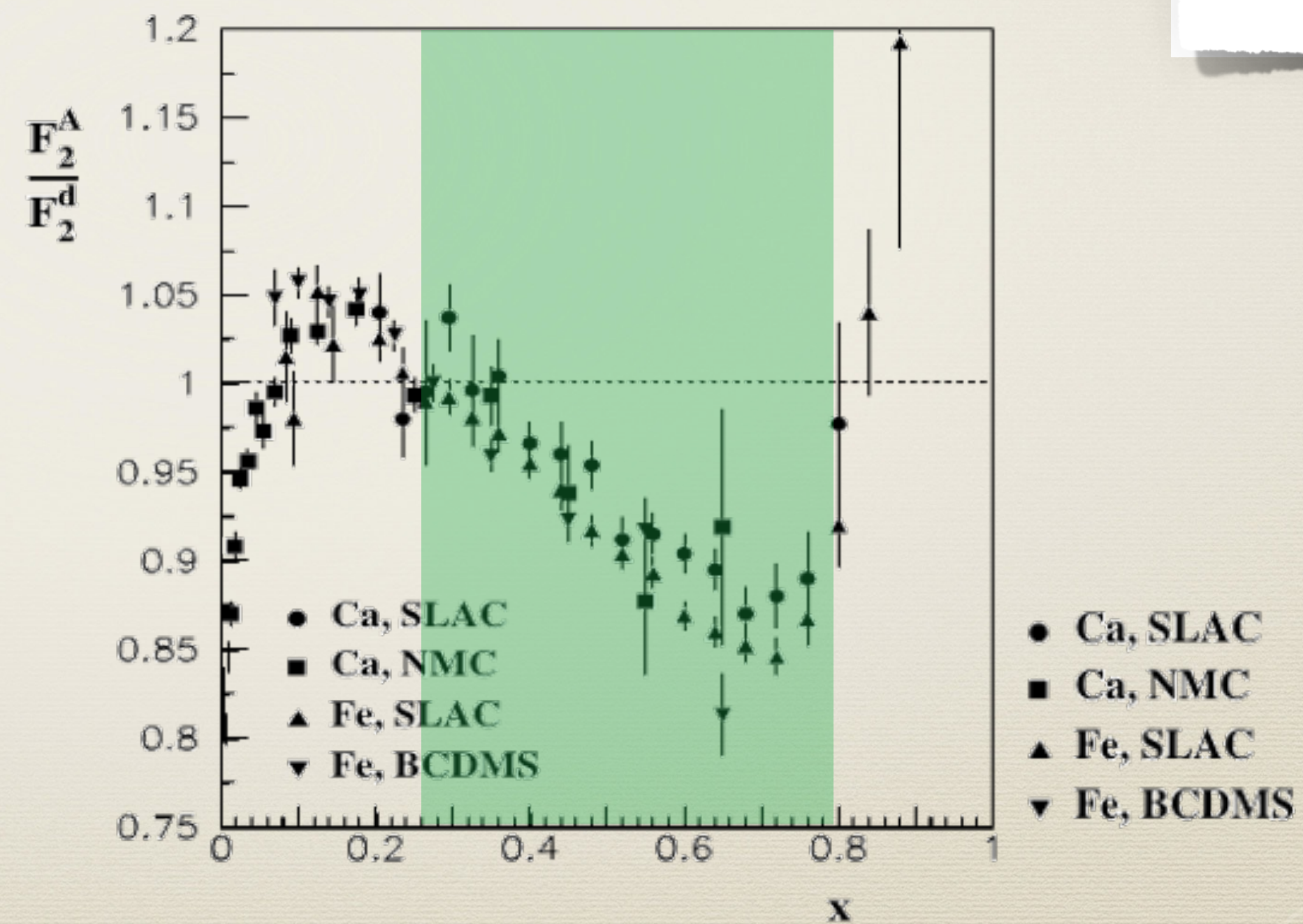
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**Explanation (exotic) advocated:** confinement radius bigger for bound nucleons, quarks in bags with 6, 9, ..., 3A quark, pion cloud effects... Alone or mixed with conventional ones...



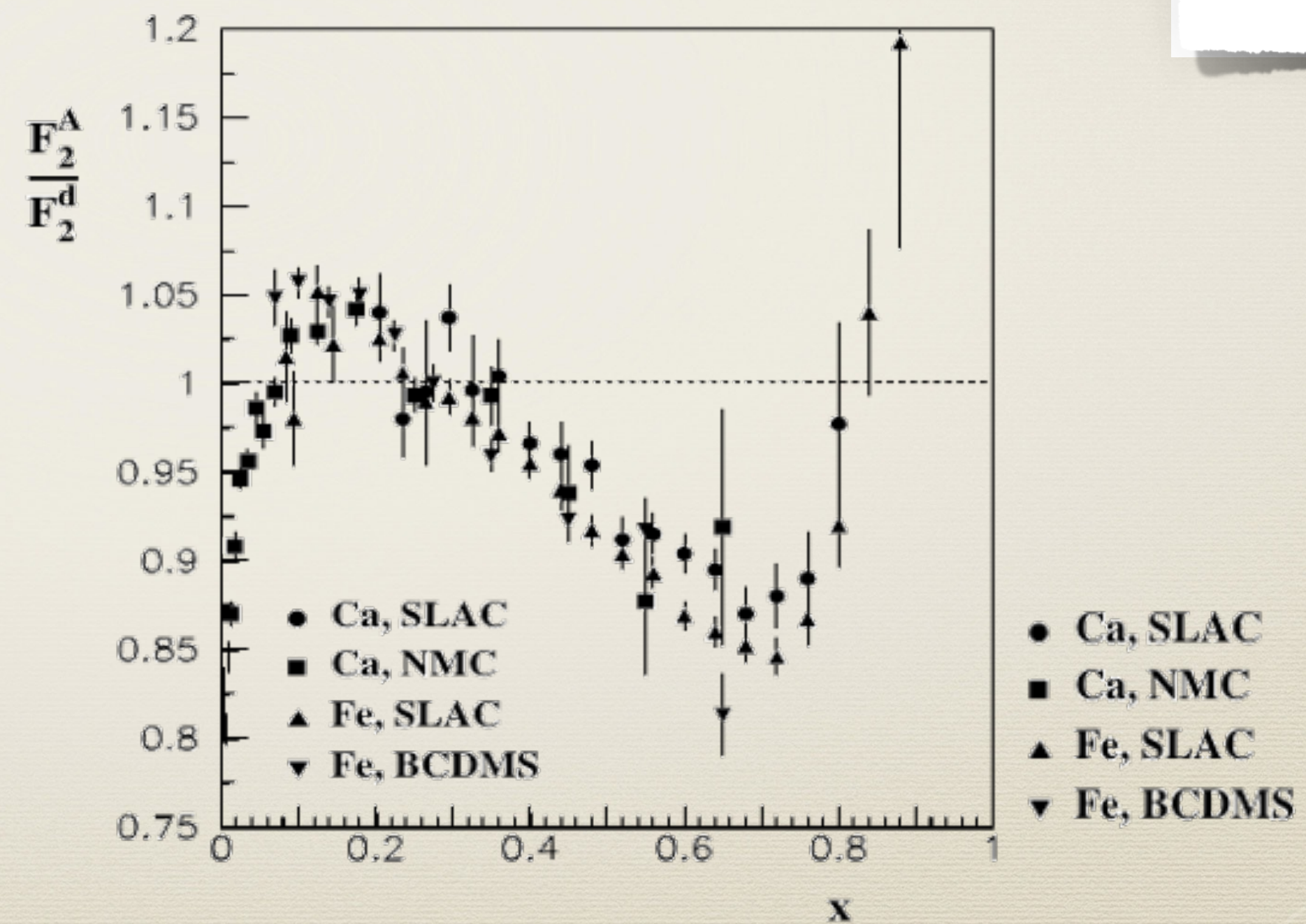
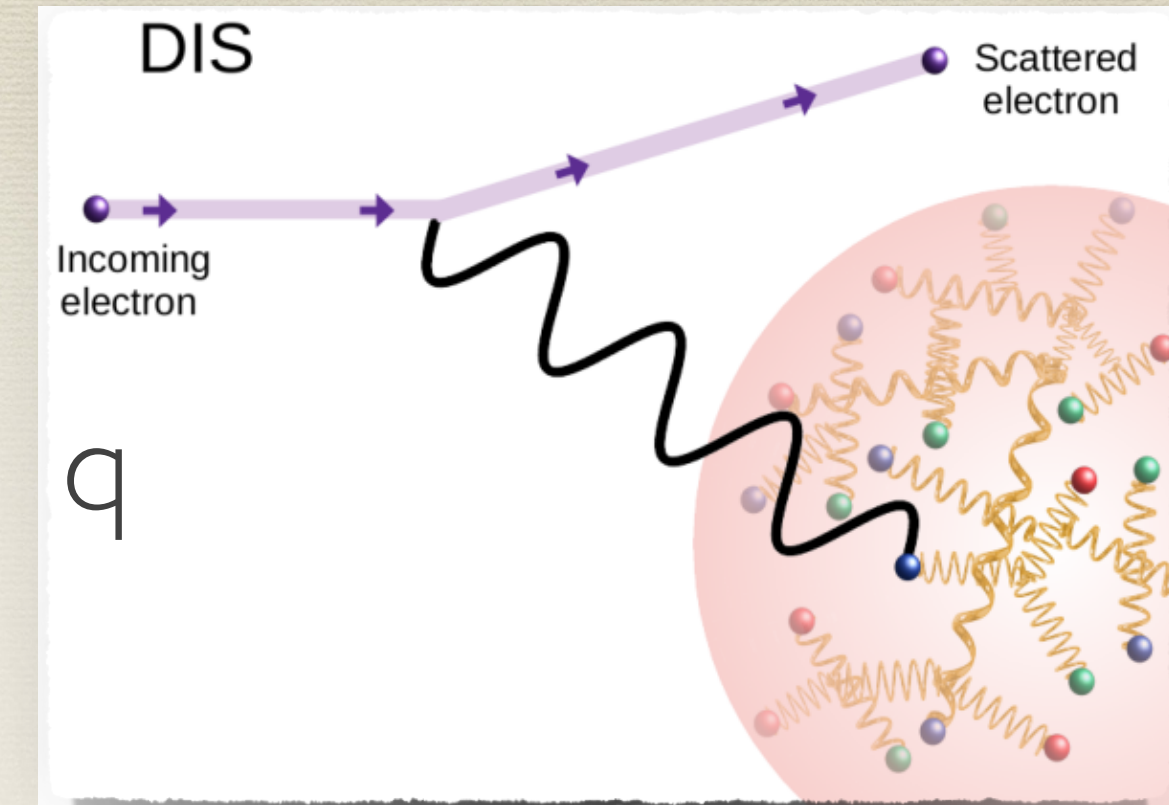
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Conventional calculations:



Qualitative agreement



No fulfillment of both particle  
and momentum sum rules





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In general, the lack of the **Poincarè covariance** and **macroscopic locality\*** generates **biases for the study of genuine QCD effects** (nucleon swelling, exotic quark configurations ...)

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**Macroscopic locality** (= **cluster separability** (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of large space like separation (i.e. causally disconnected).

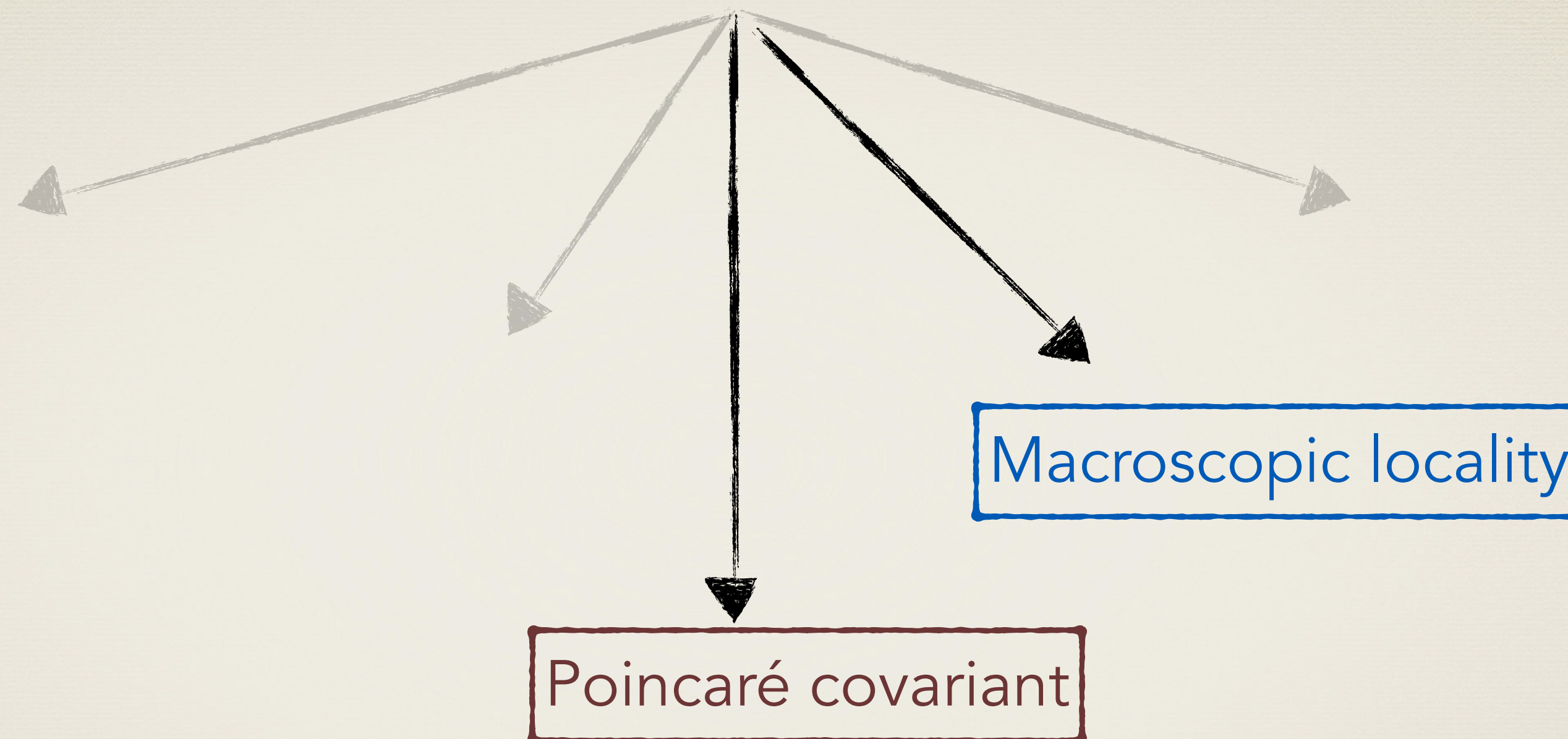
In this way, when a system is separated into disjoint subsystems by a sufficiently large space like separation, then the subsystems behave as independent systems

B.D.Keister and W.N.Polyzou, Adv.Nucl.Phys. 20 (1991), 225-479

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Our approach (Light-Front + Bakamjian-Thomas)



