



Andrea Simonelli

With A. Accardi, M. Cerutti, C. Costa, A. Signori

Unveiling the Collins-Soper kernel in  
inclusive DIS at threshold and  
implications at Jlab 22



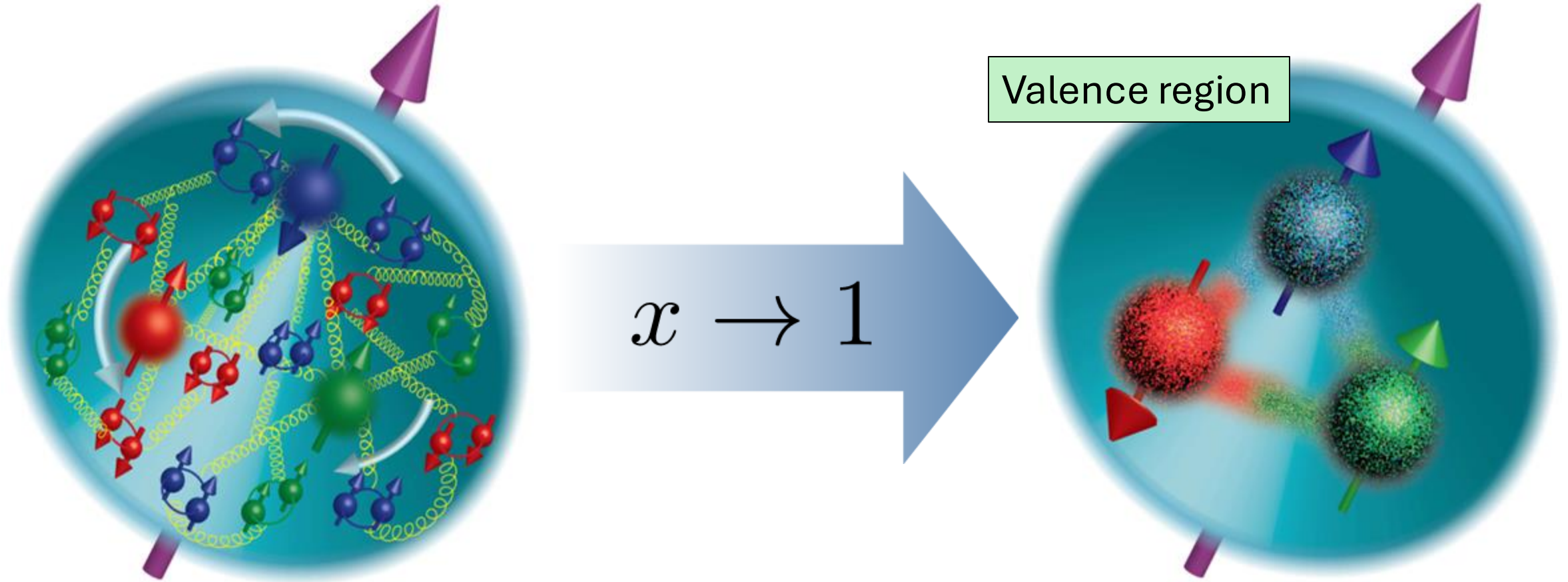
**Istituto Nazionale di Fisica Nucleare**

Sezione di Roma



# PDFs in the large-x region

Why relevant?

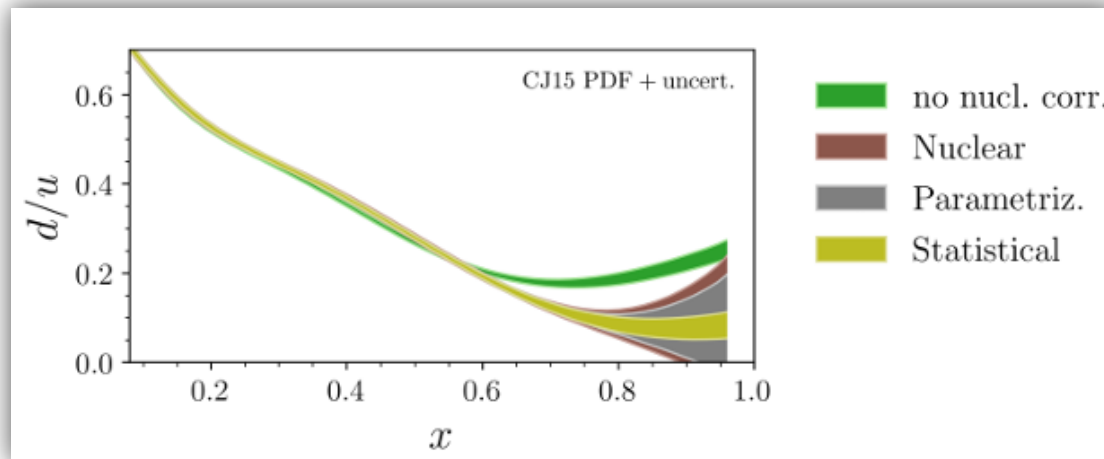


Images courtesy of A. Accardi  
et al, *Eur.Phys.J.A* 52 (2016) 9, 268

# PDFs in the large- $x$ region

## Why relevant?

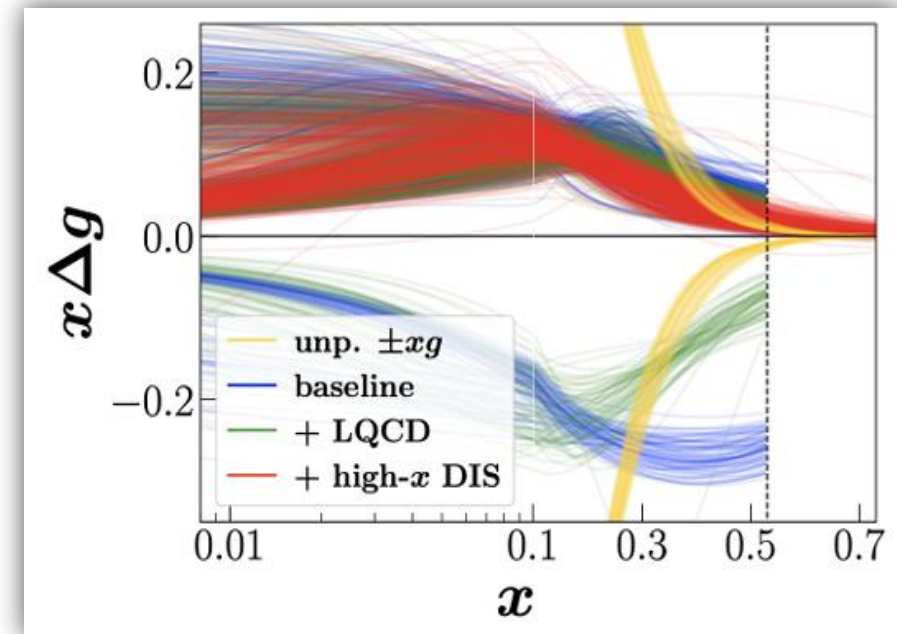
- $d/u$  ratio for investigating confinement



