XXI LNF SPRING SCHOOL “BRUNO TOUSCHEK” in Nuclear, Subnuclear and Astroparticle Physics - 8th Young Researchers' Workshop

Microscopic parametrization of the near threshold oscillations of the nucleon time-like effective electromagnetic form factors

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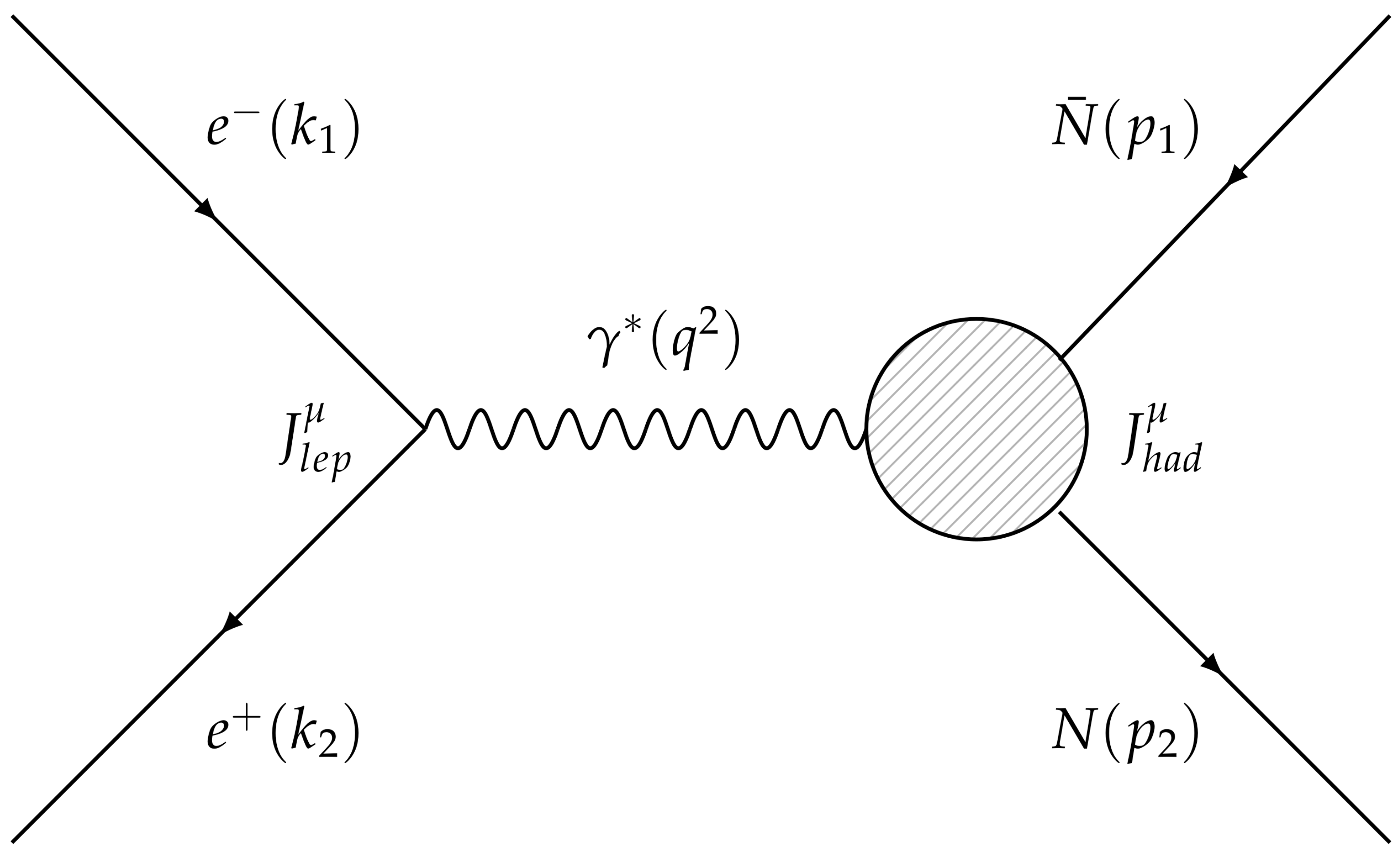
BESIII

NUCLEON FORM FACTORS



Larin, P.; Zhou, X.; Hu, J.; Maas, F.; Baldini, R.F.; Hu, H.; Huang, G. Electromagnetic Structure of the Neutron from Annihilation Reactions. Symmetry 2022, 14, 298. https://doi.org/10.3390/sym14020298Larin, P.; Zhou, X.; Hu, J.; Maas, F.; Baldini, R.F.; Hu, H.; Huang, G. Electromagnetic Structure of the Neutron from Annihilation Reactions. Symmetry 2022, 14, 298. https://doi.org/10.3390/sym14020298

$$J^\mu_{\text{had}} = \bar{u}\left(p_1, \lambda_1'\right)\left(-ie\Gamma^\mu\right) v\left(p_2, \lambda_2'\right)$$

$$\mathscr{M} = J^\nu_{\text{had}}\left(-i\frac{\eta_{\mu\nu}}{q^2}\right) J^\mu_{\text{lep}}$$

