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GR & DM

HOW DRAGGING AND GENERAL RELATIVITY COULD EXPLAIN THE MISSING MASS PROBLEM

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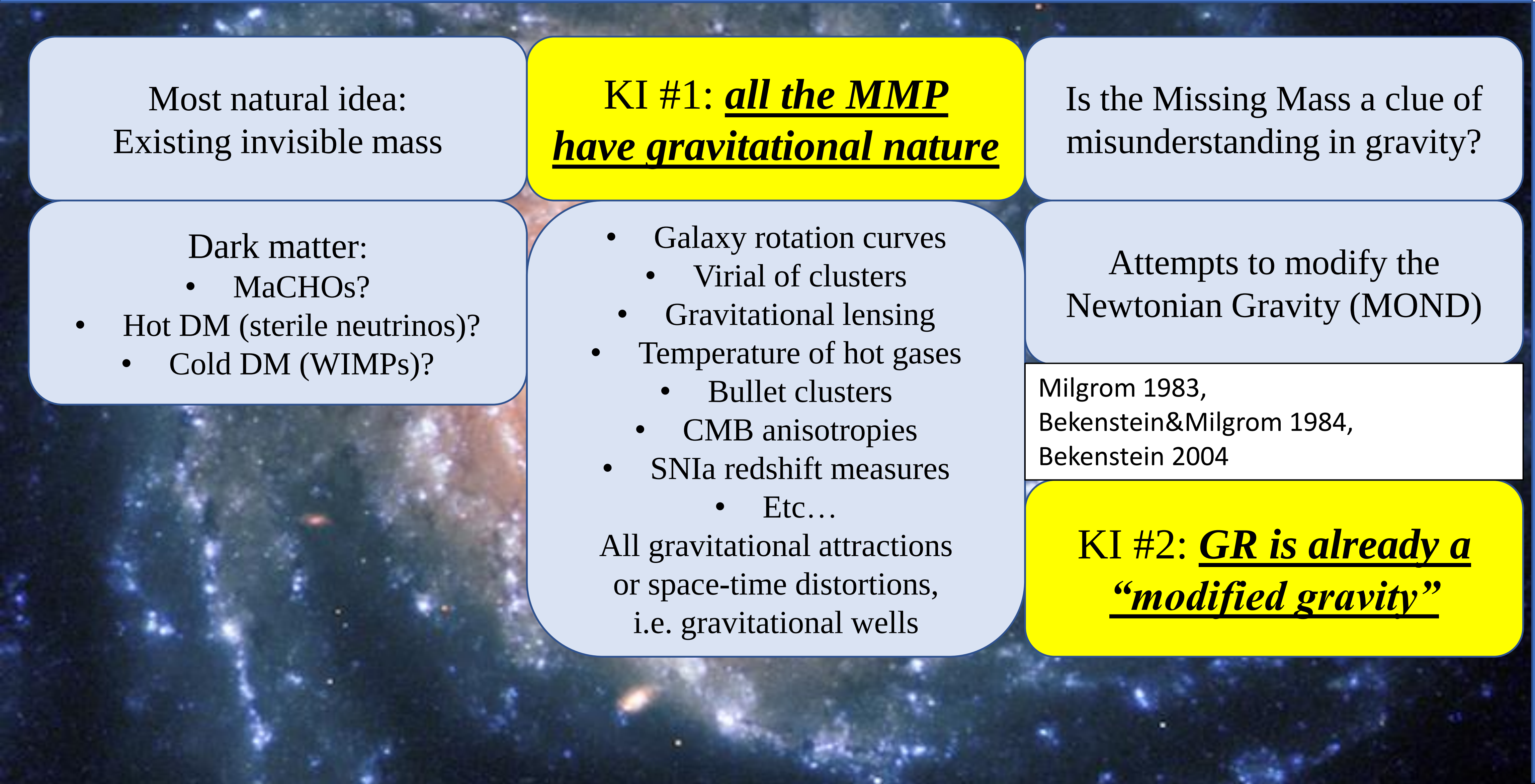
Introduction

Theoretical Framework

Empirical Measures

Conclusions

KEY IDEAS
Strategies for the Missing Mass Problem



Most natural idea:
Existing invisible mass

Dark matter:
- MaCHOs?
- Hot DM (sterile neutrinos)?
- Cold DM (WIMPs)?

KI #1: ***<u>all the MMP have gravitational nature</u>***

- Galaxy rotation curves
- Virial of clusters
- Gravitational lensing
- Temperature of hot gases
- Bullet clusters
- CMB anisotropies
- SNIa redshift measures
- Etc…

All gravitational attractions or space-time distortions, i.e. gravitational wells

Is the Missing Mass a clue of misunderstanding in gravity?

Attempts to modify the Newtonian Gravity (MOND)

Milgrom 1983,
Bekenstein&Milgrom 1984,
Bekenstein 2004

KI #2: ***<u>GR is already a “modified gravity”</u>***

KEY IDEAS
GR is more than post-Newtonian corrections

Intuition: GR = Newton +
post-Newtonian corrections

Galactic dynamics in
low energy régime:

- Sub-relativistic speeds
- Weak forces

PN terms have magnitude $\sim \frac{v^2}{c^2}$:
Negligible corrections

Ciotti 2022,
Lasenby+ 2023,
Costa+ 2023,
Glampedakis&Jones 2023
Costa&Natàrio 2023

KI #3: ***GR allows non-Newtonian phenomena in low energy régime***

Astesiano+3 2022

- Carlotto-Shoen shielding metrics
- Geons (solitonic GW)

A galaxy is an extended source:
Not *globally* Newtonian

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0i} \\ g_{0i} & g_{ij} \end{pmatrix},$$

where g_{0i} dragging term

KI #4: ***galaxy surrounded by dragging vortex supporting rotation curve***

Balasin&Grumiller 2008
Crosta+ 2020
Astesiano+5 2022
Re&Galoppo 2024

Re-weight DM amount in disc galaxies

DM phenomena =
fake DM from GR + true DM

KEY IDEAS

Low energy limit and non-commutativity

Widespread intuition:

Require small metric perturbations (classical limit): $g_{\mu\nu} = \eta_{\mu\nu} + c^{-2} h_{\mu\nu}$

Then deduce Einstein Eqs

Find in Newtonian limit: $4\pi G\rho = \Delta\Phi + O\left({}^{v^2}/_{c^2}\right), \ldots$

They don't commute!

