Pseudoscalar Mesons and Emergent Mass

Khépani Raya Montaño

Khépani Raya · Adnan Bashir · Daniele Binosi · Craig D. Roberts · José Rodríguez-Quintero

Pseudoscalar Mesons and Emergent Mass

https://doi.org/10.1007/s00601-024-01924-2

Universidad de Huelva

Science at the Luminosity Frontier: JLab at 22 GeV

Frascati, Italy. December 9-13, 2024

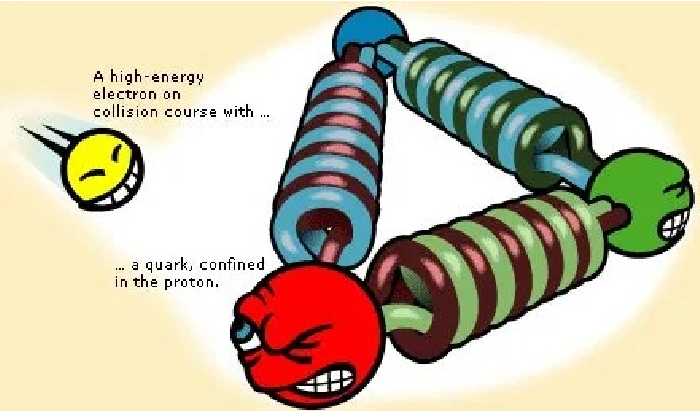
QCD: Emergent Phenomena

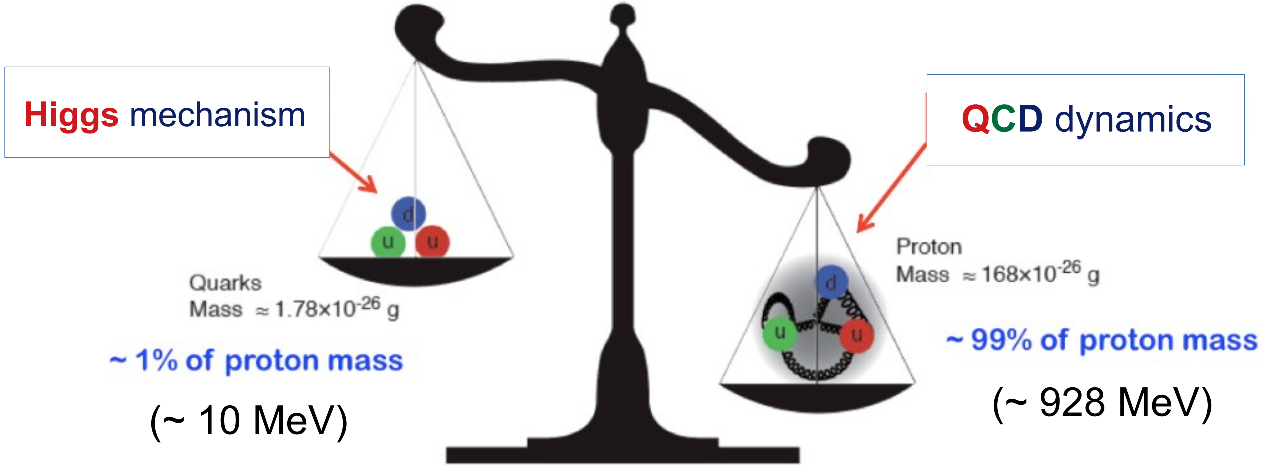
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**





QCD: Emergent Phenomena

- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

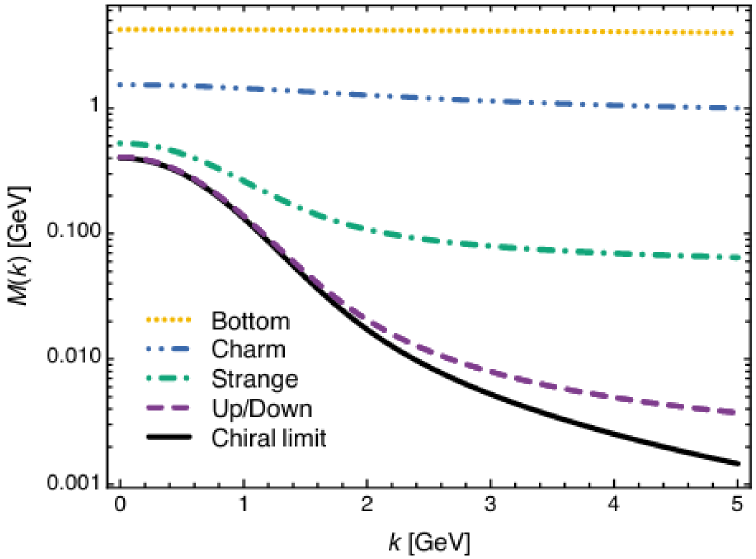
$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\frac{1}{2}\lambda^a A^a_\mu,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

Can we trace them down to fundamental d.o.f?

- Emergence of hadron masses (EHM) from QCD **dynamics**

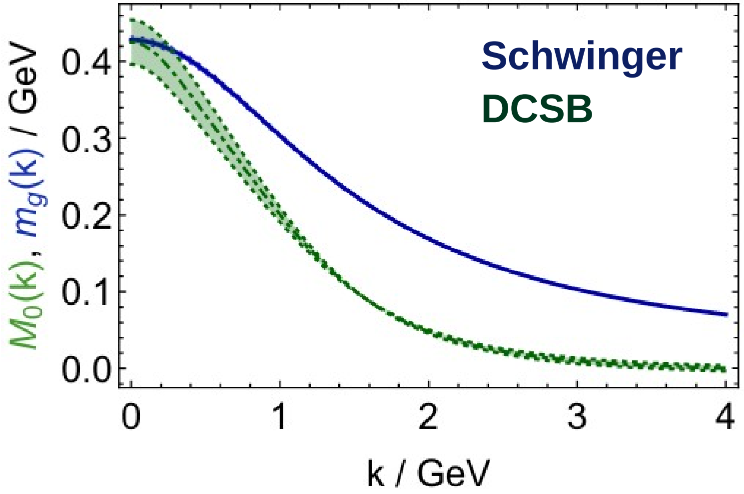
Constituent masses

Dynamical Chiral Symmetry Breaking



"Higgs" masses

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$



Gluon and quark *running masses*

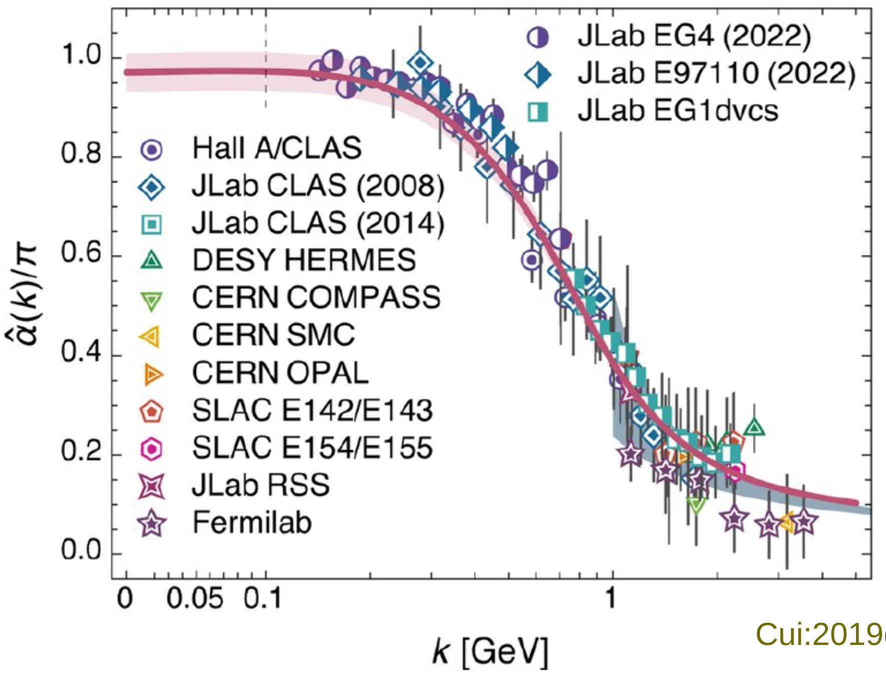
QCD: Emergent Phenomena

- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

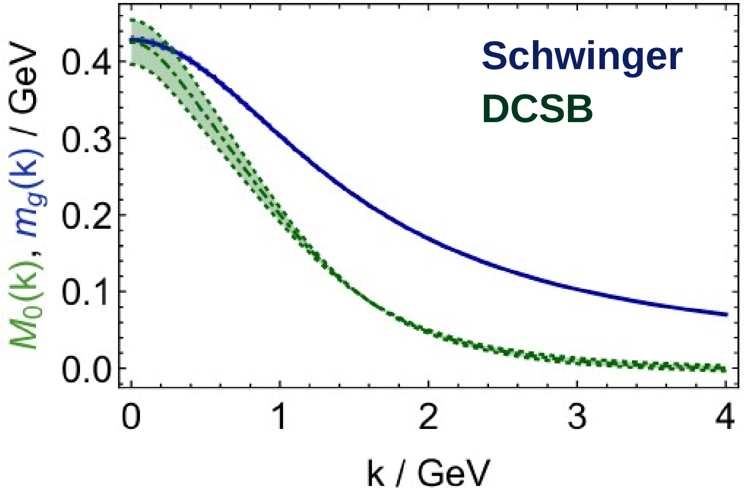
$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

Can we trace them down to fundamental d.o.f?

