MOND: An alternative to particle dark matter

Federico Lelli
INAF - Arcetri Astrophysical Observatory

INAF
ISTITUTO NAZIONALE
DI ASTROFISICA
OSSERVATORIO ASTROFISICO DI ARCETRI

MOND = MOdified Newtonian Dynamics

MOND = MOdified Newtonian Dynamics

MOND = NO DM

MOND = MOdified Newtonian Dynamics

MOND = NO DM

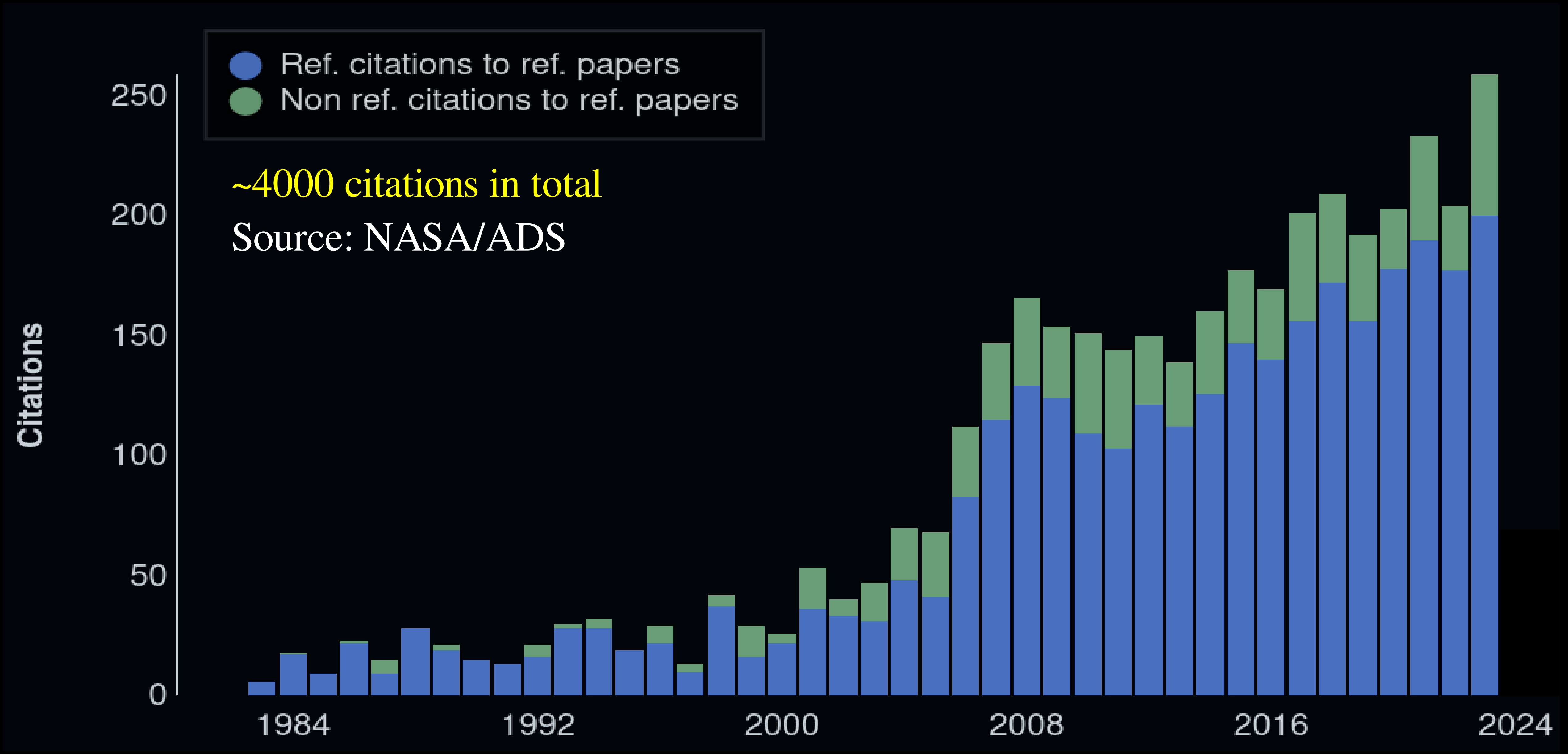
MOND = MilgrOmiaN Dynamics

Proposed by Moderhai Milgrom (1983a, b, c)

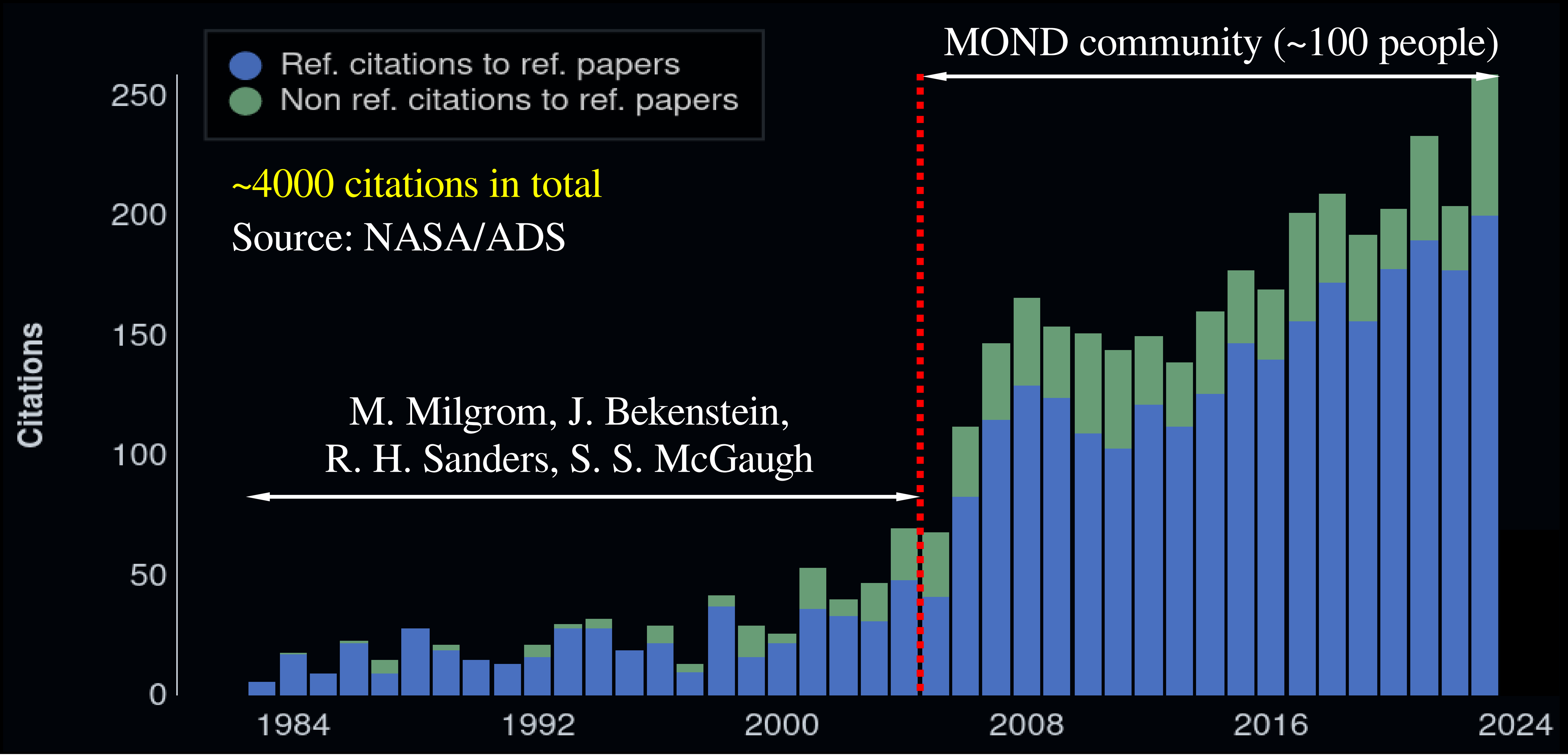
MOND still stands after more than 40 years.

Multifaceted paradigm including different theories at both the non-relativistic and relativistic levels.

Citations to the original MOND trilogy (Milgrom 1983a, b, c)



Citations to the original MOND trilogy (Milgrom 1983a, b, c)



Talk Outline

1. General MOND paradigm
2. MOND in nearby galaxies
3. MOND in galaxy clusters
4. MOND cosmology
5. Specific MOND theories

Talk Outline

1. General MOND paradigm

2. MOND in nearby galaxies

3. MOND in galaxy clusters

4. MOND cosmology

5. Specific MOND theories

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow a = g_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla}\phi_N$ Newtonian gravitational field (from the Poisson's equation)

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow a = g_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2\vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla}\phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow a = \sqrt{a_0 g_N}$

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow a = g_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow$ $a = \sqrt{a_0 g_N} \rightarrow \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow a = g_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow a = \sqrt{a_0 g_N} \rightarrow \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$ Flat rotation curve at large R!

MOND postulates at the non-relativistic level

1) New constant of Physics: $a_0 \simeq 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow a = g_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2\vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla}\phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow a = \sqrt{a_0 g_N} \rightarrow \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$ Flat rotation curve at large R!

Or impose scale invariance (Milgrom 2009, ApJ): $(\vec{x}, t) \rightarrow (\lambda\vec{x}, \lambda t)$ V is invariant!

A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES[1]

M. MILGROM
Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study

Received 1982 February 4; accepted 1982 December 28

ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density Σ falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of Σ, the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant, a_0, determined in a few independent ways is approximately $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$, which is of the order of $cH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$.

The main predictions are:

1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

2. The $V_\infty^4 = a_0 GM$ relation should hold exactly.

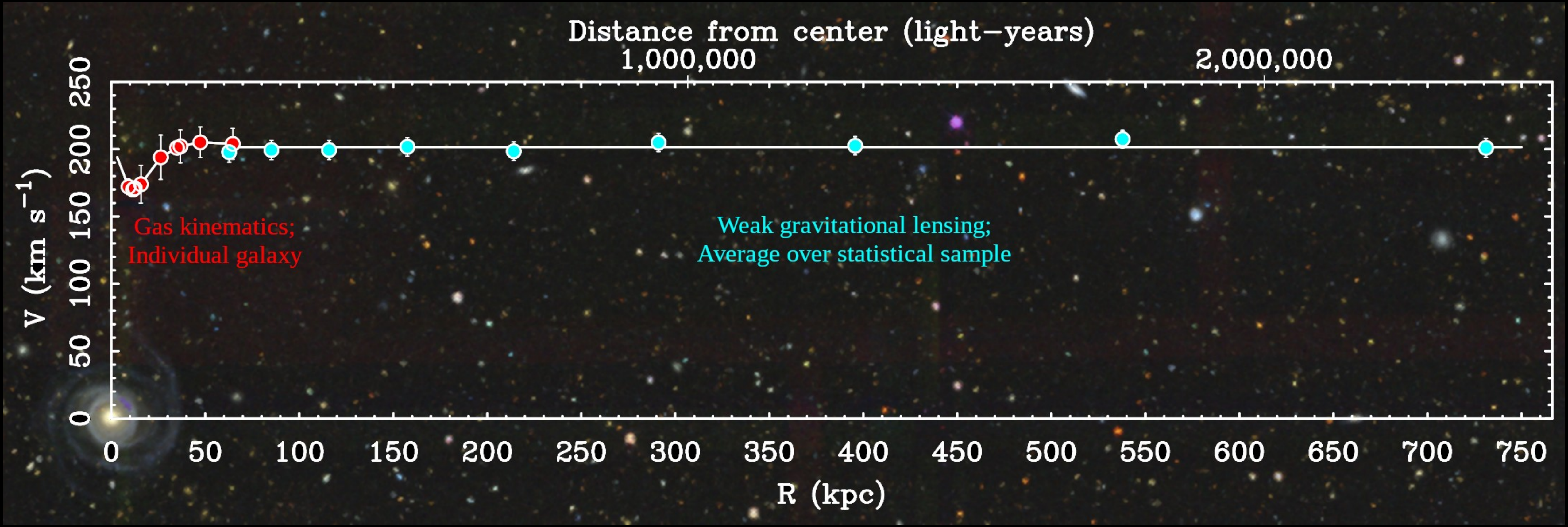
3. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing r in a predictable way.

Talk Outline

1. ~~General MOND paradigm~~
2. MOND in nearby galaxies
3. MOND in galaxy clusters
4. MOND cosmology
5. Specific MOND theories

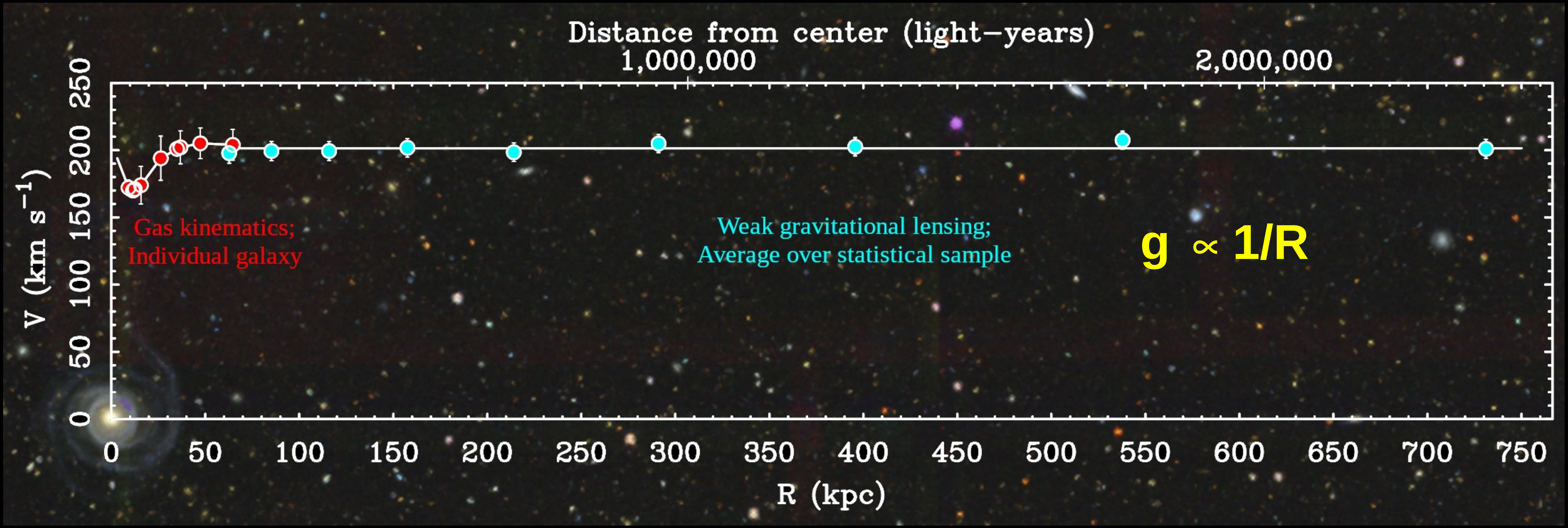
(1) $V_c = \text{const}$ at large R for isolated objects

(1) V_c = const at large R for isolated objects → weak lensing data



See Mistele+2024, ApJ

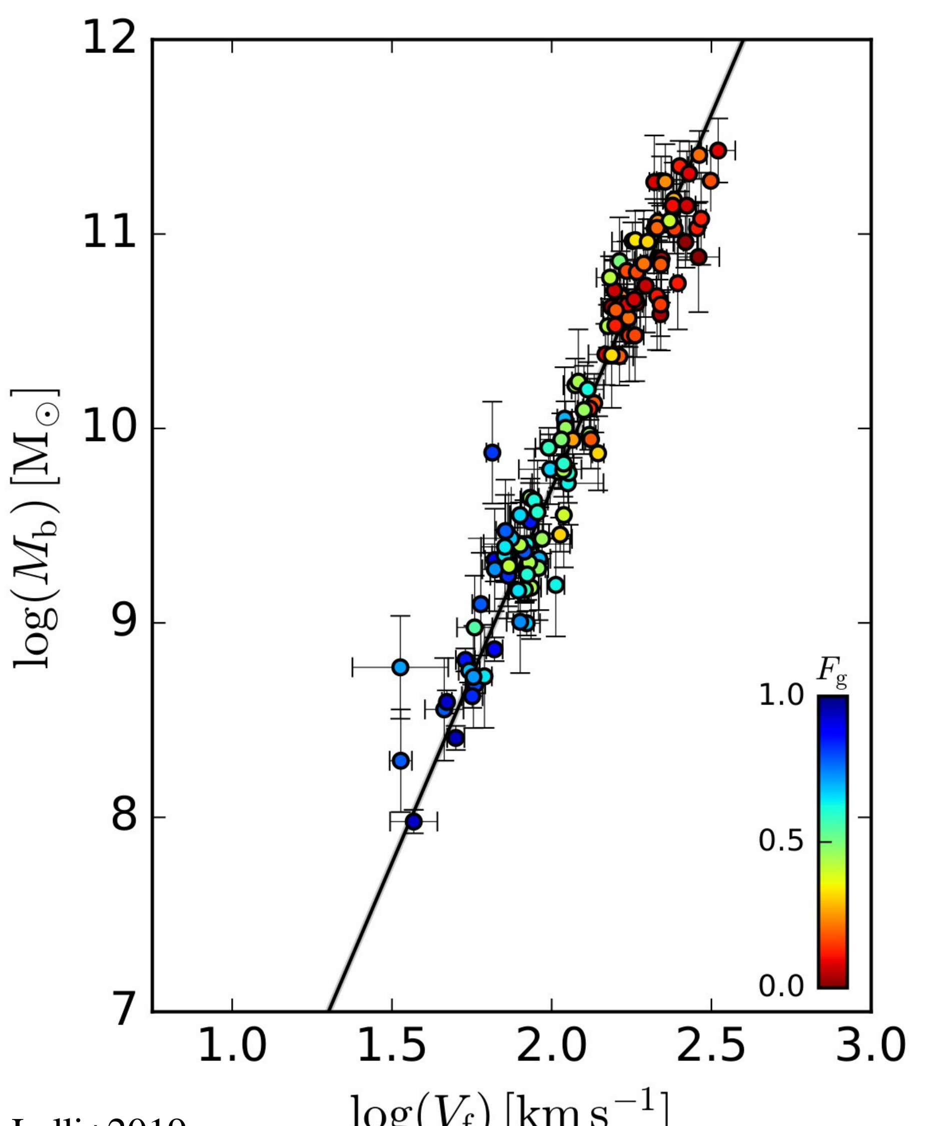
(1) V_c = const at large R for isolated objects → weak lensing data



