Status and Inconsistencies in the Standard LCDM Model: a theoretical cosmologist perspective

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InDark INFN QUAGRAP

Outline

- Introduction: history of ΛCDM
- standard ΛCDM and some alternative
  - GR+CP: FLRW+perturbations
- nonlinearity in the late Universe
  - believe in ΛCDM and use relativistic simulations
- Conclusions

University of Durham

The ΛCDM model of cosmogony

Cosmological constant

Cold dark matter

slide curtesy of
Carlos Frenk (Durham)

- Proposed in 1980s, it is an *ab initio,* fully specified model of cosmic evolution and the formation of cosmic structure
- Has strong predictive power and can, in principle, be ruled out
- Has made a number of predictions that were subsequently verified empirically (e.g. CMB, LSS, galaxy formation)

Three Nobel Prizes in Physics since 2006

Institute for Computational Cosmology

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CMB fluctuations (for COBE): George Smooth 2006
Λ inferred from SN-Ia: Nobel 2011
Peebles 2019: "for theoretical discoveries in physical cosmology", e.g. flat Universe with Ω_Λ=0.7

Institute for Computational Cosmology

History of Λ

- Myth #1: Λ proposed by Einstein to obtain a static Universe model, then rejected by Einstein after 1929 Hubble "discovery" of the expansion of the Universe

  - fact #1: Λ proposed by Einstein to obtain a static Universe model in 1916-7

  - fact #2: Einstein rejected Λ in a letter to Weyl in 1923, where he clearly say that, after the work of De Sitter that galaxies in his model move apart because of Λ, "then get rid of the cosmological term" https://einsteinpapers.press.princeton.edu/vol14-trans/71

History of Λ

- Myth #2 Λ was forgotten, until rediscovered in Cosmology after the SNae observations that inferred it from the acceleration of the Universe expansion
  - fact #3: inflationary scenario (Guth 1981 but also earlier works by Starobinsky and Grishchuk) predicts a very flat Universe, thus $\Omega_\Lambda = 1 - \Omega_m$
  - fact #4: based on this and on data available at the time Peebles (1984) inferred a value for $\Omega_m = 0.2 \pm 0.1$ and corresponding Ω_Λ

History of Λ

THE ASTROPHYSICAL JOURNAL, **284**:439–444, 1984 September 15

TESTS OF COSMOLOGICAL MODELS CONSTRAINED BY INFLATION

P. J. E. PEEBLES
Joseph Henry Laboratories, Princeton University
Received 1984 February 6; accepted 1984 March 23

ABSTRACT

The inflationary scenario requires that the universe have negligible curvature along constant-density surfaces. In the Friedmann-Lemaître cosmology that leaves us with two free parameters, Hubble's constant H_0 and the density parameter Ω_0 (or, equivalently, the cosmological constant Λ). I discuss here tests of this set of models from local and high-redshift observations. The data agree reasonably well with $\Omega_0 \sim 0.2$.

Subject heading: cosmology

Inflation -> flatness + HZ almost scale invariant spectrum

personal note: as a student in Rome in the '80s, I can testify that general-relativistic cosmological models with Λ where part of the undergraduates lectures

Λ
anticipated from theory
(flatness + CMB fluctuations + simulations),
vs data
(CMB + galaxy distribution)
in
Nature 348 (1990) 705–707.

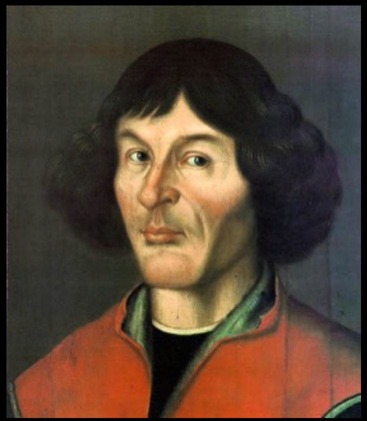
The cosmological constant and cold dark matter

G. Efstathiou, W. J. Sutherland & S. J. Maddox

Department of Physics, University of Oxford, Oxford OX1 3RH, UK

THE cold dark matter (CDM) model[1–4] for the formation and distribution of galaxies in a universe with exactly the critical density is theoretically appealing and has proved to be durable, but recent work[5–8] suggests that there is more cosmological structure on very large scales ($l > 10\ h^{-1}$ Mpc, where h is the Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1}) than simple versions of the CDM theory predict. We argue here that the successes of the CDM theory can be retained and the new observations accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant, which is dynamically equivalent to endowing the vacuum with a non-zero energy density. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. As well as explaining large-scale structure, a cosmological constant can account for the lack of fluctuations in the microwave background and the large number of certain kinds of object found at high redshift.

Cosmological Principle



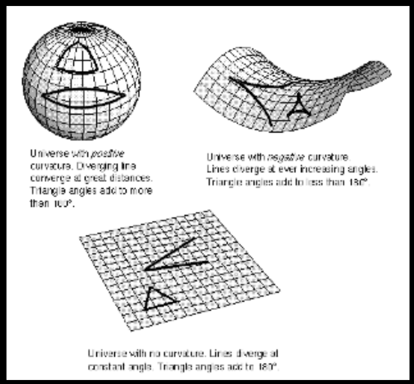
- CP: on large *enough* scales - at any given time - the universe is the same in every direction and at all locations
- generalises to cosmology the Copernican Principle
- mathematically, it translates in an assumption of
- 1) HOMOGENEITY

  "same at all locations" ⇒ symmetry under translations

  (3 in 3 dimensions: 3 Killing vectors)
- and 2) ISOTROPY

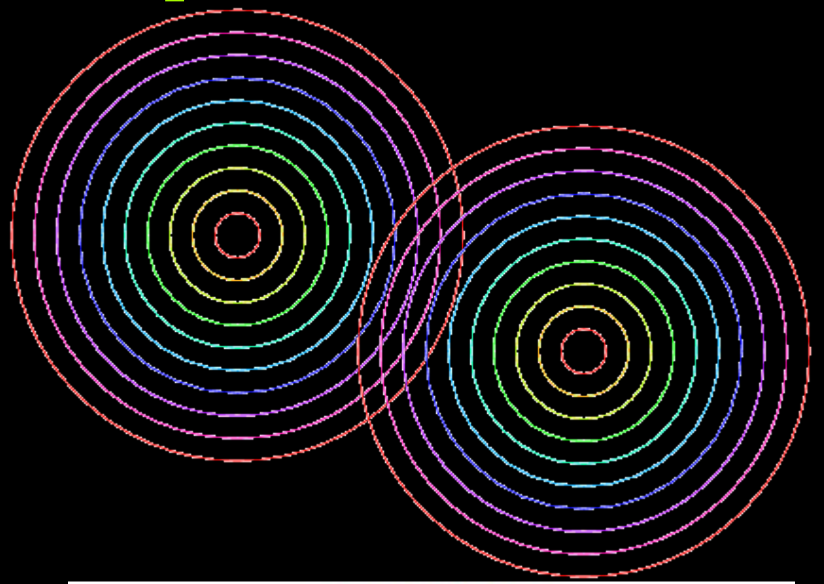
  "same in every direction" ⇒ symmetry under rotation

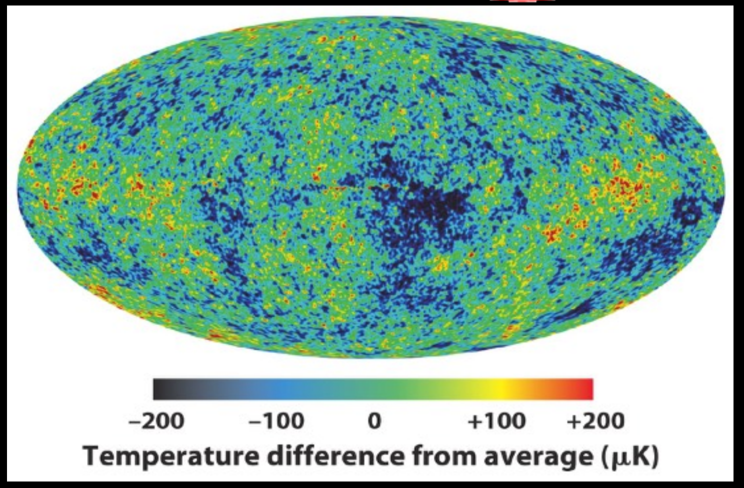
  (3 in 3 dimensions: 3 Killing vectors)
- 3-d SPACE is maximally symmetric
- Einstein Equations + symmetries: FLRW model

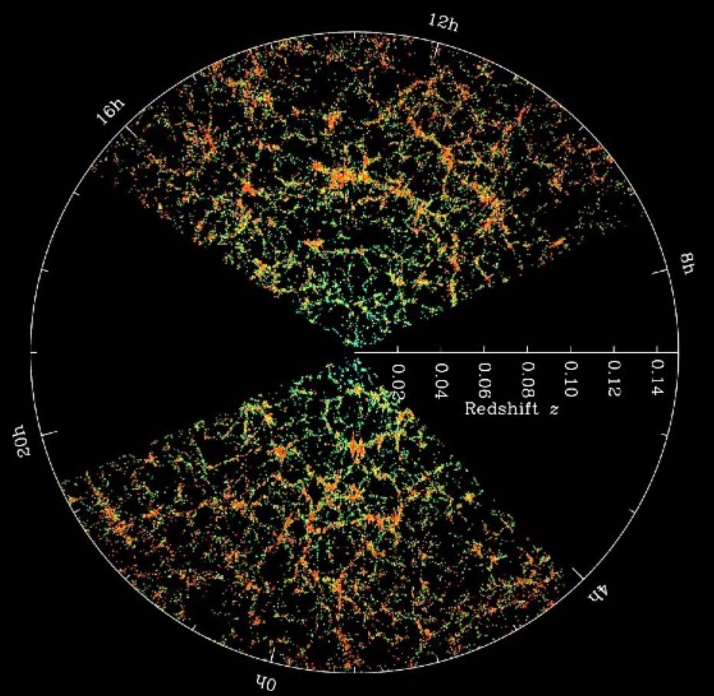


Cosmological Principle

- WHY A PRINCIPLE?
  - Isotropy around us is an observational fact: Cosmic Microwave Background (CMB)
  - homogeneity is supported by galaxy surveys, ~150 Mpc, and by matching models and observations, but it is - fundamentally - an hypothesis
- THEORY OPEN PROBLEM #1
  - we fit a FLRW to data, we should understand how to build an average model from Einstein eq (tensor averaging not known) Clarkson+, 1109.2314







Cosmological Equations

- maximal symmetry of space reduces Einstein equations (set of nonlinear PDEs admitting a hyperbolic formulation and a well posed initial value problem) to a set of ODEs

1) conservation of energy (I principle)
2) equation of motion for the "scale factor" a(t)
3) an "Hamiltonian constrain" (a first integral of the first two): the Friedmann equation

Cosmological Equations

- I principle + adiabatic expansion for equation of state P=wρ gives $\rho = \rho_0 a^{-3(1+w)}$
- equation of motion for the "scale factor" a(t):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M + \rho_R) + \frac{\Lambda}{3}$$

- the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_R) + \frac{\Lambda}{3}$$

Cosmological Equations

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Λ: simplest form of Dark Energy (DE)

- the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_R) + \frac{\Lambda}{3}$$

Dark Matter and Dark Energy

- more in general, conservation of energy with $w = P/\rho$

$$\dot{\rho} = -3H\rho(1+w)$$

- CDM: w=0, DE w< -1/3

- equation of motion for the "scale factor" a(t):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_M + \rho_{DE}(1+3w)\right]$$

- the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_{DE})$$

Cosmological Parameters

- expansion rate, Hubble parameter $H \equiv \frac{\dot{a}}{a}, \;\; H_0 = \dot{a}_0$
- Friedmann equation today

$$H_0^2 + K = \frac{8\pi G}{3}(\rho_{M0} + \rho_{R0}) + \frac{\Lambda}{3}$$

- dimensionless density parameters

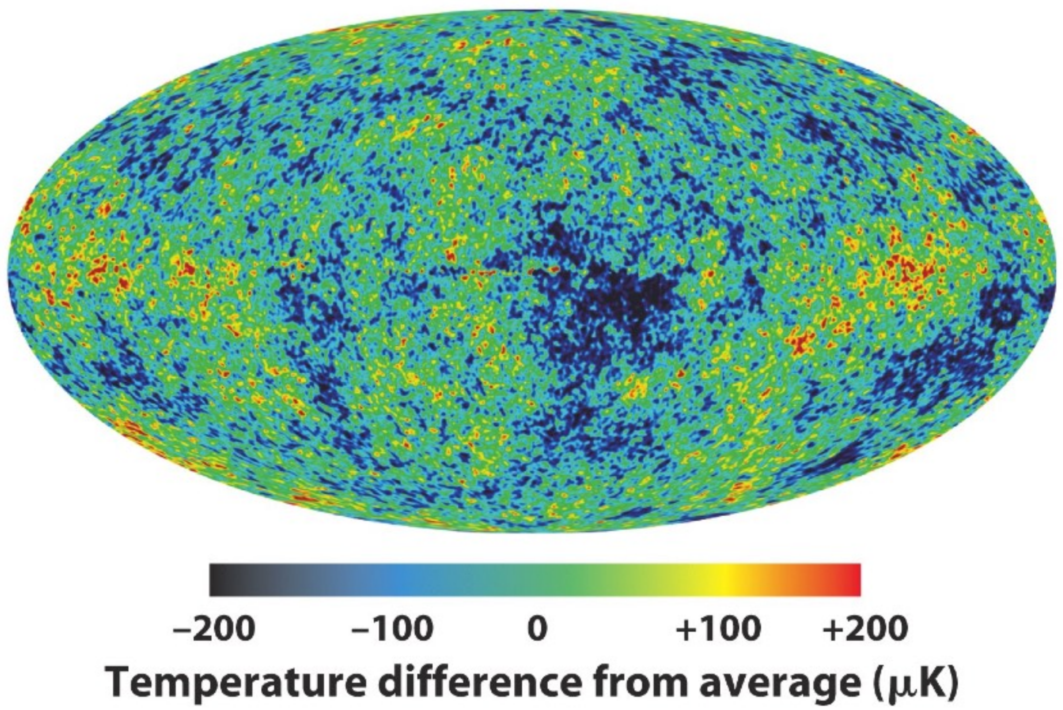
$$\rho_c \equiv \frac{3H_0^2}{8\pi G}, \;\; \Omega_{M0} \equiv \frac{\rho_{M0}}{\rho_c}, \;\; \Omega_{R0} \equiv \frac{\rho_{R0}}{\rho_c}, \;\; \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}, \;\; \Omega_K \equiv -\frac{K}{H_0^2}$$

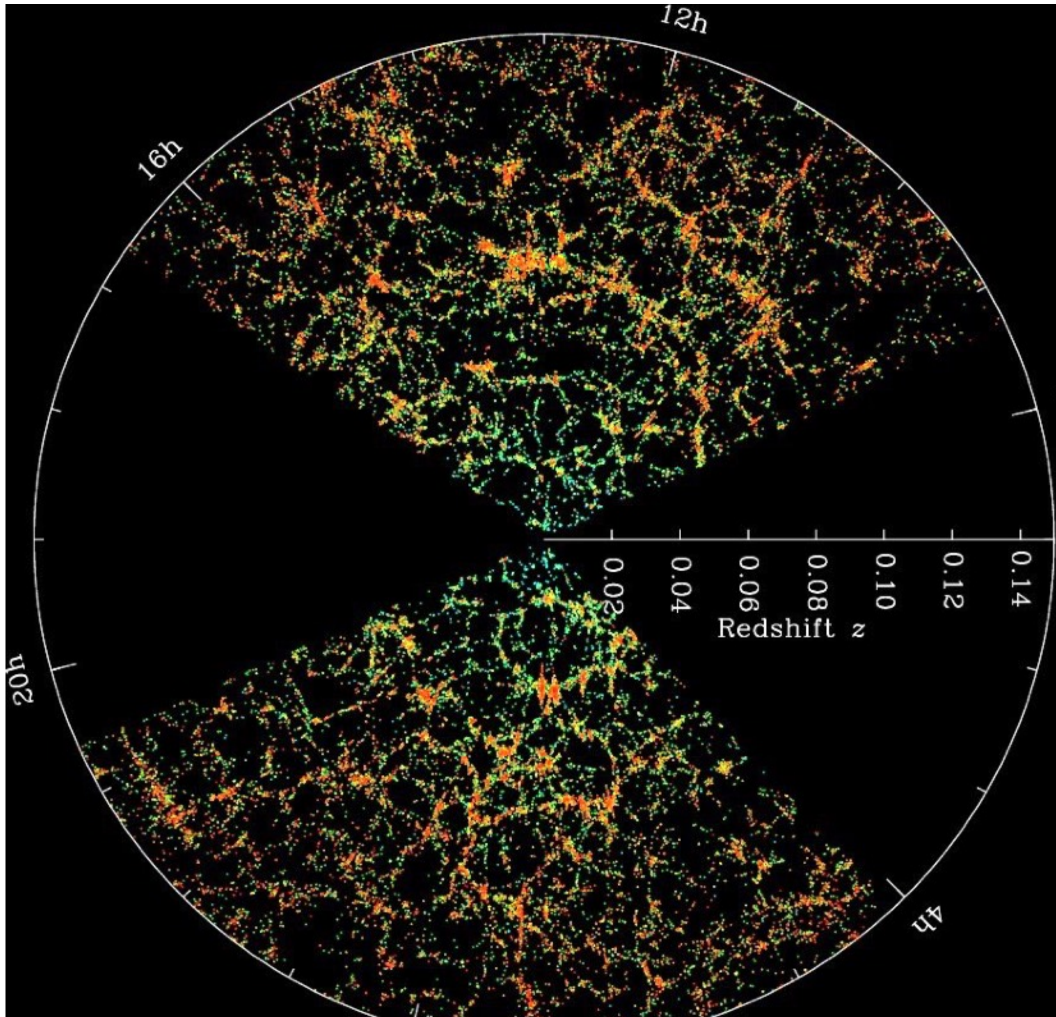
- today (neglecting radiation):

$$\Omega_{M0} + \Omega_\Lambda = 1 - \Omega_K$$

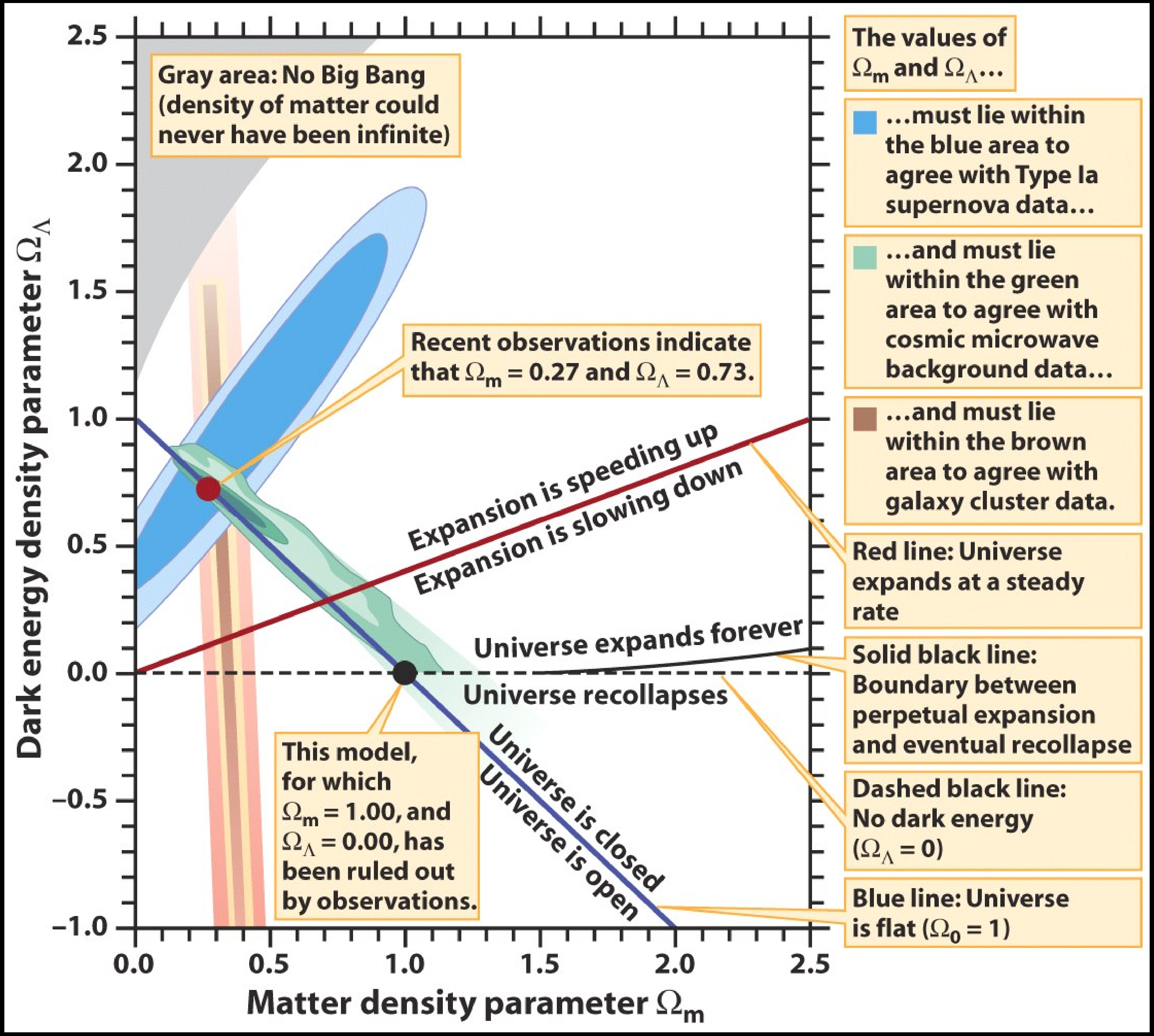
Relativistic Perturbations

- theory essential to model CMB fluctuations
- originate in inflation with a testable amplitude and almost scale-invariant spectrum
- essential to model structure formation on the largest (~Hubble) scales
- measuring features (Non-Gaussianity) should shed light on the early Universe