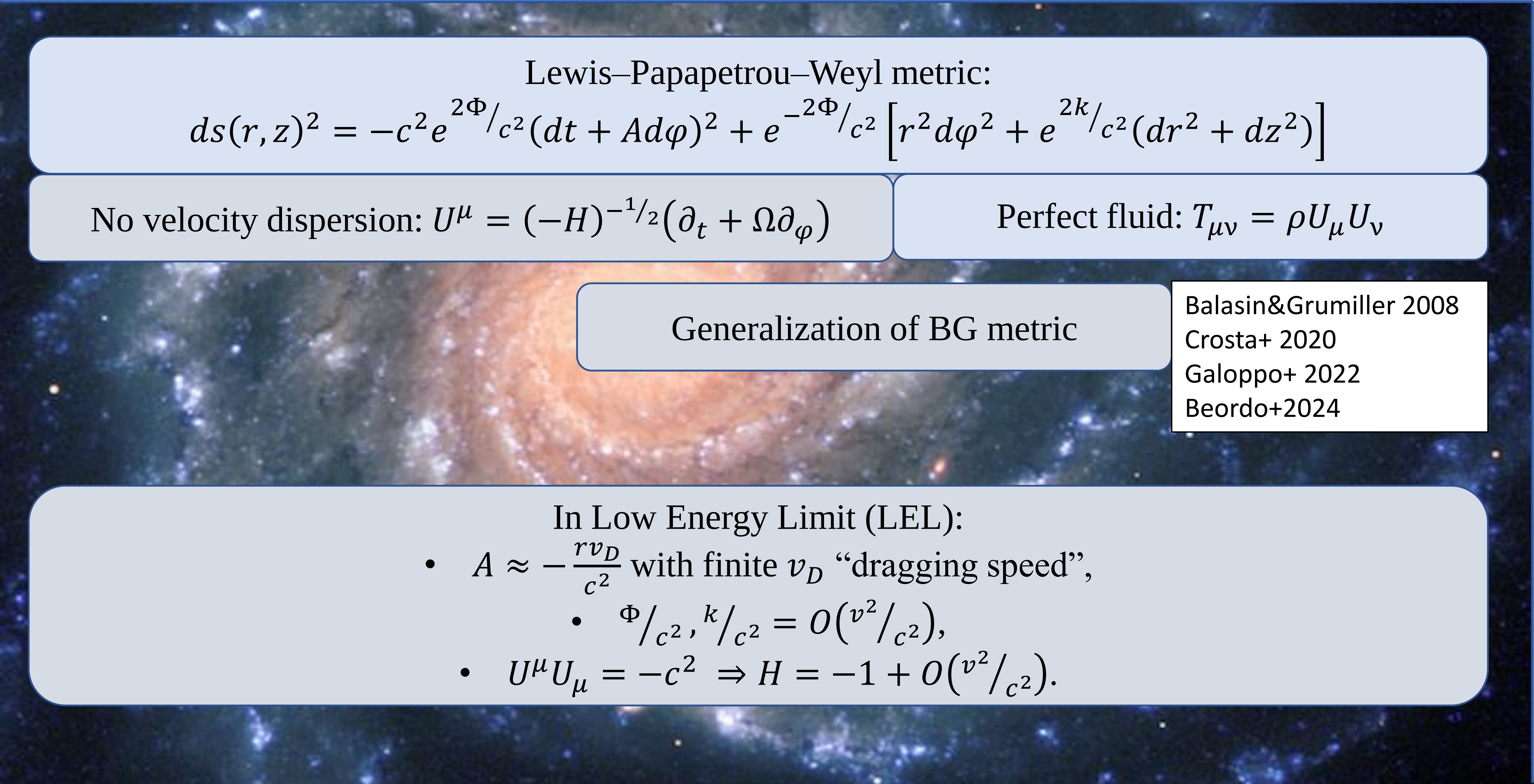
Switch the order:

Start with full Einstein Eqs

Then expand formulas at the lowest order in ${}^{v}/_{c}$: Low Energy Limit (LEL)

Find non-negligible corrections to Newtonian eqs!: $4\pi G\rho = \Delta\Phi + something, \ldots$

(η, H) MODEL
Metric and source



Lewis–Papapetrou–Weyl metric:

$$ds(r,z)^2 = -c^2 e^{2\Phi/c^2}(dt + A\,d\varphi)^2 + e^{-2\Phi/c^2}\left[r^2 d\varphi^2 + e^{2k/c^2}(dr^2 + dz^2)\right]$$

No velocity dispersion: $U^\mu = (-H)^{-1/2}\left(\partial_t + \Omega\partial_\varphi\right)$

Perfect fluid: $T_{\mu\nu} = \rho U_\mu U_\nu$

Generalization of BG metric

Balasin&Grumiller 2008
Crosta+ 2020
Galoppo+ 2022
Beordo+2024

In Low Energy Limit (LEL):

- $A \approx -\frac{r v_D}{c^2}$ with finite v_D "dragging speed",
- $\Phi/c^2, k/c^2 = O\left(v^2/c^2\right)$,
- $U^\mu U_\mu = -c^2 \Rightarrow H = -1 + O\left(v^2/c^2\right)$.

(η, H) MODEL
Metric and source

$$ds(r,z)^2 = -c^2 e^{2\Phi/c^2}(dt + A\,d\varphi)^2 + e^{-2\Phi/c^2}\left[r^2 d\varphi^2 + e^{2k/c^2}(dr^2 + dz^2)\right],$$

$$U^\mu = (-H)^{-1/2}(\partial_t + \Omega\partial_\varphi),\ T_{\mu\nu} = \rho U_\mu U_\nu$$

(η, H) family of exact solutions.
Fully generated by the choice of functions $\eta(r,0)$ and $H(\eta)$

$\hat{\Delta}\tilde{\eta} = 0$ s.t. $\hat{\Delta} = \partial_r^2 - \partial_r/r + \partial_z^2$ Grad-Shafranov operator,
and $\tilde{\eta}(r,z) := \eta + \frac{c^2}{2} r^2 \int \frac{H'}{H}\frac{d\eta}{\eta} - \frac{1}{2}\int \frac{H'}{H}\eta\, d\eta$ Velocity Field Equation (VFE)

$$\Omega(\eta) = 1/2 \int H'(\eta)\, d\eta/\eta$$

$$8\pi G\rho = \frac{v^2(2-\eta l)^2 - r^2 l^2}{4e^\mu}\frac{\eta_r^2 + \eta_z^2}{\eta^2} \text{ s.t. } l(\eta) := H'/H$$

$$g_{tt} = H - 2vr\Omega + \frac{r^2\Omega^2}{-H\gamma^2},$$
$$g_{t\varphi} = rv + \frac{r^2}{\gamma^2 H}\Omega,$$
$$g_{\varphi\varphi} = \frac{r^2}{-H\gamma^2}, \text{ s.t. } \gamma = \gamma(v_Z)$$

Astesiano+3 2022
Astesiano+5 2022
Re&Galoppo 2024

(η, H) MODEL
What speed?