Li, Accardi et al, *PRD 109* (2024)

- Theoretical constraints (positivity bounds)

N. Sato @ PDFLattice 2024

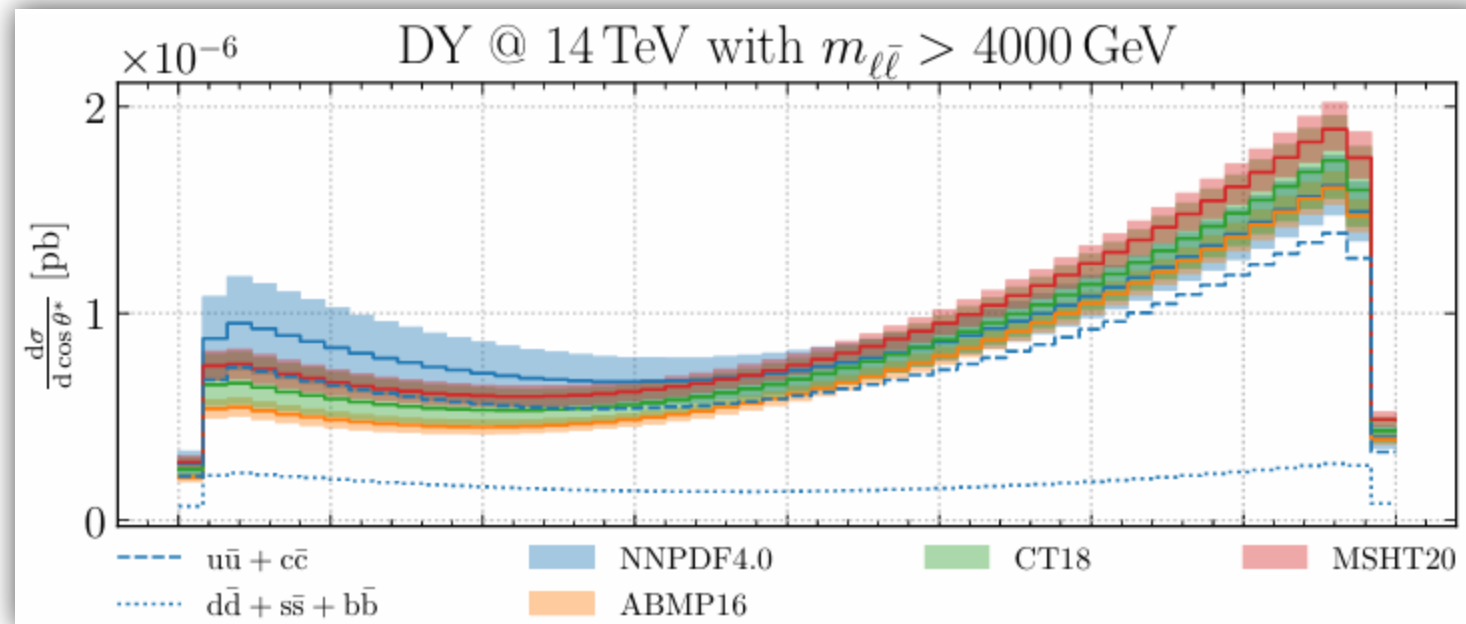


A. Simonelli

# PDFs in the large-x region

## Why relevant?

- Beyond Standard Model searches
- Forward facilities (LHC)



R. Ball et al, *Eur.Phys.J.C* 82 (2022) 12, 1160

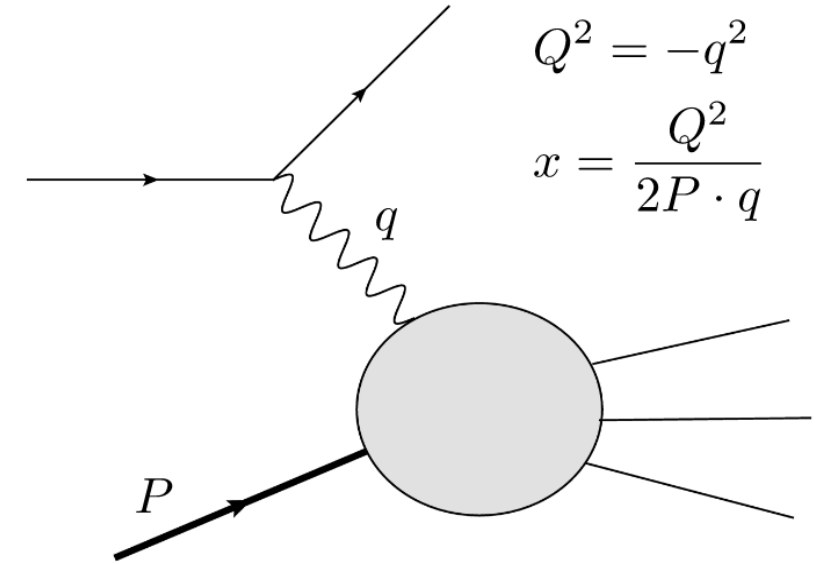


# PDFs in the large-x region

How to access them?

## DIS at threshold

(simplest case)



- High precision data from Jefferson Lab

**Strong interaction physics at the luminosity frontier with 22 GeV electrons at Jefferson Lab**

Review | Published: 04 September 2024

Volume 60, article number 173, (2024) [Cite this article](#)