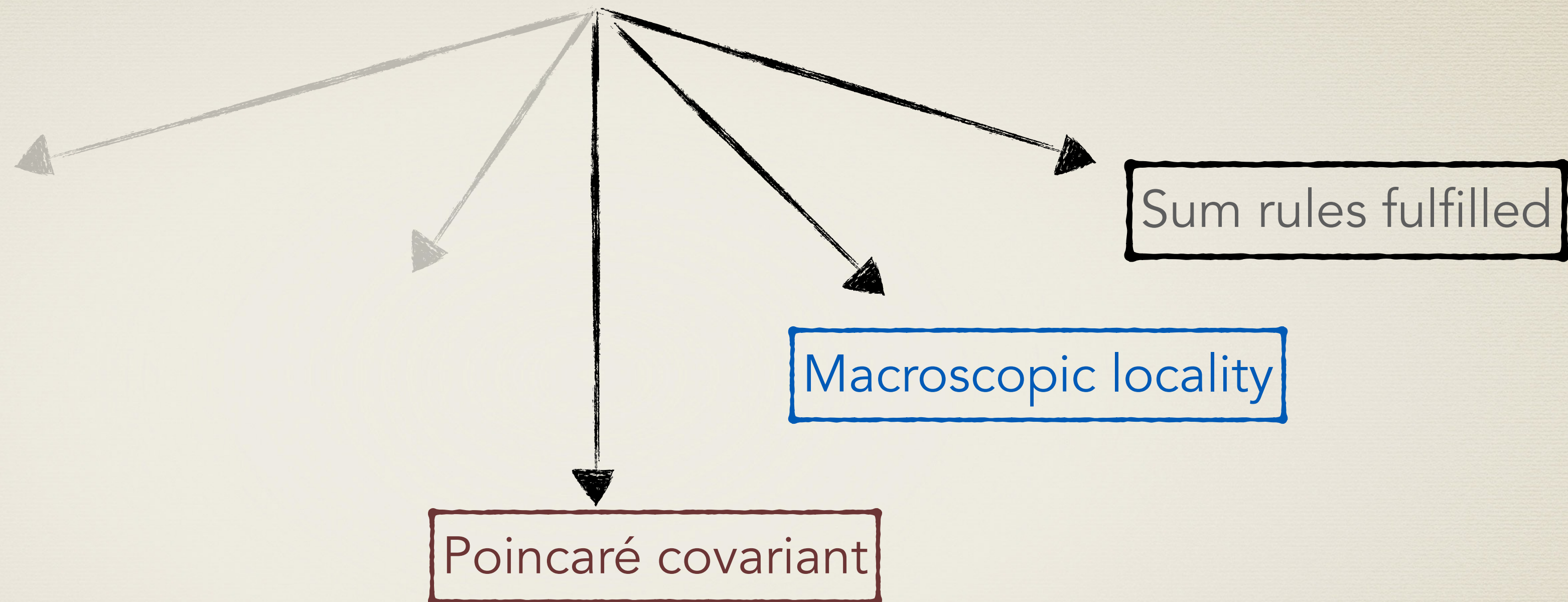
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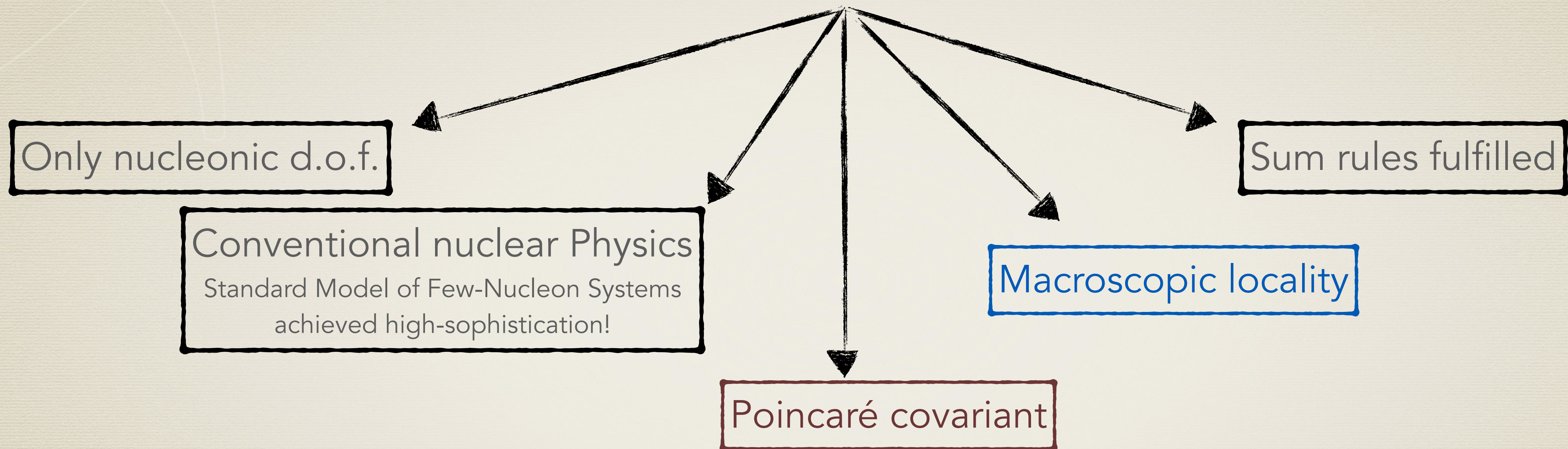
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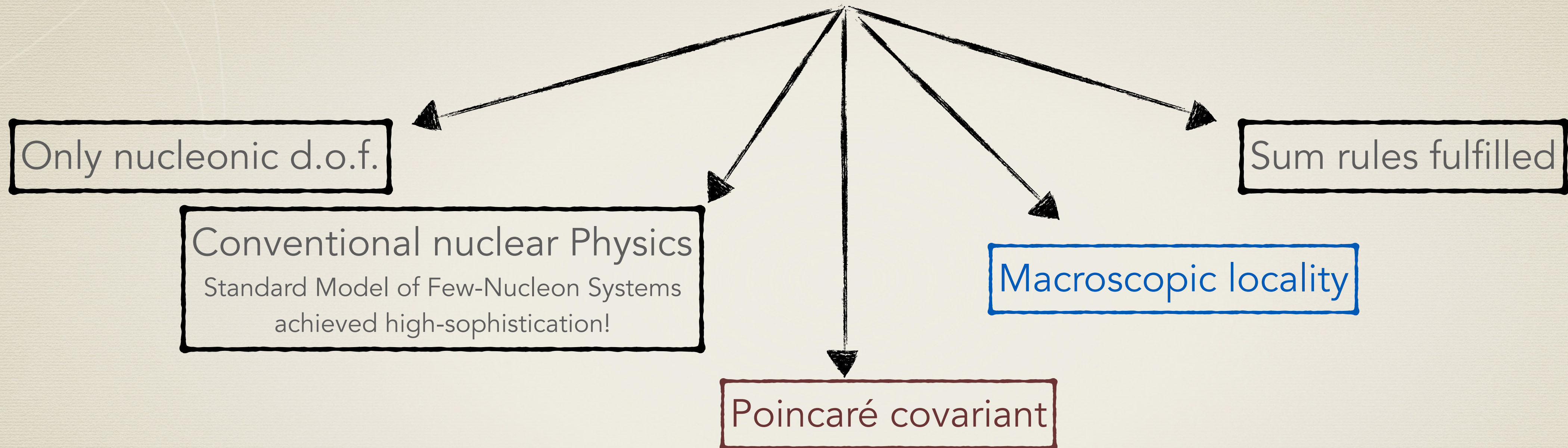
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- We provide a **reliable baseline for the calculation of the nuclear SFs** where only the well known nuclear part is considered
- This relativistic treatment is needed for the kinematics of the **JLab12, JLab22 and EIC**

# LF approach in pills

**Poincaré covariance** → Find 10 generators:

$P^\mu \rightarrow 4D$  displacements and  $M^{\nu\mu} \rightarrow$  Lorentz transformation, that fulfill:

$$[P^\mu, P^\nu] = 0; [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu)$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$



- **7 Kinematical generators** (max  $n^\circ$ ): i) 3 LF boosts (in instant form they are dynamical!); ii)  $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$ ; iii) Rotation around the z-axis
- The LF boosts have a subgroup structure: **trivial separation of intrinsic and global motion, as in the NR case**
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Such a goal can be achieved in different equivalent ways depending on the initial conditions



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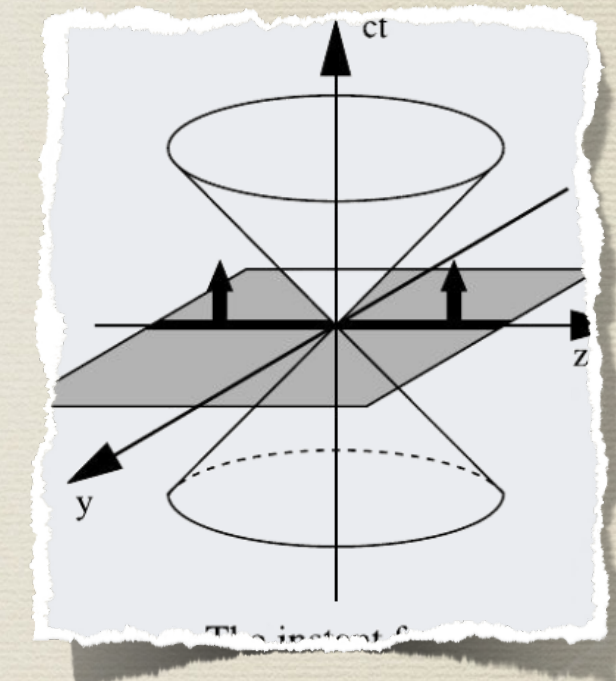
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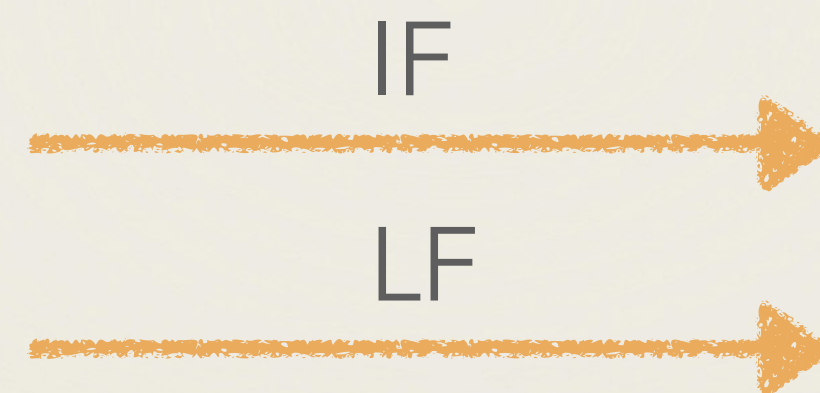
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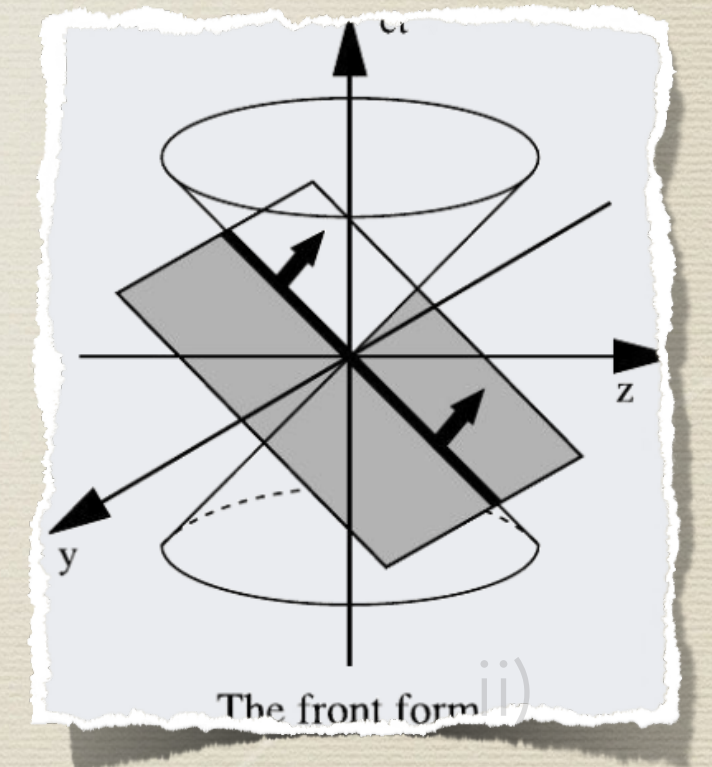
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$$t = x^0 = 0$$

