Unified model for particles and condensate Dark Matter

The Role of self-interaction

Alex Soto

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Phys.Rev.D 108 (2023) 8, 083513 (ArXiv:2303.02049 [astro-ph.CO])
Phys.Rev.D 110 (2024) 2, 023504 (ArXiv:2311.05280 [astro-ph.CO])
ArXiv:2407.xxxxx (To Appear soon)

Newcastle University

Motivation for Ultra-Light DM

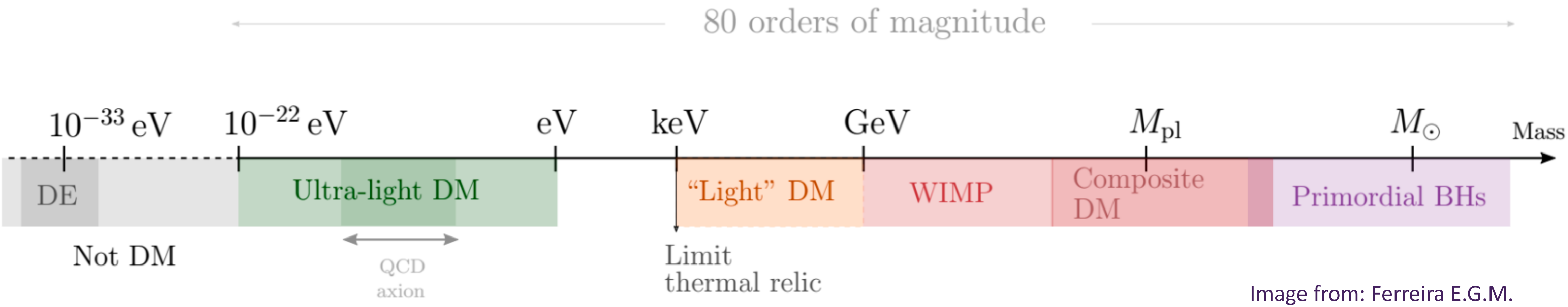
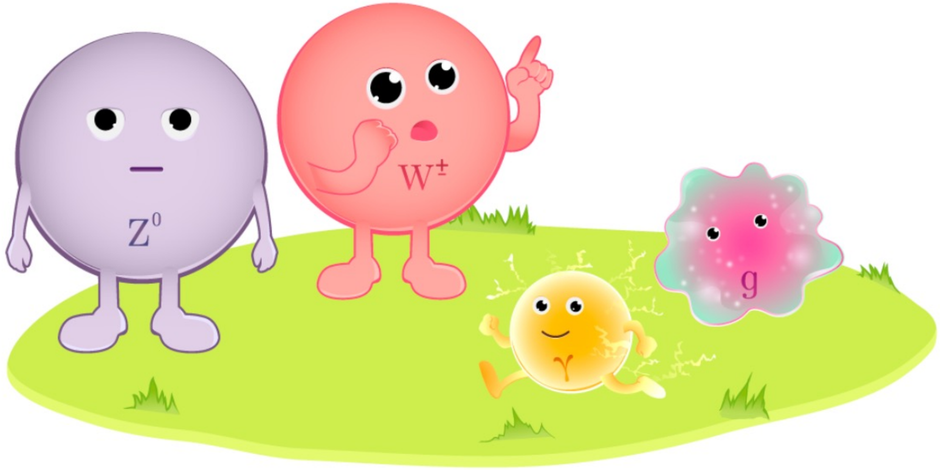


Image from: Ferreira E.G.M.
Astron Astrophys Rev **29,** 7 (2021),
arXiv:2005.03254 **[astro-ph.CO]**.

Motivation for Ultra-Light DM



Bosons

- QCD Axion
  R. D. Peccei and H. R. Quinn. (1977)
  - Scalar field (10^{-5} to 10^{-3} eV/c^2, spin-0)
  - It solves the CP problem
- Axion like particles
  A. Arvanitaki et al., arXiv:0905.4720 **[hep-th]**
  - Motivated by String models (Axiverse)
  - Wide range of masses
- Higher spin particles