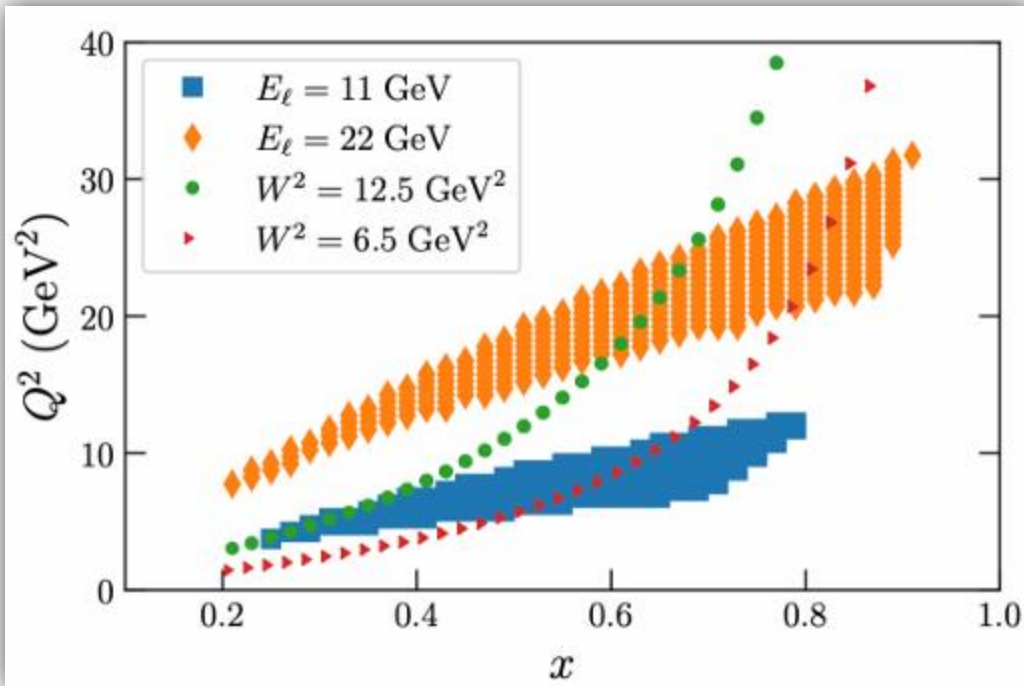
# PDFs in the large-x region

Hall C (11 GeV), 2409.15236

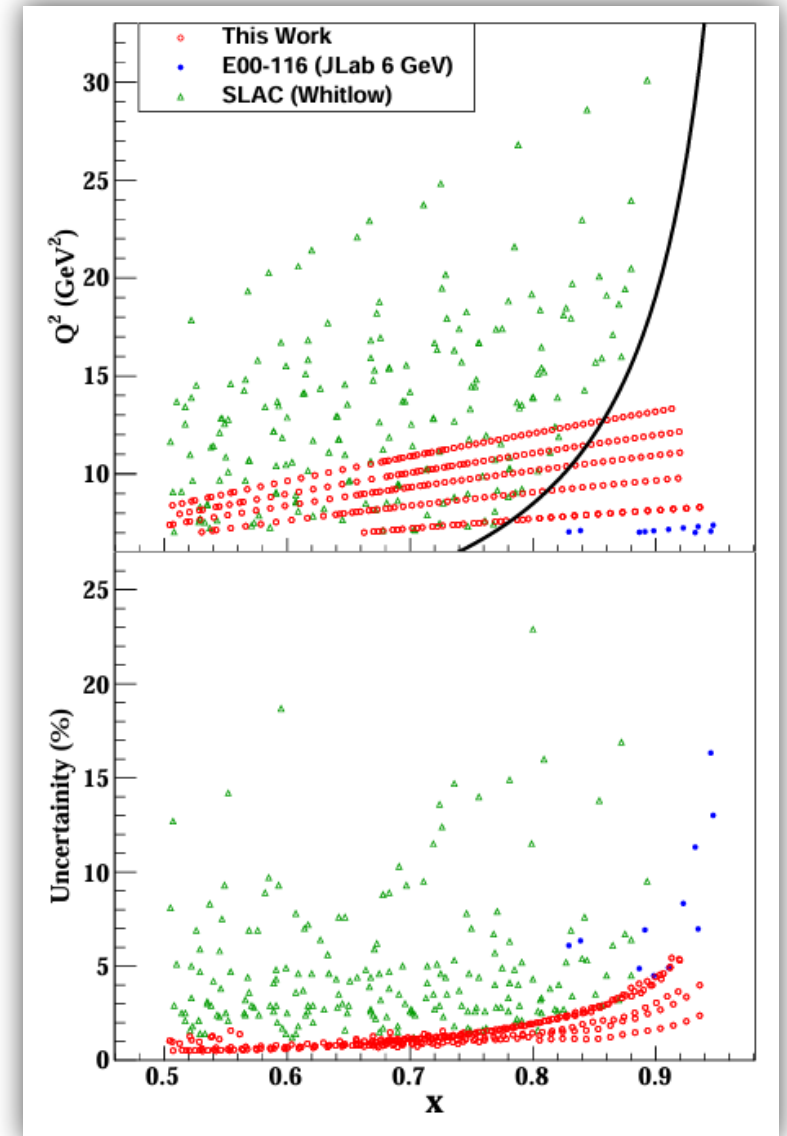
How to access them?

## DIS at threshold

(simplest case)



$$x \lesssim 0.9 !$$



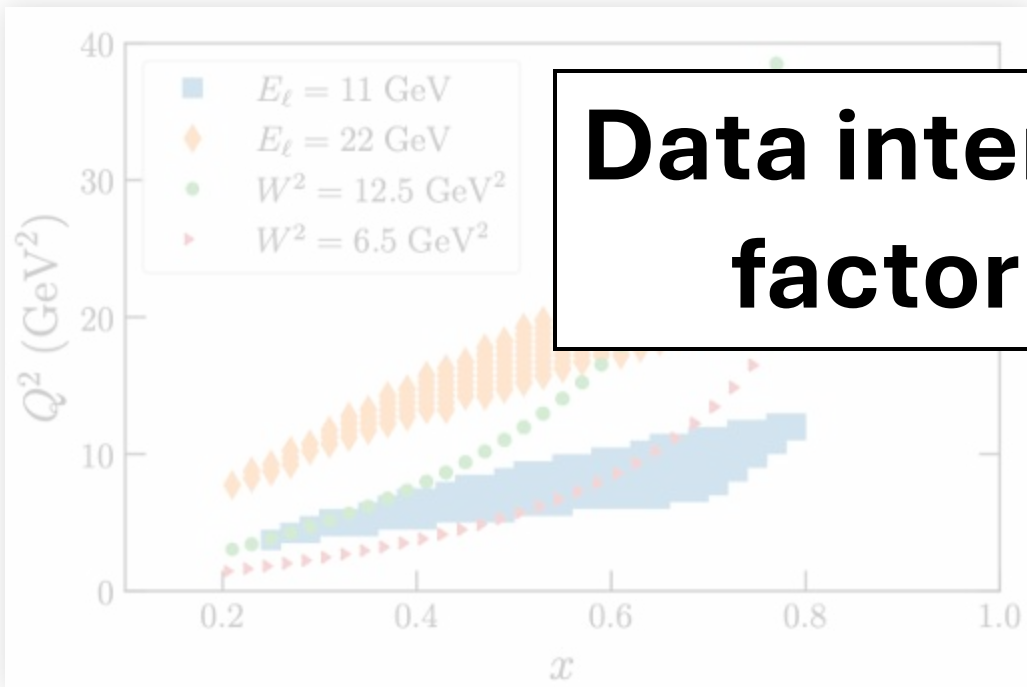
# PDFs in the large-x region

How access them?