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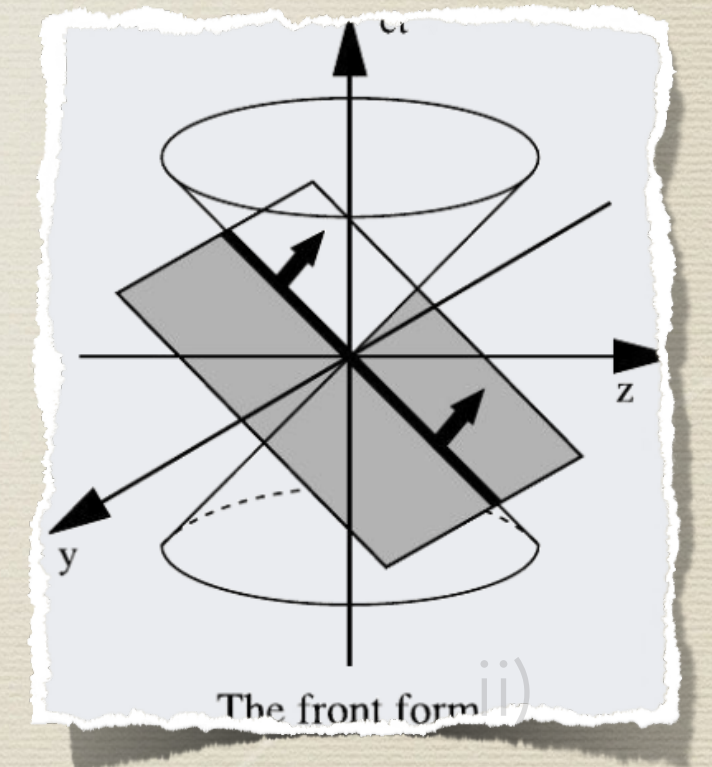
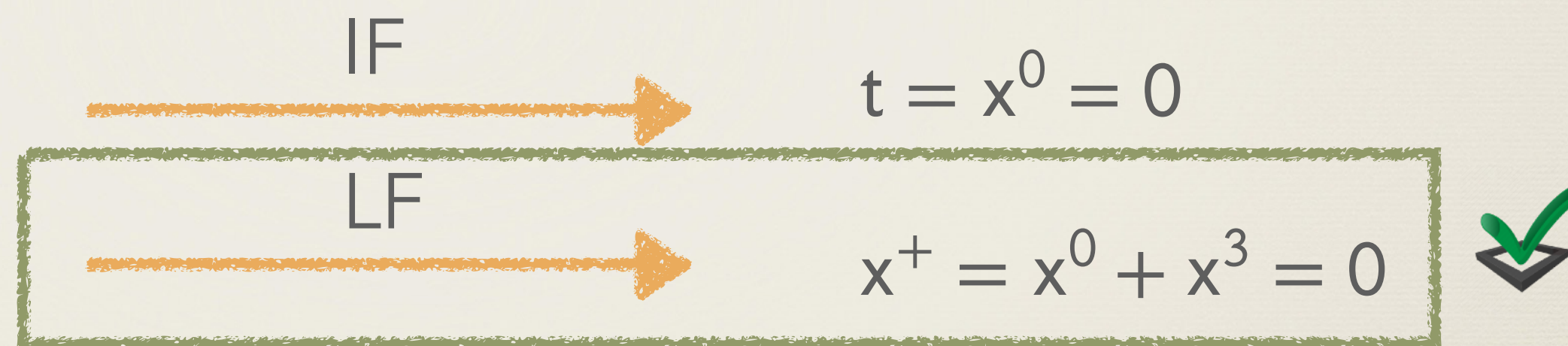
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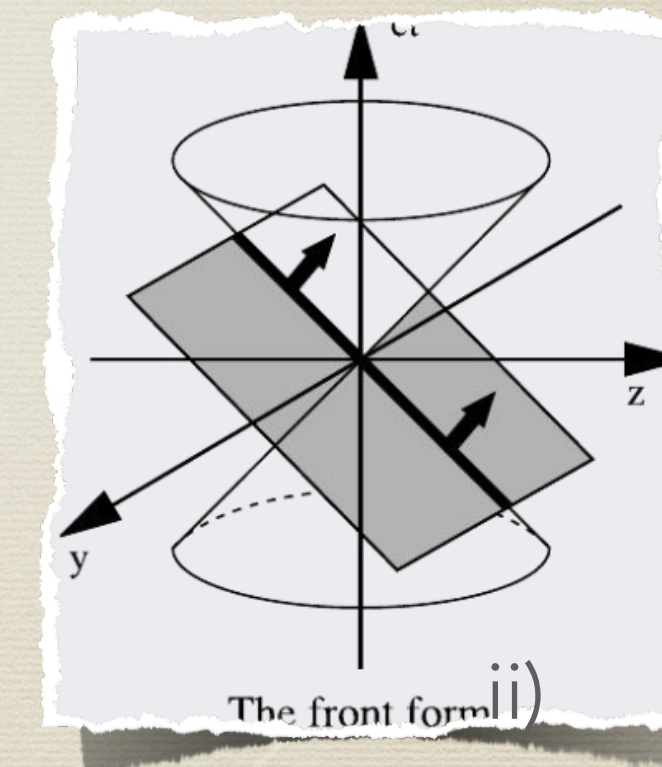
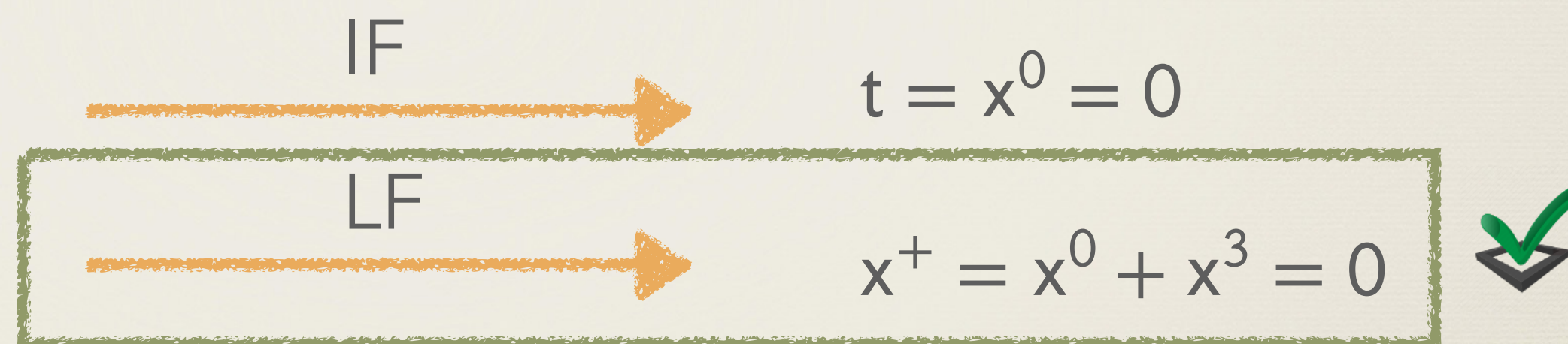
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# LF + Bakamjian-Thomas construction

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**BT properly constructed the 10 Poincaré operators in presence of interactions** following this scheme:

- i) Only the mass operator  $M$  contains the interaction
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Free mass

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Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

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# Our approach: Reference frames

In order to implement **macro-locality**, it is crucial to distinguish between different frames:

- The Lab frame, where  $\tilde{P} = (M_{BT}, \mathbf{0}_\perp)$
- The intrinsic LF frame of the whole system,  $[1,2,\dots,A]$ , where  $\tilde{P} = (M_0[1,2,\dots,A], \mathbf{0}_\perp)$  with

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# Our approach: Reference frames

In order to implement **macro-locality**, it is crucial to distinguish between different frames:

- The Lab frame, where  $\tilde{P} = (M_{BT}, \mathbf{0}_\perp)$
- The intrinsic LF frame of the whole system,  $[1,2,\dots,A]$ , where  $\tilde{P} = (M_0[1,2,\dots,A], \mathbf{0}_\perp)$  with

$$k_i^+ = \xi_i M_0[1,2,\dots,A] \text{ and } M_0[1,2,\dots,A] = \sum_{i=1}^A \sqrt{m^2 + \mathbf{k}_i^2}$$

- The intrinsic LF frame of the cluster  $[1; 2,3,\dots,(A-1)]$  where  $\tilde{P} = (\mathcal{M}_0[1; 2,3,\dots,A-1], \mathbf{0}_\perp)$  with

$$k^+ = \xi \mathcal{M}_0[1; 2,3,\dots,A-1] \text{ and } \mathcal{M}_0[1; 2,3,\dots,A-1] = \sqrt{m^2 + \kappa^2} + \sqrt{M_s^2 + \kappa^2}$$

$M_s = (A-1)m + \epsilon$  is the mass of the fully interacting spectator system

While  $\mathbf{p}_\perp^{LAB} = \mathbf{k}_{1\perp} = \kappa_\perp$

# Our approach: LF spectral function I

Since we use an **impulse approximation** assumption, we rely on the **spin-dependent LF spectral function**

$$P_{\sigma'\sigma}^{\tau}(\tilde{\kappa}, \epsilon, \mathbf{S}, M)$$

$$P_{\sigma'\sigma}^N(\tilde{\kappa}, \epsilon, \mathbf{S}, M) = \sum_{JJ_z} \sum_{TT_z} \rho(\epsilon)_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle \langle \Psi_{JM}; \mathbf{S}, T_A T_{Az} |_{LF} tT; \alpha, \epsilon; JJ_z; \tau\sigma, \tilde{\kappa} \rangle_{LF}$$



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$|tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\mathbf{k}} \rangle_{LF}$  is the **tensor product** of the plane wave of the struck nucleon and the state of the fully interacting spectator system  $[2, \dots, A - 1]$  in the intrinsic reference frame of the cluster  $[1; 2, 3, \dots, A - 1]$  when the spectator system has energy  $\epsilon$ . It fulfills the macrolocality\*

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The LF spectral function contains the determinant of the Jacobian of the transformation between the intrinsic frames  $[1; 2, 3, \dots, A - 1]$  and  $[1, 2, \dots, A]$ , connected each other by a LF boost

# Our approach: LF spectral function II

$$P_{\sigma'\sigma}^N(\tilde{k}, \epsilon, \mathbf{S}, M) = \sum_{JJ_z} \sum_{TT_z} \rho(\epsilon)_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{k} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle \langle \Psi_{JM}; \mathbf{S}, T_A T_{Az} |_{LF} tT; \alpha, \epsilon; JJ_z; \tau\sigma, \tilde{k} \rangle_{LF}$$

How do we deal with **LF states**?

# Our approach: LF spectral function II

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How do we deal with **LF states**?

1) We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations**:

$$\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\mathbf{k}} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{LF} \rightarrow \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{IF}$$

# Our approach: LF spectral function II

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2) Then we can approximate the **IF overlap** into a NR overlap by using the **NR wave function for the nucleus**, thanks to the BT construction:

$$\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{IF} \sim \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{NR}$$

# Our approach: LF spectral function II

$$P_{\sigma'\sigma}^N(\tilde{\mathbf{k}}, \epsilon, \mathbf{S}, M) = \sum_{JJ_z} \sum_{TT_z} \rho(\epsilon)_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\mathbf{k}} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle \langle \Psi_{JM}; \mathbf{S}, T_A T_{Az} |_{LF} tT; \alpha, \epsilon; JJ_z; \tau\sigma, \tilde{\mathbf{k}} \rangle_{LF}$$

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**Poincarè covariance preserved but using the successful NR phenomenology**

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How do we deal with

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$\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa}$

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thanks to the BT cons

$\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa}$

Poincarè covariance



We used wave functions of  ${}^2H, {}^3H, {}^3He, {}^4He$  calculated through 3 different potentials: **Av18+UIX\*** and 2 versions of the **Norfolk  $\chi EFT$  interactions NVIa+3N\*\*** and **NVib+3N\*\***

\*R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, **Phys. Rev. C 51 (1995) 38–51**;  
R. B. Wiringa et al., **Phys. Rev. Lett. 74 (1995) 4396–4399**

\*\*M. Viviani et al., **Phys. Rev. C 107 (1) (2023) 014314**; M. Piarulli et al., **Phys. Rev. Lett. 120 (5) (2018) 052503**; M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim, **Phys. Rev. C 107 (1) (2023) 014314**

the nucleus,



# Nuclear SFs and EMC ratio

To calculate the EMC ratio  $R_{EMC}^A(x) = \frac{F_2^A(x)}{F_2^d(x)}$  for any nucleus A, we need the nuclear SFs.

Within our approach we have:

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi F_2^N \left( \frac{mx}{\xi M_A} \right) f_A^N(\xi)$$

\* $\xi$  = longitudinal momentum fraction carried by a nucleon in the nucleus

Since our approach **fulfill both macro-locality and Poincaré covariance** the LC momentum distribution satisfies 2 essential sum rules at the same time ():

$$A = \int_0^1 d\xi [Z f_1^p(\xi) + (A - Z) f_1^n(\xi)]: \text{Baryon number SR};$$

$$1 = Z \langle \xi \rangle_p + (Z - N) \langle \xi \rangle_n; \langle \xi \rangle_N = \int_0^1 d\xi \xi f_1^N(\xi): \text{Momentum SR (MSR)}$$

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Unpolarized LF spectral function:  
 $P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P_{\sigma\sigma}^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$

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2) **The free nucleon SFs** E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, *Phys. Scr.* **95**, 064008 (2020):

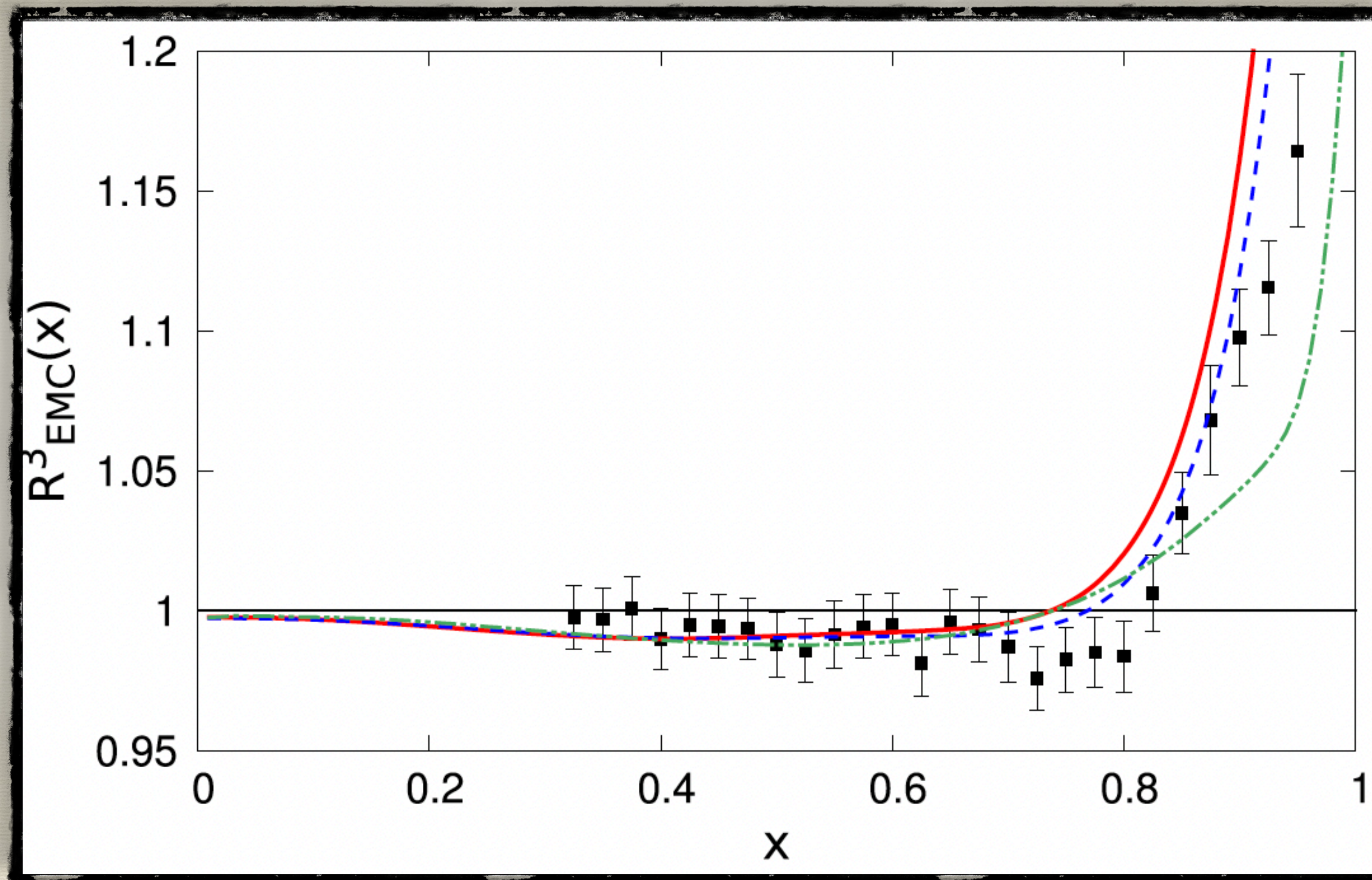
a) we choose a **parametrization** for  $F_2^p(x)$

b) we use the **MARATHON data** (MARATHON Coll., *Phys. Rev. Lett* **128** (2022) 13,132003)

for the parametrization of the ratio  $\frac{F_2^n}{F_2^p}$  to get  $F_2^n$

# The EMC effect for $^3\text{He}$

E.Pace, M.R. G.Salmè and S.Scopetta, Phys. Lett. B 839(2023) 127810



[1] J. Arrington, et al, Phys. Rev. C 104 (6) (2021) 065203

[2] S. A. Kulagin and R. Petti, Phys. Rev. C 82, 054614 (2010)