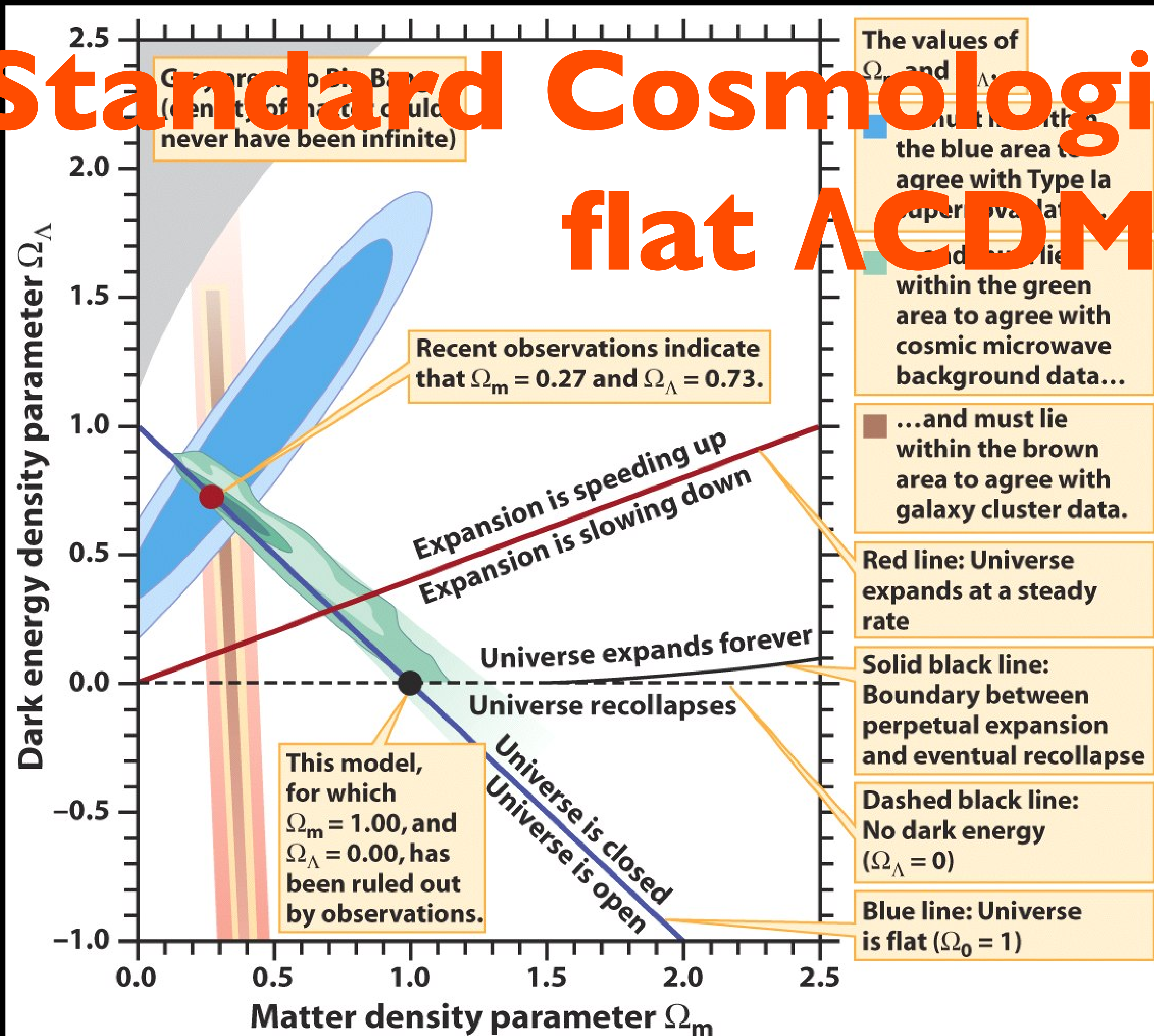


Standard Cosmology



Standard Cosmology

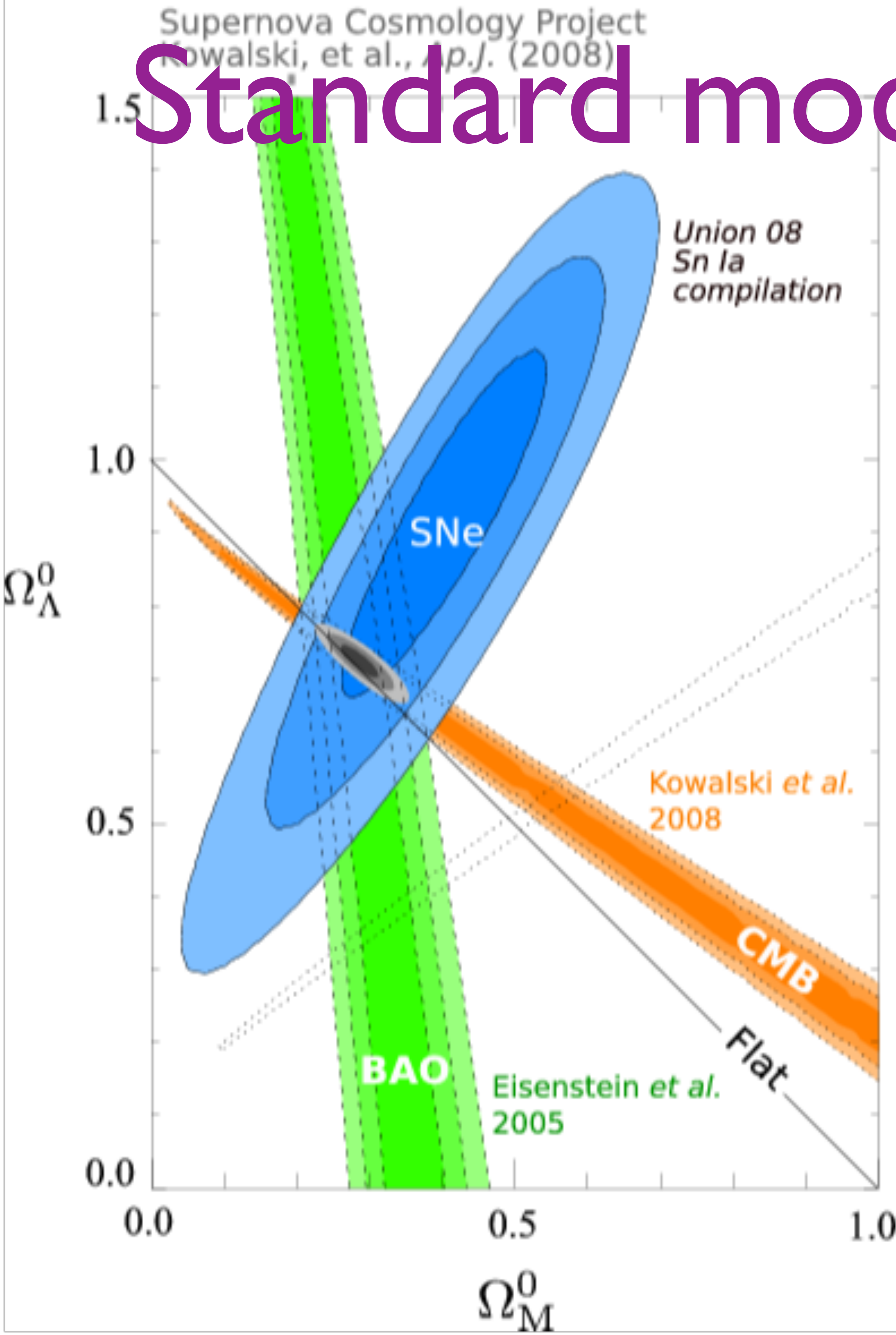
Standard Cosmological Model: flat ΛCDM



BOOMERanG!

De Bernardis (Rome)

Standard model: flat ΛCDM



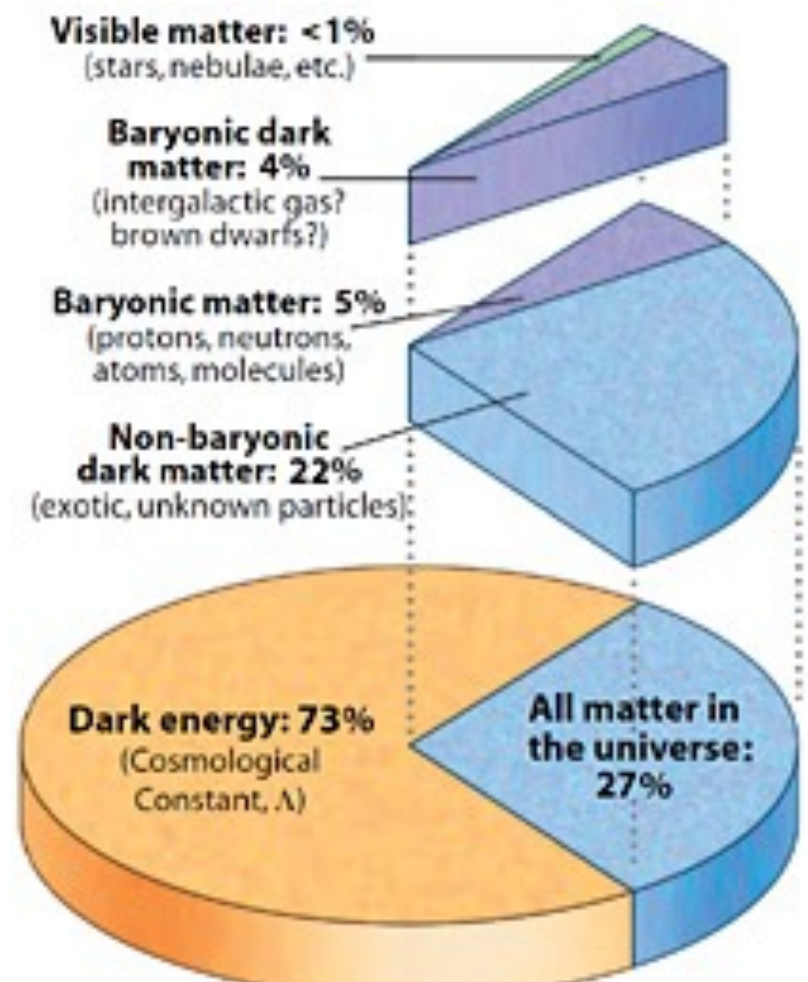
Plank 2018: $-0.095<\Omega_K<-0.007$

(Aghanim et al. [Planck], 2020, Astron. Astrophys. 641, A6)

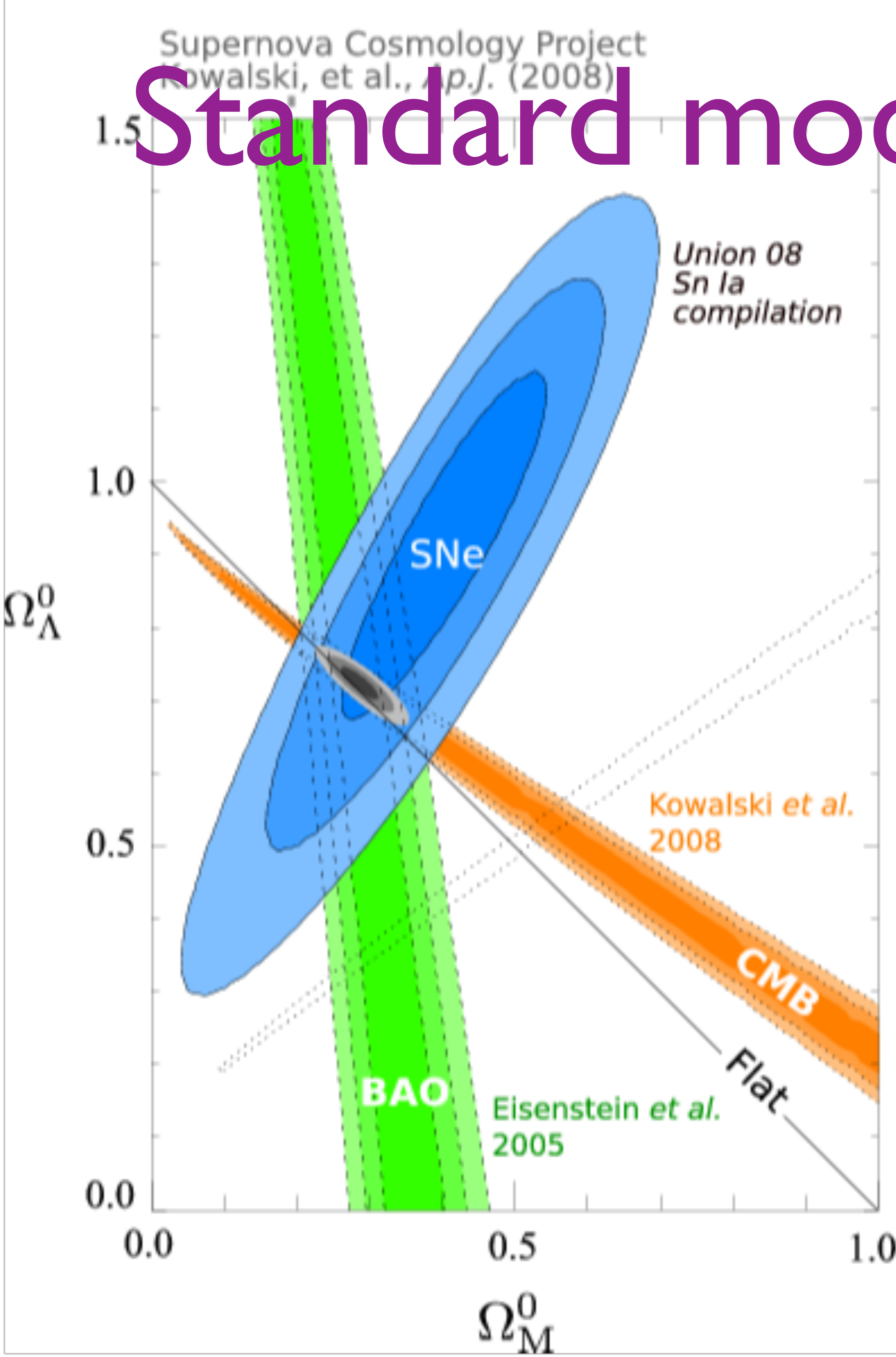
but cf. Di Valentino, Melchiorri & Silk, Nature Astron. 4 (2019), 2, 196

and Yang+ 2210.09865

$$\Omega_\Lambda = 1 - \Omega_K - \Omega_{M0}$$



Standard model: flat ΛCDM



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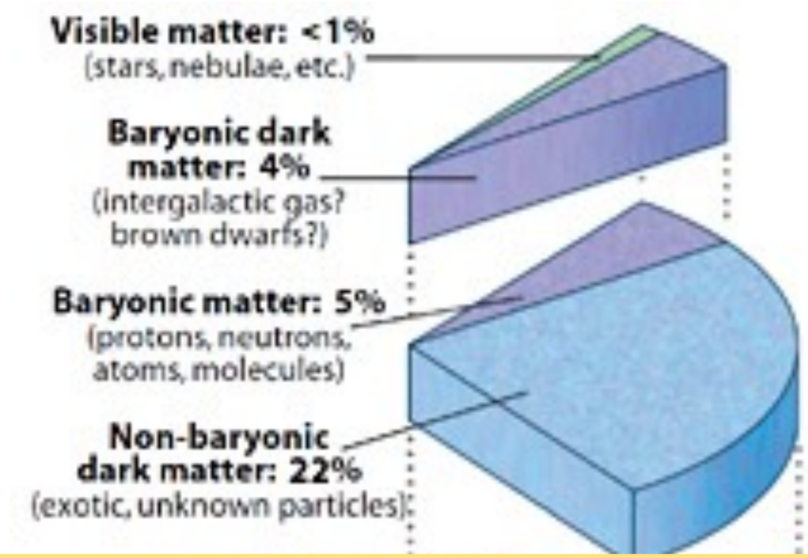
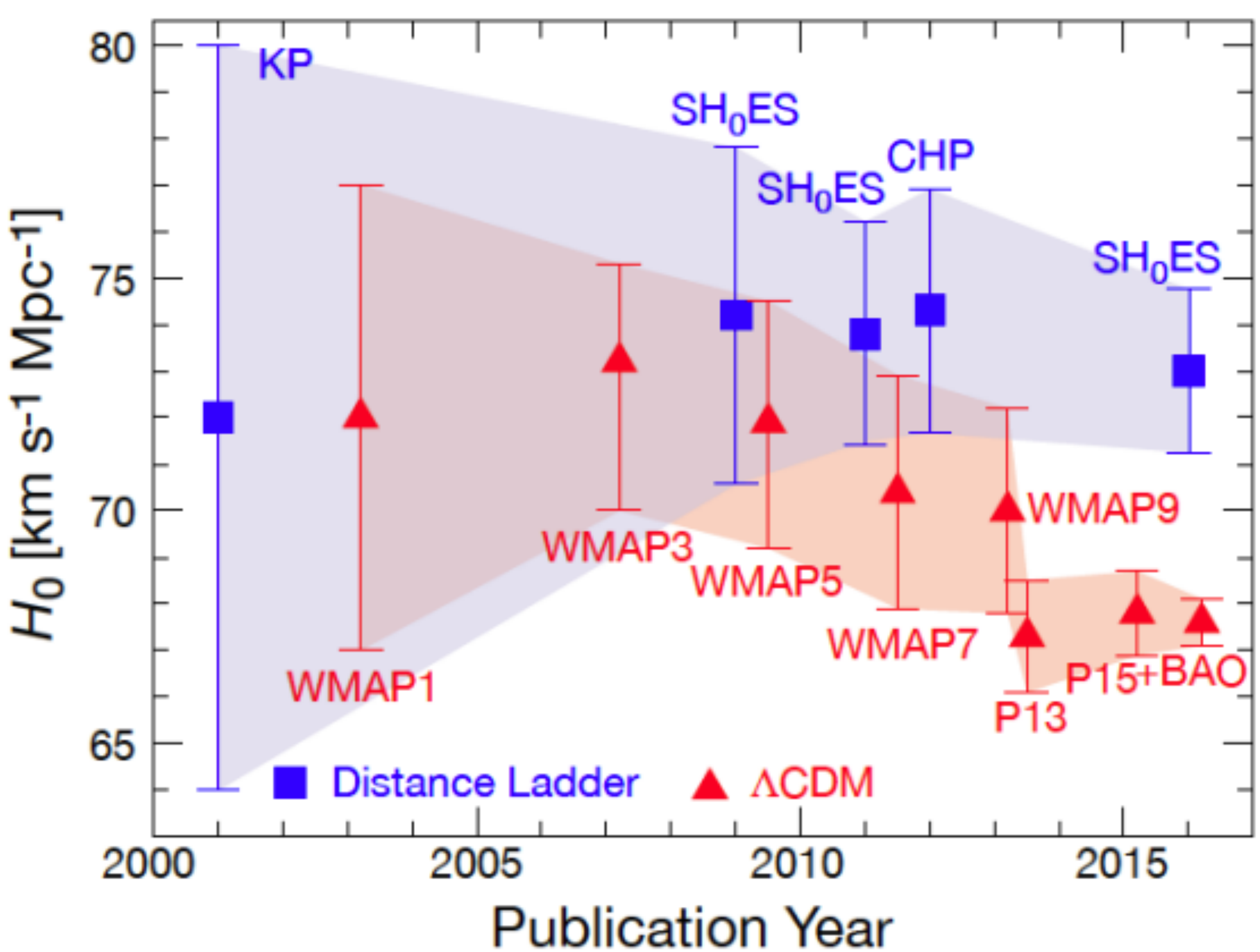


PROBLEM #2: SPATIAL CURVATURE

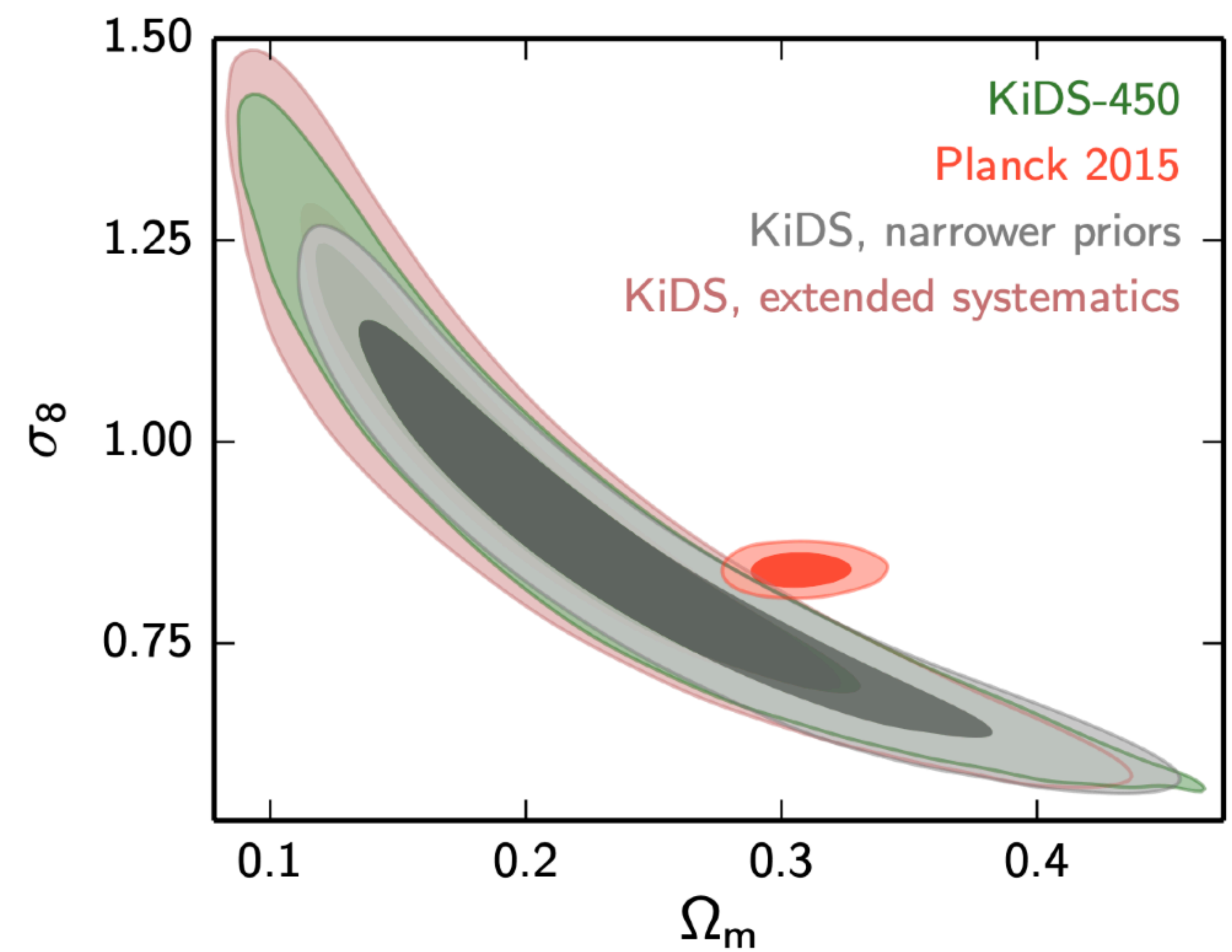
Cosmo-tensions

- tensions have been emerging from measurements of
- H_0 : tension of order ≈ 3σ (Riess et al 2016), now ≈ 5σ
- σ_8 : tension of order ≈ 2σ
- σ_8 : quantifies the amplitude of matter fluctuations on an 8 Megaparsec (Mpc) scale

plot from Freedman 2017 arXiv:1706.02739



Joudaki et al. 2016



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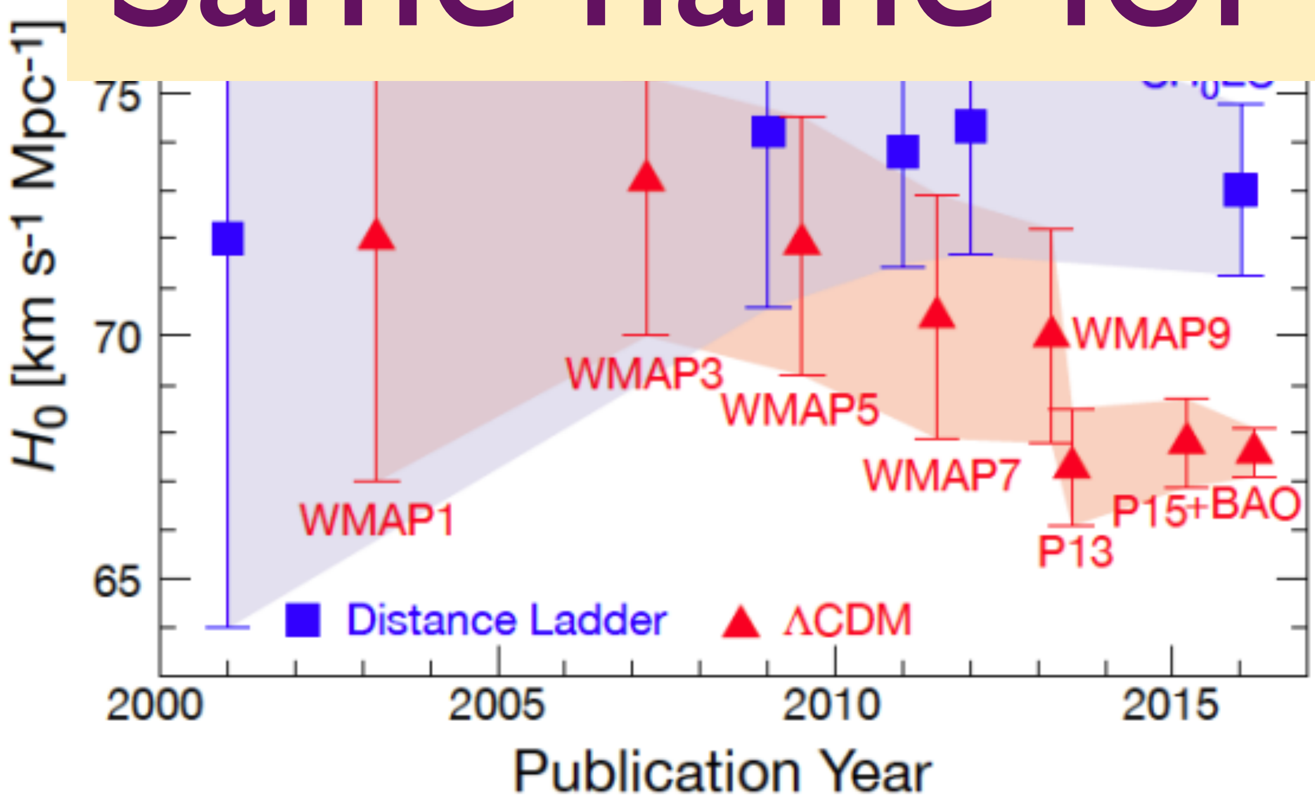
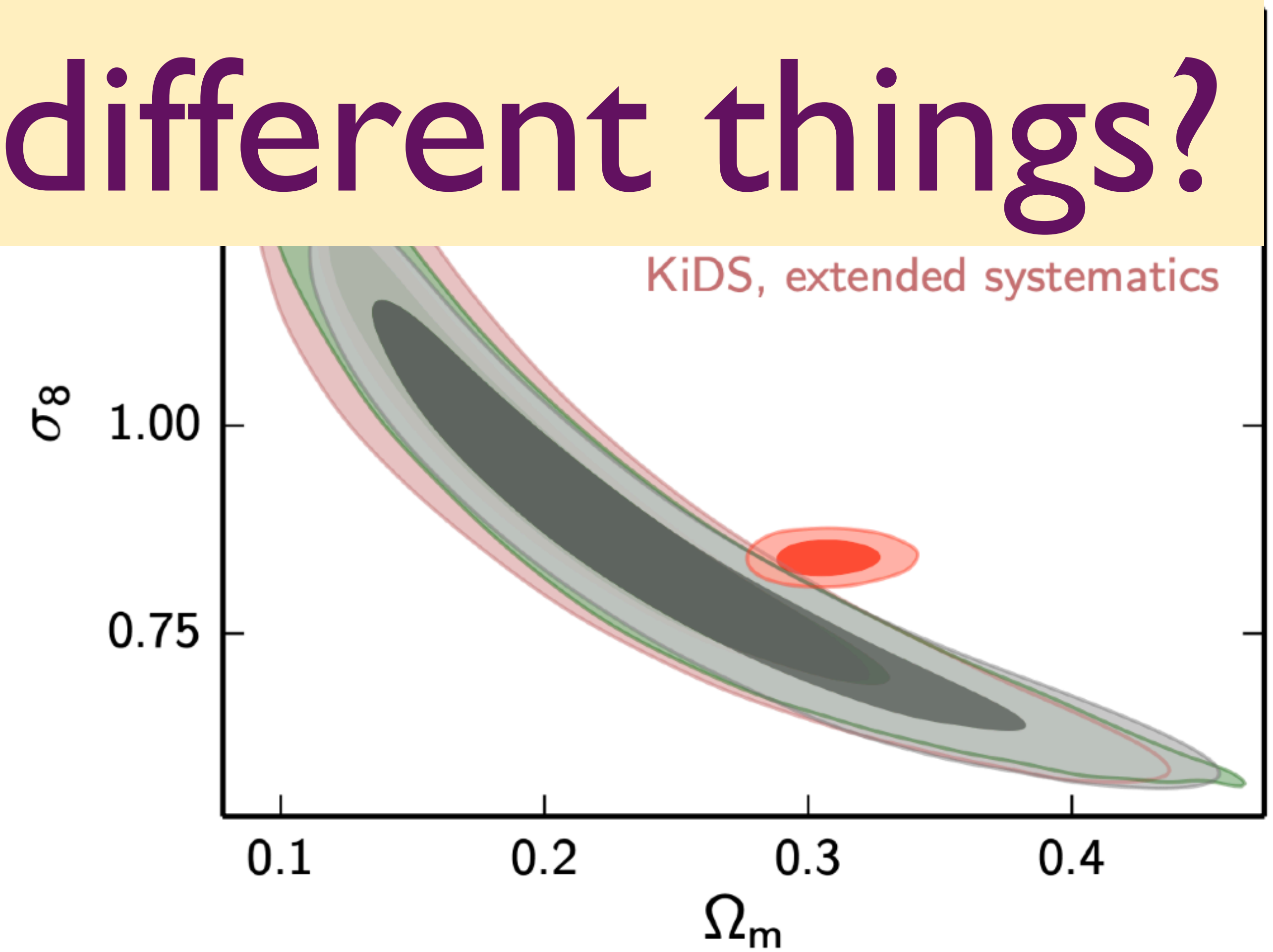
H_0 from CMB and SNae: one is global, the other is local. Same name for different things?