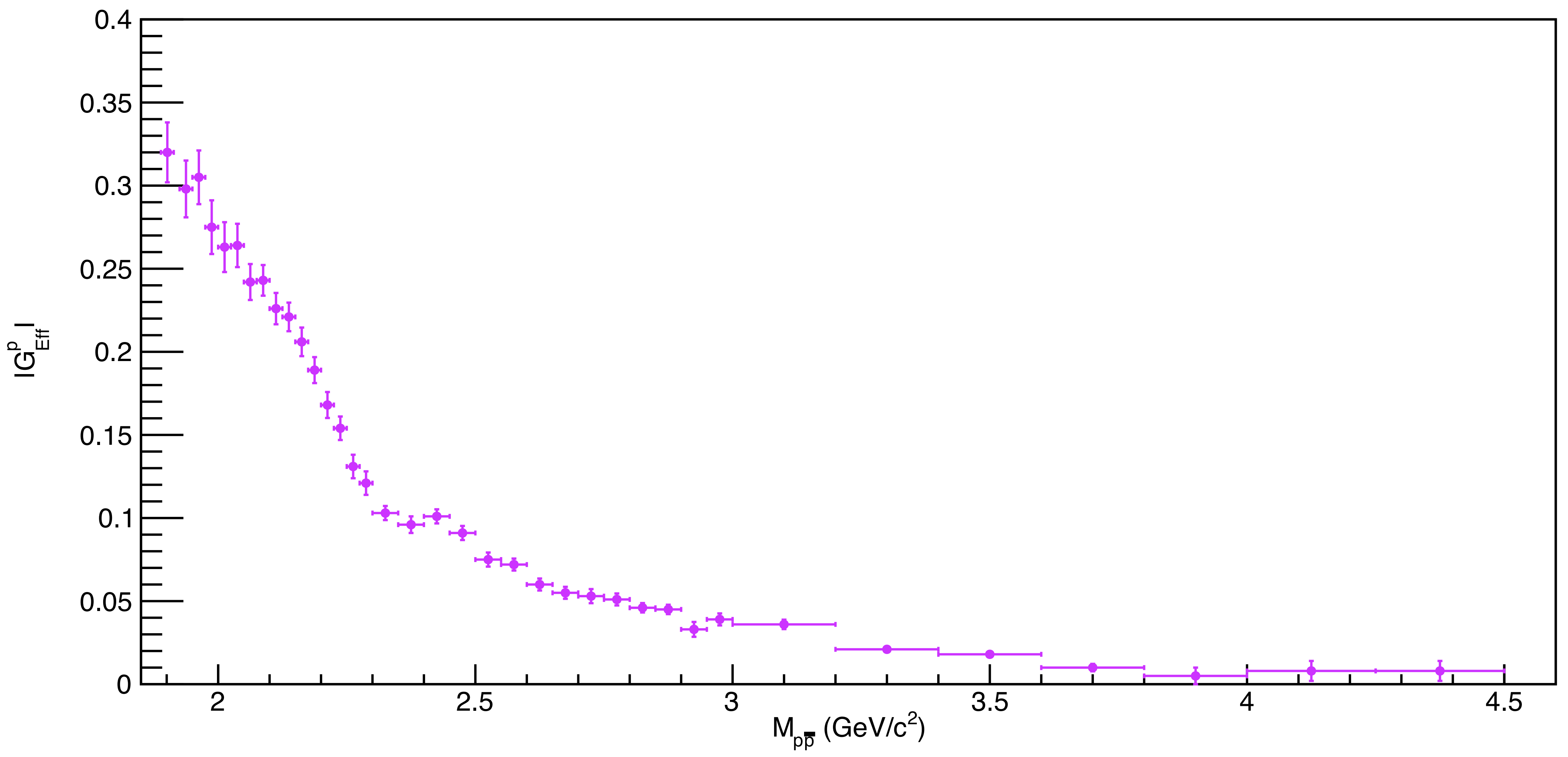
$$\Gamma^\mu = F_1\left(Q^2\right)\gamma^\mu + F_2\left(Q^2\right) i\frac{\sigma^{\mu\nu}}{2M_N}q_\nu$$

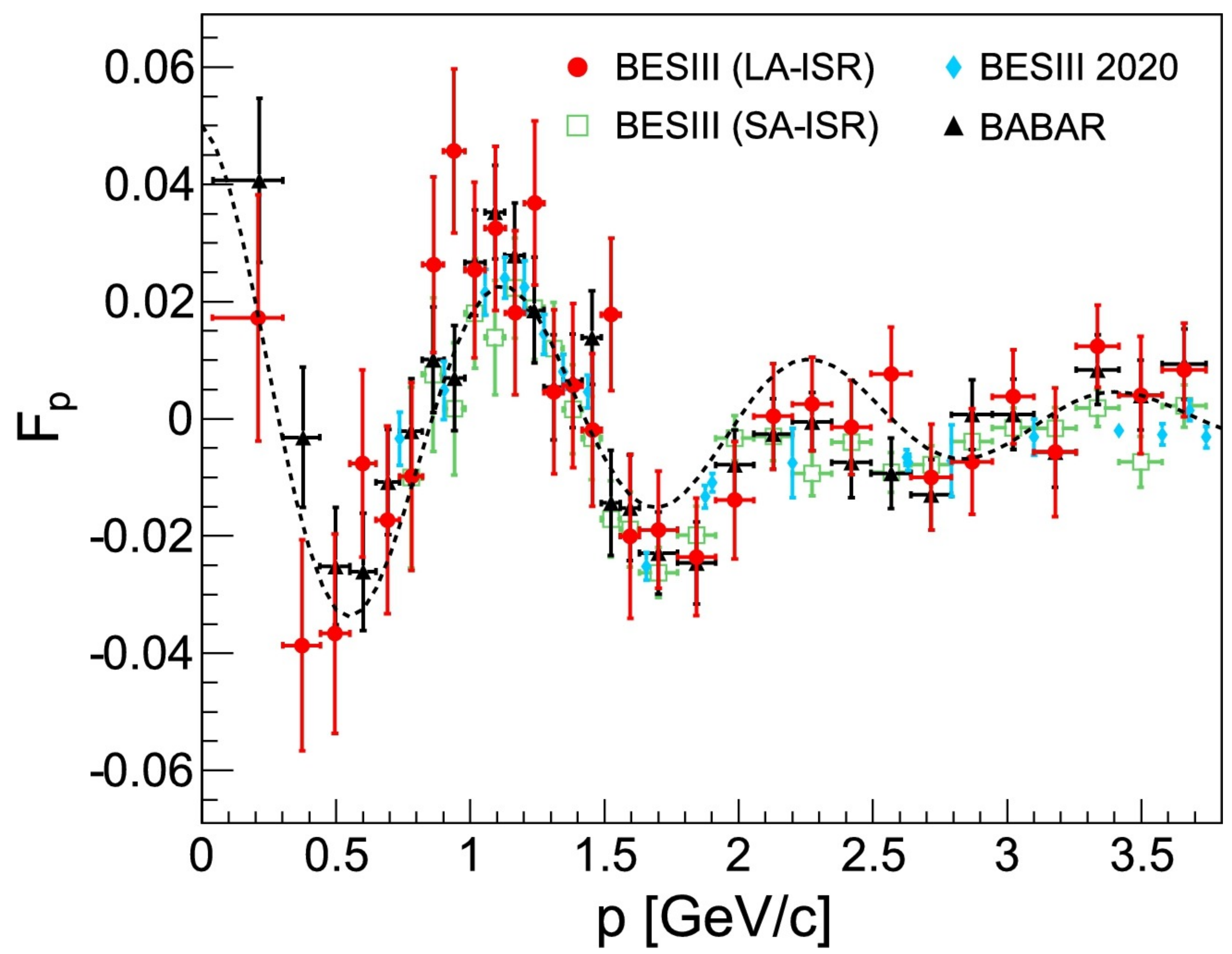
Dirac Pauli

- Baryon form factors are functions of the four momentum squared
- In the Breit frame, $G_{E(M)}$ represents the electric charge (magnetic momentum) density's Fourier Transforms

$$\begin{cases} G_E = F_1 - \tau F_2 \\ G_M = F_1 + F_2 \end{cases}, \quad \tau = \frac{Q^2}{4M_N^2} \qquad \begin{cases} G_E(0) = Q_N \\ G_M(0) = \mu_N \end{cases}$$