**Solid line: Av18/UIX + SMC\***  
**Dashed line: Av18 + SMC\***  
**Dotted-dashed: Av18/UIX + CJ15\*\***

**Full squares: JLab data from experiment E03103 [1] as reanalyzed in [2]**

\*[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]

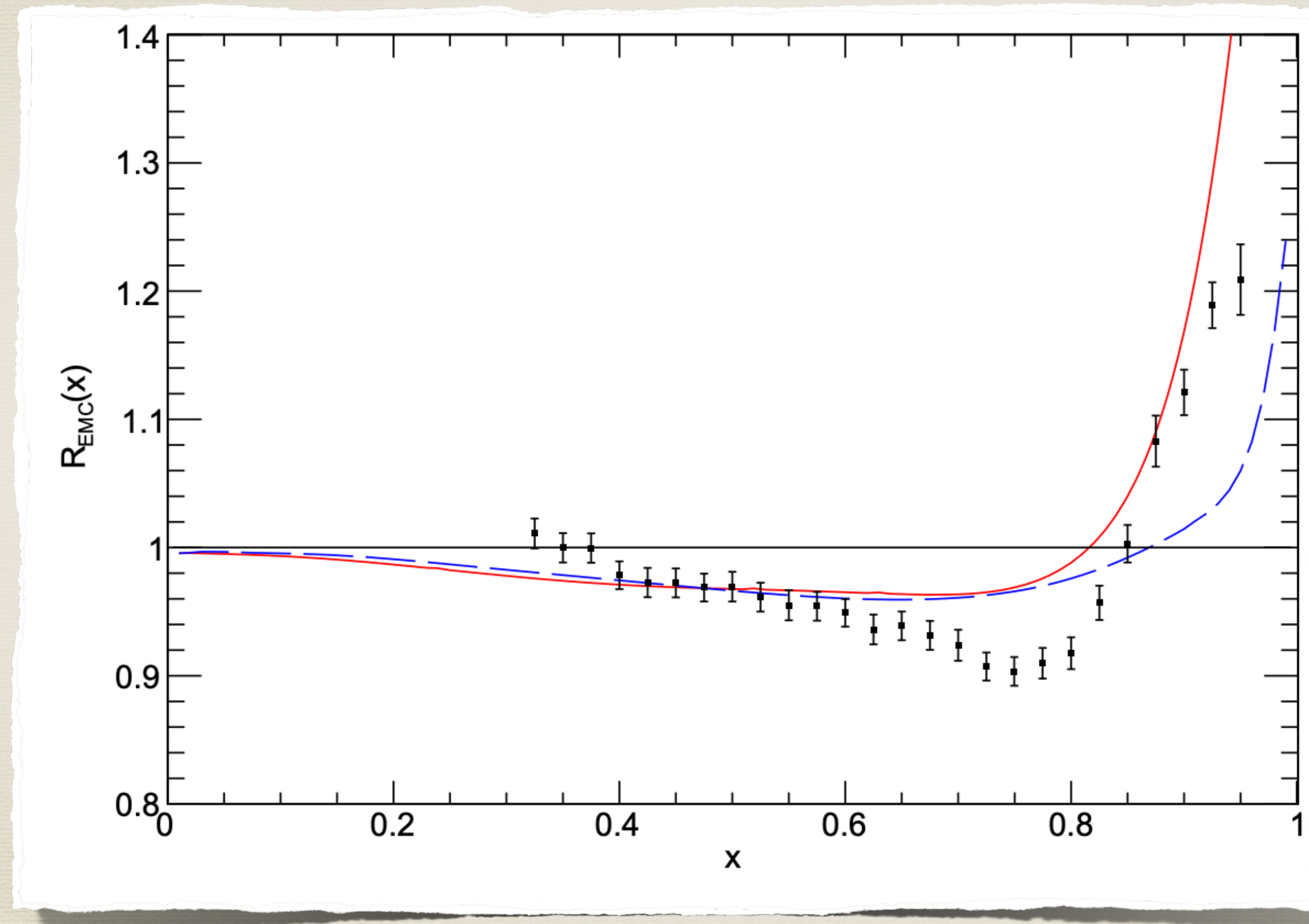
\*\*[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]

**Small but solid effect, comparable to the experimental data**

# The EMC effect for $^4\text{He}$

F.Fornetti, E.Pace, M.R., G.Salmè, S.Scopetta and M.Viviani, *Phys.Lett.B* 851 (2024) 138587

Full squares: JLab data  
from experiment  
E03103



Both lines calculated with  
Av18/UIX  
Solid line: SMC parametrization  
of  $F_2^p$  \*  
Dashed line: CJ15 +TMC  
Parametrization of  $F_2^{p**}$   
 $F_2^n$  extracted from MARATHON  
data

\*[B. Adeva, et al., *Phys. Lett. B* 412  
(1997) 414–424.]

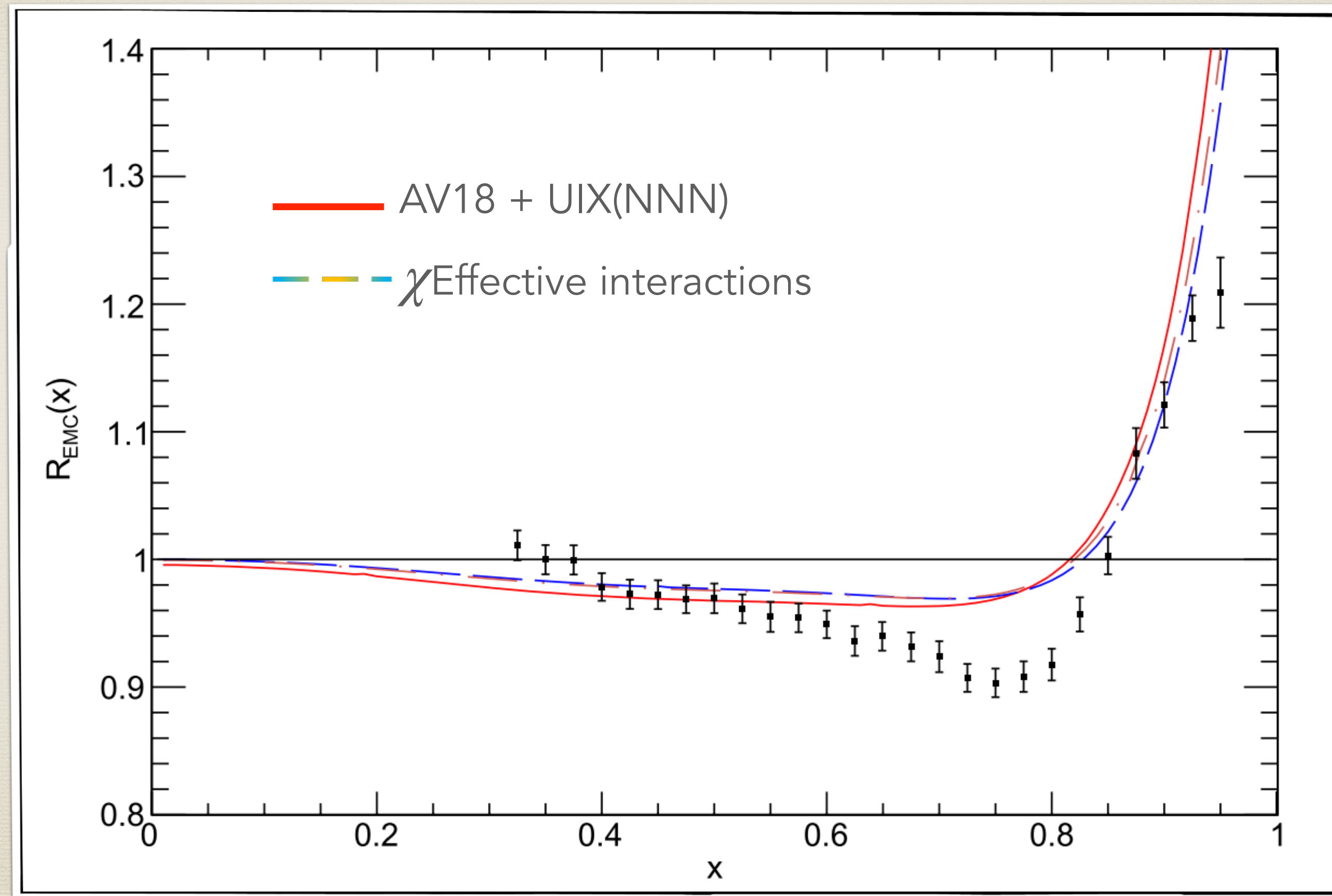
\*\*[A. Accardi, L. T. Brady, W.  
Melnitchouk, J. F. Owens, N. Sato, *Phys.  
Rev. D* 93 (11) (2016) 114017]

The dependence on the choice of the **free nucleon SFs** is largely under control in the **properly EMC region**

# The EMC effect for $^4\text{He}$

F.Fornetti, E.Pace, M.R., G.Salmè, S.Scopetta and M.Viviani, *Phys.Lett.B* 851 (2024) 138587

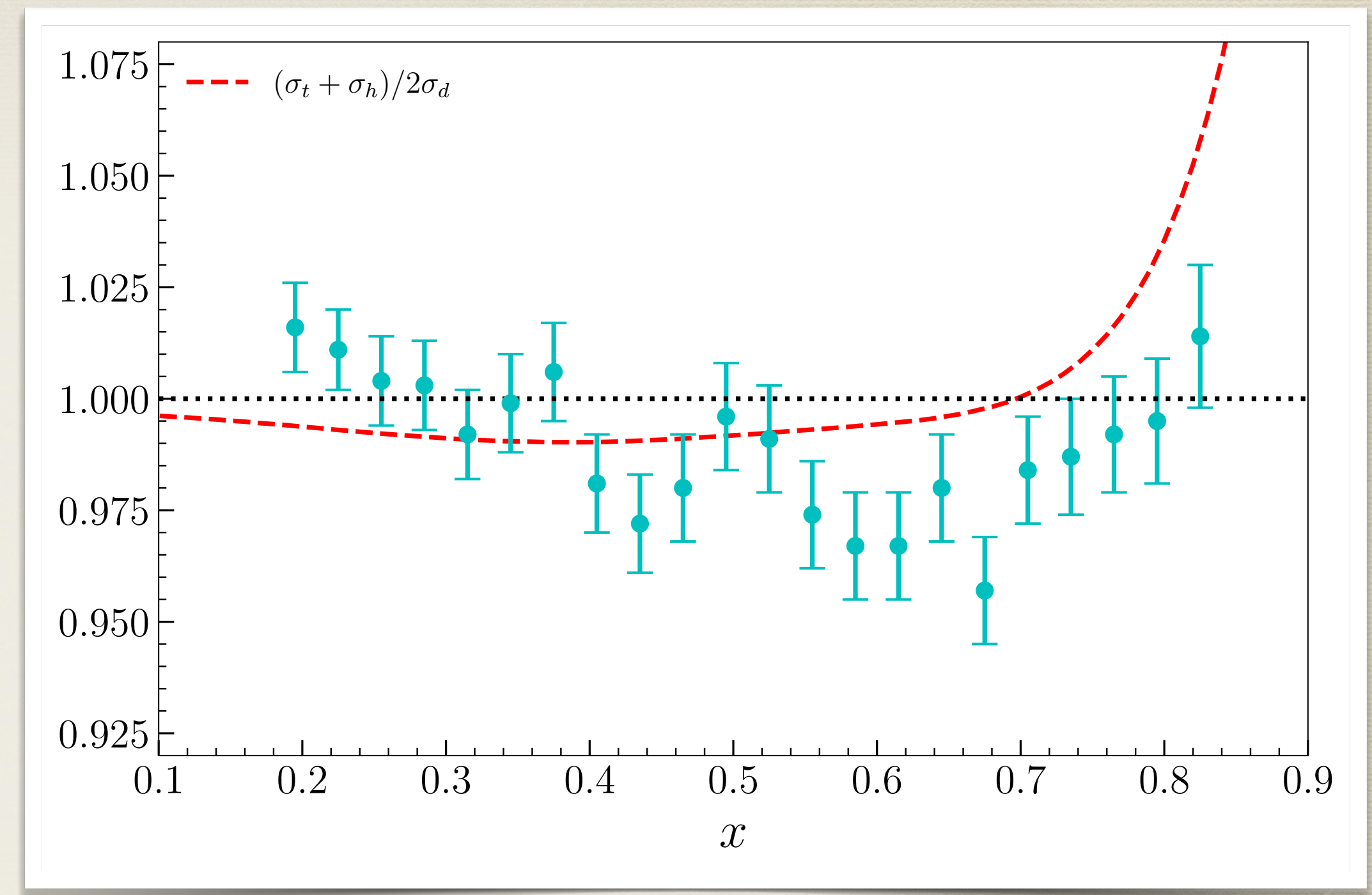
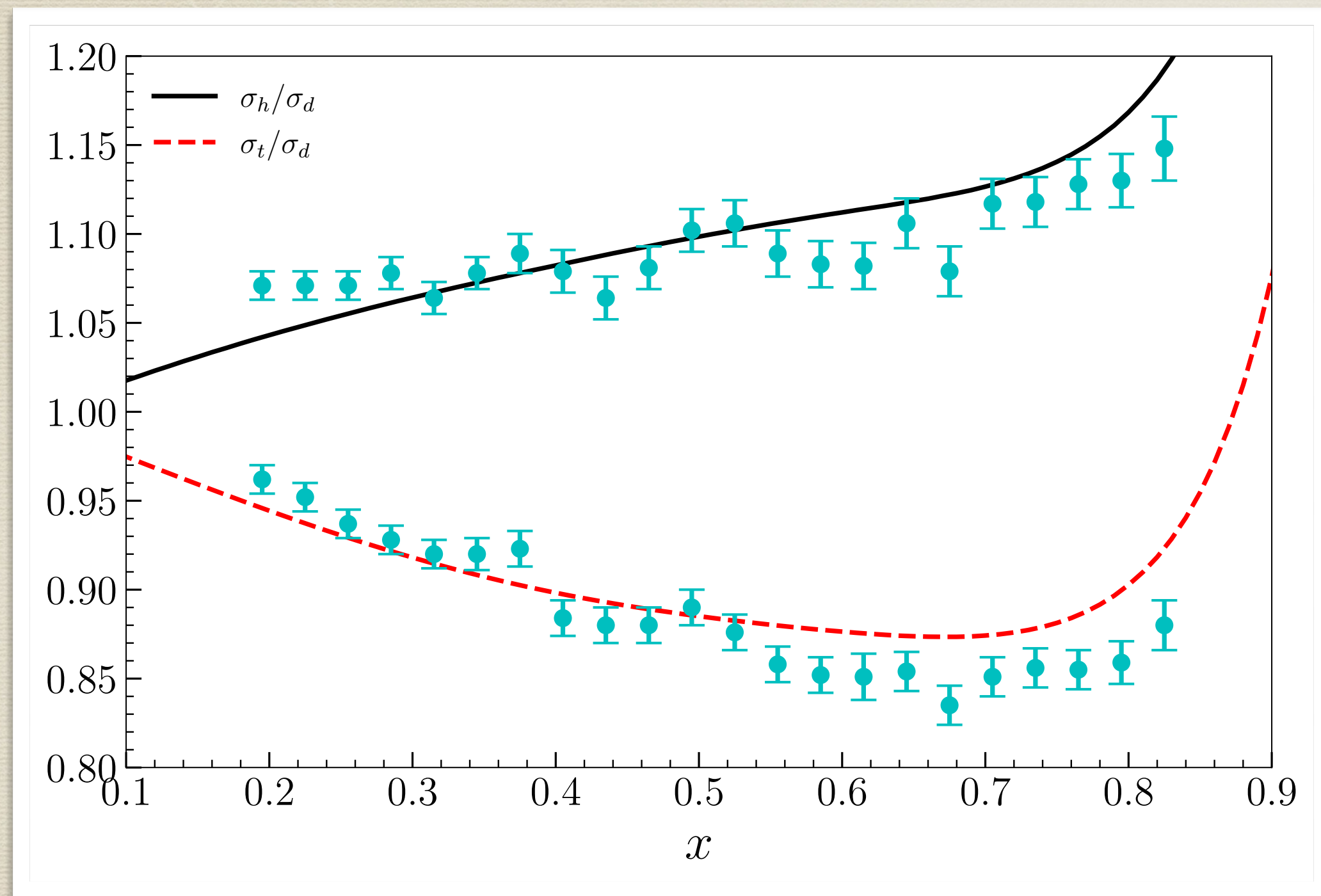
Full squares: JLab data  
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E03103