- Emergence of hadron masses (EHM) from QCD **dynamics**



Cui:2019dwv



Gluon and quark *running masses*

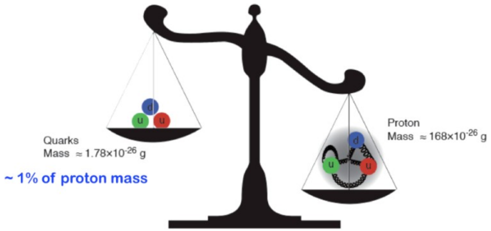
Mass **Budgets**

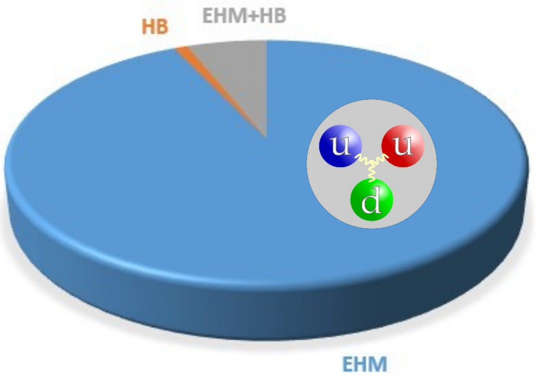
$$M_{u/d} \approx 0.3\,\mathrm{GeV}$$

- What is the origin of **EHM**?

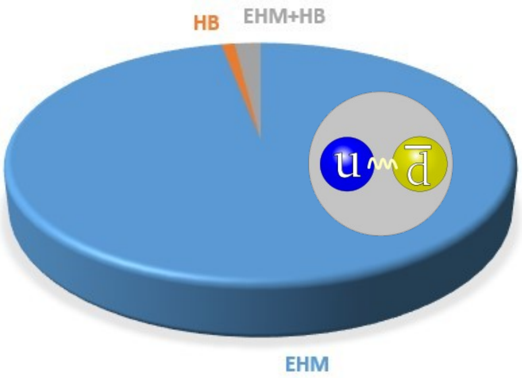
  … its connection with *e.g.* **confinement** and **DCSB**?

> - **Most** of the **mass** in the visible universe is contained within **nucleons**
>   - Which remain **pretty massive** whether there is Higgs mechanism or not...





Proton mass budget



Rho meson mass budget

$$m_p = 0.938\,\mathrm{GeV} \approx 2M_u + M_d$$

$$m_\rho = 0.775\,\mathrm{GeV} \approx M_u + M_d$$

Proton and **rho** meson mass budgets are practically **identical**

Mass **Budgets**

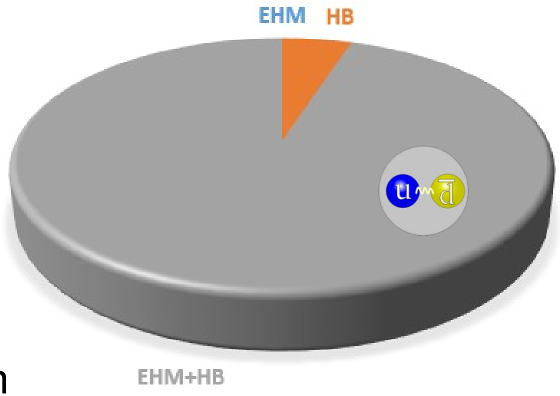
$$m_s/m_u \sim 20$$
$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

- The lightest hadrons in Nature, **pions** and **kaons**, follow an opposite pattern.

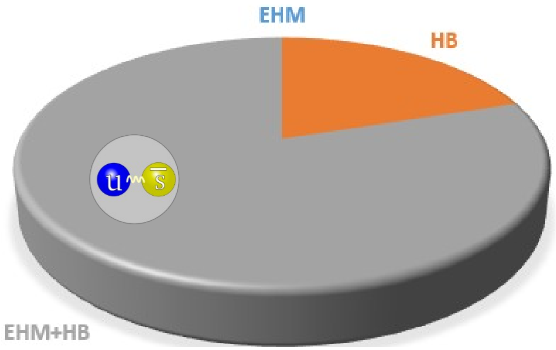
$$m_\pi = 0.14\,\text{GeV} \neq M_u + M_d$$
$$m_K = 0.49\,\text{GeV} \neq M_u + M_s$$

- In fact, the **absence of Higgs** mechanism would render these states **massless**.
  - And **structurally alike**.

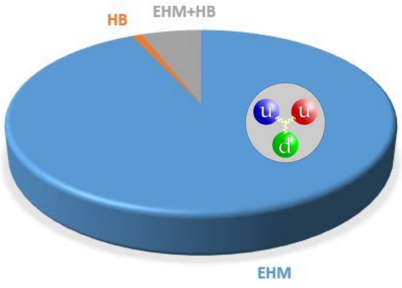
- Both are quark-antiquark **bound-states** and **NG bosons**
  - Their mere **existence** is connected with **mass** generation in the **SM**



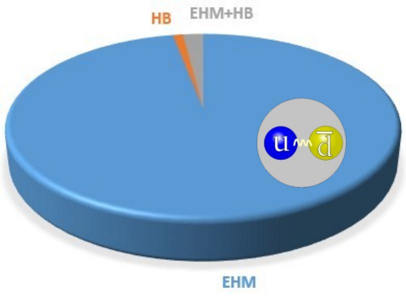
Pion mass budget



Kaon mass budget



Proton mass budget



Rho meson mass budget

Mass **Budgets**

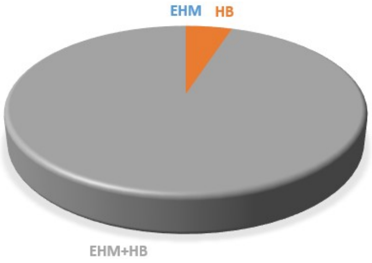
$$m_s/m_u \sim 20$$

$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

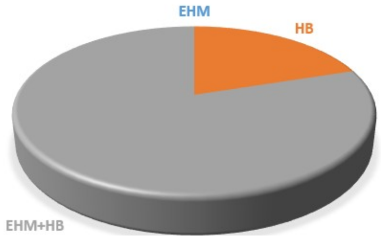
- Both are quark-antiquark **bound-states** and **NG bosons**
  - Their mere **existence** is connected with **mass** generation in the **SM**

➢ This is also true for the η **meson**, but not for its partner, η′.

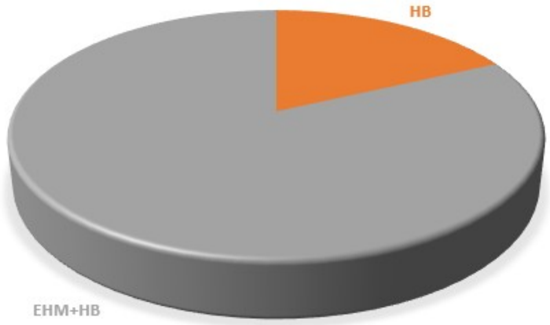
- The **U$_A$(1)** anomaly, another manifestation of **EHM**, sets it apart from the **NG-boson** family
- Yet, the η–η′ system have tightly intertwined properties



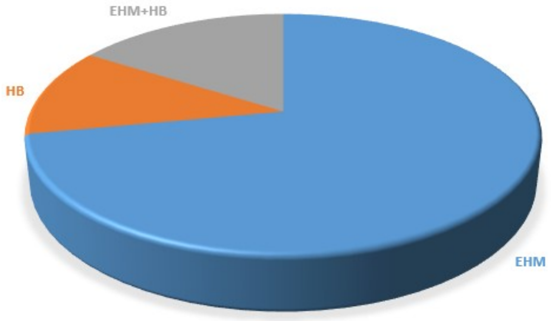
Pion mass budget



Kaon mass budget



η mass budget



η′ mass budget

Mass Budgets

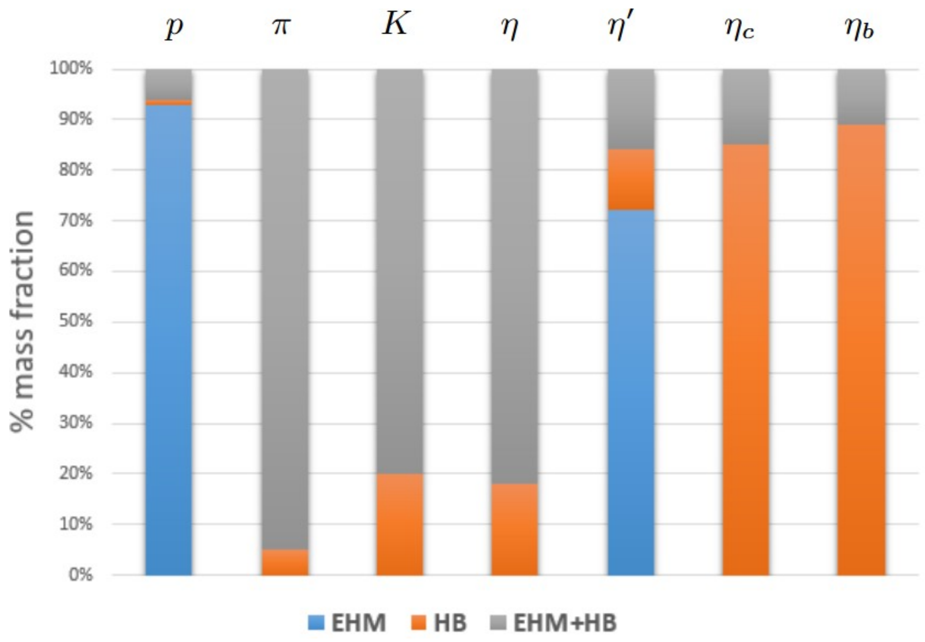
$$m_s/m_u \sim 20$$
$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

- The examination of ***all*** pseudoscalars is crucial in elucidating the role of the **mass generation** mechanisms on the structural **properties**.
  - An so the **emergent** features of the strong interactions.