See Mistele+2024, ApJ

(2) $V_f^{\,4} = a_0\, G\, M_b$ for isolated objects

(2) $V_f^{\,4} = a_0\, G\, M_b$ for isolated objects → rotation supported galaxies



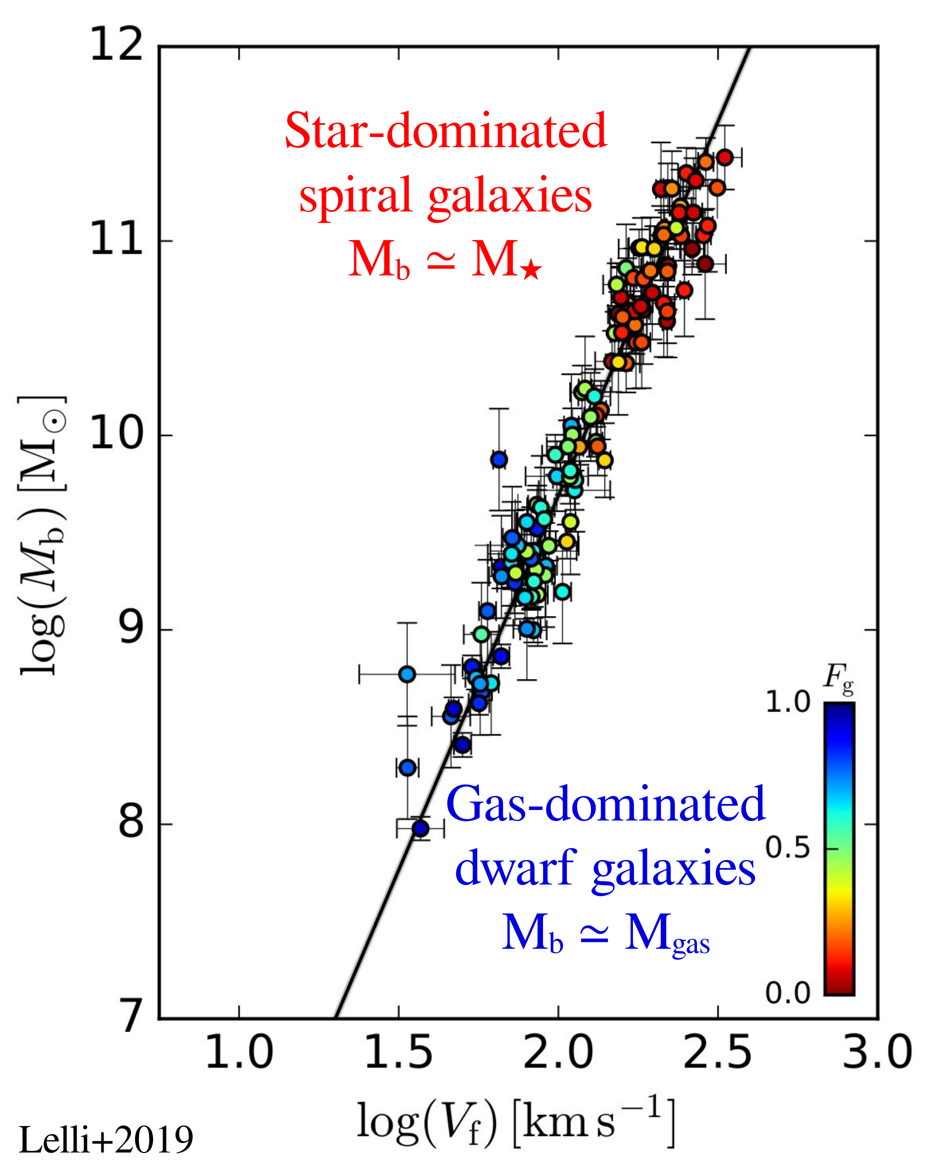
Lelli+2019

Tully-Fisher relation (1977, A&A):

HI linewidth *vs* Luminosity for bright spirals

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022), Ponomareva+2018, Schombert+(2020), Di Teodoro (2021, 2022), and many more.

(2) $V_f^{\,4} = a_0 G M_b$ for isolated objects → rotation supported galaxies



Tully-Fisher relation (1977, A&A):

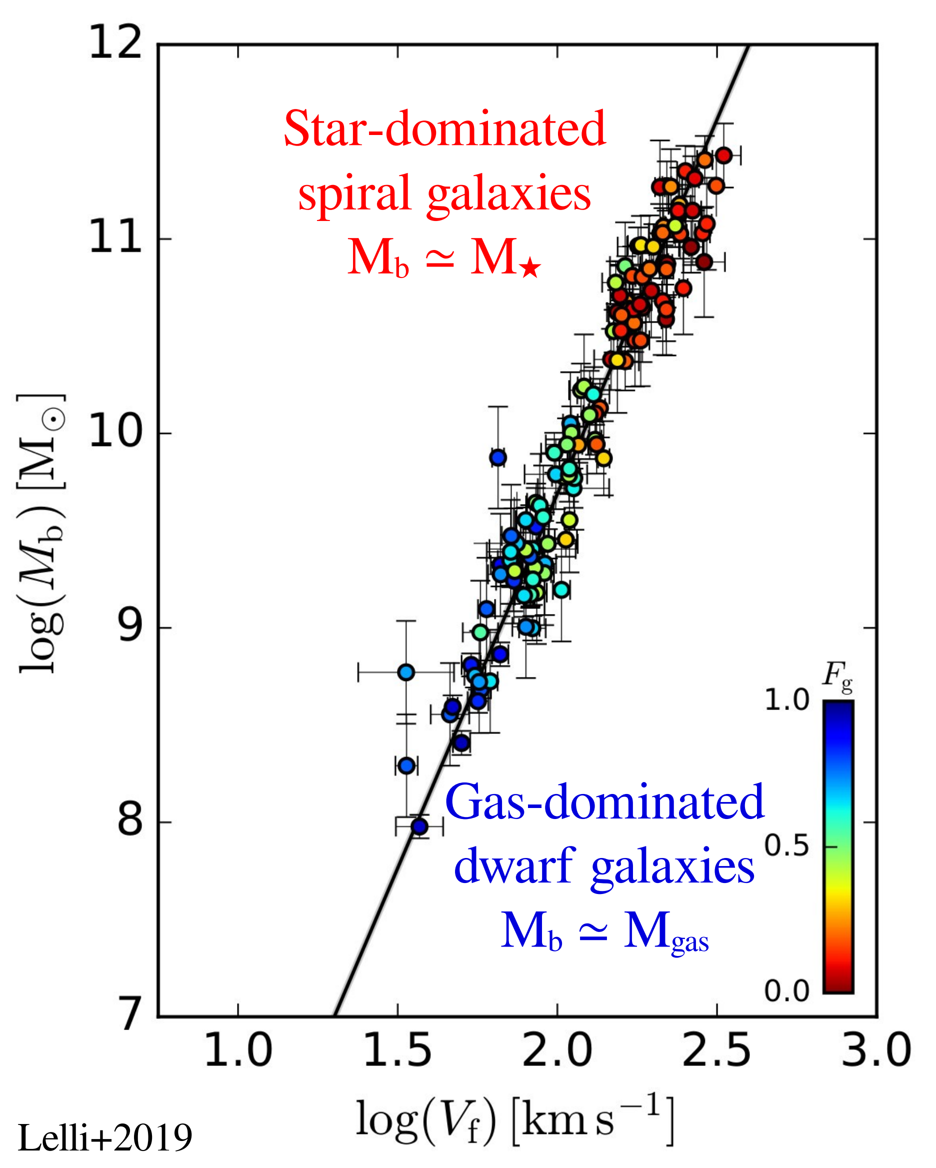
HI linewidth *vs* Luminosity for bright spirals

Four independent predictions in one equation:

(i) Key quantities are V_f and M_b (stars+gas) → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022), Ponomareva+2018, Schombert+(2020), Di Teodoro (2021, 2022), and many more.

(2) $V_f^{\,4} = a_0 G M_b$ for isolated objects → rotation supported galaxies



Tully-Fisher relation (1977, A&A):

HI linewidth *vs* Luminosity for bright spirals

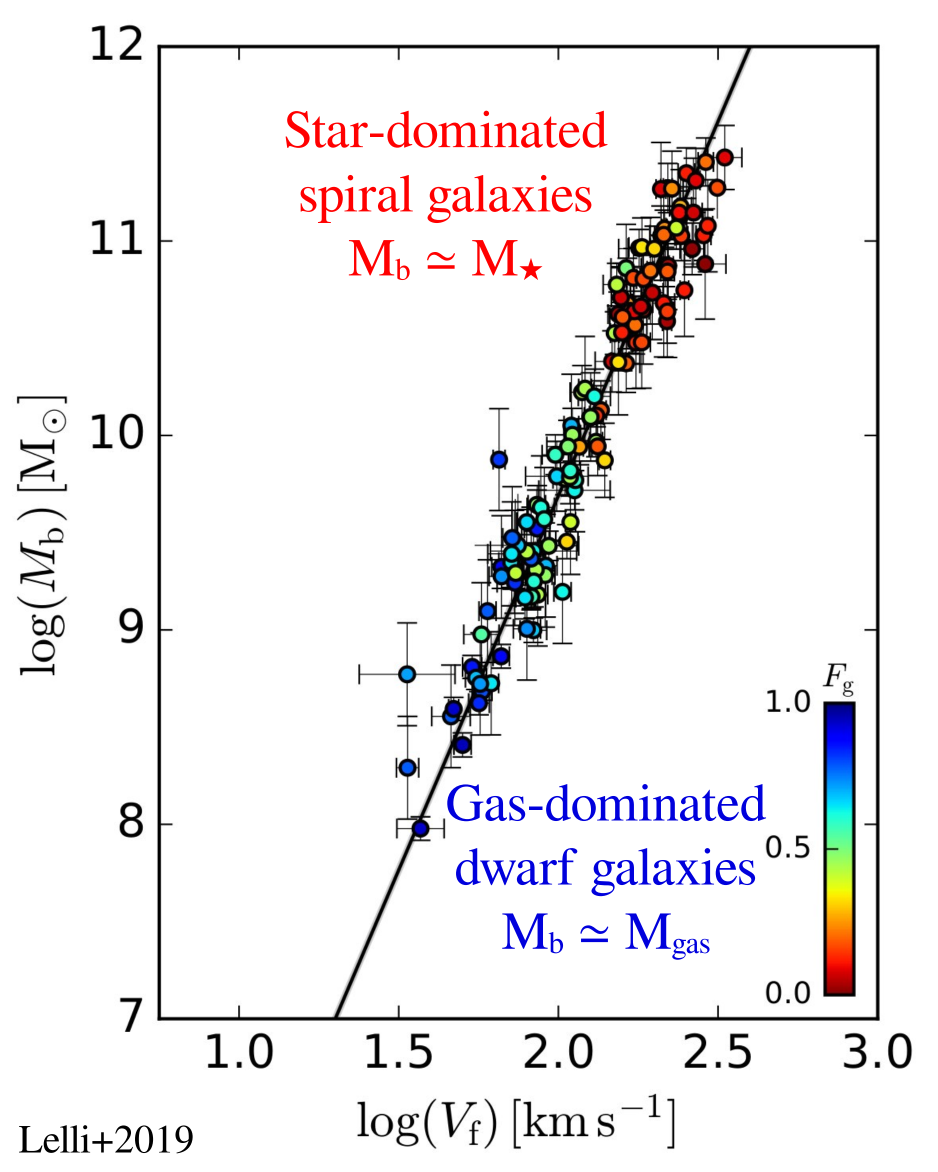
Four independent predictions in one equation:

(i) Key quantities are V_f and M_b (stars+gas) → OK

(ii) Slope should be exactly 4 → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022), Ponomareva+2018, Schombert+(2020), Di Teodoro (2021, 2022), and many more.

(2) $V_f^{\,4} = a_0 G M_b$ for isolated objects → rotation supported galaxies



Tully-Fisher relation (1977, A&A):

HI linewidth *vs* Luminosity for bright spirals

Four independent predictions in one equation:

(i) Key quantities are V_f and M_b (stars+gas) → OK

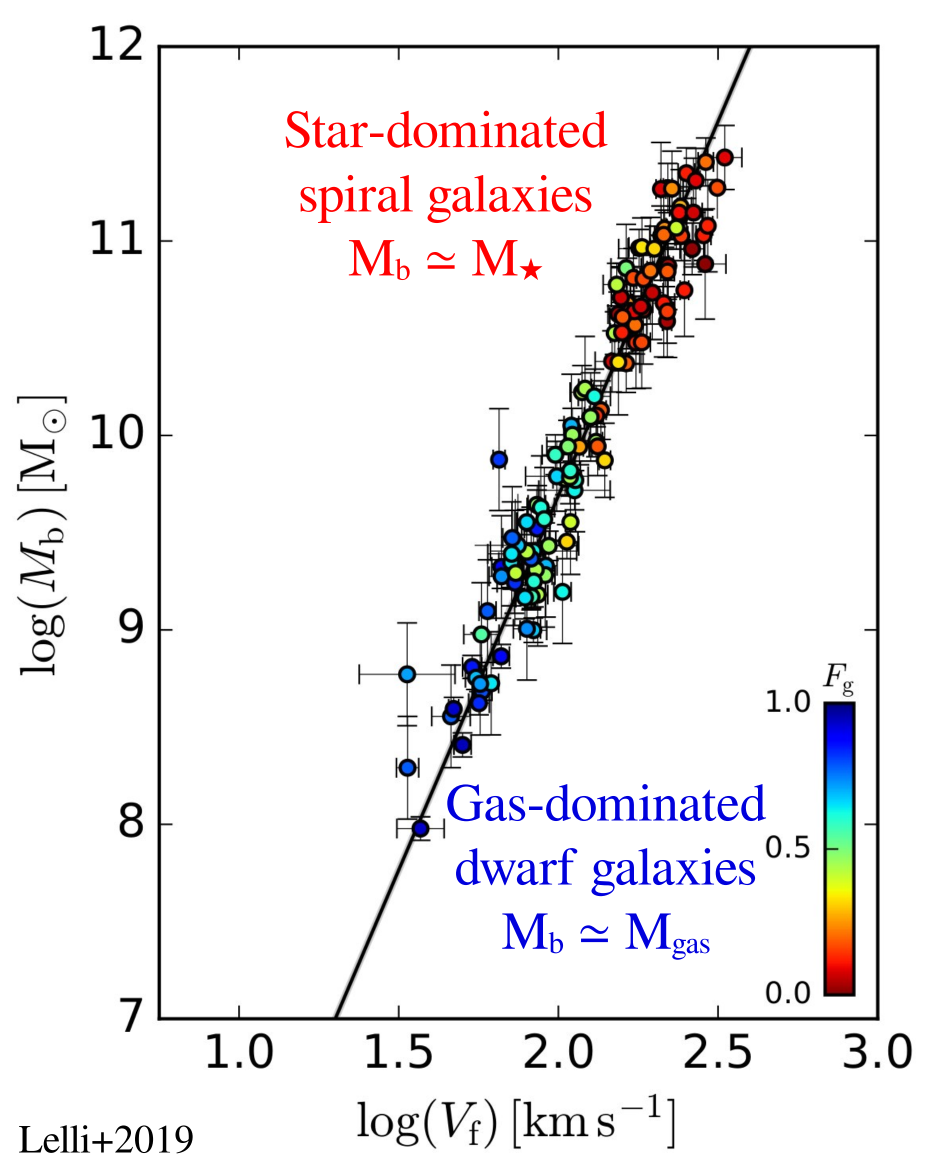
(ii) Slope should be exactly 4 → OK

(iii) Normalization is a_0G → OK with other estimates

(see Lelli 2022, Nature Astronomy)

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022), Ponomareva+2018, Schombert+(2020), Di Teodoro (2021, 2022), and many more.