Zero Angular Momentum Observers (ZAMO):

$$e_Z^0 = \frac{\sqrt{g_{\varphi\varphi}}}{W}\left(\partial_t + \chi\partial_\varphi\right),\ e_Z^1 = \frac{1}{\sqrt{g_{\varphi\varphi}}}\partial_\varphi,\ e_Z^2 = e^{-{(\Phi-k)}/{c^2}}\partial_r,\ e_Z^3 = e^{-{(\Phi-k)}/{c^2}}\partial_z;$$

where $\chi = -{g_{t\varphi}}/{g_{\varphi\varphi}}$ “ dragging angular speed”

Measures speed $v_Z = \frac{e_Z^1\cdot U^\mu}{e_Z^0\cdot U^\mu} = \frac{W(\Omega-\chi)}{-H\gamma_Z^2} = {\eta}/{r}$

In LEL:

$v_Z \approx r(\Omega-\chi)$

Stationary (not static!) Observers:

$$e_S^0 = e^{-{\Phi}/{c^2}}\partial_t,\ e_S^1 = \frac{e^{{\Phi}/{c^2}}}{W}\left(\partial_\varphi - A\partial_t\right),\ e_S^2 = e^{-{(\Phi-k)}/{c^2}}\partial_r,\ e_S^3 = e^{-{(\Phi-k)}/{c^2}}\partial_z$$

Measures speed $v_S = \frac{e_S^1\cdot U^\mu}{e_S^0\cdot U^\mu} = \gamma_S^{-1} e^{-{\Phi}/{c^2}} W\Omega$

In LEL:

$v_S \approx r\Omega \sim 10^{-3}$

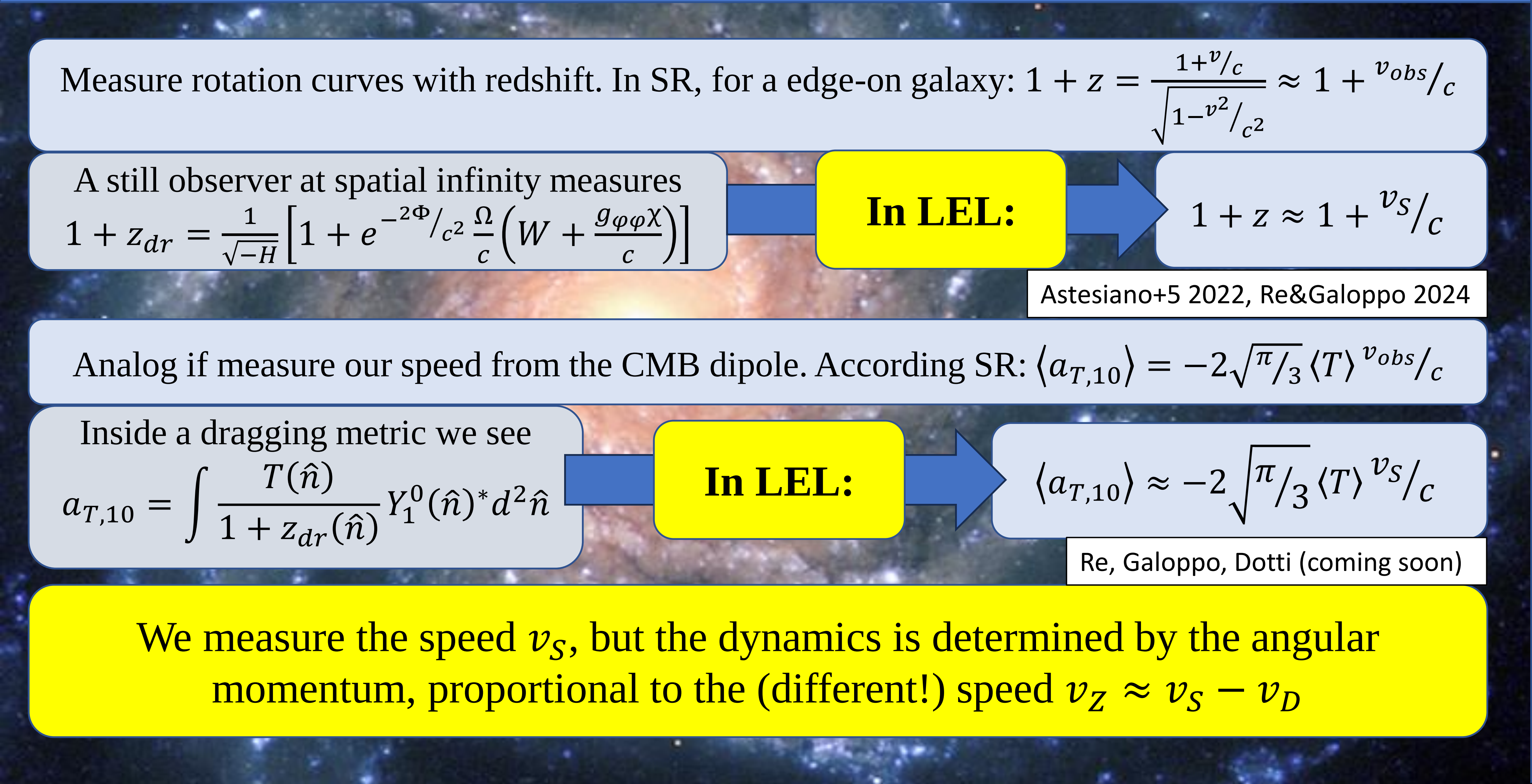
Dragging speed: $v_D = \frac{e_S^1\cdot e_Z^0}{e_S^0\cdot e_Z^0} = \frac{g_{\varphi\varphi}\chi}{W}$

In LEL:

$v_D \approx r\chi \sim 10^{-3}$

Re&Galoppo 2024

(η, H) MODEL
What speed?