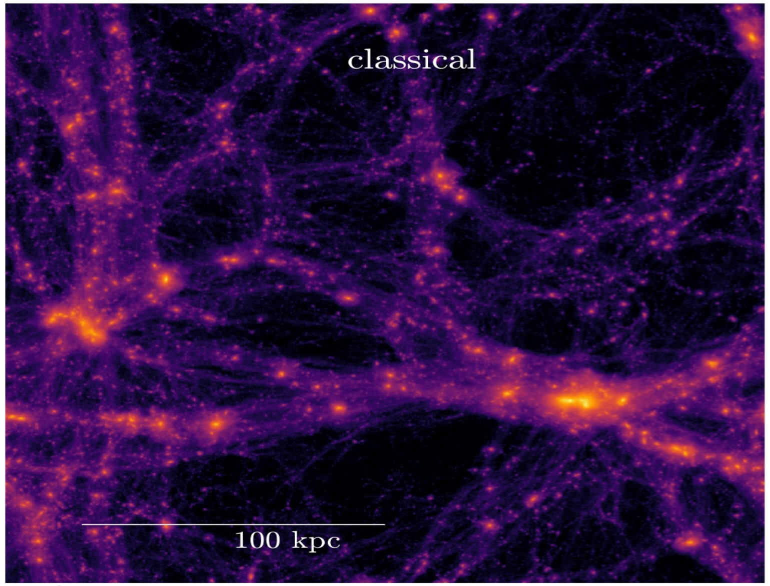
Motivation: Fuzzy Dark Matter

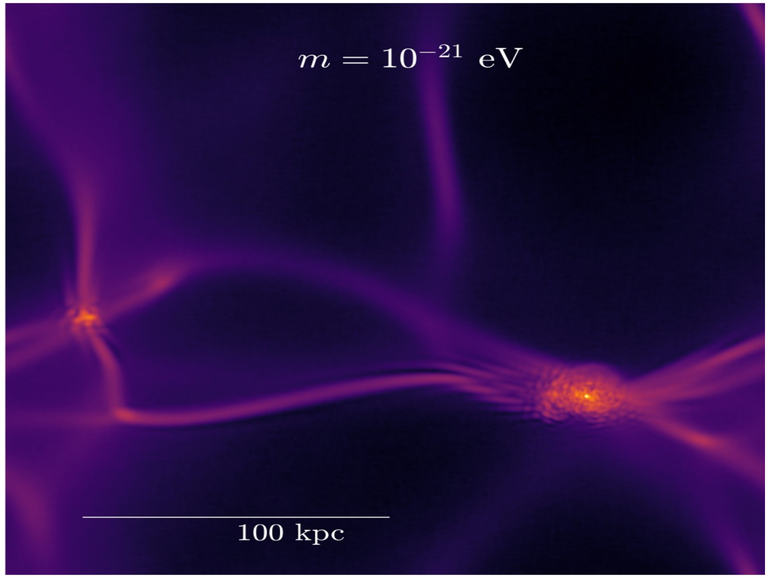
✓ The typical model is a spin zero non-relativistic ultralight bosonic particle (around 10^{-22} eV/c^2) which solves small scale problems of CDM

$$i\hbar\frac{\partial\Phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Phi + mV\Phi$$

$$\nabla^2 V = 4\pi G m|\Phi|^2$$

Bose-Einstein Condensate





Images from: Mocz *et al.*, Phys. Rev. D **97**, 083519 (2018), arXiv:1801.03507 **[astro-ph.CO]**