Planck 2018, *A&A 641*

2. The Exquisite fit of ΛCDM to CMB anisotropies

Efstathiou, 2406.12106

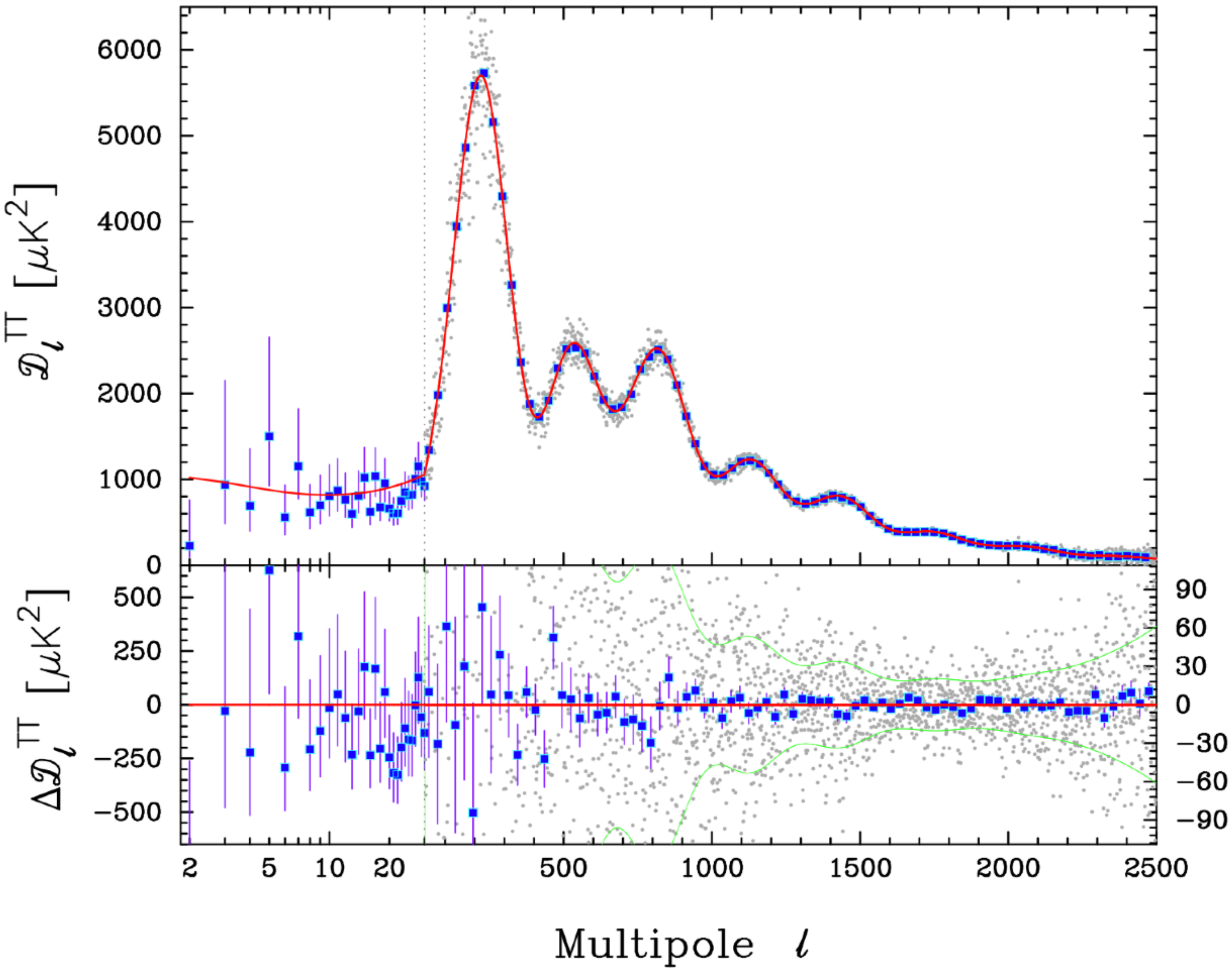
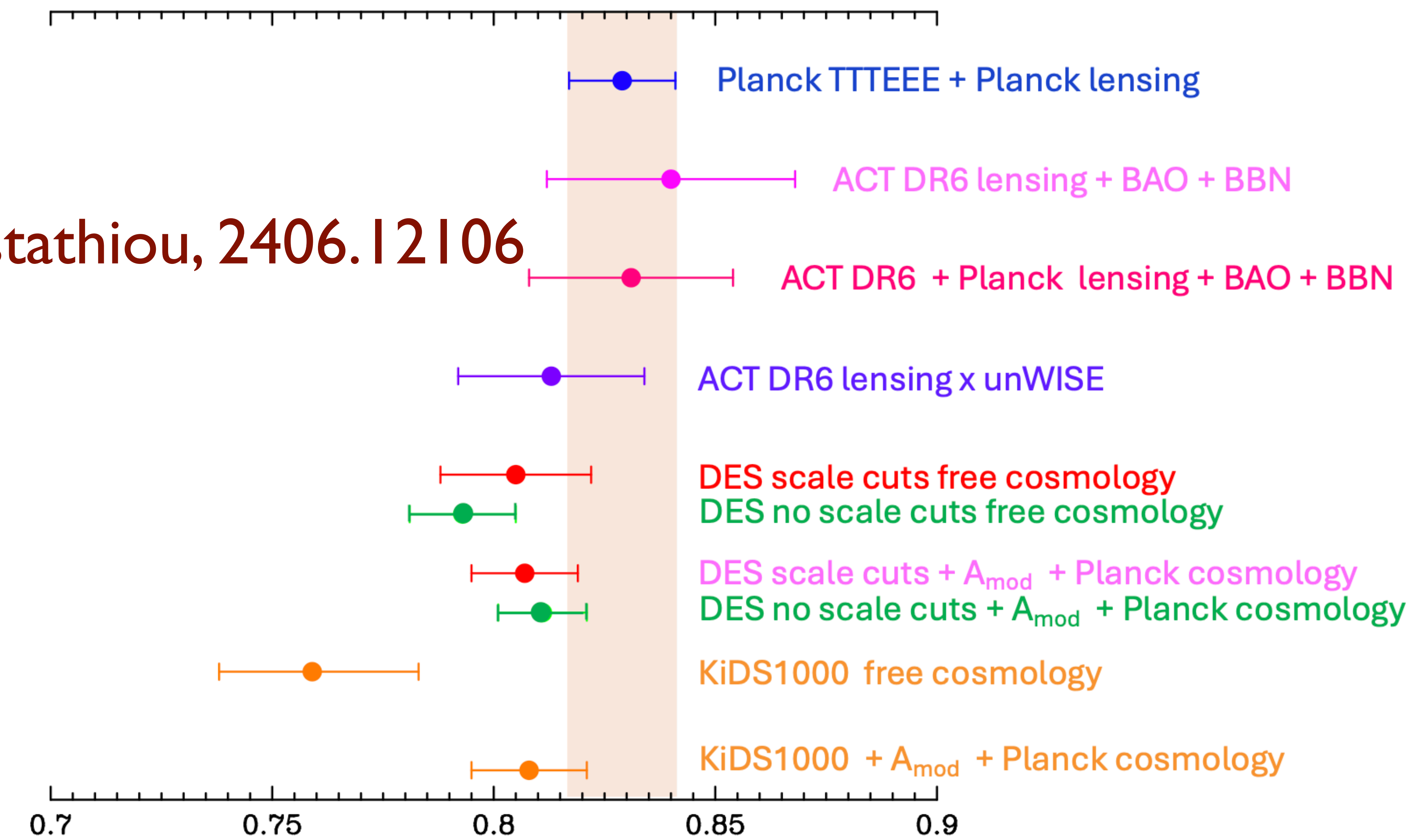


Figure 1. The upper panel shows the *Planck* CMB temperature power spectrum and the lower panel shows the residuals with respect to the power spectrum of the base six parameter ΛCDM model fitted to the TTTEEE spectra (shown by the red line in the upper panel) . The multipole scale is logarithmic

σ_8 - S_8 tension

Efstathiou, 2406.12106



$$S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$$

Figure 5. Summary of measurements of S_8 including new results from ACT DR6 CMB lensing measurements (Qu et al., 2024a) and ACT DR6 lensing cross-correlate with unWISE galaxies (Farren et al., 2024). The remaining entries show results for the DES and KiDS weak lensing surveys as described in the text.

Standard ΛCDM Cosmology

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations, good for early times and/or for large scales, e.g. CMB and LSS; I-order, II order, "gradient expansion" (aka long-wavelength approximation)
  3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.

beyond ΛCDM

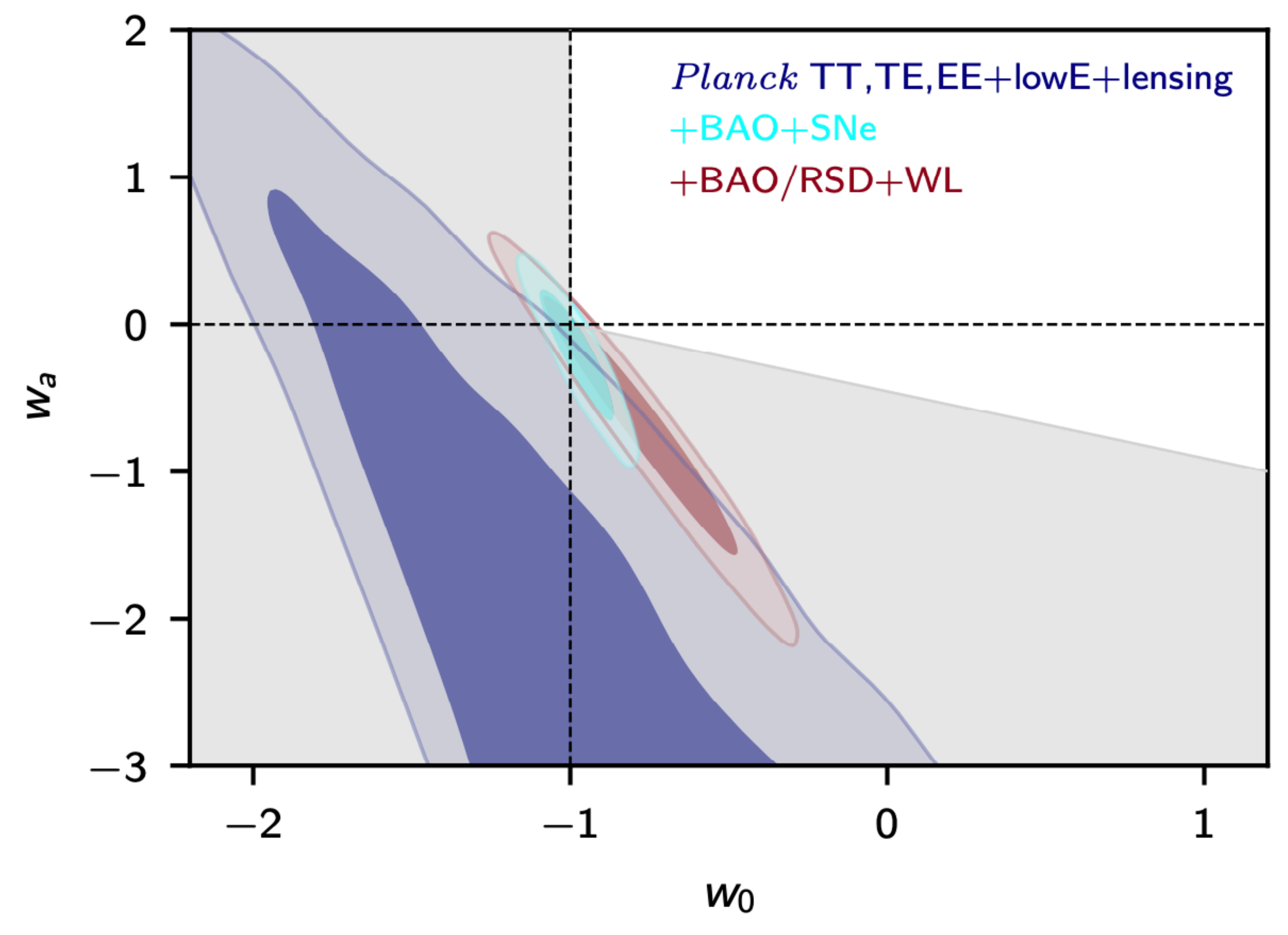
- ΛCDM is the simplest and very successful model supporting the observations that, assuming the Cosmological Principle, are interpreted as acceleration of the Universe expansion
- ΛCDM: Λ accelerates the expansion, Cold Dark Matter (CDM) drives structure formation
- Tensions in observations and theoretical considerations lead to explore alternatives

beyond standard ΛCDM: recipes

Going beyond ΛCDM, two traditional main alternatives, plus a (relativity) new one:

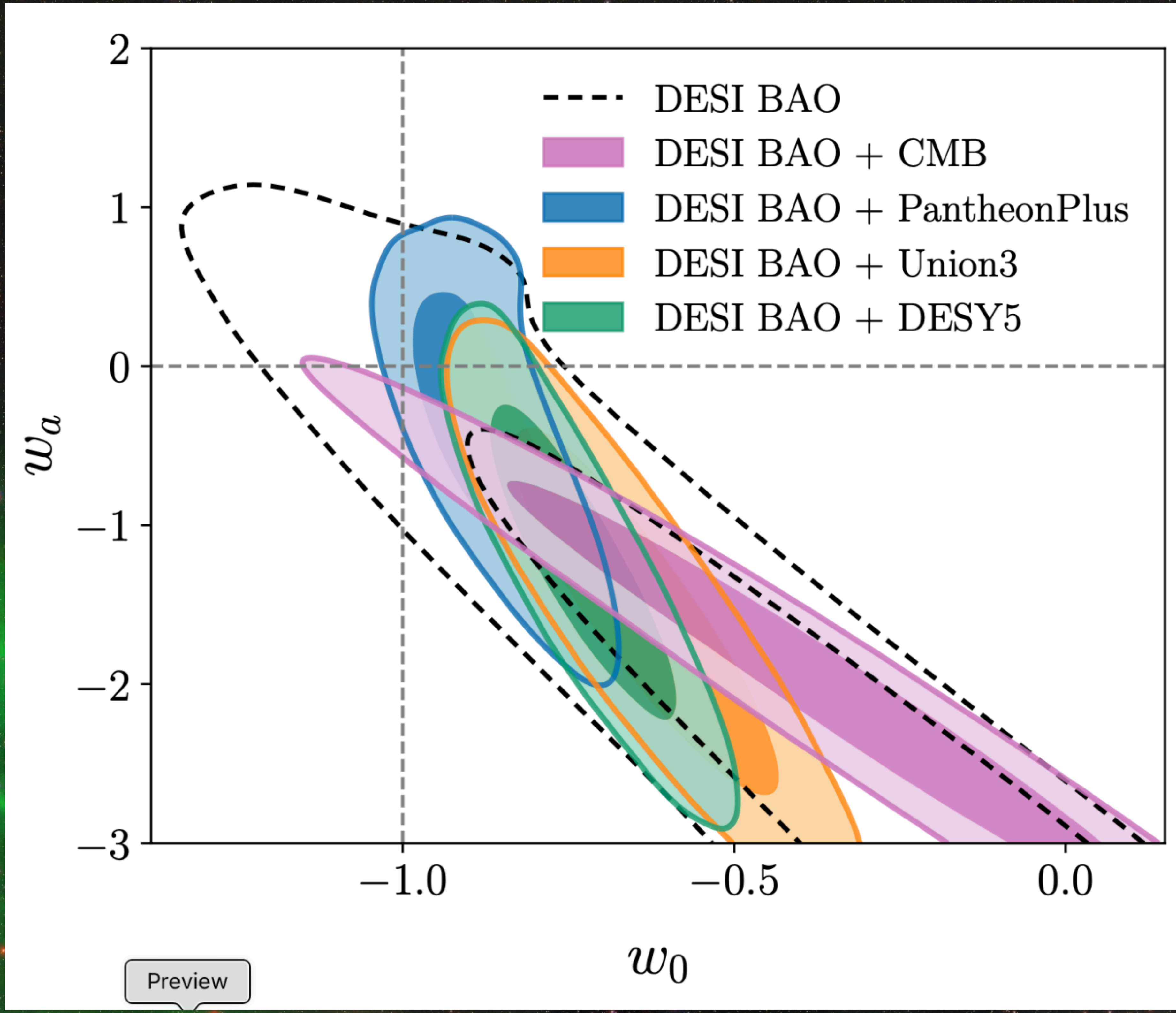
1. Maintain the Cosmological Principle (FLRW background), then either
   a) maintain GR + dark components (CDM+DE or UDM, or interacting DE)
   b) modified gravity (f(R), branes, etc…)

Planck 2018, w(z) EoS



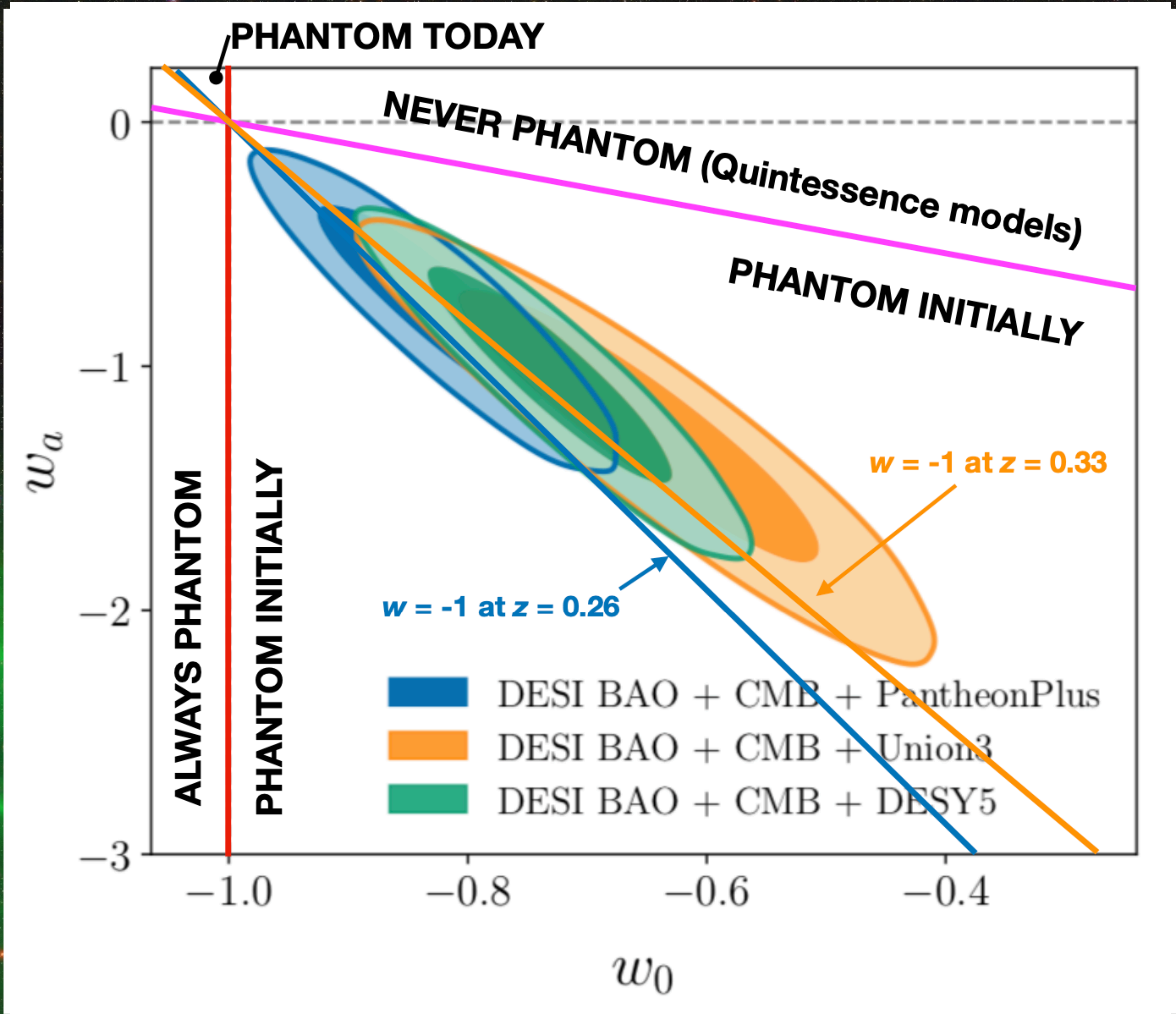
consistent with Λ, AKA w=-1, i.e. w_0=-1, w_a=0

DESI-2024 results on w_a-w_0



DESI VI: 2404.03002 — Cortes & Liddle 2404.08056

DESI-2024 results on w_a-w_0



DESI VI: 2404.03002 — Cortes & Liddle 2404.08056

DESI-2024: w_a-w_0, Phantom DE?