Measure rotation curves with redshift. In SR, for a edge-on galaxy: $1+z = \frac{1+{}^{v}/_{c}}{\sqrt{1-{}^{v^2}/_{c^2}}} \approx 1 + {}^{v_{obs}}/_{c}$

A still observer at spatial infinity measures

$$1+z_{dr} = \frac{1}{\sqrt{-H}}\left[1 + e^{-2\Phi/c^2}\frac{\Omega}{c}\left(W + \frac{g_{\varphi\varphi}\chi}{c}\right)\right]$$

In LEL:

$$1+z \approx 1 + {}^{v_S}/_{c}$$

Astesiano+5 2022, Re&Galoppo 2024

Analog if measure our speed from the CMB dipole. According SR: $\langle a_{T,10}\rangle = -2\sqrt{{}^{\pi}/_{3}}\,\langle T\rangle\,{}^{v_{obs}}/_{c}$

Inside a dragging metric we see

$$a_{T,10} = \int \frac{T(\hat{n})}{1+z_{dr}(\hat{n})} Y_1^0(\hat{n})^* d^2\hat{n}$$

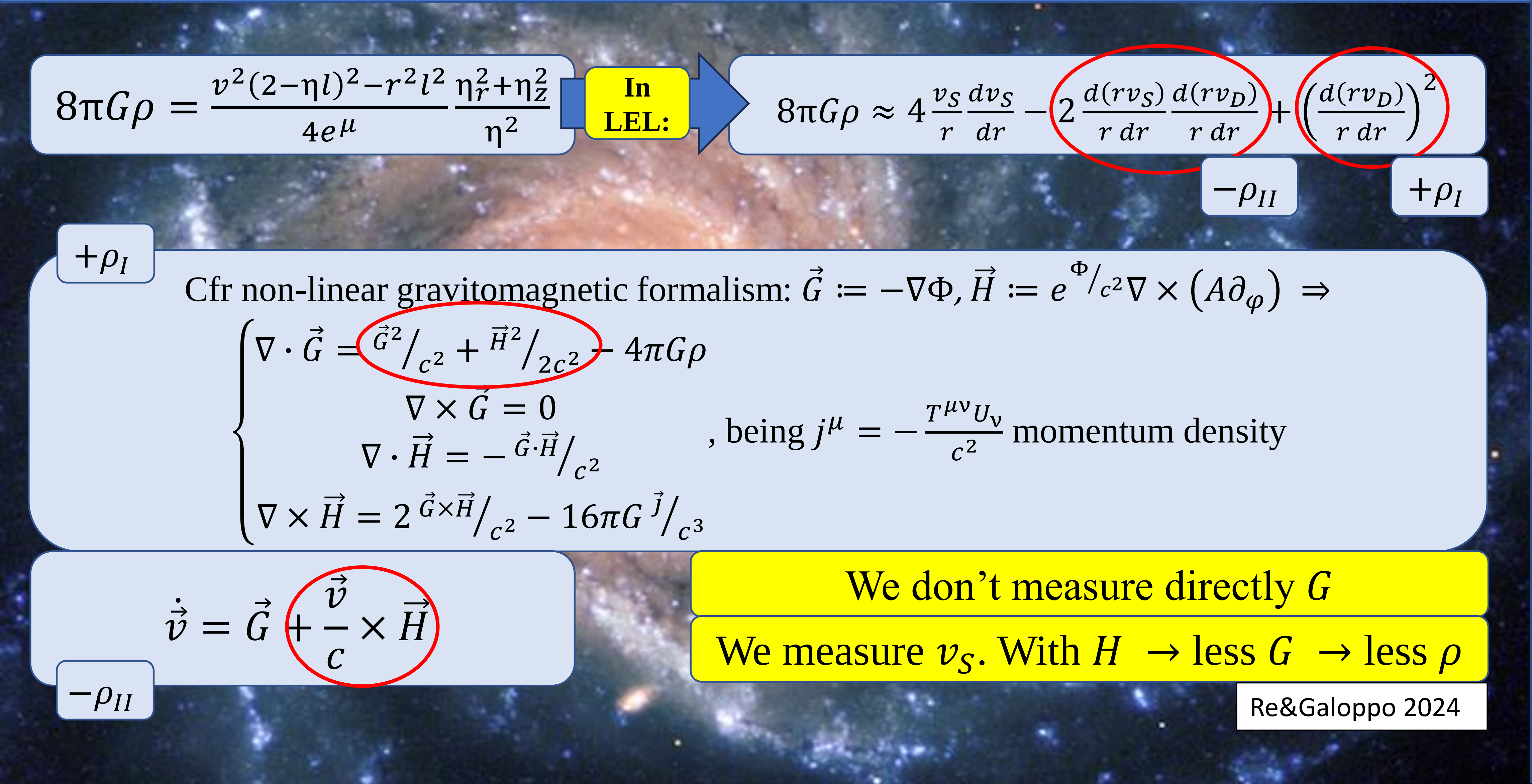
In LEL:

$$\langle a_{T,10}\rangle \approx -2\sqrt{{}^{\pi}/_{3}}\,\langle T\rangle\,{}^{v_S}/_{c}$$

Re, Galoppo, Dotti (coming soon)

We measure the speed v_S, but the dynamics is determined by the angular momentum, proportional to the (different!) speed $v_Z \approx v_S - v_D$

PHANTOM DARK MATTER
Corrections on required density



$$8\pi G\rho = \frac{v^2(2-\eta l)^2 - r^2 l^2}{4e^{\mu}}\,\frac{\eta_r^2+\eta_z^2}{\eta^2}$$

In LEL:

$$8\pi G\rho \approx 4\frac{v_S}{r}\frac{dv_S}{dr} \underbrace{- 2\frac{d(rv_S)}{r\,dr}\frac{d(rv_D)}{r\,dr}}_{-\rho_{II}} + \underbrace{\left(\frac{d(rv_D)}{r\,dr}\right)^2}_{+\rho_I}$$

Cfr non-linear gravitomagnetic formalism: $\vec{G} \coloneqq -\nabla\Phi, \vec{H} \coloneqq e^{\Phi/c^2}\nabla\times\left(A\partial_{\varphi}\right) \Rightarrow$

$$\begin{cases} \nabla\cdot\vec{G} = \underbrace{\vec{G}^2/c^2 + \vec{H}^2/2c^2}_{+\rho_I} - 4\pi G\rho \\ \nabla\times\vec{G} = 0 \\ \nabla\cdot\vec{H} = -\vec{G}\cdot\vec{H}/c^2 \\ \nabla\times\vec{H} = 2\,\vec{G}\times\vec{H}/c^2 - 16\pi G\,\vec{j}/c^3 \end{cases}$$

, being $j^{\mu} = -\frac{T^{\mu\nu}U_{\nu}}{c^2}$ momentum density

$$\dot{\vec{v}} = \vec{G} + \underbrace{\frac{\vec{v}}{c}\times\vec{H}}_{-\rho_{II}}$$

We don’t measure directly G

We measure v_S. With H → less G → less ρ

Re&Galoppo 2024

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PHANTOM DARK MATTER

Corrections on gravitational lensing

Gauss-Bonnet ($\chi(S) = 1$)

$$\theta_R + \theta_S = \int\int_S K\, da + \int_{\partial S} \kappa_g\, d\lambda$$