NUCLEON FORM FACTORS OSCILLATIONS





M. Ablikim et al. "Measurement of proton electromagnetic form factors in the time- like region using initial state radiation at BESIII". In: Physics Letters B 817 (2021), p. 136328. issn: 0370-2693. doi: https://doi.org/10.1016/j.physletb. 2021.136328. url: https: www.sciencedirect.com/science/article/pii/ S0370269321002689.

$$\left|G^{p,n}_{\text{Eff}}\right| = \sqrt{\frac{2\tau\left|G^{p,n}_{M}\right|^2 + \left|G^{p,n}_{E}\right|^2}{2\tau + 1}}$$

- High precision data are required
- The subtraction of the time-like dipole behaviour shows the oscillations

BARYONIC STATES IN LIGHT-FRONT QUANTIZATION

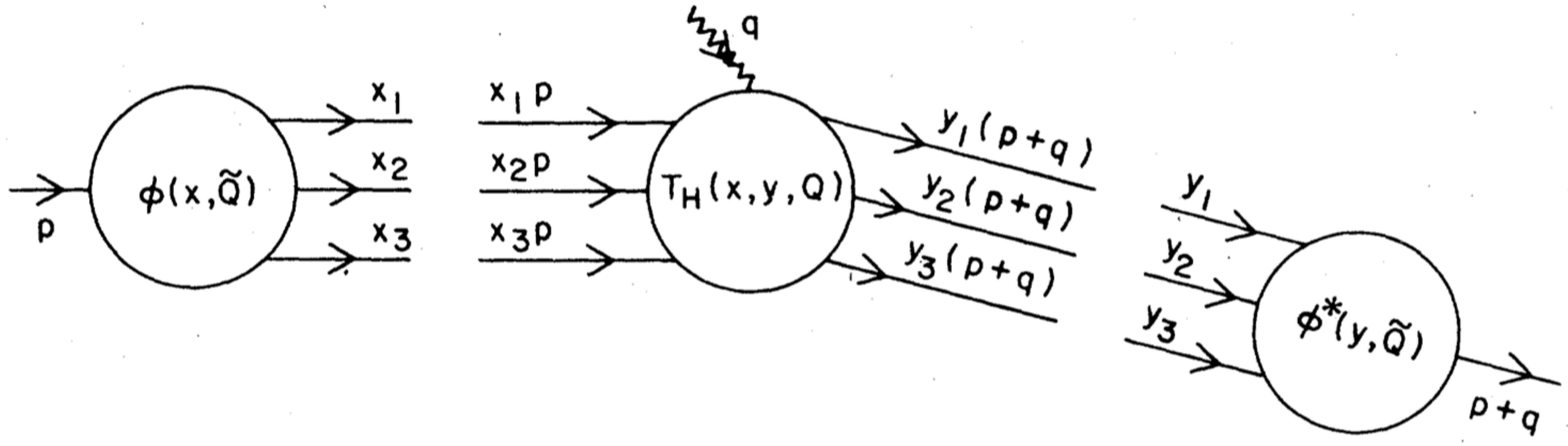
$$\left|\psi\right\rangle = \sum_n \int [d\mu_n] \left\langle \mu_n \mid \psi;\, M, P^+, \vec{P}_\perp, S^2, S_z;\, h \right\rangle \left|\mu_n\right\rangle$$

$$\left|\text{baryon}\right\rangle = \left|0\right\rangle + \left|qqq\right\rangle + \left|qqqg\right\rangle + \cdots$$

Leading Order

$$[d\mu_n] = [dx_n]\,[d^2k_{n\perp}] = \delta\left(1 - \sum_{j=1}^{N_n} x_j\right)\delta^{(2)}\left(\sum_{j=1}^{N_n} \vec{k}_{j\perp}\right)\prod_{i=1}^{N_n} dx_i d^2k_{i\perp}$$

$$\left\langle P' \left| J^+ \right| P \right\rangle = \int [dx] \int [dy]\, \varphi^*\left(y, Q^2\right) T_H\left(x, y, Q^2\right) \varphi\left(x, Q^2\right) \Rightarrow \text{Factorization}$$



G. Peter Lepage and Stanley J. Brodsky, Phys. Rev. D 22, 2157 – Published 1 November 1980

- At the leading order, the baryonic state is represented by three collinear quarks
- The hard scattering kernel T_H describes the subnuclear process

THREE QUARK OPERATORS

$$4\left\langle 0\right|\varepsilon^{ijk}u^i_\alpha\left(a_1 z\right)u^j_\beta\left(a_2 z\right)d^k_\gamma\left(a_3 z\right)\left|P\left(P,\lambda\right)\right\rangle$$

$$\stackrel{L.T.}{=} V_1\left(P^\nu\gamma_\nu C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + A_1\left(P^\nu\gamma_\nu\gamma_5 C\right)_{\alpha\beta}N_\gamma + T_1\left(P^\nu i\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma$$

- The three quark matrix element can be parametrized
- The parametrising functions are called Light Cone Distribution Amplitudes (LCDAs)
- The LCDAs are functions for the light like four-vector z

- It is best to operate in the four-momenta space, thus defining the LCDAs Fourier transforms as

$$F\left(a_i p\cdot z\right) = \int\left[dx\right]\tilde{F}\left(x_i\right)e^{-ipz\sum_i a_i x_i}$$

$$x_i = \frac{k_i^+}{p^+},\quad \sum_{i=1}^{3}x_i = 1$$

THREE QUARK OPERATORS

- By using u and d quarks symmetry, it remains only one independent twist-3 LCDA

Isospin 1/2 requirement $\Rightarrow 2T_1\left(x_1, x_2, x_3\right) = \left[V_1 - A_1\right]\left(x_1, x_3, x_2\right) + \left[V_1 - A_1\right]\left(x_2, x_3, x_1\right)$

$$u - d \text{ symmetry} \Rightarrow \begin{cases} V_1\left(x_1, x_2, x_3\right) = V_1\left(x_2, x_1, x_3\right) \\ A_1\left(x_1, x_2, x_3\right) = -A_1\left(x_2, x_1, x_3\right) \\ T_1\left(x_1, x_2, x_3\right) = T_1\left(x_2, x_1, x_3\right) \end{cases}$$