(2) $V_f^4 = a_0 G M_b$ for isolated objects → rotation supported galaxies



Tully-Fisher relation (1977, A&A):

HI linewidth *vs* Luminosity for bright spirals

Four independent predictions in one equation:

(i) Key quantities are V_f and M_b (stars+gas) → OK

(ii) Slope should be exactly 4 → OK

(iii) Normalization is a_0G → OK with other estimates

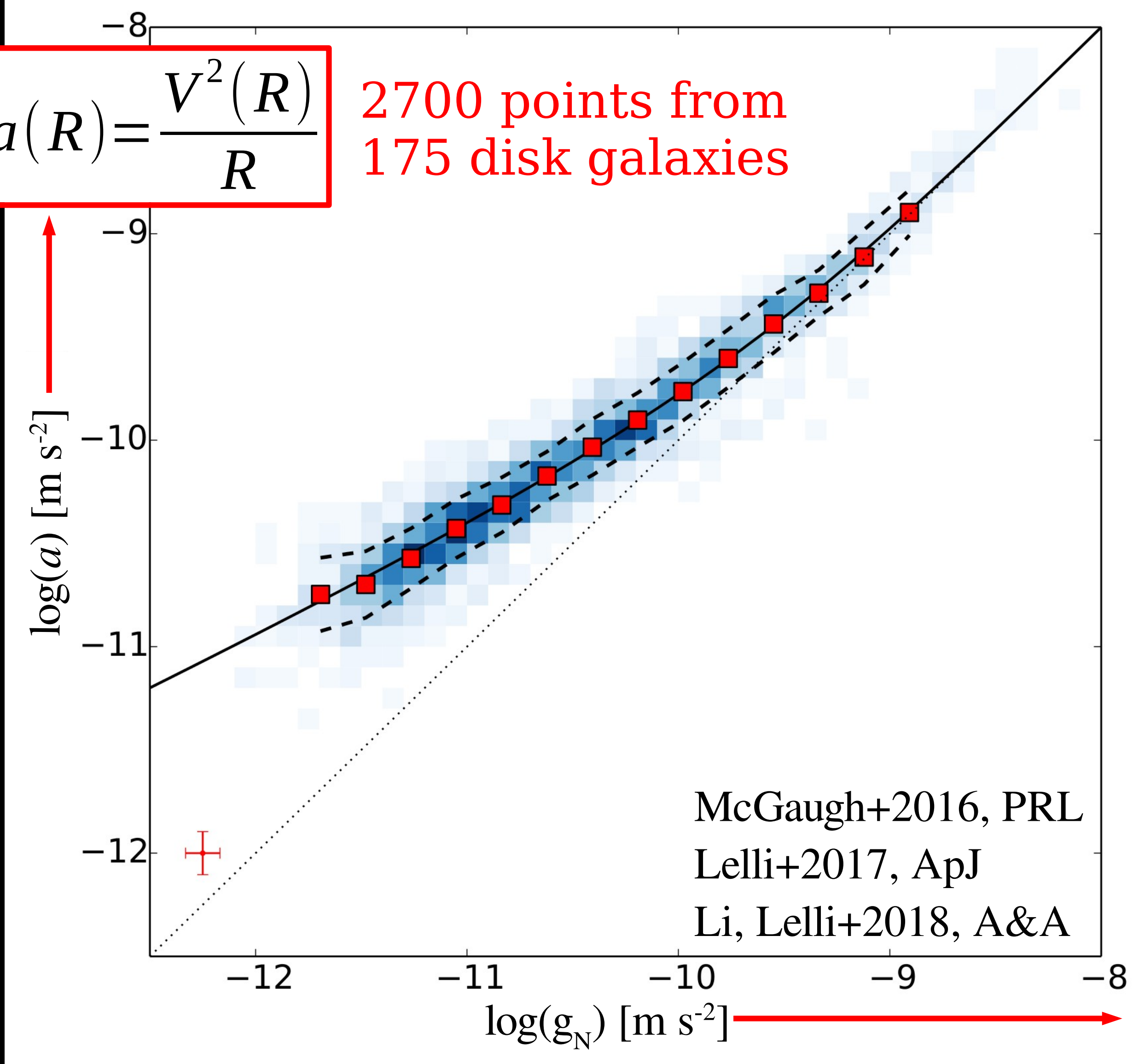
(see Lelli 2022, Nature Astronomy)

(iv) NO dependence on other properties (e.g., R_e, Σ_e), so NO intrinsic scatter along the relation → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022), Ponomareva+2018, Schombert+(2020), Di Teodoro (2021, 2022), and many more.

(3) Rotation curves can be predicted from the baryon distribution

$$a(R) = \frac{V^2(R)}{R}$$

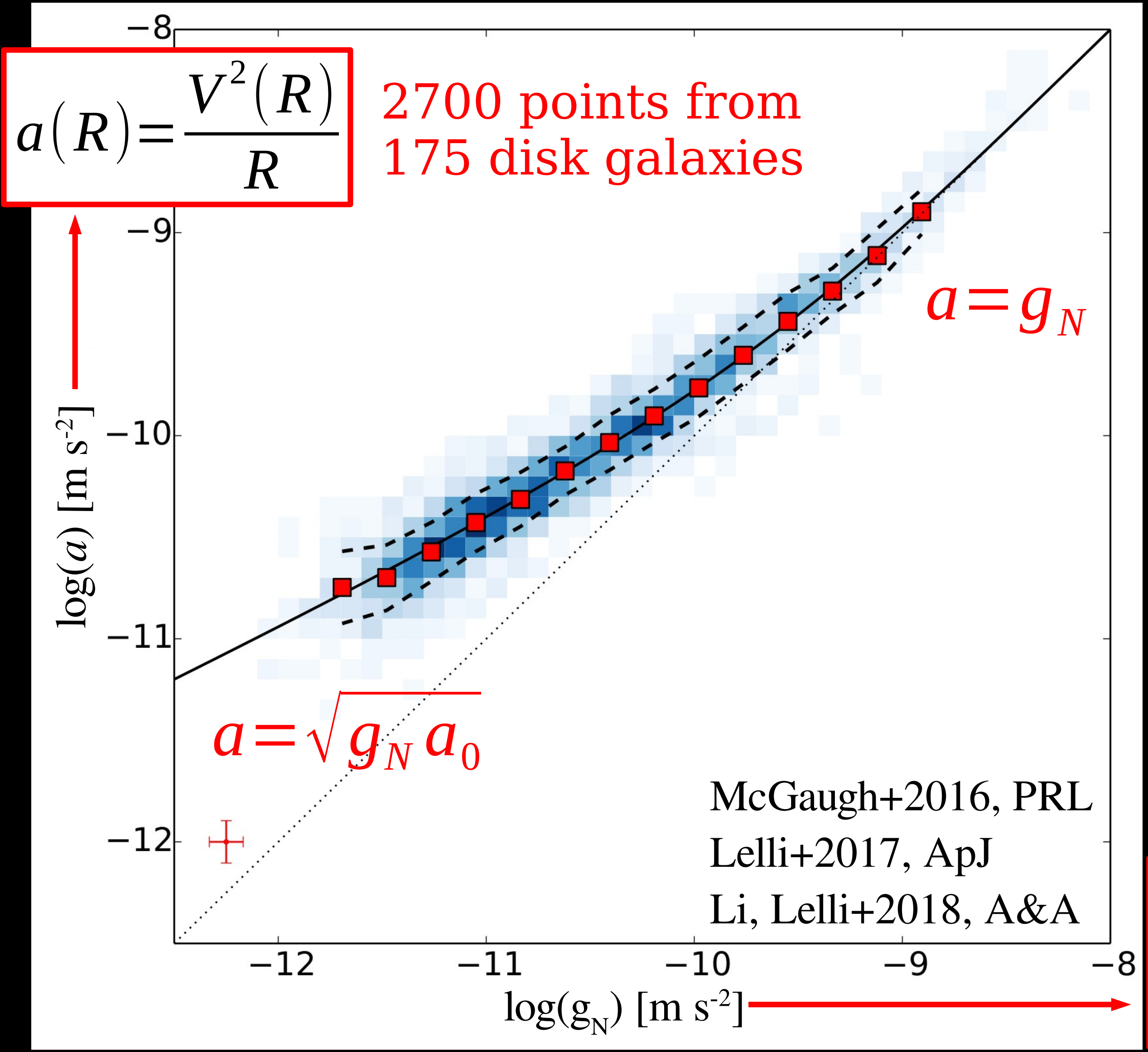


Radial Acceleration Relation (RAR)

(i) Fully empirical, independent of MOND

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

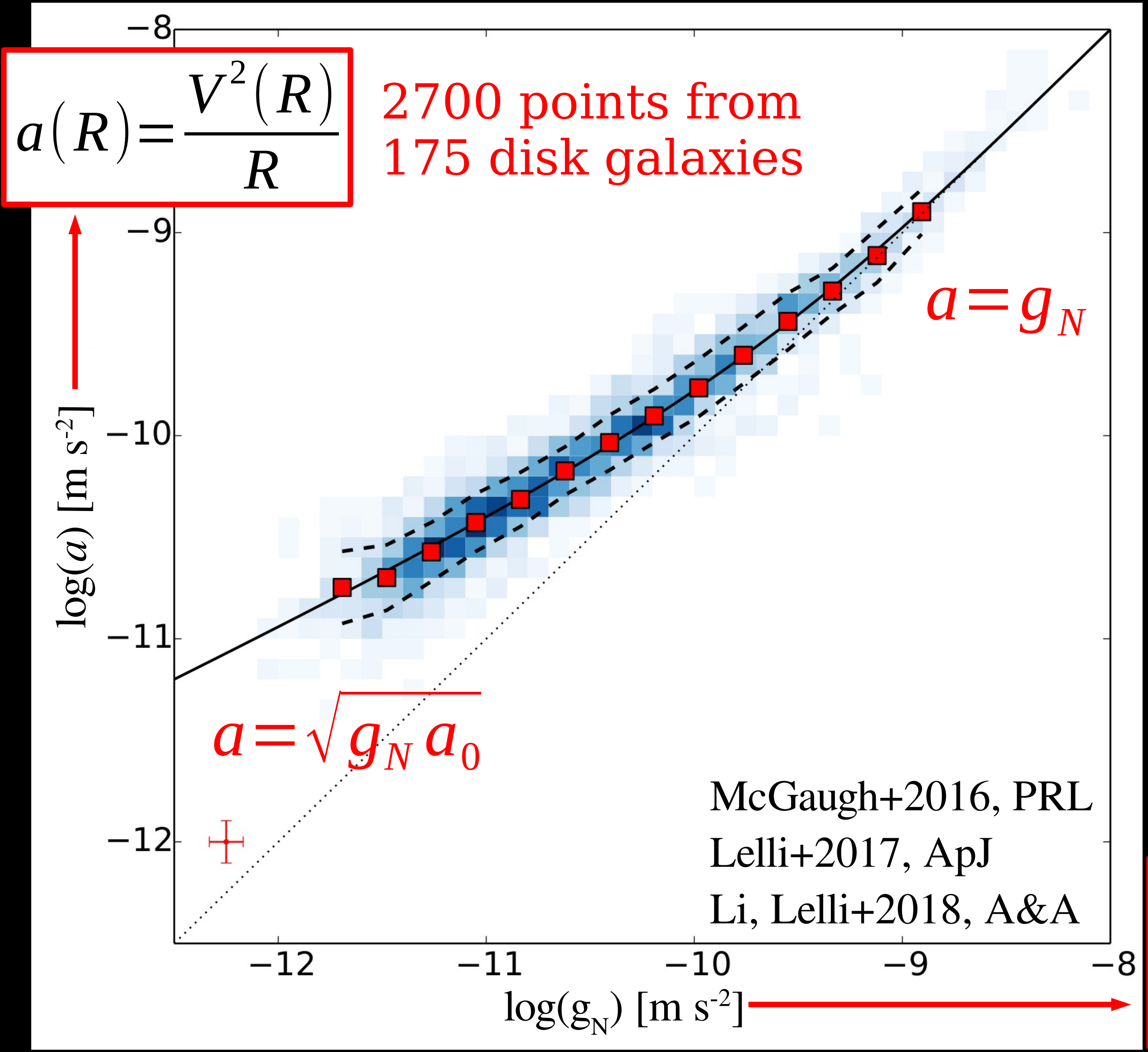
(i) Fully empirical, independent of MOND

(ii) MOND asymptotic limits → OK

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$

$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

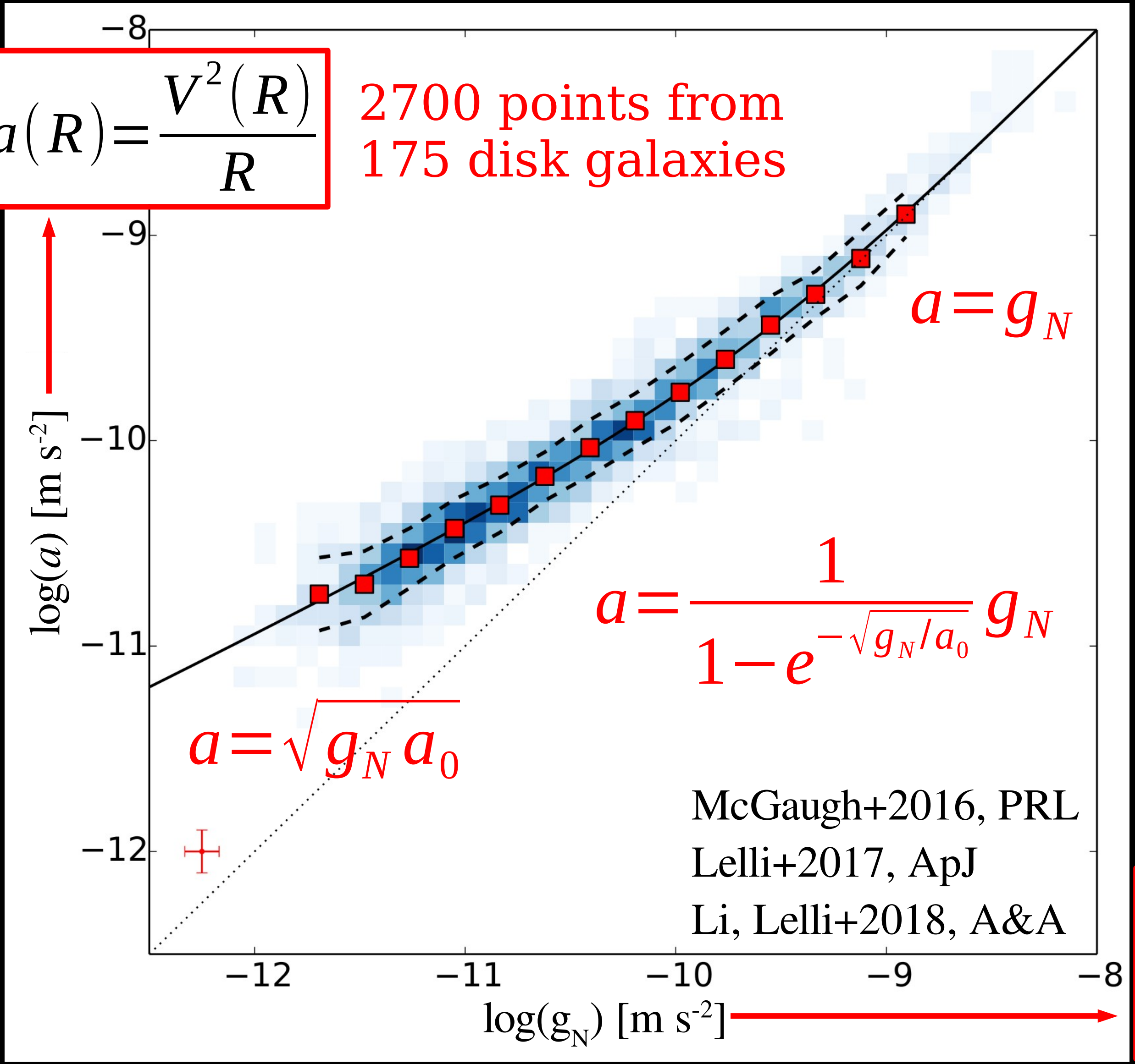
(i) Fully empirical, independent of MOND