Gravitomagnetism:

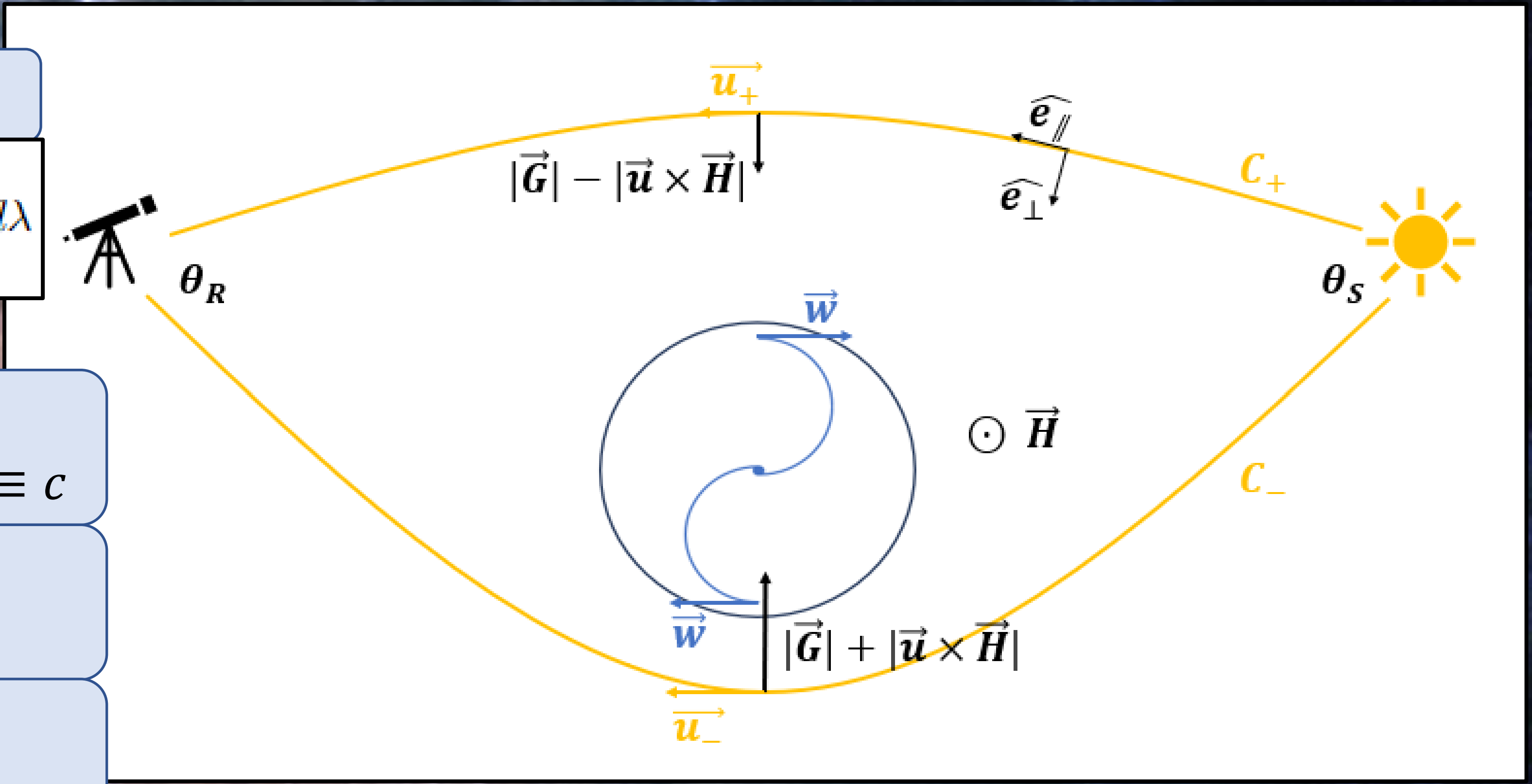
$$\kappa_g = \left(\vec{G} + \vec{u} \times \vec{H}\right) \cdot \hat{e}_\perp,\ |\vec{u}| \equiv c$$

Asymmetric geodesics:

$$r_\pm \approx r_0 \pm \Delta r$$

Far from the centre:

$$G_{,r}, H_{,r} < 0$$



$$\theta_R + \theta_S \approx (\theta_R + \theta_S)|_{Newt} - 2c \int_{C_+} H_{,r}(r_0)\, \Delta r\, d\lambda$$

Galoppo+ 2022
Re&Galoppo 2024

ESTIMATION OF DRAGGING SPEED
With Newtonian ad-hoc term

$$8\pi G(\rho_B + \alpha\rho_{DM}) \coloneqq 8\pi G\rho \coloneqq \frac{\eta_{,r}^2}{r^2} - r^2\Omega_{,r}^2 + 2\frac{v_S^2}{r^2}$$

Fraction $1 - \alpha$ of DM explained by dragging $v_D = v_S - {}^{\eta}/_{r}$

$$4\pi G(\rho_B + \rho_{DM}) = 2\frac{v_S v_{S,r}}{r} + \frac{v_S^2}{r^2}$$

$\alpha = 1 \Leftrightarrow v_D \equiv 0$: spherically symmetric Newtonian model with 100% of DM

Evaluate for MW: $\rho_B \coloneqq \rho_{B0} e^{-r/r_B}$ exponential, $\rho_{DM} \coloneqq \rho_{DM0}\frac{r_{DM}}{r}\left(1 + \frac{r}{r_{DM}}\right)^{-2}$ NFW,
$r_B \cong 2.1$ kpc, $r_{DM} \cong 5.69$ kpc, $v_\infty \cong 220$ km/s, $\rho_{DM0} \cong 6.4\%\ \rho_{B0}$