- Only one independent LCDA remaining: $\varphi_N\left(x_1, x_2, x_3\right) = V_1\left(x_1, x_2, x_3\right) - A_1\left(x_1, x_2, x_3\right)$

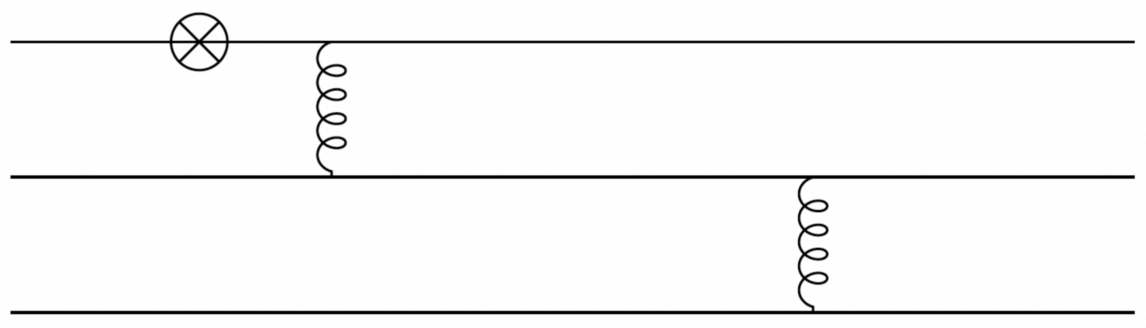
CHERNYAK-ZHITNITSKY FORMULA

Initial State

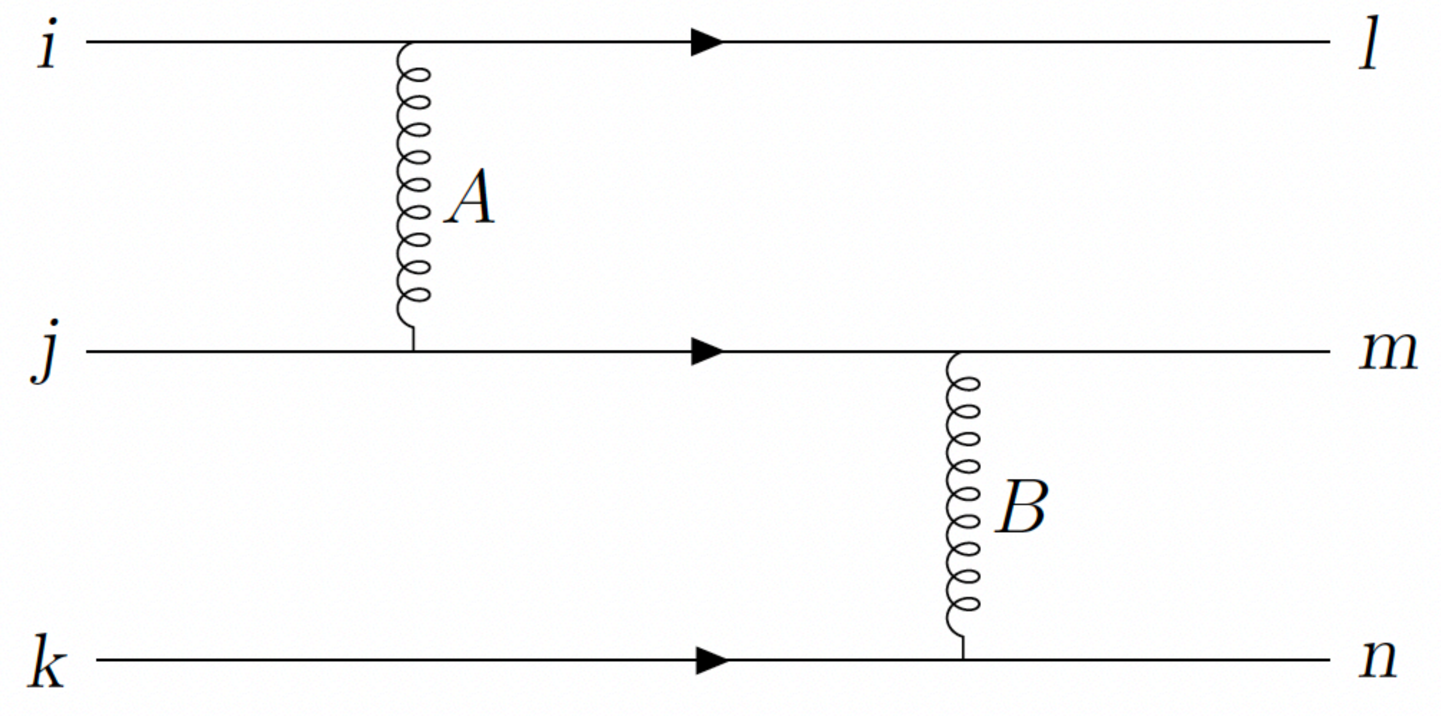
$$q^4 G_M\left(q^2\right) = \frac{\left(4\pi\bar{\alpha}_s\right)^2}{54}\left|f_N\right|^2 \int [dx] \int [dy] \left\{ 2\sum_{i=1}^{7} e_i T_i\left(x,y\right) + \sum_{i=8}^{14} e_i T_i\left(x,y\right) \right\}$$

Final State

Es: $i = 1 \Rightarrow T_1\left(x,y\right) = \dfrac{\varphi_N(x)\,\varphi_N\left(y\right) + 4T(x)\,T\left(y\right)}{\left(1-x_1\right)^2 x_3 \left(1-y_1\right)^2 y_3}$



- Form factor behaviour determined mostly by the running coupling constant $\bar{\alpha}_s\left(q^2\right)$
- Through symmetry relations, the minimal number of leading twist contributing diagrams is 14