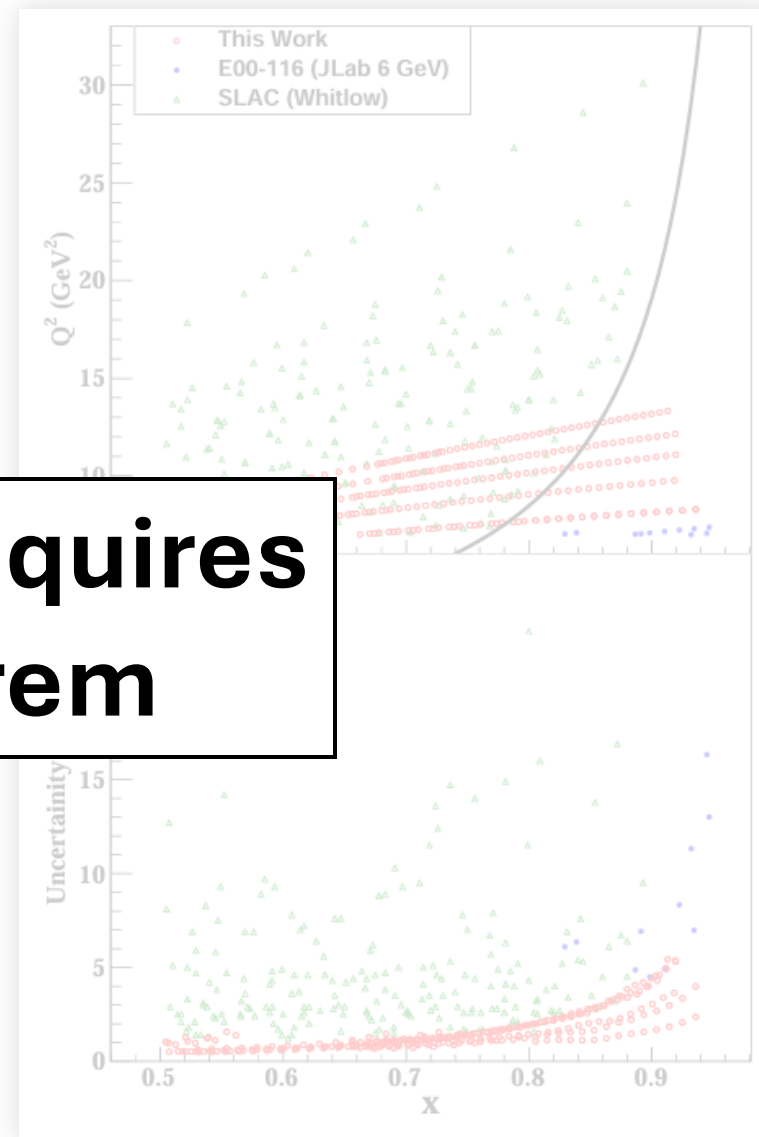
## DIS at threshold

(simplest case)

**Data interpretation requires factorization theorem**



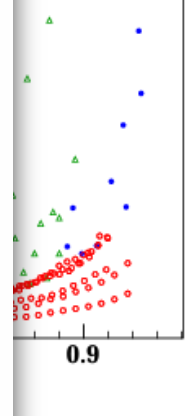
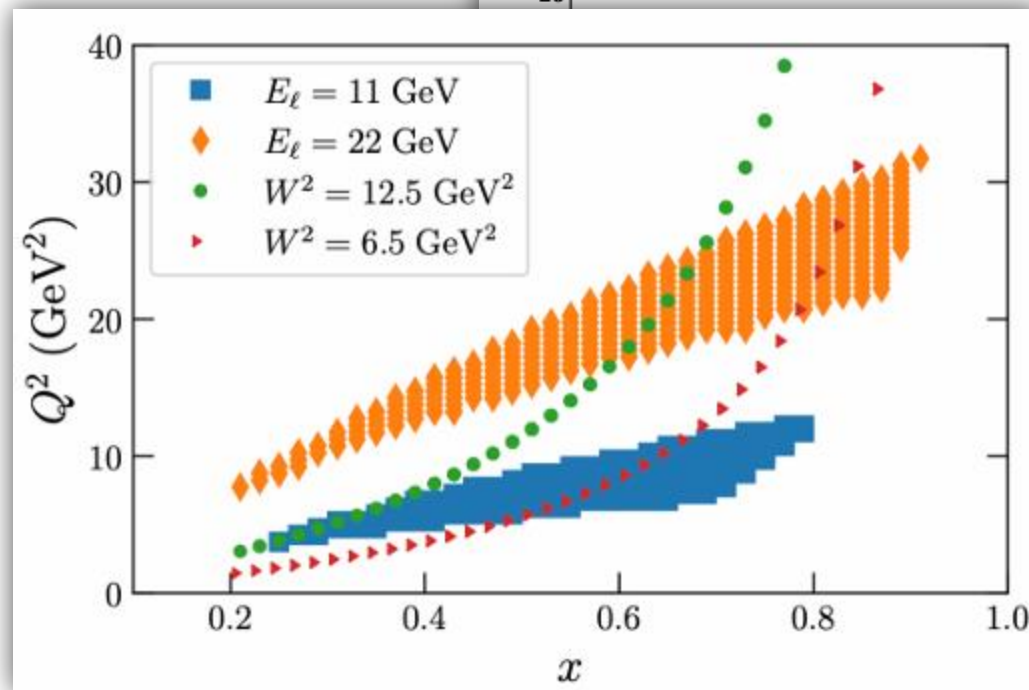
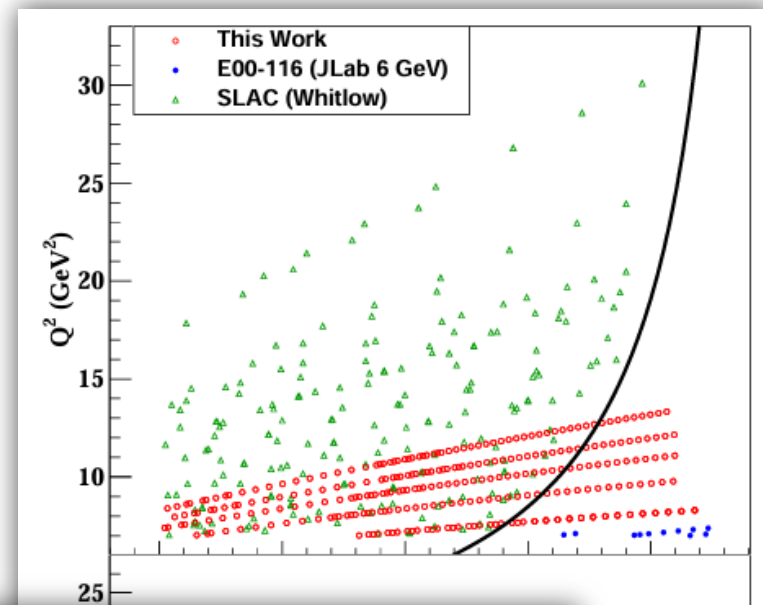
$x \lesssim 0.9!$



Can't we just use the usual DIS collinear factorized cross section?