(ii) MOND asymptotic limits → OK

(iii) No other dependencies → OK

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

(i) Fully empirical, independent of MOND

(ii) MOND asymptotic limits → OK

(iii) No other dependencies → OK

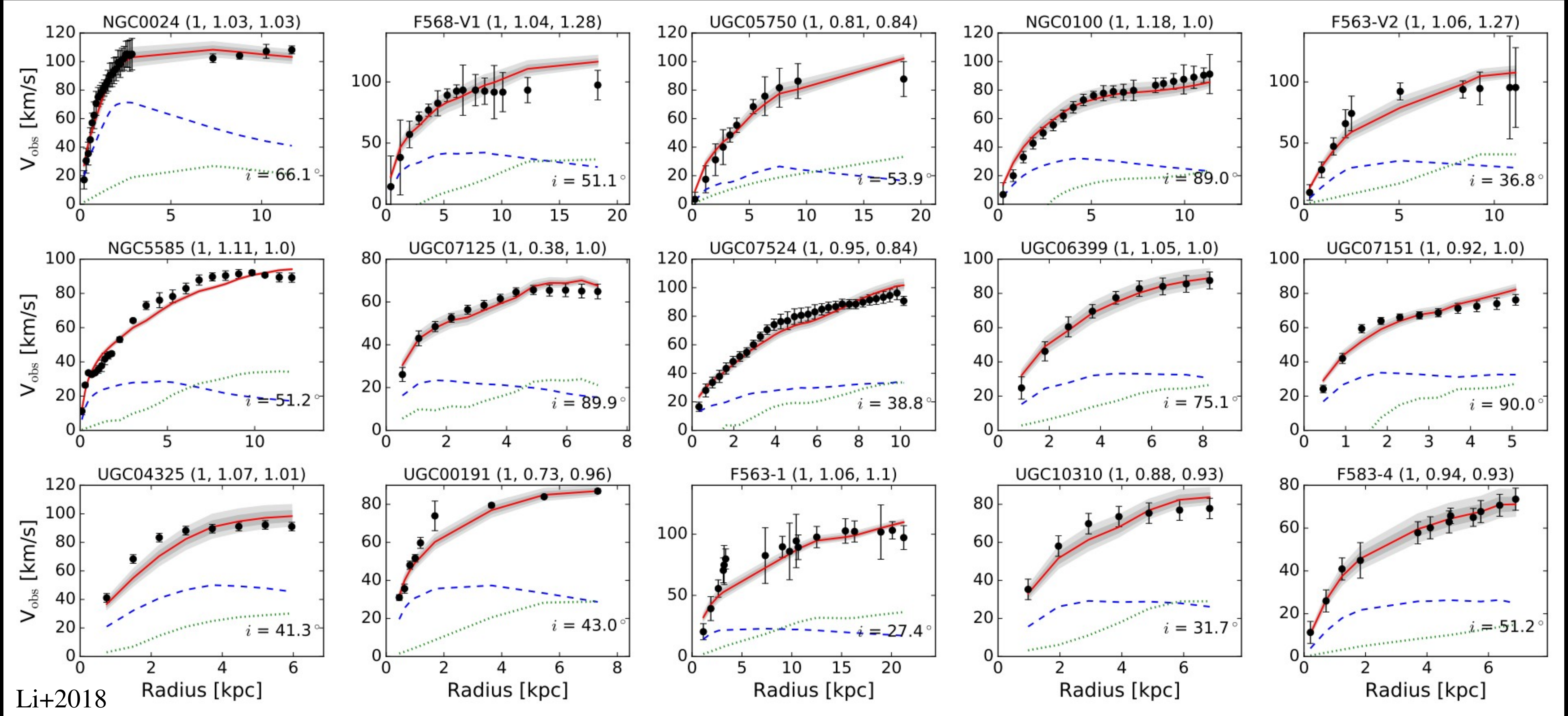
(iv) MOND interpolation function μ or ν

$$a\mu\left(\frac{a}{a_0}\right)=g_N \iff a=\nu\left(\frac{g_N}{a_0}\right)g_N$$

We can now assume $\nu(g_N/a_0)$ and predict rotation curves given ρ_b (within the errors)

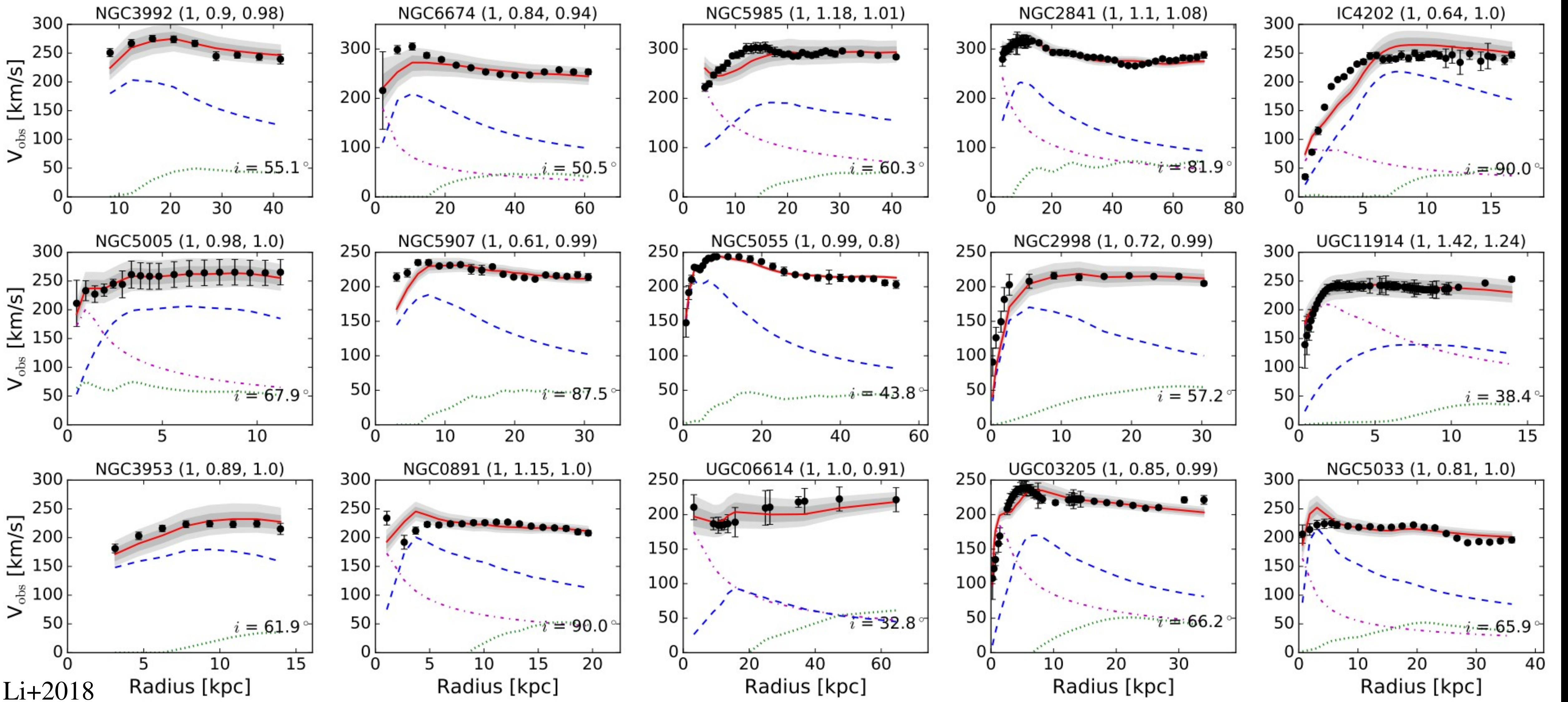
$$\nabla^2\Phi_N(R,z)=4\pi G\rho_b(R,z)$$
$$g_N(R,z=0)=-\nabla\Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



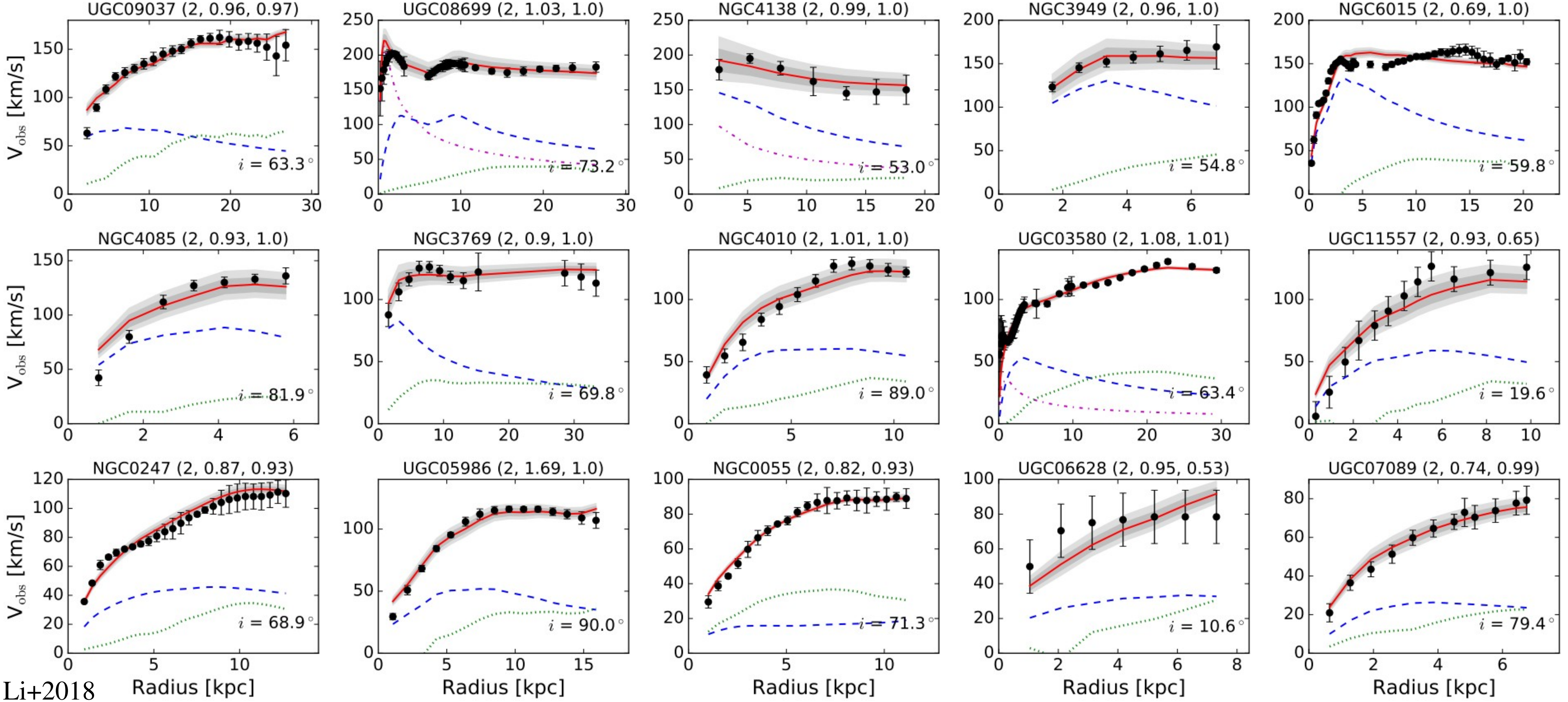
Li+2018

(3) Rotation curves can be predicted from the baryon distribution



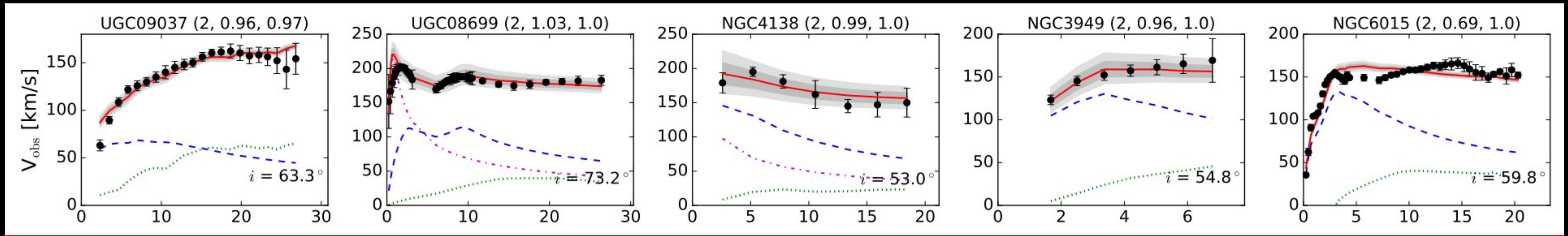
Li+2018

(3) Rotation curves can be predicted from the baryon distribution

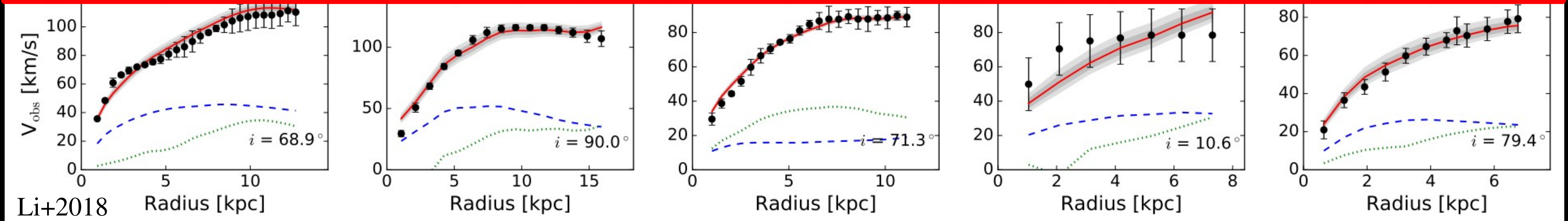


Li+2018

(3) Rotation curves can be predicted from the baryon distribution



For the full sample of 175 galaxies, see Li, Lelli, McGaugh et al. (2018) or the SPARC database (http://astroweb.cwru.edu/SPARC/)