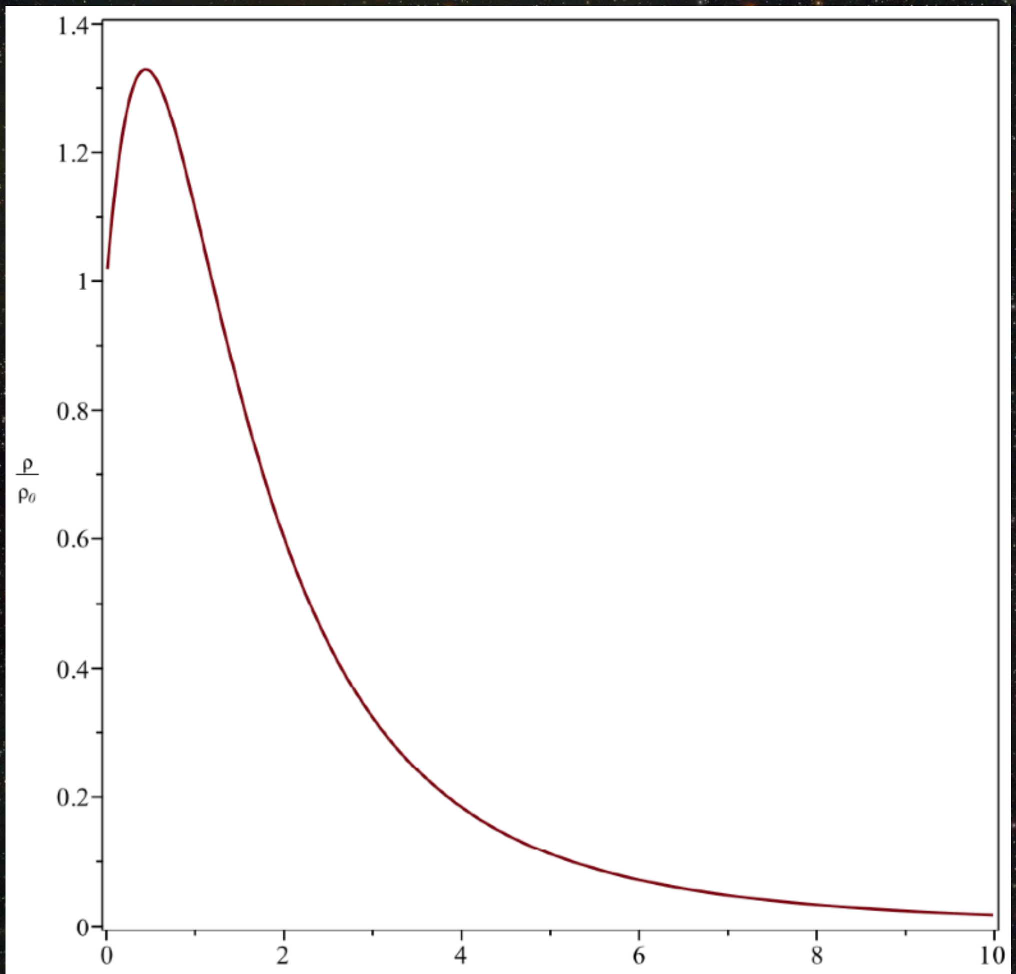
- Cortes & Liddle: "We argue that conclusions on dark energy evolution are strongly driven by the assumed parameter priors"

$$w(a) = w_0 + w_a(1-a)$$

- problem: the w_a-w_0 parametrisation was invented for low redshift, but has been extend up to the CMB

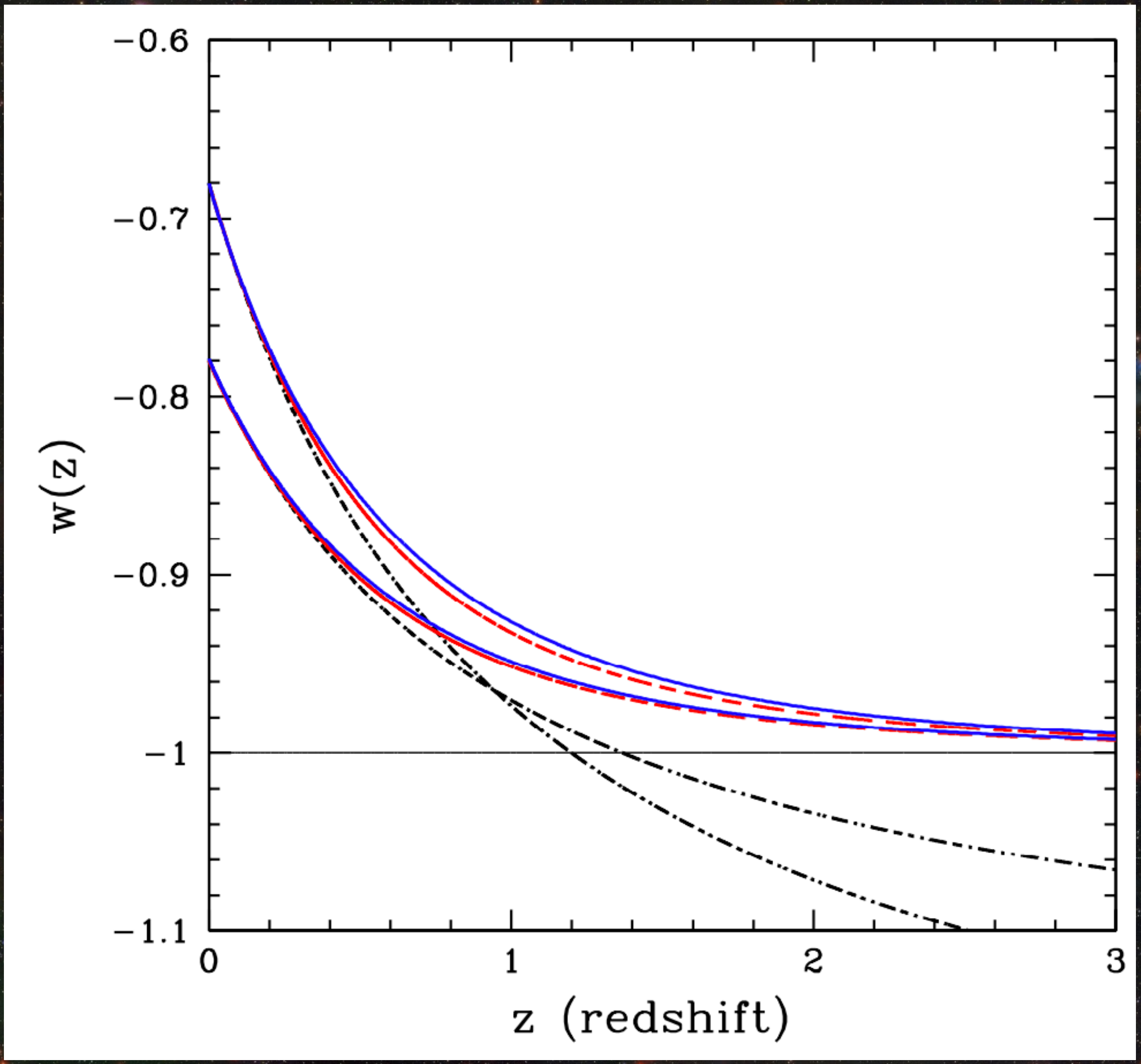
$$\dot{\rho} = -3H\rho(1+w)$$

- integrating conservation of energy with this w(a) gives a DE growing in the past, with a max when w=-1
- not possible with a canonical scalar field or an adiabatic perfect fluid



w_a-w_0, Phantom DE?

- Way out #1: model by model comparison (unpractical)
- Way out #2: use a different parametrisation mimicking quintessence scalar fields
- Way out #3: CDM interacting with Dark Energy



different parametrizations avoiding the w=-1 crossing vs the w_a-w_0 one, Crittenden+ astro-ph/0702003

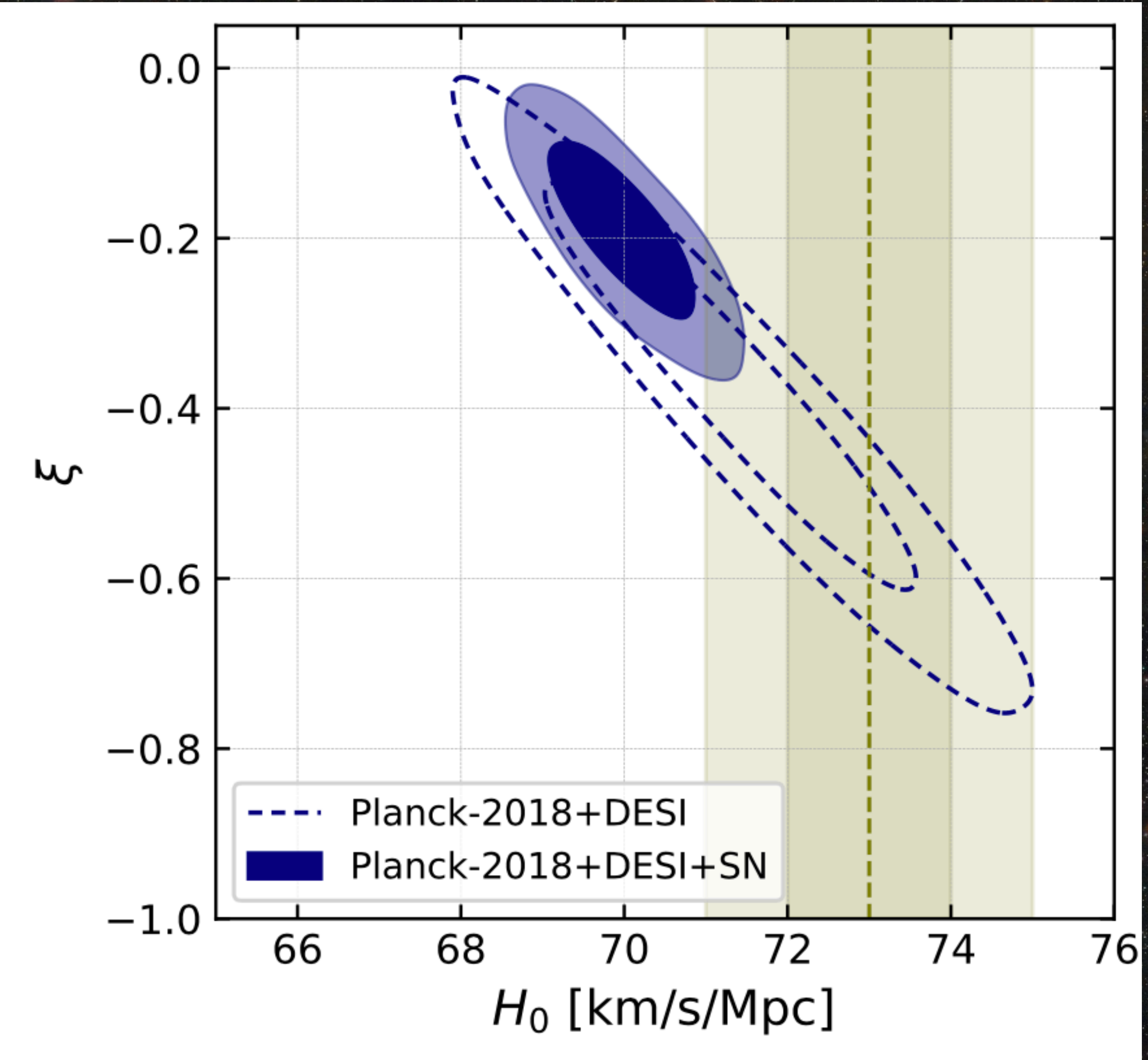
CDM interacting with DE

- Giare'+, 2404.15232
- Assume interaction

$$\nabla_\mu T_i^{\mu\nu} = Q_i^\nu, \quad \sum_i Q_i^\mu = 0$$

$$Q_i^\mu = (Q_i + \delta Q_i)u^\mu + a^{-1}(0, \partial^\mu f_i),$$

$$Q = \mathcal{H}\xi\rho_{\rm DE}$$



The joint Planck+DESI analysis yields a preference for a non-vanishing $\xi = -0.38^{+0.18}_{-0.16}$, well exceeding the 95% CL. Additionally, it provides a value $H_0 = 71.4 \pm 1.5$ km/s/Mpc, in perfect agreement with local distance ladder estimates. Therefore, focusing on Planck-2018 and DESI-BAO altogether, IDE can fully resolve the Hubble tension, see also Fig. 1. Adding CC does not change this result, see also Tab. I.

Figure 1. 2D contours at 68% and 95% CL for the coupling parameter ξ and the Hubble parameter H_0, as inferred by the different combinations of Planck-2018, DESI, and SN data listed in the legend. The olive-green band represents the value of H_0 measured by the SH0ES collaboration.

beyond standard ΛCDM: recipes

2. Maintain GR, drop CP, then either

   a) try to construct an homogeneous isotropic model from averaging, possibly giving acceleration: dynamical back-reaction (uncompleted programme)

   b) consider inhomogeneous models, e.g. LTB (violating the CP) or Szekeres (not necessarily violating the CP): back-reaction on observations (for LTB see e.g. Kenworthy et al 1901.08681 and Camarena et al 2205.05422)

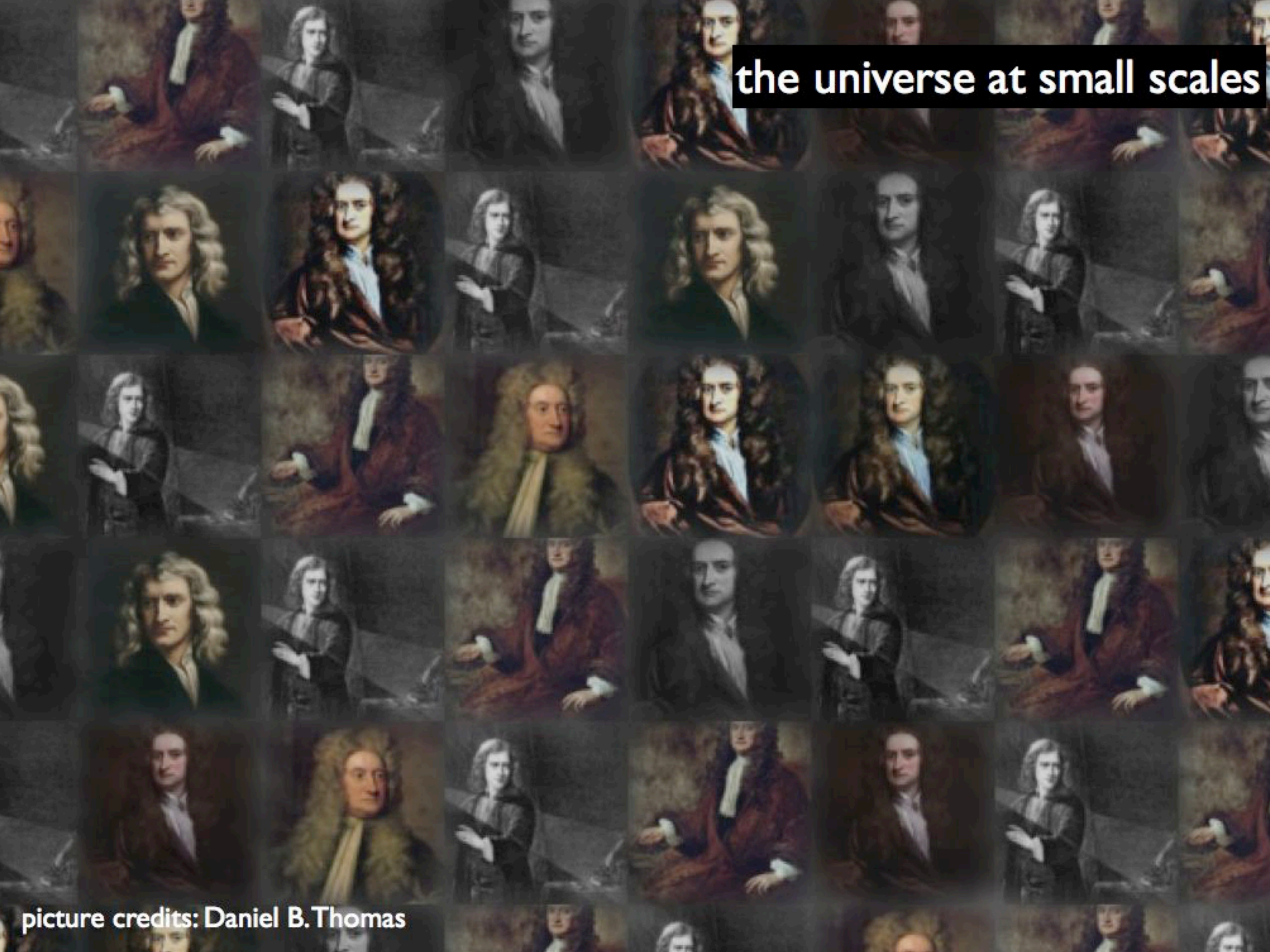
Questions on ΛCDM

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations (e.g. CMB; linear, nonlinear)
  3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)
- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - It is timely to bridge the gap between 2 and 3

the universe at very large scales: GR

picture credits: Daniel B. Thomas

the universe at small scales



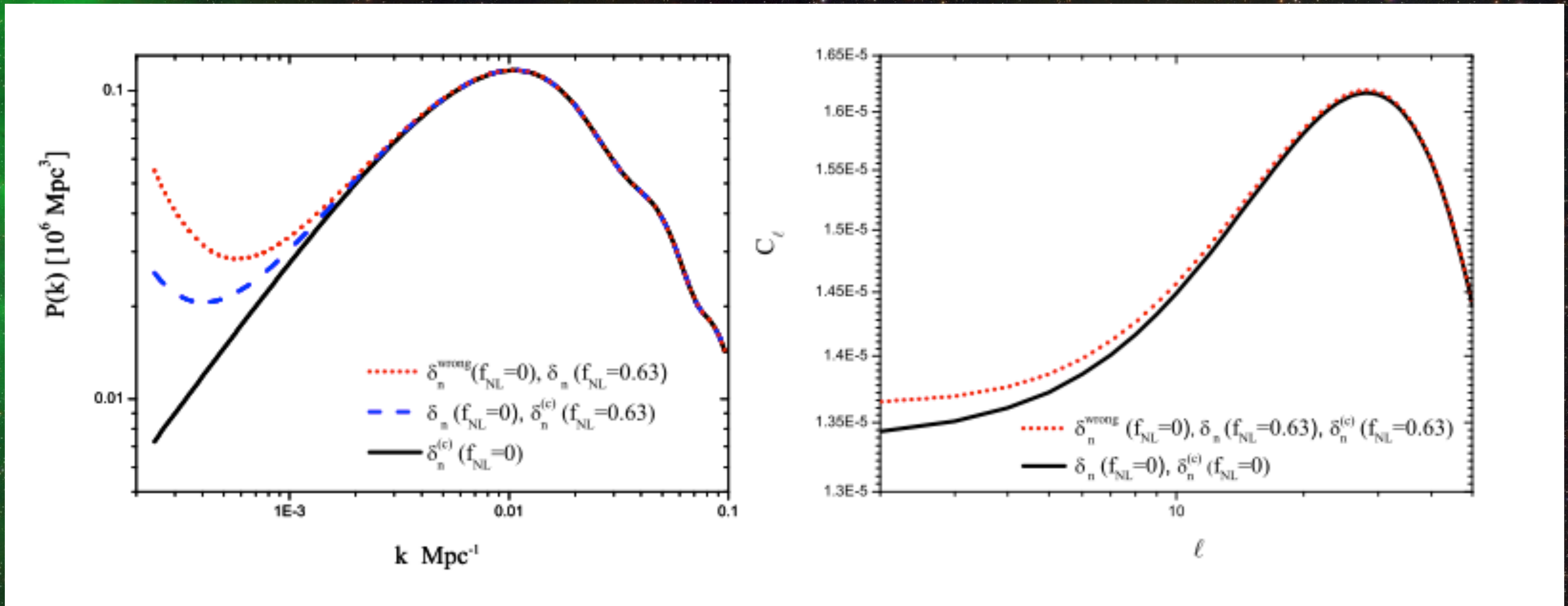
picture credits: Daniel B. Thomas

“take home message #1”

- in view of EUCLID, SKA and other very large scale galaxy surveys, it is important to consider perturbative relativistic effects in structure formation (1st and 2nd order)
- at large scales: matter power spectrum

MB, Crittenden, Koyama, Maartens, Pitrou & Wands, *Disentangling non-Gaussianity, bias and GR effects in the galaxy distribution*, arXiv:1106.3999, PRD 85 (2012)

see Bonvin & Durrer PRD 84 (2011) and Challinor & Lewis PRD 84 (2011) and many papers that followed



fresh from the press...

Blanco, Bonvin, Clarkson & Maartens 2406.19908

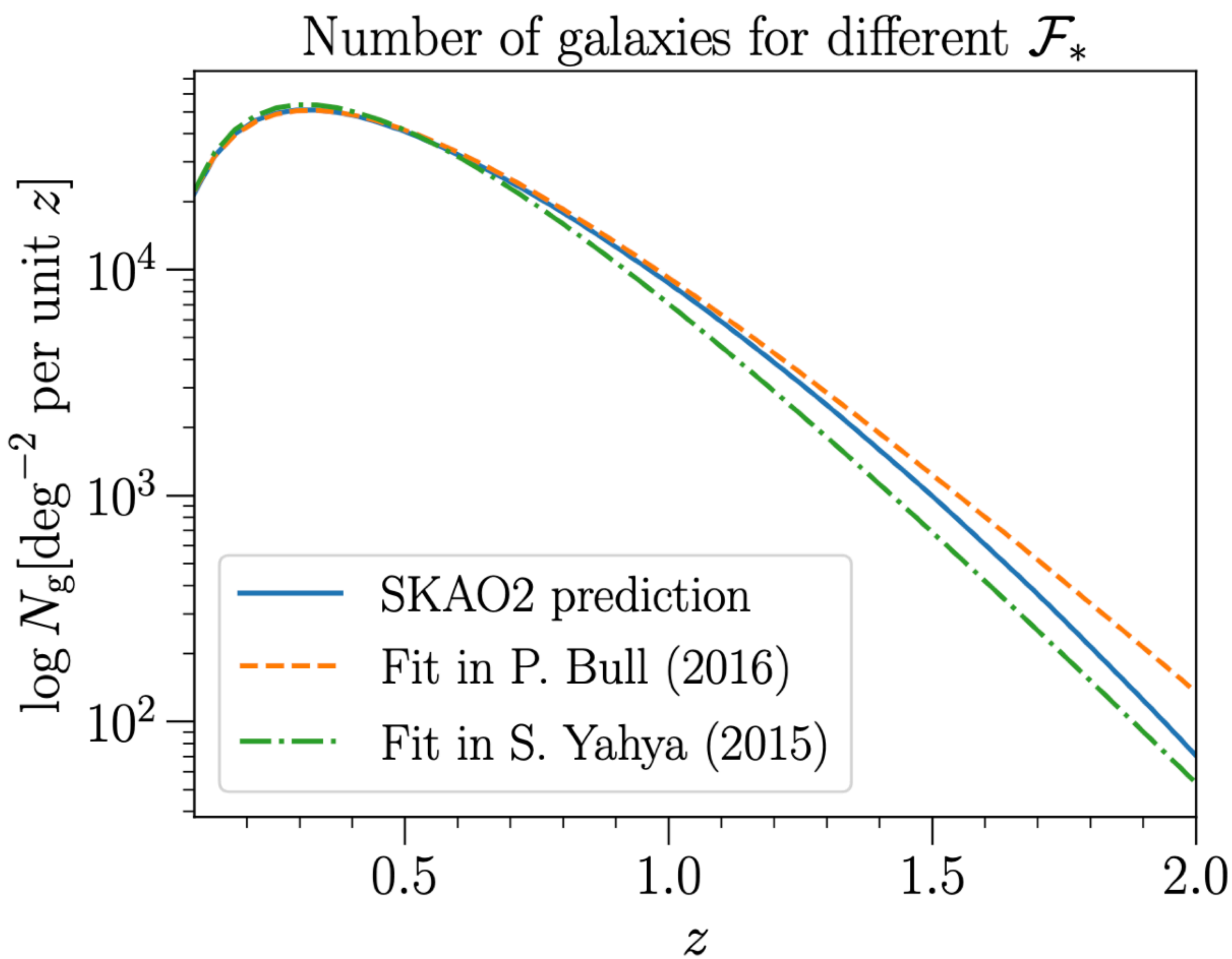


Figure 1: Logarithm of the number of galaxies per unit of solid angle and redshift for different choices of the flux limit. The solid blue line is obtained by using the z-dependent $\mathcal{F}_*$ from [64]. The dashed orange line is obtained by using the fit in [66]. Finally, the dashdot green line corresponds to the fit from [65] with redshift-independent flux limit $\mathcal{F}_* = 5.0\,\mu$Jy.

beyond standard ΛCDM: recipes

3. NEW: stick with ΛCDM, but use fully nonlinear GR, i.e. *Numerical Relativity* simulations

   i) Full GR equations: Bentivegna & MB, 1511.05124, Giblin et all 1511.01105, Macpherson et al 1807.01714, Heinesen et al 2111.14423, Dhawan et al 2205.12692 (this last 3 related to H_0)

   ii) Full GR N-body, with some approximation (post-Friedmann, or neglect tensor modes): MB, Thomas and Wands 1306.1562; Adamek et al 1509.01699, 1604.06065, Barrera-Hinojosa, Li, MB & He 2010.08257

   iii) more recent work, also motivated by JWST, focus on gravitational collapse of structures

Nonlinearity, Gravito-Magnetism, Numerical Relativity

beyond relativistic perturbation theory:
it is all down to the Raychaudhuri equation!

$$\dot{\Theta} = -\frac{\Theta^2}{3} - 4\pi G \rho_M - 2\sigma^2 + 2\omega^2 + \Lambda$$

$\Theta \to 3H = 3\frac{\dot{a}}{a}$, $\sigma = \omega = 0$ for homogeneous and isotropic case

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda}{3}$$

Gravito-magnetism and frame-dragging

- In GR, a moving mass generates a gravito-magnetic field (cf. moving charges in EM)
- associated with rotation, e.g. frame-dragging of neutron stars and black holes
- also relevant in weak-field: leading order $1/c^3$ post-newtonian correction
- measured by satellites around the Earth
    - C. W. F. Everitt et al, Phys. Rev. Lett. 106, 221101 (2011).
    - I. Ciufolini and E.C. Pavlis, Nature (London) 431, 958 (2004).