**NO!**

The kinematics is different!





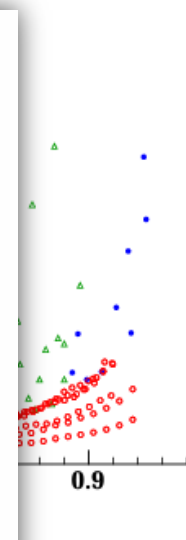
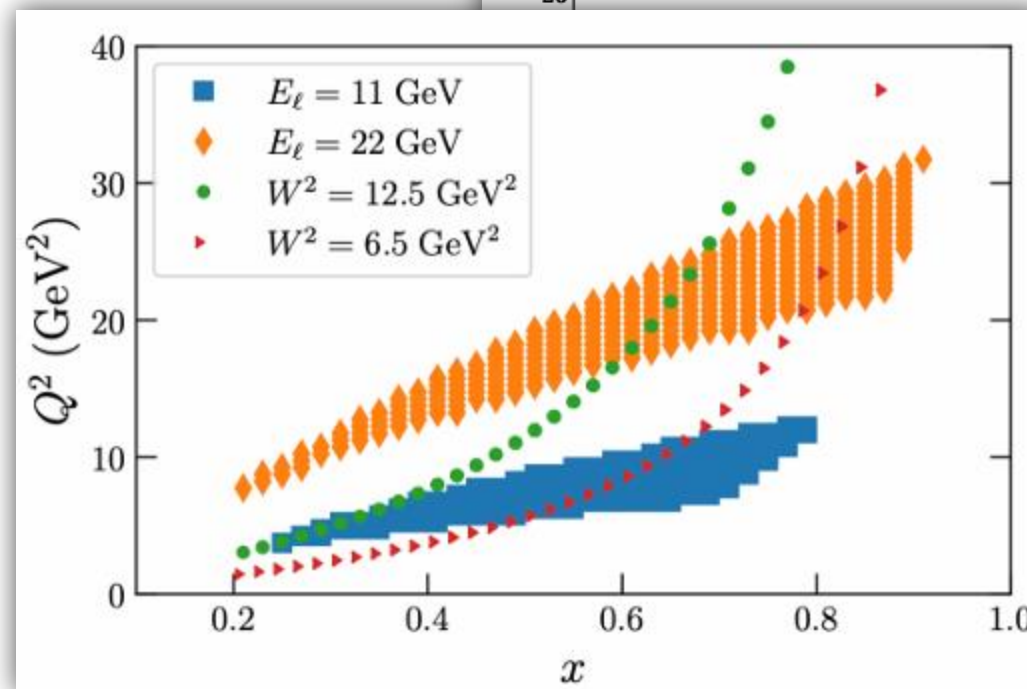
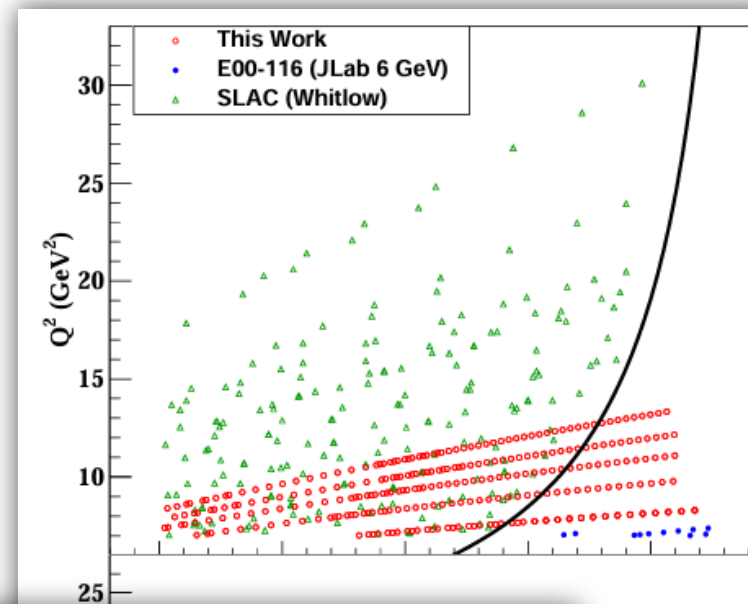
Can't we just use the usual DIS collinear factorized cross section?

Same question

**NO!**

The kinematics is different!

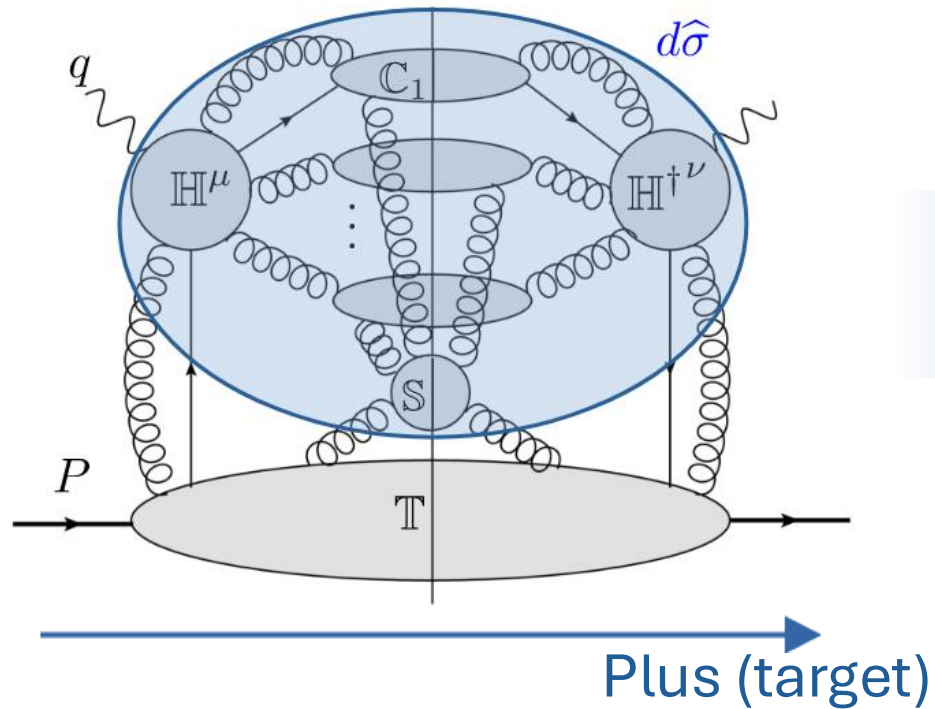
Can't we just use the collinear factorization (SIDIS) for describing low  $q_T$  data?