Both lines calculated with  
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Solid line: SMC parametrization  
of  $F_2^p$  \*  
 $F_2^n$  extracted from MARATHON  
data

\*[B. Adeva, et al., *Phys. Lett. B* 412  
(1997) 414–424.]

# Recent (ongoing) calculations



Data from:

D. Abrams, H. Albataineh, B.~S. Aljawrneh,...,et al,

"The EMC Effect of Tritium and Helium-3 from the JLab MARATHON Experiment,"

[arXiv:2410.12099 [nucl-ex]].

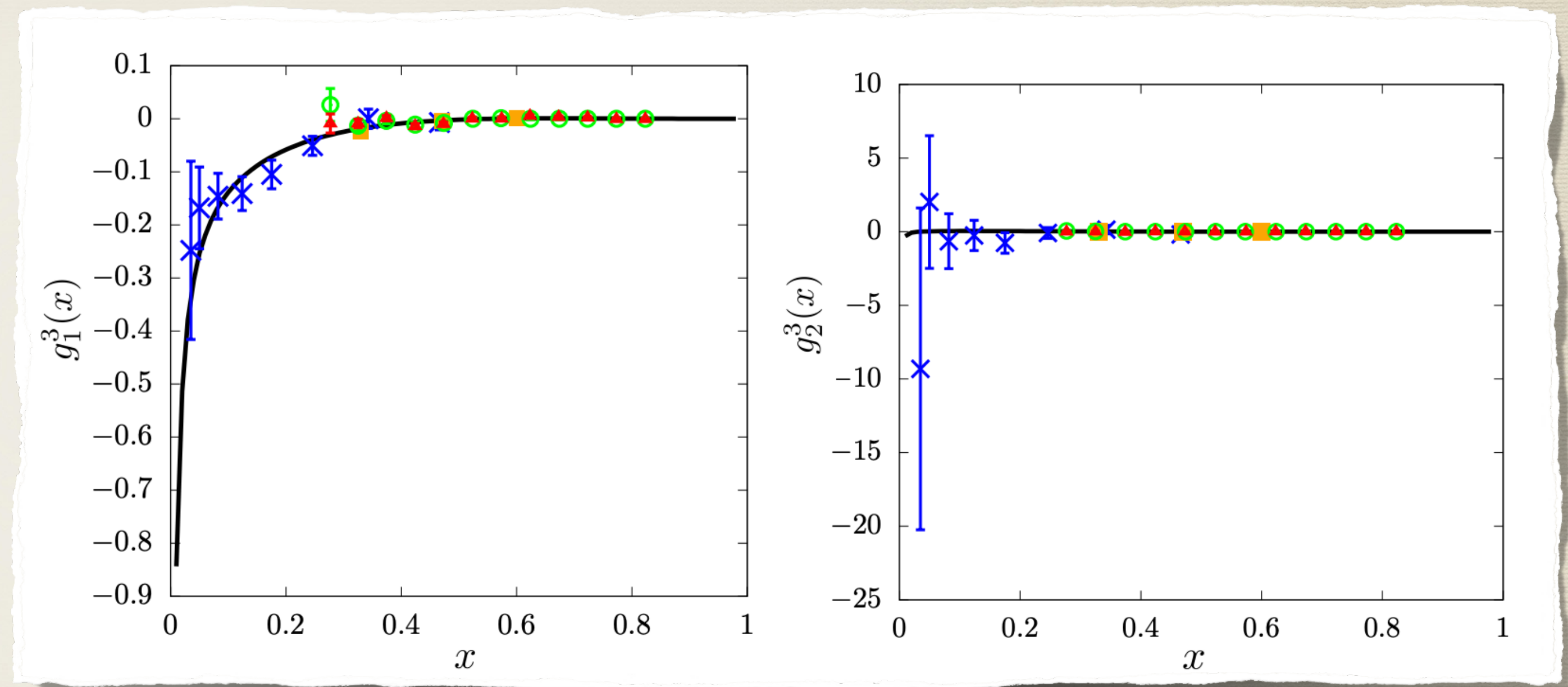


# Some recent works...

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

Calculation of the spin dependent  $^3\text{He}$  structure functions within the Light-Front covariant approach:

Also in this case there are no free parameters and the  $^3\text{He}$  w.f. corresponding to the Av18 potential has been used



# Some recent works...

F. Fornetti, E. Proietti..., M. R. et al, PRC 110 (2024), 3, L031303

Extraction of the neutron spin dependent structure from  $^3\text{He}$  data:

$$\bar{g}_j^n(x) = \frac{1}{p_j^n} [g_j^3(x) - 2p_j^p g_j^p(x)] \quad (j = 1, 2)$$

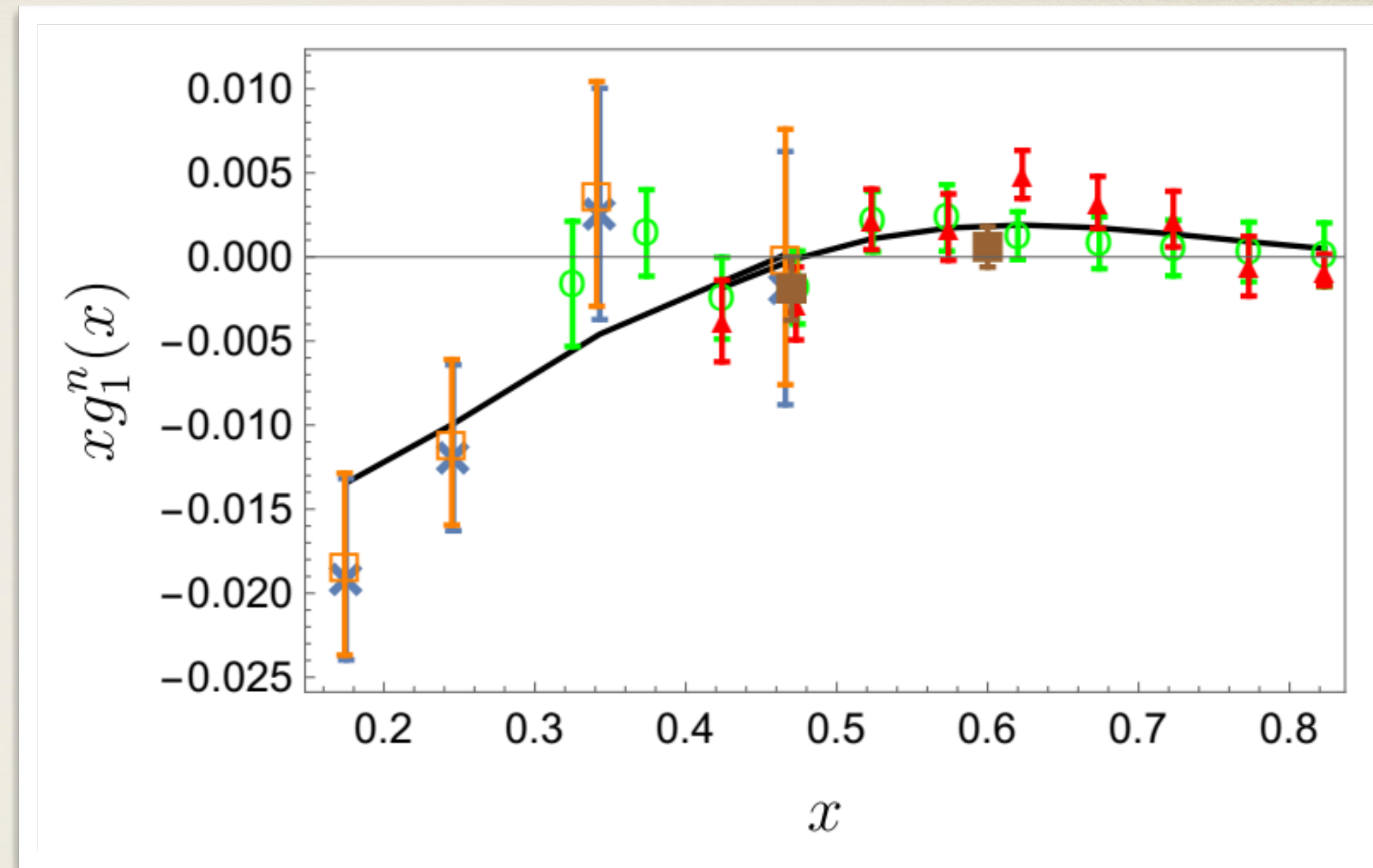
with the effective polarizations obtained from the  $^3\text{He}$  w.f.

$$p_1^n \simeq 0.873$$

$$p_2^n \simeq 0.873$$

$$p_1^p \simeq -0.0230$$

$$p_2^p \simeq -0.0245$$



**Points:** extracted from  $^3\text{He}$  data using our formula

**Line:** M. Gluck, et al, Phys. Rev. D 63, 094005 (2001).

The  $^3\text{He}$  spin structure is makes this nucleus unique to extract the neutron distributions!

# Conclusions

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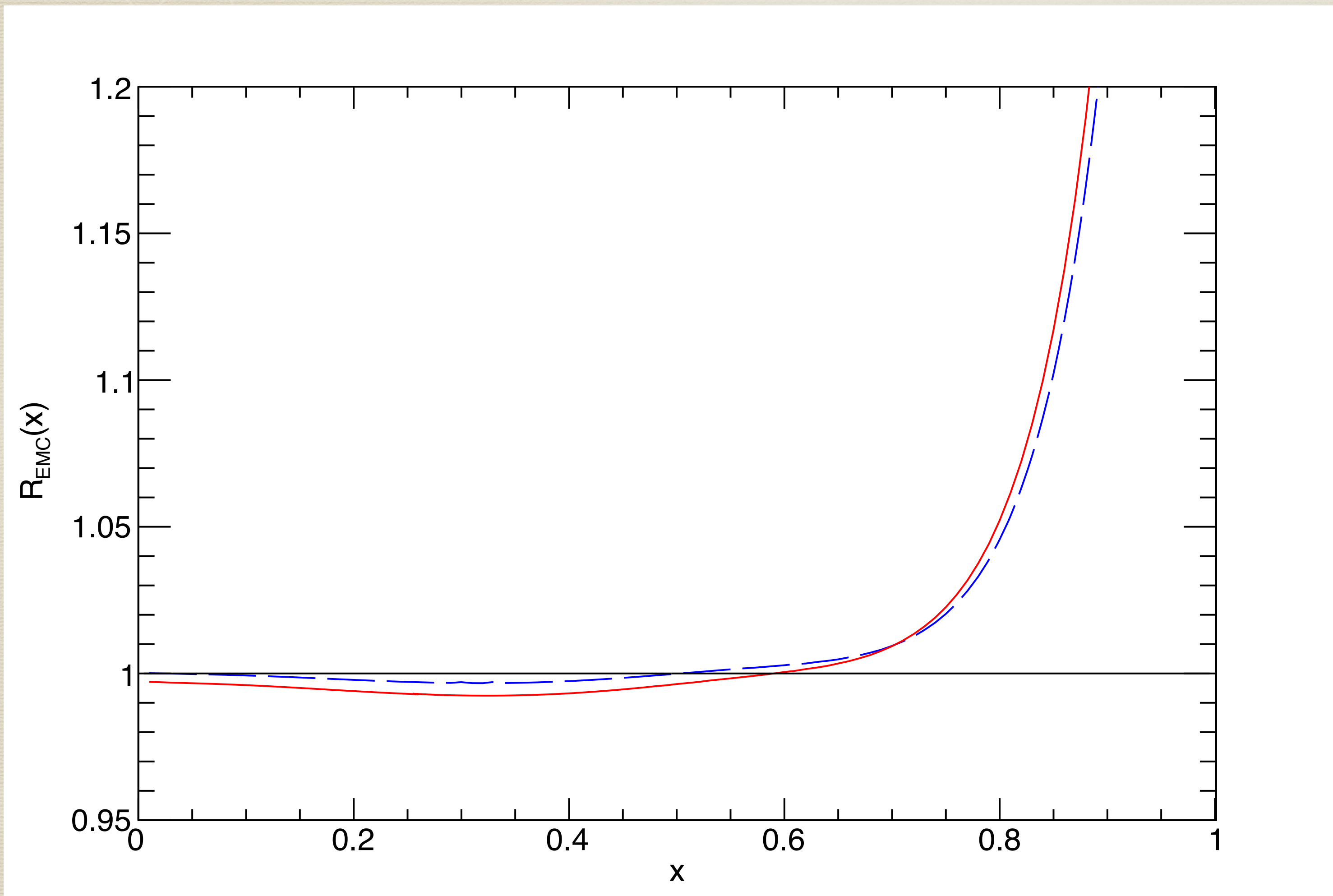
## ■ EMC of light-nuclei within a Poincaré covariant LF approach

- ☑ We developed a rigorous formalism for the calculation of nuclear SFs (also for TMDs) involving only nucleonic DOF with the conventional nuclear physics
- ☑ For  ${}^3\text{He}$  we obtain results in agreement with experimental data for the EMC effect. Useful analysis for planned experiments in future facilities
- ☑ For the deviations from experimental data could be ascribed to genuine QCD effects: **our results provide a reliable baseline to study exotic phenomena**