Numerical Relativity N-body with GRAMSES: gravity-magnetism and Frame Dragging

Cristian Barrera Inojosa, Baojiu Li, Marco Bruni & Jian-hua He,
MNRAS 501, 5697–5713 (2021)

based on the Gramses N-body code developed in

Barrera-Hinojosa C., Li B., 2020a, J. Cosmol. Astropart. Phys., 2020, 007
Barrera-Hinojosa C., Li B., 2020b, J. Cosmol. Astropart. Phys., 2020, 056

Vector modes in ΛCDM: the gravitomagnetic potential in dark matter haloes from relativistic *N*-body simulations

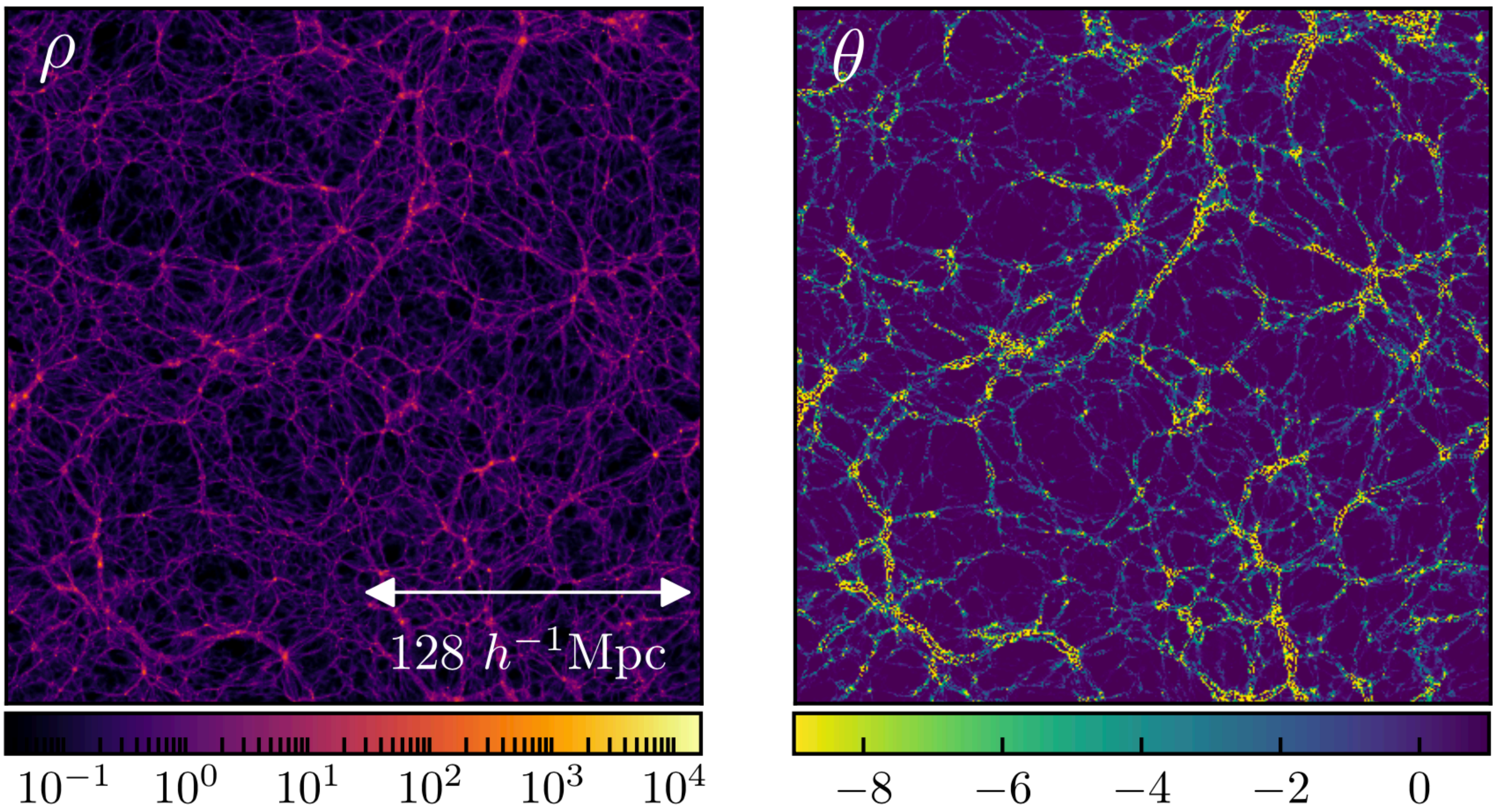
MNRAS 501, 5697–5713 (2021)

Cristian Barrera-Hinojosa[1★] Baojiu Li[1] Marco Bruni[2,3] and Jian-hua He[4,5]

a slice of the simulation box at z=0

velocity fields normalised to aHf

matter density field   divergence of the velocity field [1]



vector fields

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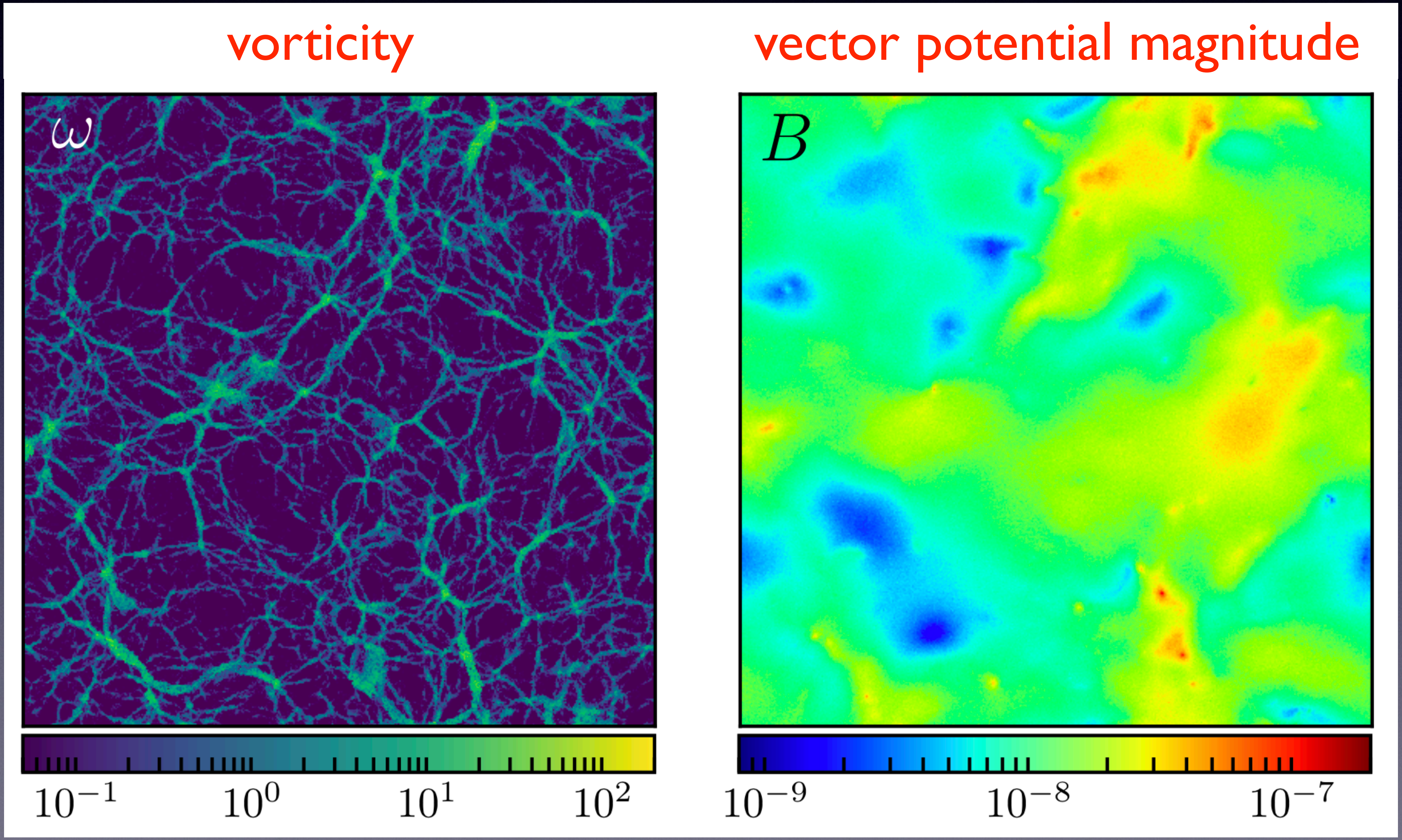
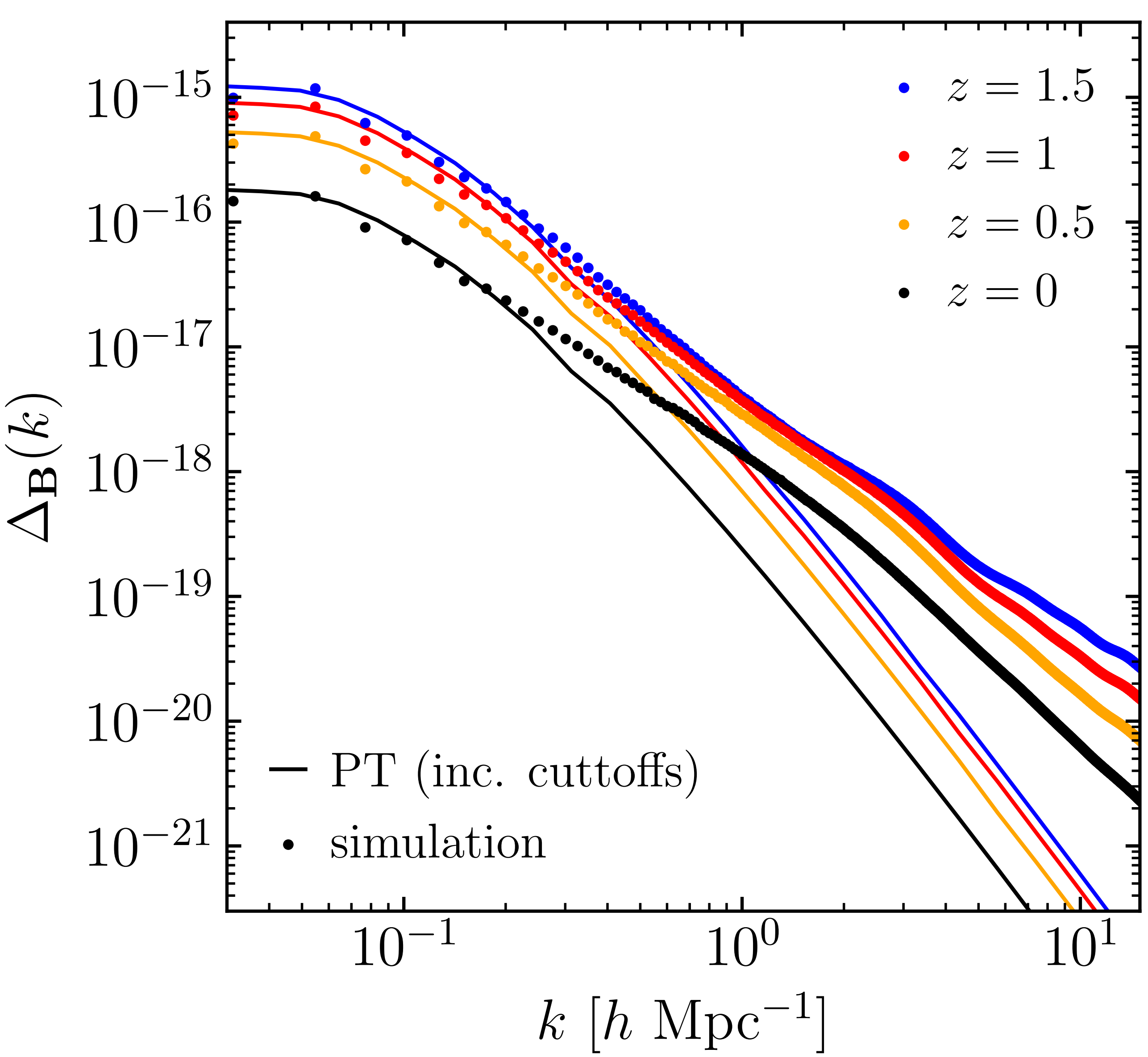


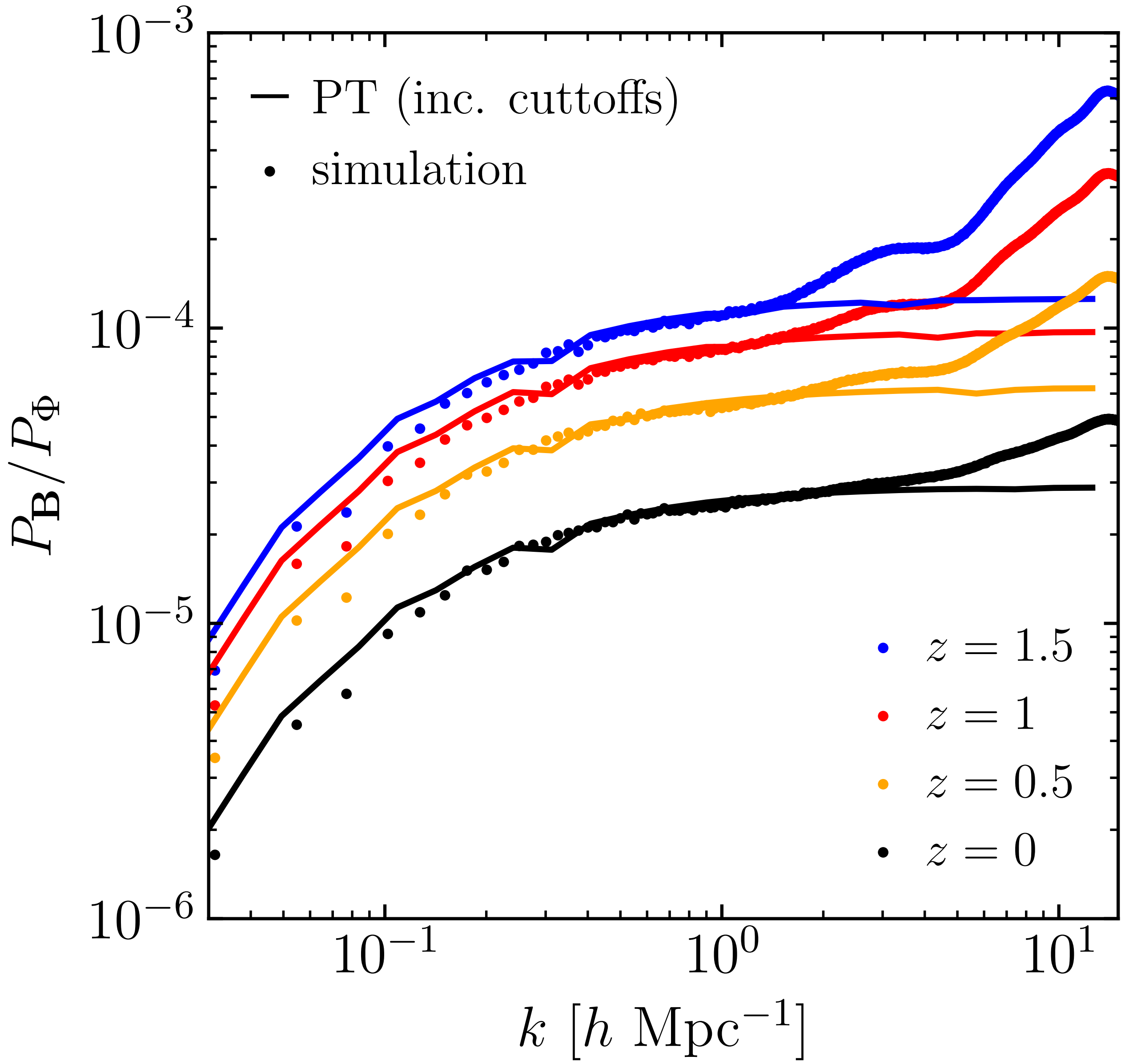
power spectra

The solid lines represent the corresponding second-order perturbation theory predictions (Lu et al. 2009), in which cutoffs have been introduced in the convolution calculation to accommodate the lack of power in the simulation results on large scales due to box size.

dimensionless power spectrum of the vector potential at different redshifts



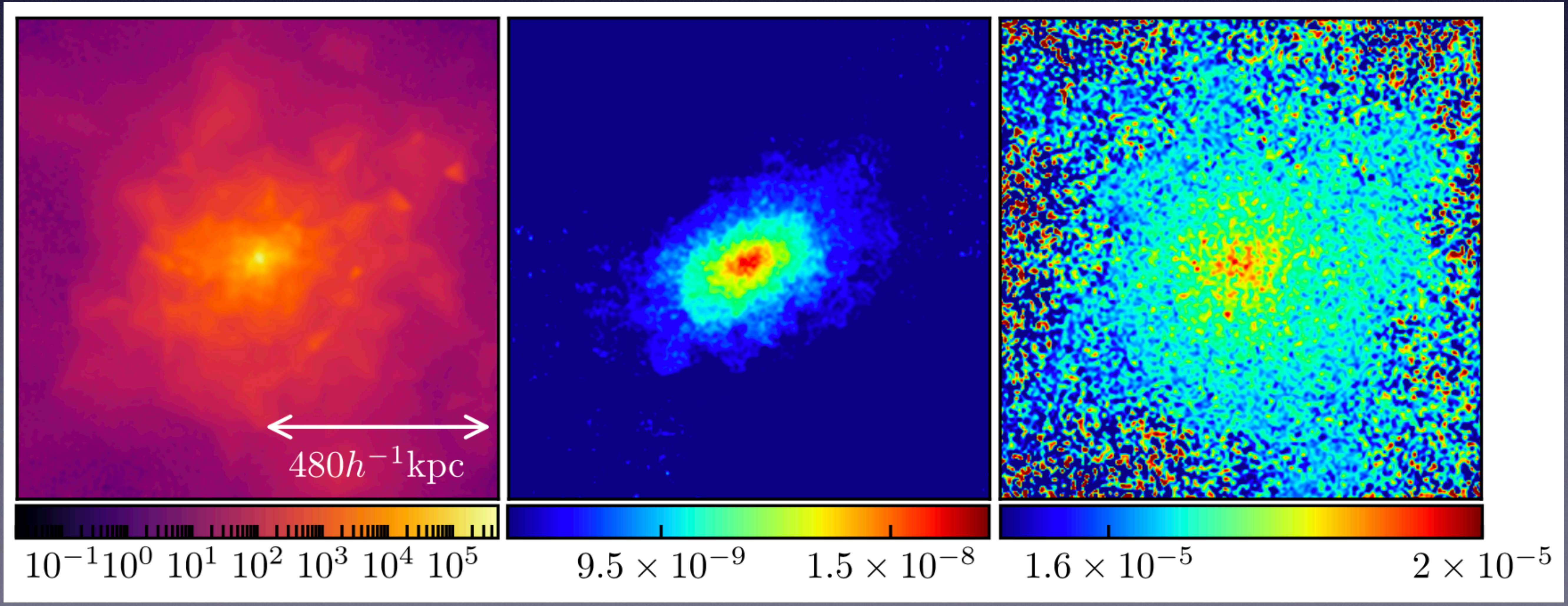
ratio of the vector and scalar potentials at different redshifts



Dark Matter Halos: $3.1 \times 10^{12}\ h^{-1}\ M_\odot$

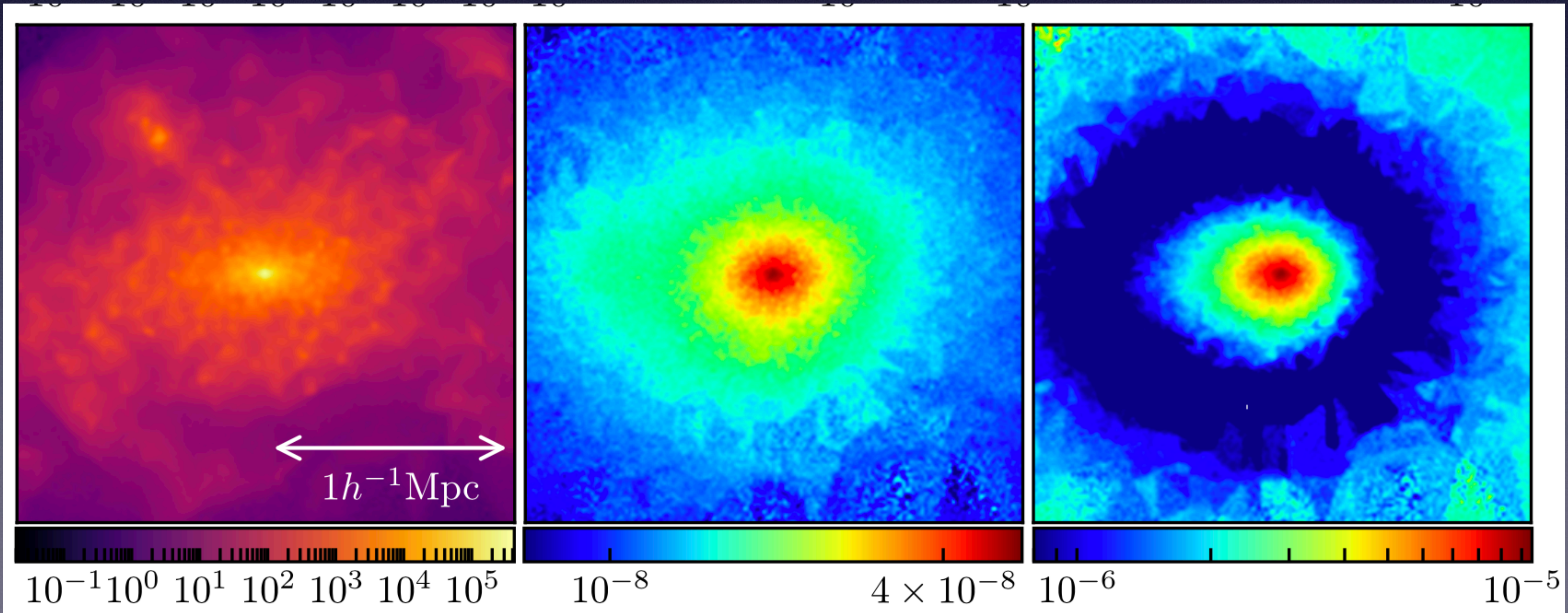
- visualisation at z=0, or matter density field, the gravito-magnetic vector potential *B* and the scalar gravitational field |Φ| (dimensionless units)

- in smaller halos the effect is much more concentrated and weaker



Dark Matter Halos: $3.0\times10^{13}\ h^{-1}\ M_\odot$

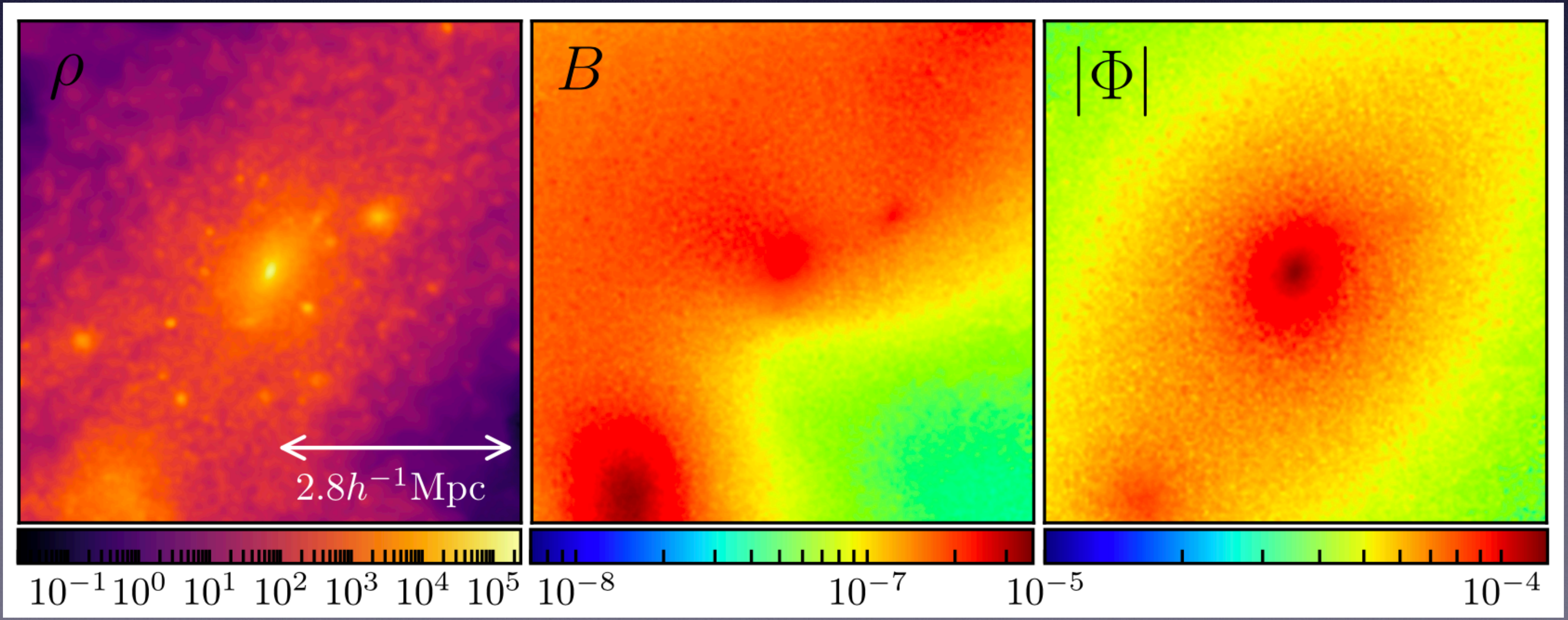
- visualisation at z=0, or matter density field, the gravito-magnetic vector potential *B* and the scalar gravitational field |Φ| (dimensionless units)
- in medium size halos the effect is more extended and stronger