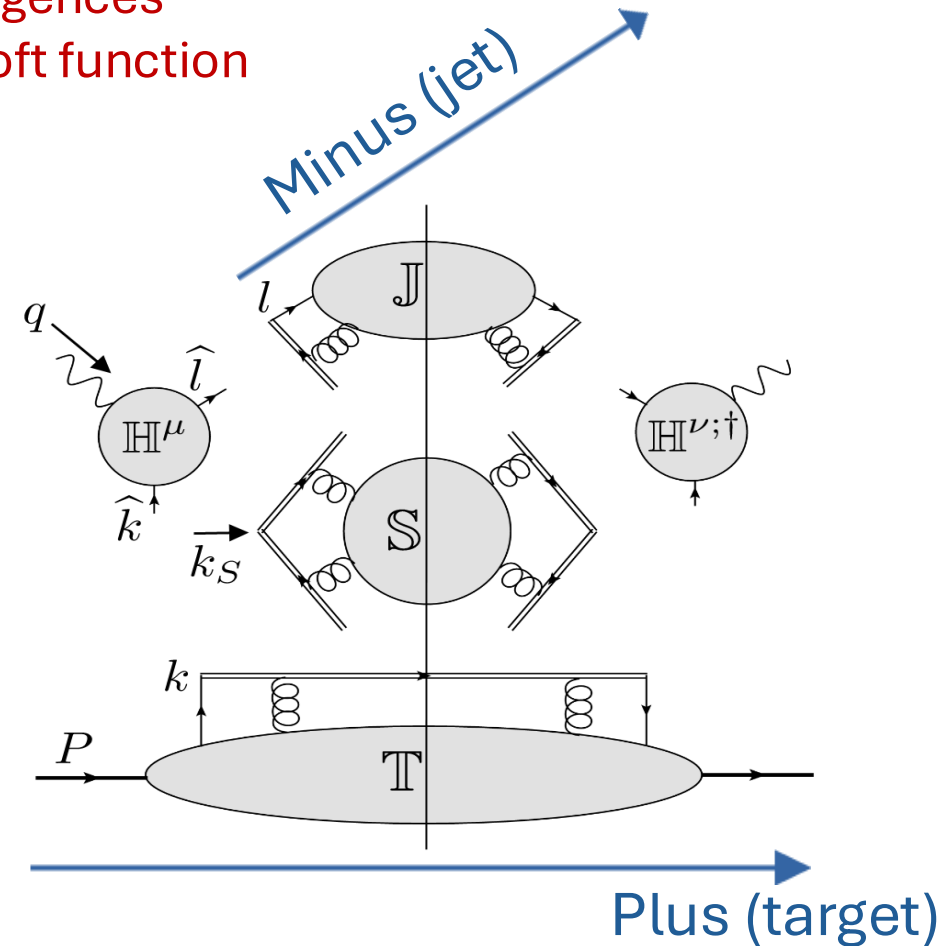
# Analogies with TMD case

Same problems of TMD case:

- Rapidity divergences
- Correlating soft function



$$x \rightarrow 1$$



- One single (approximately) light-cone direction
- One hard scale  $Q^2$

- Two (approximately) light-cone directions
- One hard scale  $Q^2$  + One IR scale  $W$

# Analogies with TMD case

Same problems of TMD case:

- Rapidity divergences
- Correlating soft function

The validity of factorization *was* not clear

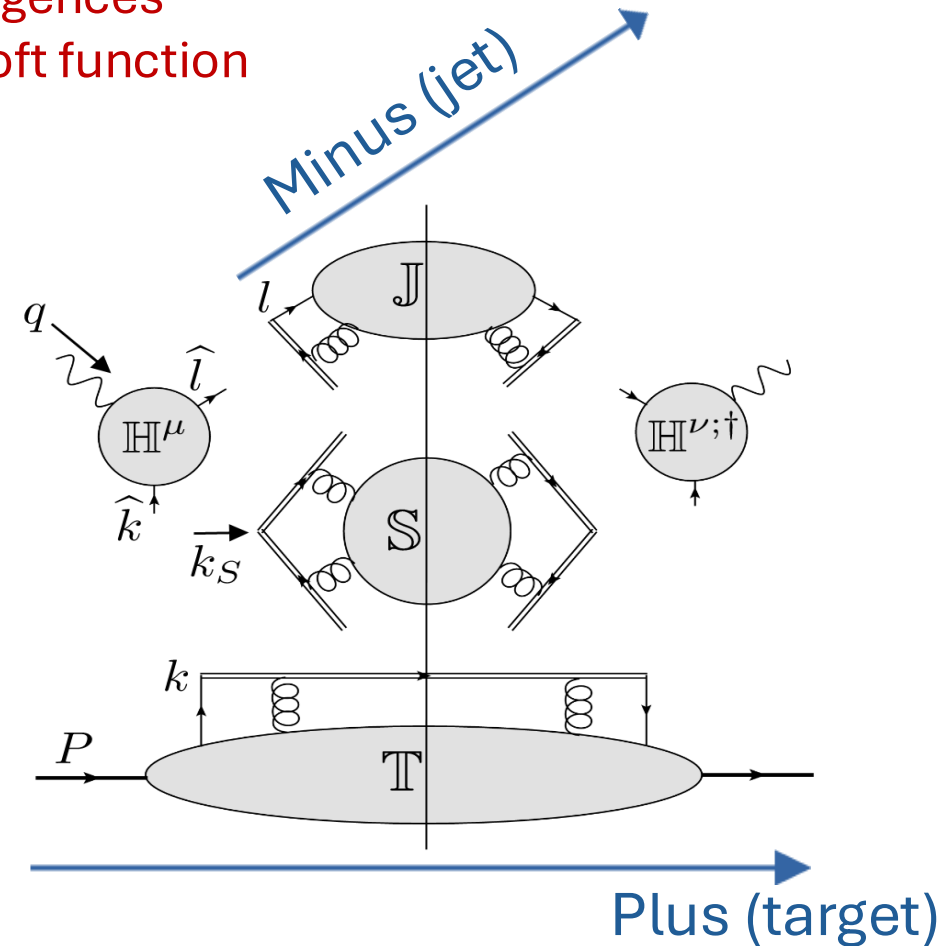
[rapidity anomalous dimensions] reveal sensitivity to IR scales, which may signal a **breakdown** of rapidity factorization in SCET<sub>II</sub>.