Motivation: Condensate and Non-Condensate

Similarities between Fuzzy Dark Matter and Ultracold Atom gases

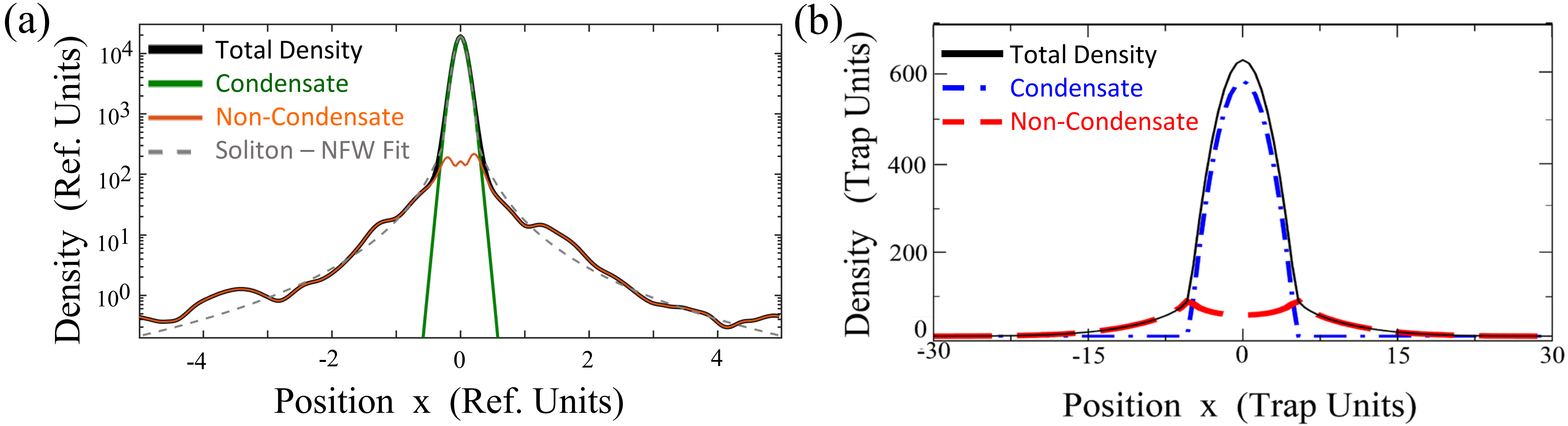


Image from: N. Proukakis, G. Rigopoulos and A.S., *Phys. Rev. D* 108 (8 2023), p. 083513 2303.02049 [astro-ph.CO]

The Generalised Model

We can go to the non-relativistic limit in this action

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{3}\phi^4 \right)$$

After splitting in higher and low modes

$$\phi = \Phi_0 + \varphi$$

Low momentum

High momentum

And using Schwinger-Keldysh formalism, Wigner transforms and more…

The Generalised Model

…We get our equations:

$$i\frac{\partial\Phi_0(x)}{\partial t}=\Big(-\frac{1}{2m}\nabla^2+V_c(x)\Big)\Phi_0(x)-i\,R\Phi_0(x)+\xi_1(x)-2g\int d^4x'\Pi^R(x',x)V_{\mathrm{nc}}(x')\Phi_0(x)+g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t}+\frac{\boldsymbol{k}}{m}\cdot\nabla f-\nabla V_{\mathrm{nc}}\cdot\nabla_{\boldsymbol{k}}f=\frac{1}{2}(I_a+I_b)$$

$$\nabla^2V^{\mathrm{cl}}(x)=4\pi G\,m\Big(n_c(x)+\tilde{n}(x)+\frac{1}{2}\xi_2(x)\Big)-4\pi G\,m\int d^4x'\,\Pi^R(x',x)V_{\mathrm{nc}}(x')$$

N. Proukakis, G. Rigopoulos and A.S.,
Phys. Rev. D 108 (8 2023), p. 083513
2303.02049 [astro-ph.CO]
And to Appear

$$V_c(x)=m\,V^{\mathrm{cl}}(x)+g(n_c(x)+2\tilde{n}(x))$$

$$V_{\mathrm{nc}}(x)=m\,V^{\mathrm{cl}}(x)+2g(n_c(x)+\tilde{n}(x))$$

Mean field potentials

$$n_c=|\Phi_0|^2 \qquad \tilde{n}=\int\frac{d^3k}{(2\pi)^3}f$$

Condensate and particle number densities

The Generalised Model

$$i\frac{\partial\Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\mathrm{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\nabla f - \nabla V_{\mathrm{nc}}\cdot\nabla_{\boldsymbol{k}} f = \frac{1}{2}(I_a + I_b)$$

Collisional terms

$$\nabla^2 V^{\mathrm{cl}}(x) = 4\pi G m\left(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\right) - 4\pi G m\int d^4x'\,\Pi^R(x',x)V_{\mathrm{nc}}(x')$$

$$I_b = 4g^2\int \frac{d^3p_2d^3p_3d^3p_4}{(2\pi)^5\hbar^7}\delta(\varepsilon_{\boldsymbol{p}_3} + \varepsilon_{\boldsymbol{p}_4} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}})\delta(\boldsymbol{p} + \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{p}_4)$$
$$\times[f_3f_4(f+1)(f_2+1) - ff_2(f_3+1)(f_4+1)]$$

Particle-Particle collisions

The Generalised Model

$$i\frac{\partial \Phi_0(x)}{\partial t} = \Big(-\frac{1}{2m}\nabla^2 + V_c(x)\Big)\Phi_0(x) - i\,R\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\rm nc}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\nabla f - \nabla V_{\rm nc}\cdot\nabla_{\boldsymbol{k}} f = \frac{1}{2}(I_a + I_b)$$

Collisional terms

$$\nabla^2 V^{\rm cl}(x) = 4\pi G m\Big(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\Big) - 4\pi G m\int d^4x'\,\Pi^R(x',x)V_{\rm nc}(x')$$

$$\left.\begin{aligned}I_a = {}& 4g^2 n_c\int \frac{d^3p_1\,d^3p_2\,d^3p_3}{(2\pi)^2\hbar^4}\delta(\varepsilon_{\boldsymbol{q}}+\varepsilon_{\boldsymbol{p}_1}-\varepsilon_{\boldsymbol{p}_2}-\varepsilon_{\boldsymbol{p}_3})\delta(\boldsymbol{p}_2-\boldsymbol{p}_1-\boldsymbol{q}+\boldsymbol{p}_3)\\ &\times(\delta(\boldsymbol{p}_1-\boldsymbol{p})-\delta(\boldsymbol{p}_2-\boldsymbol{p})-\delta(\boldsymbol{p}_3-\boldsymbol{p}))[(1+f_1)f_2f_3 - f_1(1+f_2)(1+f_3)]\end{aligned}\right\}$$