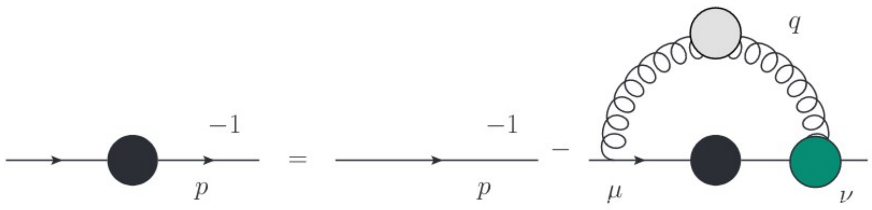
(confinement, mass generation)

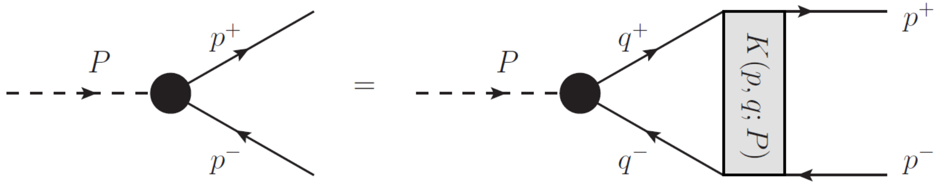
- Light **pseudoscalars** hold a special role due to their link with **symmetries and anomalies** in the Standard Model.
- **Modern facilities** are set to scrutinize their properties at **unprecedented** depth.

Accardi:2023chb Arrington:2021biu Chen:2020ijn



Continuum Schwinger Methods (CSM)





Dyson-Schwinger Equations

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions
  - **Infinite** tower of coupled equations.
  - ✗ Systematic **truncation** required

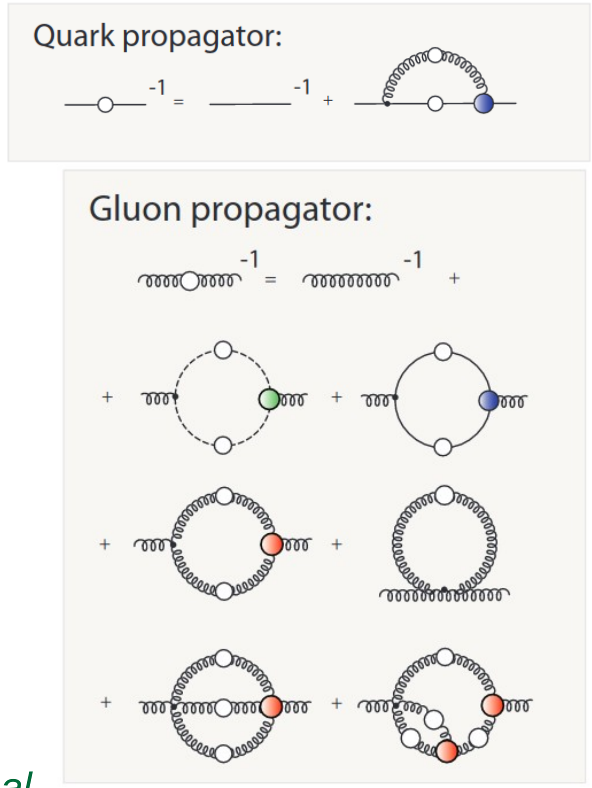
✓ **No assumptions** on the **coupling** for their derivation.

- ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**

✓ **Not limited** to a certain domain of current **quark masses**

✓ Maintain a **traceable connection** to QCD.

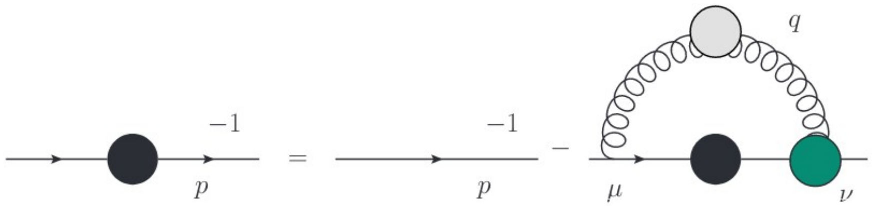
Example DSEs



C.D. Robert and A.G. Williams, Prog.Part.Nucl.Phys. 33 (1994) 477-575

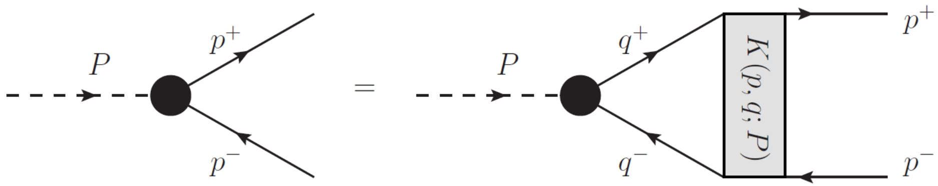
G. Eichmann, H. Sanchis-Alipus *et al.* Prog.Part.Nucl.Phys. 91 (2016) 1-100

DSE-BSE approach



Quark DSE

- Relates the quark propagator with **QGV** and **gluon propagator**.



Meson BSE

- Contains **all interactions** between the valence quark and antiquark

- Any <u>sensible truncation</u> must preserve the **Goldstone's Theorem**, whose most fundamental expression is captured in:

> "**Pions** exists, if and only if, **DCSB** occurs."

$$f_\pi E_\pi(k; P=0) = B(k^2)$$

$E_\pi(k; P=0)$ → **Leading** BSA

$B(k^2)$ → "***Mass*** Function"

L. Chang *et al.*,
Phys.Rev.Lett. 110 (2013) 13, 132001

Valence-quark distribution amplitudes (DAs)

$$f_{\mathbf{P}}\varphi_{\mathbf{P}}^{q}(x;\zeta) = \mathrm{tr}_{\mathrm{CD}}\int_{dk}^{\Lambda}\delta_{n,P}^{x}(k)\gamma_5\gamma\cdot n(k;P)\chi_{\mathbf{P}}(k_-;P)$$

Light-front momentum fraction

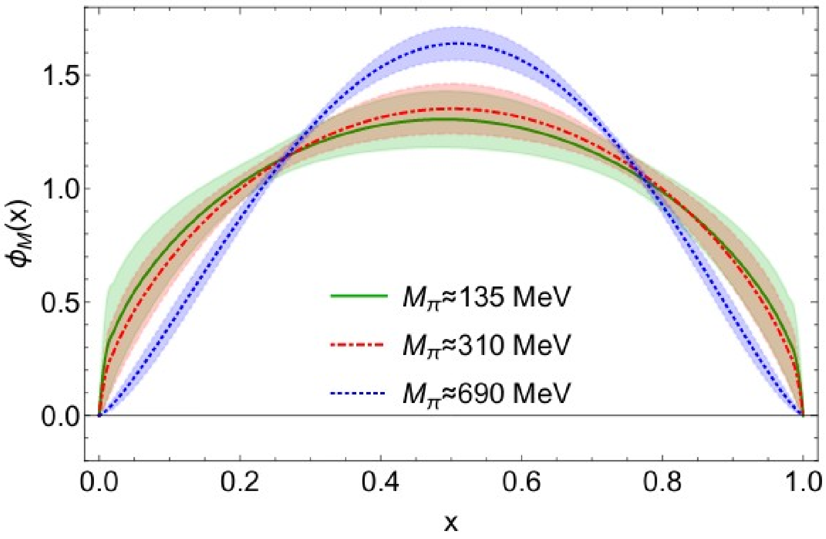
Expressed in terms of **BSWF**

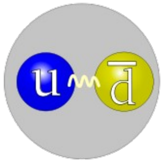
- **1-dimensional** projection of the **light-front wavefunction**.
- Clear **probe of EHM**, related with hard **exclusive processes**, etc.

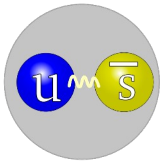
π-K DAs

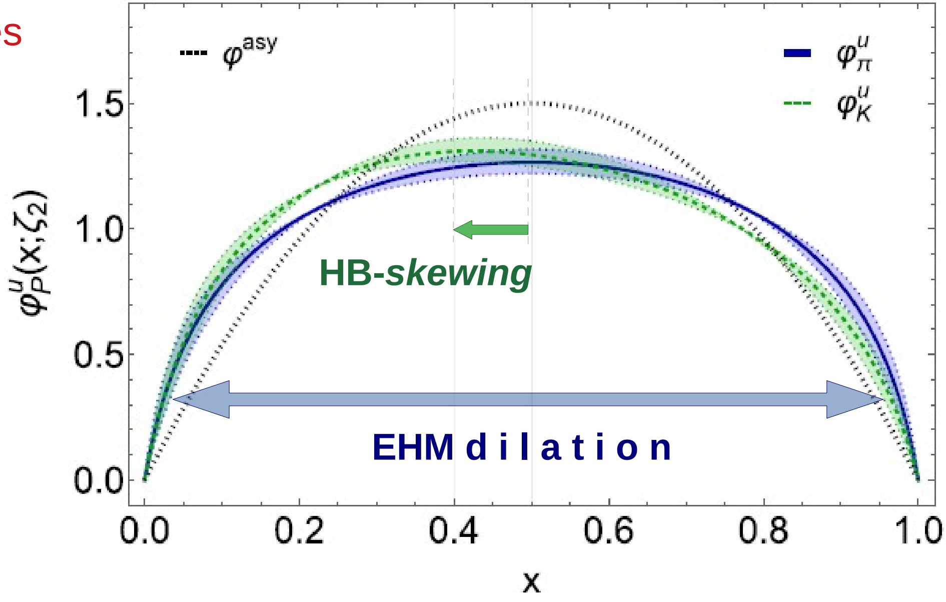
$$m_s/m_u \sim 20$$
$$f_K/f_\pi \sim M_s/M_u \sim 1.2$$

- **Broad** and concave **DAs**. @ real-life scales
- **Mild** skewing in **Kaon**: **strong** & **weak** interplay.