## ■ To do next

- ☐ Inclusion of the fully Poincaré relativistic approach to Generalized Parton Distributions
- ☐ Include off-shell effects
- ☐ Application of the approach to heavier nuclei ( ${}^6\text{Li}$  starting project)
- ☐ Studying Double Parton Distributions of light-nuclei (in preparation)

# Backup slides



Results similar to  ${}^3He$  and  ${}^4He$

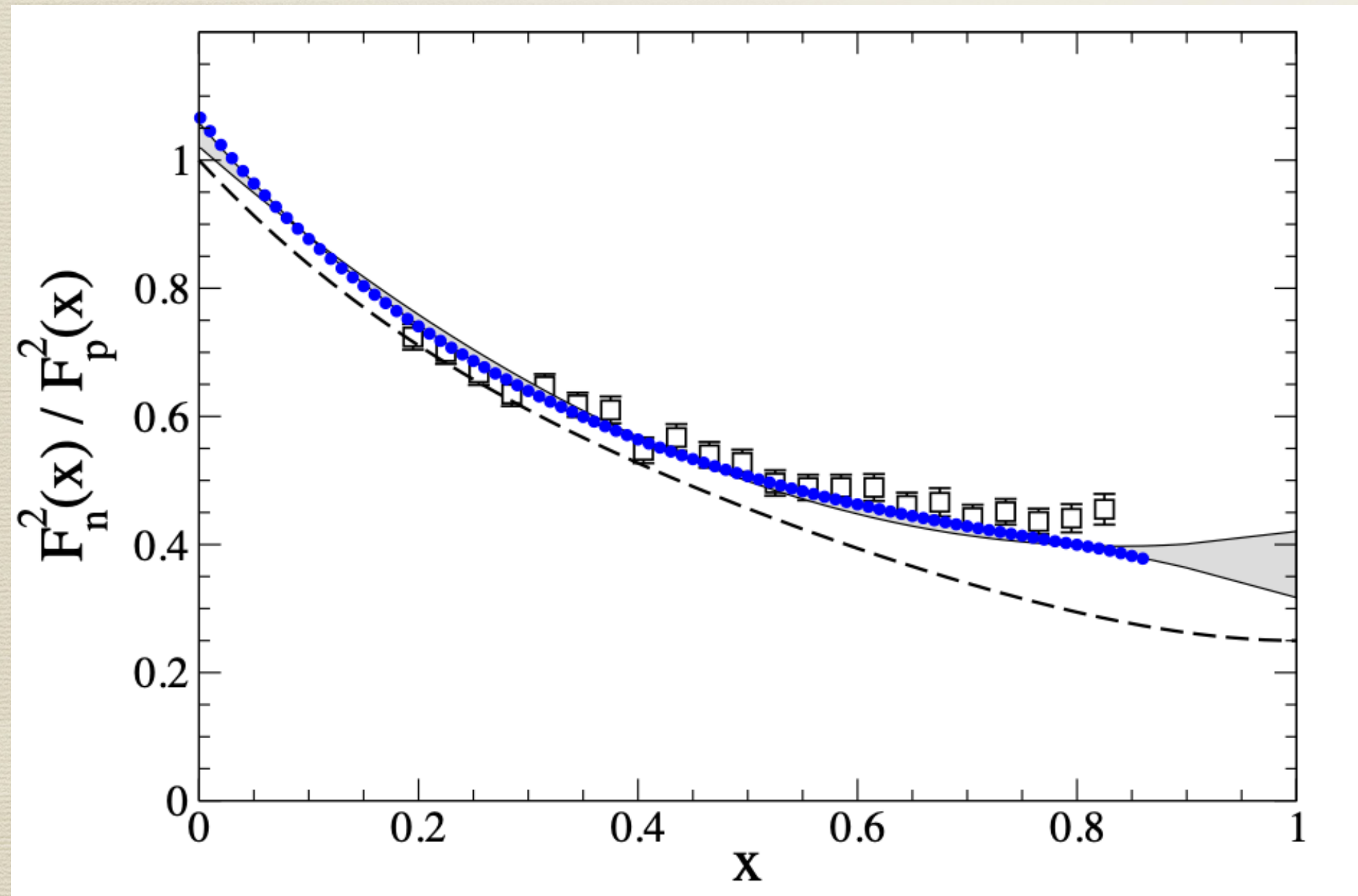
Solid line: Av18/UIX; Dashed-line: NV1b/UIX

# Backup slides

MARATHON coll. : experimental data of the super-ratio  $R^{ht}(x) = F_2^{3He}(x)/F_2^{3H}(x)$

${}^3He$ : 2p + n;  ${}^3H$ : n + 2p

Is possible to extract the ratio  $F_2^n(x)/F_2^p(x)$  through the super-ratio



**E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810**

Dashed line: ratio from SMC collaboration

Empty squares: MARATHON extraction

Solid line: cubic and conic extractions from  $F_2^p$  SMC parametrization, fitted to MARATHON data

# Backup slides

- In Instant form (initial hyperplane  $t=0$ ), one can couple spins and orbital angular momenta via Clebsch-Gordan (CG) coefficients. In this form the three rotation generators are independent of the interaction.
- To embed the CG machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum

$$\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D_{\sigma', \sigma}^{1/2}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the } \mathbf{J}=1/2 \text{ case}} |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

Wigner rotation for the  $\mathbf{J}=1/2$  case

- $R_M(\tilde{\mathbf{k}})$  is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi_{\sigma'}^\dagger \frac{m + k^+ - i\sigma \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m + k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma \rangle_c$$

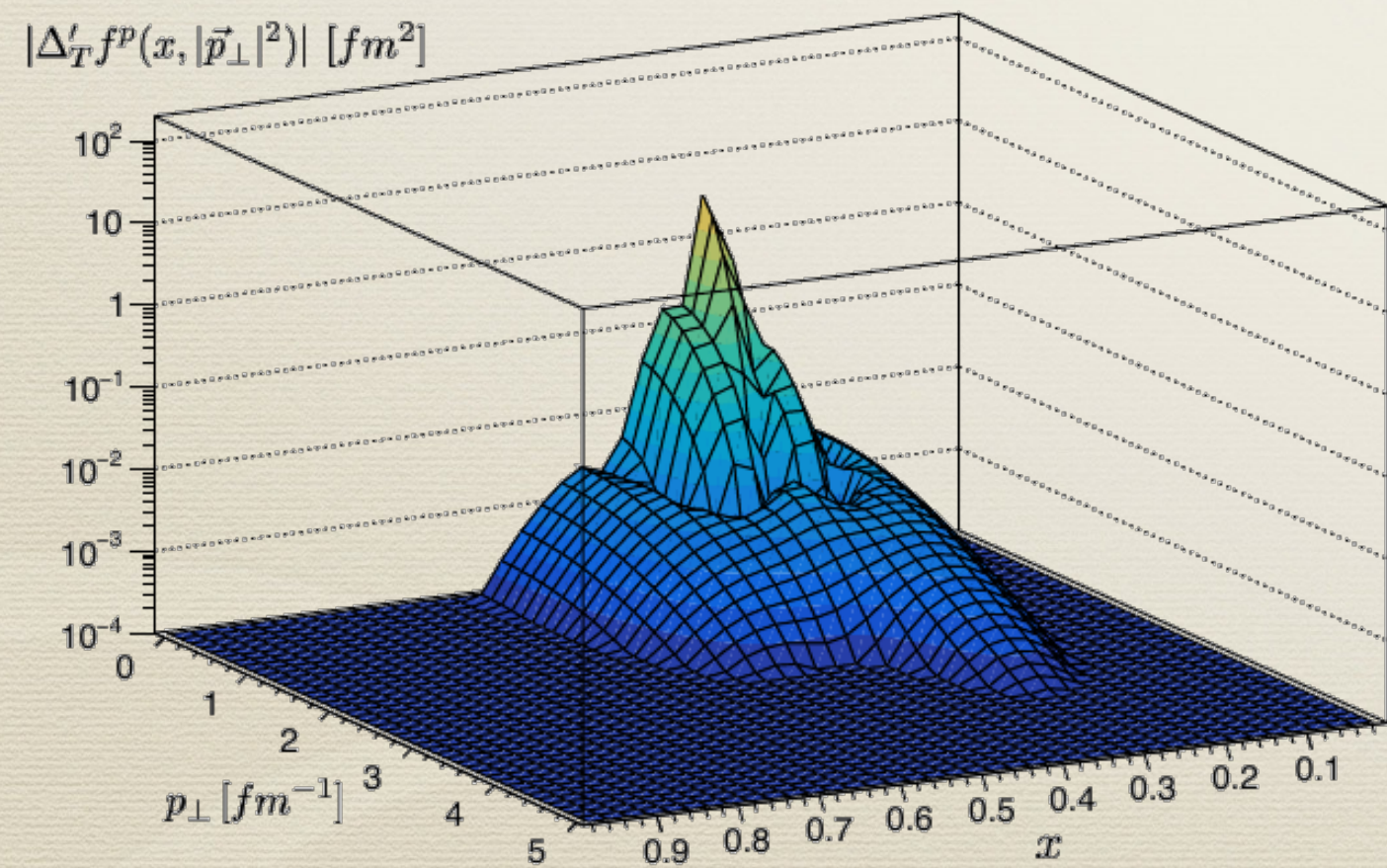
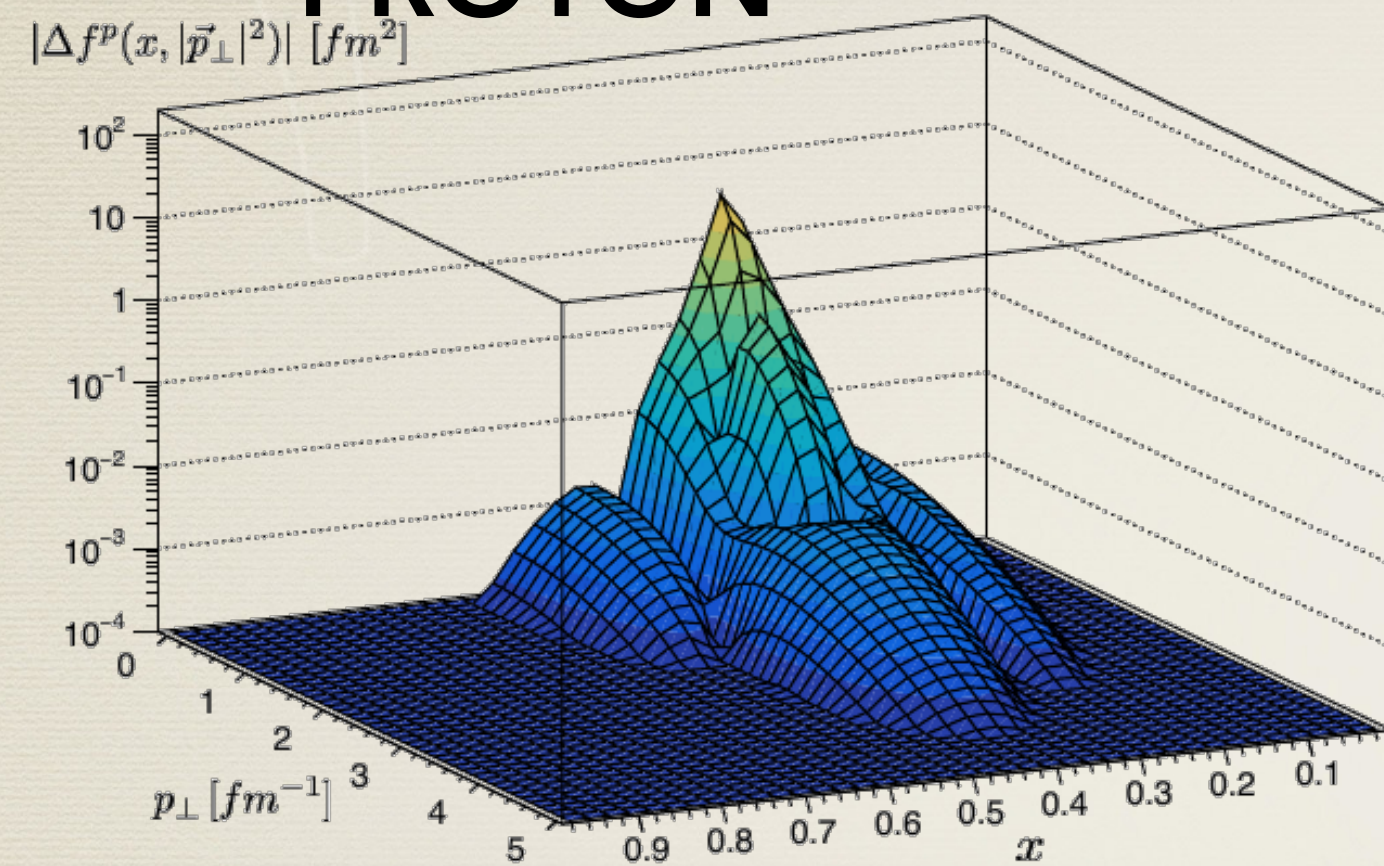
→ two-dimensional spinor

N.B. If  $|\mathbf{k}_\perp| \ll k^+, m \rightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