Particle-Condensate collisions

The Generalised Model

Collisional terms

$$i\frac{\partial\Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - \boxed{iR}\,\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\mathrm{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\nabla f - \nabla V_{\mathrm{nc}}\cdot\nabla_{\boldsymbol{k}} f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V^{\mathrm{cl}}(x) = 4\pi G m\left(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\right) - 4\pi G m\int d^4x'\,\Pi^R(x',x)V_{\mathrm{nc}}(x')$$

$$R = \frac{1}{4n_c}\int\frac{d^3p}{(2\pi)^3}I_a$$

→ *Particle-Condensate collisions*

The condensate can grow or decrease

The Generalised Model

Complex Noise

$$i\frac{\partial \Phi_0(x)}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V_c(x)\right)\Phi_0(x) - i\,R\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\text{nc}}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\nabla f - \nabla V_{\text{nc}}\cdot\nabla_{\boldsymbol{k}} f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V^{\text{cl}}(x) = 4\pi G m\left(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\right) - 4\pi G m\int d^4x'\,\Pi^R(x',x)V_{\text{nc}}(x')$$

$$\langle \xi_1^*(x')\xi_1(x)\rangle = \frac{i}{2}\Sigma^K_{(c)}(x)\delta(x - x')$$

→ Noise correlation

Noise can induce condensate production

The Generalised Model: Limits

$$i\frac{\partial\Phi_0(x)}{\partial t} = \Big(-\frac{1}{2m}\nabla^2 + V_c(x)\Big)\Phi_0(x) - iR\Phi_0(x) + \xi_1(x) - 2g\int d^4x'\Pi^R(x',x)V_{\rm nc}(x')\Phi_0(x) + g\xi_2(x)\Phi_0(x)$$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\nabla f - \nabla V_{\rm nc}\cdot\nabla_{\boldsymbol{k}} f = \frac{1}{2}(I_a + I_b)$$

$$\nabla^2 V^{\rm cl}(x) = 4\pi G m\Big(n_c(x) + \tilde{n}(x) + \frac{1}{2}\xi_2(x)\Big) - 4\pi G m\int d^4x'\,\Pi^R(x',x)V_{\rm nc}(x')$$

For g = 0, order m and all condensed → We recover FDM

For g = 0, order m and all non-condensed → We recover Vlasov-Poisson equations for CDM

No gravity → We recover Cold Atom Physics models:
ZNG and Stochastic Projected Gross-Pitaevskii

Hydrodynamic equations

We work up to order g with universe expansion: $\nabla^2 V = 4\pi G a^2(\rho_c + \tilde{\rho})$

Condensate → Madelung transformation

$$\frac{\partial \rho_c}{\partial t} + 3H\rho_c + \frac{1}{a}\nabla\cdot(\rho_c \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{a}\boldsymbol{v}\cdot\nabla\boldsymbol{u} = -\nabla\left(-\frac{\hbar^2}{2m^2a^3}\frac{\nabla^2\sqrt{\rho_c}}{\sqrt{\rho_c}} + \frac{1}{a}V + \frac{g}{m^2 a}(\rho_c + 2\tilde{\rho})\right)$$

Non-condensed particles → Moments and truncation

$$\frac{\partial \tilde{\rho}}{\partial t} + 3H\tilde{\rho} + \frac{1}{a}\nabla\cdot(\tilde{\rho}\tilde{\boldsymbol{v}}) = 0$$

$$\frac{\partial \tilde{\boldsymbol{u}}}{\partial t} + \frac{1}{a}\tilde{\boldsymbol{v}}\cdot\nabla\tilde{\boldsymbol{u}} = -\nabla\left(\frac{1}{a}V + \frac{2g}{m^2 a}(\rho_c + \tilde{\rho})\right) - \frac{1}{a\tilde{\rho}}\nabla P$$

$$\frac{\partial P}{\partial t} + 5HP + \frac{1}{a}\nabla\cdot(\tilde{\boldsymbol{v}}P) = -\frac{2}{3a}P\nabla\cdot\tilde{\boldsymbol{v}}$$