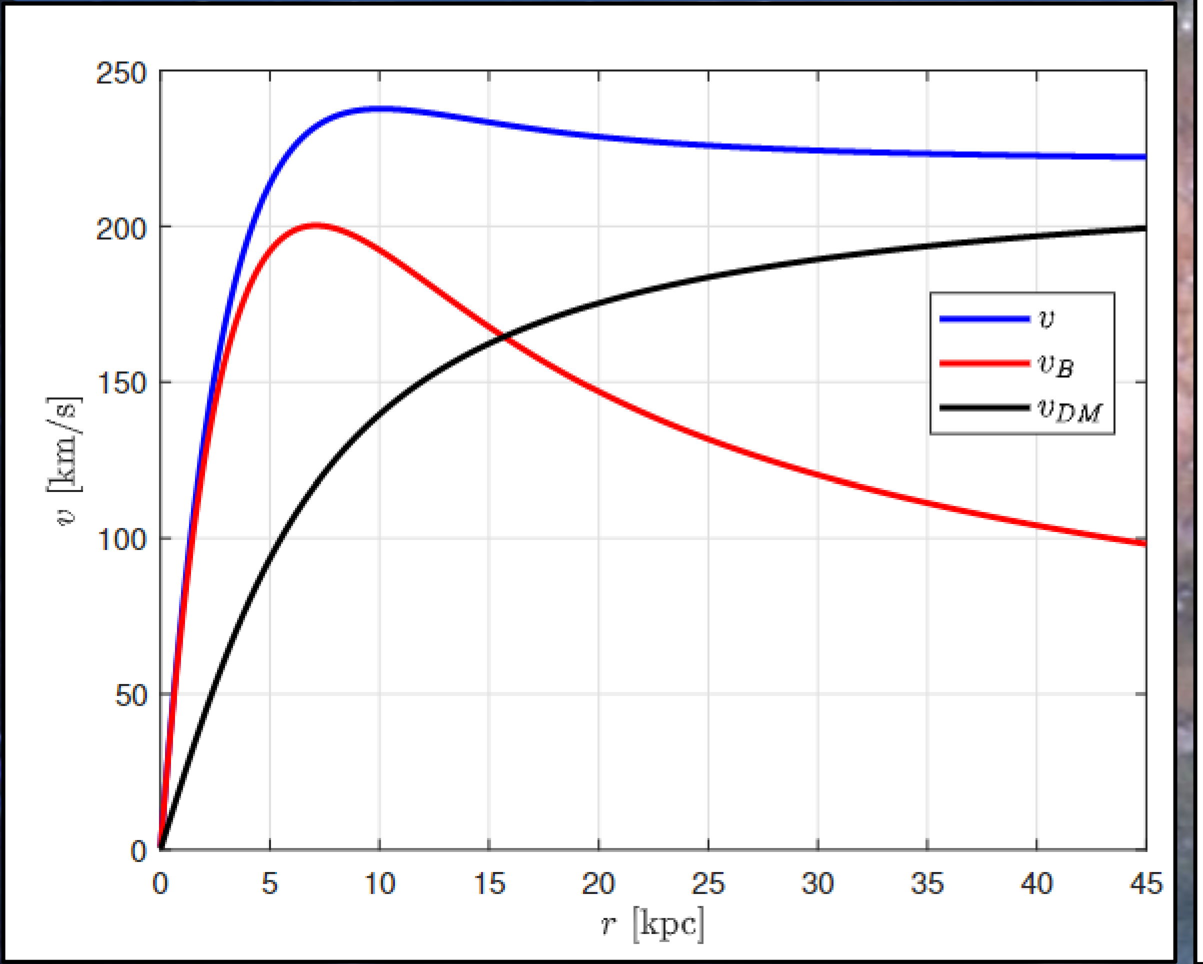
At $\boldsymbol{r_\odot \cong 8.6}$ kpc: $\boldsymbol{w(r_\odot, 0) \cong (1-\alpha)\cdot 71}$ km/s

Example: $\alpha = {}^{1}/_{2} \Rightarrow v_D(r_\odot, 0) \approx 35$ km/s in our neighborhood

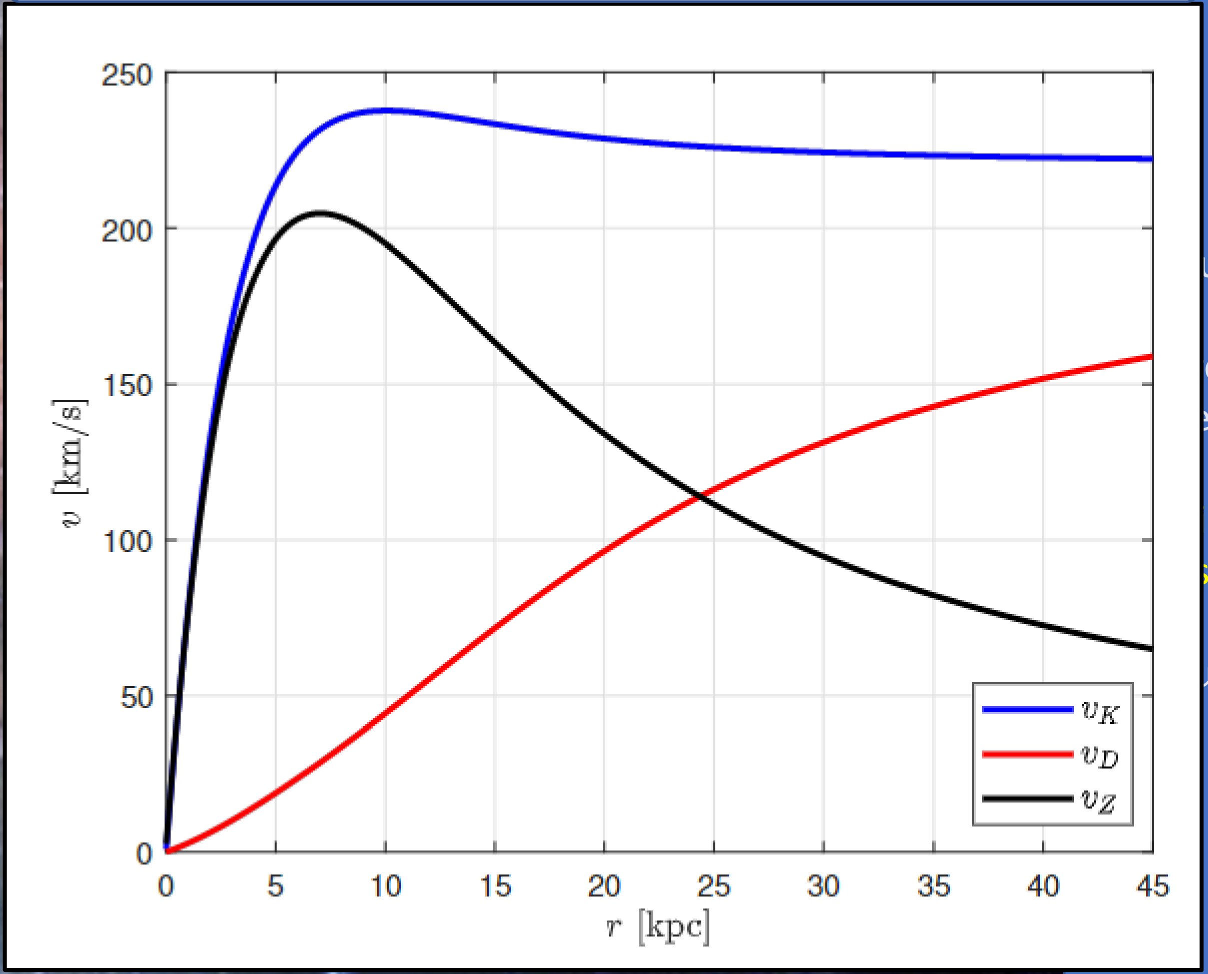
Re&Galoppo 2024

ESTIMATION OF DRAGGING SPEED
With Newtonian ad-hoc term

Newtonian description



General-relativistic description



ESTIMATION OF DRAGGING SPEED
Coming soon: more precise formulas with pressure!