# Backup slides

**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

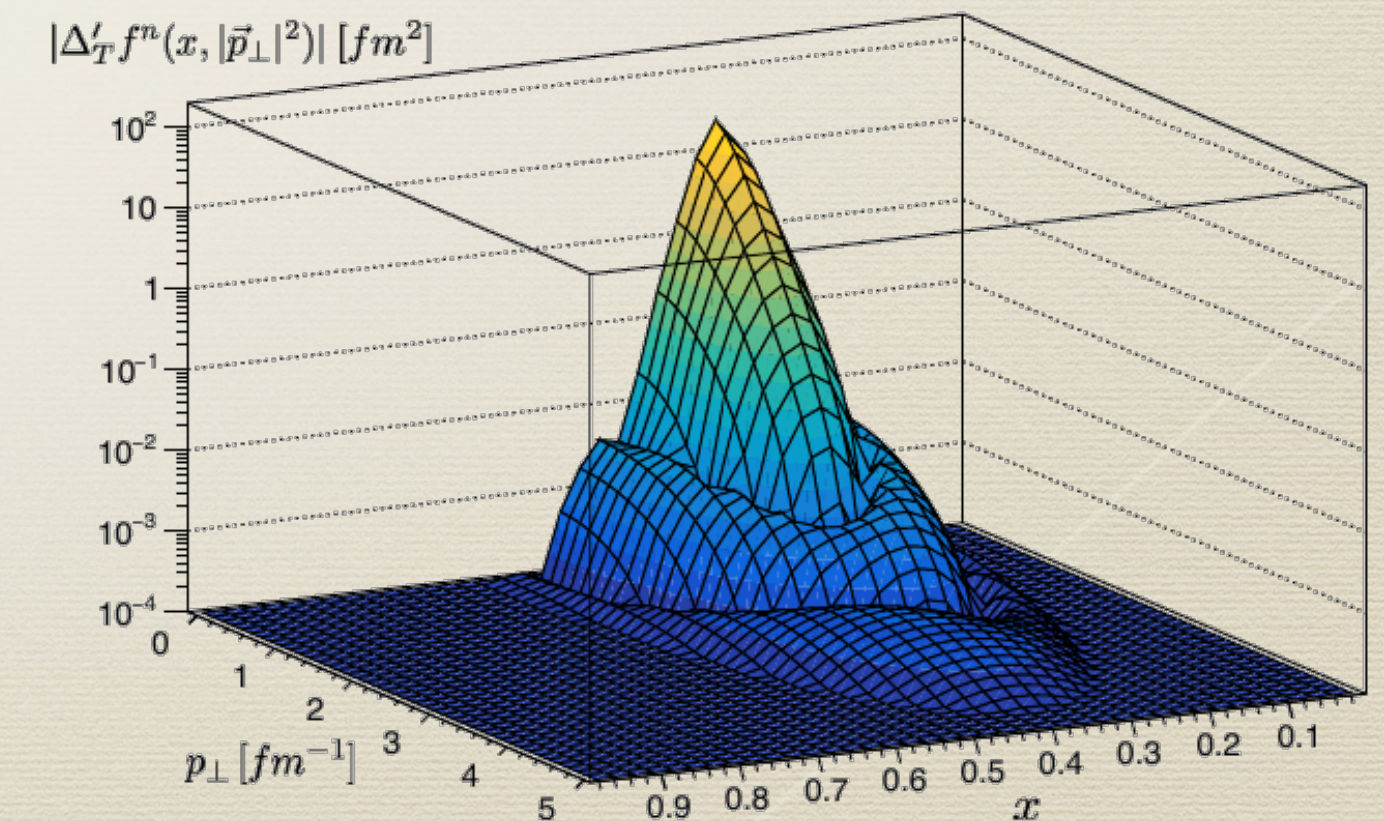
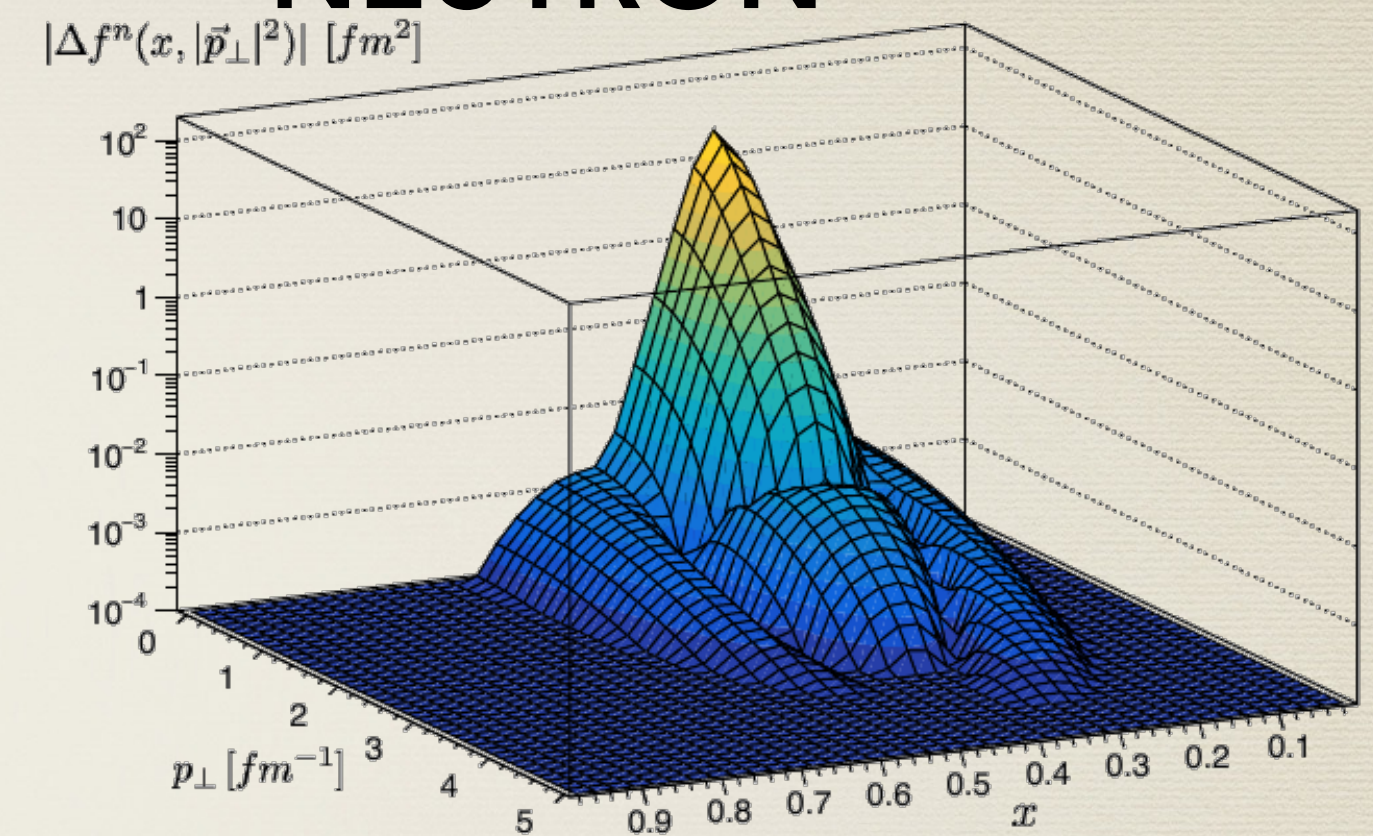
## PROTON



$$\Delta f^T(x, |\mathbf{p}_\perp|^2)$$

$$\Delta'_T f^T(x, |\mathbf{p}_\perp|^2)$$

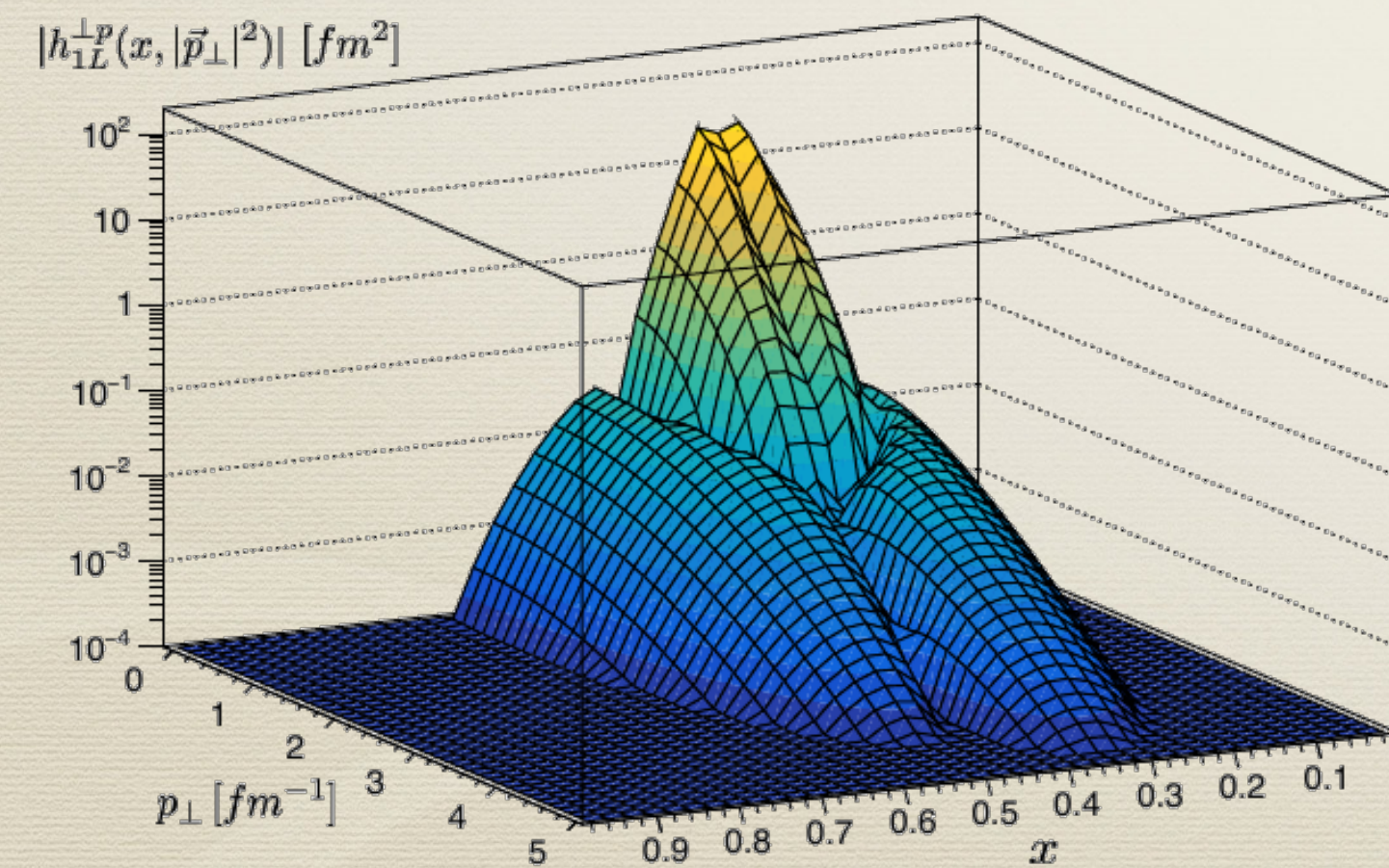
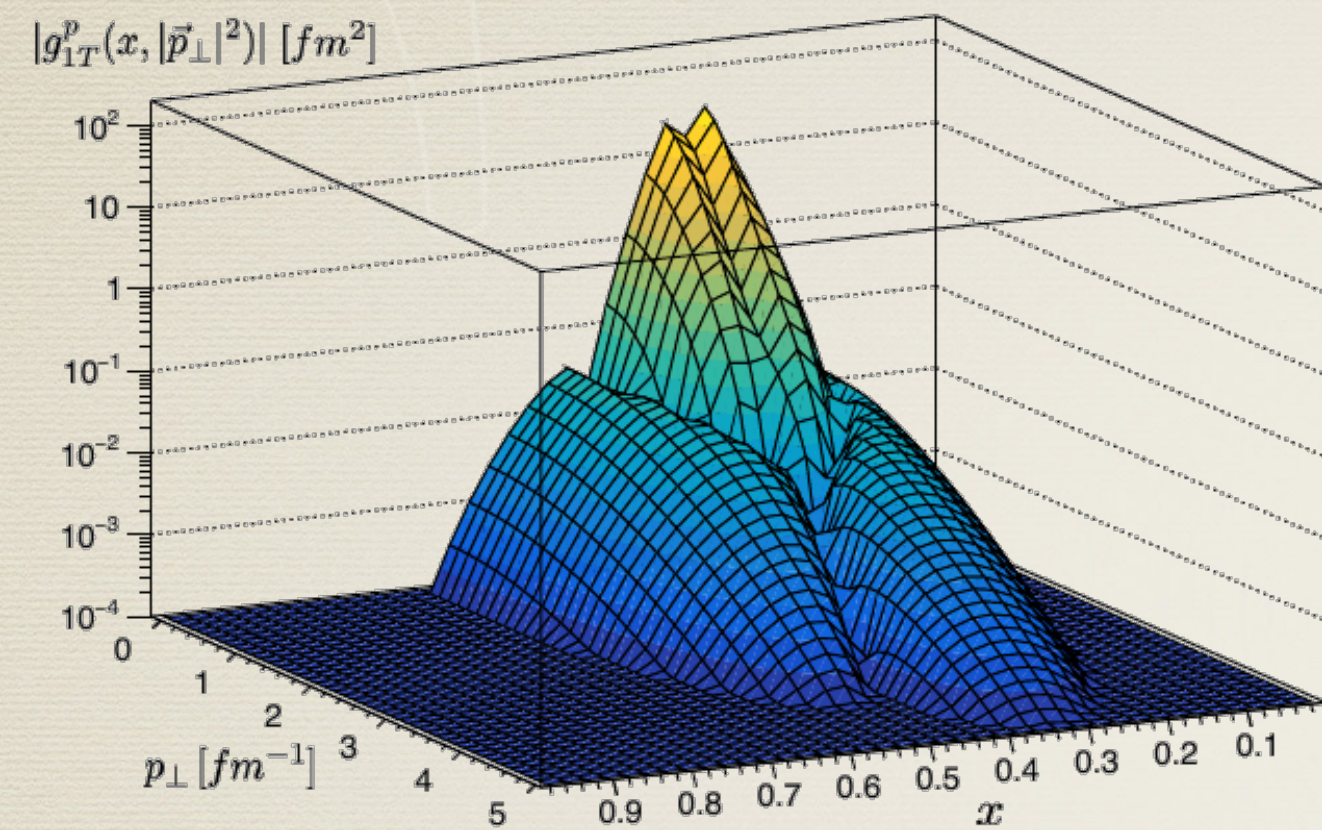
## NEUTRON