[Fleming, Labun (2012)]

...but now it is! (coming very soon)

Unveiling the Collins-Soper kernel in inclusive DIS at threshold

Andrea Simonelli,<sup>1,2,\*</sup> Alberto Accardi,<sup>3,1,†</sup> Matteo Cerutti,<sup>4,1,‡</sup> Caroline S. R. Costa,<sup>1,5,§</sup> and Andrea Signori<sup>6,7,¶</sup>



- Two (approximately) light-cone directions
- One hard scale  $Q^2$  + One IR scale  $W$

# Factorization for DIS at threshold

$$W^{\mu\nu} = N_C \mathbf{u}^{\mu\nu} H(\mu, Q) \sum_j \int \frac{dN}{2\pi i} x^{-N} \times \hat{\phi}_{1,j/h}^{\text{sqrt.}}(N, \mu, y_n) \hat{\mathcal{J}}_j^{\text{sqrt.}}(N, \mu, y_n)$$

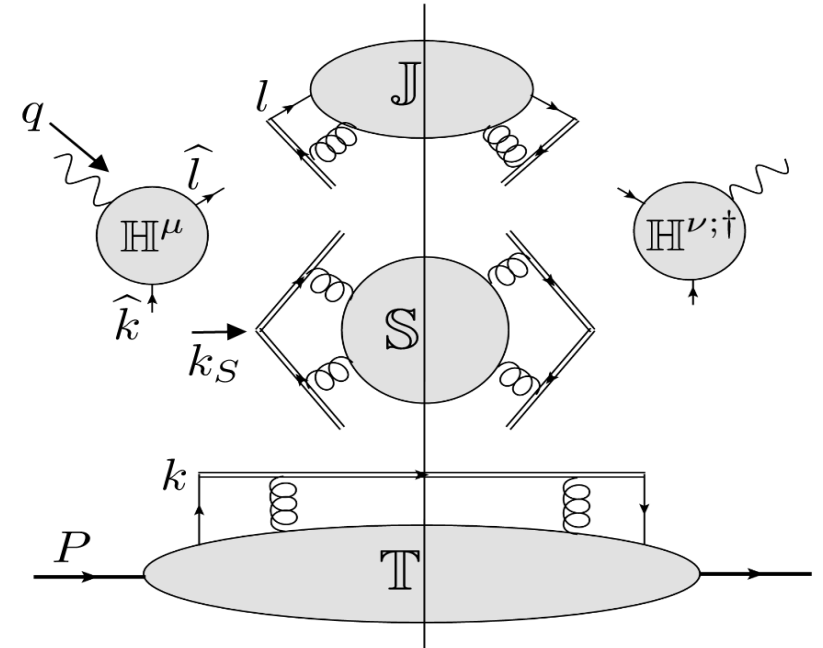
Square root of soft factors included (subtractions, rapidity divergences)

Off light-cone target function

Off light-cone Jet function

$$\frac{\partial}{\partial y_n} \log \hat{\mathcal{J}}^{\text{sqrt.}}(N, \mu, y_n) = +K(a_S(\mu), L_N + y_n)$$

$$\frac{\partial}{\partial y_n} \log \hat{\phi}^{\text{sqrt.}}(N, \mu, y_n) = -K(a_S(\mu), L_N + y_n)$$



The Collins-Soper kernel appears  
New venues to access it (lattice)

# Factorization for DIS at threshold

$$W^{\mu\nu} = N_C \mathbf{u}^{\mu\nu} H(\mu, Q) \sum_j \int \frac{dN}{2\pi i} x^{-N} \\ \times \hat{\phi}_{1,j/h}^{\text{sqrt.}}(N, \mu, y_n) \hat{\mathcal{J}}_j^{\text{sqrt.}}(N, \mu, y_n)$$

**Off light-cone**  
target function

**Off light-cone**  
Jet function

$$\frac{\partial}{\partial y_n} \log \hat{\mathcal{J}}^{\text{sqrt.}}(N, \mu, y_n) = +K(a_S(\mu), L_N + y_n) \\ \frac{\partial}{\partial y_n} \log \hat{\phi}^{\text{sqrt.}}(N, \mu, y_n) = -K(a_S(\mu), L_N + y_n)$$

How to get back to usual  
light-cone operators  
(threshold PDF)?

The Collins-Soper kernel appears  
New venues to access it (lattice)



# Factorization for DIS at threshold

$$W^{\mu\nu} = N_C \mathbf{u}^{\mu\nu} H(\mu, Q) \sum_j \int \frac{dN}{2\pi i} x^{-N} \\ \times \hat{\phi}_{1,j/h}^{\text{sqrt.}}(N, \mu, y_n) \hat{\mathcal{J}}_j^{\text{sqrt.}}(N, \mu, y_n)$$

**Off light-cone**  
target function

**Off light-cone**  
Jet function

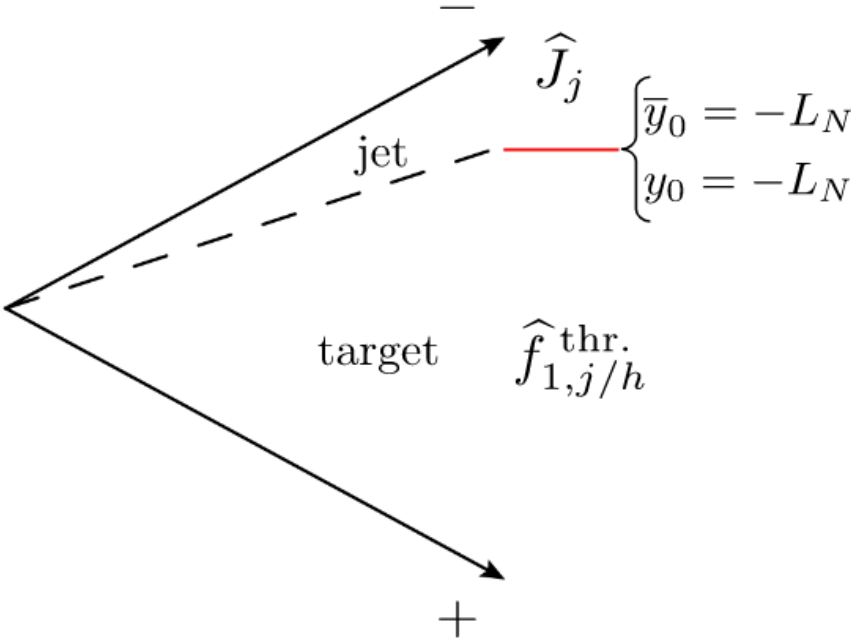
How to get back to usual light-cone operators (threshold PDF)?

$$y_n = -L_N$$

$$\frac{\partial}{\partial y_n} \log \hat{\mathcal{J}}^{\text{sqrt.}}(N, \mu, y_n) = +K(a_S(\mu), L_N \oplus y_n) \\ \frac{\partial}{\partial y_n} \log \hat{\phi}^{\text{sqrt.}}(N, \mu, y_n) = -K(a_S(\mu), L_N \oplus y_n)$$