Li+2018

Talk Outline

1. ~~General MOND paradigm~~

2. ~~MOND in nearby galaxies~~

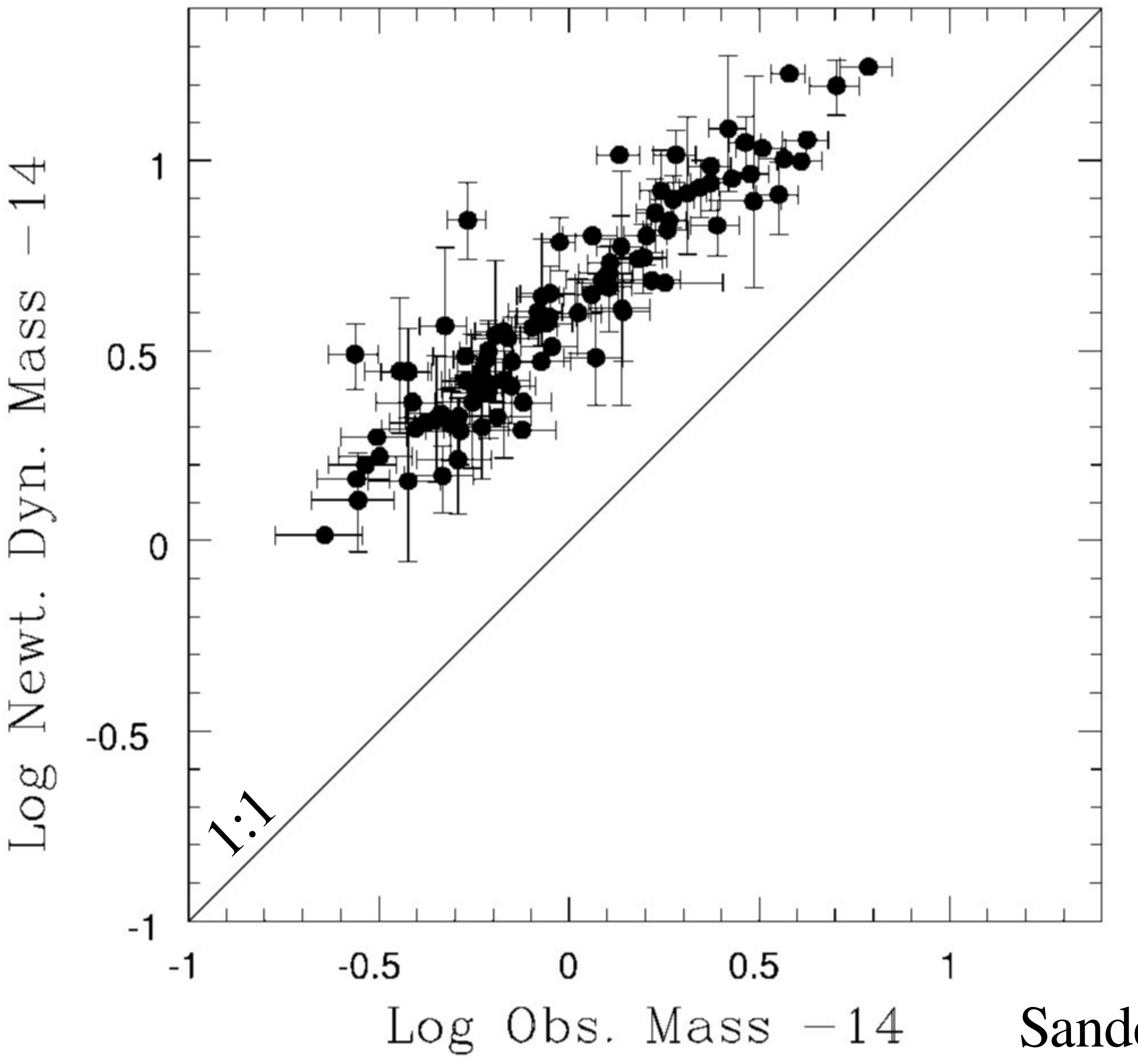
3. MOND in galaxy clusters

4. MOND cosmology

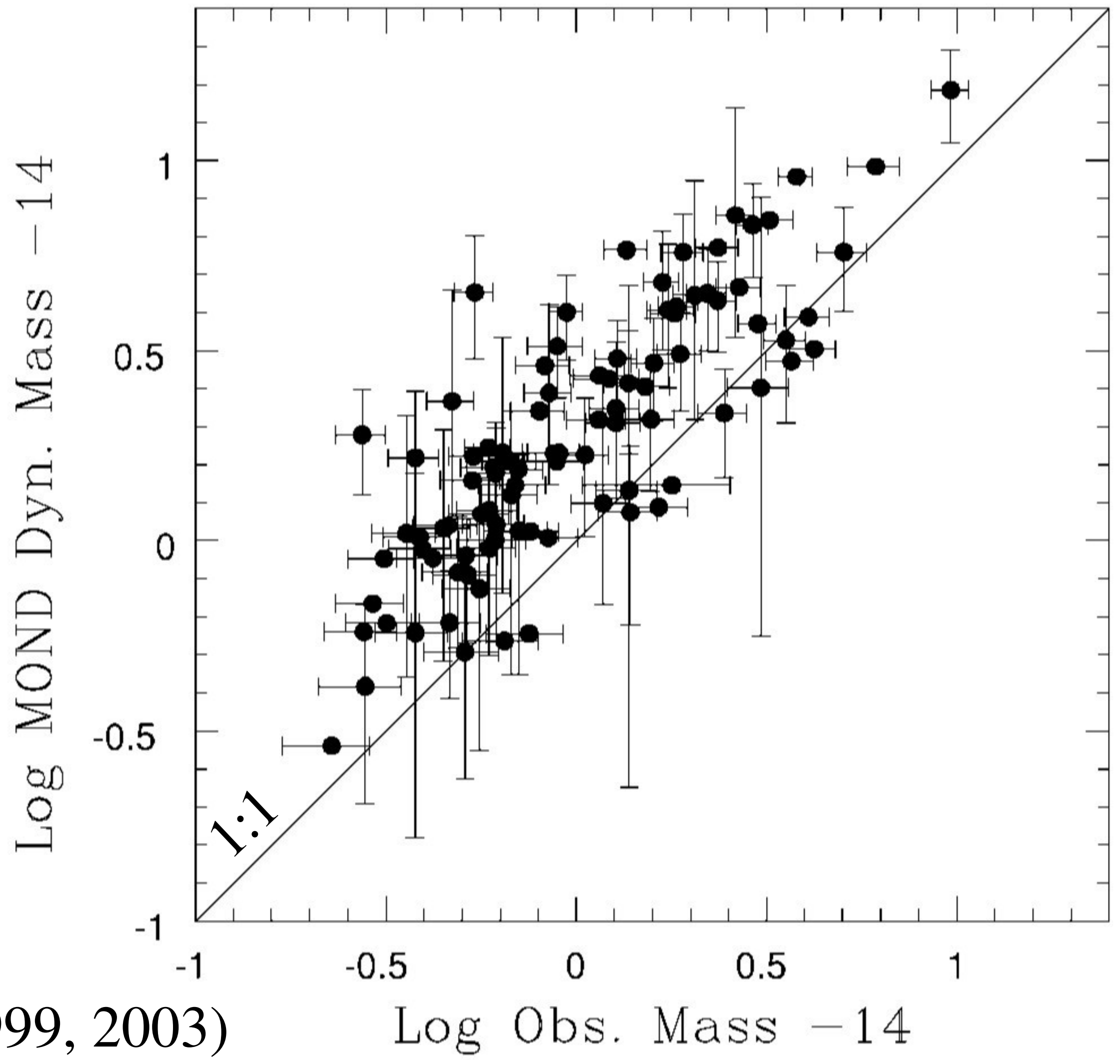
5. Specific MOND theories

Galaxy Clusters: Long-standing problem for MOND

Newtonian analysis: $M_{dyn}/M_{bar} \simeq 5$



MOND analysis: $M_{dyn}/M_{bar} \simeq 2$



Sanders (1999, 2003)

Nature of the missing mass?

- Dark baryons such as compact clouds of cold gas (Milgrom 2008)

For HI clouds (T ~10^4 K) → $M_{cl} < 10^5\ M_\odot$ and $R_{cl} < 50$ pc

(Kelleher & Lelli 2024)

Below current HI detection limits; SKA may detect them.

Nature of the missing mass?

- Dark baryons such as compact clouds of cold gas (Milgrom 2008)

  For HI clouds (T ~10^4 K) → $M_{cl} < 10^5\ M_\odot$ and $R_{cl} < 50$ pc

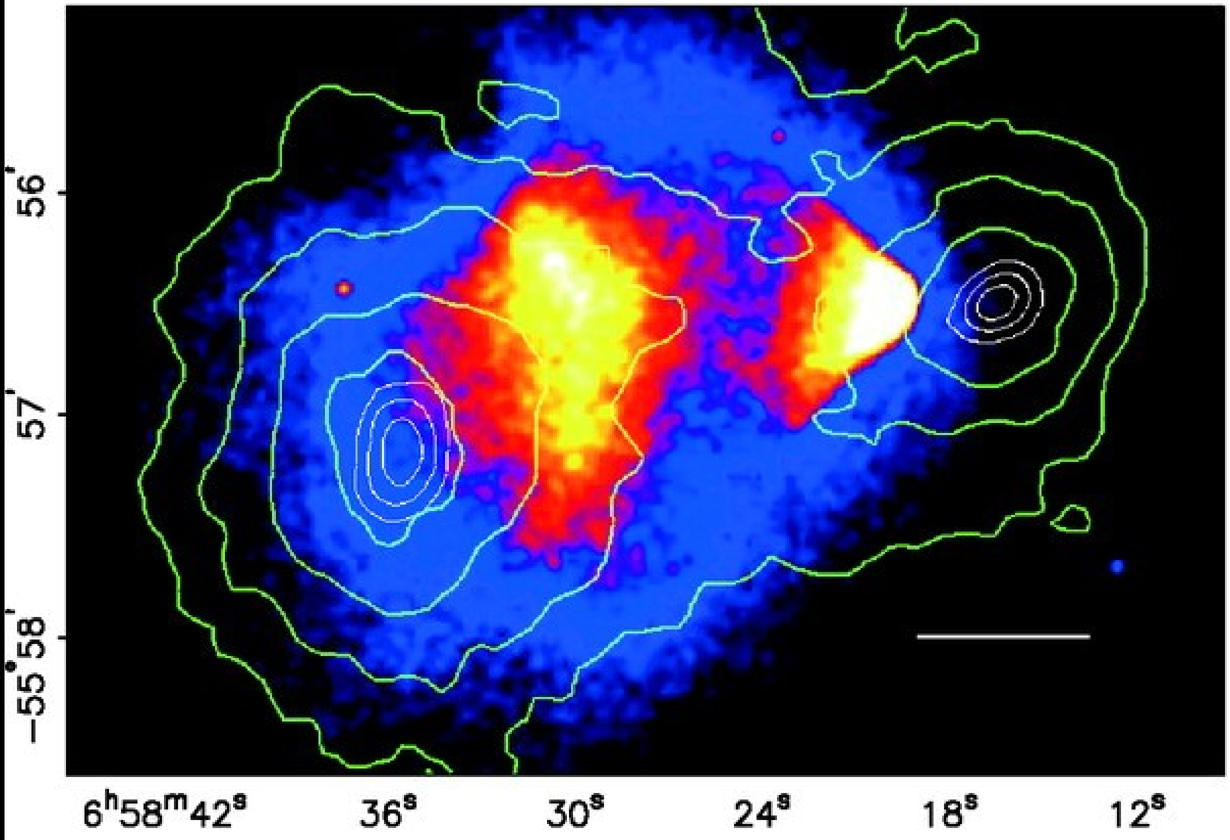
  (Kelleher & Lelli 2024)

  Below current HI detection limits; SKA may detect them.

- Sterile neutrinos of ~10 eV (Angus 2008, 2009) but CMB power spectrum may be problematic (Thomas+2016, Kopp+2018, Ilic+2021)

Bullet cluster in MOND: missing mass must be collisionless

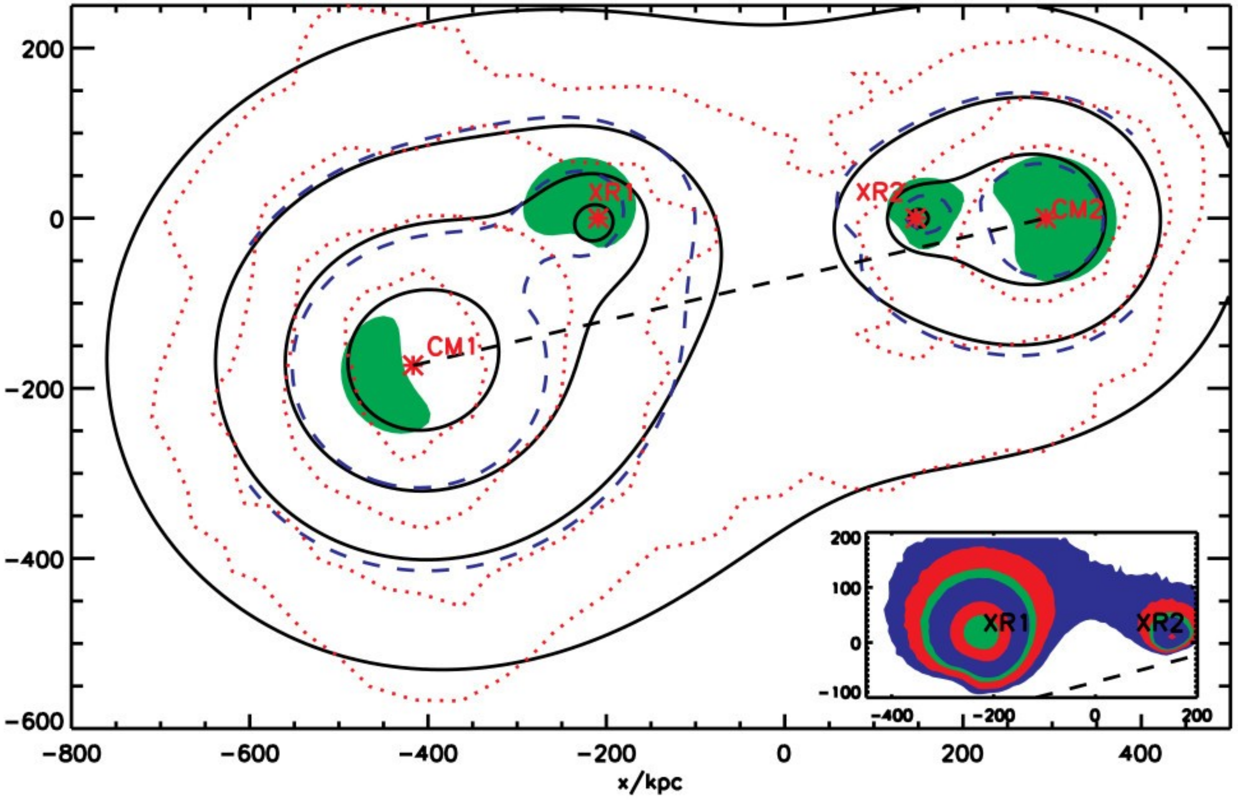
OBSERVATIONS (Clowe+2004, ApJ)



Green: Observed lensing map (total mass)

Blue/Red/Yellow: X-ray emission (hot gas)

MOND (Angus+2006, MNRAS; Angus+2007, ApJ)



Red: Observed lensing convergence map

Black: TeVeS model with 2eV neutrinos

Blue: total surface densities (baryons+υ)

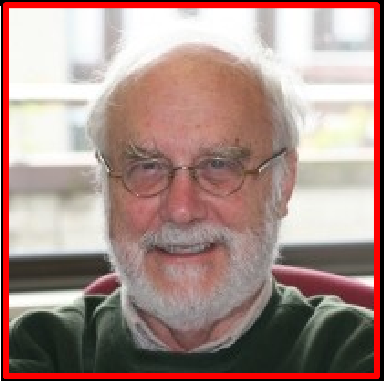
Talk Outline

1. ~~General MOND paradigm~~
2. ~~MOND in nearby galaxies~~
3. ~~MOND in galaxy clusters~~
4. MOND cosmology
5. Specific MOND theories

Cosmology with modified Newtonian dynamics (MOND)

R. H. Sanders

Kapteyn Astronomical Institute, Groningen, The Netherlands



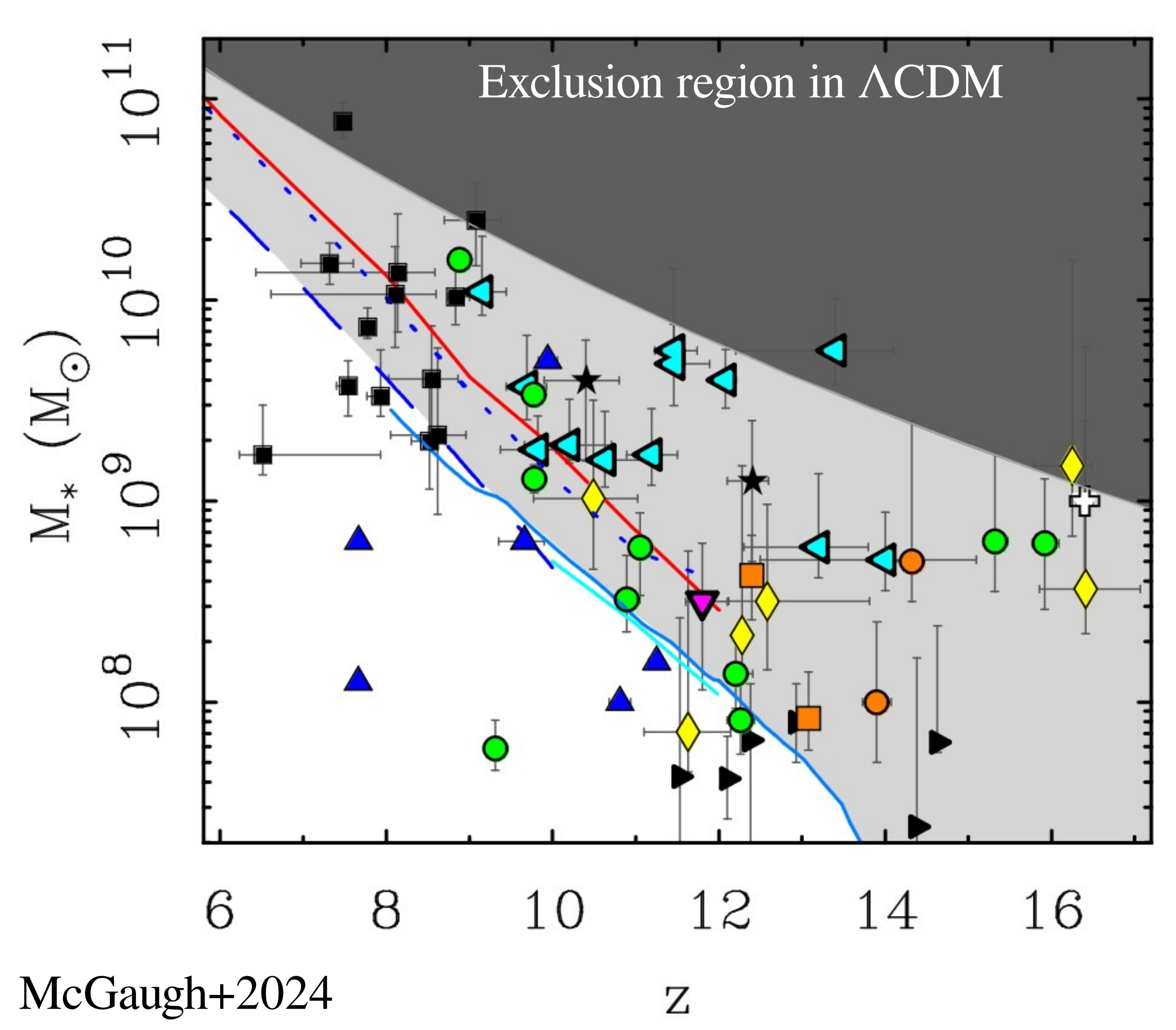
Accepted 1998 January 13. Received 1997 December 22; in original form 1997 October 17