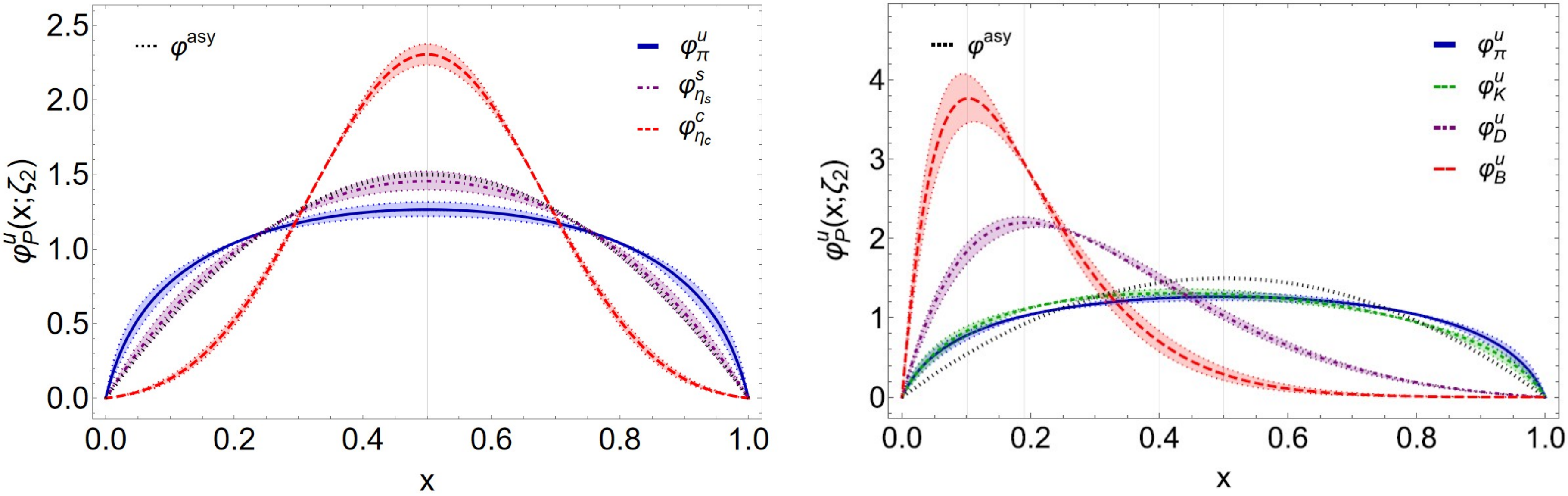
- Lattice QCD supports those findings:

Zhang:2020gaj Bali:2019dqc Segovia:2013eca

Pointwise form of the **PDA**!.

‘Heavy’ mesons **DAs**

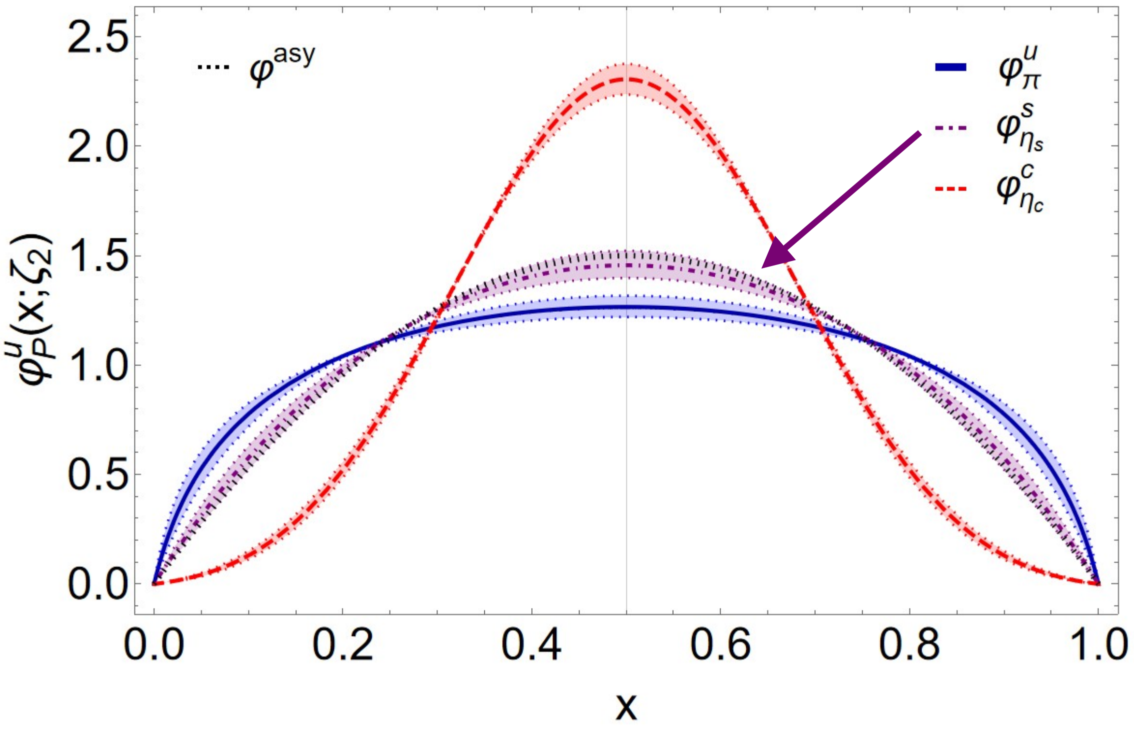
- In systems with heavy quarks, the **DAs** become **narrow**.

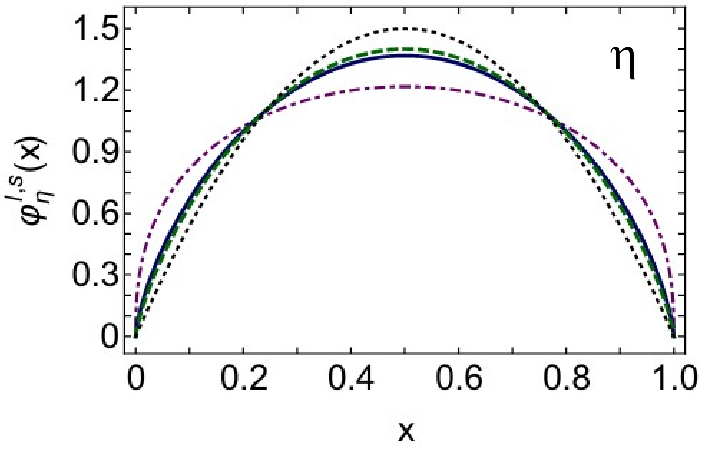


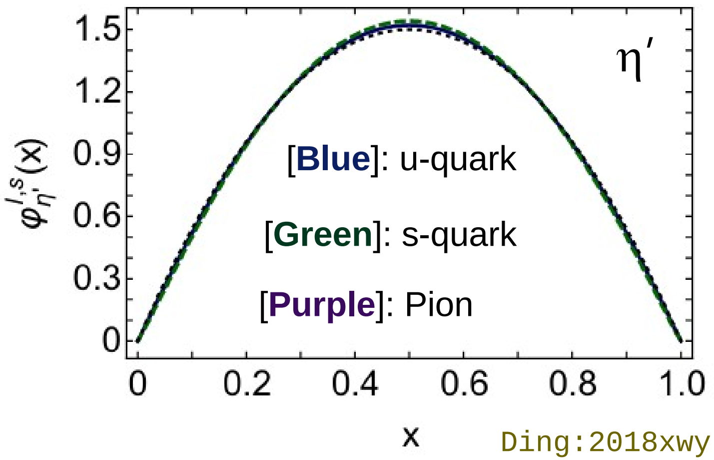
- Unlike the Kaon, heavy-light systems **DAs** are markedly ***skewed***
  - The **peaks** located at: $x_{\max}^{\pi,K,D,B} = 0.5,\ 0.4,\ 0.18,\ 0.1$

Drawing boundaries

- Systems with ss-bar components draw the line between **strong** and **weak** mass generation being dominant.



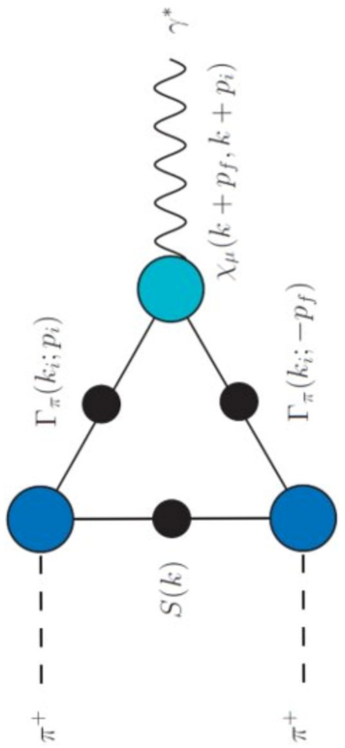




Ding:2018xwy

L. Chang *et al.*,
Phys.Rev.Lett. 111 (2013) 14, 141802

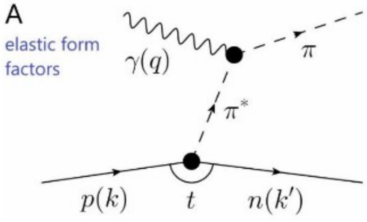
Electromagnetic Elastic Form Factors (EFFs)



$$P_\mu F^q_{\mathbf{P}}(Q^2) = \mathrm{tr}_{\mathrm{CD}} \int_{dk}^{\Lambda} \chi^q_\mu(k+p_o, k+p_i)\Gamma_{\mathbf{P}}(k_i;p_i)\, S_h(k)\, \Gamma_{\mathbf{P}}(k_o;-p_o)$$

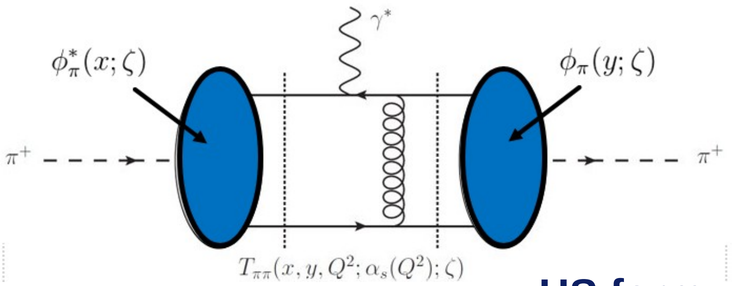
All can be written in terms of **propagators** and **vertices**

- Gives information on **momentum/charge** distribution.
- **Pion EFF** highly relevant for contemporary physics.