# Backup slides

**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204

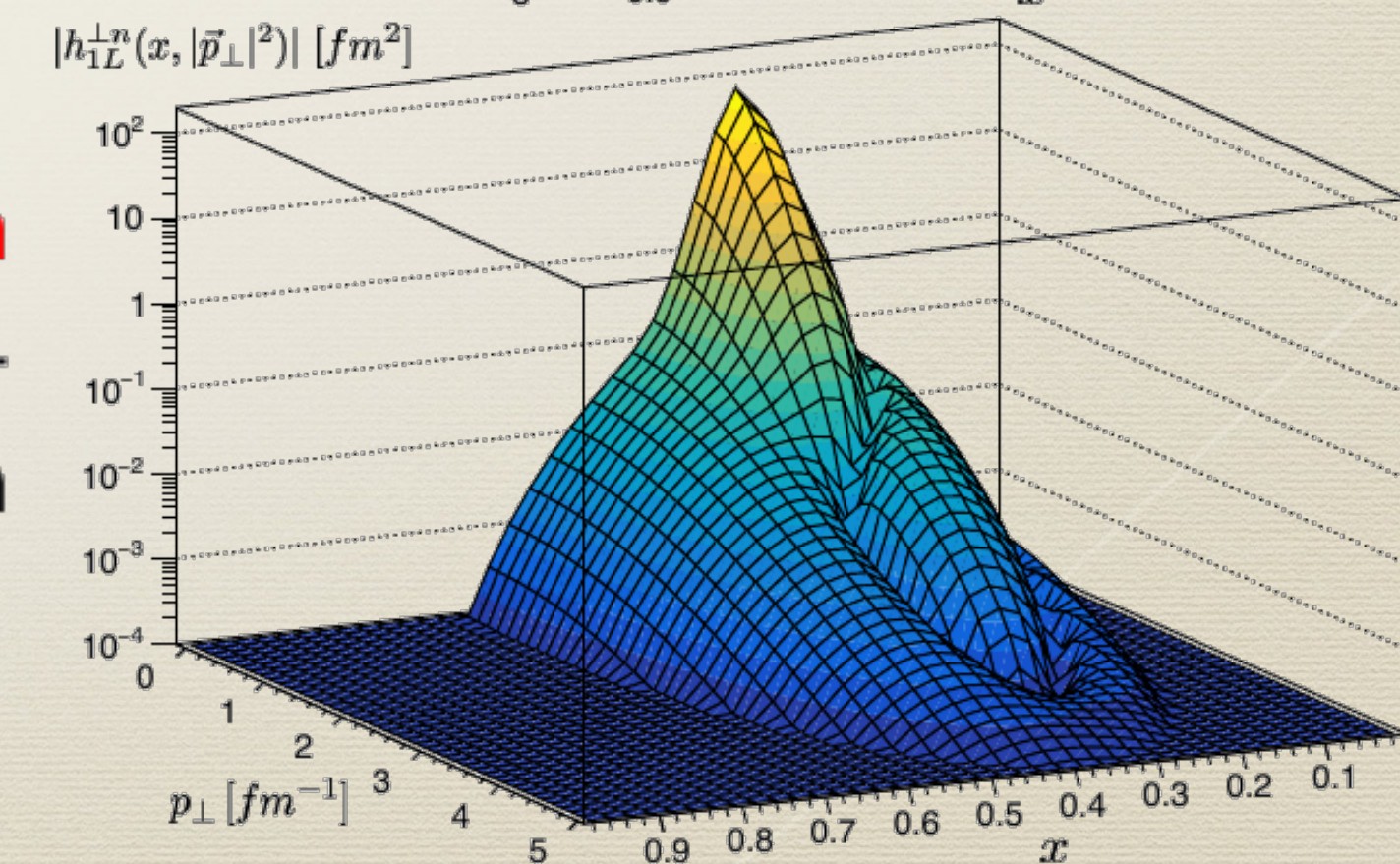
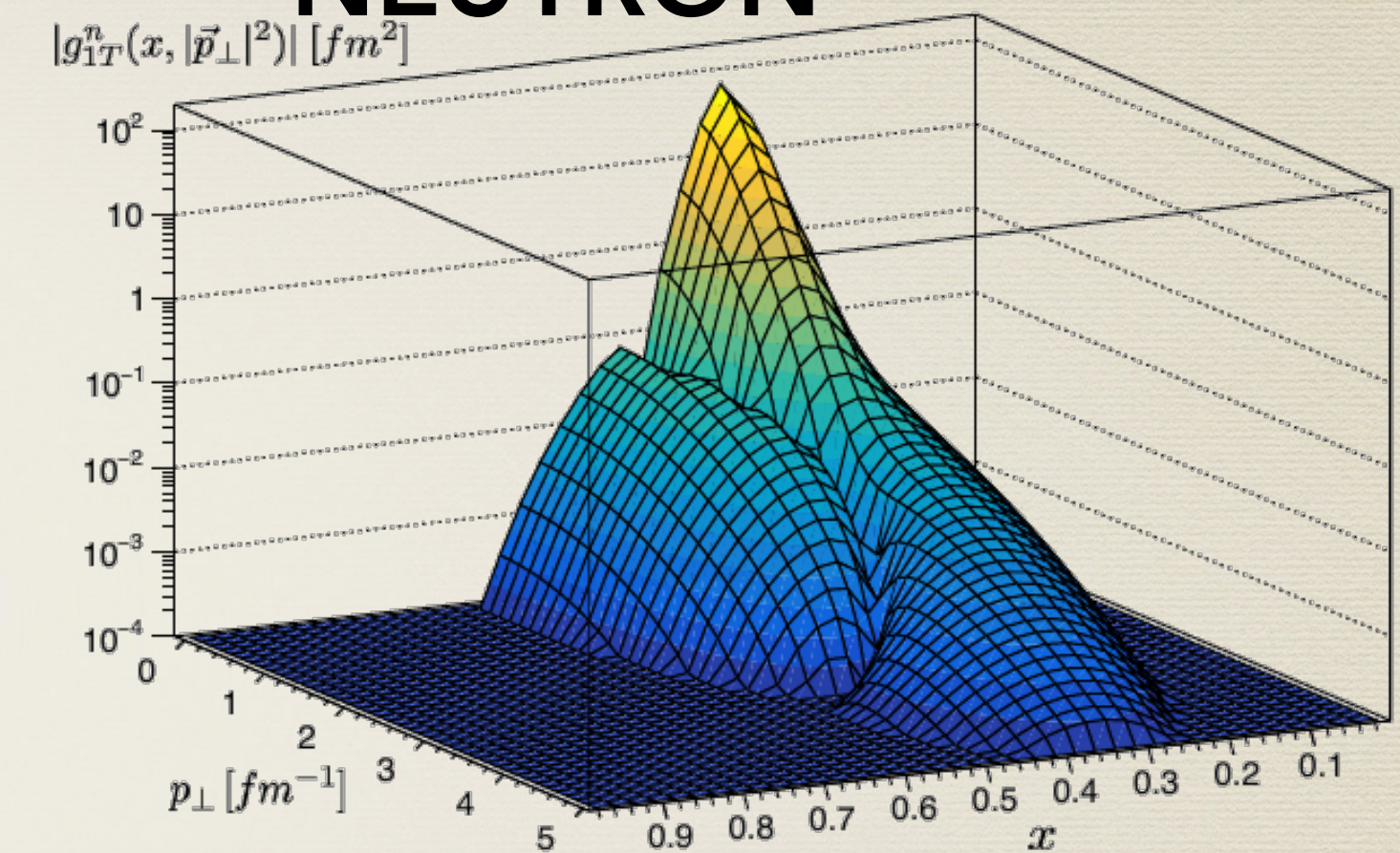
## PROTON



Absolute value of the **nucleon longitudinal-polarization** distribution,  $g_{1T}^\tau(x, |\mathbf{p}_\perp|^2)$ , in a transversely polarized  ${}^3\text{He}$ .

Absolute value of the **nucleon transverse-polarization** distribution,  $h_{1L}^{\perp \tau}(x, |\mathbf{p}_\perp|^2)$  in a longitudinally polarized  ${}^3\text{He}$ .

## NEUTRON





# Backup slides

\* E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, **Phys. Scr.** 95, 064008 (2020)

$$W_A^{S,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P^N(\tilde{\kappa}, \epsilon) w_{N,\sigma}^{S,\mu\nu}(p, q)$$

hadronic tensor of the nucleon

$W_A^{S,\mu\nu}$  is parametrized by the SFs  $F_2^A(x)$  and  $F_1^A(x)$ :

Unpolarized LF spectral function:

$$P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P_{\sigma\sigma}^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{S,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

Free nucleon SF

Where  $x = \frac{Q^2}{2P_A \cdot q}$  and  $\xi = \frac{\kappa^+}{\mathcal{M}_0[1; 2, 3, \dots, A-1]}$  with  $z = \frac{Q^2}{2p \cdot q} = \frac{p \cdot x}{P_A^+ \xi}$

# Backup slides

\* A. Del Dotto, E.Pace, G. Salmè and S.Scopetta, **Phys. Rev. C 95,014001 (2017)**

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W_A^{S,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\mathbf{k}}, \epsilon) F_2^N(z)$$

In the Bjorken limit  $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$  so we can use the **light-cone momentum distribution (LCMD)** instead of the **LF spectral function** \*

$$\text{LCMD: } f_1^N(\xi) = \int d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\mathbf{k}}, \epsilon) \frac{E_s}{1-\xi} = \int d\mathbf{k}_\perp n^n(\xi, \mathbf{k}_\perp)$$

**LF momentum distribution:**

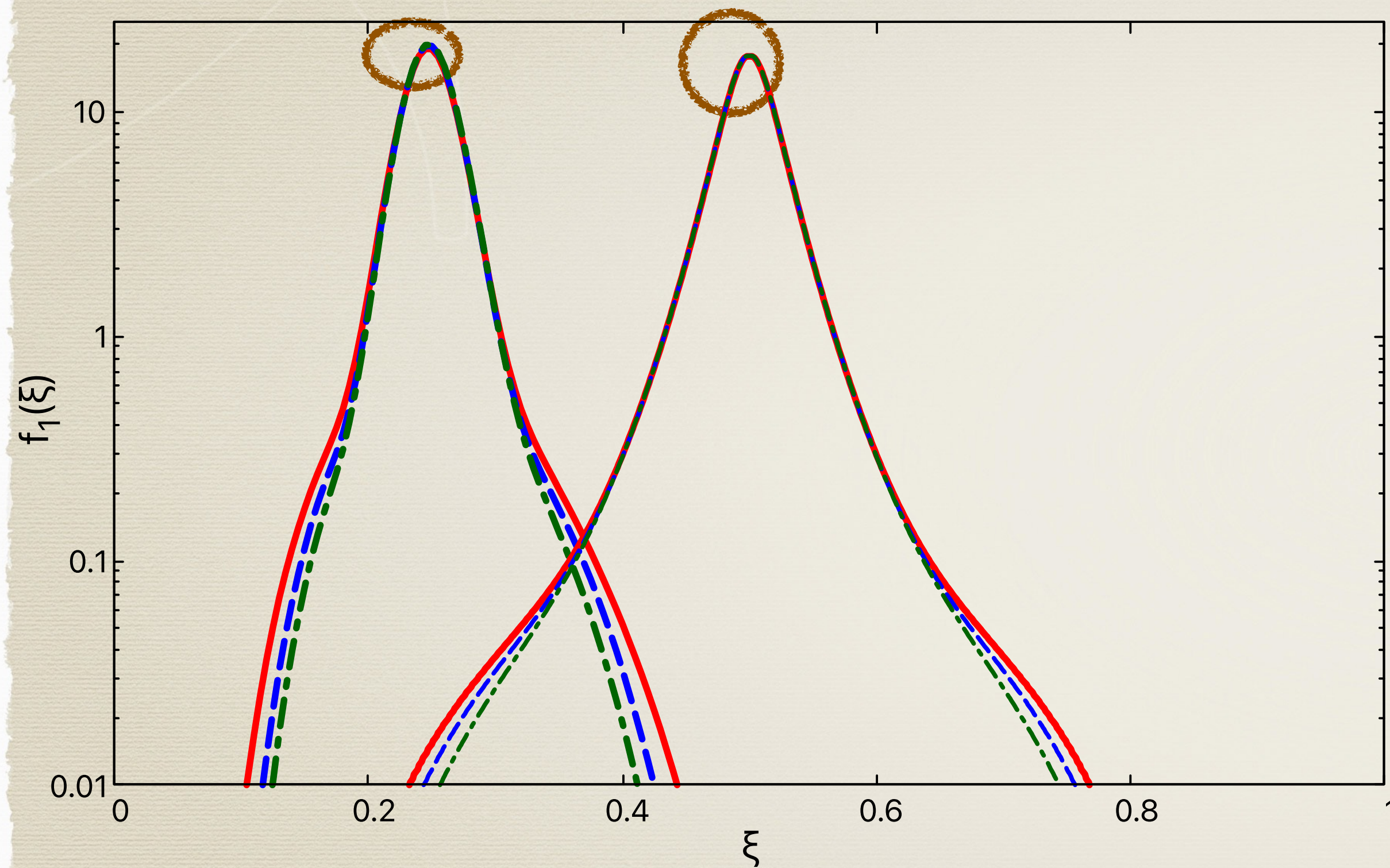
$$n^N(\xi, \mathbf{k}_\perp) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i] \left| \frac{\partial k_z}{\partial \xi} \right| \mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \dots, \mathbf{k}_{A-1})$$

Squared nuclear wave function. Thanks to the BT construction, one is allowed to use the NR one

Determinant of the Jacobian matrix. LF boost: effect of a Poincaré covariance approach

# Backup slides

The distributions are peaked at  $1/A$  with an accuracy of  $1/1000$ :



F.F. Pace, M. Rinaldi, G. Salmè, S. Scopetta and M. Viviani, *Phys.Lett.B* 851 (2024)

LC momentum distribution for  ${}^4\text{He}$  (peaked at 0.25) and deuteron (peaked at

- The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the **high-momentum content** of the 1-body momentum distribution)
- The tails of the LC momentum distribution calculated by the **Av18/UIX** potential is larger than the ones obtained by the  **$\chi$ EFT** potentials for both  ${}^4\text{He}$  and deuteron

# Backup slides

For the **polarized DIS** we need to calculate the **antisymmetric** part of the **hadronic tensor**:

$$W_A^{a,\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P_\sigma^N(\tilde{\mathbf{k}}, \epsilon, \mathbf{S}, \mathcal{M}) w_{N,\sigma}^{a,\mu\nu}(p, q)$$

hadronic tensor of the nucleon

Spin-dependent LF spectral function

$W_A^{a,\mu\nu}$  is parametrized by the the **spin-dependent SFs (SSFs)**  $g_1^A(x, Q^2)$  and  $g_2^A(x, Q^2)$

As for the unpolarized case, in the **Bjorken limit** we can write a **convolution formula** for the **SSFs**:

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[ g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right], j = 1, 2$$

# Backup slides

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi [g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi)], j = 1, 2$$

The **spin-dependent LCMD**  $l_j^N(\xi)$  and  $h_j^N(\xi)$  are related to the **transverse momentum-**

We used the **TMDs** for  ${}^3\text{He}$  calculated with the **Av18** potential in Ref. **[1]**

**GRSV** parametrization [2] for the  $g_1^N(x)$  SSF

$g_2^N(x)$  extracted by  $g_1^N(x)$  with the **Wandzura-Wilczek** formula [3]:

$$g_2^N(x) = -g_1^N(x) + \int_x^1 dy \frac{g_1^N(y)}{y}$$

**[1]** R.Alessandro. A.Del Dotto. E.Pace. G.Perna. G.Salmè and S.Scopetta. **Phvs.Rev.C 104(2021) 6.065204**

**[2]** M. Glück. E. Reva. M. Stratmann. and W. Vogelsand. **Phvs. Rev. D 63. 094005 (2001)**

**[3]** S. Wandzura and F. Wilczek. **Phvs. Lett. B 72. 195 (1977)**

# Backup slides

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi \left[ g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi) \right]$$

One can **approximate** this equation using that  $l_j^N(\xi), h_j^N(\xi)$  are **peaked** around  $\xi \simeq 1/A$  and so

$$g_j^{\bar{n}}(x) = \frac{1}{p_j^n} \left[ g_j^{3He}(x) - 2p_j^p g_j^p(x) \right]$$

Where the **effective polarization**  $p_j^N$  are **integral** of the **TMDs**  $\Delta f(\xi, k_\perp)$  and  $\Delta'_T f(\xi, k_\perp)^*$

$$p_1^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta f(\xi, k_\perp) \text{ and } p_2^N = \int_0^1 d\xi \int d\mathbf{k}_\perp \Delta'_T f(\xi, k_\perp)$$

We compared our extraction of the **neutron SSFS** with the one of the **GRSV parametrization** and with the **NR extraction**, obtained through the effective polarizations calculated from a NR