Leading twist diagrams contributing to the hard scattering kernel T_H

CONFORMAL EXPANSION

$$\varphi_N\left(\mathbf{x}, Q^2\right) = \underbrace{120 x_1 x_2 x_3}_{\varphi_{as}(\mathbf{x})} \sum_n B_n P_n(\mathbf{x}) \left(\frac{\alpha_s\left(Q^2\right)}{\alpha_s\left(\mu^2\right)}\right)^{\gamma_n/\beta_0}$$

$$\langle 0 \left| \varepsilon^{ijk} u_i^{\uparrow}\left(a_1 z\right) C \gamma^{\mu} n_{\mu} u_j^{\downarrow}\left(a_2 z\right) \gamma^{\mu} n_{\mu} d_k^{\uparrow}\left(a_3 z\right) \right| P(P, \lambda) \rangle$$

$$= -\frac{1}{2} f_N (pn)\, N^{\uparrow}(p) \int [dx] \exp\left[-ipn\left(a_1 z x_1 + a_2 z x_2 + a_3 z x_3\right)\right] \varphi_N\left(x_1, x_2, x_3, \mu^2\right)$$

Nucleonic Function @ the origin

- The expression of the polynomials $P_n(\mathbf{x})$ and of the anomalous dimensions γ_n have to be determined, using the $\mathscr{L}_{\text{QCD}}$ conformal invariance

CONFORMAL BASIS: THE APPELL POLYNOMIALS

n	M	P_n	γ_n/β_0	N_n
0	0	1	2/27	120
1	1	$x_1 - x_3$	26/81	1260
2	1	$-2 + 3x_1 + 3x_3$	10/27	420
3	2	$2 - 7x_1 - 7x_3 + 8x_1^2 + 8x_3^2 + 4x_1x_3$	38/81	1756
4	2	$x_1 - x_3 - \frac{4}{3}x_1^2 + \frac{4}{3}x_3^2$	46/81	34020
5	2	$2 - 7x_1 - 7x_3 + \frac{14}{3}x_1^2 + \frac{14}{3}x_3^2 + 14x_1x_3$	16/27	1944

- The conformal basis is identified in the normalised Appell polynomials
- They are defined in the triangle $T = \left\{ \left(x_1, x_3\right) \in \left[0,1\right]^2 \subset \mathbb{R}^2 : x_1 + x_3 \leq 1 \right\}$

NEAR THRESHOLD PARAMETERS

$$B_0 = 1$$

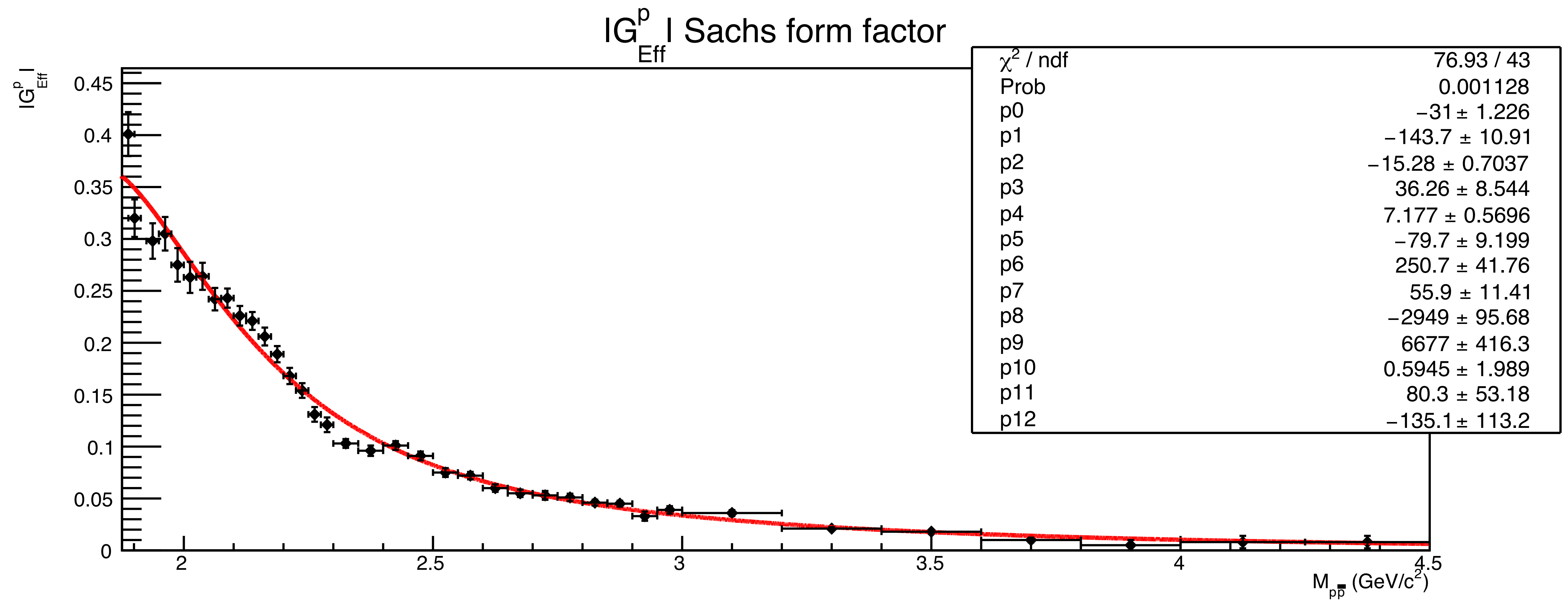
$$B_n\left(Q^2\right) = b_0^{(n)} + \frac{b_1^{(n)}}{Q^2}, \quad n = 1, 2$$