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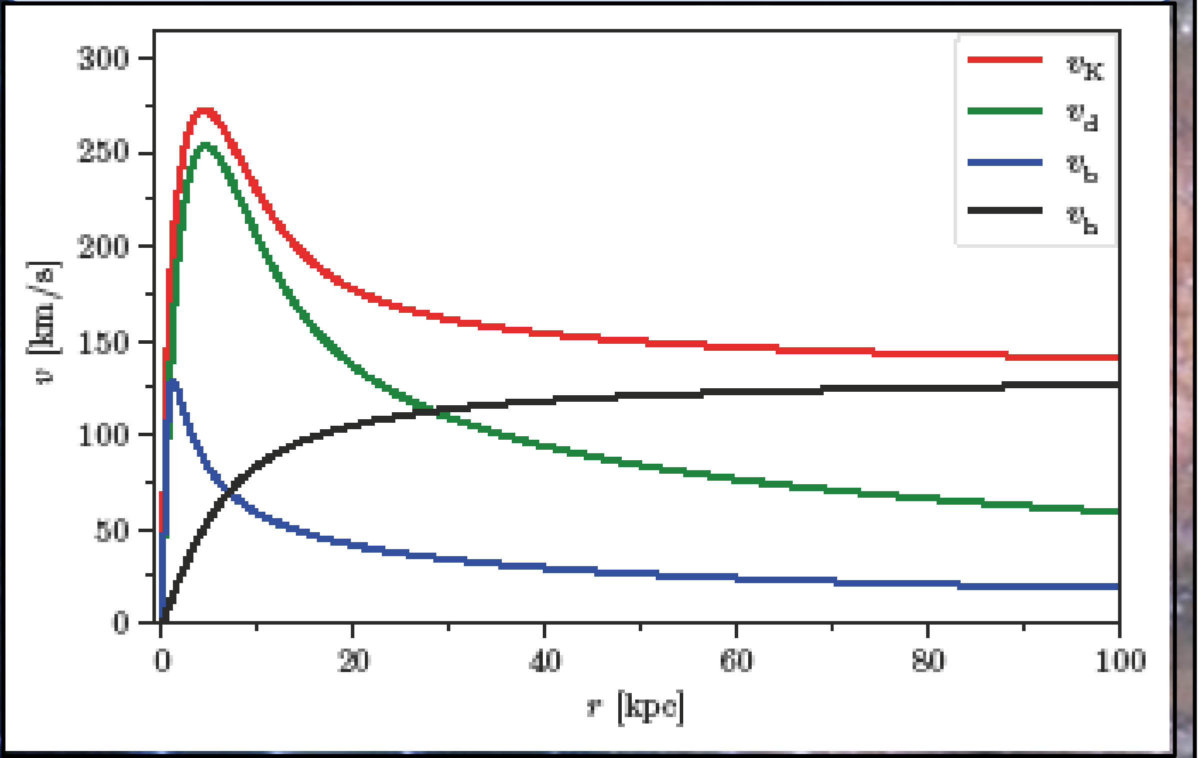
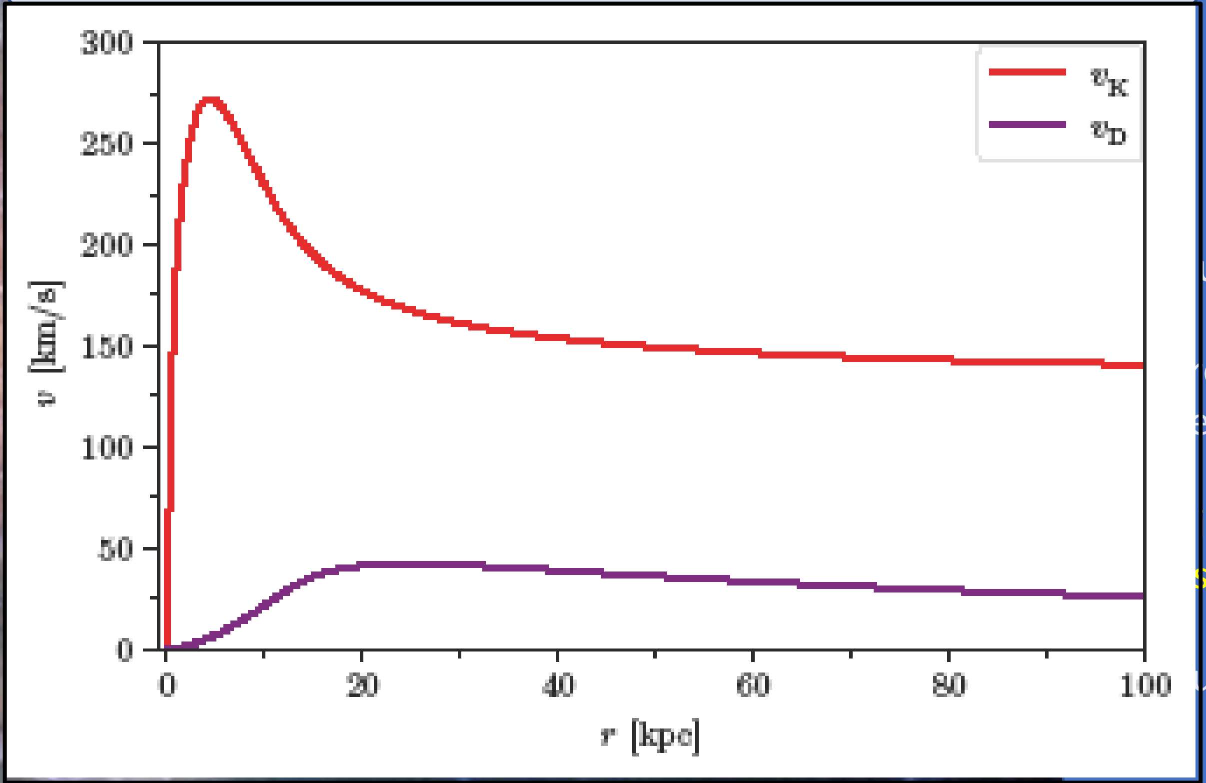
Newtonian description