Dark Matter Halos: $6.5 \times 10^{14}\ h^{-1}\ M_{\odot}$

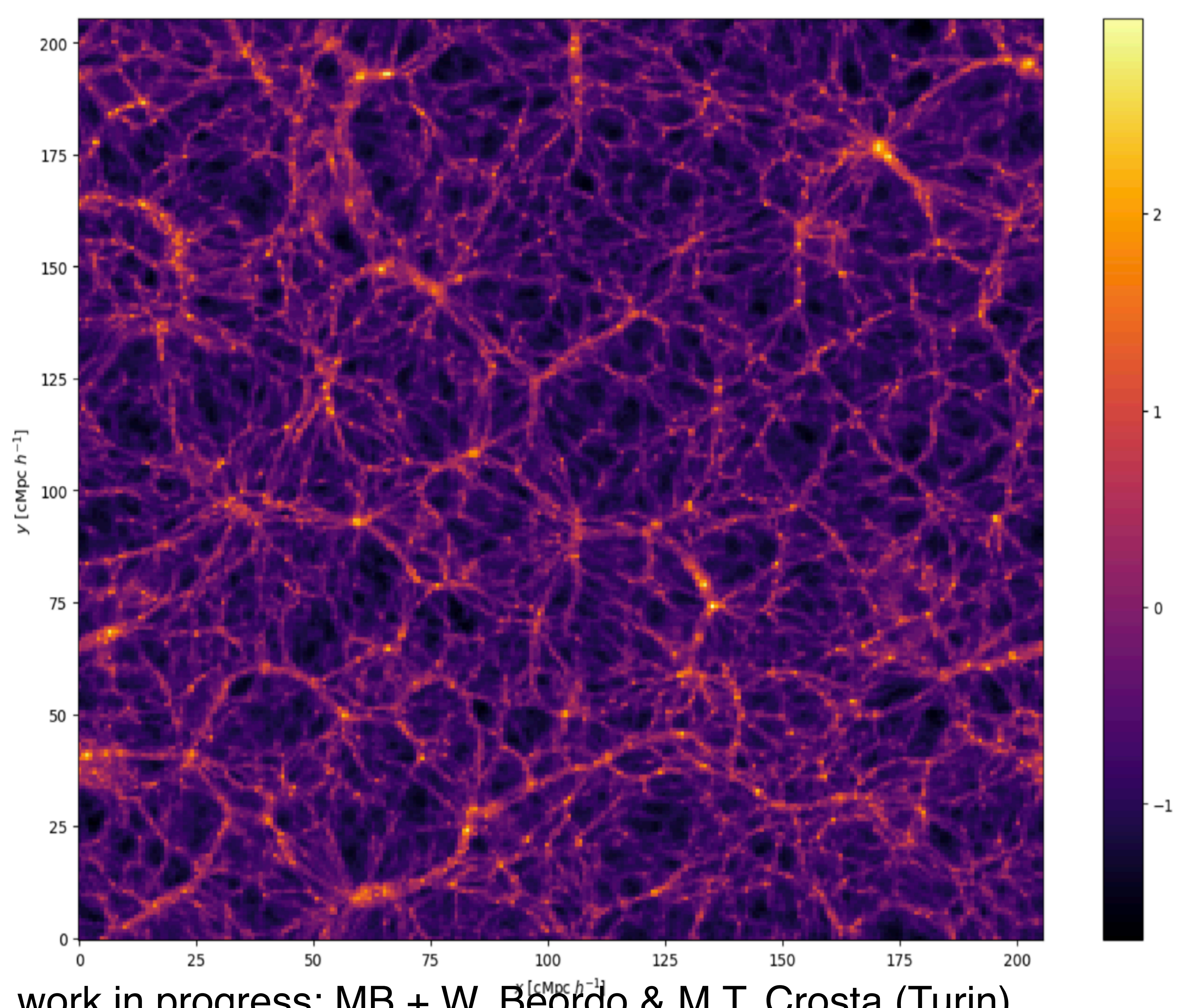
- visualisation at z=0, or matter density field, the gravito-magnetic vector potential *B* and the scalar gravitational field |Φ| (dimensionless units)

- in larger and less virtualised halos the effect is even more extended, stronger and diffused



extending down to galactic scales

- visualisation at z=0, of matter density field, the scalar gravitational field |Φ| and the gravito-magnetic vector potential *B*



work in progress: MB + W. Beordo & M.T. Crosta (Turin)

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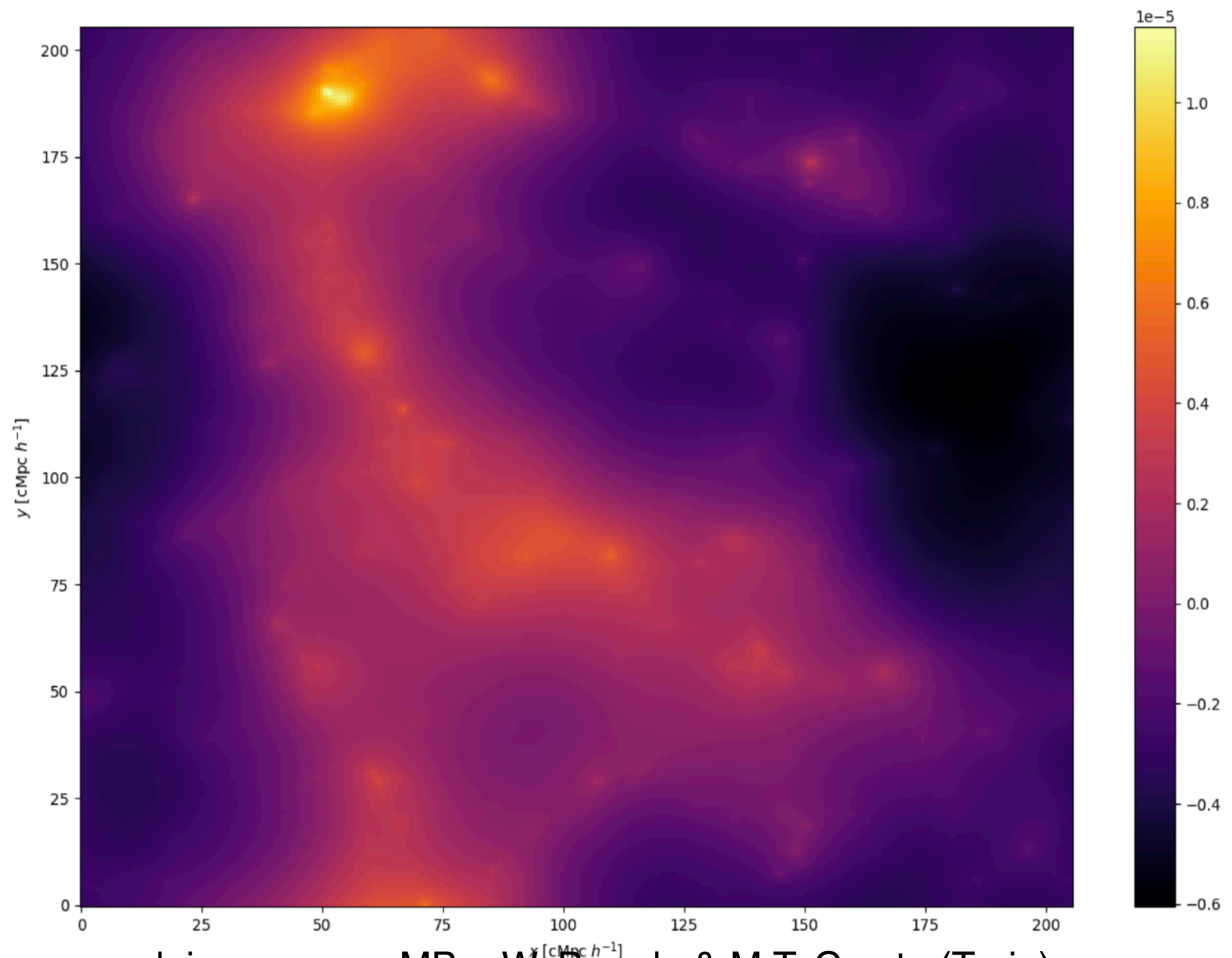

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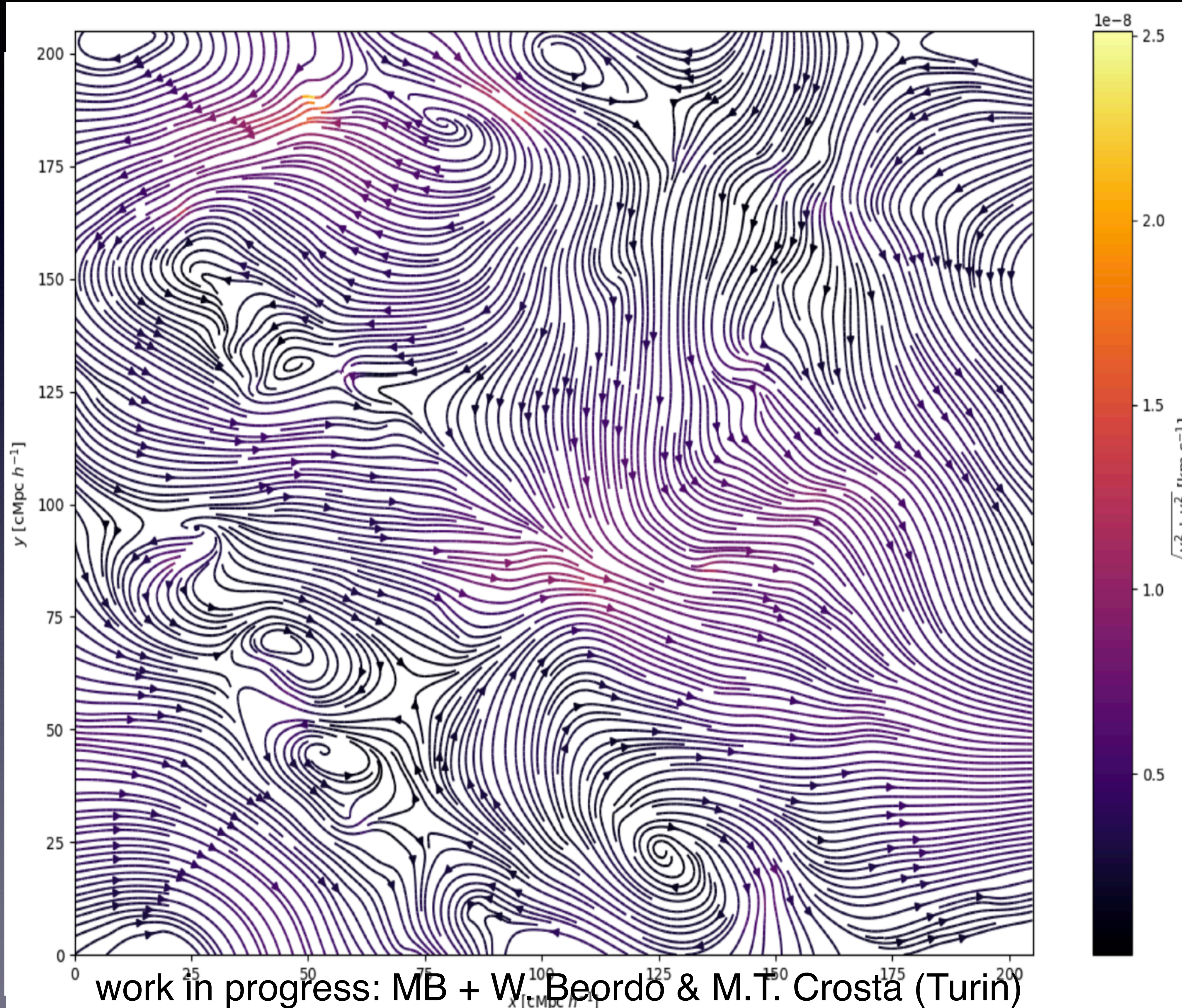


work in progress: MB + W. Beordo & M.T. Crosta (Turin)

Cosmological Numerical Relativity with Einstein Toolkit: quasi-spherical collapse

Robyn Munoz & Marco Bruni, Phys. Rev. D 107, 123536 (2023)
based on the Einstein Toolkit fluid code, plus the EBWeyl code presented in
Robyn Munoz & Marco Bruni, Class. Quantum Grav. 40 135010 (2023)

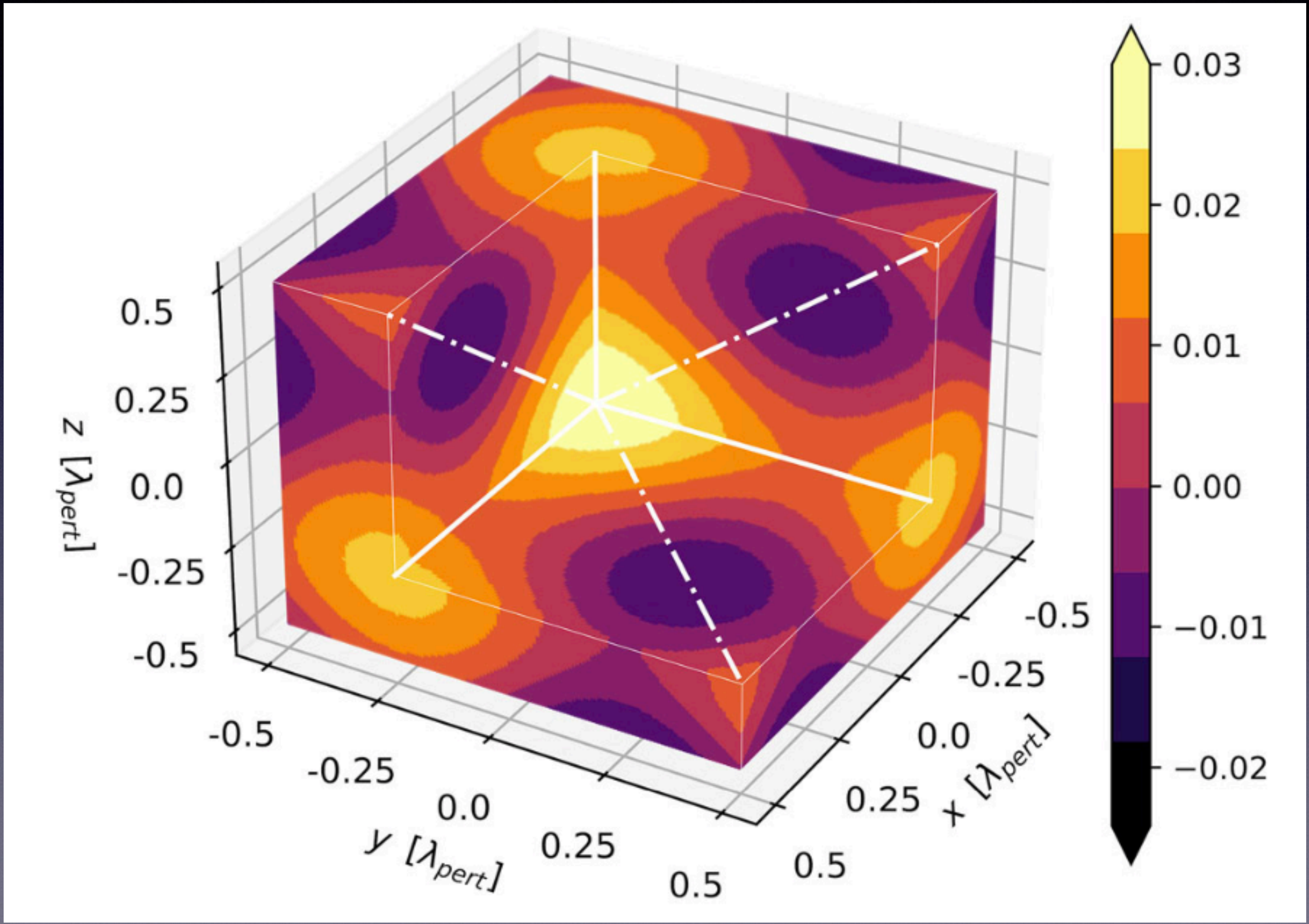
full GR with ET: Reductionism

- Cosmic web: a network of peaks connected by filaments and separated by voids, see J. R. Bond, L. Kofman, and D. Pogosyan, Nature (London) 380, 603 (1996).
- For our purposes, i.e. understand gravity-electro-magnetism in non-linear structure formation with Einstein Toolkit, a reductionist approach is useful.
- Fluid simulations valid up to first shall crossing, starting at z~300
- initial conditions based on inflationary curvature perturbation variable with simple spatial distribution

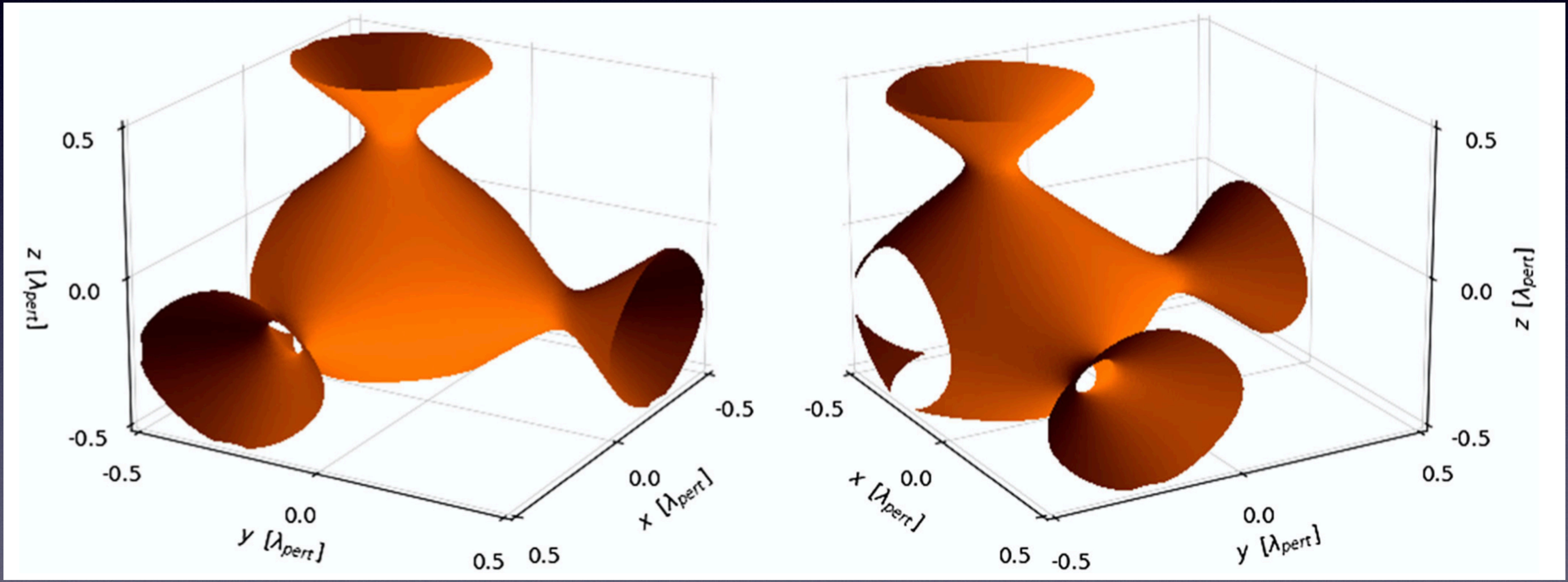
$$\mathcal{R}_c = A_{\text{pert}}(\sin(x k_{\text{pert}}) + \sin(y k_{\text{pert}}) + \sin(z k_{\text{pert}})),$$

- quasi-spherical around peak, but with filaments and voids

δ: density distribution



iso-density surface δ=0.01 at z_{in}=302

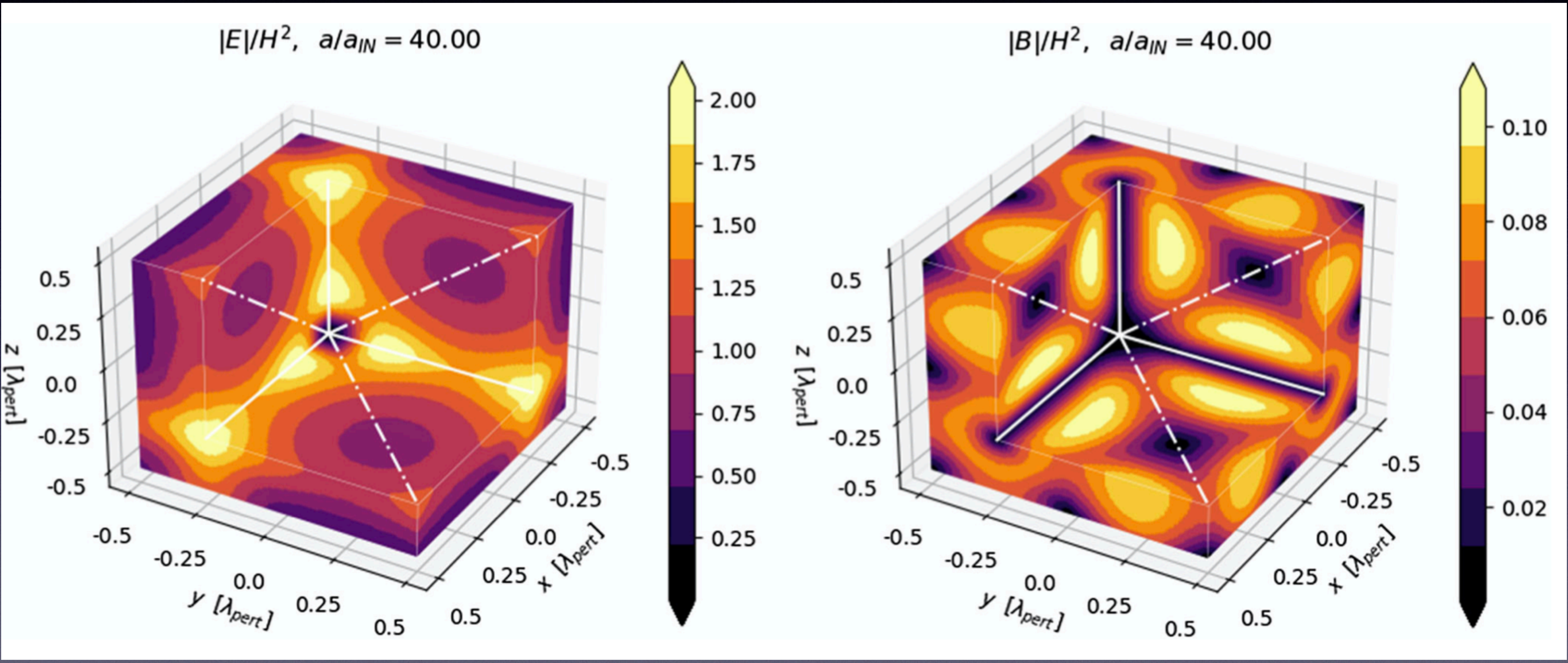


two different points of view

frame-dragging vector potential

- transverse vector part of the metric
- Weyl Curvature tensor can be split in electric part $E^{\mu}{}_{\nu}$ and magnetic part $H^{\mu}{}_{\nu}$
- the spit of the Weyl curvature tensor into E and H parts depends on the frame/observer, in analogy with EM and the split of the EM tensor, but for GW $E^2 - H^2 = 0$ in all frames

E and B parts of the Weyl curvature



simple structure makes evident that E is stronger around the peaks and along filaments, while B is stronger around filaments

Top-hat collapse model

- the so-called Top-hat collapse model described the detachment of an overdensity from the Hubble expansion, and it is based on a spherically symmetric closed model (negative energy, or positive curvature in GR) that expands, reaches a Turn Around, then recollapses.
- It is at the base of the mass function theory of Press-Schechter mass function and the Sheth-Tormen extension
- linear value of $\delta^{(1)}$ used as benchmark, conventional values of nonlinear δ used to flag virialization, collapse time predict first shell-crossing when $\delta^{(1)}=1.69$