Linear Regime

Our system admits consistently a particle pressure $P = \kappa\tilde{\rho}^{\,5/3}$

$$\ddot{\delta}_c + 2H\dot{\delta}_c + \left(\frac{\hbar^2 k^4}{4m^2a^4} - 4\pi G\bar{\rho}f + \frac{g\bar{\rho}f k^2}{m^2a^2}\right)\delta_c - (1-f)\left(4\pi G\bar{\rho} - \frac{2g\bar{\rho}\,k^2}{m^2a^2}\right)\delta_{\rm nc} = 0$$

$$\ddot{\delta}_{\rm nc} + 2H\dot{\delta}_{\rm nc} - \left(4\pi G\bar{\rho}(1-f) - \frac{1}{a^2}\left(\frac{2g\bar{\rho}(1-f)}{m^2} + \frac{5\kappa\bar{\rho}^{2/3}(1-f)^{2/3}}{3}\right)k^2\right)\delta_{\rm nc} - f\left(4\pi G\bar{\rho} - \frac{2g\bar{\rho}\,k^2}{m^2a^2}\right)\delta_c = 0$$

Parameters →

m: boson mass

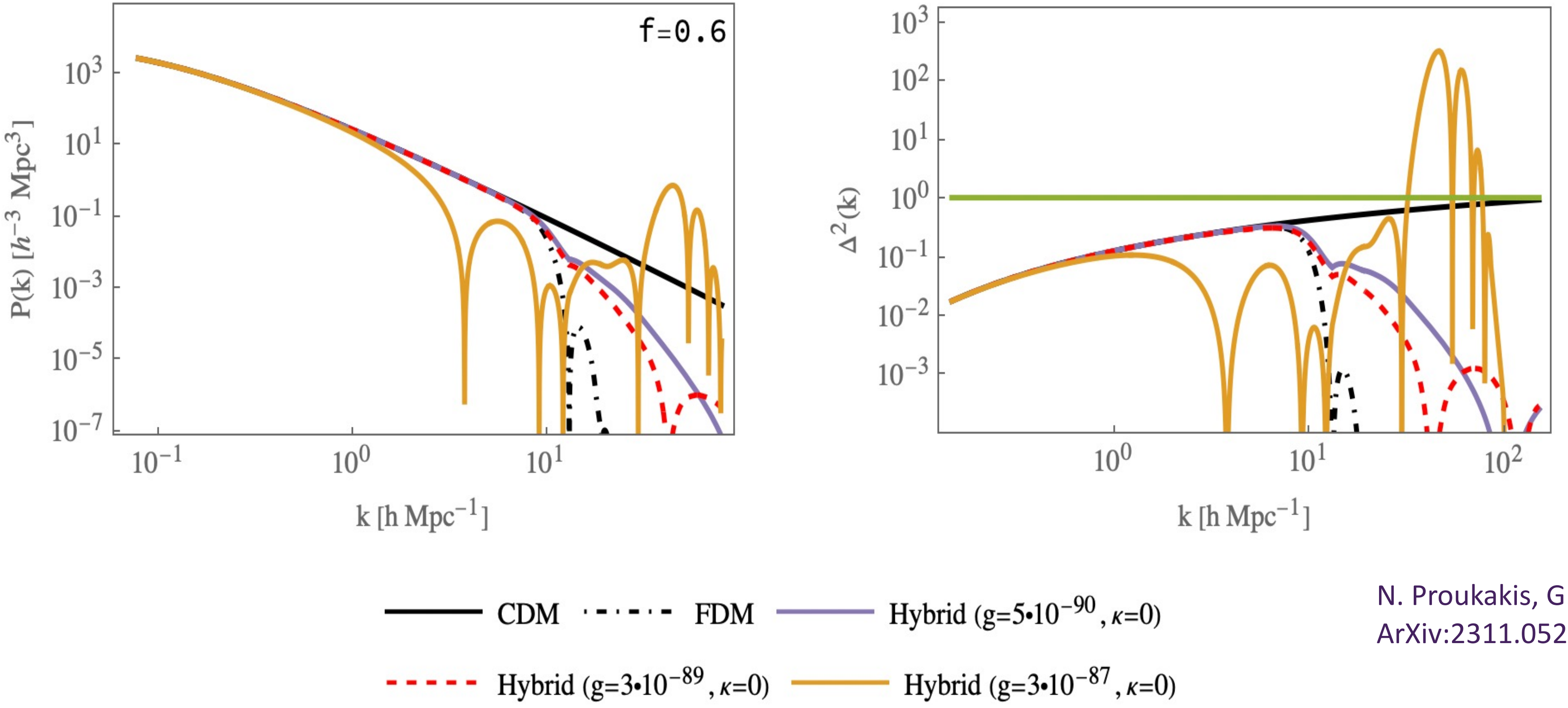
f : condensate fraction

g : self-interaction

κ : particle pressure $(P = \kappa\tilde{\rho}^{5/3})$

Linear Regime

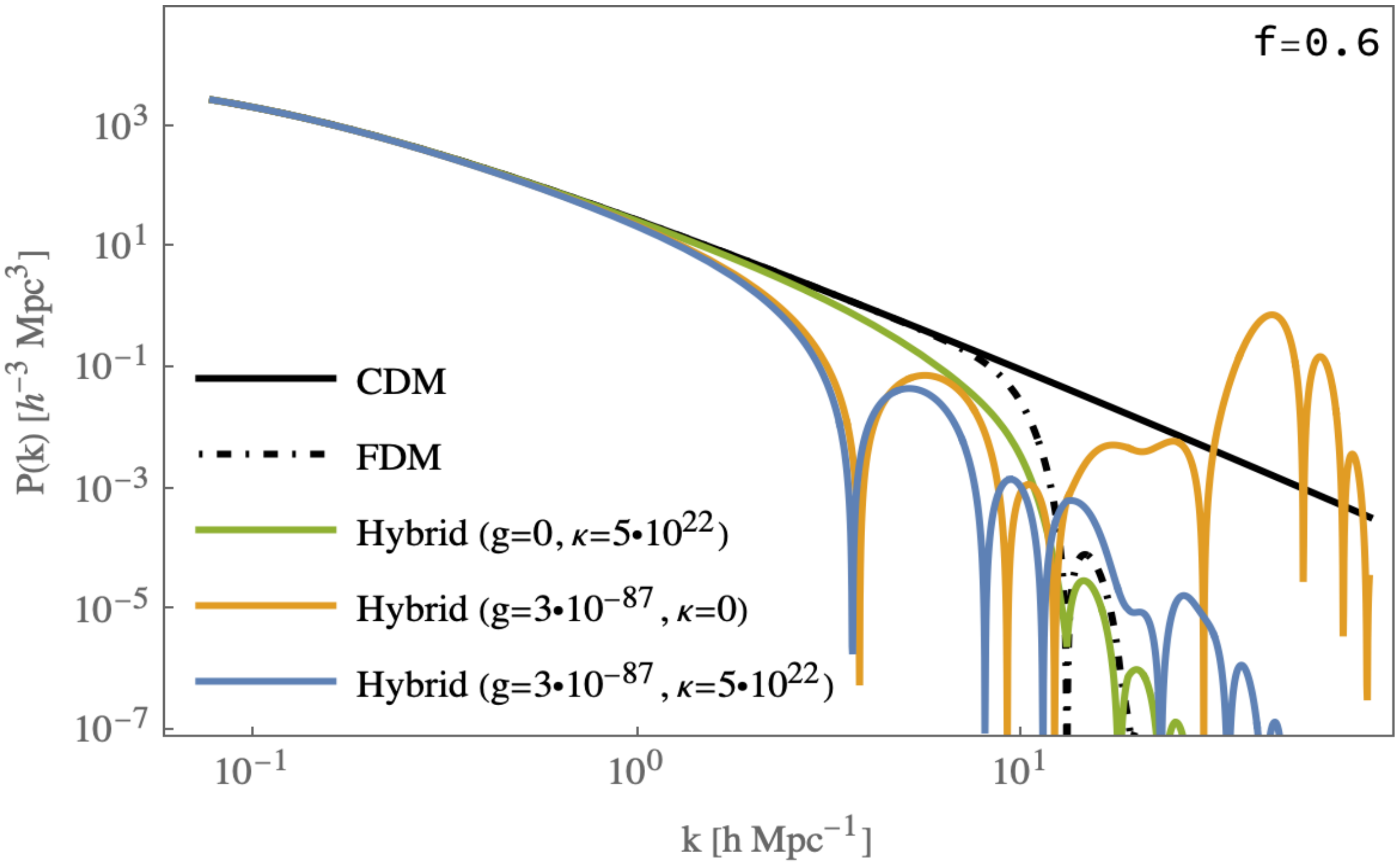
Plot with $\kappa = 0$, for a mass $m = 2 \cdot 10^{-22} \frac{eV}{c^2}$ and redshift $z = 3$. Mixed case shows an enhancement in the power spectrum