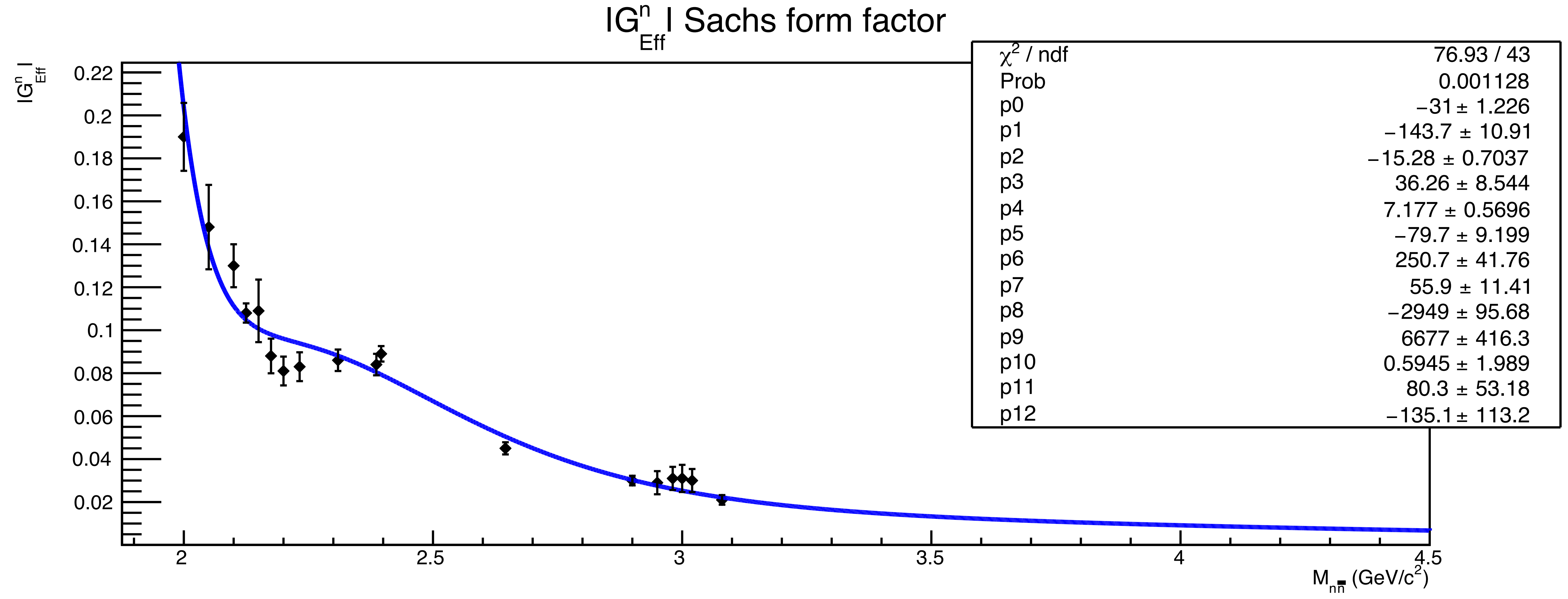
$$B_m\left(Q^2\right) = b_0^{(m)} + \frac{b_1^{(m)}}{Q^2} + \frac{b_2^{(m)}}{\left(Q^2\right)^2}, \quad m = 3, 4, 5$$

- The procedure encodes the effective form factor's oscillations to the three quark model
- The evolution equation's effect is negligible in comparison to the α_s and B_n coefficients evolution

- A dependance over the negative powers of q^2 is proposed
- B_0 is fixed since it is related to the normalization of $\varphi_N(\mathbf{x})$