To … el

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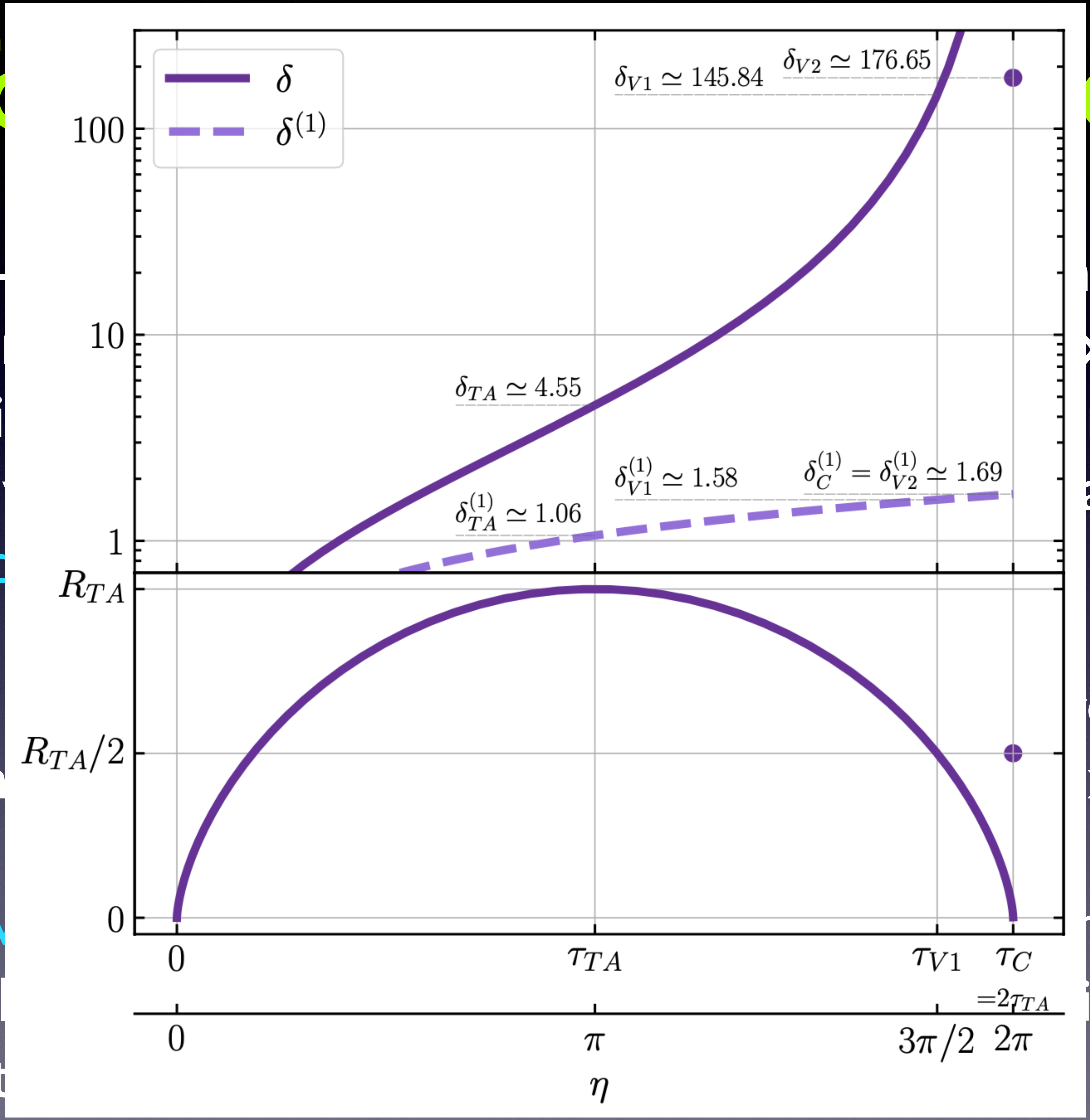
- linear v … nal values of nonl … ime predict …



validity of Top-hat collapse

		Top-Hat, $\Lambda = 0$	Here, $\Lambda = 0$	Here, $\Lambda \neq 0$
Initially	$z_{\rm IN}$		205.4	302.5
	$a/a_{\rm IN}$	35.4137	35.24467 $\pm$ 7e-5	35.195 $\pm$ 3e-3
	z		4.85620 $\pm$ 1e-5	7.6234 $\pm$ 7e-4
Turn Around (TA)	$\gamma_{\rm OD}^{1/6}/\gamma_{\rm IN,OD}^{1/6}$		20.10169 $\pm$ 3e-5	20.0600 $\pm$ 1e-4
$K_{\rm OD} = 0$	$\langle\gamma^{1/6}\rangle_{\mathcal{D}}/\langle\gamma^{1/6}\rangle_{\mathcal{D},\rm IN}$		35.2064 $\pm$ 1e-4	35.154 $\pm$ 3e-3
	$\delta_{\rm OD}^{(1)}$	1.06241	1.05734 $\pm$ 2e-6	1.05584 $\pm$ 8e-5
	$\delta_{\rm OD}$	4.55165	4.55164 $\pm$ 1e-5	4.5626 $\pm$ 5e-4
	$a/a_{\rm IN}$	56.22	55.9 $\pm$ 1e-1	55.87 $\pm$ 8e-2
	z		2.692 $\pm$ 7e-3	4.432 $\pm$ 8e-3
Collapse	$\gamma_{\rm OD}^{1/6}/\gamma_{\rm IN,OD}^{1/6}$		0.4 $\pm$ 6e-1	0.8 $\pm$ 2e-1
/Crash	$\langle\gamma^{1/6}\rangle_{\mathcal{D}}/\langle\gamma^{1/6}\rangle_{\mathcal{D},\rm IN}$		55.8 $\pm$ 1e-1	55.77 $\pm$ 2e-2
	$\delta_{\rm OD}^{(1)}$	1.686	1.678 $\pm$ 3e-3	1.676 $\pm$ 2e-3
	$\delta_{\rm OD}$	$+\infty$	$2e+6 \pm 2e+6$	$4e+5 \pm 4e+5$
Virialization	$a/a_{\rm IN}$	52.64	52.5055 $\pm$ 9e-4	52.469 $\pm$ 2e-3
$R = R_{\rm TA}/2$	$\delta_{\rm OD}$	145.84	145.84	145.84
Virialization	$a/a_{\rm IN}$	56.22	52.83625 $\pm$ 7e-5	52.801 $\pm$ 2e-3
$R = R_{\rm TA}/2$ & $\tau = \tau_C$	$\delta_{\rm OD}$	176.65	176.65	176.65

Summary and Outlook

- Take Home message #1: squeeze more from ΛCDM by going beyond FLRW + perturbations, nonlinearity and full GR:
    - Numerical Relativity simulations can lead to new predictions in ΛCDM: field is in its infancy, more is needed, e.g. ray tracing
    - main challenges are computational, as well of interpretation: Newtonian vs GR, observable effects. etc..
- Take Home message #2: in cosmology we rarely make specific assumptions on CDM:
    - perhaps more specific models should be tested vs observations
    - PBH is an interesting option
- Take Home message #3: exploring DE beyond Λ, both in the late and early Universe, is important:
    - late Universe DE, possible interacting with CDM, may explain some tensions
    - in the early universe, it may solve the singularity problem of GR

Conclusions

- Studying GR effects in structure formation is important: now new code developed by Monaco & Co. in Trieste merges Gadget with gevolution (code by Adamek et al) to study effects on large scale
- Full GR codes like Einstein Toolkit (fluid) GRAMSES (N-body) now available to study effects on smaller scales where nonlinear is stronger
- Our results show that gravito-magnetism is more important at higher redshifts and for larger masses, when and where there is more dynamics
- the B field is stronger around filaments and could be possibly be detected in lensing in future
- top-hat model excellent to predict turn-around and collapse of peaks, i.e. first shall-crossing, for quasi-spherical peaks
- work in progress with shear at the peak shows that its effect in the Raychaudhuri equations remains subdominant, hence this eq. reduces to the Fridmann equation and therefore the top-hat model remains an excellent approximation

DE with quadratic EoS:
1) + CDM and radiation
2) interacting DE & CDM

- Dynamical System analysis
- once we assume that Dark Energy exists,
- let's explore the possibility of avoiding singularities

Burkmar and Bruni, PRD 107, 083533 (2023)
2302.03710
work with PhD student Molly Burkmar
+ work in progress

From Λ to nonlinear Dark Energy

- Dark Energy: anything giving acceleration in the Raychaudhuri acceleration, now with pressure:

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 + A - 2(\sigma^2 - \omega^2) - 4\pi G(\rho + 3P)$$

$$\Theta \to 3H = 3\frac{\dot{a}}{a}, \;\; A = \sigma = \omega = 0 \text{ for homogeneous and isotropic case}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

  - we call DE a component with $P < -\rho/3$
  - this violates energy conditions assumed in the '60 by singularity theorems (Penrose & Hawking)
- general idea: explore DE effects in the early Universe, to see if we can avoid the Big-Bang singularity

quadratic EoS + CDM and radiation

- we assume that two effective cosmological constants exist:

$$\dot{\rho_x} = -3H(\rho_x - \rho_\Lambda)\left(1 - \frac{\rho_x}{\rho_*}\right).$$

- use dimensionless variables

$$x = \frac{\rho_x}{\rho_*} \quad y = \frac{H}{\sqrt{\rho_*}} \quad z = \frac{\rho_m}{\rho_*} \quad r = \frac{\rho_r}{\rho_*} \quad \mathcal{R} = \frac{\rho_\Lambda}{\rho_*} \quad \eta = \sqrt{\rho_*}t.$$

- several different possibilities in phase space, with R<x<1, depending on parameters

- positive curvature crucial to have bounces and cycles

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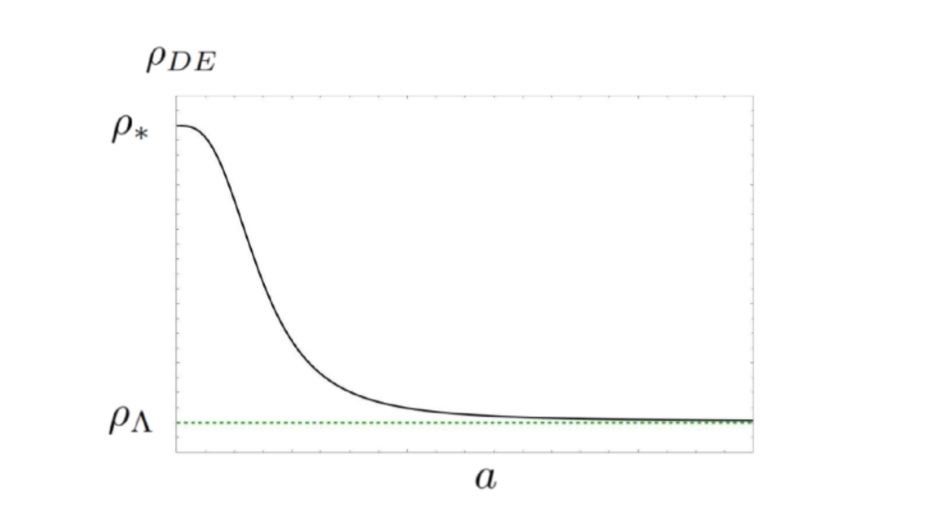

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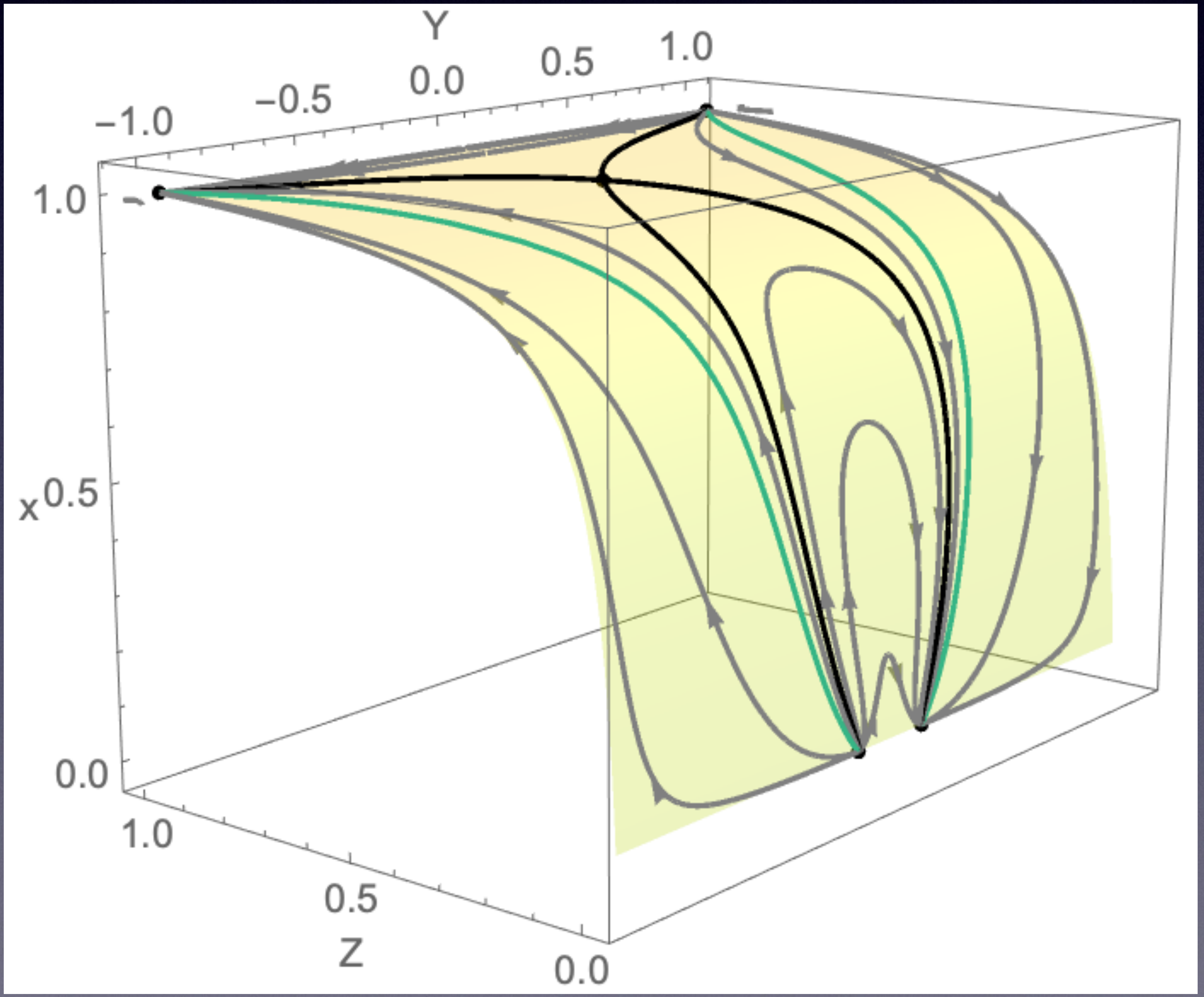
- several different possibilities in phase space, with R<x<1, depending on parameters
- positive curvature crucial to have bounces and cycles

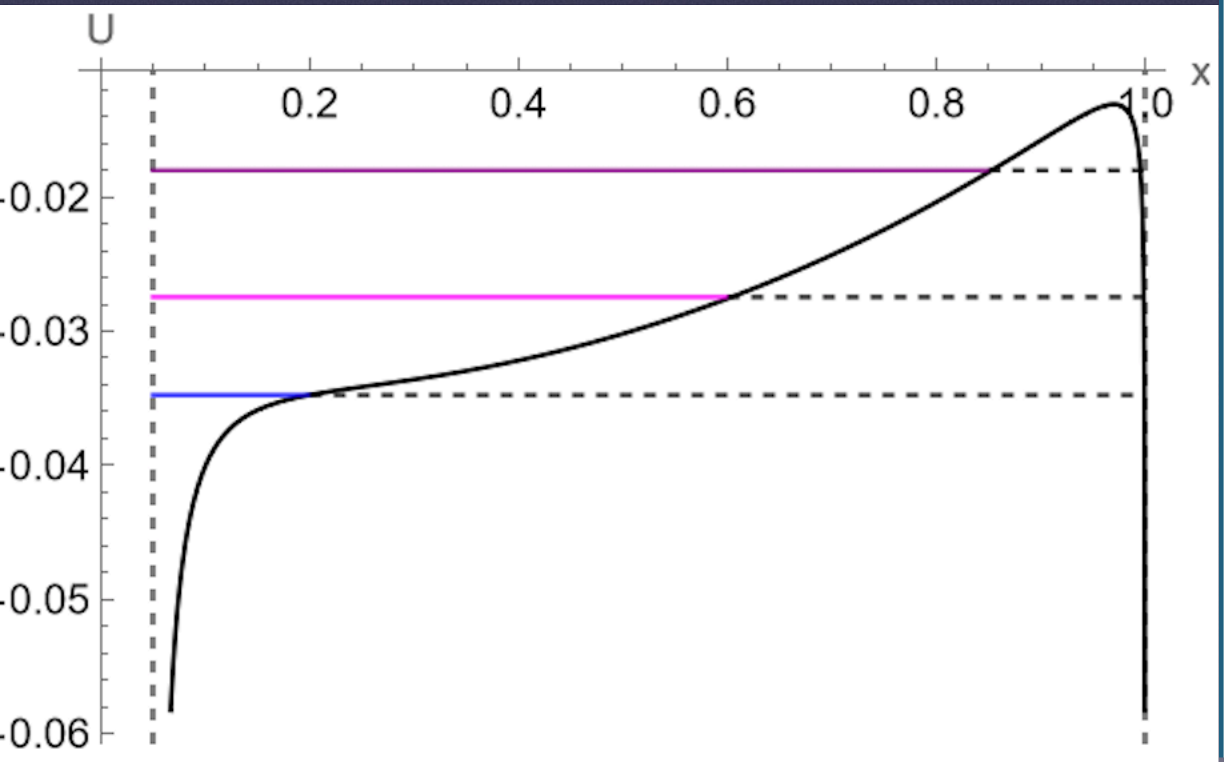
quadratic EoS + CDM and radiation

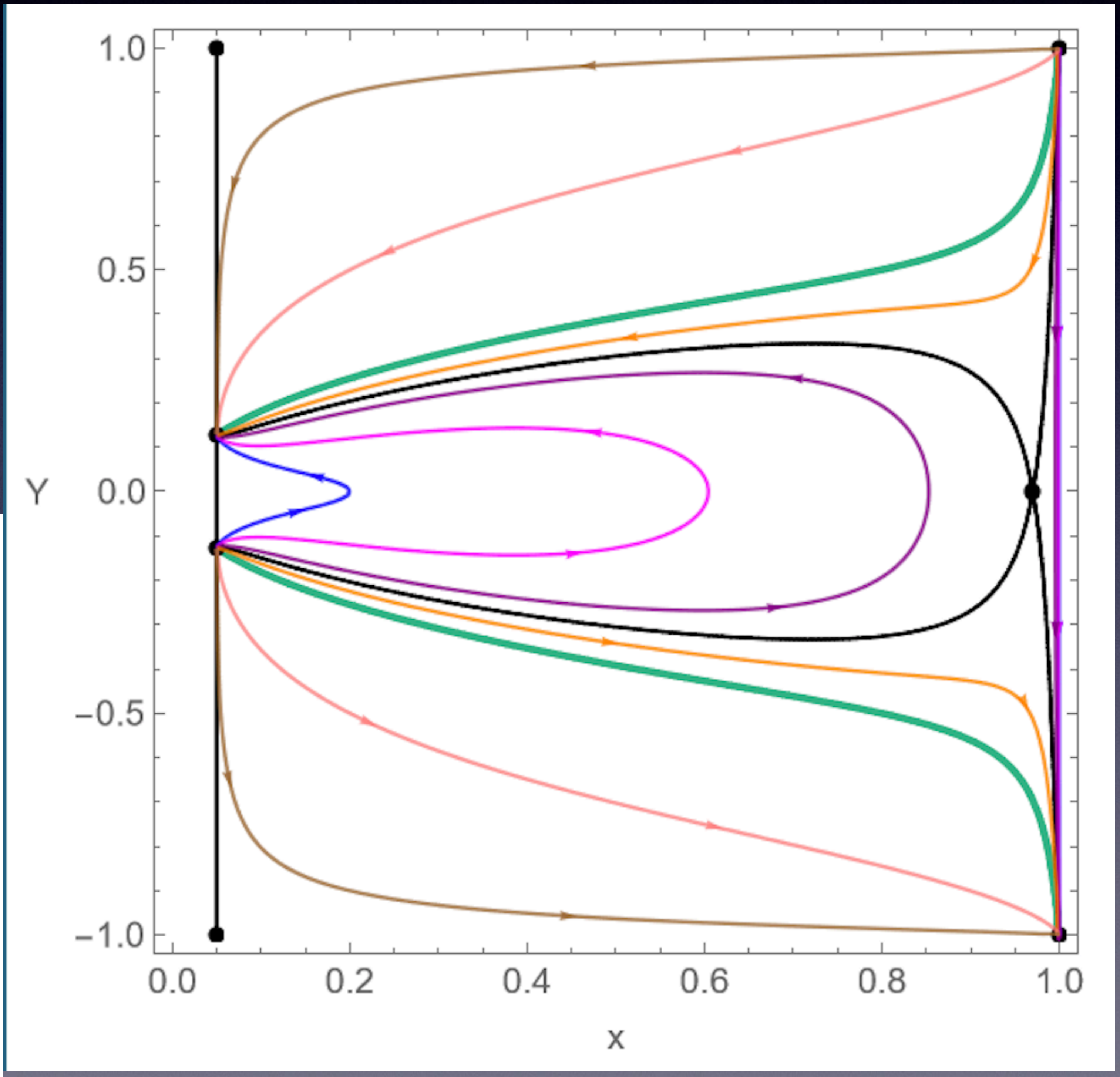
- with DE x, Hubble expansion rate y and a matter component z dynamics is in 3D, but first integral reduces motion to be 2D: projection on x-y plane
- phase space plot in terms of x and compactified variables Y and Z
- case with a single Einstein point: no cycle trajectories, only bounces



quadratic EoS + CDM and radiation

- finite potential barrier implies that there are closed-models trajectories with enough "energy" to have a past singularity





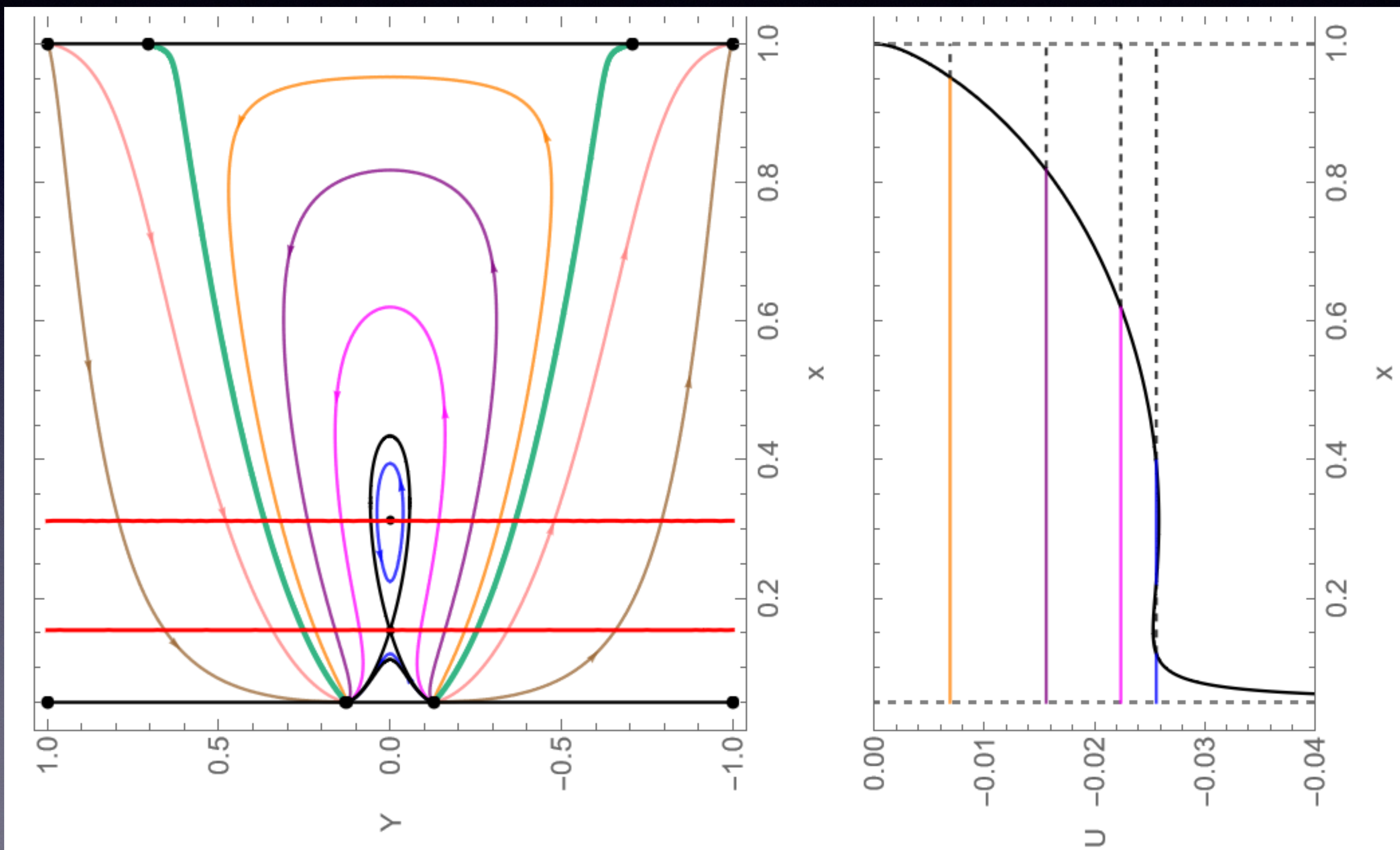
quadratic EoS + CDM and radiation

- for closed (positive curvature) models with quadratic EoS DE + unbounded CDM and radiation bounces are not generic