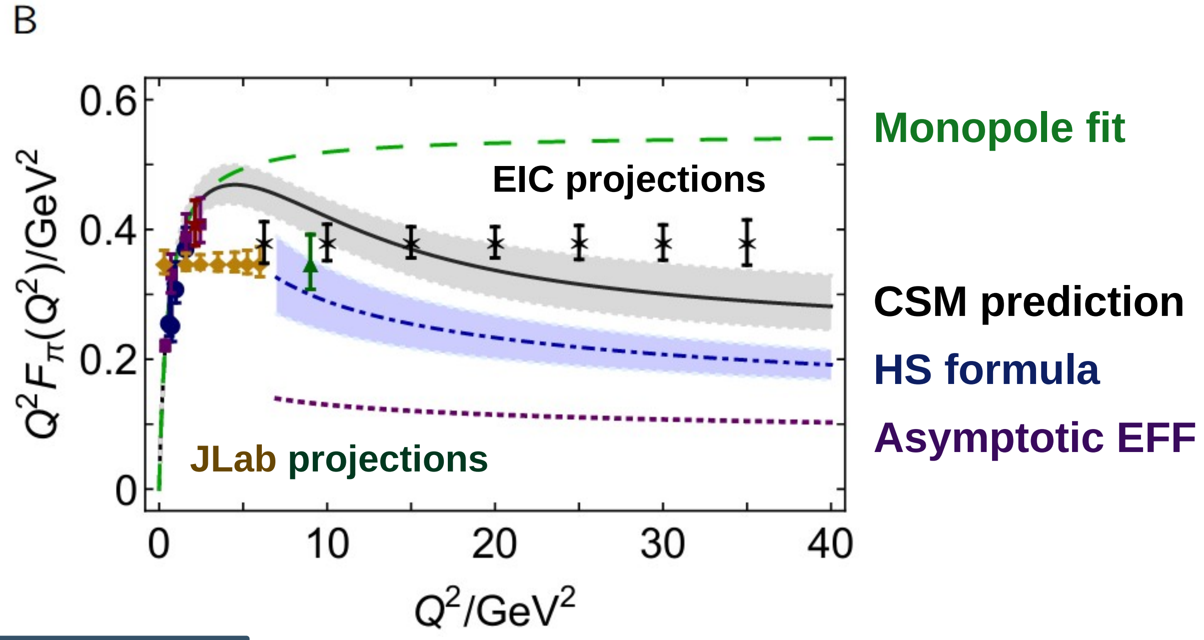
Elastic **Form Factors**

- In the **large-Q^2** regime, **QCD** connects the **EFF** and **DA**:



(at sufficiently large Q^2)

HS formula



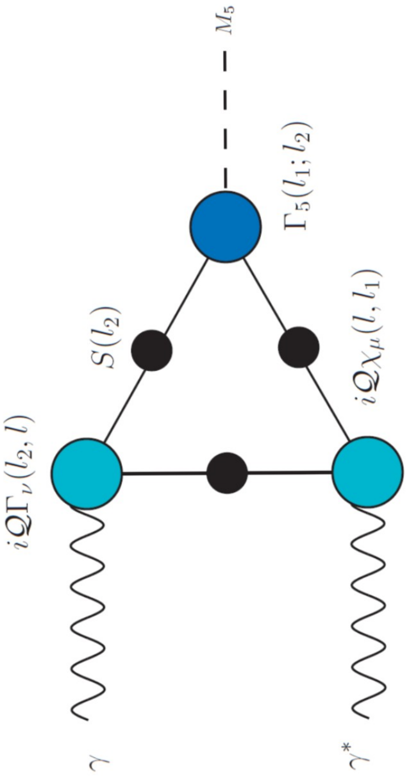
- At **leading-order**:

$$Q^2F_{\mathbf{P}}(Q^2) \overset{Q^2\gg m_p^2}{\approx} 16\pi\alpha_s(Q^2) f_{\mathbf{P}}^2 w_{\mathbf{P}}^2(Q^2)$$

$$w_{\mathbf{P}} = \frac{1}{3}\int_0^1 dx\,\frac{1}{x}\,\varphi_{\mathbf{P}}(x;Q^2) \longleftarrow \textbf{PDA} \longrightarrow$$

- The asymptotic behavior is weighted by f_P, a measure of **EHM**.
- **Factorization/scaling violations** are proof of the validity of **QCD itself.**

Two-photon Transition Form Factors (TFFs)

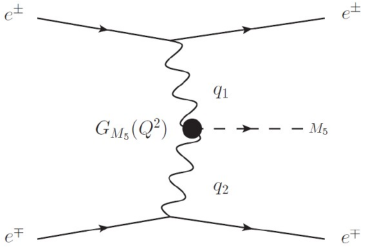


$$\mathcal{T}_{\mu\nu}(k_1,k_2) = \frac{e^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G_{\mathbf{P}}(k_1^2,k_2^2,k_1\cdot k_2)$$

$$\mathcal{T}_{\mu\nu}(k_1,k_2) = e^2\mathcal{Q}_{\mathbf{P}}^2\mathrm{tr}\int_l i\chi_\mu^q(l,l_1)\Gamma_{\mathbf{P}}(l_1,l_2)S(l_2)i\Gamma_\nu^q(l_2,l)$$

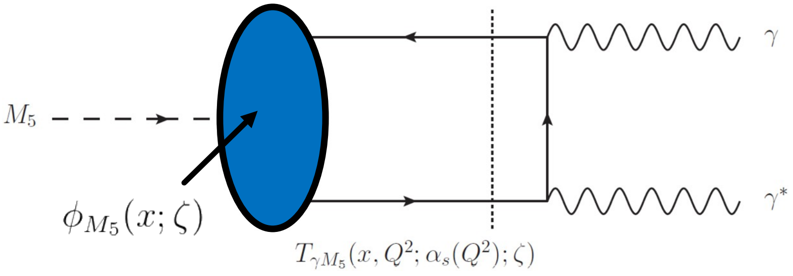
All can be expressed in terms of **propagators** and **vertices**

- Gives information on **momentum**/**charge** distribution.
- **Pion TFF** highly relevant for contemporary physics.



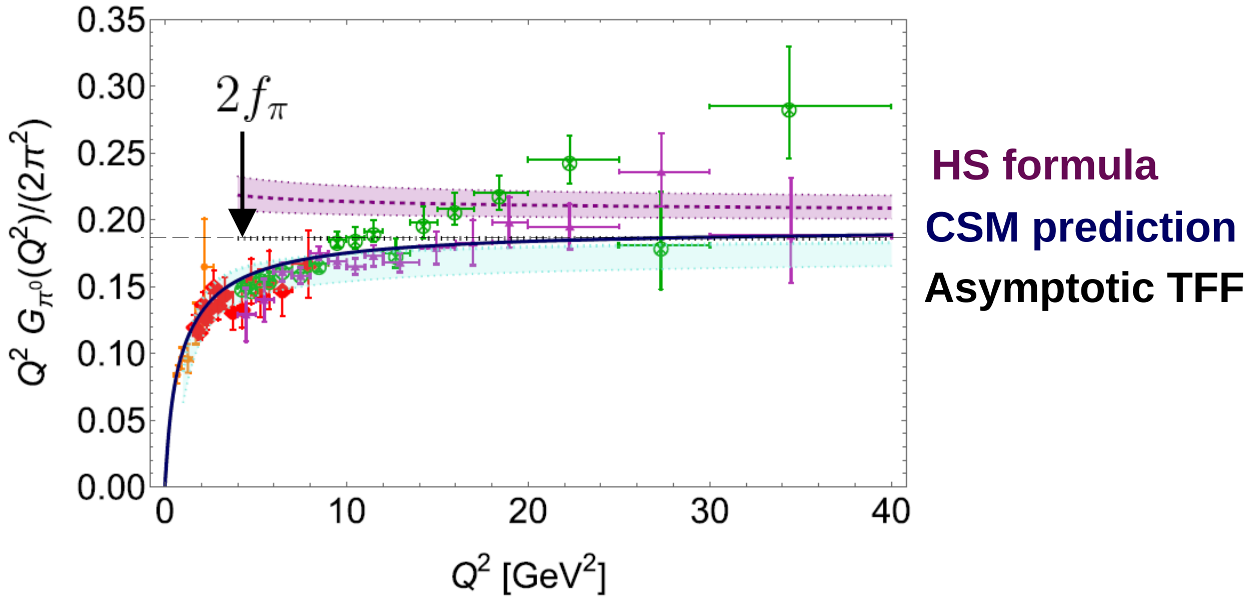
Two-photon **TFFs**

- In the **large-Q^2** regime, **QCD** connects the **EFF** and **DA**:



(at sufficiently large Q^2)

HS formula



- At **leading-order**:

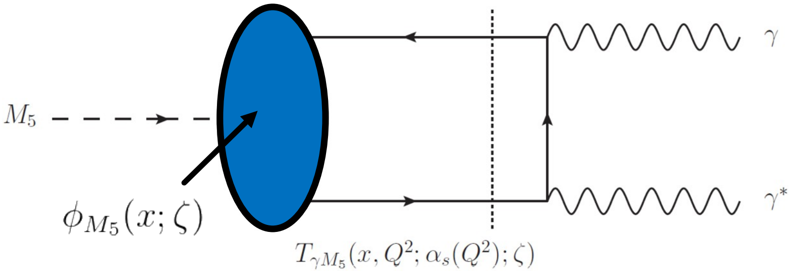
$$Q^2 G_{\mathbf{P}}^q(Q^2) \overset{Q^2 \gg m_p^2}{\approx} 12\pi^2\, f_{\mathbf{P}}^q\, e_q^2\, w_q(Q^2)$$

$$w_{\mathbf{P}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_{\mathbf{P}}(x;Q^2)$$ ← **PDA** →

- The asymptotic behavior is weighted by f_P, a measure of **EHM**.
- **Factorization/scaling violations** are proof of the validity of **QCD itself.**