General-relativistic description



Galoppo, Wiltshire, Re (coming soon)

EMPIRICAL MEASURES
Counter-rotating matter

Some disc galaxies have counter-rotating stars or gas

Kuijken+ 1996,
Corsini 2014

The counter-rotating component is also dragged! $v_+ + v_- \propto v_D$

Re (coming soon)

Consider geodesics for a test particle with tangent motion $\frac{\partial\varphi}{\partial\tau} \cong \tilde{\Omega} \cong \tilde{v}/r$

Without dragging ($v_D \equiv 0$): depends only on the potential Φ s.t. $g_{tt} = -e^{2\Phi/c^2}, \frac{\partial\Phi}{\partial r} = \frac{v_S^2}{r}$

Geodesic $\ddot{r} \cong \frac{\tilde{v}^2 - v_S^2}{r}$ $\Rightarrow$ symmetric $v_\pm \cong \pm v_S$

With dragging w:
$\frac{\partial\Phi}{\partial r} \cong v_S\left(\frac{v_Z}{r} - \frac{\partial v_D}{\partial r}\right)$

Geodesic
$\ddot{r} \cong \frac{\tilde{v}^2 - v_S^2}{r} + \frac{v_S - \tilde{v}}{r}\frac{\partial(r v_D)}{\partial r}$

Asymmetric
$v_+ \cong v_S, v_- = -v_S + \frac{\partial(r v_D)}{\partial r}$

Non-negligible deviation from Newton!

CONCLUSIONS
What has been done and what remains to be done

What we know:

- GR non-linearities allow solitonic solutions for the dragging terms
- Strong dragging implies non-negligible deviations from Newton
- A quite small dragging vortex sustains flat rotation curve
- It returns also a suitable correction on the gravitational lensing
- The dragging speed can be measured with counter-rotating matter components

Future perspectives:

- Generalize equations (in LEL) for non-negligible pressure (bulge, elliptical galaxies…)
- Consequences of the dragging on CMB
- Measure the actual dragging with the counter-rotating matter
- Apply GR to other MMP (gravitational lensing, universe expansion…)

Galoppo, Wiltshire, Re (coming soon)

Re, Galoppo, Dotti (coming soon)

Re (coming soon)

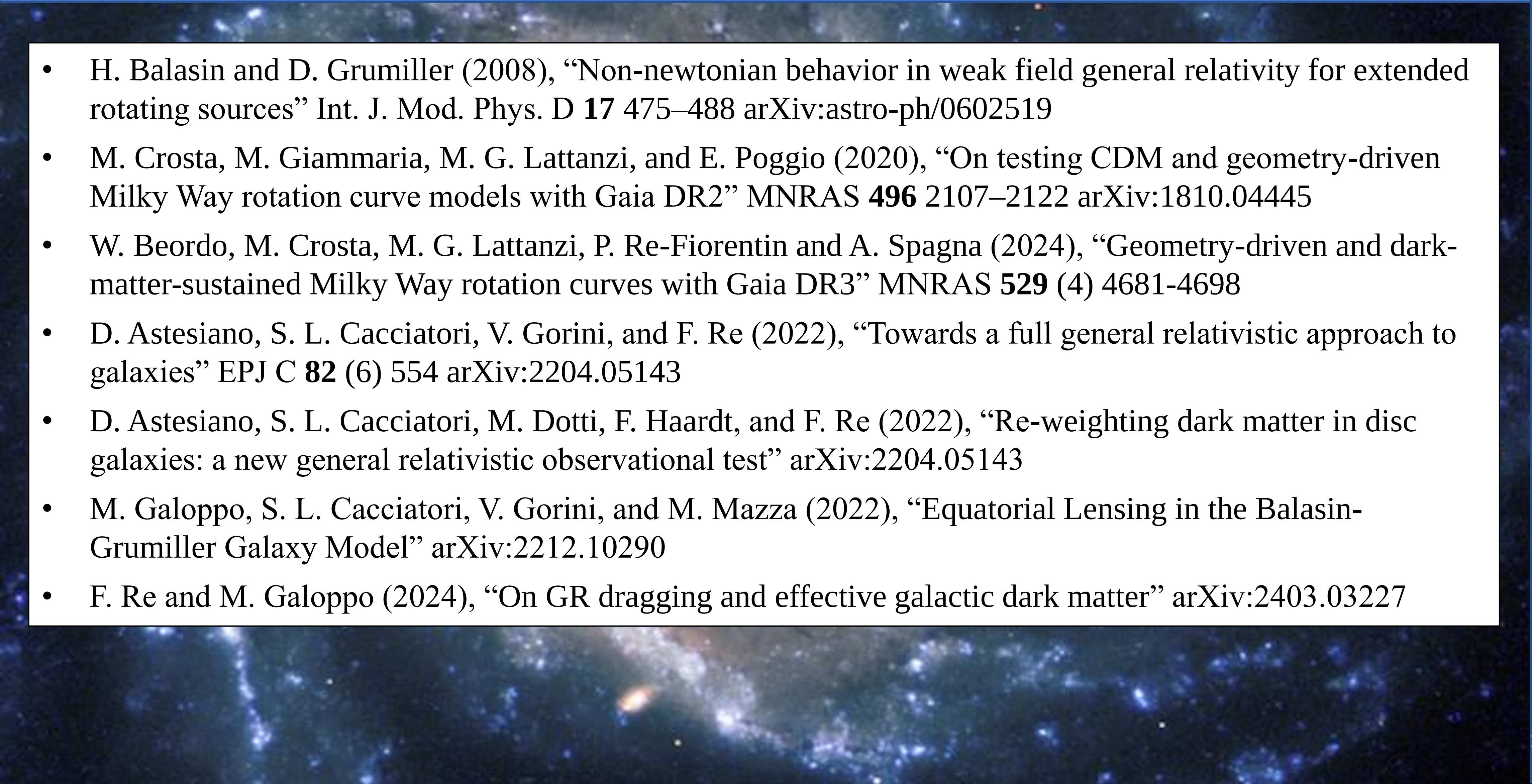
Galoppo+ 2022; Re 2020, Re 2021, Vigneron&Buchert 2019, Buchert 2008

Stay tuned!

CONCLUSIONS

Thanks for your attention!

CONCLUSIONS
Minimal bibliography



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