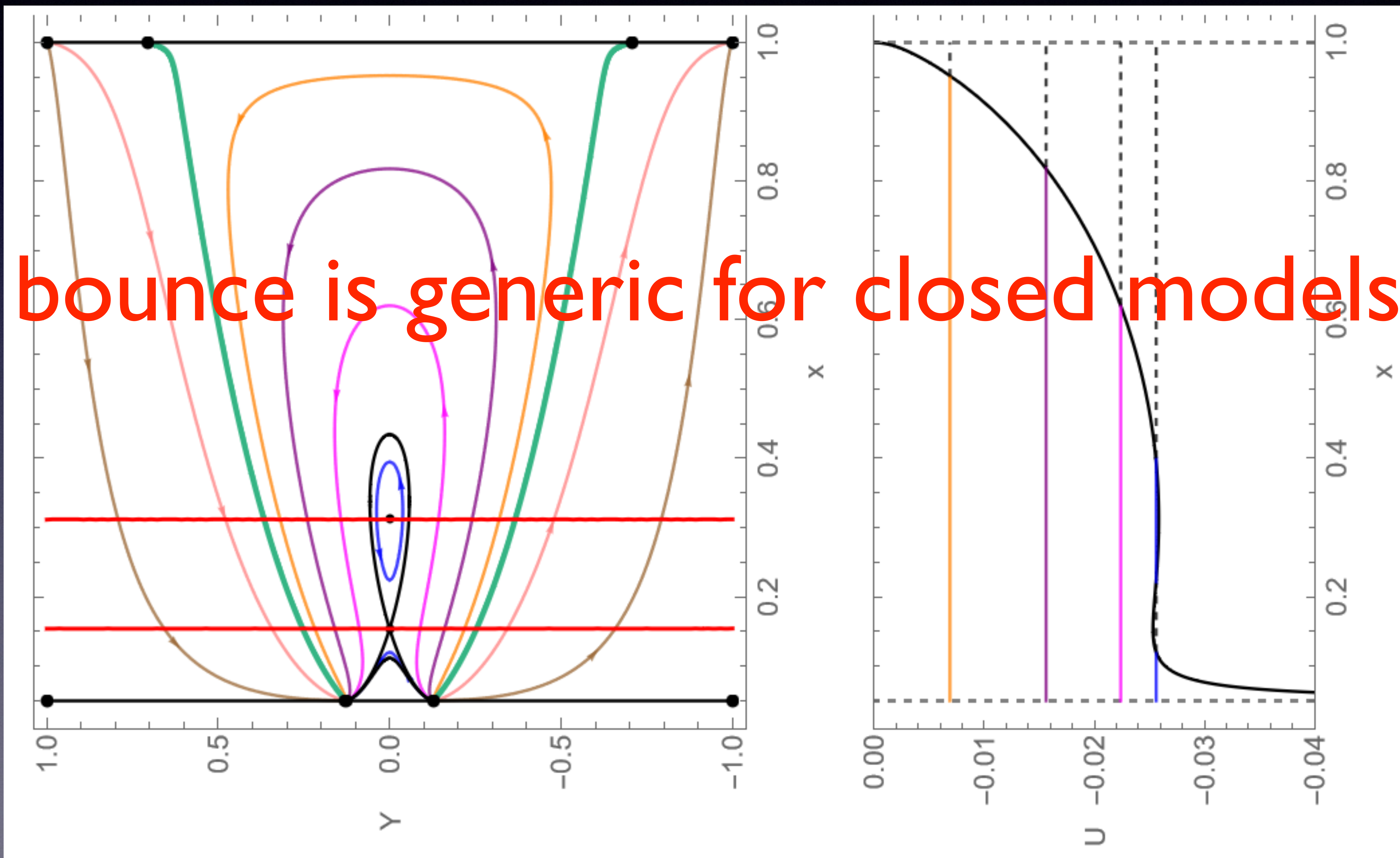
- in a realistic scenario for the early Universe radiation and CDM should arise at a later stage after the bounce, e.g. as in a standard post-inflation reheating phase

- in this light, we can then introduce an upper bound for CDM and radiation in our qualitative analysis

quadratic EoS DE + radiation and CDM with an upper bound



quadratic EoS DE + radiation and CDM with an upper bound



bounce is generic for closed models

coupled DE & CDM

- quadratic EoS DE now nonlinearly coupled to CDM
- work in progress with PhD student Molly Burkmar
- Dynamical System approach and dimensionless variables and parameters introduced as before

coupled DE & CDM

- Dynamical System approach and dimensionless variables and parameters introduced as before
- explicitly use w for the linear part of DE

$$\dot{\rho}_m = -3H\rho_m + \frac{qH\rho_x\rho_m}{\rho_i}$$

$$\dot{\rho_x} = -3H(\rho_x - \rho_\Lambda)\left(1 + w_x + \epsilon\frac{\rho_x}{\rho_*}\right) - \frac{qH\rho_x\rho_m}{\rho_i}$$

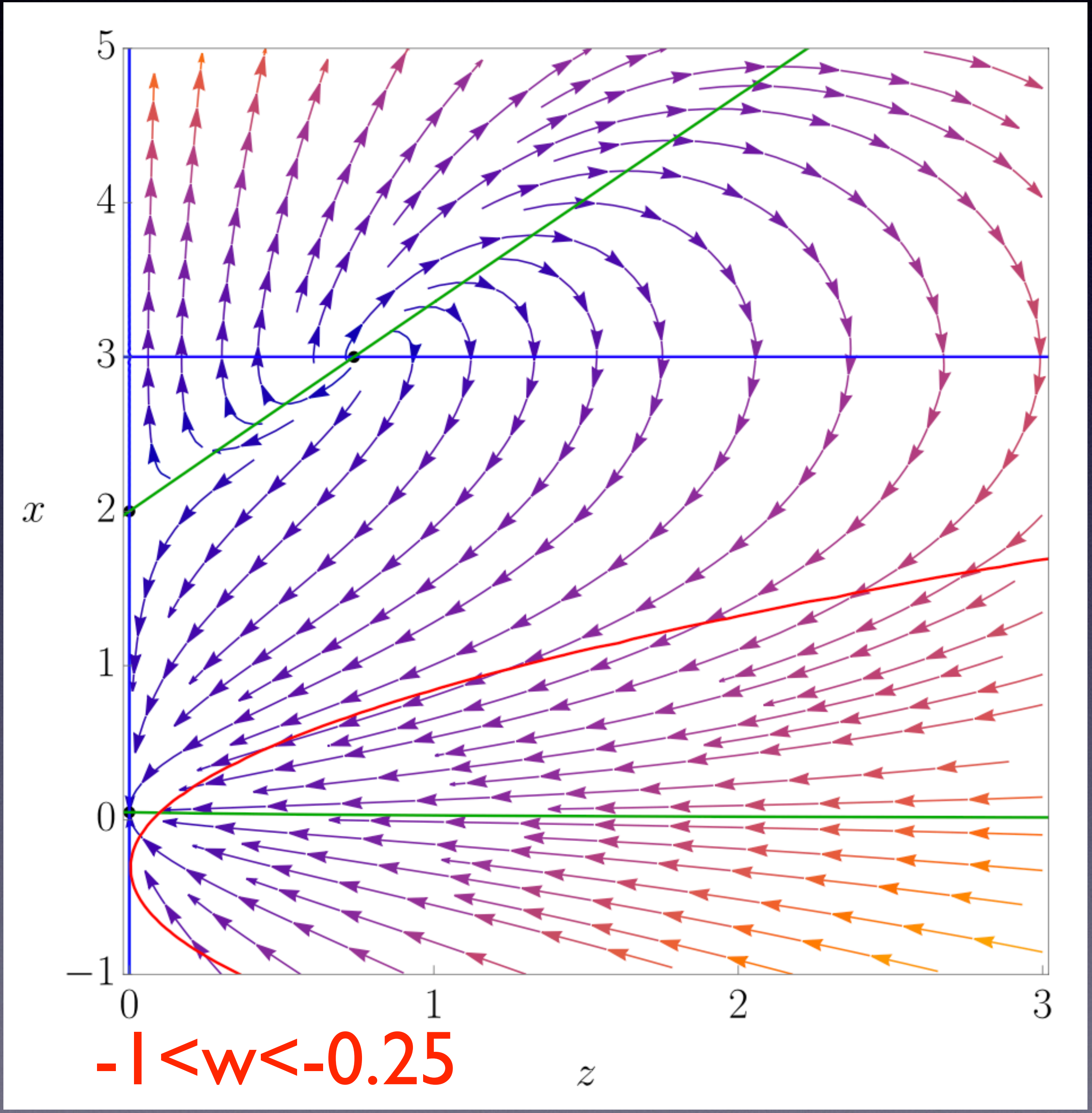
$$\dot{H} = -H^2 - \frac{1}{6}(\rho_m + \rho_x(1 + 3w_x - 3\epsilon\mathcal{R}) - 3\rho_\Lambda(1 + w_x) + 3\epsilon\frac{\rho_x^2}{\rho_*}),$$

$$H^2 = \frac{\rho_m}{3} + \frac{\rho_x}{3} - \frac{k}{a^2}.$$

we fix ε=-1 and use dimensionless variables as before

DE+CDM dynamics

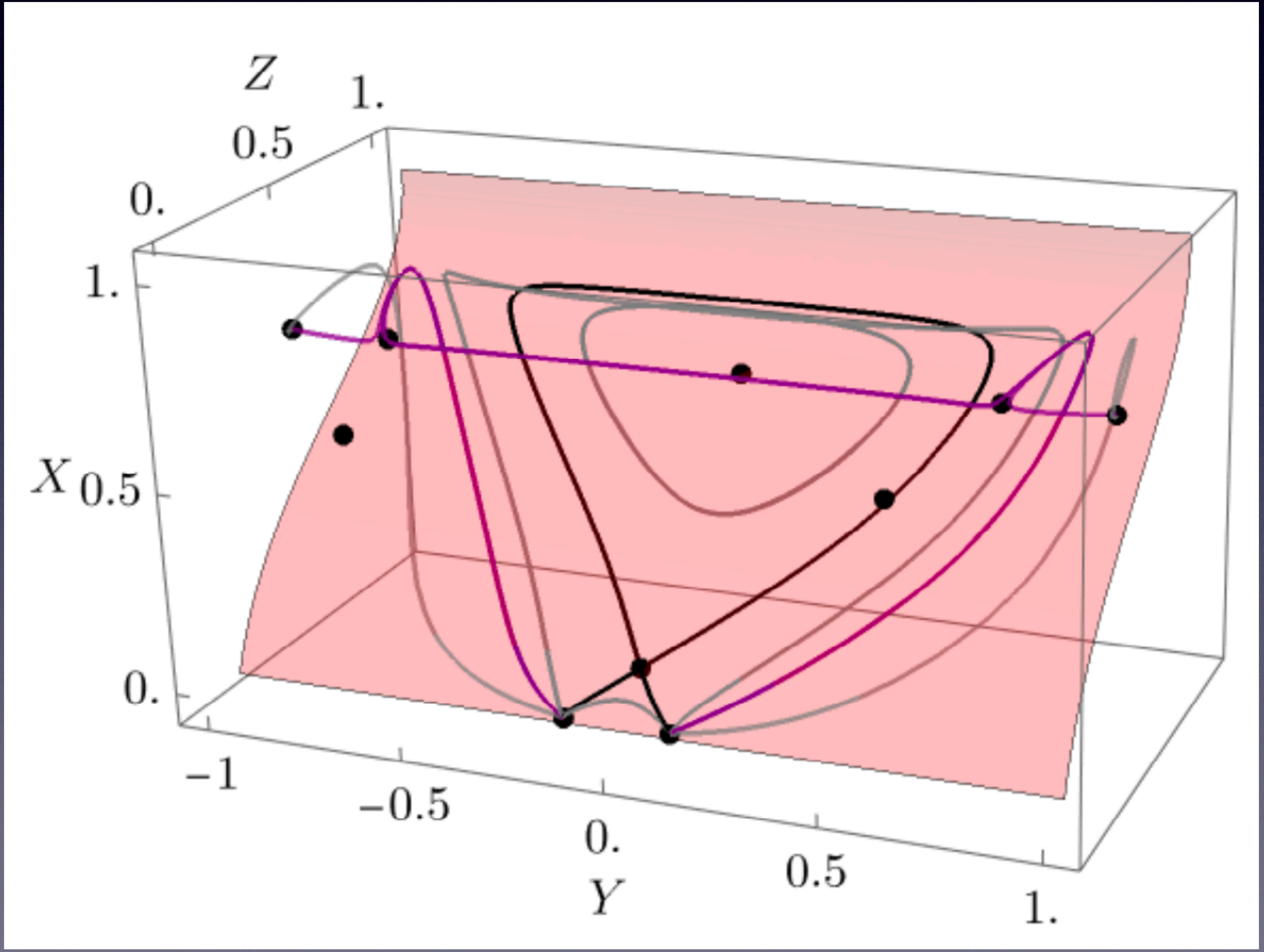
- assuming expansion H>0, eliminate H from continuity equations and use e-folding number N=ln(a) as time
- past attractor for expanding models (or transition phase for K>0) is a repelling spiral S, a high energy unstable "cosmological constant"
- all trajectories for expansion emerge from S
- all trajectories but some special cases end in a low energy "cosmological constant" attractor
- the trick is to have trajectories that have a decelerated phase, below the red line



DE+CDM+H 3-D dynamics

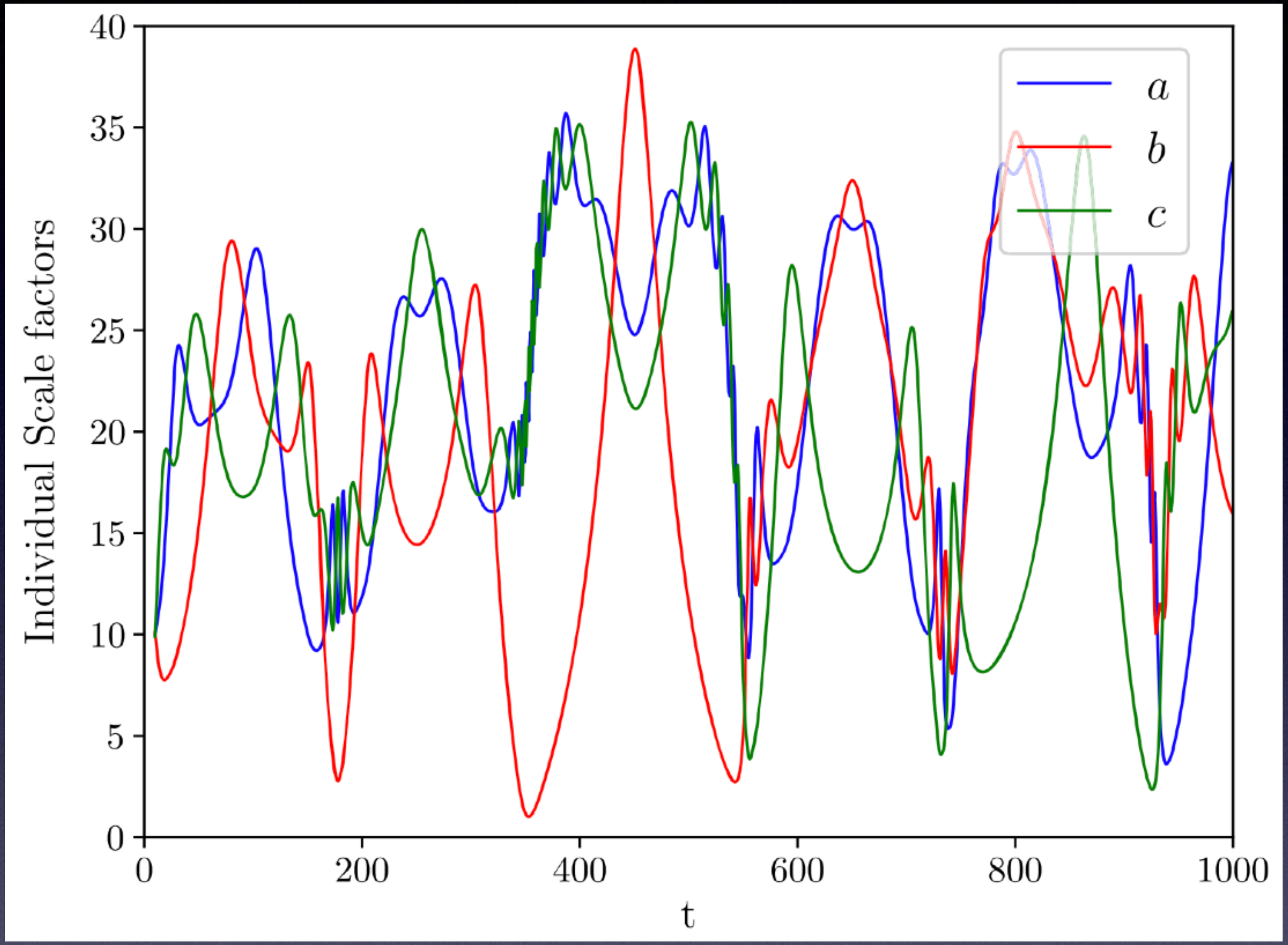
purple trajectories represents curved models within Planck values for ΩK, 4 de Sitter points and 2 Einstein points

- flat and open models EMERGE from the expanding de Sitter point, asymptotically in the past
- closed models EMERGE from the contracting de Sitter point, asymptotically in the past, go through a transient de Sitter phase with a bounce
- all go through deceleration and then a final accelerated phase toward the de Sitter future attractor



Biachi IX with quadratic EoS

- Bianchi IX: most general homogenous anisotropic geometry
- general for super-horizon scales
- should emerge as average geometry
- focus on cycle case
- cycles survive by they are chaotic, i.e. in general always anisotropic
- isotropy emerges as an attractor if we introduce a friction anisotropic pressure term



C. Ganguly and M. Bruni, Physical Review Letters 123 201301 (2019). [arXiv:1902.06356]

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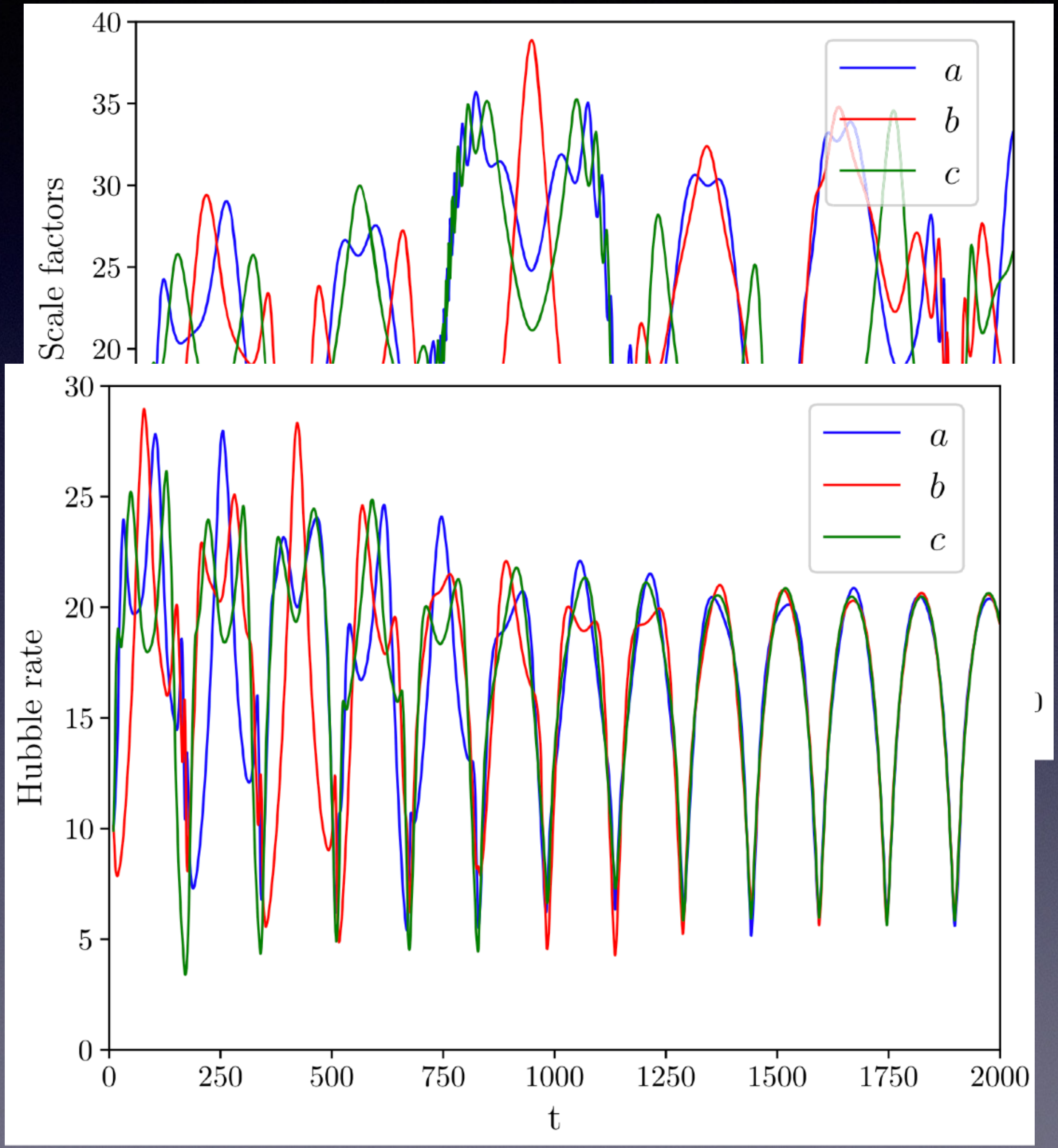


C. Ganguly and M. Bruni, Physical Review Letters 123 201301 (2019). [arXiv:1902.06356]

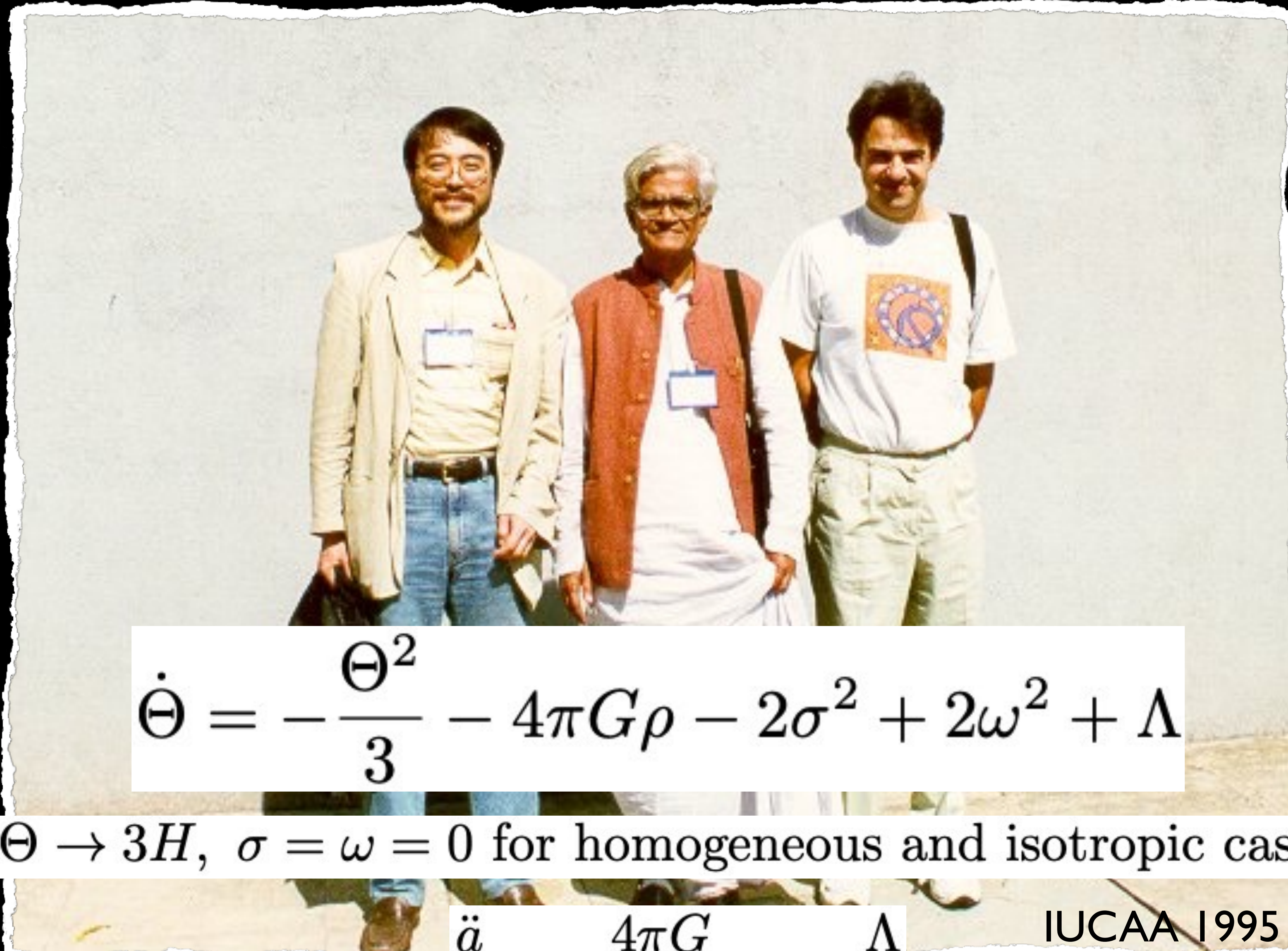
why the top-hat model works so well?

- It is all down to the Raychaudhuri equation!

$$\dot{\Theta} = -\frac{\Theta^2}{3} - 4\pi G\rho_M - 2\sigma^2 + 2\omega^2 + \Lambda$$

$\Theta \to 3H = 3\frac{\dot{a}}{a}$, $\sigma = \omega = 0$ for homogeneous and isotropic case

reduces to Friedmann equation $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda}{3}$$



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$\Theta \rightarrow 3H, \ \sigma = \omega = 0$ for homogeneous and isotropic case

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_M + \frac{\Lambda}{3}$$

IUCAA 1995

contributions to Raychaudhuri

- in our quasi-spherical collapse the shear σ is negligible at the peak initially
- at Turn-Around (Top panel) Θ=0 and the shear remains negligible at the peak, and subdominant in general
- at collapse (Bottom panel) the shear remains negligible at the peak
- 3-Ricci curvature (not shown) becomes important at the peak
- Raychaudhuri is very well approximated by the Friedmann equation for closed models
- Top-Hat works very well to predict first shell-crossing