Two-photon **TFFs**

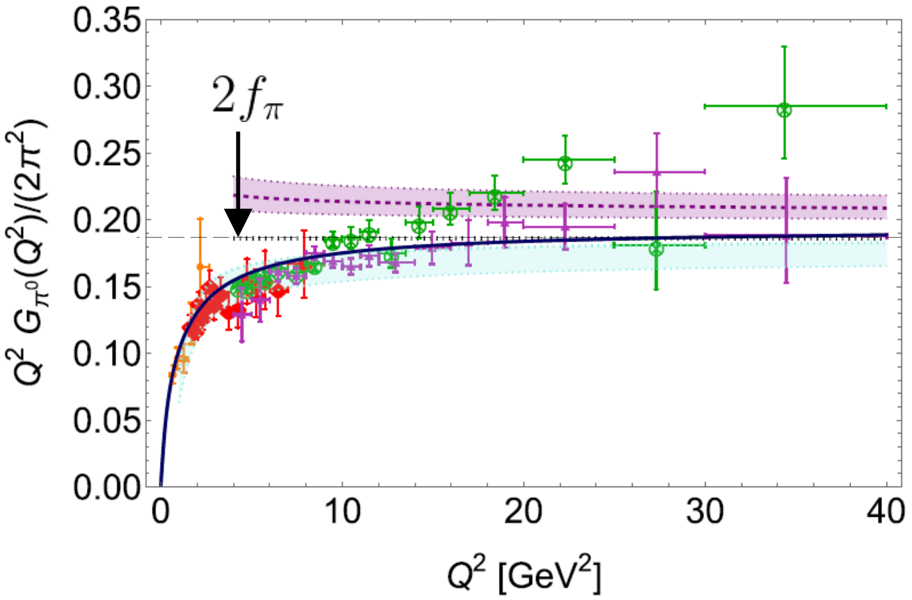
- In the **large-Q²** regime, **QCD** connects the **EFF** and **DA**:



HS formula

(at sufficiently large Q^2)

$$Q^2 G^q_{\mathbf{P}}(Q^2) \overset{Q^2 \gg m_p^2}{\approx} 12\pi^2 \, f^q_{\mathbf{P}} \, e_q^2 \, w_q(Q^2)$$



HS formula

CSM prediction

Asymptotic TFF

- In the **opposing** end, the **chiral *anomaly*** entails:

$$2 f^0_{\mathbf{P}} G^0_{\mathbf{P}^0}(Q^2 = 0) = 1$$

- Thus **EHM** (via **DCSB**) sets the infrared scale as well.
- Any deviations from this result are a measure of **EHM**+**HB** interplay.

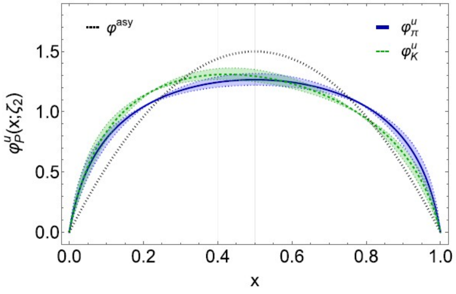
Pion **TFFs**

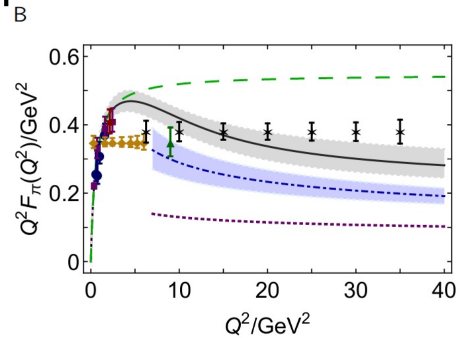
KR, L. Chang, A. Bashir *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

- The **CSM prediction** satisfies the **Abelian anomaly**, $2 f_\pi^0 G_{\pi^0}^0(Q^2=0)=1$

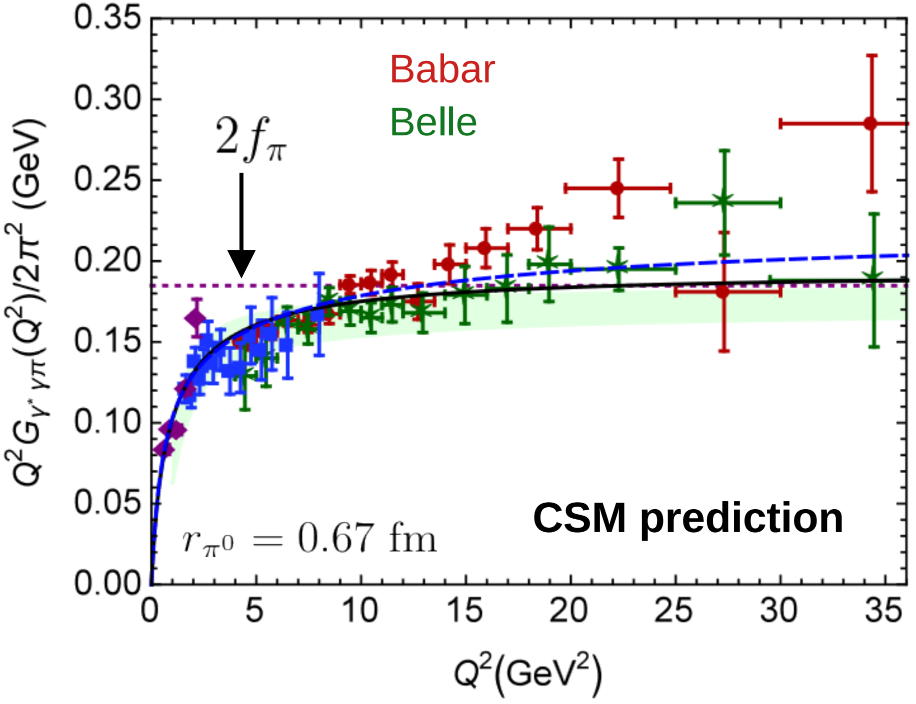
  … while faithfully recovering the **asymptotic limit**.

- A **dilated+concave DA**, at the hadronic scale, connects both pion **EFF** and **TFF**.





- Precise **agreement** with ***all*** experimental data; except for Babar at large **Q²**.



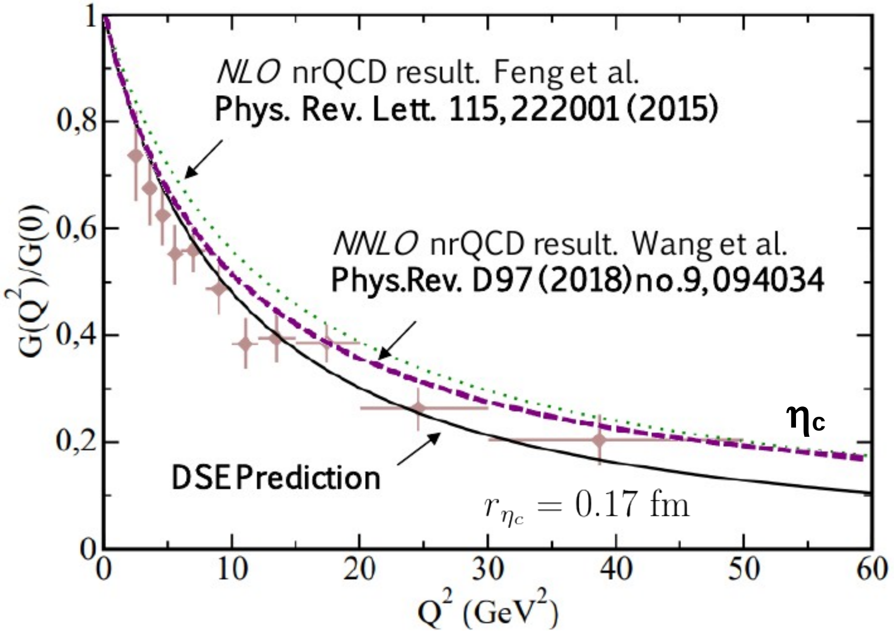
$$Q^2 G_{\mathbf{P}}^q(Q^2) \overset{Q^2 \gg m_p^2}{\approx} 12\pi^2 f_{\mathbf{P}}^q e_q^2 w_q(Q^2)$$

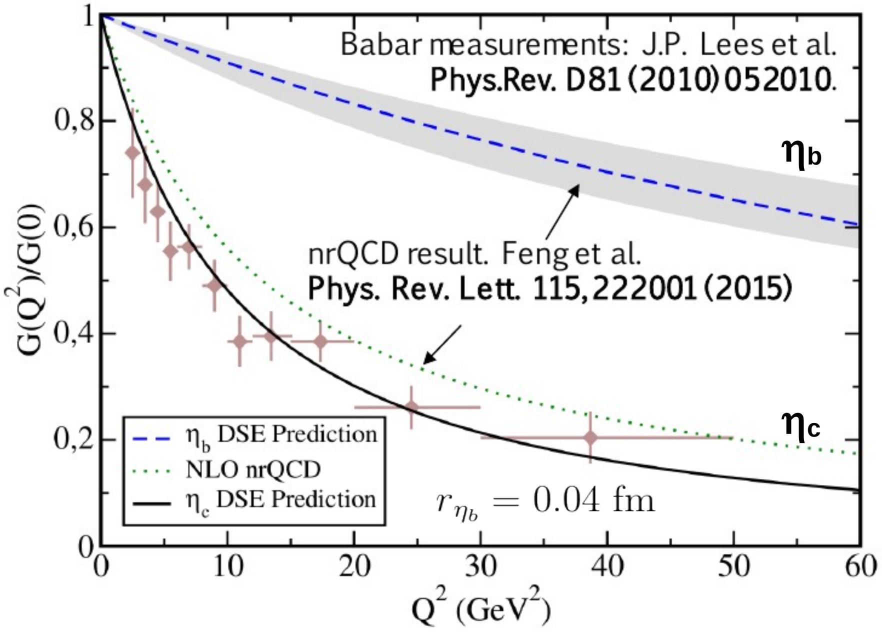
η_c–η_b TFFs

KR, M. Ding *et al.*
Phys.Rev.D 95 (2017) 7, 074014

- Studying the **heavy quarkonia** provide a complementary perspective.

(recall the narrow DAs)





- The η_c prediction shows agreement with available **experimental** data.
- In both η_c and η_b cases, there is compatibility with **nrQCD**.