ABSTRACT

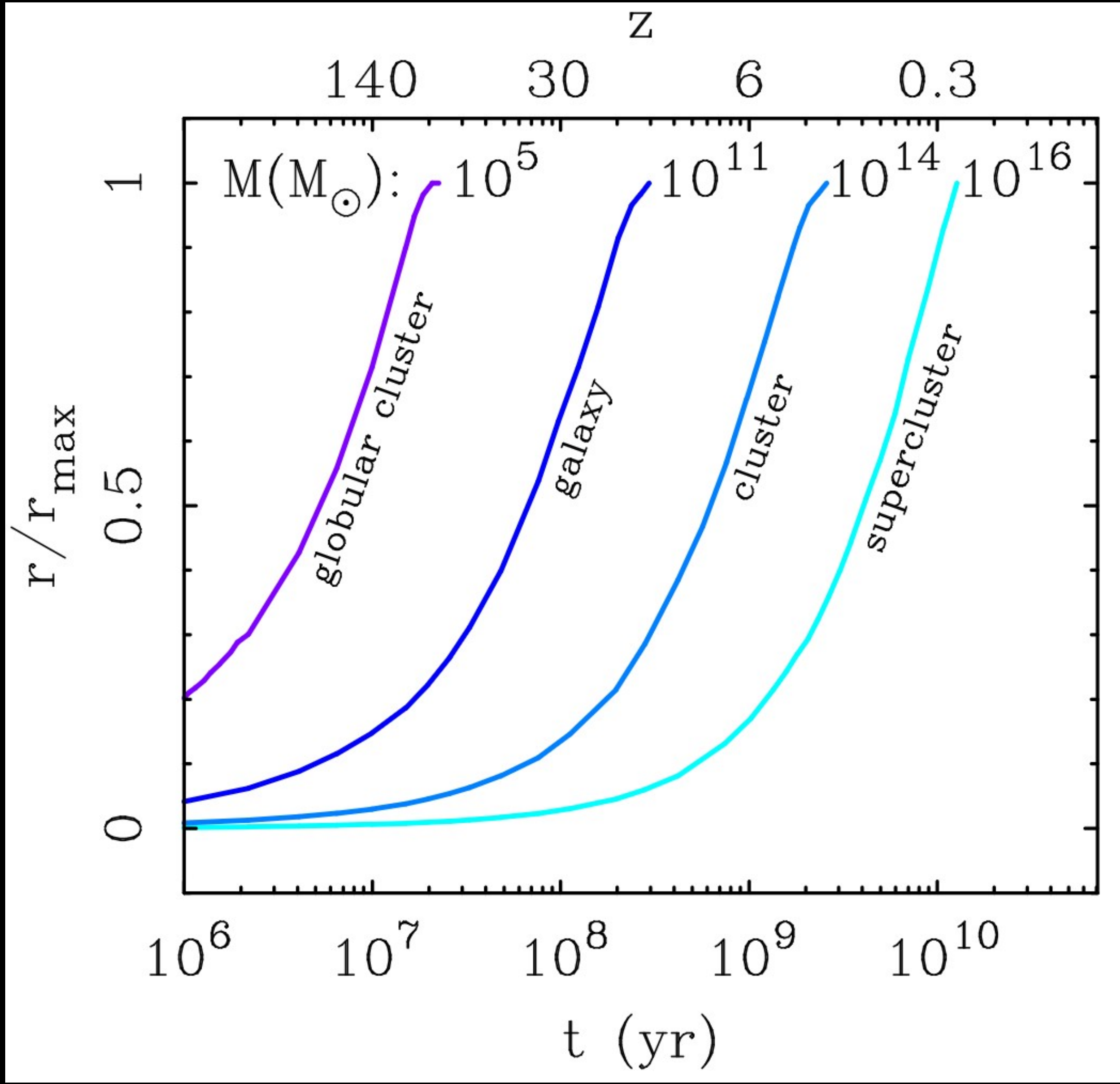
It is well known that the application of Newtonian dynamics to an expanding spherical region leads to the correct relativistic expression (the Friedmann equation) for the evolution of the cosmic scalefactor. Here, the cosmological implications of Milgrom's modified Newtonian dynamics (MOND) are considered by means of a similar procedure. Earlier work by Felten demonstrated that in a region dominated by modified dynamics the expansion cannot be uniform (separations cannot be expressed in terms of a scalefactor) and that any such region will eventually recollapse regardless of the initial expansion velocity and mean density. Here I show that, because of the acceleration threshold for the MOND phenomenology, a region dominated by MOND will have a finite size which, in the earlier Universe ($z > 3$), is smaller than the horizon scale. Therefore, uniform expansion and homogeneity on the horizon scale are consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on smaller scales. In the radiation-dominated era, the amplitude of MOND-induced inhomogeneities is much smaller than that implied by observations of the cosmic background radiation, and the thermal and dynamical history of the Universe is identical to that of the standard big bang model. In particular, the standard results for primordial nucleosynthesis are retained. When matter first dominates the energy density of the Universe, the cosmology diverges from that of the standard model. Objects of galaxy mass are the first virialized objects to form (by $z = 10$), and larger structure develops rapidly. At present, the Universe would be inhomogeneous out to a substantial fraction of the Hubble radius.

Massive galaxies at z >10 is the new normal!

Compilation of JWST observations



MOND prediction (following Sanders 1998)



Talk Outline

~~1. General MOND paradigm~~

~~2. MOND in nearby galaxies~~

~~3. MOND in galaxy clusters~~

~~4. MOND cosmology~~

5. Specific MOND theories

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Modified Inertia ($\rightarrow F = ma$)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Modified Inertia ($\rightarrow F = ma$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Modified Inertia ($\rightarrow F = ma$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

TeVeS: Tensor-Vector-Scalar (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Modified Inertia ($\rightarrow F = ma$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c\,\Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c\,\Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

MOND-like DM models:

Bipolar DM (Blanchet+2008, 2009, 2015, 2017)

Superfluid DM (Berezhiani & Khoury 2015)

Baryon-interacting DM (Famaey+2018, +2020)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c\,\Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

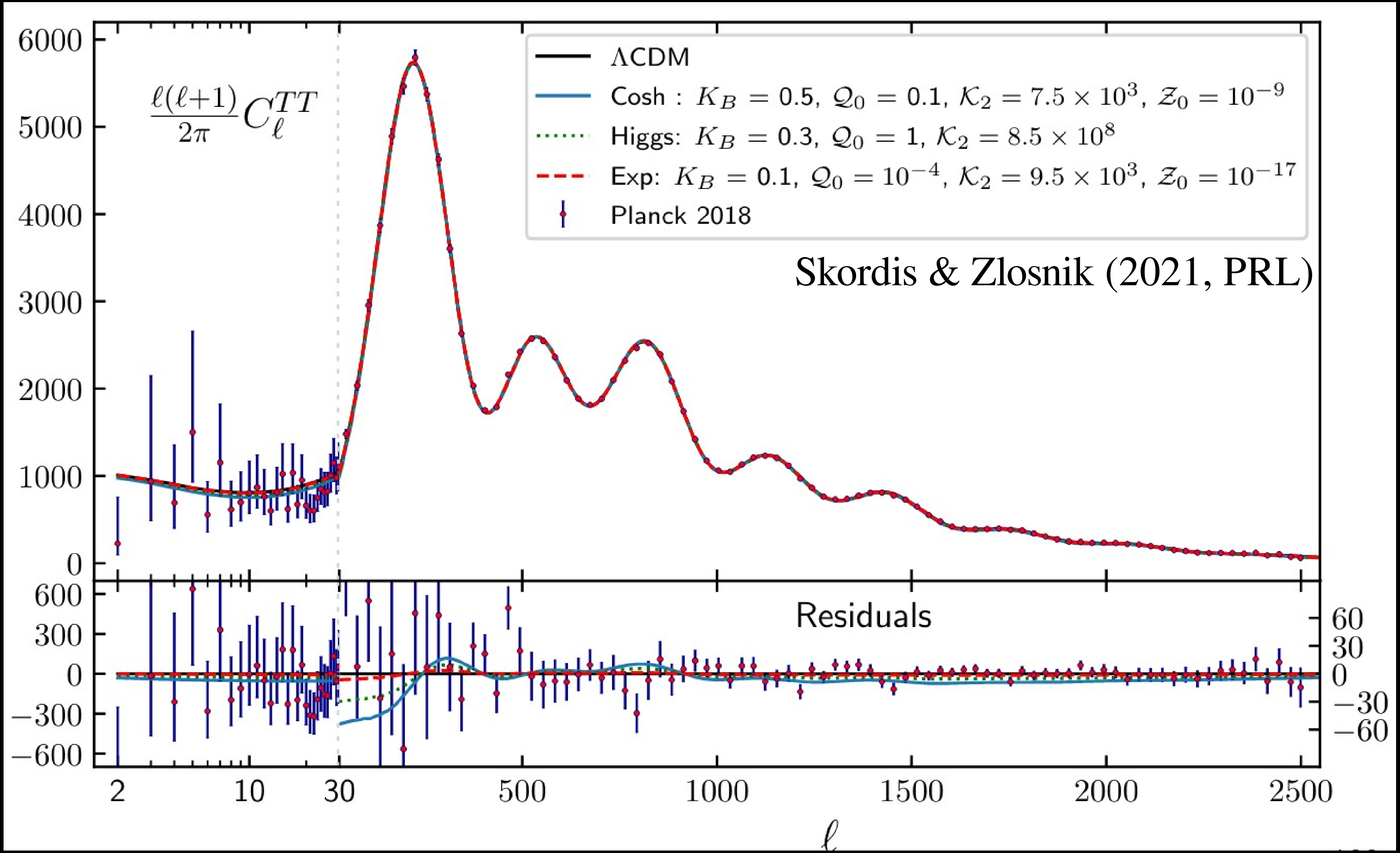
MOND-like DM models:

Bipolar DM (Blanchet+2008, 2009, 2015, 2017)

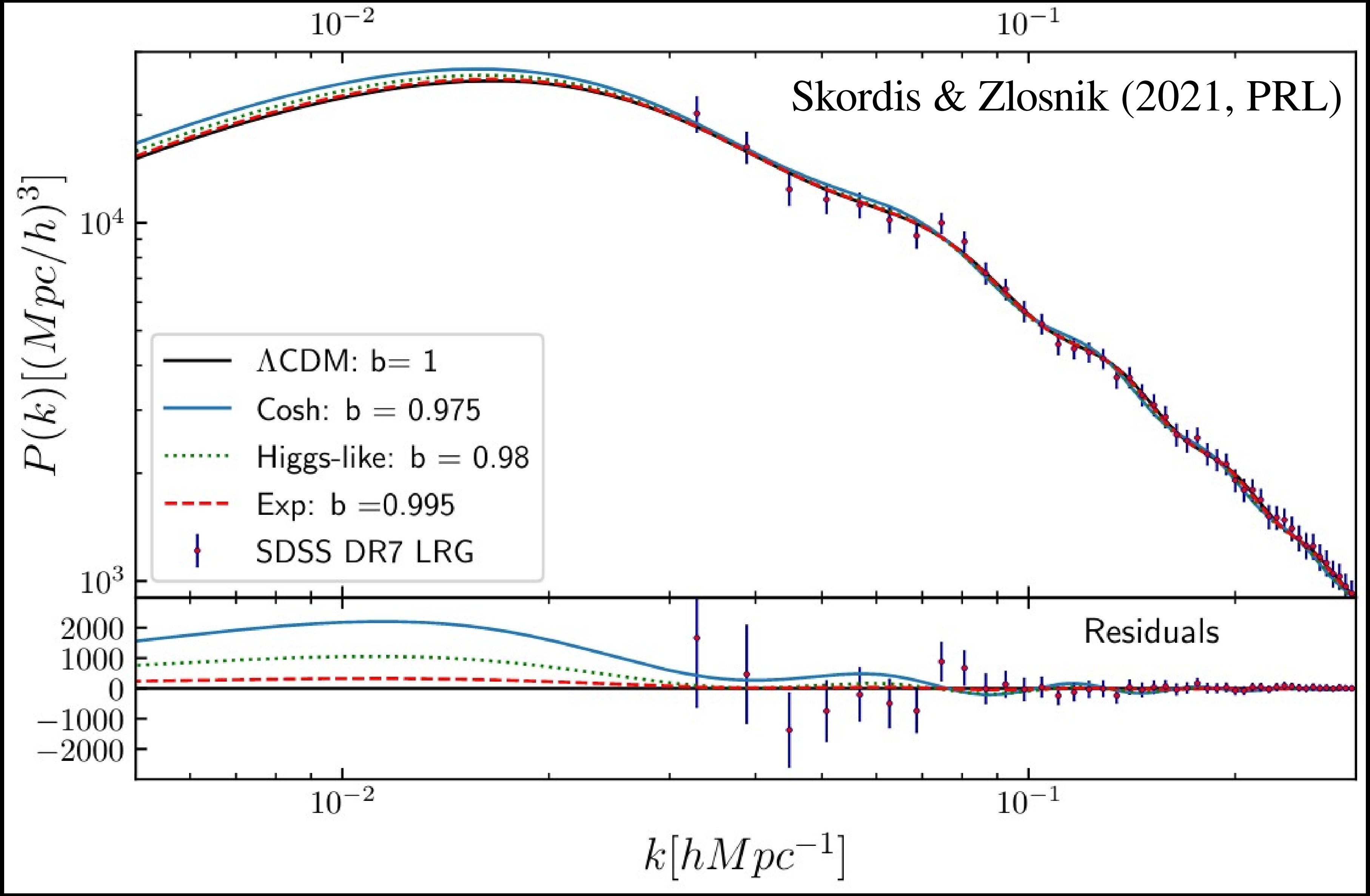
Superfluid DM (Berezhiani & Khoury 2015)

Baryon-interacting DM (Famaey+2018, +2020)

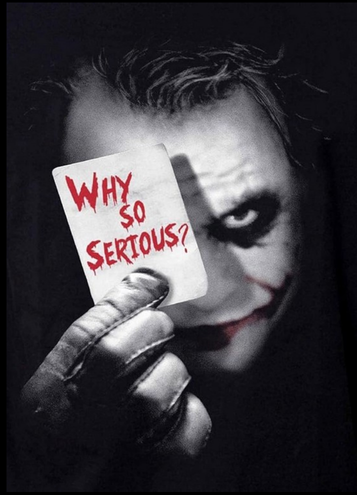
AeST theory: CMB Power Spectrum



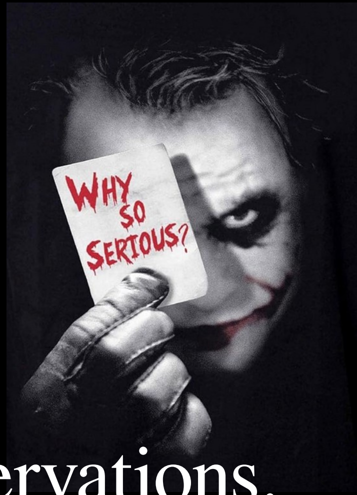
AeST theory: Linear Matter Power Spectrum



Why taking MOND seriously?

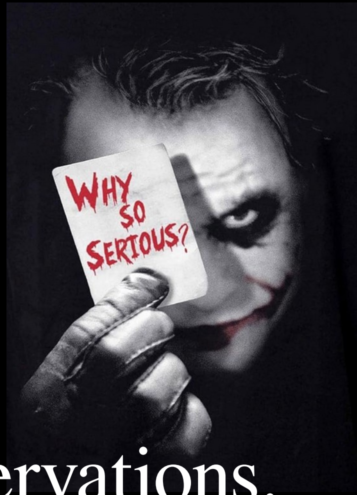


Why taking MOND seriously?



- MOND made multiple *a-priori* predictions, later confirmed by observations.
  A-priori predictions are the core of the scientific method.

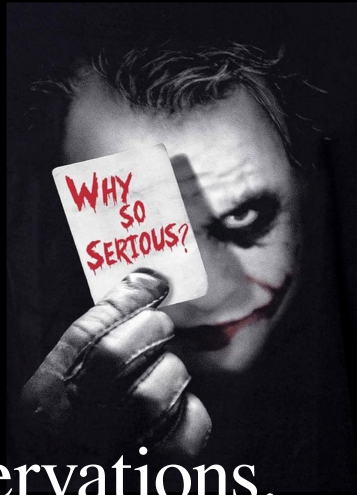
Why taking MOND seriously?



- MOND made multiple *a-priori* predictions, later confirmed by observations. A-priori predictions are the core of the scientific method.

- Even if MOND is wrong (or DM is detected), there must be a baryon – DM coupling in galaxies acting in a MOND way. It tells us on the nature of DM.