N. Proukakis, G. Rigopoulos and A.S., ArXiv:2311.05280 [astro-ph.CO]

Linear Regime

Particle pressure



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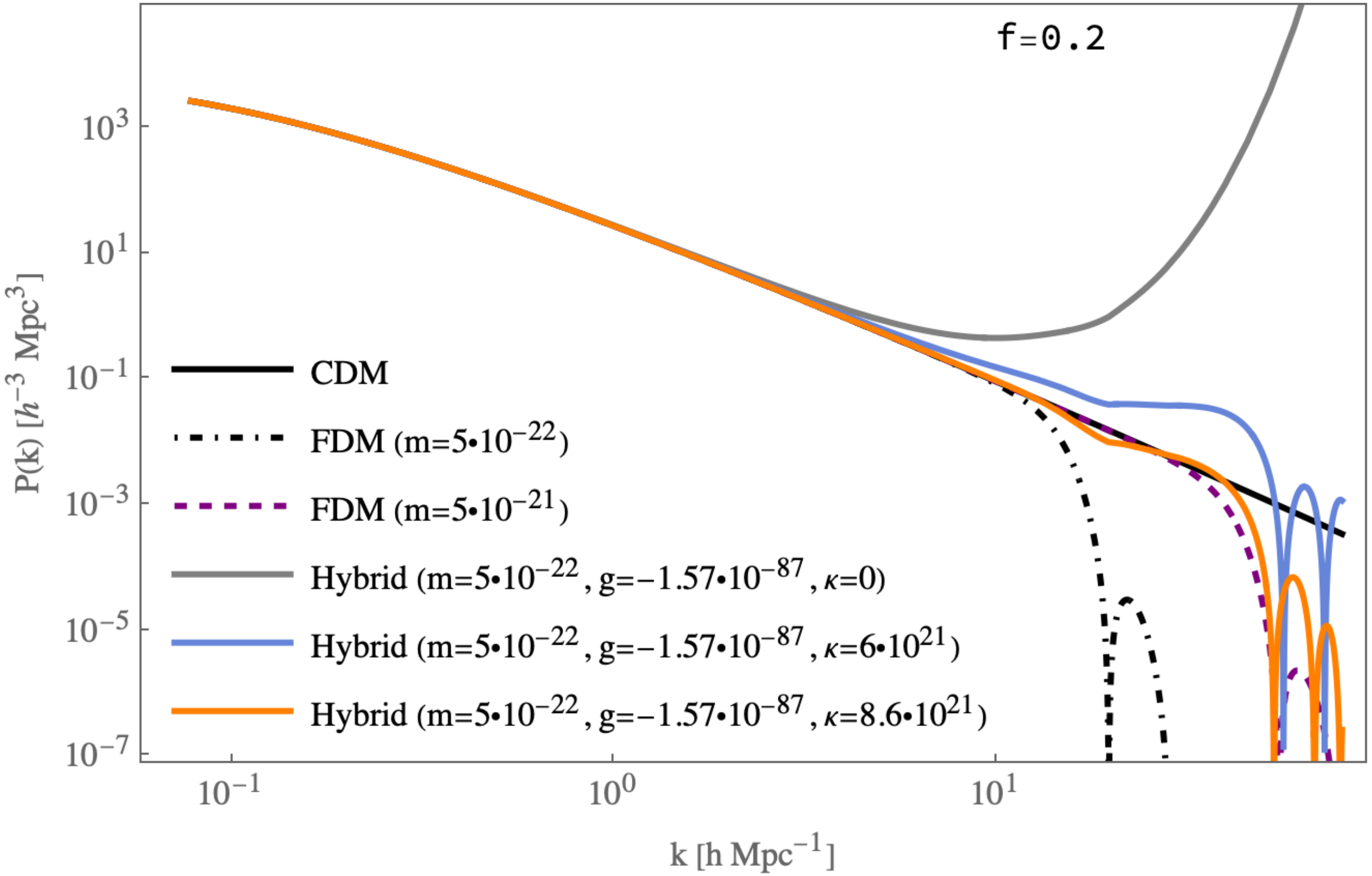
Linear Regime



Hybrid model looks like FDM of higher mass.

We must be careful with statements about constraints on the mass.