Exploiting the inclusivity of DIS

# Factorization for DIS at threshold



How to get back to usual light-cone operators (threshold PDF)?

$$y_n = -L_N$$

$$\hat{\mathcal{J}}_i^{\text{sqrt}}(N, \mu, -L_N) = C(a_S(\mu), L_N) \hat{J}_j(N; \mu)$$

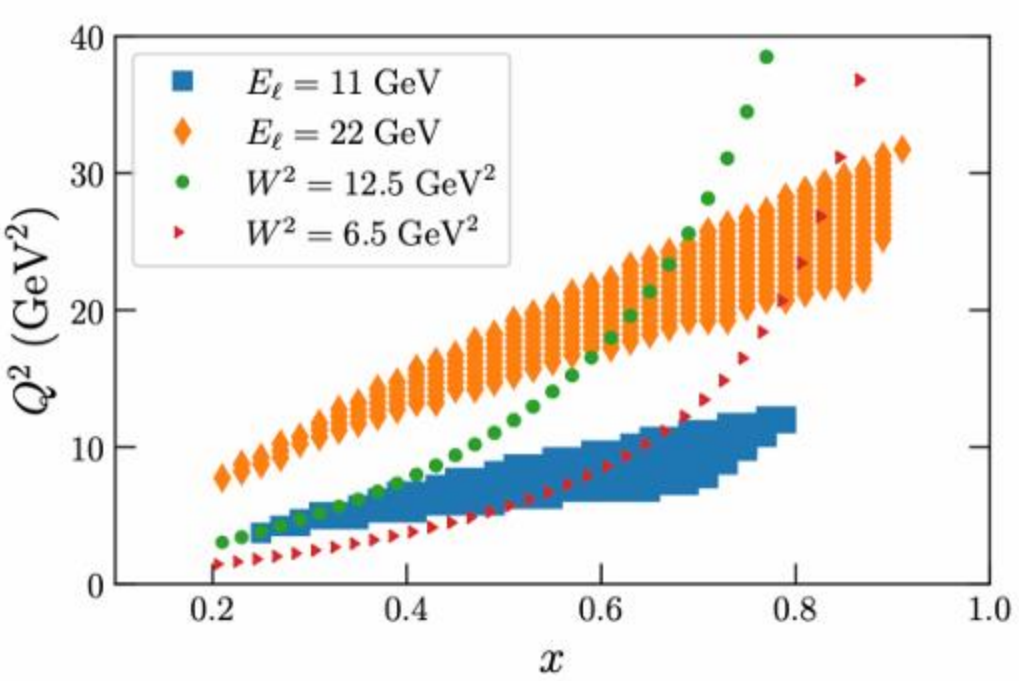
$$\hat{\phi}_j^{\text{sqrt}}(N, \mu, -L_N) = \frac{\hat{f}_j(N, \mu)}{C(a_S(\mu), L_N)}$$

Matching to light-cone operators

# Factorization for DIS at threshold

$$W^{\mu\nu} = N_C \mathbf{u}^{\mu\nu} H(\mu, Q) \sum_j \int \frac{dN}{2\pi i} x^{-N} \hat{f}_j(N, \mu) \hat{J}_j(N, \mu)$$

Naïve intuition on factorization confirmed



**Threshold PDF**

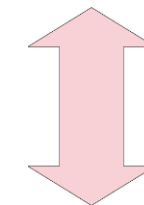
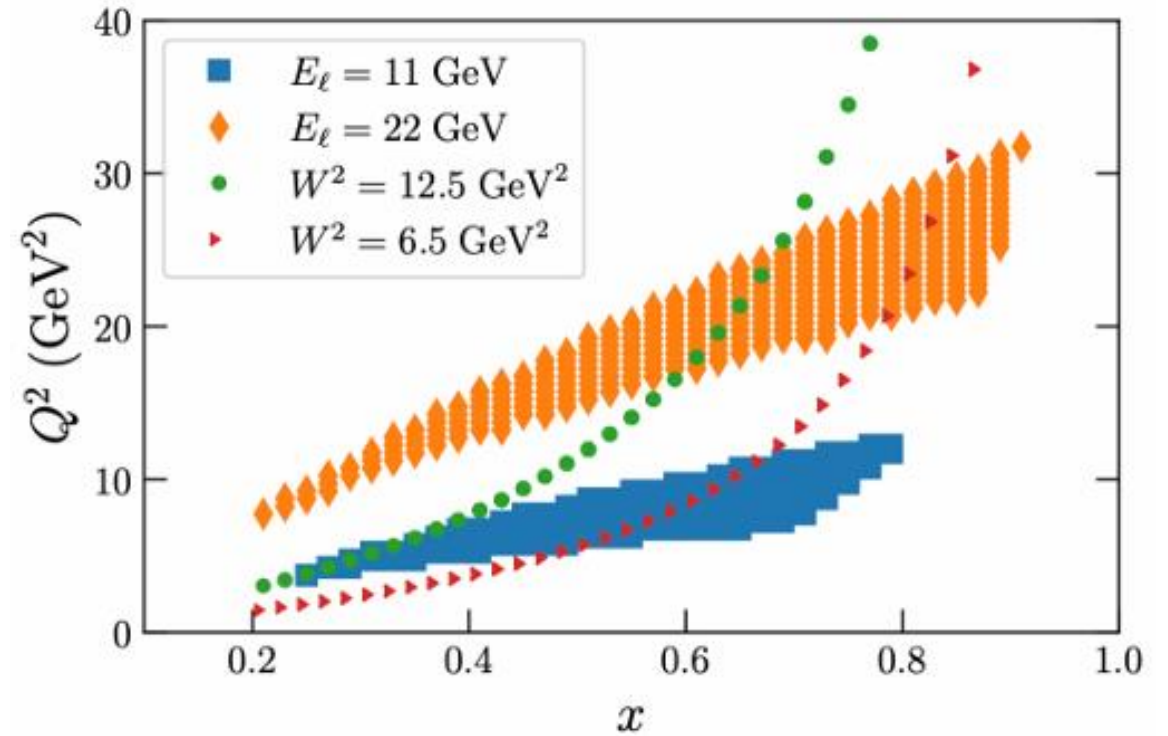
**Jet Function**

Now (and only now) we are ready for phenomenology

# Conclusions

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- All the problems/issues with the factorization of DIS at threshold are now solved.
- The analogies with the TMD case are made transparent and used as tools to get the final result.
- The Collins-Soper kernel appears!
- The theory ground for threshold DIS is now solid and ready for phenomenology to be built on it.



Determination of large- $x$   
behavior of PDFs