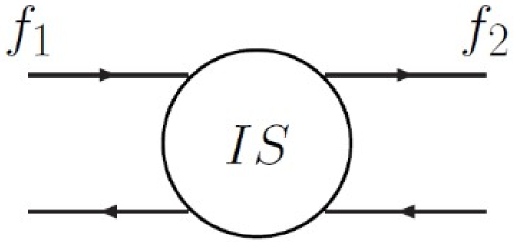
η–η′ TFFs

M. Ding, KR *et al.*
Phys.Rev.D 95 (2017) 7, 074014

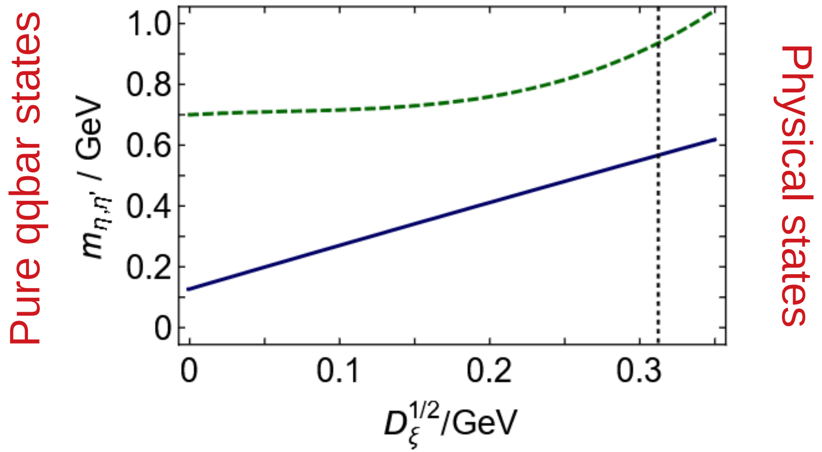
- In addressing the **η–η′** properties, interaction kernels need to consider the **non-Abelian** anomaly. Bhagwat:2007ha

$$[\Gamma_{\eta,\eta'}(P)]_{l_1 l_2} = \int_q [\mathcal{K}_{\rm L} + \mathcal{K}_N]^{l_1' l_2'}_{l_1 l_2}(P)[\chi_{\eta,\eta'}(q;P)]_{l_1' l_2'}$$

Ladder kernel — Anomaly kernel



- The strength of the **non-Abelian** anomaly dictates the **η–η′ mass splitting**, and the degree of **mixing**.



- Masses, decay **widths** and **TFFs** are sensitive to the effects of the **anomalies**.

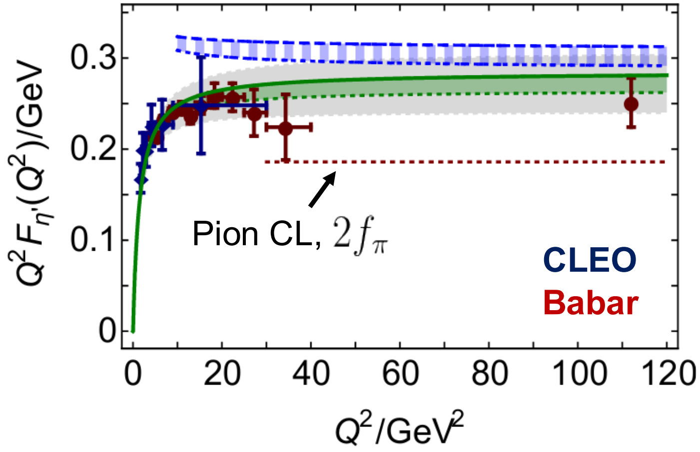
Meson	This Work	Experiment [5]
π^0	0.2753 (31)	0.2725 (29)
η	0.2562 (170)	0.2736 (60)
η'	0.3495 (60)	0.3412 (76)
η_c	0.0705 (40)	0.0678 (30)
η_b	0.0038 (2)	−−

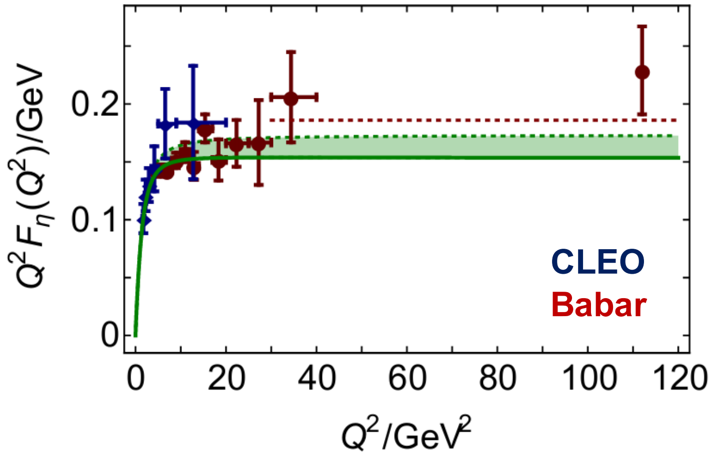
$$F_M(0,0) = \sqrt{\frac{4\Gamma_M^{\gamma\gamma}}{\pi\alpha_{em}^2 m_M^3}}$$

Raya:2019dnh

η–η' TFFs

M. Ding, KR *et al.*
Phys.Rev.D 95 (2017) 7, 074014



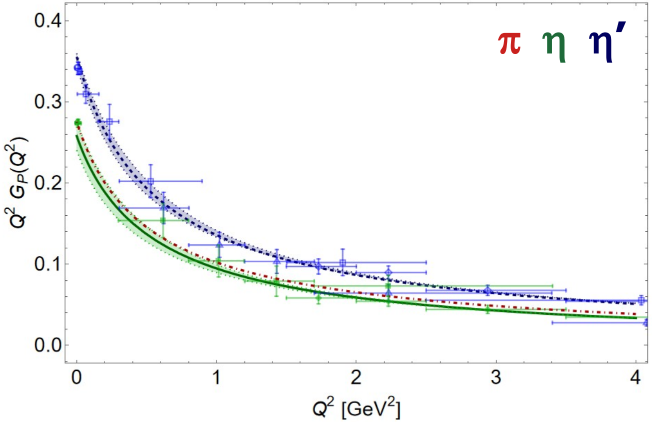


- The produced **TFFs** meet the **expectations** of the **chiral anomaly**

$$F_M(0,0) = \sqrt{\frac{4\Gamma_M^{\gamma\gamma}}{\pi\alpha_{em}^2 m_M^3}}$$

Meson	This Work	Experiment [5]
π^0	0.2753 (31)	0.2725 (29)
η	0.2562 (170)	0.2736 (60)
η'	0.3495 (60)	0.3412 (76)

- Are **plainly compatible** with the **experimental** data, in both **low** and **large-Q²** regions.



Domain of **interest** for **muon g-2** HLbL contributions !

$$a_\mu^{\pi^0-\text{pole}} = (6.14 \pm 0.21) \times 10^{-10}$$
$$a_\mu^{\eta-\text{pole}} = (1.47 \pm 0.19) \times 10^{-10}$$
$$a_\mu^{\eta'-\text{pole}} = (1.36 \pm 0.08) \times 10^{-10}$$

Raya:2019dnh

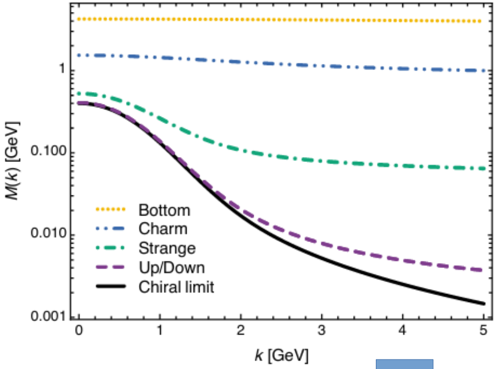
- **TFFs** also feature the **trends** expected from **pQCD**.

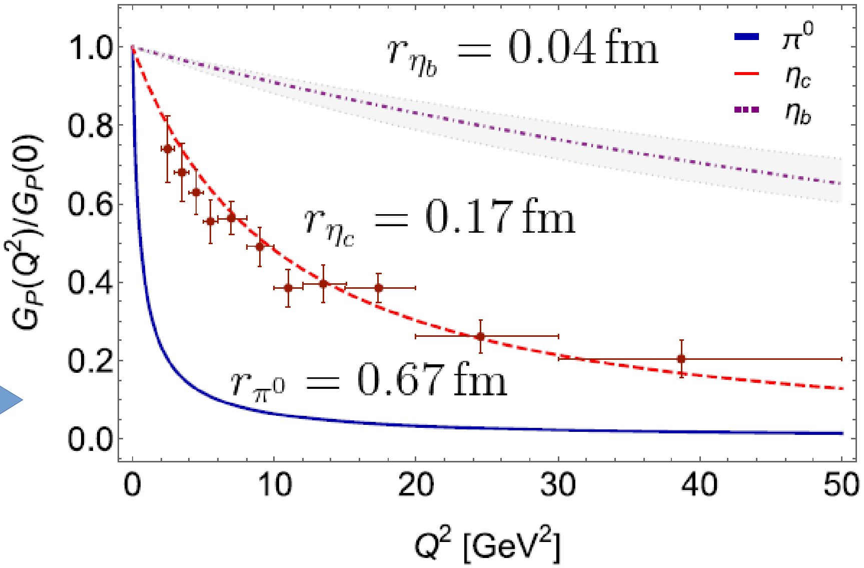
Two-photon TFFs

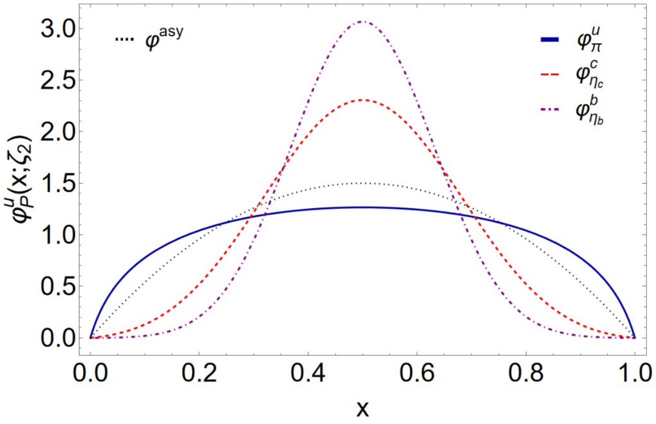
KR, A. Bashir, P. Roig
Phys.Rev.D 101 (2020) 7, 074021