MODEL'S RESULTS

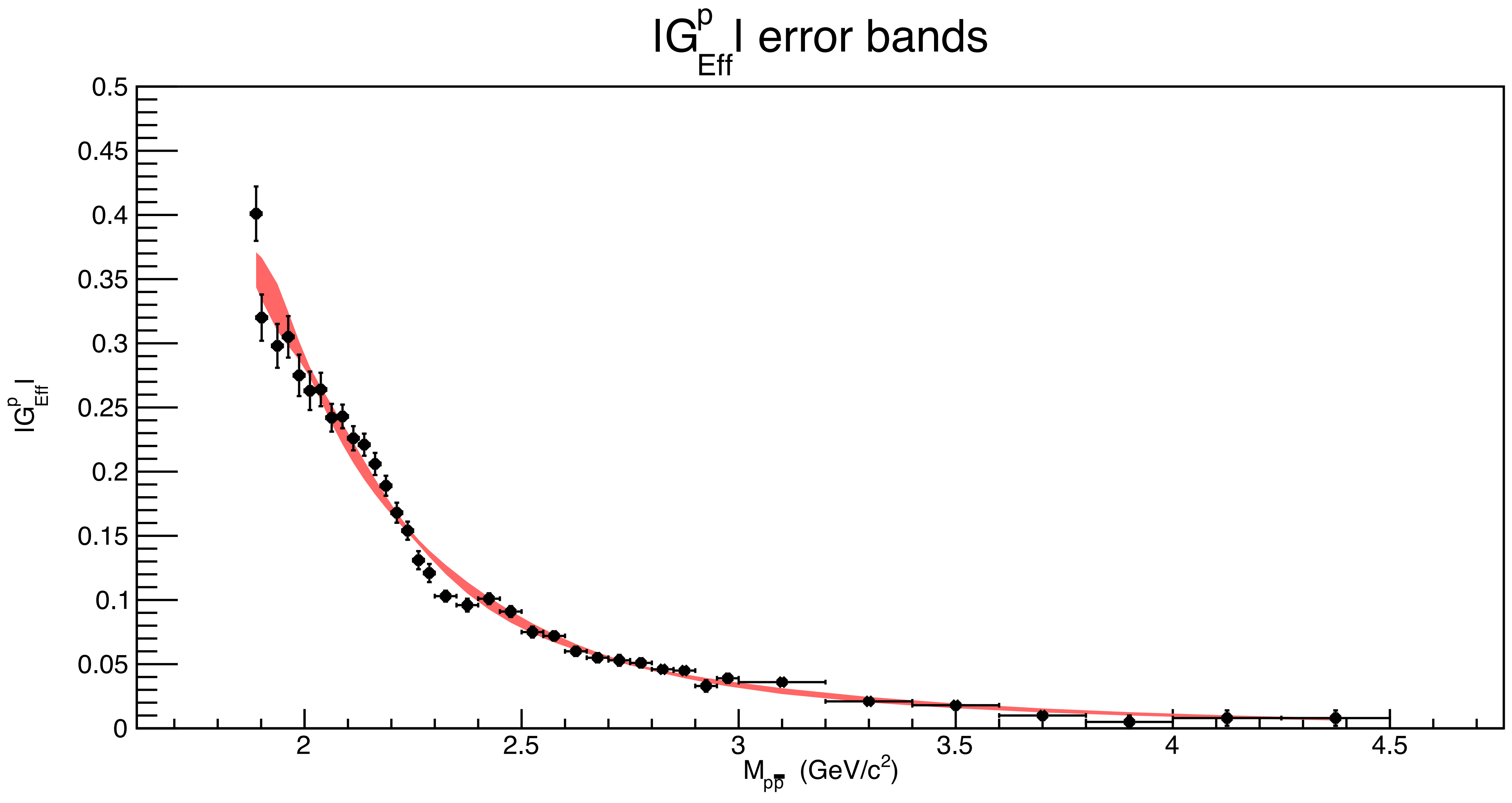




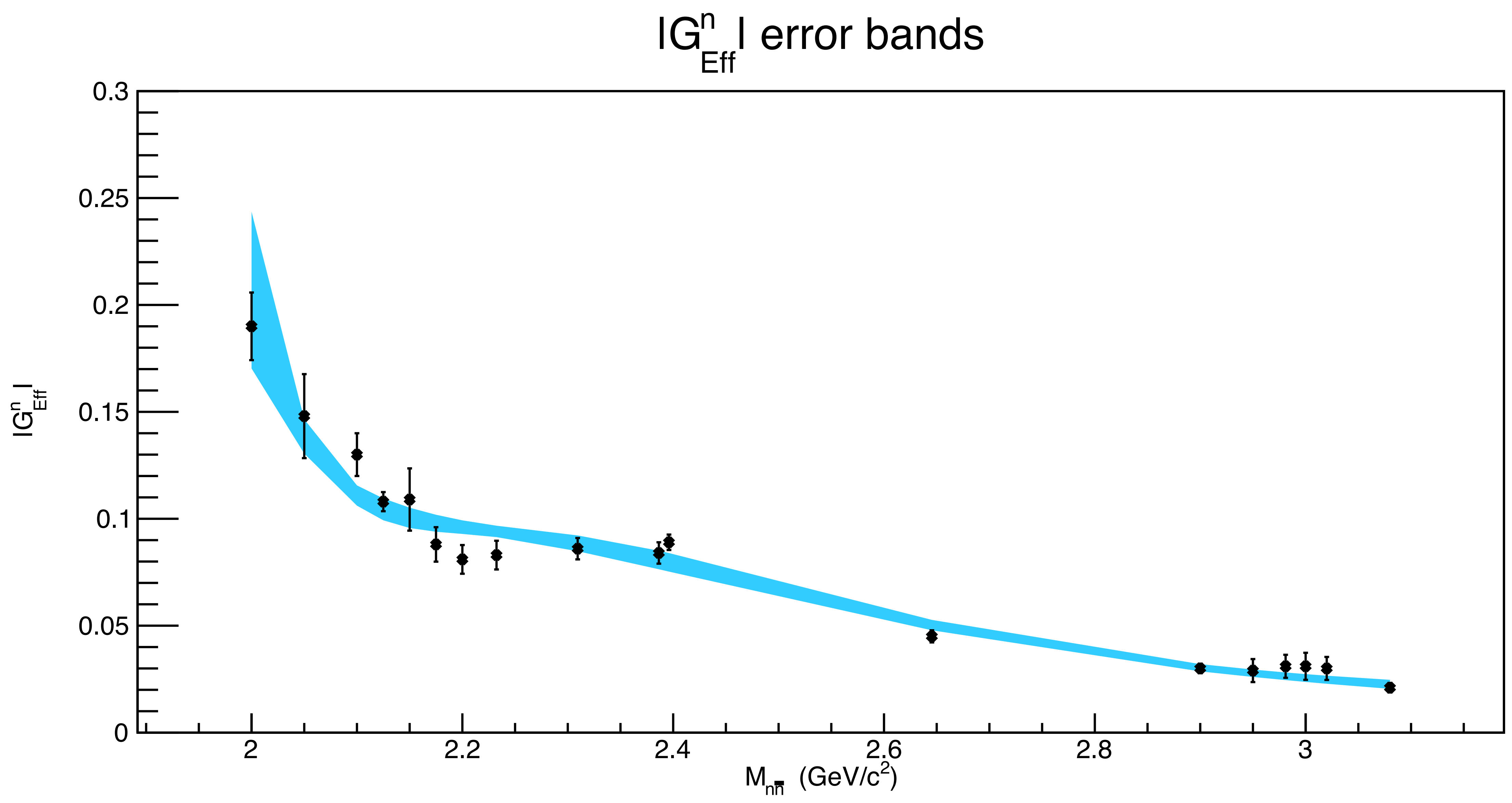
Simultaneous fit for proton and neutron using our model's parameters

- Isospin symmetry of the nucleons
- Simultaneous fit for proton and nucleon, sharing the same parameters
- Near threshold oscillations for the neutron