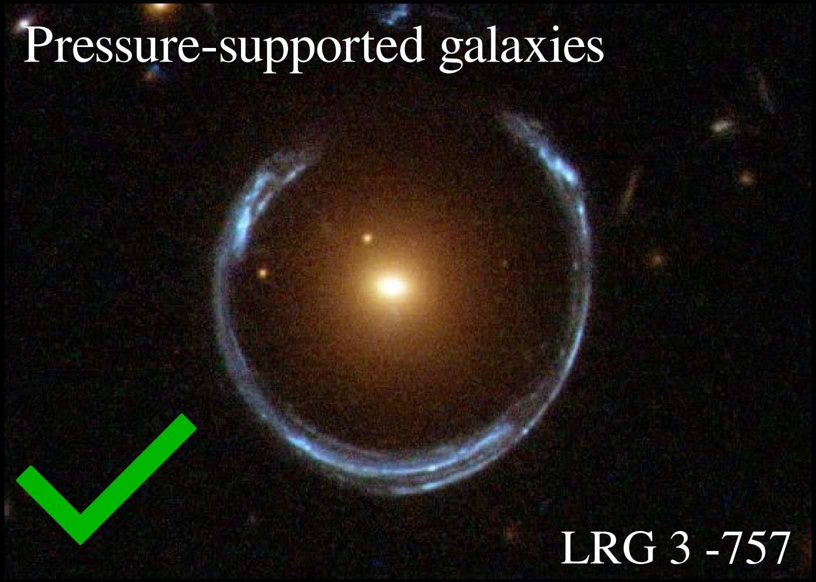
Why taking MOND seriously?



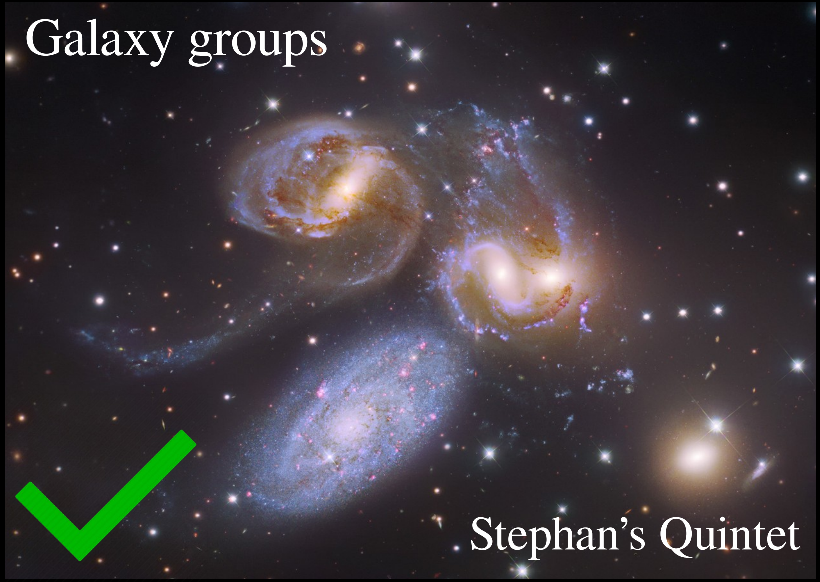
- MOND made multiple *a-priori* predictions, later confirmed by observations. A-priori predictions are the core of the scientific method.
- Even if MOND is wrong (or DM is detected), there must be a baryon – DM coupling in galaxies acting in a MOND way. It tells us on the nature of DM.
- Over the past ~10 years, substantial progress in MOND research in both observations and theory (new tests, new ideas, new relativistic theories).

Summary: Status of MOND at Various Scales

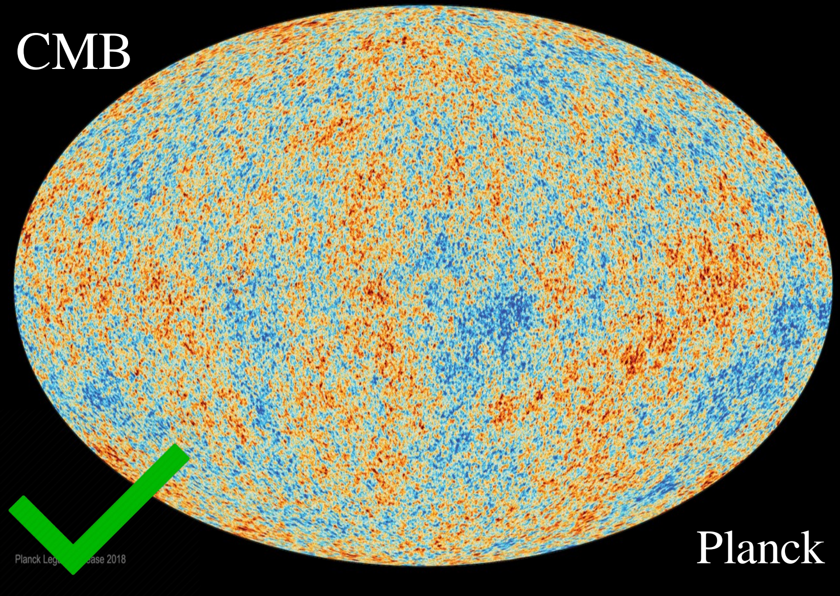
Galaxy Scales (~1-100 kpc)

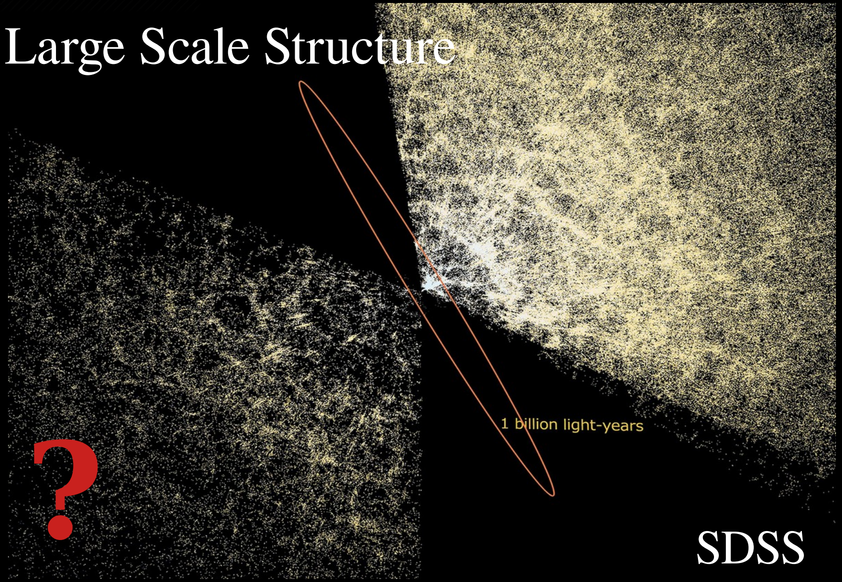


Groups/Clusters Scales (~1-5 Mpc)



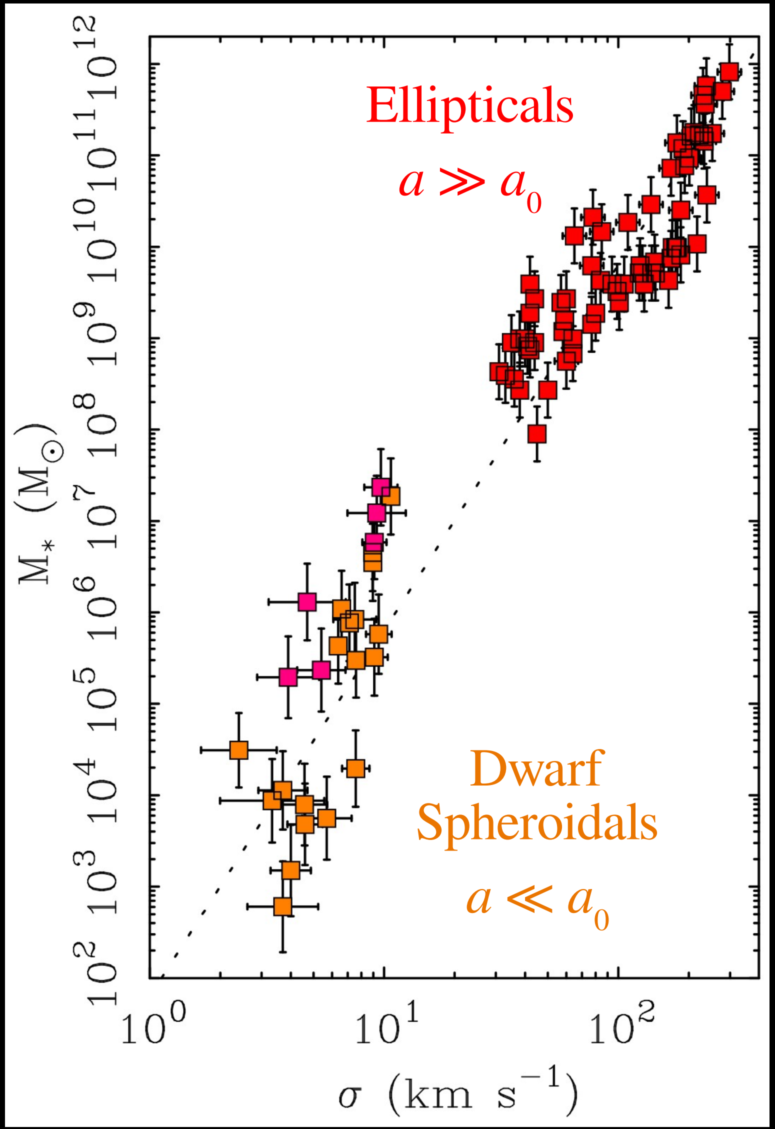
Cosmological Scales (>100 Mpc)





More Slides

(4) $\sigma_V^4 \simeq a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals



Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(ii) Normalization is $a_0 G$ → OK with BTFR estimate!

(iii) No dependence on other quantities IF $a \ll a_0$ → OK

σ_V is measured at $R < R_e$ (containing half luminosity):

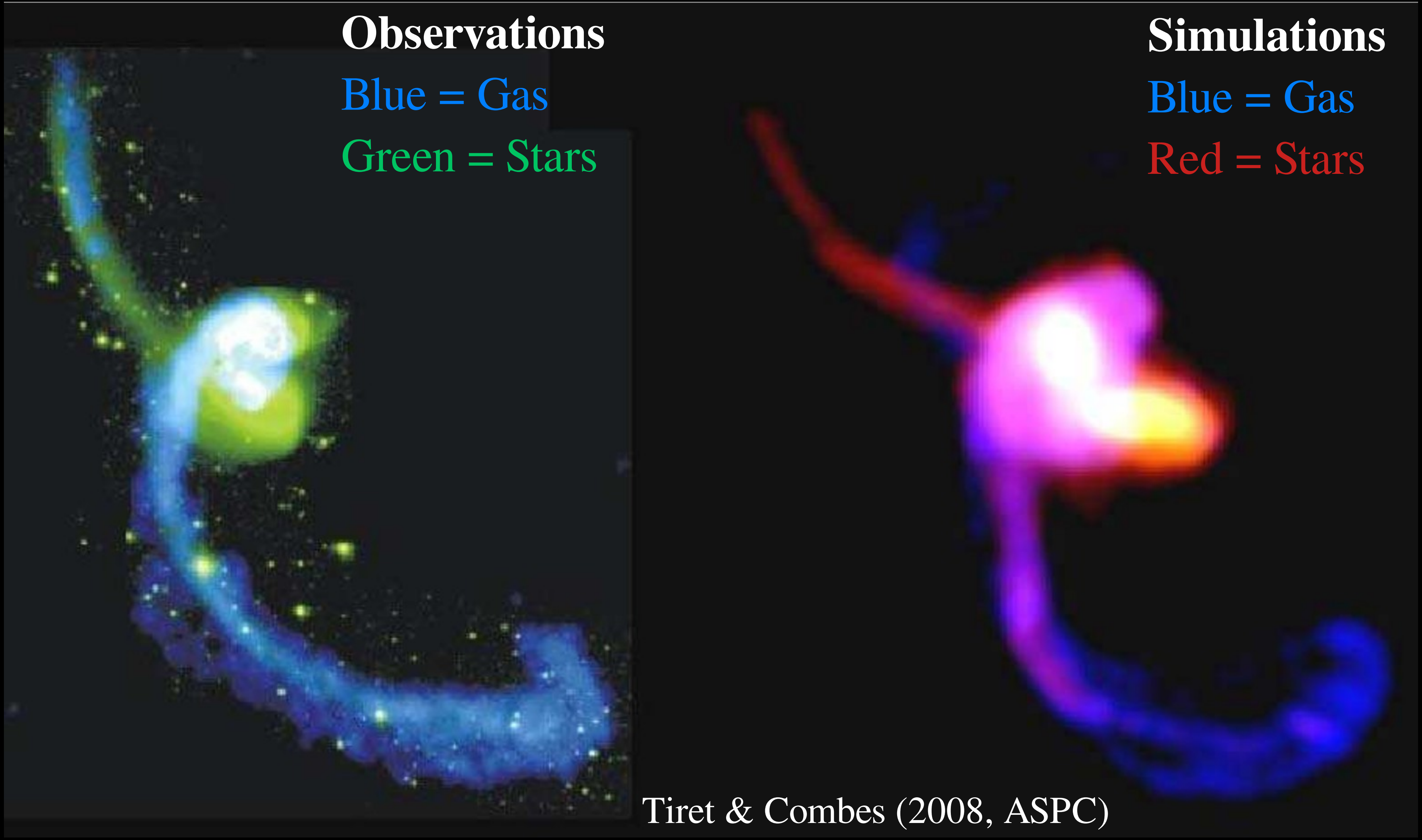
For dwarf spheroidals: $a \ll a_0$ at $R<R_e$ → MOND regime

For giant ellipticals: $a \gg a_0$ at $R<R_e$ → Newtonian regime

$$\frac{\sigma_V^2}{R} \simeq \frac{GM}{R^2} \Rightarrow M \simeq \sigma_V^2 R_e$$

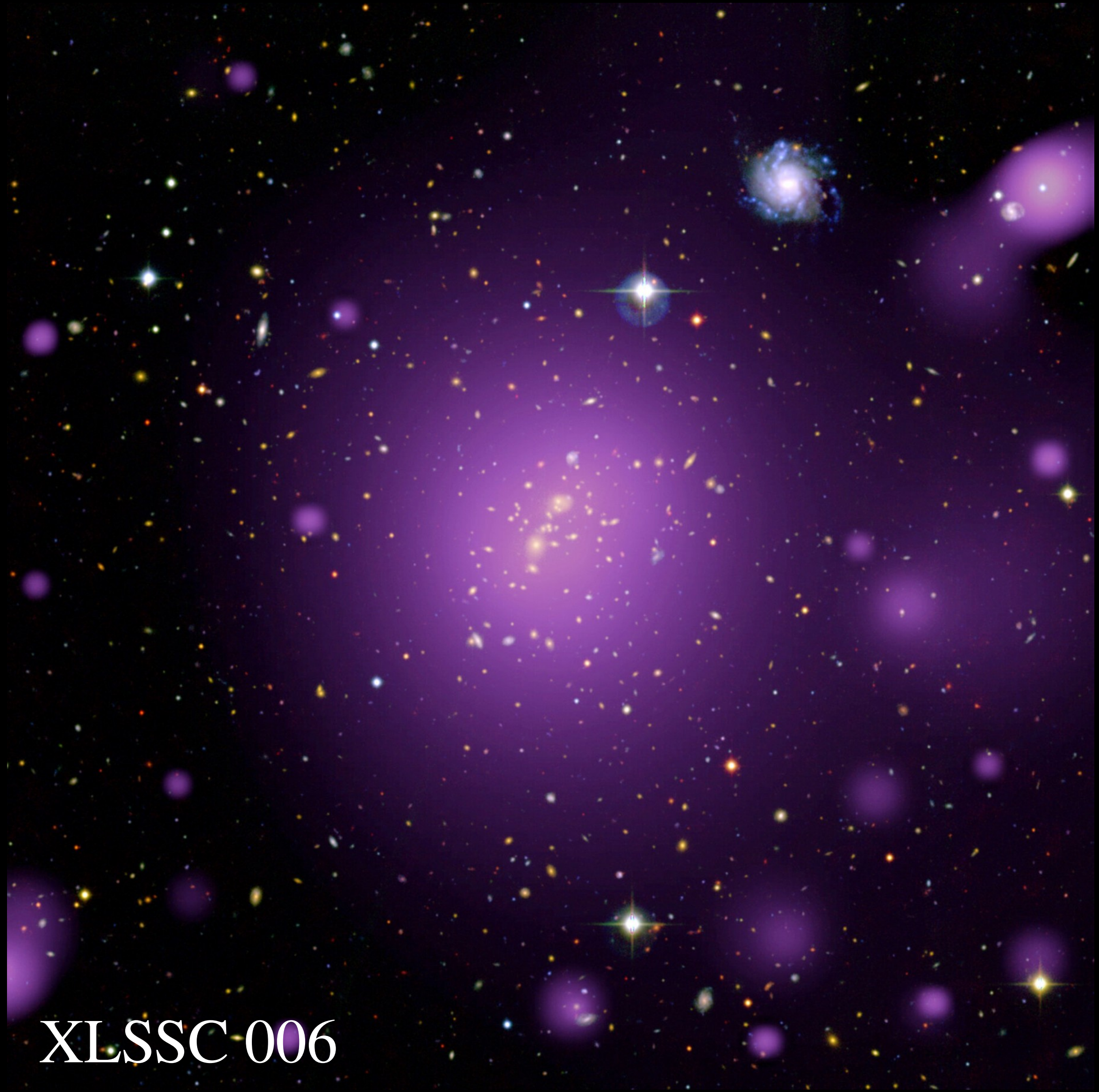
Fundamental plane of ellipticals
(Djorgovski & Davis 1987; Dressler 1987)