- ***All*** two-photon **TFFs** involving ground-state neutral pseudoscalars are within reach:
  - Invariably, **agreement** with the **experimental** data is found, with the exception of the **large-Q^2** Babar data for the pion.
- Clearly, the shape of **M(k)** echoes in **TFFs** and **DAs.**

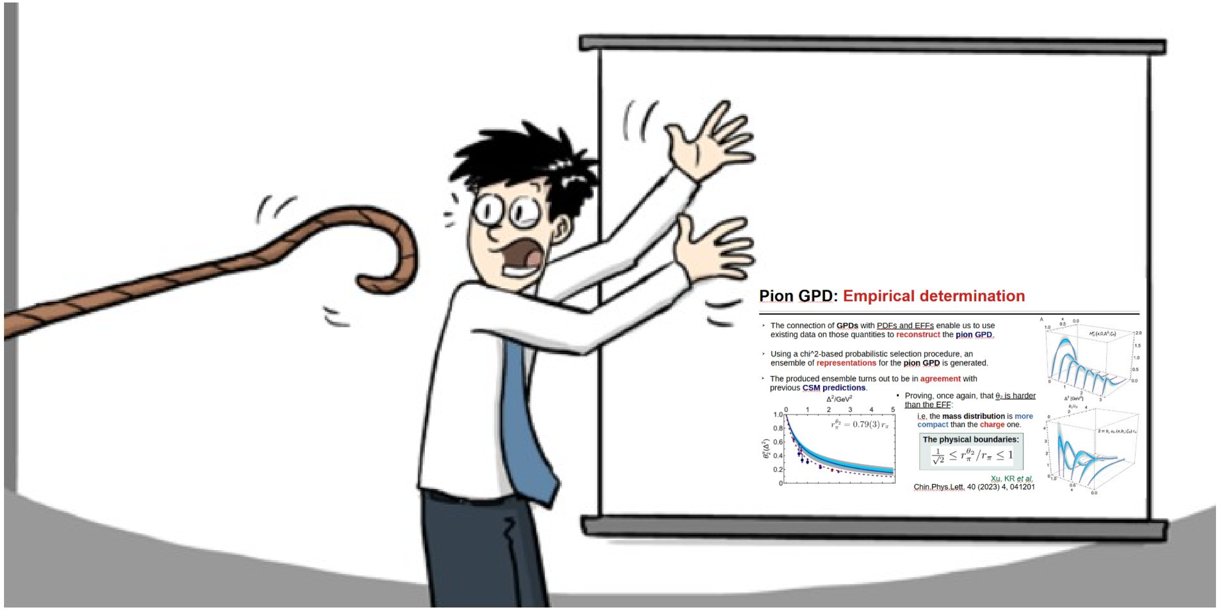
Exposing the **impact** of the **mass generation** mechanisms







Final **Highlights**

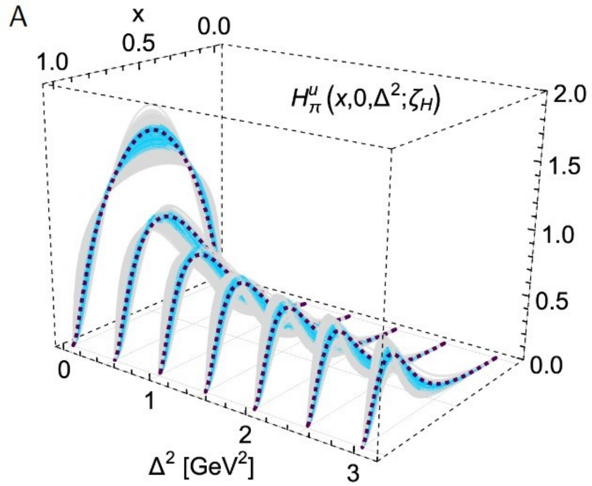


Final **Highlights**

- The **emergent phenomena** in **QCD** produces unique outcomes:
  - The degrees-of-freedom are not directly accessible, we get to observe hadrons (**confinement**).
  - Through their own mechanisms, **dynamical mass generation** is present in both **matter** and **gauge** sectors of QCD; the later yielding a running **coupling** that saturates at infrared momenta.

- **Pseudoscalar** mesons are an ideal platform to inquire on these facets of **QCD**:
  - Their mere **existence and properties** are connected with the **mass generation** in the Standard Model and, potentially, **confinement**.
  - Modern facilities are **capable** to address the properties of **NG bosons** and it's connection with QCD's emergent phenomena.

- Theory has evolved to the point where **all sorts of** parton **distributions** of <u>pseudoscalar</u> mesons are **within reach**.
  - TFFs are valuable as they encode symmetries, their breaking, scaling violations, and the transition between soft and hard scales.



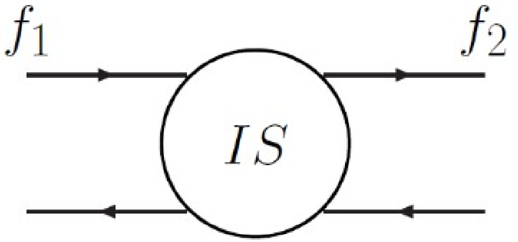
η–η′ BSE

M. Ding, KR *et al.*
Phys.Rev.D 95 (2017) 7, 074014

- In addressing the **η–η′** properties, interaction kernels need to consider the **non-Abelian** anomaly. Bhagwat:2007ha

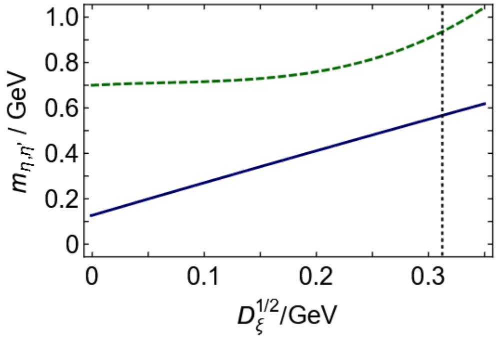
$$[\Gamma_{\eta,\eta'}(P)]_{l_1 l_2} = \int_q [\mathcal{K}_{\rm L} + \mathcal{K}_N]_{l_1 l_2}^{l_1' l_2'}(P)[\chi_{\eta,\eta'}(q;P)]_{l_1' l_2'}$$

Ladder kernel Anomaly kernel



$$\mathcal{K}_N = \xi(s)\left[\cos^2\theta_\xi [z\gamma_5]_{l_2' l_1'}[z\gamma_5]_{l_2 l_1} + \sin^2\theta_\xi [z\gamma_5\gamma\cdot P]_{l_2' l_1'}[z\gamma_5\gamma\cdot P]_{l_2 l_1}/M_l^2\right]$$

$$\xi(s) = \frac{8\pi^2}{\omega_\xi^4} D_\xi e^{-s/\omega_\xi^2} \qquad z = {\rm diag}[1, 1, M_l/M_s]$$



$$\phi = 42.8^\circ,$$
$$f^l = 0.101\,{\rm GeV} = 1.08\,f_\pi,$$
$$f^s = 0.138\,{\rm GeV} = 1.49\,f_\pi,$$

	f_η^l	f_η^s	$f_{\eta'}^l$	$f_{\eta'}^s$
herein - direct	0.072	−0.092	0.070	0.104
herein - Eq. (28)	0.074	−0.094	0.068	0.101
phen. [36, 81, 82]	0.090(13)	−0.093(28)	0.073(14)	0.094(8)

Quark-photon Vertex

KR, L. Chang *et al.*,
Phys.Rev.D 93 (2016) 7, 074017

- The **QPV** should fulfill (at least) the following:
  - ✔ Longitudinal WGTI
  - ✔ Free of kinematic singularities
  - ✔ Recover the point-particle limit
  - ✔ Produce the abelian anomaly
- Should **expedite** the computation of the **TFFs**

- We adopt the following **Ansatz**:

$$\begin{aligned}\chi_\mu(k_f,k_i) =&\gamma_\mu \Delta_{k^2\sigma_V} \\ &+ [s\,\gamma\cdot k_f \gamma_\mu \gamma\cdot k_i + \bar{s}\gamma\cdot k_i\gamma_\mu\gamma\cdot k_f]\Delta_{\sigma_V} \\ &+ [s\,(\gamma\cdot k_f\gamma_\mu + \gamma_\mu\gamma\cdot k_i) \\ &\quad + \bar{s}\,(\gamma\cdot k_i\gamma_\mu + \gamma_\mu\gamma\cdot k_f)]\, i\Delta_{\sigma_S}\,,\end{aligned}$$

- Where: $\Delta_F = [F(k_f^2) - F(k_i^2)]/[k_f^2 - k_i^2]$

- The **transverse** pieces makes it possible to recover the **Abelian anomaly**, via:

$$s = 1 + s_0 \exp(-\mathcal{E}_\pi/M_E)$$

$$\mathcal{E}_\pi = Q/2$$

$$M_E = \{p | p^2 = M^2(p^2), p > 0\}$$

- The value $\mathbf{s_0}$ is fixed so that the TFF is properly normalized in line with the **predicted decay width**:

$$\Gamma^{\gamma\gamma}_{\eta_{c,b}} = \frac{8\pi\alpha_{em}^2 \mathrm{c}^4_{\eta_{c,b}} f^2_{\eta_{c,b}}}{m_{\eta_{c,b}}}$$

$$\Gamma^{\gamma\gamma}_{\eta,\eta'} = \frac{9\alpha_{em}^2 m^3_{\eta,\eta'}}{64\pi^3}\left[c_l \frac{f^l_{\eta,\eta'}}{(f^l)^2} + c_s \frac{f^s_{\eta,\eta'}}{(f^s)^2}\right]^2$$

$$F_M(0,0) = \sqrt{\frac{4\Gamma_M^{\gamma\gamma}}{\pi\alpha_{em}^2 m_M^3}}$$