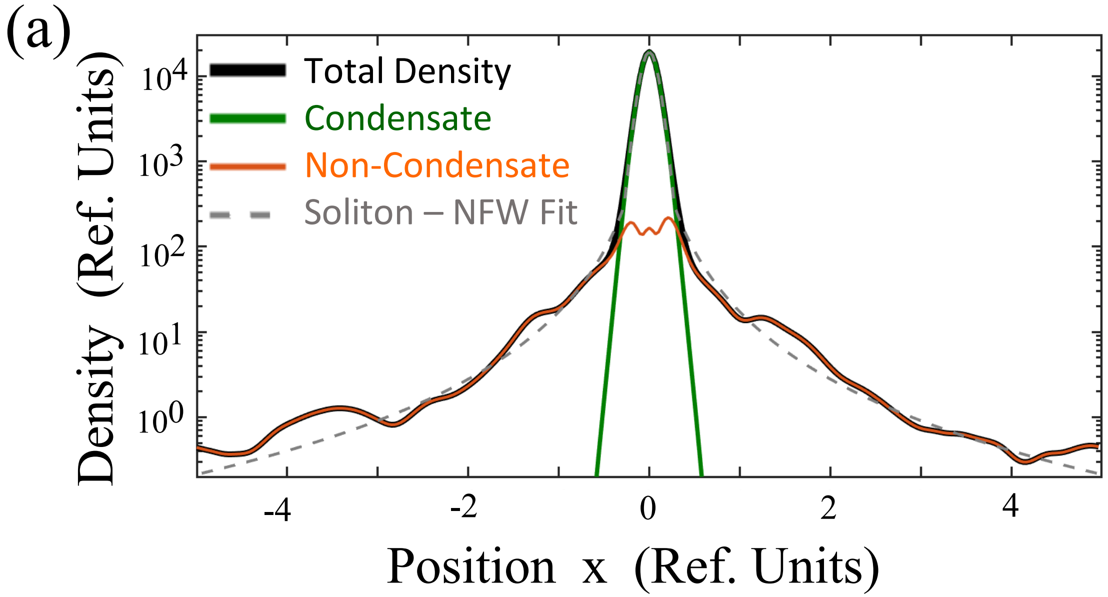
N. Proukakis, G. Rigopoulos and A.S., ArXiv:2311.05280 [astro-ph.CO]

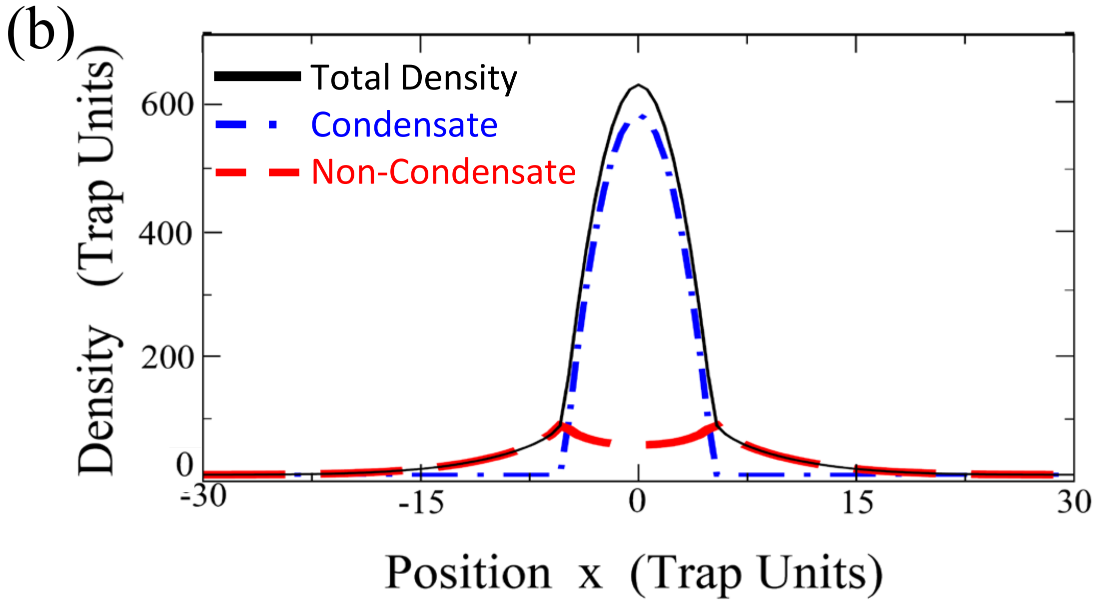
Summary and Comments

- We have a general model for bosonic Dark Matter (condensate + particles). Known models are recovered under the appropriate limits.

- Self-Interaction can enrich the dynamics of the system and it could have an importance in the generation of the condensate. We observe in the linear regime interesting features, but a non-linear regime simulation is crucial.

- The model with both components with self-interaction can mimic FDM, so we need to be careful with placing constraints.


(a)
Total Density
Condensate
Non-Condensate
Soliton – NFW Fit
Density (Ref. Units)
10^4
10^3
10^2
10^1
10^0
-4
-2
0
2
4
Position x (Ref. Units)


(b)
Total Density
Condensate
Non-Condensate
Density (Trap Units)
600
400
200
0
-30
-15
0
15
30
Position x (Trap Units)

Thanks!