MODEL'S RESULTS - ERROR BANDS



Proton and neutron effective form factors error bands

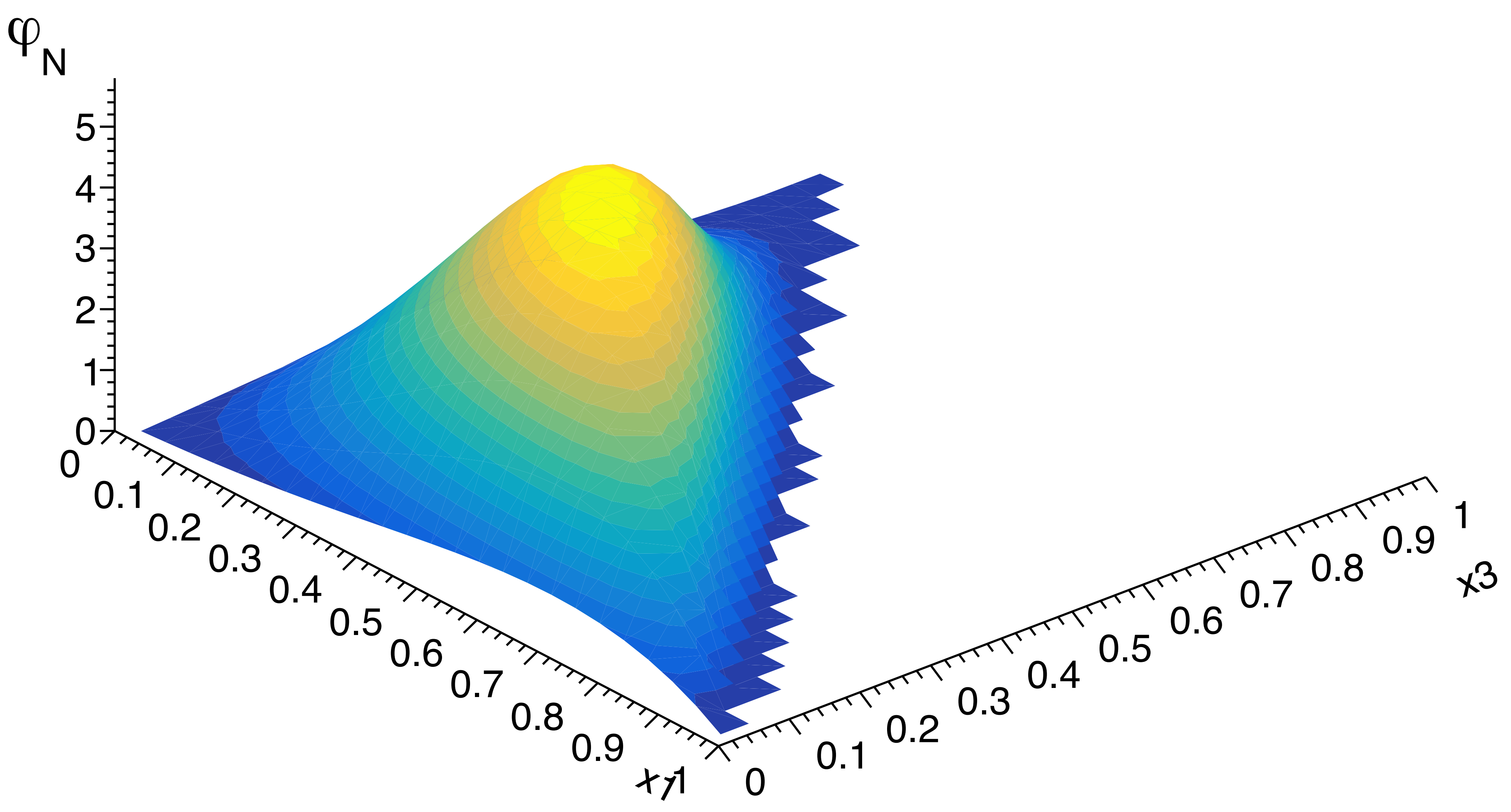


- Error bands containing both statistical and systematic errors

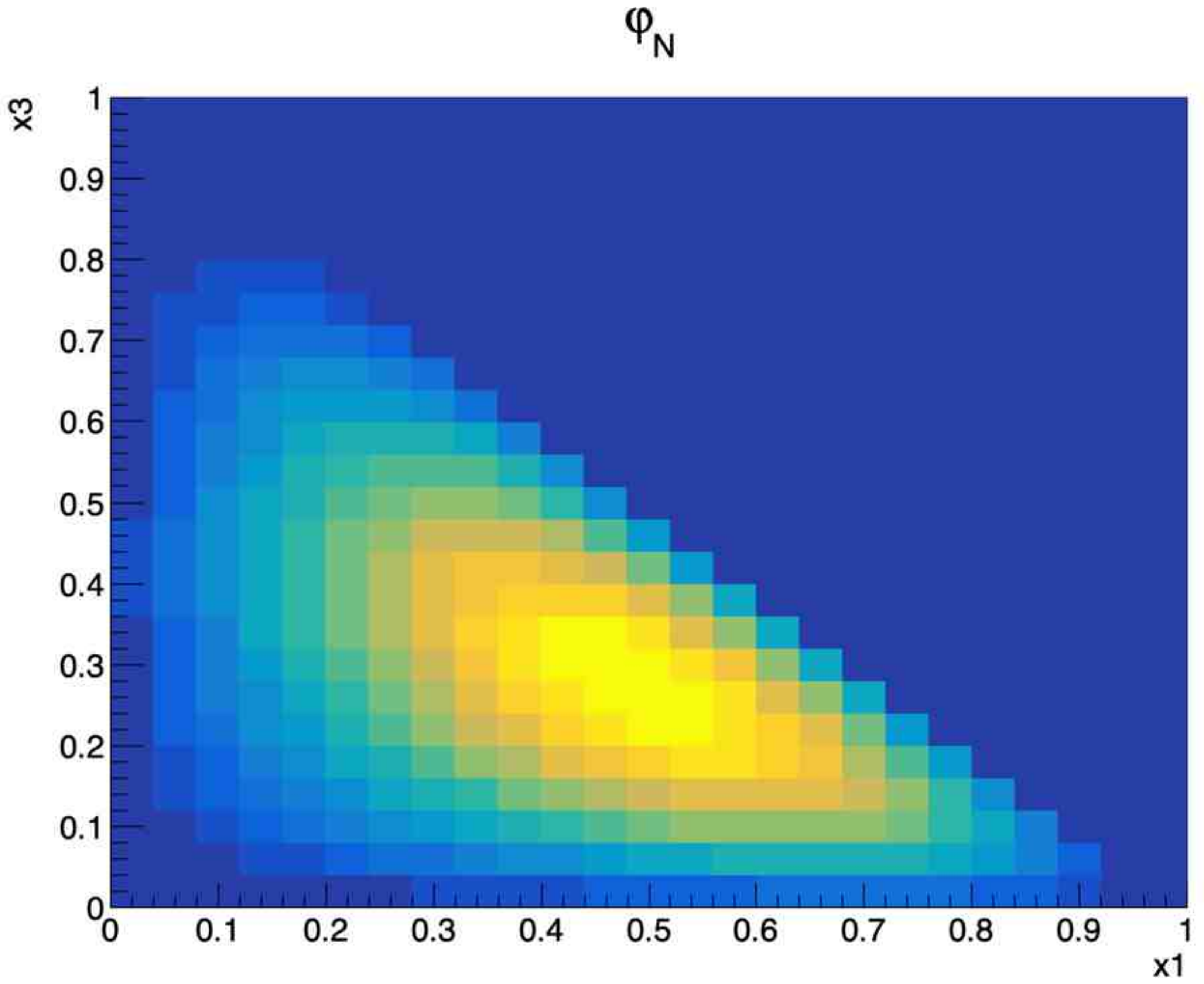
Gaussian Variation of the data points

Variation of the parameters number and q^2 dependance

MODEL'S RESULTS - ERROR BANDS



- The nucleon distribution function $\varphi_N(x)$ presents a global maximum



- The quark's momenta distribution is compatible with a quark - diquark interaction

$$\varphi_N^{\text{max}} \,@\, \mathbf{x} = \left(0.492873, 0.237073, 0.270055\right)$$

SUMMARY

- The proposed model provides a suitable description of the nucleon effective form factor but same can not be said for the proton, where the oscillations are barely visible
- The proton behaviour can be explained by the charge limited order momenta used in the description
- Still, this work can shed light over the connection between nucleon form factors oscillations and the internal structure of the hadrons
- Further search of a light-front model for nucleon form factors oscillations is still in act

Thank you giving me the opportunity to speak