Interacting & Merging Galaxies: The Antennae



Tiret & Combes (2008, ASPC)

Galaxy Clusters: bound systems with ~100-1000 galaxies



Observed baryon budget:

~10% galaxies (optical & NIR)

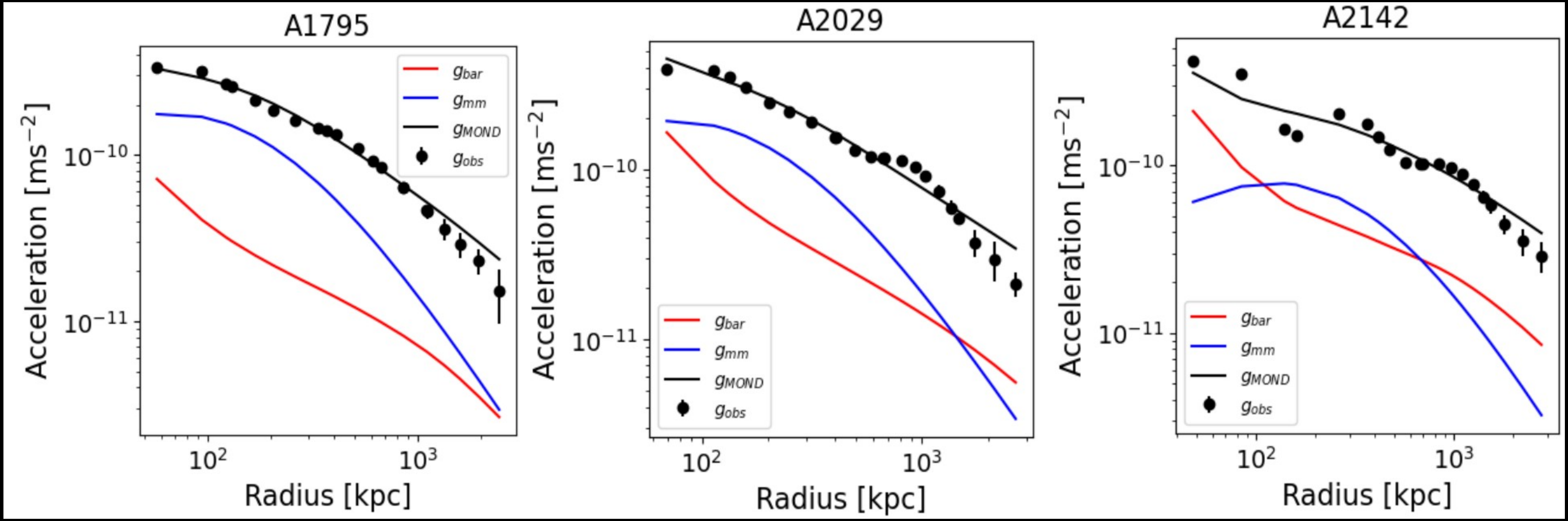
~90% hot ionized gas (X rays)

Sphere in Hydrostatic Equilibrium

$$g_{obs} = \frac{-1}{\rho_{gas}} \frac{\partial P_{gas}}{\partial r} \qquad P_{gas} = \frac{k_B \rho_{gas} T_{gas}}{w\, m_p}$$

$$\Rightarrow \quad g_{obs} = \frac{-k_B}{w\, m_p \rho_{gas}} \frac{\partial}{\partial r} \left(\rho_{gas} T_{gas} \right)$$

MOND fits with a missing mass component (Kelleher & Lelli 2024)



Density profile of missing mass:

$$\rho_{mm}(r) = \frac{\rho_0}{(1 + r/r_s)^4}$$

Converged (physical) total mass:

$$\Rightarrow \quad M_{mm,tot} = \int_0^\infty \rho(r) 4\pi r^2 dr = \frac{4\pi}{3} \rho_0 r_s^3$$

MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \simeq \frac{H_0 \cdot c}{2\pi}$$

H_0 = Hubble constant → maybe $a_0(t) \sim H(t)$?

$$a_0 \simeq \frac{c^2 \sqrt{\Lambda/3}}{2\pi}$$

Λ = Cosmological constant → relation to Dark Energy?

IF this numerology has some deeper, fundamental meaning:

Either the state of the Universe at large enters in local dynamics, or

the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984, ApJ)

$$S_N = \int dt\, L_N = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Lagrangian is quadratic in $\nabla\Phi$

↓

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla}\Phi|^2}{a_0^2} \right)$$

$F(z) \rightarrow z$ for $z = |\nabla\Phi|^2/a_0^{\,2} \gg 1$

$F(z) \rightarrow z^{3/2}$ for $z = |\nabla\Phi|^2/a_0^{\,2} \ll 1$

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla \cdot \left[\mu\left(\frac{|\vec{\nabla}\Phi|}{a_0} \right) \vec{\nabla}\Phi \right] = 4\pi G \rho$$

Modified Poisson's Equation

$$\mu(x) = \frac{dF(z)}{dz} \qquad z = x^2$$

$F(z)$ provides the interpolation function $\mu = \upsilon^{-1}$

QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S_N = \int dt\, L_N = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Single gravitational potential Φ

$$\frac{-1}{8\pi G}\left[2\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi_N - a_0^2 Q\left(\frac{|\vec{\nabla}\Phi_N|^2}{a_0^2} \right) \right]$$

Two potentials: Φ and Φ_N!

QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S_N = \int dt\, L_N = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$ Single gravitational potential Φ

↓

$$\frac{-1}{8\pi G}\left[2\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi_N - a_0^2 Q\left(\frac{|\vec{\nabla}\Phi_N|^2}{a_0^2} \right) \right]$$ Two potentials: Φ and Φ_N!

Principle of least Action varying Φ, Φ_N and $\bar{x}$ → set of 3 equations

$\nabla^2 \Phi_N = 4\pi G\rho$ ⟶ Standard, linear Poisson's equation for Φ_N

$\nabla^2 \Phi = \vec{\nabla} \cdot \left[\nu\left(|\vec{\nabla}\Phi_N| / a_0 \right) \vec{\nabla}\Phi_N \right]$ ⟶ Non-linear step: get Φ from Φ_N $\quad \nu(\sqrt{x}) = \frac{dQ(x)}{x}$

$\vec{a} = -\vec{\nabla}\Phi$ ⟶ Acceleration/force set by second potential Φ

Relativistic MOND → add degrees of fredoom (new fields) to GR

- Tensor $g_{\mu\nu}$ → Einstein's metric
- Scalar ϕ → for the DM effect in the NR limit (Bekenstein & Milgrom 1984, ApJ)
- Vector A^{μ} → for gravitational lensing (Sanders 1997, ApJ; Bekenstein 2004, PRD)
- Free function(s) → interpolation function(s) (non-fundamental effective theories?)

AeST: Aether Scalar-Tensor theory (Skordis & Zlosnik 2019, PRD; 2021, PRL; 2022, PRD)

(1) Grav. Waves: $B^{\mu} = e^{-2\phi} A^{\mu}$ (timelike) → theories $\{B_{\mu}, g_{\mu\nu}\}$ → $c_{GW} = c_{EM}$

(2) Cosmology: *k*-essence term for ϕ → $\rho_{\phi} \propto a(t)^{-3}$ (like dust or CDM)

MOND as Modified Inertia (Milgrom 1994, 1999, 2022)

$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla}\Phi_N$ $\bar{A}$ is a functional of the full trajectory $\bar{x}(t)$

For $a \gg a_0$, $A \rightarrow a = d^2x/dt^2$ (Newton's 2nd law)

No full theory yet, but two general results:

(1) Imposing Newtonian and MOND limits + Galilei Invariance $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

Theory is time non-local: $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i \vec{x}}{dt^i}; i = 1, 2, \ldots N\right)$

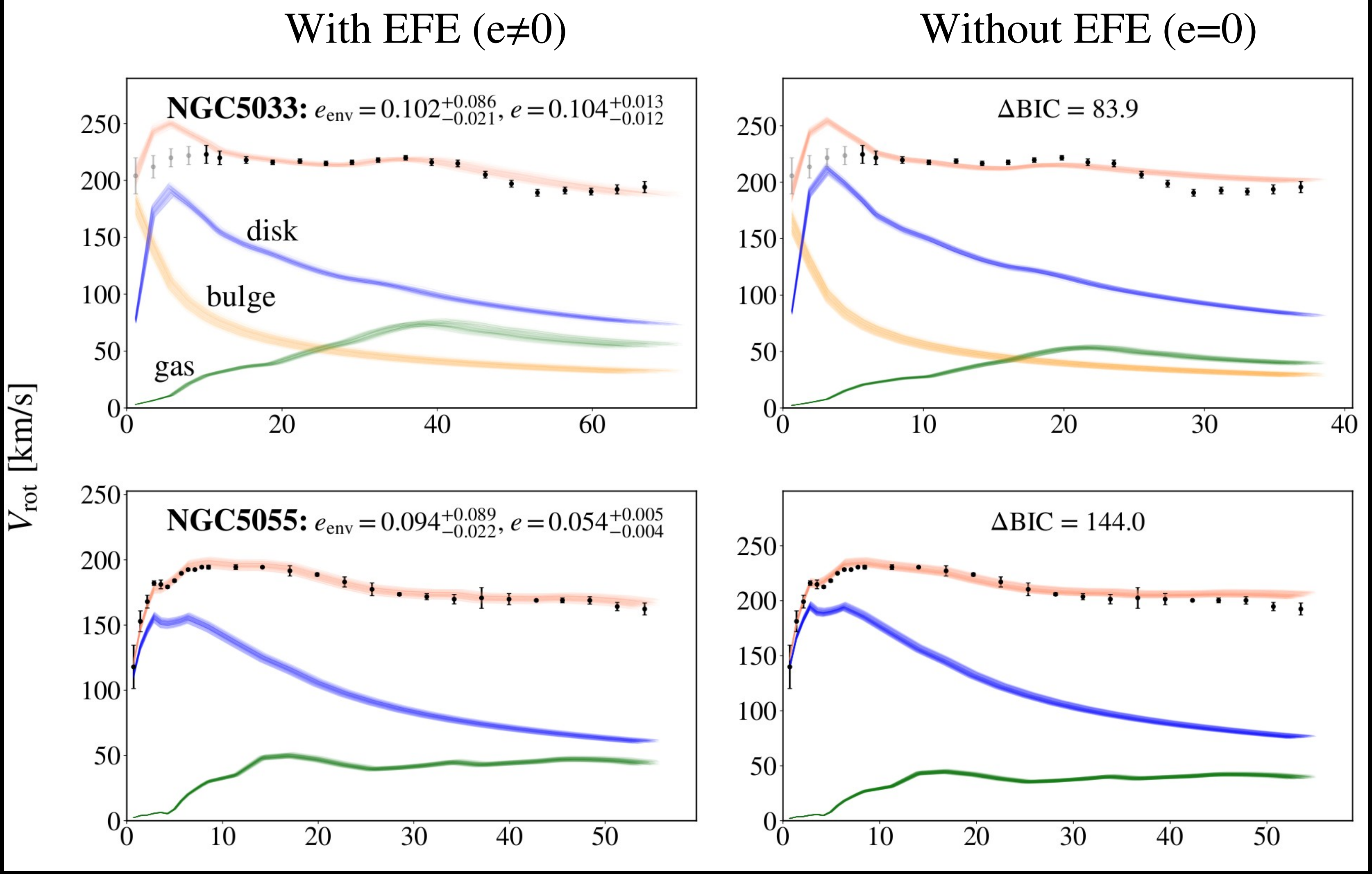
Accelerations at $(\bar{x}, t)$ depend on the full orbital history!

(2) For purely circular orbits: $\vec{a}\,\mu\left(\frac{a}{a_0}\right) = \vec{g}_N$ holds exactly (RAR for disk galaxies)

The interpolation function is a derived concept valid for circular orbits!

EFE is weak: individual detections only in extreme cases

NGC 5055

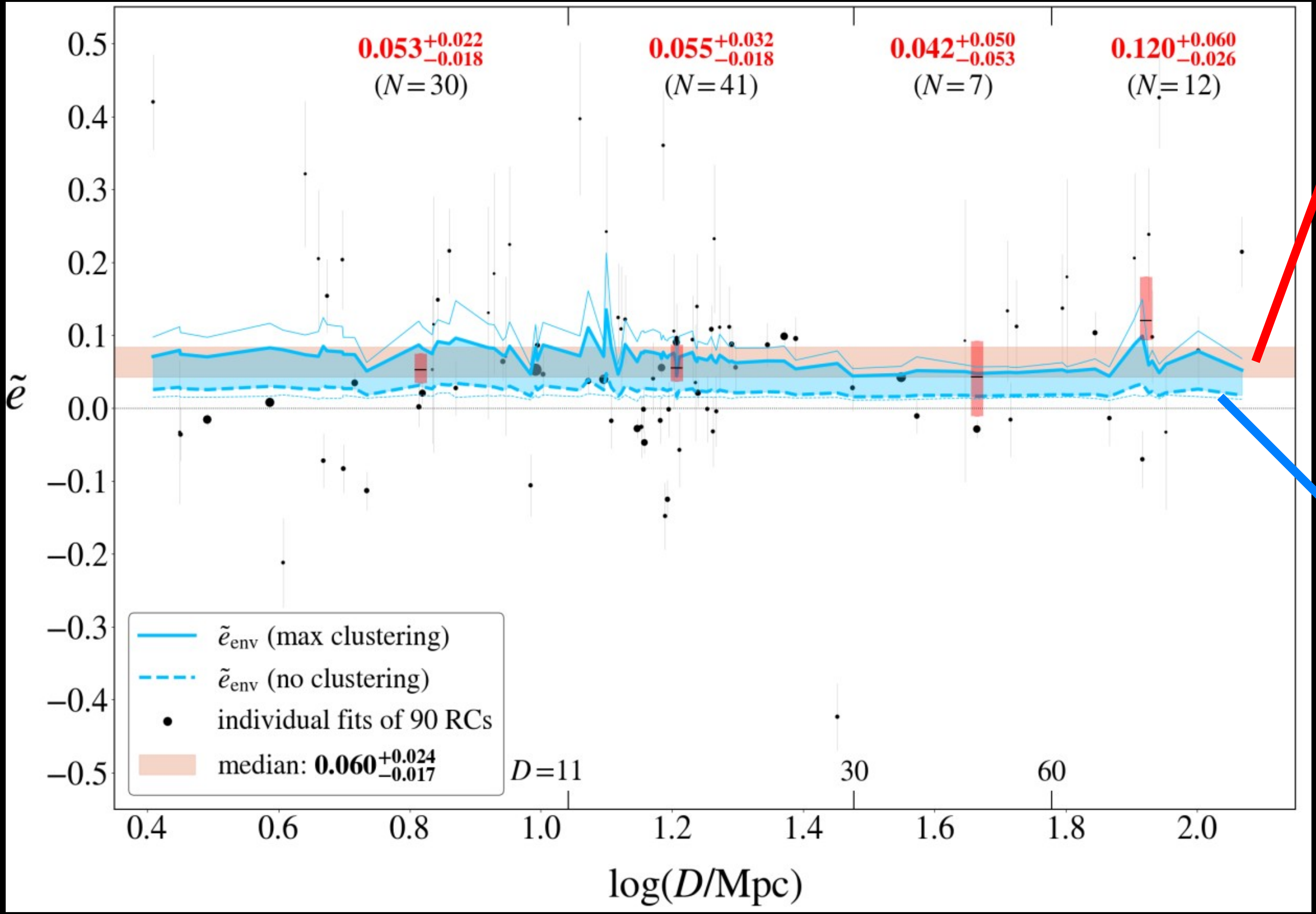


NGC 5033

Chae+2020, 2021

Statistical approach: EFE>0 at >4σ and agrees with LSS

$$\frac{g_{ext}}{a_0}$$



From Rotation Curve Fits

From Baryon Large Scale Structure

Chae+2020, 2021

Galaxy Clusters on the Radial Acceleration Relation